جامعة الأزهر – غزة
كلية العلوم – قسم الفيزياء

محاضرات الفيزياء الطبية

Medical Physics Lecture

إعداد
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المحتويات

مقدمة

منذ نشـأة الخليقة على سـطح الأرض شرع الإنسـان يتسـاءل عن كيفية وجود الأشـياء وعن سـبب وجودها. هذا التساؤل كان دافعه الطبيعة الفضولية لدى البشر الذين متعهم الله بنعمة العقل والتفكير. ولكن بسـبب جهل الإنسان القديم وخوفه فإنه كان يعزو ظواهر الطبيعة إلى وجود قوى خارقة مجهولة، فعبـدها ظاناً أنها المسـئولة عن بقائه. ولكن مع مزيد من الملاحظات والاكتشـافات ودافع الحاجة إلى الاختراع والابتكار أدرك أن الطبيعة تحكمها قوانين مترابطة تربط بين نشـاطاته كإنسـان وعلاقته بالعالم الجامد والعالم الحي على حد سواء.

إن العلم الذي يدرس الطبيعة التي خلقها الله سـبحانه وتعالى في تكامل وتناسـق إبداعي رائع هو "علم الفيزياء" الذي سـمى قديما "علم الطبيعة". ولقد اسـتطاع الإنسـان البدائي التوصل إلى بعض مفاهيم هذا العلم نتيجة لحاجته للحصول على الطعام فبابتكاره لعملية الصيد بالرمح كان عليه أن يكون مدركاً لكميتين فيزيائيتين أسـاسـيتين هما المسافة والزمن ومن ثم كان عليه معرفة سرعة الجسم النسبية ليتمكن من إصابة هدفه.

في الحقيقة يمكن أن نقول أن علم الفيزياء هو العلم الذي يهتم بالإجابة عن أي سـؤال يبدأ بــــ "لماذا" فعلى سـبيل المثال تسـاءل العالم نيوتن "لماذا سـقطت التفاحة إلى أسـفل؟" وبالإجابة عن هذا السـؤال توصـل نيوتن إلى وضـع القـانون العـام للجاذبية الذي يربط القوى التجاذبية بين الكواكب والأقمار. وبعد فهم طبيعة هذه القوى تمكن الإنسـان من وضع أقمار صناعية تدور حول الأرض في مدارات ثابتة أفادت البشرية في مجالات عديدة وعلى رأسها مجال الاتصالات.

أهمية علم الفيزياء

إن تطور علم الفيزياء هو نتيجة طبيعية لحاجة الإنسـان إلى إيجاد تفسـير للظواهر الطبيعية وفهم سلوكها والقوى المؤثرة عليها من خلال اسـتنباط قوانين ترتبط ببعضـها. إن التطور التكنولوجي الملحوظ في جميع المجالات سـواء في الطب أو الهندسـة أو الفضـاء أو الاتصـالات أو الكمبيوتر وغيرها ما هو إلا تطبيقات لنتائج أبحاث واكتشـافات فيزيائية. فعلى سـبيل المثال علم الفيزياء هو علم أسـاسي في مجال الطب يسـتخدم في تشـخيص المرض سـواء كان باسـتخدام أشـعة اكس أو النظائر المشـعة أو الرنين المغناطيسي_ أو الأمواج فوق الصـوتية حيث تعتبر جميعها تطبيقات لأبحاث واكتشـافات فيزيائية ولا يمكن أن يكون هناك علاج بدون تشخيص فكلما تطورت وسائل التشخيص أمكن القضاء على أمراض كـانت فتاكة، أما الهندسـة بجميع فروعها ومجالاتها فهي تطبيق عملي لعلم الفيزياء فمثلا تحويل الطاقة

الحرارية إلى طاقة حركية أساسها قوانين فيزيائية استخدمها المهندسون الميكانيكيون في تصميم وسيلة النقل والمحرك منذ زمن بعيد. أما بالنسبة لمجال الاتصالات فقد شهد تطورا ملحوظا مع تطور الاكتشافات الفيزيائية فقد أدى اكتشاف الكهرباء وفهم قوانينها إلى استخدامها كوسيلة للاتصالات عن طريق إرسال المعلومات على شكل نبضات كهربائية خلال الأسلاك النحاسية. وبعد اكتشاف الفيزيائيين لأشعة الليزر والألياف الضوئية تحولت تكنولوجيا الاتصالات من استخدام الكهرباء إلى استخدام الضوء لما في ذلك من ميزات تفوق سابقتها بكثير. أما بالنسبة إلى علم الكمبيوتر فهو مثال واضح للتطبيقات الفيزيائية فبعد فهم طبيعة المواد وخواصها الكهربية ومن ثم اكتشاف أشباه الموصلات أصبحت هذه المواد البنية الأساسية للدوائر الإلكترونية للكمبيوتر، ولا شك أن التقدم الملحوظ في تكنولوجيا صناعة الكمبيوتر هو نتيجة للتقدم في الأبحاث والاكتشافات الفيزيائية فمثلا احتلت الشاشات التي تستخدم البلورات السائلة محل الشاشات التقليدية فأصبح الكمبيوتر بكل إمكاناته بحجم كتاب صغير.

من هذه الأمثلة ندرك أن علم الفيزياء هو علم أساسي لفهم باقي العلوم وتطويرها وقد أدركت الدول المتقدمة أهمية علم الفيزياء فشجعت على دراسته وأولته اهتماما كبيرا من حيث دعم الأبحاث العلمية وتشجيعها في مختلف المجالات الفيزيائية.

الشكوى من صعوبة دراسة الفيزياء

من الشائع بين الناس عامة والطلبة خاصة أن مادة الفيزياء صعبة ومعقدة جداً وهذا في حد ذاته غير صحيح فإن دراسة علوم الفيزياء تحتاج من الدارس إلى استخدام مهارته في إمعان الفكر وربط المعلومات السابقة والحديثة مع بعضها ببعض والخروج باستنتاج منطقي مقنع، ولأن علم الفيزياء هو علم تجريبي يعتمد على القياس وبالتالي يحتاج إلى معادلات وقوانين رياضية تربط الكميات الفيزيائية، وغالبا ما يتحول تركيز الدارس لموضوع الفيزياء من الفهم الفيزيائي إلى التعامل مع أرقام مجردة فلا يستطيع فهم معناها ومن هنا يصبح التعامل مع كل مسألة على أنها موضوع درس جديد وقوانين جديدة بالرغم من أن تلك المسألة ما هي إلا تطبيق آخر للقانون نفسه ولكن الشيء الجديد ما هو إلا مجهول آخر فيصاب الدارس بالإحباط لفشله في حل السؤال. ولكن إذا ما اتبع الأسلوب الصحيح في دراسة هذا العلم فستكون دراسته ميسرة وشيقة جدا.

الفيزياء ليست رياضيات فهناك فرق شاسع بين الاثنتين. الفيزياء تستعين بالمعادلات الرياضية فقط بعد تحديد الكميات الأساسية التي تؤثر في النموذج الفيزيائي تحت الدراسة وباستخدام المفاهيم الفيزيائية

يمكن إهمال تـأثير بعض تلـك الكميـات وبعـدها يأتي دور الرياضـيات لتحويل العلاقة الفيزيائية إلى معادلة رياضية تحل وتبسط صورتها.

سبل الاستفادة من هذه المحاضرات في حل المسائل والتمارين

- حاول حل الأمثلة المحلولة في الكتاب دون الاستعانة بالنظر إلى الحل الموجود.
- اقرأ صيغة السؤال للمثال المحلول عدة مرات حتى تستطيع فهم السؤال جيداً.
- حدد المعطيات ومن ثم المطلوب من السؤال.
- حدد الطريقة التي ستوصلك إلى إيجاد ذلك المطلوب على ضوء المعطيات والقوانين.
- قارن حلك مع الحل الموجود في الكتاب مستفيداً من أخطائك.

آمل أن أكون قد قدمت لأبنائنا الدارسـين من خلال هذا العمل المتواضـع ما يعينهم على فهم واستيعاب هذا الفرع من فروع المعرفة. كما أتقدم بالشكر لكل من يقدم نصيحة حول هذا الكتاب وموضوعاته.

والله من وراء القصد

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نبذة عن المحاضر

د. حازم فلاح سكيك

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- ✶ مؤسس وعميد كلية الدراسات المتوسطة بجامعة الازهر - غزة من الفترة 1996-2005
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الوحدة الأولى: علم الفيزياء والقياس

Physics and Measurements

Selected Topics in Physics for Medical Sciences Students

www.physicsacademy.org

Unit 1
Units, Dimensions, and Vectors
Lecture 1

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Units, Dimensions, and Vectors

- Introduction
- Unit systems
- Dimensional Analysis
- Vector and Scalar
- Coordinate system
- Properties of Vectors
- The unit vector
- Components of a vector
- Product of a vector

About Physics

- Explains Nature
- Fundamental science
- Most technology of today (cell phones, DVD player, etc) are result of discoveries that happened in physics last century.
- Develop problem solving and logical reasoning skills – very important in any field of work!!!

What is Physics?

Physics tries to explain processes we observe in terms of a set of 'laws' or rules

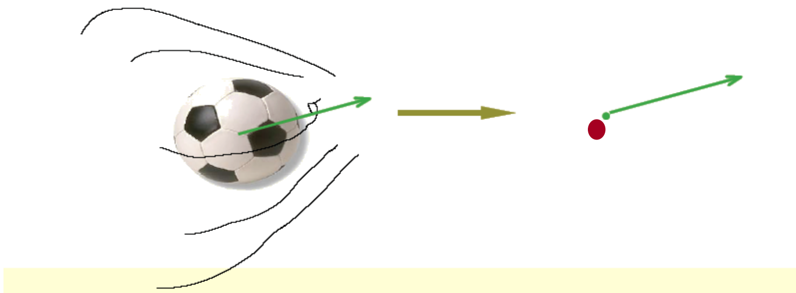
These laws are normally expressed in terms of mathematical equations with which we can predict things

These predictions can be compared to what we see

"Theory & Experiment"

Idealized Models

- A physical system is often too complicated to analyze all at once
- A **model** is a simplified version of a physical system
- Example: We neglect the size and shape of a *ball* and represent it as a *particle* (completely localized at a single point in space)



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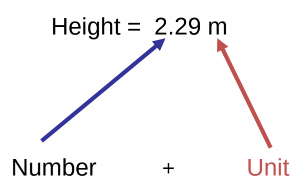
Physics and Measurements

- Physics is based on experimental observations and quantitative measurements. These observations have described by numbers and units.
- **Numbers** give us how large our measurement was, and
- **the units** tell us the nature of this measurement.



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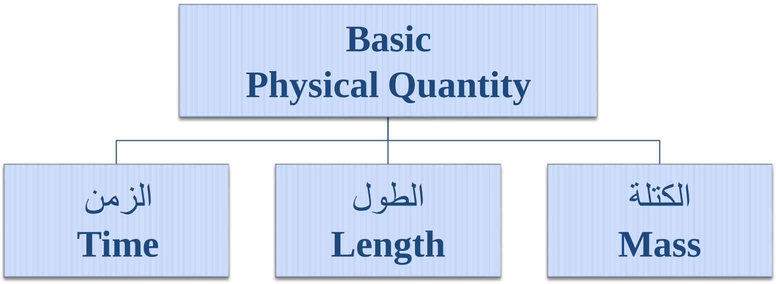
Physical Quantity



Unit systems

Two systems of units are widely used in the world, the metric and the British systems.

The metric system measures the length in meters whereas the British system makes use of the foot, inch, …..

The metric system is the most widely used. Therefore the metric system will be used in this course.

By international agreement the metric system was formalized in 1971 into the *International System of Units* (SI). There are seven basic units in the SI as shown in table Below.

"*For this Course only three units are used, the meter, kilogram, and second*".

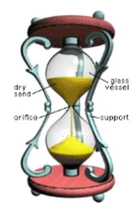
Basic units in the SI

Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Number of particles	mole	mol
Luminous intensity	candela	cd

Definition of Basic Quantities

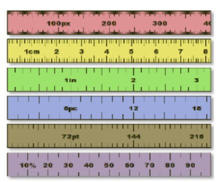
Mass

The SI unit of mass is the *Kilogram*, which is defined as the mass of a specific platinum-iridium alloy cylinder.



Time

The SI unit of time is the *Second*, which is the time required for a cesium-133 atom to undergo 9,192,631,770 vibrations.



Length

The SI unit of length is *Meter*, which is the distance traveled by light in vacuum during a time of 1/299,792,458 second.

Power of ten prefixes

- 3,000 m = 3 × 1,000 m
  = 3 × 10^3 m = 3 km
- 1,000,000,000 = 10^9 = 1G
- 1,000,000 = 10^6 = 1M
- 1,000 = 10^3 = 1k

- 141 kg = ? g
- 1 GB = ? Byte = ? MB

10^x	Prefix	Symbol
x=18	exa	E
15	peta	P
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deca	da

Power of ten prefixes

- 0.003 s = 3 × 0.001 s
  = 3 × 10^{-3} s = 3 ms
- 0.01 = 10^{-2} = centi
- 0.001 = 10^{-3} = milli
- 0.000 001 = 10^{-6} = micro
- 0.000 000 001 = 10^{-9} = nano
- 0.000 000 000 001 = 10^{-12}
  = pico = p

- 3 cm = ? m = ? mm

10^x	Prefix	Symbol
x=-1	deci	d
-2	centi	c
-3	milli	m
-6	micro	μ
-9	nano	n
-12	pico	p
-15	femto	f
-18	atto	a

Role of "UNITS" in problem solving

- Need to know conversion.
- Do problems with all units in the same system.
- Only quantities with same units can be added or subtracted.

Example

$$40\,\text{m} + 11\,\text{cm} = ?$$

The above expression yields:

a) 40.11 m
b) 4011 cm
c) A or B
d) Impossible to evaluate (dimensionally invalid)

$$1.5\,\text{m} \cdot 3.0\,\text{kg} = ?$$

The above expression yields:

a) 4.5 m kg
b) 4.5 g km
c) A or B
d) Impossible to evaluate (dimensionally invalid)

Express a speed of 50 kilometers per hour as meters per second.

Derived Quantities

All physical quantities measured by physicists can be expressed in terms of the three basic unit of length, mass, and time. For example, ***speed*** is simply length divided by time, and the ***force*** is actually mass multiplied by length divided by time squared.

$$[\text{Speed}] = L/T = LT^{-1}$$
$$[\text{Force}] = ML/T^2 = MLT^{-2}$$

where [Speed] is meant to indicate the unit of speed, and M, L, and T represents mass, length, and time units.

- **Multiply and divide units just like numbers**

- **Derived quantities: area, speed, volume, density ……**
  - Area = Length × Length — unit for area = m^2
  - Volume = Length × Length × Length — unit for volume = m^3
  - Speed = Length / time — unit for speed = m/s
  - Density = Mass / Volume — unit for density = kg/m^3

- In 2008 Olympic Game, Usain Bolt sets world record at 9.69 s in Men's 100 m Final. What is his average speed?

$$\text{speed} = \frac{100\ \text{m}}{9.69\ \text{s}} = \frac{100}{9.69} \cdot \frac{\text{m}}{\text{s}} = 10.32\ \text{m/s}$$

Dimensional Analysis

- The word **dimension** in physics indicates the physical nature of the quantity. For example the distance has a dimension of ***length***, and the ***speed*** has a dimension of ***length/time***.

- The dimensional analysis is used to **check the formula**, since the dimension of the left hand side and the right hand side of the formula must be the same.

Example

Using the dimensional analysis check that this equation

$x = ½\ at^2$ is correct, where x is the distance, a is the acceleration and t is the time.

Solution

$$x = ½\ at^2$$

The right hand side [x] = L

The left hand side $[at^2] = \frac{L}{T^2} \times T^2 = L$

This equation is correct because the dimension of the left and right side of the equation have the same dimensions.

Example

Show that the expression $v = v_o + at$ is dimensionally correct, where v and v_o are the velocities and a is the acceleration, and t is the time.

Solutions:

The right hand side

$$[v] = \frac{L}{T}$$

The left hand side

$$[at] = \frac{L}{T^2} \times T = \frac{L}{T}$$

Therefore, the expression is dimensionally correct

Example

Suppose that the acceleration of a particle moving in circle of radius *r* with uniform velocity *v* is proportional to the r^n and v^m. Use the dimensional analysis to determine the power n and m.

Solution

Let us assume *a* is represented in this expression

$$a = k\, r^n\, v^m$$

Where *k* is the proportionality constant of dimensionless unit.

The Left hand side

$$[a] = \frac{L}{T^2}$$

- **The Right hand side**

$$[k\, r^n\, v^m] = L^n \left(\frac{L}{T}\right)^m = \frac{L^{n+m}}{T^m}$$

Therefore

$$\frac{L}{T^2} = \frac{L^{n+m}}{T^m}$$

Hence, $n+m=1$ and $m=2$

Therefore. n =-1 and the acceleration a is

$$a = k\, r^{-1}\, v^2$$

$k = 1$

$$a = \frac{v^2}{r}$$

Problem

- **Which of the following equations are dimensionally correct?**

(a) $v_f = v_i + ax$

(b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$

•The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position $s = k\, a^m\, t^n$,

where k is a dimensionless constant.

Show by dimensional analysis that this expression is satisfied if m = 1 and n =2. Can this analysis give the value of k?

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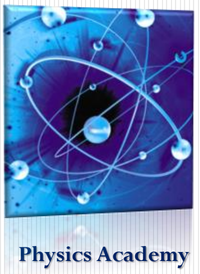

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Unit 1
Units, Dimensions, and Vectors
Lecture 2

Dr. Hazem Falah Sakeek
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Units, Dimensions, and Vectors

- **Introduction**
- **Unit systems**
- **Dimensional Analysis**
- **Vector and Scalar**
- **Coordinate system**
- **Properties of Vectors**
- **The unit vector**
- **Components of a vector**
- **Product of a vector**

Trigonometry

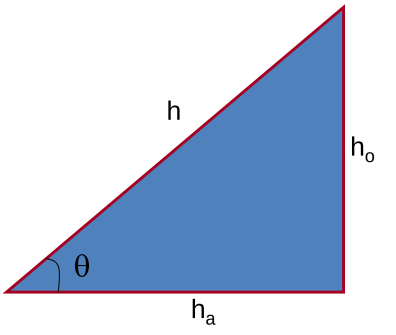
- Sine, Cosine, Tangent

$\text{Sin}\ \theta = h_o/h$
$\text{Cos}\ \theta = h_a/h$
$\text{Tan}\ \theta = h_o/h_a$

$\theta = \text{Sin}^{-1}(h_o/h)$
$\theta = \text{Cos}^{-1}(h_a/h)$
$\theta = \text{Tan}^{-1}(h_o/h_a)$

NOTE: $\text{Tan}^{-1} \neq 1/\text{Tan}$

$h^2 = h_o^2 + h_a^2$



Vector and Scalar

- **Scalars:** One that can be described by a single number (along with the unit)
  - Water freezes at a temperature of 0° C or 32° F
  - The mass of a book is 198.2 g
  - The length of room is 5 m
  - The car kinetic energy was 0.345 J
- **Vectors**: A quantity that deals with magnitude and direction is called a vector quantity.
  - The wind had a velocity of 25 km/h from the North
  - The momentum was 1.234 kg m/s to the left
- Textbooks use either **A** or $\vec{A}$

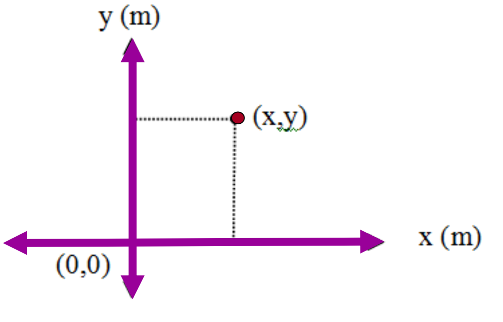
Vector and Scalar Quantities

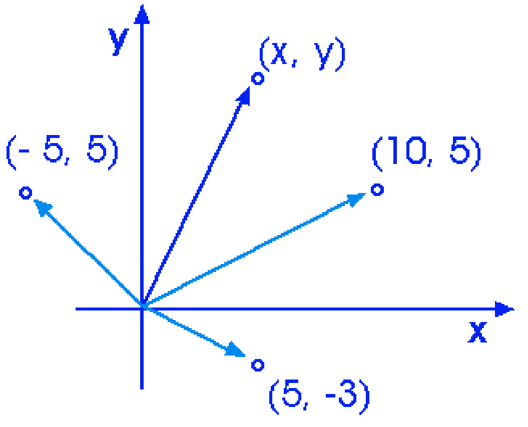
Vector Quantity	Scalar Quantity
Displacement	Length
Velocity	Mass
Force	Speed
Acceleration	Power
Field	Energy
Momentum	Work

Coordinate system (1)

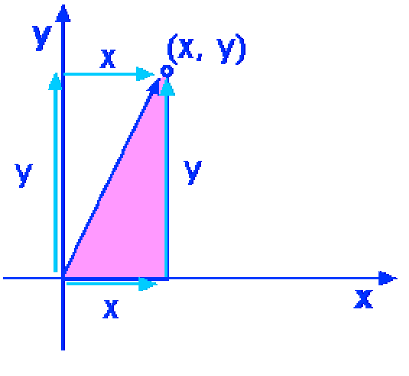
(1) The rectangular coordinates

- This coordinate system is consist of a fixed reference point (0,0) which called the origin.
- A set of axis with appropriate scale and label.





The x- and y-coordinates may be either positive or negative



We can call this x-component, a vector along the x-direction of length x, and indicate that it is a vector by **x**.

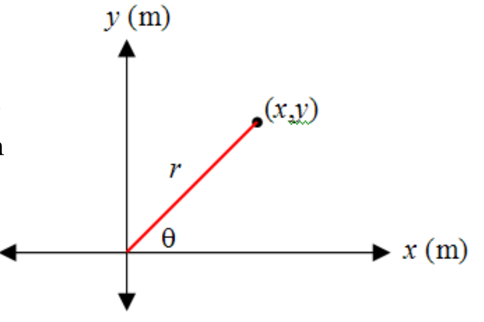
Likewise, we can call this y-component, a vector along the y-direction of length y, and indicate that it is a vector by **y** .

$$\mathbf{r} = \mathbf{x} + \mathbf{y}$$

Coordinate system (2)

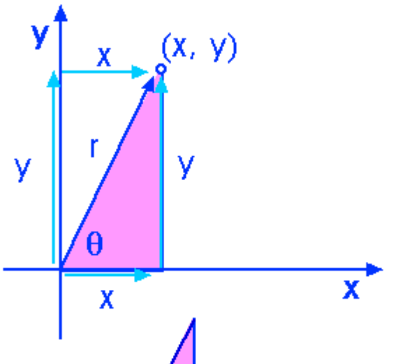
(2) The polar coordinates

Sometimes it is more convenient to use the polar coordinate system (r, θ), where r is the distance from the origin to the point of rectangular coordinate (x,y), and θ is the angle between r and the x axis.



The relation between coordinates

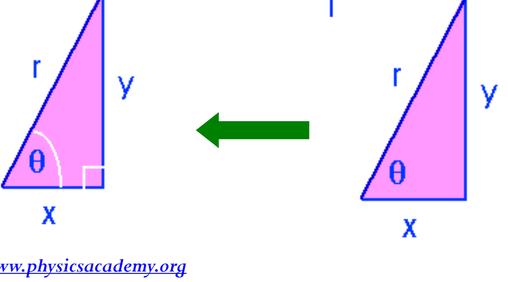
The relation between the rectangular coordinates (x,y) and the polar coordinates (r, θ) is shown in Figure,

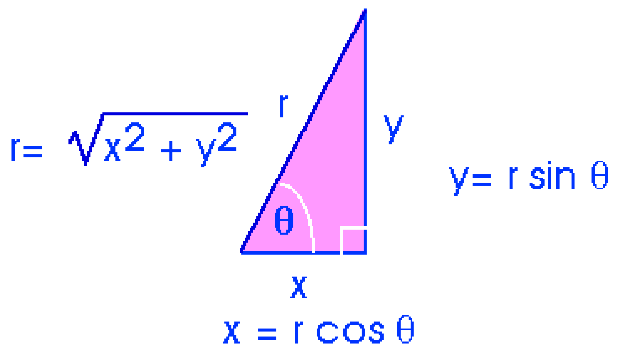


$\sin\theta = y/r$

$\cos\theta = x/r$

$\tan\theta = y/x$





It is common practice to measure the angle from the positive x-axis and to measure it **positive** for a **counterclockwise** direction.

Example 1

The example shown below might be for an angle of θ = 53° . Then, if r = 10, the components will be

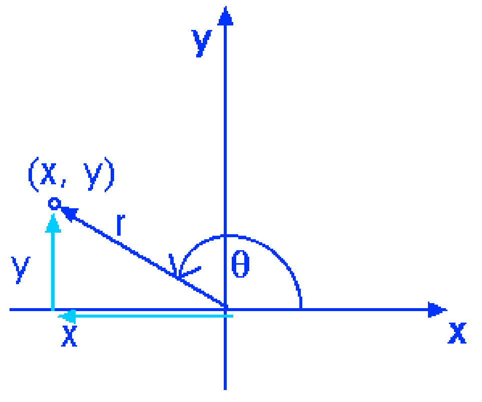
$x = r \cos \theta = (0.6)(10) = 6$

$y = r \sin \theta = (0.8)(10) = 8$

Of course, angle θ does not need to be limited to the first quadrant. for θ = 150°, r = 10 for this numerical example. For that case,

$x = r \cos \theta = (10)(\cos 150^{o}) = (10)(-0.866) = -8.66$

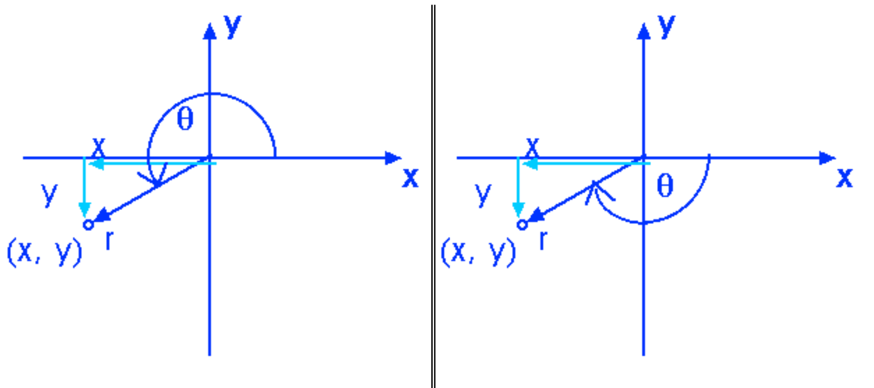
$y = r \sin \theta = (10)(\sin 150^{o}) = (10)(0.500) = 5.00$



notice the signs and compare them with the diagram.

x = - 8.66 is located to the **left** and y = 5.00 is located **up** .

من الممكن ان نقيس الزاوية لتحديد الاتجاه من محور x مع عقارب عكس عقارب الساعة أو مع عقارب الساعة ولكن يجب ان نميز ذلك باشارة الزاوية.

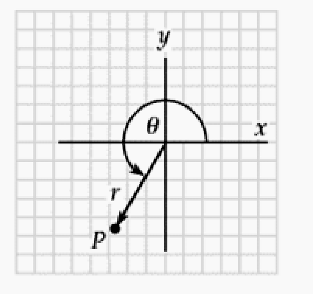


$r = 10,\ \theta = 210^{o}$

$r = 10,\ \theta = -150^{o}$

Example 2

- The polar coordinates of a point are r = 5.5m and θ=240°. What are the Cartesian coordinates of this point?



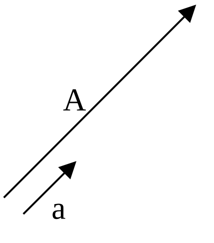
- **Solution**

$x = r \cos \theta = 5.5\times\cos 240^{o} = -2.75$ m

$y = r \sin \theta = 5.5\times\sin 240^{o} = -4.76$ m

The unit vector

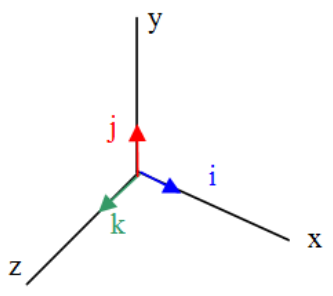
- A unit vector is a vector having a magnitude of unity and its used to describe a direction in space.



المتجه ***A*** يمكن تمثيله بمقدار المتجه ضرب متجه الوحدة ***a*** كالتالي:

A= ***a*** *A*

- كذلك يمكن تمثيل متجهات وحدة (i, j, k) لمحاور الإسناد المتعامدة (x, y, z) كما في الشكل التالي:-



i ≡ a unit vector along the *x*-axis

j ≡ a unit vector along the *y*-axis

k ≡ a unit vector along the *z*-axis

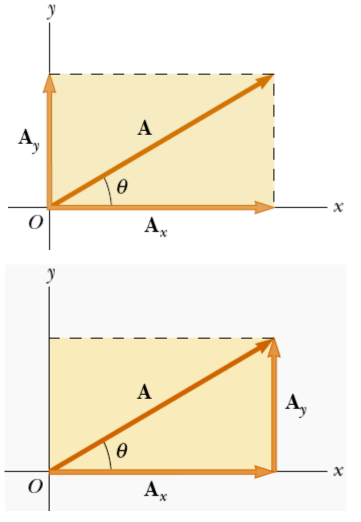
- Instead of explicitly writing $A_x = 5, A_y = 0$; $B_x = 5, B_y = 5$; $C_x = -10, C_y = 0$; and $D_x = -5, D_y = 5$, we can write this same information in a different form. We can write

$$A = 5\,i + 0j$$
$$B = 5i + 5j$$
$$C = -10i + 0j$$
$$D = -5i + 5j$$

$$R = A + B + C + D$$
$$R = (5\,i + 0j) + (5i + 5j) + (-10i + 0j) + (-5i + 5j)$$
$$R = (5+5-10-5)i + (0+5+0+5)j$$
$$R = -5i + 10j$$

Components of a vector

Any vector lying in *xy* plane can be resolved into two components one in the *x*-direction and the other in the *y*-direction as shown in Figure



$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

- عند التعامل مع عدة متجهات فإننا نحتاج إلى تحليل كل متجه إلى مركباته بالنسبة إلى محاور الإسناد (x,y) مما يسهل إيجاد المحصلة بدلاً من استخدام الطريقة البيانية لإيجاد المحصلة.

The magnitude of the vector **A**

$$A = \sqrt{A_x^2 + A_y^2}$$

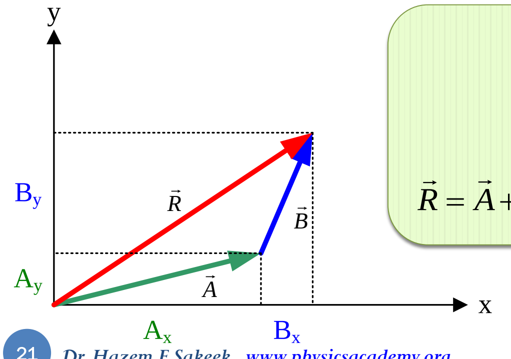
The direction of the vector to the *x*-axis

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

A vector **A** lying in the *xy* plane, having rectangular components A_x and A_y can be expressed in a unit vector notation

$$\mathbf{A} = A_x\mathrm{i} + A_y\mathrm{j}$$

ملاحظة: يمكن استخدام طريقة تحليل المتجهات في جمع متجهين A و B كما في الشكل التالي:



$$\vec{A} = A_x i + A_y j$$

$$\vec{B} = B_x i + B_y j$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)i + (A_y + B_y)j$$

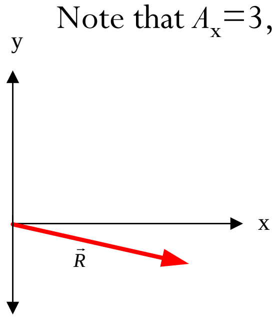
Example 3

Find the sum of two vectors **A** and **B** and given by

$$\vec{A} = 3i + 4j \text{ and } \vec{B} = 2i - 5j$$

Solutions

Note that A_x=3, A_y=4, B_x=2, and B_y=-5



$$\vec{R} = \vec{A} + \vec{B} = (3+2)i + (4-5)j = 5i - j$$

The magnitude of vector **R** is

$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{25+1} = \sqrt{26} = 5.1$$

The direction of **R** with respect to *x*-axis is

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-1}{5} = -11^o$$

Example 4

Two vectors are given by $\vec{A} = 3i - 2j$ and $\vec{B} = -i - 4j$. Calculate (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$, (c) $|\vec{A} + \vec{B}|$, (d) $|\vec{A} - \vec{B}|$, and (e) the direction of $\vec{A} + \vec{B}$ and $|\vec{A} - \vec{B}|$.

Solutions

(a) $\vec{A} + \vec{B} = (3i - 2j) + (-i - 4j) = 2i - 6j$

(b) $\vec{A} - \vec{B} = (3i - 2j) - (-i - 4j) = 4i + 2j$

(c) $|\vec{A} + \vec{B}| = \sqrt{2^2 + (-6)^2} = 6.32$

(d) $|\vec{A} - \vec{B}| = \sqrt{4^2 + 2^2} = 4.47$

(e) For $\vec{A} + \vec{B}$, $\theta = \tan^{-1}(-6/2) = -71.6^\circ = 288^\circ$

For $\vec{A} - \vec{B}$, $\theta = \tan^{-1}(2/4) = 26.6^\circ$

Example 5

- A particle moves from a point in the *xy* plane having rectangular coordinates (-3,-5)m to a point with coordinates (-1,8)m. (a) Write vector expressions for the position vectors in unit vector form for these two points. (b) What is the displacement vector?

Solution

(a) $\vec{R}_1 = x_1 i + y_1 j = (-3i - 5j)m$

$\vec{R}_2 = x_2 i + y_2 j = (-i + 8j)m$

(b) Displacement = $\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$

$$\Delta\vec{R} = (x_2 - x_1)i + (y_2 - y_1)j = -i - (-3i) + 8j - (-5j) = (2i + 13j)m$$

Example 6

A vector A has a negative *x* component 3 units in length and positive *y* component 2 units in length.

(a) Determine an expression for **A** in unit vector notation.

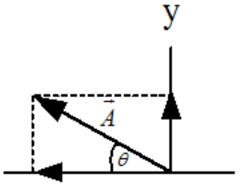
(b) Determine the magnitude and direction of **A**.

(c) What vector **B** when added to **A** gives a resultant vector with no *x* component and negative *y* component 4 units in length?

Solution

A_x = -3 units & A_y = 2 units

(a) **A**= A_xi+A_yj=-3i+2j units



(b) $|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3)^2 + (2)^2} = 3.61\ units$

$\theta = \tan^{-1}(2/-3) = 33.7^o$ (relative to the –x axis)

(c) What vector **B** when added to **A** gives a resultant vector with no *x* component and negative *y* component 4 units in length?

$R_x = 0$ & $R_y = -4$; since $\vec{R} = \vec{A} + \vec{B}$, $\vec{B} = \vec{R} - \vec{A}$

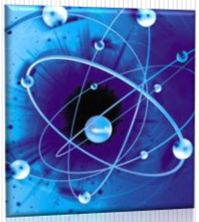
$B_x = R_x - A_x = 0-(-3) = 3$

$B_y = R_y - A_y = -4-2 = -6$

Therefore

$$\vec{B} = B_x i + B_y j = (3i - 6j)\ units$$

Selected Topics in Physics for Medical Sciences Students



Physics Academy
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Unit 1
Units, Dimensions, and Vectors
Lecture 3

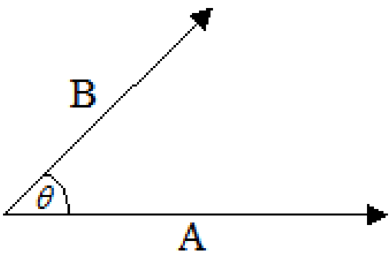
Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Units, Dimensions, and Vectors

- Introduction
- Unit systems
- Dimensional Analysis
- Vector and Scalar
- Coordinate system
- Properties of Vectors
- The unit vector
- Components of a vector
- Product of a vector

Product of a vector

- There are two kinds of vector product the first one is called **scalar product or dot product** because the result of the product is a scalar quantity. The second is called **vector product or cross product** because the result is a vector perpendicular to the plane of the two vectors.



The scalar product

- يعرف الضرب القياسي scalar product بالضرب النقطي dot product وتكون نتيجة الضرب القياسي لمتجهين كمية قياسية، وتكون هذه القيمة موجبة إذا كانت الزاوية المحصورة بين المتجهين بين 0 و 90 درجة وتكون النتيجة سالبة إذا كانت الزاوية المحصورة بين المتجهين بين 90 و 180 درجة وتساوي صفراً إذا كانت الزاوية 90.

A.B = +ve when $0 \le \theta < 90^o$
A.B = -ve when $90^o < \theta \le 180^o$
A.B = zero when $\theta = 90$

- يعرف الضرب القياسي لمتجهين بحاصل ضرب مقدار المتجه الأول في مقدار المتجه الثاني في جيب تمام الزاوية المحصورة بينهما.

$$\vec{A}.\vec{B} = |A||B|\cos\theta$$

- يمكن إيجاد قيمة الضرب القياسي لمتجهين باستخدام مركبات كل متجه كما يلي:

$$\vec{A} = A_x i + A_y j + A_z k$$
$$\vec{B} = B_x i + B_y j + B_z k$$

- **The scalar product is**

$$\vec{A}.\vec{B} = (A_x i + A_y j + A_z k).(B_x i + B_y j + B_z k)$$

- بضرب مركبات المتجه **A** في مركبات المتجه **B** ينتج التالي:

$$\vec{A}.\vec{B} = (A_x i.B_x i + A_x i.B_y j + A_x i.B_z k$$
$$+ A_y j.B_x i + A_y j.B_y j + A_y j.B_z k$$
$$+ A_z k.B_x i + A_z k.B_y j + A_z k.B_z k)$$

Therefore

$$\therefore \vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The angle between the two vectors is

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$$

Example 1

- Find the angle between the two vectors

$$\vec{A} = 2i + 3j + 4k \qquad \vec{B} = i - 2j + 3k$$

Solution

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$$

$$A_x B_x + A_y B_y + A_z B_z = (2)(1)+(3)(-2)+(4)(3)=8$$

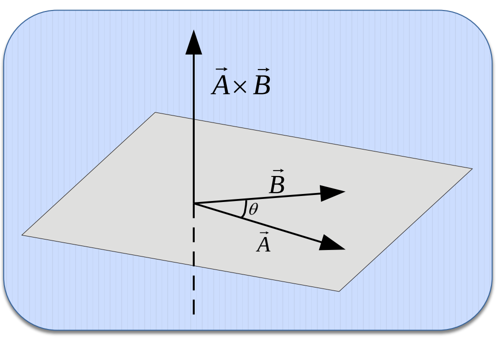
$$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$|B| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$\cos\theta = \frac{8}{\sqrt{29}\sqrt{14}} 0.397 \Rightarrow \theta = 66.6^o$$

The vector product

- يعرف الضرب الاتجاهي *vector product* بـ *cross product* وتكون نتيجة الضرب الاتجاهي لمتجهين كمية متجهة. قيمة هذا المتجه $\vec{C} = \vec{A} \times \vec{B}$ واتجاهه عمودي على كل من المتجهين A و B وفي اتجاه دوران بريمة من المتجه A إلى المتجه B كما في الشكل التالي:



$$\vec{A} \times \vec{B} = AB\sin\theta$$

$$\vec{A}\times\vec{B} = AB\sin\theta$$

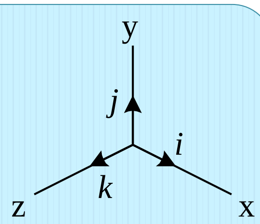
$$\vec{A}\times\vec{B} = (A_x i + A_y j + A_z k)\times(B_x i + B_y j + B_z k)$$

To evaluate this product we use the fact that the angle between the unit vectors *i*, *j* , *k* is 90°.

$$i\times i = 0 \qquad i\times j = k \qquad i\times k = -j$$

$$j\times j = 0 \qquad j\times k = i \qquad j\times i = -k$$

$$k\times k = 0 \qquad k\times i = j \qquad k\times j = -i$$



$$\vec{A}\times\vec{B} = (A_y B_z - A_z B_y)i + (A_z B_x - A_x B_z)j + (A_x B_y - A_y B_x)k$$

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If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$

$$= (a_2\times b_3 - a_3\times b_2)\mathbf{i} - (a_1\times b_3 - a_3\times b_1)\mathbf{j} + (a_1\times b_2 - a_2\times b_1)$$

If $\vec{C} = \vec{A}\times\vec{B}$, the components of C are given by

$$C_x = A_y B_z - A_z B_y$$

$$C_y = A_z B_x - A_x B_z$$

$$C_z = A_x B_y - A_y B_x$$

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Example 2

- If $\vec{C} = \vec{A} \times \vec{B}$, where $\vec{A} = 3i - 4j$, and $\vec{B} = -2i + 3k$, what is $\vec{C}$?

Solution

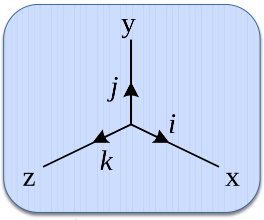
$$\vec{C} = \vec{A} \times \vec{B} = (3i - 4j) \times (-2i + 3k)$$

- which, by distributive law, becomes

$$\vec{C} = -(3i \times 2i) + (3i \times 3k) + (4j \times 2i) - (4j \times 3k)$$

- Using equation $\vec{A} \times \vec{B} = AB \sin\theta$ to evaluate
- each term in the equation above we get

$$\vec{C} = 0 - 9j - 8k - 12i = -12i - 9j - 8k$$



- The vector **C** is perpendicular to both vectors **A** and **B**.

Unit 1
Problems

[1] Show that the expression $x=vt+1/2at^2$ is dimensionally correct, where x is a coordinate and has units of length, v is velocity, a is acceleration, and t is time.

[2] Which of the equations below are dimensionally correct?
(a) $v = v_o+at$
(b) $y = (2\text{m})\cos(kx)$,
where $k = 2\ \text{m}^{-1}$.

[3] Show that the equation $v^2 = v_o^2 + 2at$ is dimensionally correct, where v and v_o represent velocities, a is acceleration and x is a distance.

[4] The period T of simple pendulum is measured in time units and given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and g is the acceleration due to gravity. Show that the equation is dimensionally correct.

[5] Suppose that the displacement of a particle is related to time according to the expression $s = ct^3$. What are the dimensions of the constant c?

[6] Two points in the xy plane have Cartesian coordinates (2, -4) and (-3, 3), where the units are in m. Determine (a) the distance between these points and (b) their polar coordinates.

[7] The polar coordinates of a point are r = 5.5m and $\theta = 240^o$. What are the cartisian coordinates of this point?

[8] A point in the xy plane has cartesian coordinates (-3.00, 5.00) m. What are the polar coordinates of this point?

[9] Two points in a plane have polar coordinates (2.5m, 30^o) and (3.8, 120^o). Determine (a) the cartisian coordinates of these points and (b) the distance between them.

[10] A point is located in polar coordinate system by the coordinates r = 2.5m and θ =35^o. Find the x and y coordinates of this point, assuming the two coordinate system have the same origin.

[11] Vector $\vec{A}$ is 3.00 units in length and points along the positive x axis. Vector $\vec{B}$ is 4.00 units in length and points along the negative y axis. Use graphical methods to find the magnitude and direction of the vectors (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$.

[12] A vector has x component of -25 units and a y component of 40 units. Find the magnitude and direction of this vector.

[13] Find the magnitude and direction of the resultant of three displacements having components (3,2) m, (-5, 3) m and (6, 1) m.

[14] Two vector are given by $\vec{A}$ = 6i –4j and $\vec{B}$ = -2i+5j. Calculate (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$, $|\vec{A}+\vec{B}|$, (d) $|\vec{A}-\vec{B}|$, (e) the direction of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.

[15] Obtain expressions for the position vectors with polar coordinates (a) 12.8m, 150^o; (b) 3.3cm, 60^o; (c) 22cm, 215^o.

[16] Find the x and y components of the vector $\vec{A}$ and $\vec{B}$ shown in Figure 1.1. Derive an expression for the resultant vector $\vec{A}+\vec{B}$ in unit vector notation.

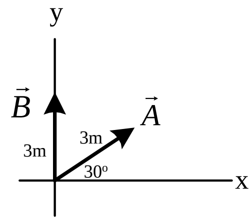


Figure 1.1

[17] A vector $\vec{A}$ has a magnitude of 35 units and makes an angle of 37^o with the positive *x* axis. Describe (a) a vector $\vec{B}$ that is in the direction opposite $\vec{A}$ and is one fifth the size of $\vec{A}$, and (b) a vector $\vec{C}$ that when added to $\vec{A}$ will produce a vector twice as long as $\vec{A}$ pointing in the negative y direction.

[18] Find the magnitude and direction of a displacement vector having *x* and *y* components of -5m and 3m, respectively.

[19] Three vectors are given by $\vec{A}$=6i, $\vec{B}$=9*j*, and $\vec{C}$=(-3*i*+4*j*). (a) Find the magnitude and direction of the resultant vector. (b) What vector must be added to these three to make the resultant vector zero?

[20] A particle moves from a point in the *xy* plane having Cartesian coordinates (-3.00, -5.00) m to a point with coordinates (-1.00, 8.00) m. (a) Write vector expressions for the position vectors in unit-vector form for these two points. (b) What is the displacement vector?

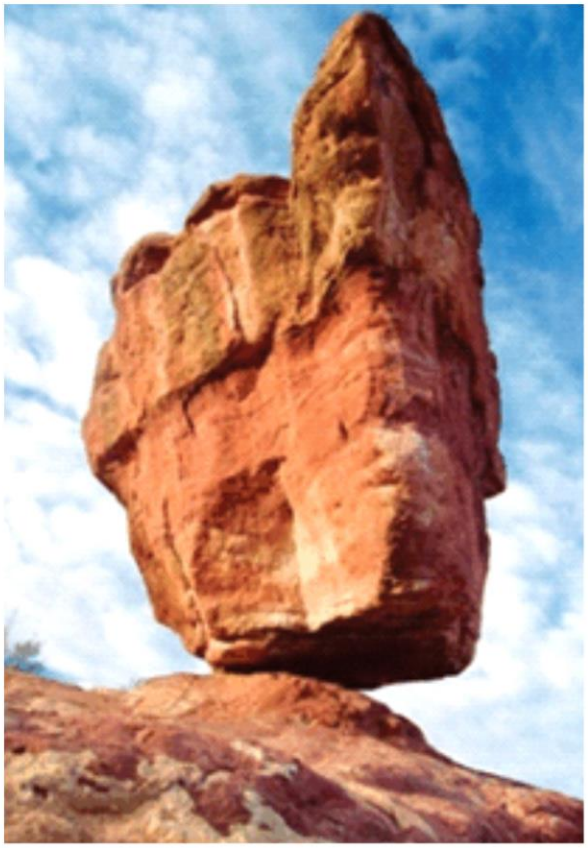
[21] Two vectors are given by $\vec{A}$= 4i+3j and $\vec{B}$= -i+3j. Find (a) $\vec{A}.\vec{B}$ and (b) the angle between $\vec{A}$ and $\vec{B}$.

[22] A vector is given by $\vec{A}$= -2i+3j. Find (a) the magnitude of $\vec{A}$ and (b) the angle that $\vec{A}$ makes with the positive *y* axis.

[23] Vector $\vec{A}$ has a magnitude of 5 units, and $\vec{B}$ has a magnitude of 9 units. The two vectors make an angle of 50^o with each other. Find $\vec{A}.\vec{B}$.

[24] For the three vectors $\vec{A}$=3i+j-k, $\vec{B}$= -i+2j+5k, and $\vec{C}$= 2j-3k, find $\vec{C}.(\vec{A}-\vec{B})$

[25] The scalar product of vectors $\vec{A}$ and $\vec{B}$ is 6 units. The magnitude of each vector is 4 units. Find the angle between the vectors.



الوحدة الثانية: السكون والاتزان
Static and Equilibrium

Dr. Hazem Falah Sakeek | www.hazemsakeek.net | 2014

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Unit 2

Static and Equilibrium

Lecture 4

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

Statics & Equilibrium

Introduction

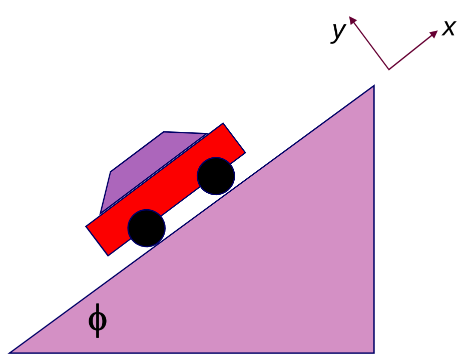
- **What is a "Force?"**
- A **force** causes something with **mass** to move (accelerate).
- **From Newton's Second Law of Motion**

  F = m x a
- The unit of force is Newton "N"

Example

- What are all of the forces acting on a car (mass m) parked on a hill?

Solution



Car on Hill:

Use Newton's 2nd Law: $F_{NET} = ma = 0$

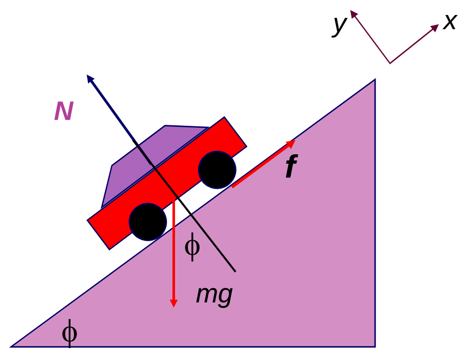
Resolve this into x and y components:

x: $f - mg \sin \phi = 0$

➡ $f = mg \sin \phi$

y: $N - mg \cos \phi = 0$

➡ $N = mg \cos \phi$



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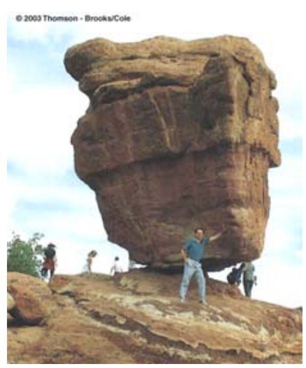
Statics

- **What is a "Statics?"**
- Statics is the study of forces acting on an object that is in **equilibrium** and at **rest**.

- *e.g. Ladders, sign-posts, balanced beams,*
- *buildings, bridges, some parts of*
- *Human body, etc…*

- *Statics can help to understand levers in our body.*
- *Statics study the balance and stability of*
- *many structures.*



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The Conditions of Equilibrium

- Any **rigid Body** is in equilibrium when two conditions are satisfied:

Condition (1)

- The net force acting on the body is zero.

$$\sum \mathbf{F} = 0$$

Condition (2)

- The net torque about any axis must be zero

$$\sum \tau = 0$$

An object extend in space that does not change its size or shape when subjected to a force.

Static Equilibrium

Translational Equilibrium

$$\sum \mathbf{F} = 0$$

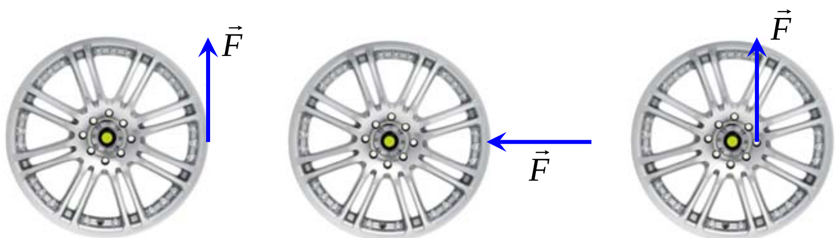
Rotational Equilibrium

$$\sum \tau = 0$$

هذا ما سنركز عليه في هذه الوحدة

Torques

Torque makes things spin!

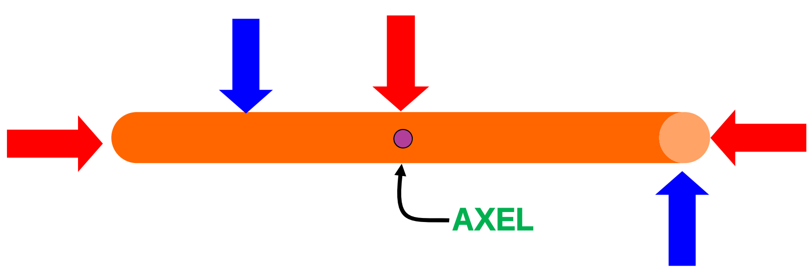


which applied force will cause the wheel to spin the fastest?

- **What makes something rotate in the first place?**

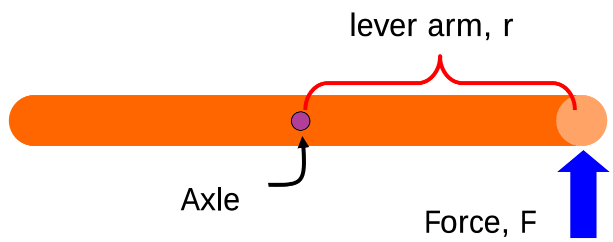
TORQUE

How do I apply a force to make the rod rotate about the axel? Not just anywhere!



To make an object rotate, a force must be applied in the right place.

the combination of force and point of application is called **TORQUE**



Net torque = 0, net force ≠ 0

The rod will accelerate upward under these two forces, but will not rotate.

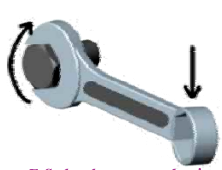
Net Force = 0 , Net Torque ≠ 0

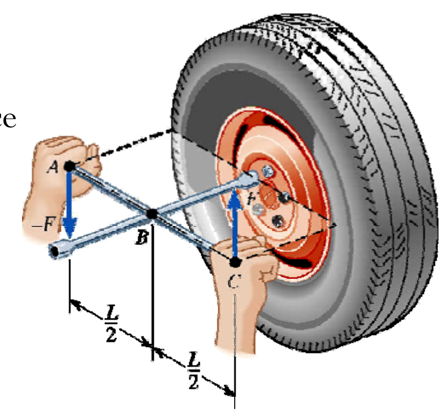
- The net force = 0, since the forces are applied in opposite directions so it will not accelerate.
- However, together these forces will make the rod rotate in the clockwise direction.

Torque

- When object is subjected to equal opposite forces. The net force is zero (the object is in translational equilibrium) But it may not in rotational equilibrium.

For example when the line of action of the two opposite force are not on the same line, then the object will rotate.





Force and Torque

How are force and torque related?

A force can create a torque by acting through a **moment arm**.

moment arm.

...produces a torque here.

A force here...

...but you don't move very far.

...but your hand moves a long way.

If you hold the wrench here, you need a lot of force...

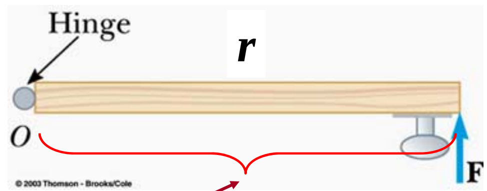
If you hold the wrench here, you don't need as much force...

Torque

Torque, τ, is the force to rotate an object about some axis

Door example:

$$\tau = Fr$$



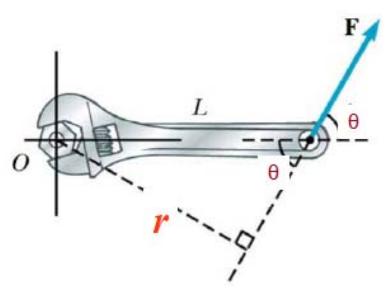
SI unit: [N m]

- τ is the torque
- r is the *lever arm* (or moment arm)
- F is the force

•The lever arm is the distance between the axis of rotation and the "line of action".

The lever arm, r, is the shortest *(perpendicular)* distance from the axis of rotation to a line drawn along the direction of the force

r = L sin θ



It is not necessarily the distance between the axis of rotation and point where the force is applied

Example

- **A mechanic holds a wrench 0.3m from the center of a nut. How large is the torque applied to the nut if he pulls at right angles to the wrench with a force of 200N?**

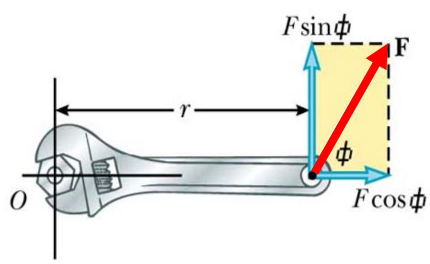
- Since he exerts the force at right angles to the wrench, the angle θ = 90º, and torque is

$$\tau = Fr\sin\theta = 0.3\times 200 = 60\,Nm$$

An Alternative Look at Torque

The force could also be resolved into its x- and y-components

- The x-component, F cosθ, produces 0 torque
- The y-component, F sinθ, produces a non-zero torque



$$\tau = rF\sin\theta$$

r is the distance along the object

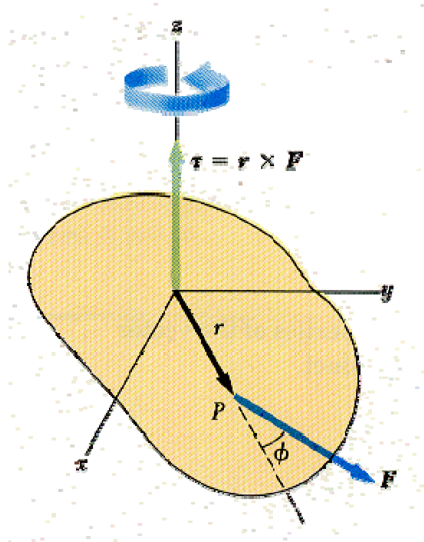
F is the force

θ is the angle between force and object

$$\tau = rF\sin\theta$$

- we can write the equation as

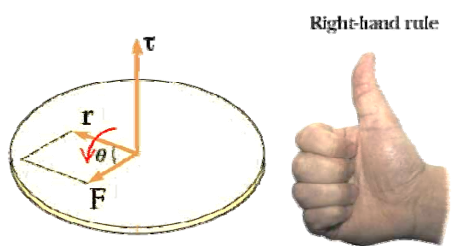
$$\vec{\tau} = \vec{r} \times \vec{F}$$



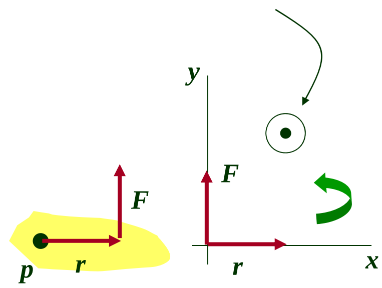
- The torque is a vector quantity and its direction is determine by the right hand rule,

If the turning tendency of the force is counterclockwise, the torque will be positive

If the turning tendency is clockwise, the torque will be negative

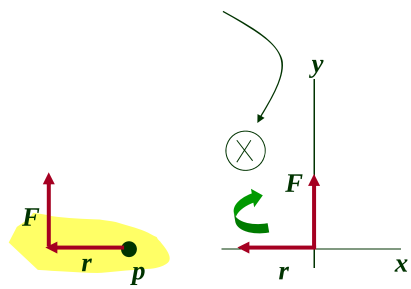


Torque is out of Page



Torque is +ve

Torque is into Page



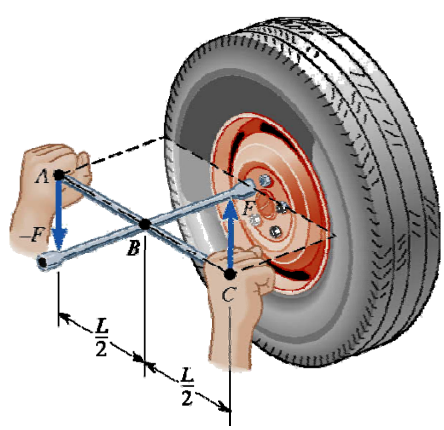
Torque is -ve

Couples

- Is a pair of forces with equal magnitudes but opposite directions acting along different lines of action.

Couples do not exert net force on an object

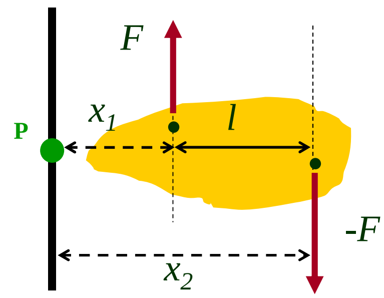
Couples produce torque, that is independent of the point of the point P



Example

- Two forces with equal magnitudes but opposite directions act on an object with different lines of action as shown in the figure.

Find the net torque on the object resulting from these forces.



Solution

نقوم بحساب الازدواج الناتج عن كل قوة عند النقطة P

- The torque resulting from the force at x_2 is

$$\tau_2 = -x_2 F$$

- The torque resulting from the force at x_1 is

$$\tau_1 = x_1 F$$

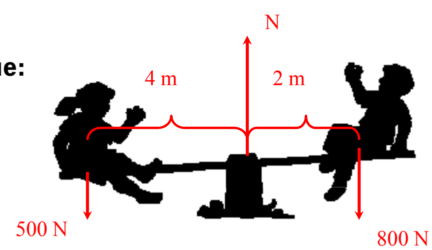
- The net torque is

$$\tau = \tau_1 + \tau_2 = x_1 F - x_2 F$$
$$\tau = (x_1 - x_2)F = -lF$$

- The minus sign means that the net torque tends to cause clockwise rotation.
- The torque is directed into the page.
- The torque is independent on the location of the point **P**, since the distance between the lines of forces is important.

Example 1

Determine the net torque:



Given:

weights: $w_1 = 500\ N$
$w_2 = 800$ N
lever arms: $d_1 = 4\ m$
$d_2 = 2\ m$

Find:

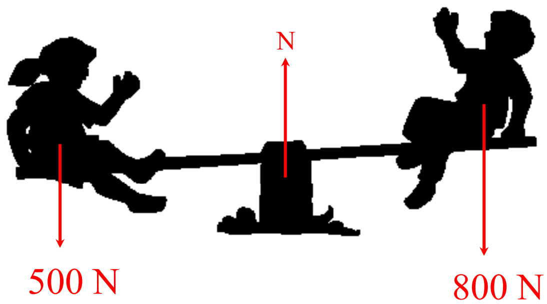
$\Sigma\tau = ?$

1. Draw all applicable forces

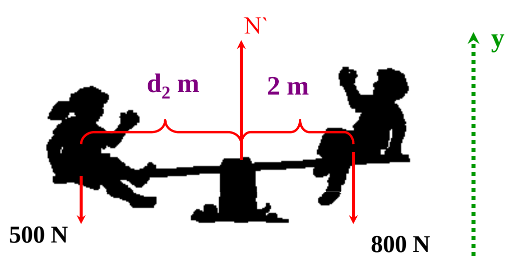
$$\sum \tau = (500\,N)(4\,m) + (-)(800\,N)(2\,m)$$
$$= +2000\,N\cdot m - 1600\,N\cdot m$$
$$= +400\,N\cdot m \quad ✓$$

Rotation would be CCW

Where would the 500 N person have to be relative to pivot for zero torque?



Example 2



Given:

weights: $w_1 = 500\ N$
$w_2 = 800$ N
lever arms: $d_1 = 4\ m$
$\Sigma\tau = 0$

Find:

$d_2 = ?$

1. Draw all applicable forces and moment arms

$$\sum \tau_{RHS} = -(800\,N)(2\,m)$$

$$\sum \tau_{LHS} = (500\,N)(d_2\,m)$$

$$-800\cdot 2\,[N\cdot m] + 500\cdot d_2\,[N\cdot m] = 0 \quad \Rightarrow \quad d_2 = 3.2\,m$$ ✓

According to our understanding of torque there would be no rotation and no motion!

What does it say about acceleration and force?

Thus, according to 2nd Newton's law ΣF=0 and a=0!

$$\sum F_i = (-500\,N) + N' + (-800\,N) = 0$$

$$N' = 1300\,N$$

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Unit 2
Static and Equilibrium
Lecture 5

Dr. Hazem Falah Sakeek
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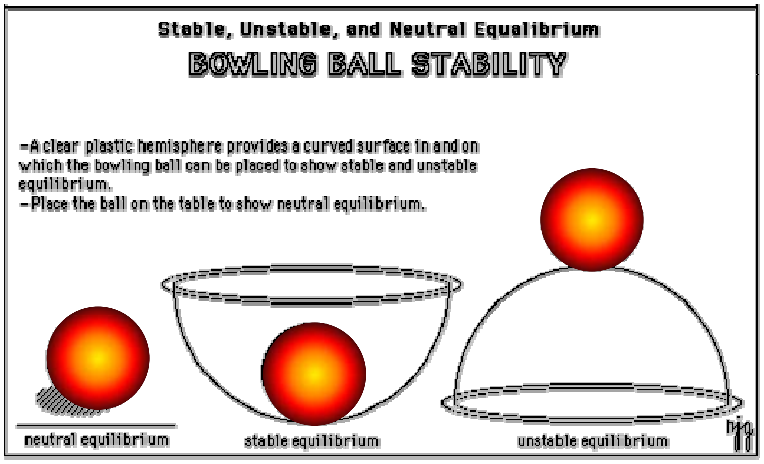
Contents

Statics & Equilibrium

- Introduction
- Torques
- Couples
- Equilibrium of Rigid Bodies
- Selected Topics
- Problems

Equilibrium of Rigid Bodies



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Equilibrium of Rigid Bodies

First Condition of Equilibrium

- The net external force must be zero

$$\Sigma \vec{F} = 0$$

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

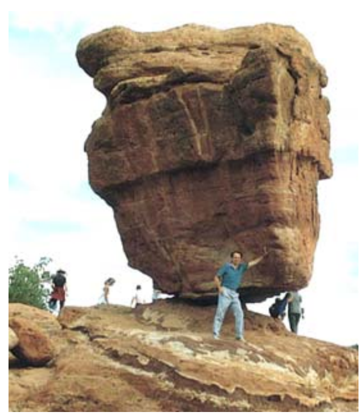
- – This is a necessary, but not sufficient, condition to ensure that an object is in complete mechanical equilibrium
- – This is a statement of translational equilibrium

Second Condition of Equilibrium

- The net external torque must be zero

$$\Sigma \tau = 0$$

- This is a statement of rotational equilibrium



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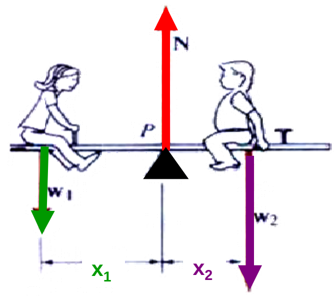
Example3

- **Two children of weights w_1 and w_2 are balanced on a board pivoted about its center.**

(a) What is the ratio of their distances x_2/x_1 from the pivot?

(b) If w_1=200N, w_2=400N, and x_1=1m, what is x_2?

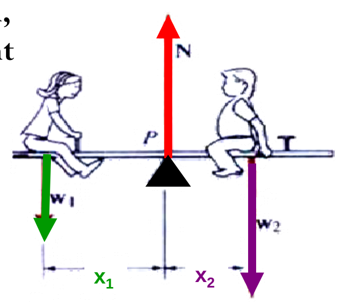


Solution

(a) **From the first equilibrium condition, the force N must balance their weight so that the net force is zero.**

$$N - w_1 - w_2 = 0 \qquad N = w_1 + w_2$$

- **From the second condition the torque about the pivot P is zero**



$$\Sigma\tau = 0$$
$$\tau = \tau_1 + \tau_2 = 0$$

$$\tau_1 = x_1 w_1$$
$$\tau_2 = -x_2 w_2$$

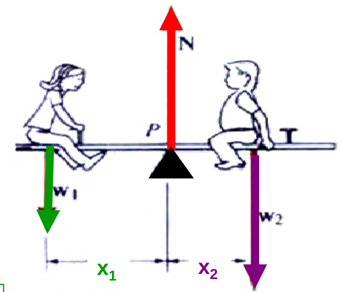
$$x_1 w_1 - x_2 w_2 = 0 \rightarrow \frac{x_2}{x_1} = \frac{w_1}{w_2}$$

(b) **If w_1=200N, w_2=400N, and x_1=1m, what is x_2?**

- From the equation

$$\frac{x_2}{x_1} = \frac{w_1}{w_2}$$

$$x_2 = x_1 \frac{w_1}{w_2} = (1m)(\frac{200N}{400N}) = 0.5m$$

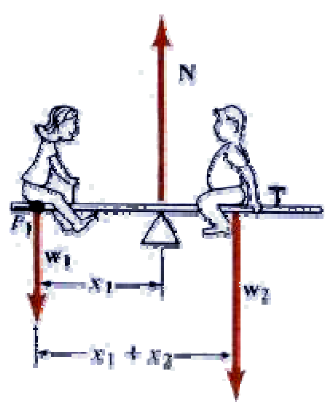


وهذا متفق مع افتراضنا أن احد الطفلين أثقل من الأخر بمرتين ولذلك هو أو هي يجب أن يجلس عند منتصف المسافة من نقطة الارتكاز

Example 4

- **Find x_2/x_1 for the seesaw of the preceding example, calculating the torques about P_1, where the child of w_1 is seated.**

- **Computing the torque about P_1**
- **From the force diagram shown in the figure we get**



Force w_1 produce torque = 0

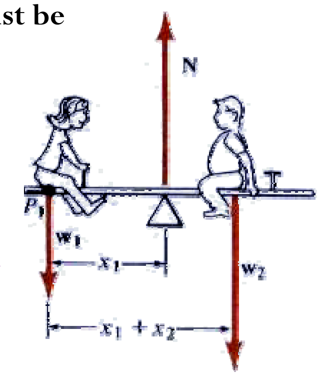
Force w_2 produce torque = $-(x_1+x_2)w_2$

Force N produce torque = x_1N

In equilibrium the sum of these torques must be zero

$$\sum \tau = 0$$

$$-(x_1 + x_2)w_2 + x_1 N = 0$$



And the Net forces must be zero

- $N - w_1 - w_2 = 0 \rightarrow N = w_1 + w_2$

- Substitute for N we get,

$$-(x_1 + x_2)w_2 + x_1(w_1 + w_2) = 0$$

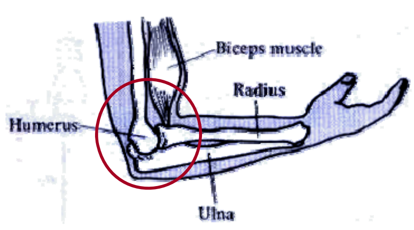
$$x_2 w_2 = x_1 w_1$$

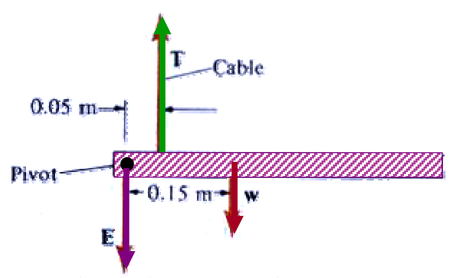
$$\frac{x_2}{x_1} = \frac{w_1}{w_2}$$

وهي نفس النتيجة التي حصلنا عليها سابقاً وهذا يعني انه يمكننا ان نبسط الحل باختيار جيد لنقطة المحور.

Example 5

- A model for the forearm in the position shown in the figure is pivoted bar supported by a cable.





The weight w of the forearm is 12N and can be treated as concentrated at the point shown.

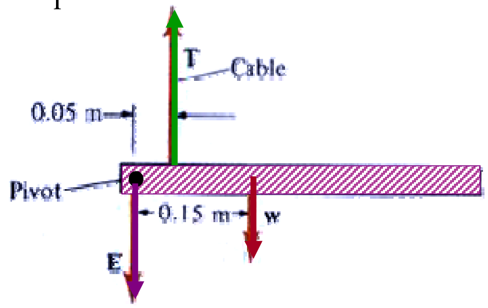
Find the tension T exerted by the biceps muscle and the force E exerted by the elbow joint.

Applying the First condition of equilibrium

$F = 0$

Then

$T - E - w = 0$



Applying the Second condition of equilibrium

$$\Sigma\tau = 0$$

Force E produce torque = 0

Force w produce torque = -(0.15)w

Force T produce torque = (0.05)T

In equilibrium the sum of these torques must be zero

$$\Sigma\tau = 0$$

$-0.15\ w + 0.05\,T = 0$

or

$T = 3\ w = 3x12 = 36\ N$

From the First equation we get the value of E

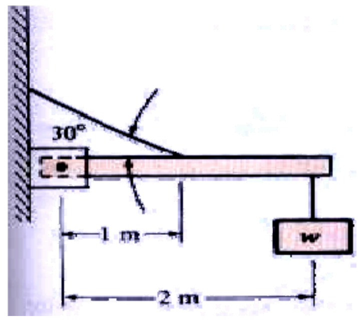
$E = T - w = 36 - 12 = +24N$

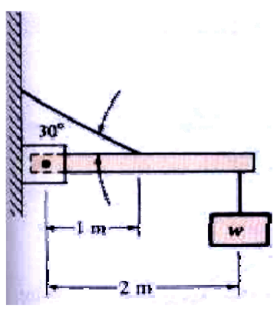
حيث ان اشارة E موجبة فهذا يعني اننا افترضنا اتجاهها كان صحيح واذا ظهرت اشارة E سالبة فيعني ذلك ان اتجاه E عكس الاتجاه المفترض.

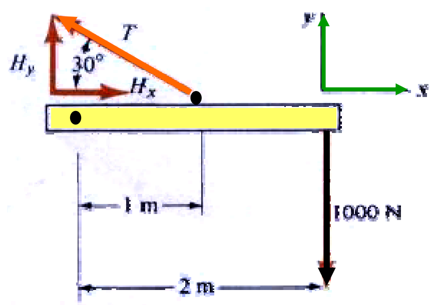
لماذا كانت قوة الشد T المبذولة بواسطة العضلة والقوة المبذولة علـى الكـوع E أكبر كثيرا من قوة الوزن w؟

Example 6

- **An advertising sign is hung from a hinged beam that is supported by a cable (see the figure).**
- The sign has a weight w=1000N, and the weights of the beam and the cable are negligible.
- Find the **tension** in the cable and the **forces exerted by the hinge**.







- **Draw a free body-diagram for all the forces.**
- قوة وزن اليافطة w للأسفل بفعل قوة الجاذبية الأرضية، وقوة الشد T على الحبل في اتجاه الحبل، أما القوة الناتجة عن تثبيت الحامل في الحائط H فهي غير معلومة ولكن نعلم أن لهذه القوة مركبة H_x ومركبة H_y وتربطهما العلاقة

$$H = [H_x^2 + H_y^2]^{1/2}$$

$$\tan\theta = \frac{H_y}{H_x}$$

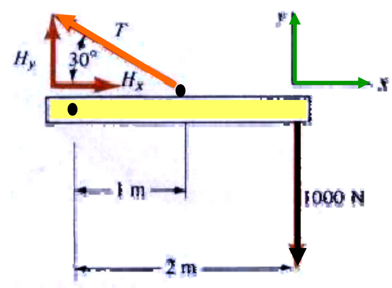
•نحتاج إلى 3 معادلات لإيجاد القوى T و H_x و H_y

•**Applying the First condition of equilibrium**

$$\Sigma \vec{F} = 0$$
$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

$H_x - T\cos 30 = 0$ (1)

$H_y + T\sin 30 - 1000 = 0$ (2)



•**Applying the Second condition of equilibrium**

$$\Sigma \tau = 0$$

يمكننا اختيار أي نقطة لتطبيق شرط الاتزان ولكن من الأنسب أن نختار النقطة عند نقطة التعليق لان الازدواج الناتج عن H_x و H_y يساوي صفر.

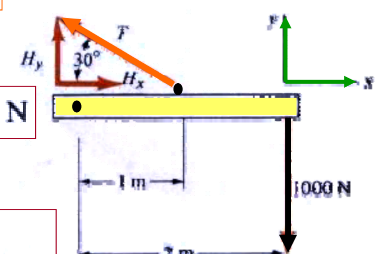
T sin30 x 1m – 1000N x 2m = 0 (3)

From eqn (3) The tension in the cable T is

$$T = \frac{2(1000\ \text{N})}{\sin 30°} = \frac{2000\ \text{N}}{0.5} = 4000\ \text{N}$$

From eqn (1) and (2) we get

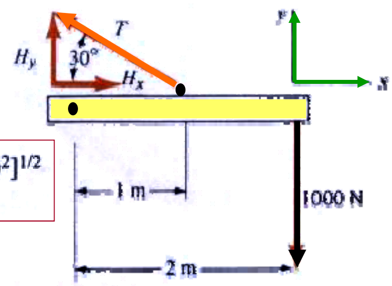
$$H_x = T\cos 30° = (4000\ \text{N})(0.866) = 3464\ \text{N}$$



$$H_y = 1000\ \text{N} - T\sin 30° = (1000\ \text{N}) - (4000\ \text{N})(0.5) = -1000\ \text{N}$$

الاشارة السالبة للمركبة H_y تشير إلى ان الاتجاه الصحيح هو عكس الاتجاه المفترض.

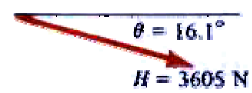
We can now find H and θ



$$H = [H_x^2 + H_y^2]^{1/2} = [(3464\ \text{N})^2 + (1000\ \text{N})^2]^{1/2} = 3605\ \text{N}$$

$$\tan\theta = \frac{H_y}{H_x} = \frac{-1000}{3464} = -0.2887$$

$$\theta = -16.1^\circ$$



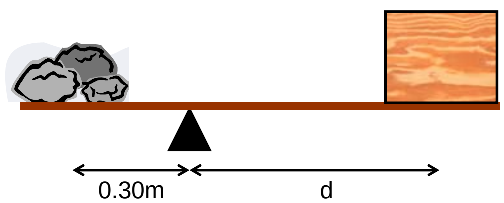
لاحظ إن قيمة الشد والقوة المؤثرة على الحامل اكبر بكثير من وزن اليافطة.

كيف يمكن تقليل الشد على الحبل؟

Exercise

Exercise 1

- A 30kg pile of rocks is balanced by a 5.0kg block of wood on a scale, as shown below. The rocks are 0.30m from the pivot point. How far, to the other side of the pivot point, must the block of wood be placed (ignore the mass of the scale)?

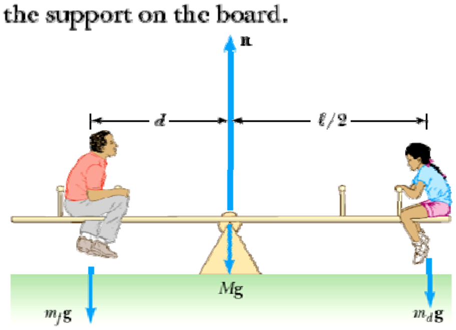


Answer: d=1.8m

Exercise 2

A seesaw consisting of a uniform board of mass M and length ℓ supports a father and daughter with masses m_f and m_d, respectively, as shown in Figure 12.8. The support (called the *fulcrum*) is under the center of gravity of the board, the father is a distance d from the center, and the daughter is a distance $\ell/2$ from the center.

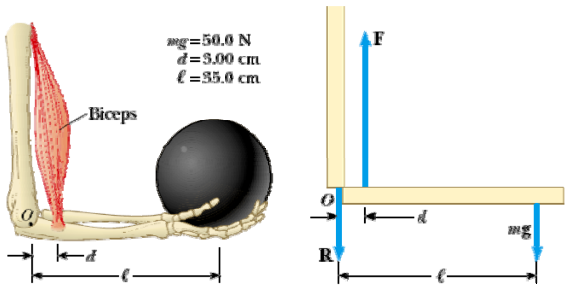
(A) Determine the magnitude of the upward force $\mathbf{n}$ exerted by the support on the board.



$$d = \left(\frac{m_d}{m_f}\right)\frac{1}{2}\ell$$

Exercise 3

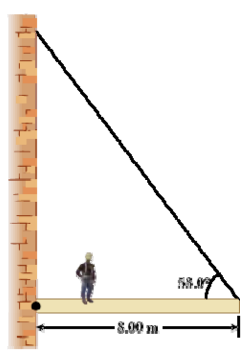
A person holds a 50.0-N sphere in his hand. The forearm is horizontal, as shown in Figure 12.9a. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0 cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of the forearm.



F=583N
R=533N

Exercise 4

A uniform horizontal beam with a length of 8.00 m and a weight of 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the beam (Fig. 12.10a). If a 600-N person stands 2.00 m from the wall, find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



T=313N
R=580N
Θ=71.1°

Selected Topics in Physics for Medical Sciences Students

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Unit 2
Static and Equilibrium
Lecture 6
Examples

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

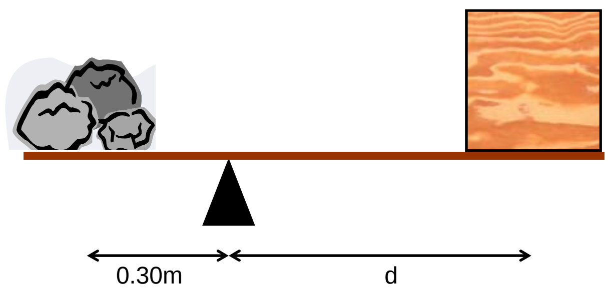
Contents

Statics & Equilibrium

- Introduction
- Torques
- Couples
- Equilibrium of Rigid Bodies
- Selected Topics
- Problems

Exercise 1

- A 30kg pile of rocks is balanced by a 5.0 kg block of wood on a scale, as shown below. The rocks are 0.30 m from the pivot point. How far, to the other side of the pivot point, must the block of wood be placed (ignore the mass of the scale)?



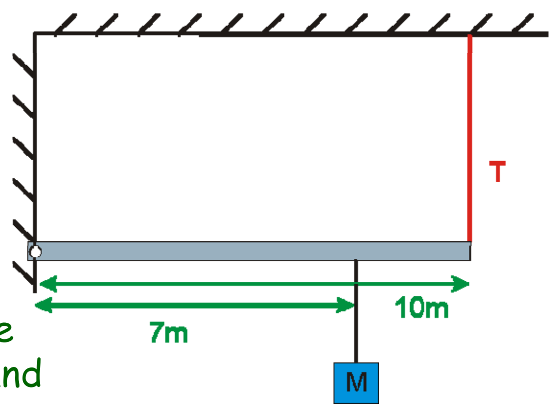
- Answer: d=1.8m

Example 1

Given M = 120 kg.
Neglect the mass of the beam.

a) Find the tension in the cable

b) What is the force between the beam and the wall



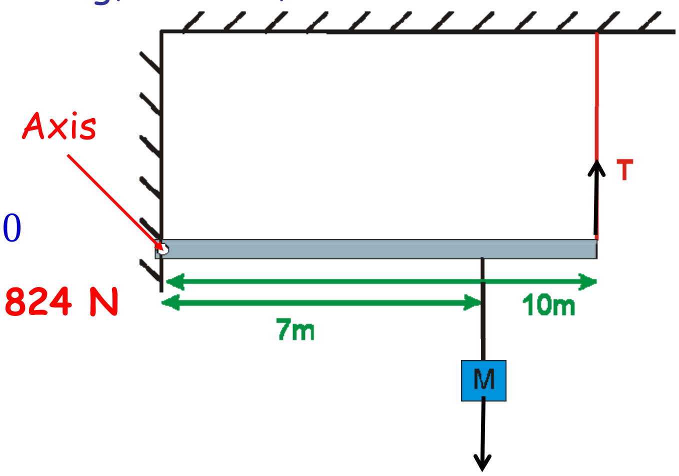
Solution

a) Given: M = 120 kg, L = 10 m, x = 7 m
Find: T

Basic formula
$$\sum \tau = 0$$

$$TL - Mgx = 0$$

Solve for T = **824 N**



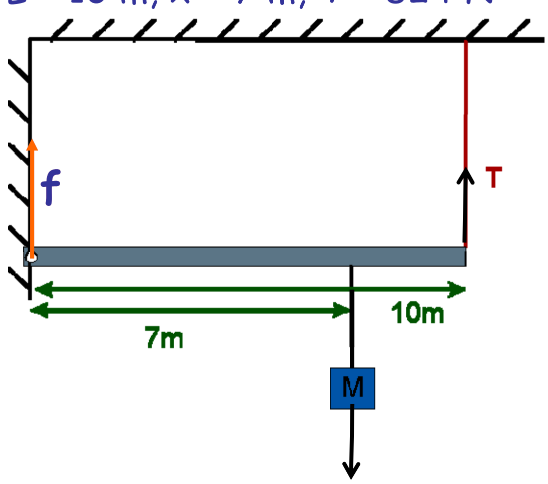
Solution

b) Given: M = 120 kg, L = 10 m, x = 7 m, T = 824 N
Find f

Basic formula
$$\sum F_y = 0$$

$$T - Mg + f = 0$$

Solve for f = **353 N**

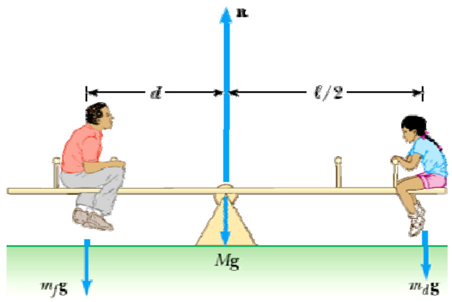


Exercise 2

A seesaw consisting of a uniform board of mass *M and* length *l* supports a father and daughter with masses m_f *and md, respectively, as shown in Figure 12.8. The support* (called the *fulcrum) is under the center of gravity of the* board, the father is a distance *d from the center, and the* daughter is a distance !/2 from the center.

(A) Determine the magnitude of the upward force n exerted by the support on the board.

(B) Determine where the father should sit to balance the system.



$$n = m_f g + m_d g + Mg$$

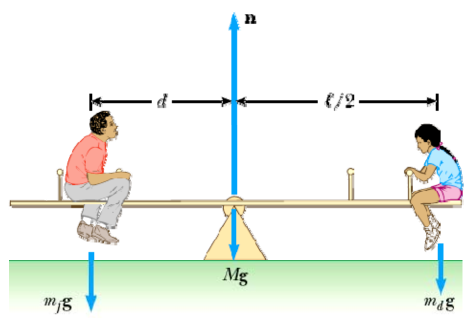
$$d = \left(\frac{m_d}{m_f}\right)\frac{1}{2}\ell$$

- Because the system in equilibrium then the net force = zero

$$\Sigma F_y = 0$$

$$n - m_f g - m_d g - Mg = 0$$

$$n = m_f g + m_d g + Mg$$

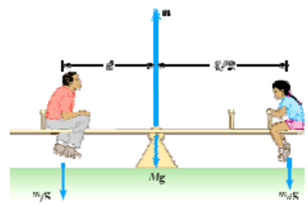


- Applying the second condition of the equilibrium

$$\Sigma\tau = 0$$

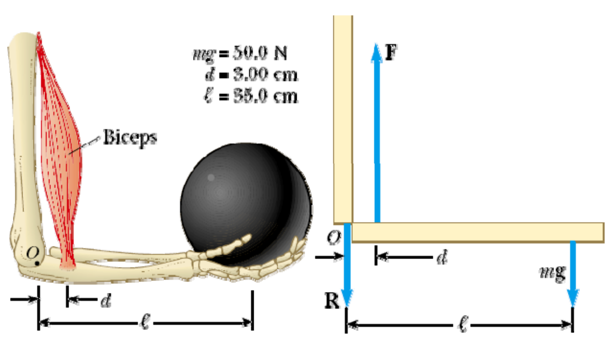
$$(m_f g)(d) - (m_d g)\frac{\ell}{2} = 0$$

$$d = \left(\frac{m_d}{m_f}\right)\frac{1}{2}\ell$$



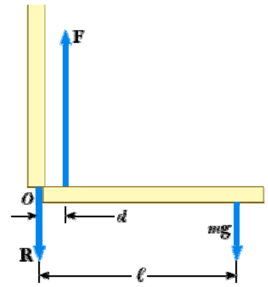
Exercise 3

A person holds a 50.0-N sphere in his hand. The forearm is horizontal, as shown in Figure. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0 cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of the forearm.



F=583N
R=533N

- **Solution**:
- We simplify the situation by modeling the forearm as shown in the figure, where F is the upward force exerted by the biceps and R is the downward force exerted by the upper arm at the joint. From the first condition of equilibrium we have,



$$\sum F_y = F - R - 50.0\text{ N} = 0$$

- From the second condition of equilibrium about point O we have,

$$\sum \tau = Fd - mg\ell = 0$$

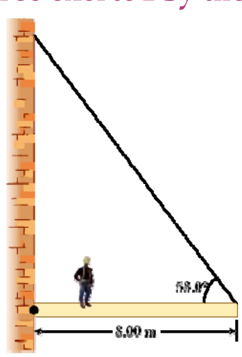
$$F(3.00\text{ cm}) - (50.0\text{ N})(35.0\text{ cm}) = 0$$

$$F = 583\text{ N}$$

$$R = 533\text{ N}$$

Exercise 4

A uniform horizontal beam with a length of 8.00 m and a weight of 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the beam. If a 600-N person tands 2.00 m from the wall, find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



T=313N
R=580N
Θ=71.1°

- Applying the first condition of the equilibrium

$$\sum F_x = R\cos\theta - T\cos 53.0° = 0$$

$$\sum F_y = R\sin\theta + T\sin 53.0° - 600\text{ N} - 200\text{ N} = 0$$

- From the second condition of equilibrium about point O we have,

$$\sum\tau = (T\sin 53.0°)(8.00\text{ m}) - (600\text{ N})(2.00\text{ m}) - (200\text{ N})(4.00\text{ m}) = 0$$

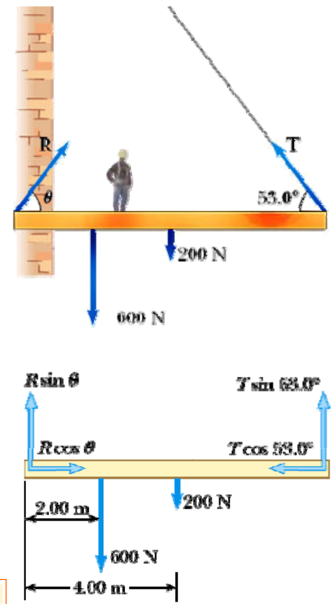
$$T = 313\text{ N}$$

$$R\cos\theta = 188\text{ N}$$
$$R\sin\theta = 550\text{ N}$$

$$\tan\theta = \frac{550\text{ N}}{188\text{ N}} = 2.93$$

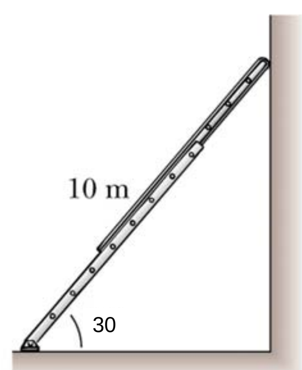
$$\theta = 71.1°$$

$$R = \frac{188\text{ N}}{\cos\theta} = \frac{188\text{ N}}{\cos 71.1°} = 580\text{ N}$$

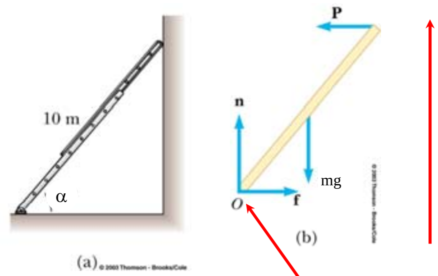


Exercise 5

Suppose that you placed a 10 m ladder (which weights 100 N) against the wall at the angle of 30°. What are the forces acting on it and when would it be in equilibrium?



Solution:



Given:

weights: $w_1 = 100\ N$
length: $l = 10\ m$
angle: $\alpha = 30°$
$\Sigma\tau = 0$

Find:

$f = ?$
n=?
P=?

1. Draw all applicable forces
2. Choose axis of rotation at bottom corner (τ of *f* and *n* are 0!)

Torques:

$$\sum \tau = -mg\frac{L}{2}\cos 30^\circ + PL\sin 30^\circ = 0$$

$$0 = -100\,N \cdot \frac{1}{2} \cdot 0.866 + P \cdot 1 \cdot \frac{1}{2}$$ ✓

$$P = -86.6\,N$$

Forces:

$$\sum F_x = f - P = 0$$

$$f = 86.6\,N$$

$$\sum F_y = n - mg = 0$$

$$n = 100\,N$$ ✓

Note: f = μ_s n, so $$\mu_s = \frac{f}{n} = \frac{86.6\,N}{100\,N} = 0.866$$

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Unit 2

Static and Equilibrium

Lecture 7

Center of Gravity

Dr. Hazem Falah Sakeek

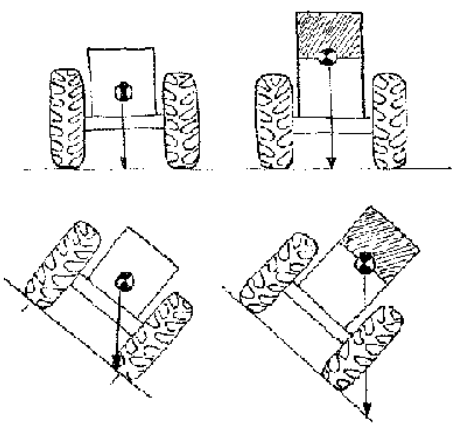
Al-Azhar University - Gaza

Contents

Statics & Equilibrium

Center of Gravity

- **The force of gravity acting on an object must be considered.**
- **In finding the torque produced by the force of gravity, all of the weight of the object can be considered to be concentrated at one point**



Calculating the Center of Gravity

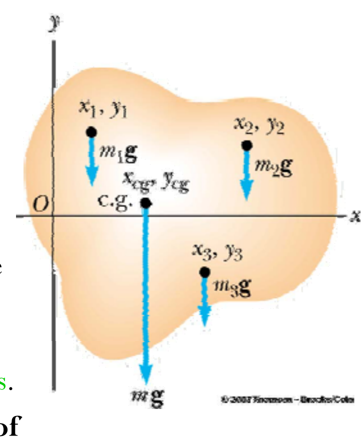
The object is divided up into a large number of very small particles of weight (mg).

Each particle will have a set of coordinates indicating its location (x,y).

The torque produced by each particle about the axis of rotation is equal to its weight times its lever arm.

We wish to locate the point of application of the *single force*, whose magnitude is equal to the weight of the object, and whose effect on the rotation is the same as all the individual particles.

- **This point is called the *center of gravity* of the object.**



Coordinates of the Center of Gravity

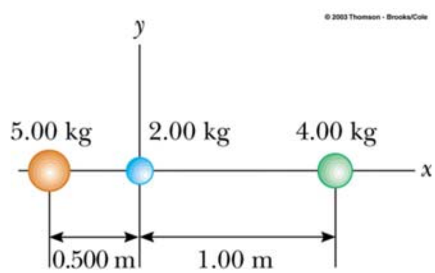
- The coordinates of the center of gravity can be found from the sum of the torques acting on the individual particles being set equal to the torque produced by the weight of the object

$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} \quad and \quad y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

- The center of gravity of a homogenous, symmetric body must lie on the axis of symmetry.
- Often, the center of gravity of such an object is the *geometric* center of the object.

Example:

Find center of gravity of the following system:



Given:

masses: $m_1 = 5.00$ kg
$m_2 = 2.00$ kg
$m_3 = 4.00$ kg
lever arms: $d_1 = 0.500\ m$
$d_2 = 1.00\ m$

Find:

Center of gravity

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{5.00\,kg(-0.500\,m) + 2.00\,kg(0\,m) + 4.00\,kg(1.00\,m)}{11.0\,kg}$$

$$= 0.136\,m$$

Experimentally Determining the Center of Gravity

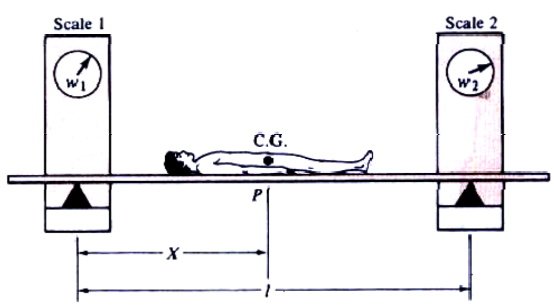
- The wrench is hung freely from two different pivots
- The intersection of the lines indicates the center of gravity

Center of Gravity of Humans

Another technique used to determine the center of gravity of humans is described in the figure below.

A board of length l is supported at its ends resting on scales adjusted to read zero with the board alone.

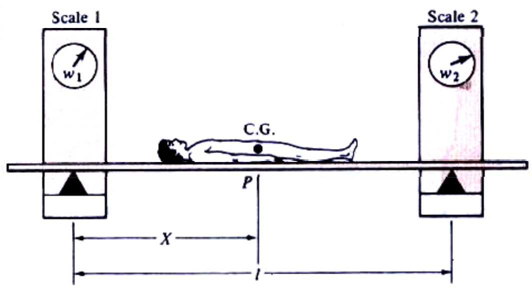
When a person lies on the board the scales read w_1 and w_2.



- The condition for the torque $\sum \tau = 0$ can be used to Find X.
- The torque about point P is

$$Xw_1 - (l - X)w_2 = 0$$

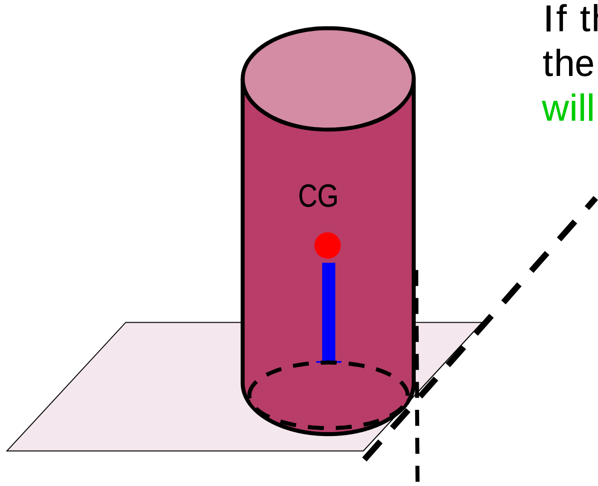
$$X = \frac{lw_2}{w_1 + w_2}$$



Why things fall over

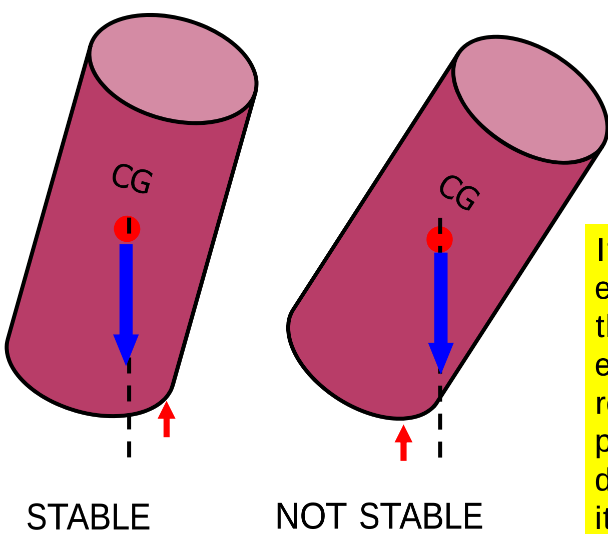
- **Every object has a special point called the center of gravity (CG). The CG is usually right in the center of the object.**
- if the center of gravity is **supported**, the object will not fall over.
- You generally want a running back with a low CG→ then it's harder to knock him down.
- The lower the CG the more **stable** an object is. stable → not easy to knock over!

Condition for stability



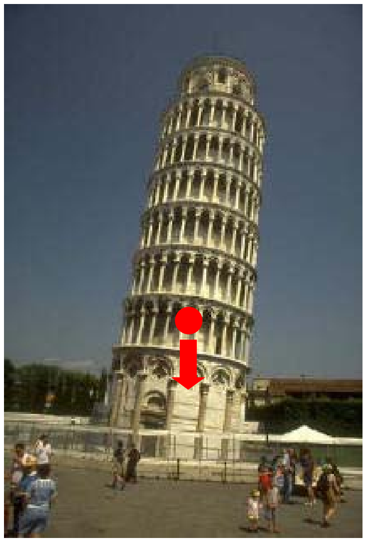
If the CG is above the edge, the object will not fall

when does it fall over?



If the vertical line extending down from the CG is inside the edge the object will return to its upright position → the torque due to gravity brings it back.

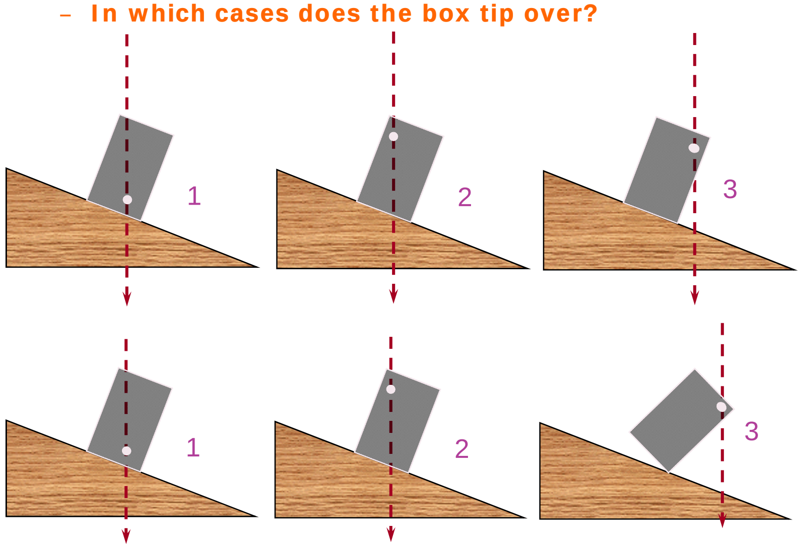
Stable structures

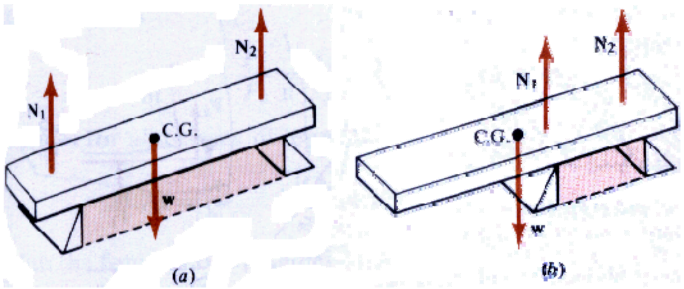


Structures are wider at their base to lower their center of gravity

A box is placed on a ramp in the configurations shown below. Friction prevents it from sliding. The center of mass of the box is indicated by a dot in each case.

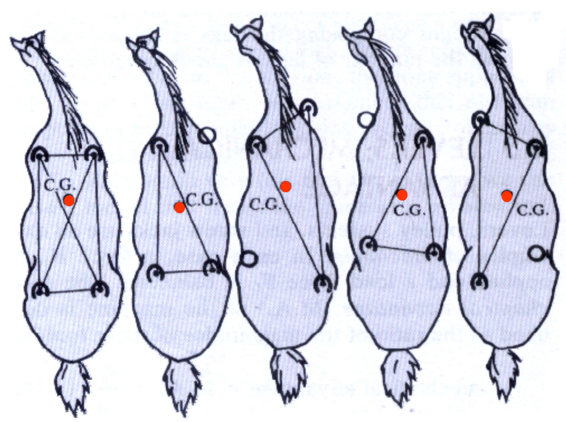
- **In which cases does the box tip over?**





The beam is in equilibrium

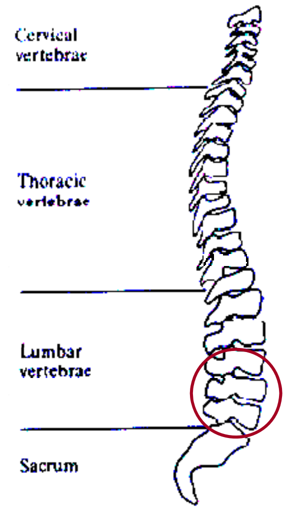
The beam is not in equilibrium

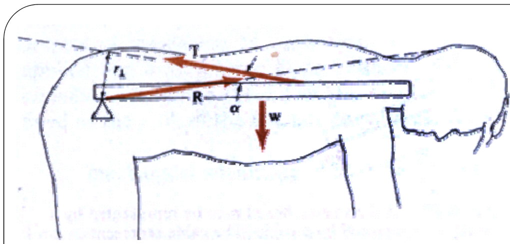


The diagram of a quadruped walking as seen from above. The open circle represents the foot that is off the ground. Notice that the center of gravity is always within the triangle formed by the three feet on the ground.

The Spinal Column

- The human spinal column is made of 24 vertebrae separated by fluid-filled disks.
- When the person bends, the spine is consider as lever, producing a very large force on the lumbrosacral disk, which separate the last vertebra from the sacrum.



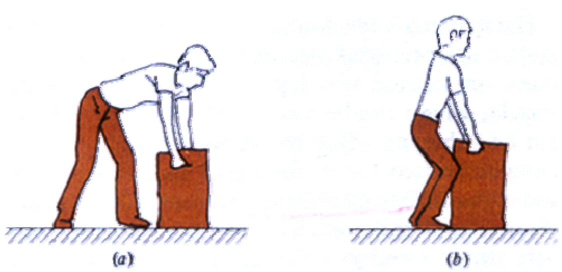


- The pivot exert force R
- The muscles of the back produce a force T
- When the back is horizontal $\alpha=12^o$.
- w is 65% of the total body weight.

Notice that because α is small, the line of action of **T** passes close to the pivot, so its lever arm $r_\perp$ is small. However, the weight **w** acts at right angles to the spine, and its lever arm is much longer. Hence for their torques to balance, the muscle force **T** must be much greater than the weight. Because **T** is large, its horizontal component is also large. In equilibrium, the force **R** due to the sacrum must have an equal but opposite horizontal component,

so this force due to the sacrum is also much larger than the weight.

If this calculation is carried out in detail, the numbers obtained are impressively large. For a man weighing 750 N (a mass of 77 kg), **T** and **R** are each close to 2200 N! If the man is also lifting a 175-N (18-kg) child, so that there is an extra 175-N weight at the right end of the bar in Fig. 4.30, **T** and **R** are each about 3300 N! Such forces in the muscles and on the disk are potentially quite hazardous.

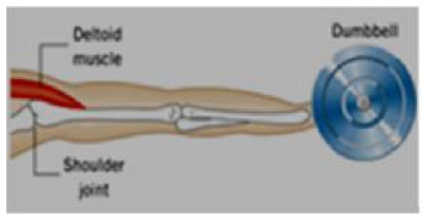


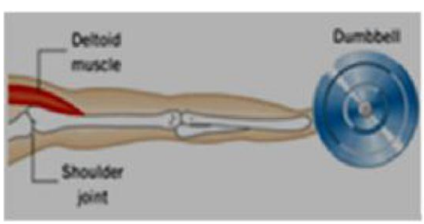
Incorrect way to lift a weight

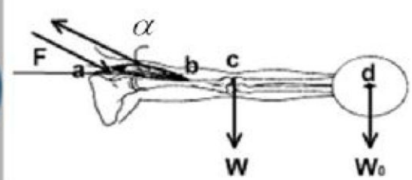
Correct way to lift a weight

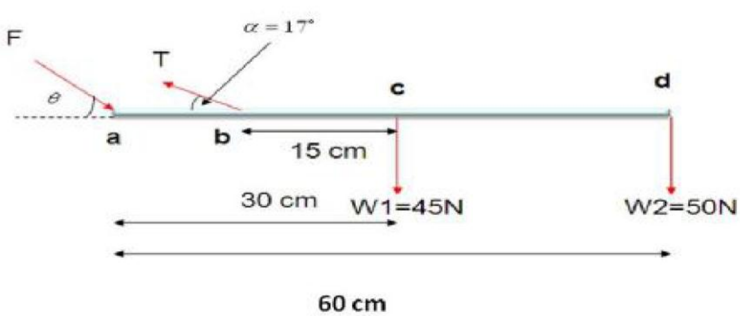
Example

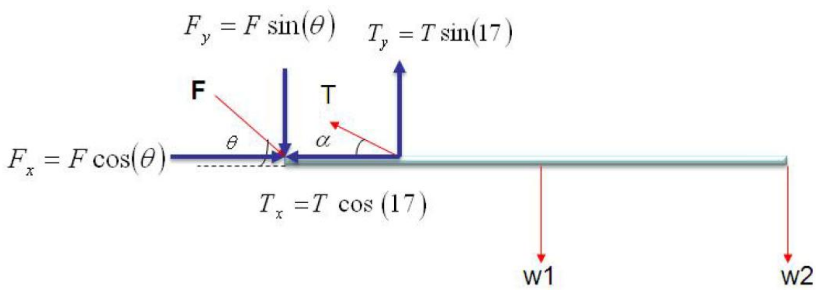
- The illustration below shows an athlete's outstretch arm (length 60cm) holding w2 = 5kg. The tension T in the deltoid muscle is applied at an angle of 17 degree. The mass of the arm w1 is 4.5 kg (at 30 cm from shoulders joint). Determine the tension T in the muscle, which is 15 cm away from w1, and the force exerted by the shoulders joint F and its direction. Consider g = 10 m. s











- Since the outstretched arm is at equilibrium start applying the laws of equilibrium, that is, apply the first law of equilibrium
- $\sum F_x = 0$, *and* $\sum F_y = 0$, notice the arrow head of each vector or vector's component.

$$F_x - T\cos(17) = 0 \Rightarrow F_x = T\cos(17)$$

$$-F_y + T\sin(17) - w_1 - w_2 = 0 \Rightarrow F_y = T\sin(17) - w_1 - w_2$$

$$\sum \tau = 0$$

$$-w_1(0.3m) - w_2(0.60m) + T\sin(17)(0.15m) = 0,$$

- Solve for , we have

$$T = \frac{+w_1(0.3m) + w_2(0.60m)}{\sin(17)(0.15m)} = \frac{45\times 0.3 + 50\times 0.60}{0.15\times\sin(17)} = 991.888N$$

- From equation 1, we have

$$F_x = T\cos(17) = 948.547N ,$$

- and from equation 2, we have

$$F_y = T\sin(17) - w_1 - w_2 = 194.999N$$

$|F| = \sqrt{F_x^2 + F_y^2} = 968.38N$ and its direction is $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 11.6163^\circ$.

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Unit 2
Static and Properties of Matter

Selected Problems

Dr. Hazem Falah Sakeek

1. Define and Explain
    - Rigid Body
    - Torque
    - Couple
    - Equilibrium
    - Static
    - Center of Gravity

2. Find the magnitude and sign of the torque due to each of the weights in Figure 1 relative to point P.

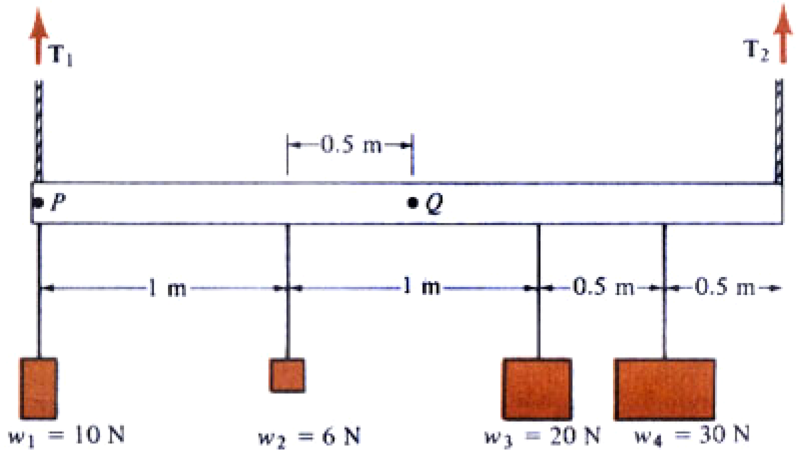


Figure 1

3. Find the magnitude and sign of the torque due to each of the weights in Figure 1 relative to point Q.

4. Using a wrench 0.4m long. A force of a 100N is needed to turn a nut. (a) How large a torque is required? (b) How large a force is needed to turn the nut using wrench 0.15m long?

5. In figure 2 find the torque due to F_1 and F_2 relative to point P.

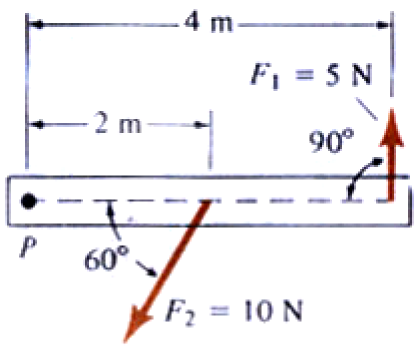


Figure 2

6. The bar in Figure 2 is pivoted at point P. Will it tend to start rotating if it is initially at rest? Explain, and indicate in which direction it would rotate if your answer is yes.

7. A weightless bar supported by two vertical ropes has four weights hung from it as shown in figure 1 Find the tensions T_1 and T_2 in the ropes.

8. Figure 3 shows the forearm when the person is holding a 12N weight (w1) in the hand (w is the weight of the forearm). (a) Find the force T exerted by the elbow joint.

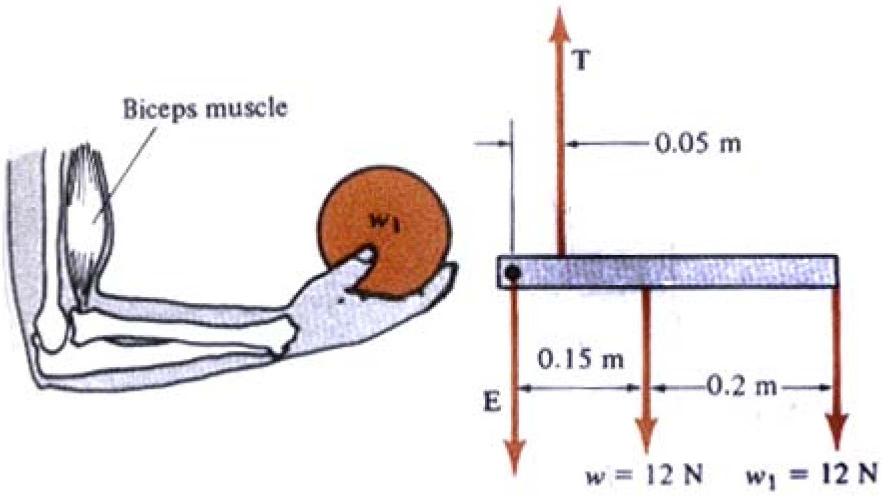


Figure 3

9. Find the tension in the ropes in figure 4

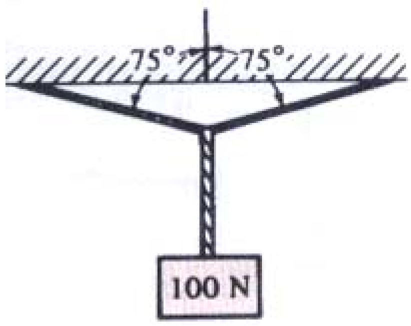


Figure 4

10. In figure 5 an object is supported by a hinged, weightless rod and a cable. Find the tension in the cable and the force exerted by the hinge.

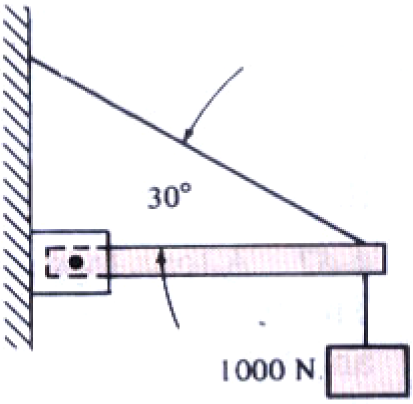


Figure 5

11. In figure 6 the hinged rod and cable are weightless. The cable will break when the tension exceeds 2000N. What is the maximum weight w that can be supported?

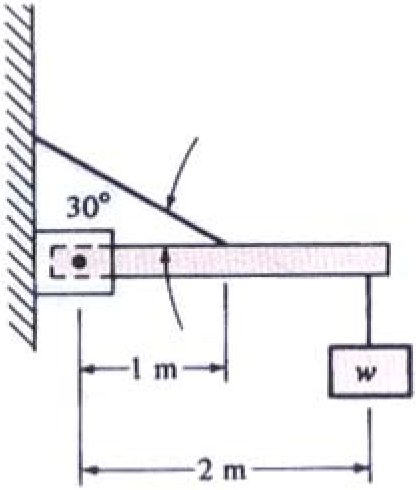


Figure 6

12. In figure 7, the weight of the upper body is w=490N. Find the force T exerted by the spinal muscles and the components R_x and R_y of the force R exerted by the pivot (sacrum) if the weight w_1 is (a) zero; (b) 175N

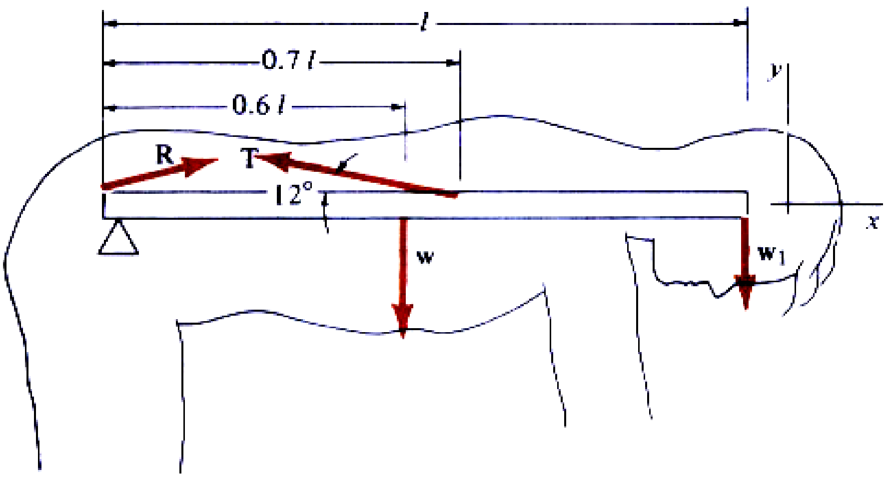
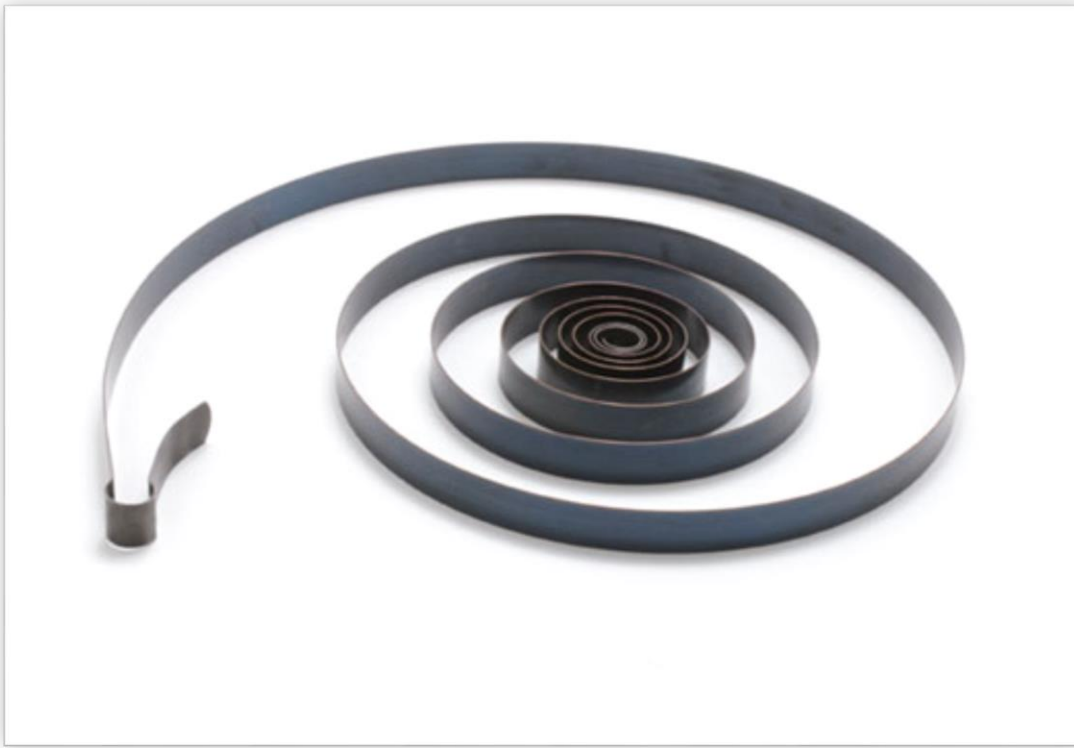
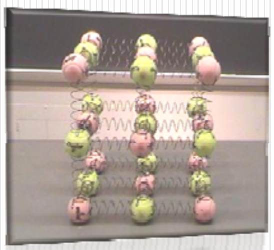


Figure 7



الوحدة الثالثة: خواص المادة
Properties of Matter

Selected Topics in Physics for Medical Sciences Students



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Unit 3

Elastic Properties of Materials

Lecture 8

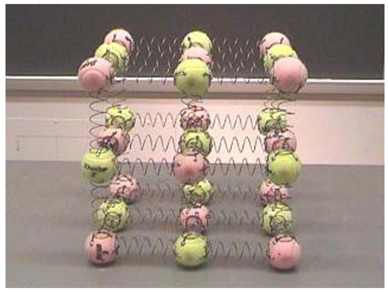
Dr. Hazem Falah Sakeek

Al-Azhar University - Gaza

Elastic Properties of Materials

- لقد افترضنا في الدروس السابقة ان القوة التي يتعرض لها جسم لا تؤثر على شكله أو حجمه، وسميت الأجسام في هذه الحالة بـ Rigid Body وهي تلك الأجسام التي لا يتغير فيها شكلها أو حجمها نتيجة لتعرضها إلى قوة خارجية.
- ولكن في الواقع العملي فإن هذا غير صحيح حيث انه إذا تعرض الجسم إلى قوة فإنها تؤدي إلى تغيير في شكله أو حجمه، فمثلا يتقوس لوح من الخشب تحت تأثير قوة عليه، وإذا زادت القوة المؤثرة لتتجاوز قوة تحمل اللوح فينكسر. كذلك يطول أو يقصر ساق من الحديد إذا تعرض إلى قوة شد أو قوة ضغط على امتداد طوله.

- وكما نعلم ان الأجسام مكونة من جزيئات تترابط مع بعضها البعض بواسطة قوى كهربية ويمكن من خلال إجراء بعض الحسابات أن نقوم بتقدير سمك اللوح من الخشب أو من أي مادة أخرى ليتحمل وزن معين.

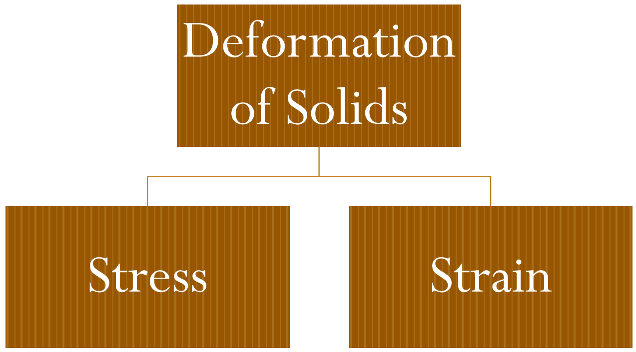


سوف ندرس المواضيع التالية:

- **Stress**
- **Strain**
- **Young's Modulus**

Introduction

- In our first topic, Static Equilibrium, we examined structures in which we assumed the object were rigid
- Rigid means that the object did not deform due to the applied force.
- In real, we have deformation "change in shape and size ".

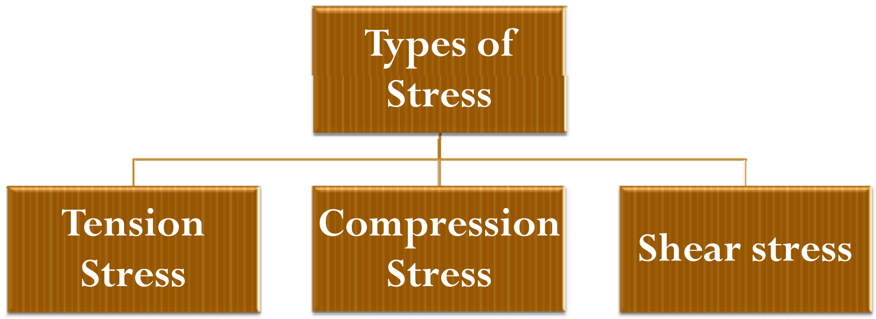


Stress and Strain

- **Stress is a quantity that is proportional to the force causing a deformation**

- **Stress is the external force acting on an object per unit cross-sectional Area.**

- **Strain is the result of stress, which is the measure of the degree of deformation**

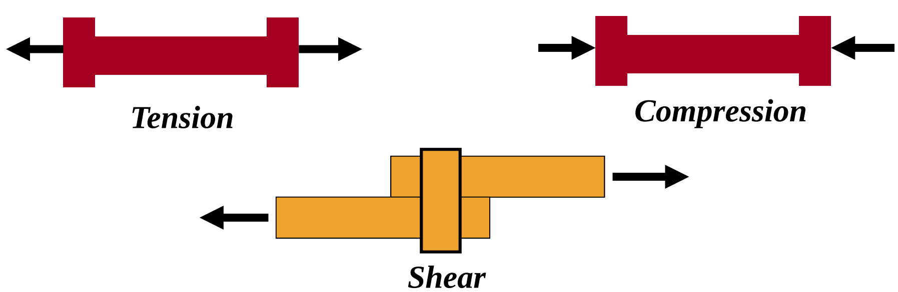


Types of Stress

***Tension Stress*: is the force per unit area producing elongation of an object.**

***Compression Stress*: is the force per unit area producing compression of an object.**

Shear stress: is the opposite force "sliding forces " applied to parallel faces of the object. Producing change in shape of material without changing its volume



Stress

- **Stress :** Force per unit Area

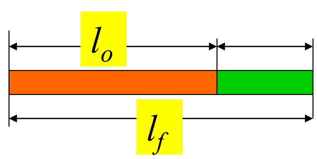
$$\sigma = \frac{F}{A}$$

F : Force applied in Newton
A : cross sectional area in m^2
σ : stress in N/m^2

Strain

Ratio of elongation of a material to the original length

$$\varepsilon = \frac{\Delta l}{l_o}$$



Δl : elongation (m)
l_o : original length of a material (m)
ε : strain Dimensionless

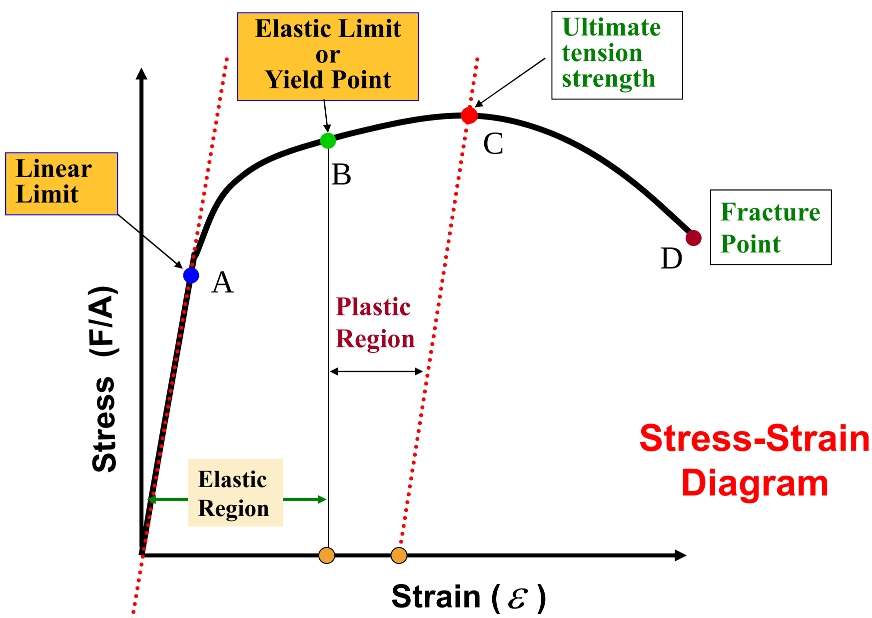
Elongation $\Delta l = l_f - l_o$

l_f : loaded length of a material (m)

Relation Between Stress and Strain

- The relation between the stress and the strain for a material under tension can be found experimentally.

- A plot of Strain vs. Stress.
- The diagram gives us the behavior of the material and material properties.
- Each material produces a different stress-strain diagram.



Elastic Limit

For small values of the strain, the stress-strain graph is a straight line; the stress σ is linearly proportional to the strain ε. This is called the *linear region* for a material. Beyond the *linear limit A*, the stress is no longer linearly proportional to the strain. However, from *A* to the *elastic limit* or *yield point B*, the object still returns to its original dimensions when the applied force *F* is removed.

The deformation up to *B* is said to be elastic.

If the applied force is further increased, the strain increases rapidly. In this region, if the applied force is removed, the object does not return completely to its original dimensions; it retains a permanent deformation. The highest point *C* on the stress-strain graph is the *ultimate tension strength* σ_t of the material or its maximum stress. Beyond this point, additional strain is produced even by a reduced applied force, and *fracture* occurs at point *D*. From *B* to *D* the material is said to undergo *plastic deformation*. If the ultimate tension strength and fracture points *C* and *D* are close together,
the material is *brittle*; if they are far apart,
the material is said to be *ductile*.

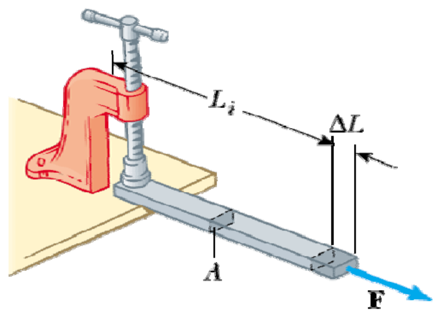
Example

- A bar has dimensions 1cm by 1cm by 20cm. It is subjected to a 10000N tension force and stretches 0.01cm. Find

(a) the stress;

(b) the strain;

(c) If the stress-strain graph is straight line, how much does the bar stretch when the applied force is increased to 50000N?



Solution

(a) The stress is the ratio of the force applied and cross-sectional area. Thus,

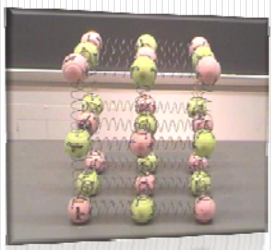
$$\sigma = \frac{F}{A} = \frac{10000N}{(0.01m)^2} = 10^8 N/m^2$$

(b) The strain is the fractional elongation

$$\varepsilon = \frac{\Delta l}{l_o} = \frac{0.01cm}{20cm} = 5\times10^{-4}$$

(c) If the stress-strain graph is a straight line, the two are proportional. When the applied force increased by a factor of 5 (50000/10000) then the elongation increases to 0.05cm

Selected Topics in Physics for Medical Sciences Students



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Unit 3
Elastic Properties of Materials
Lecture 9
Hook's Law

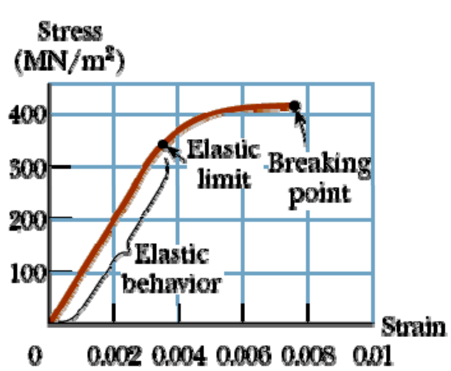
Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Young's Modulus

- In the linear region of the Strain vs. Stress diagram, the slope equals the stress-to-strain ratio and is called Young' Modulus or Elastic Modulus (E) of the material

$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{\sigma}{\varepsilon}$$



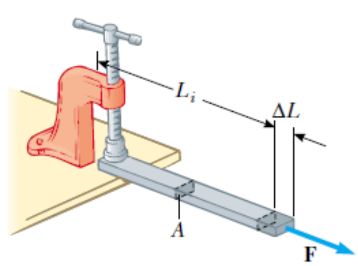
Deformation types and define an elastic modulus

1. **Young's modulus,** which measures the resistance of a solid to a change in its length.
2. **Shear modulus,** which measures the resistance to motion of the planes within a solid parallel to each other.
3. **Bulk modulus,** which measures the resistance of solids or liquids to changes in their volume.

Young's Modulus: Elasticity in Length

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L / L_0}$$

Young's modulus



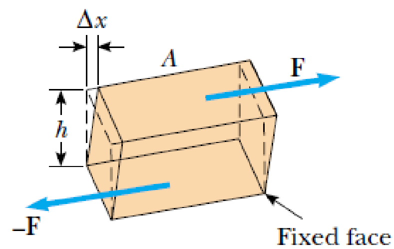
- Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression.

Shear Modulus: Elasticity of Shape

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta x/h}$$

shear modulus

Where Δx is the horizontal distance that the sheared face moves and h is the height of the object.

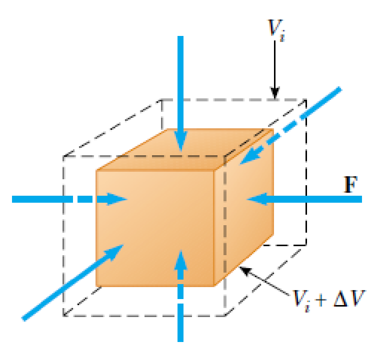


Bulk Modulus: Volume Elasticity

$$Y = \frac{\sigma}{\varepsilon} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$

bulk modulus

- A negative sign is inserted in this defining equation so that *Y is a positive number. This* maneuver is necessary because an increase in pressure (positive ΔP) *causes a decrease* in volume (negative ΔV) *and vice versa*.



Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Example

How much pressure is needed to compress the volume of an iron block by 0.10 percent? Express answer in N/m^2, and compare it to atmospheric pressure (1.0×10^5 N/m^2).

Solution:

Y for iron = 90×10^{10} N/m^2

$$Y = -\frac{\Delta P}{\Delta V / V_i}$$

$$\frac{\Delta V}{V_i} = \frac{0.1}{100} = 10^{-3}$$

$$90 \times 10^{10} = -\frac{\Delta P}{10^{-3}}$$

$$\therefore \Delta P = 90 \times 10^{6} \, N/m^2$$

Hook's Law

- The linear stress-strain region in the diagram is also called **Hook's Law** region. In this region, since the stress is linearly related to the strain. The force is linearly related to the elongation.

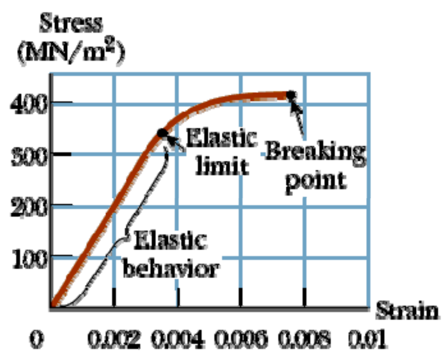
$$Y = \frac{\sigma}{\varepsilon}$$

$$\sigma = Y\varepsilon$$

$$\varepsilon = \frac{\Delta l}{l_o}$$

$$\sigma = \frac{F}{A}$$

$$\frac{F}{A} = Y\frac{\Delta l}{l_o}$$



$$\frac{F}{A} = Y\frac{\Delta l}{l_o}$$

- Thus, in tension or compression the force on an object is proportional to its elongation.

$$F = k\Delta l$$ Hook's law

- Where **k** is the spring constant

$$k = \frac{YA}{l_o}$$

ينطبق قانون هوك على المواد التي في منطقة التغير الخطي بين الإجهاد stress والانفعال strain وتكون قيمة k كبيرة كلما زاد مساحة المقطع وقل طوله.

Example 10-2

In example 10-1, a $10^8 N/m^2$ stress produces a strain of $5x10^{-4}$.

What is Young's modulus for this bar?

Solution

$$Y = \frac{\sigma}{\varepsilon} = \frac{10^8}{5\times10^{-4}} = 20\times10^{10}\, N/m^2$$

Example

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1.0 \text{ x} 10^5 \text{ N/m}^2$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is $2.0 \text{ x } 10^7 \text{ N/m}^2$. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

Solution

- From the definition of bulk modulus, we have

$$Y = -\frac{\Delta P}{\Delta V / V_i}$$

$$\Delta V = -\frac{V_i \Delta P}{Y}$$

$$\Delta V = -\frac{(0.50m^2)(2.0\times10^7\,N/m^2 - 1.0\times10^5\,N/m^2)}{6.1\times10^{10}\,N/m^2}$$

$$\Delta V = -1.6\times10^{-4}\,m^3$$

The negative sign indicates that the volume of the sphere decreases.

Example

A vertical steel girder (عارضة خشبية) with a cross-sectional area of 0.15 m^2 has a 1550 kg sign hanging from its end. (Ignore the mass of the girder itself.)

- **(a) What is the stress within the girder?**
- **(b) What is the strain on the girder?**
- **(c) If the girder is 9.50 m long, how much is it lengthened?**

Solution

- $A = 0.15\ m^2$, $m = 1550\ kg$, $l_o = 9.5m$
- $F = mg = 1550 \times 9.8 = 15200\ N$

$$\sigma = \frac{F}{A} = \frac{mg}{A} = \frac{15200}{0.15} = 1.0 \times 10^5\ N/m^2$$

$$Y = \frac{\sigma}{\varepsilon}$$

$$\therefore \varepsilon = \frac{\sigma}{Y} = \frac{1.0 \times 10^5}{200 \times 10^9} = 5.0 \times 10^{-7}$$

$$\Delta l = \varepsilon \times l_o = 4.8 \times 10^{-6}\ m$$

Example

A 15 cm long animal tendon (وتر) was found to stretch 3.7 mm by a force of 13.4 N. The tendon was approximately round with an average diameter of 8.5 mm.

- **Calculate the elastic modulus of this tendon.**

Solution

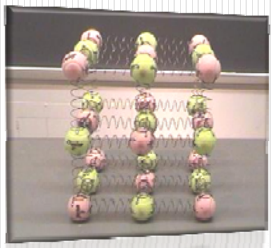
- $\Delta l = 3.7$ mm, $l_0 = 15$ cm, $2r = 8.5$ mm and $\mathbf{F = 13.4\ N}$
- $\Delta l = 3.7 \times 10^{-3}$ m, $l_0 = 0.15$ m, $r = 4.25 \times 10^{-3}$ m
- The cross sectional area $A = \pi r^2 = 5.7 \times 10^{-5}\ m^2$

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta l / l_0} = 9.5 \times 10^6\ N/m$$

Problem

- **A nylon tennis string on a racquet is under a tension of 250 N. If its diameter is 1.00 mm, by how much is it lengthened from its un-tensioned length?**

Selected Topics in Physics for Medical Sciences Students



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Unit 3

Elastic Properties of Materials

Lecture 10

Elastic Strain Energy

Dr. Hazem Falah Sakeek

Al-Azhar University - Gaza

Example 1

A 200-kg load is hung on a wire having a length of 4.00 m, cross-sectional area 0.200 x 10^{-4} m^2, and Young's modulus 8.00 x10^{10} N/m^2. What is its increase in length?

$$\frac{F}{A} = Y\frac{\Delta L}{L_i}$$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{\left(0.200 \times 10^{-4}\right)\left(8.00 \times 10^{10}\right)} = \boxed{4.90 \text{ mm}}$$

Example 2

A child slides across a floor in a pair of rubber-soled shoes. The friction force acting on each foot is 20.0 N. The footprint area of each shoe sole is 14.0 cm^2, and the thickness of each sole is 5.00 mm. Find the horizontal distance by which the upper and lower surfaces of each sole are offset. The shear modulus of the rubber is 3.00 MN/m^2.

$$Y = \frac{F/A}{\Delta x/h} \qquad \Delta x = \frac{hF}{YA}$$

$$\Delta x = \frac{(5.00 \times 10^{-3}\ \text{m})(20.0\ \text{N})}{(3.0 \times 10^{6}\ \text{N/m}^2)(14.0 \times 10^{-4}\ \text{m}^2)} = 2.38 \times 10^{-5}\ \text{m}$$

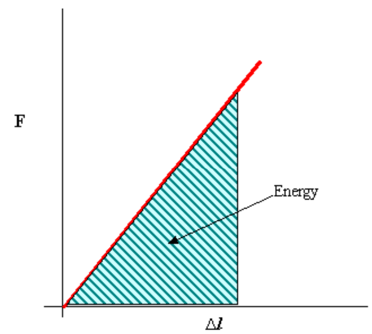
$$\Delta x = \boxed{2.38 \times 10^{-2}\ \text{mm}}$$

Elastic Strain Energy

When we stretch a wire, a work done on the wire. We are stretching the bonds between the atoms. If we release the wire, we can recover the energy stored in the wire due to stretch, which is called the **elastic strain energy**.

Ideally we recover all of it, but in reality a certain amount is lost as heat. This lost energy is called hysteresis. The energy is the area under the force-extension graph (the area of the triangle). So we can use this result to say:

$$E = \frac{1}{2} F \Delta l$$



$$E = \frac{1}{2} F \Delta l$$

- Substituting for F, we get

$$E = \frac{YA}{2l} \Delta l^2$$

- ***Example,*** *What is the elastic strain energy contained in a copper wire of diameter 0.8 mm and 16 cm length that has stretched by 4 mm under a load of 400 N?*

$$E = \frac{1}{2} F \Delta l = 0.5 \times 400 N \times 4 \times 10^{-3} m = 0.8 J$$

Bone Fracture: Energy Considerations

- Knowledge of the maximum energy that parts of the body can safely absorb allows us to estimate the possibility of injury under various circumstances. **We shall first calculate the amount of energy required to break a bone of area *A* and length *l*.**
- Assume that the bone remains elastic until fracture. Let us designate the breaking stress of the bone as σ_B. The corresponding force F_B that will fracture the bone is, from Eq.,

$$\frac{F_B}{A} = Y \frac{\Delta l}{l_o} \quad \Rightarrow \quad F_B = \frac{YA}{l_o} \Delta l$$

$$F_B = \sigma_B A = \frac{YA}{l_o}\Delta l$$

- The compression Δl at the breaking point is, therefore,

$$\Delta l == \frac{\sigma_B l_o}{Y}$$

- From $E = \frac{YA}{2l}\Delta l^2$, the energy stored in the compressed bone at the point of fracture is

$$E = \frac{YA}{2l}\left(\frac{\sigma_B l_o}{Y}\right)^2 \rightarrow E = \frac{A l_o \sigma_B{}^2}{2Y}$$

Example

Discuss the possibility of fracture of two leg bones that have a combined length of about 90 cm and an average area of about 6 cm^2 when a 70 kg person jump from a height of 60 cm.

Solution

First we calculate the maximum energy the two legs can absorb before fracture:

The breaking stress of the bone σ_B is 1.5×10^8 N/m^2, and Young's modulus for the bone is 1.5×10^{10} N/m^2.

- **The total energy absorbed by the bones of one leg at the point of compressive fracture is**

$$E = \frac{Al_o \sigma_B{}^2}{2Y}$$

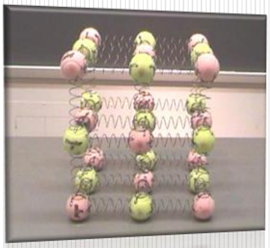
$$E = \frac{(6\times10^{-4})(0.9)(1.5\times10^{8})^2}{1.5\times10^{10}} = 192.5J$$

- **The combined energy in the two legs is twice this value, or 385 J**
- **Now we have to calculate the energy gained by jumping**

E = *m g h* = 70 (9.8) (0.6) = 412 J

The energy produced by jumping is higher than the maximum energy can be safely absorbed by the two legs so if all this energy is absorbed by the leg bones, they may fracture.

Selected Topics in Physics for Medical Sciences Students



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Unit 3
Elastic Properties of Materials
Lecture 11
Problems

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Problems

- (1) A 2 m long bar has a rectangular cross section. 0.02 m by 0.04 m. If it is subjected to a 10,000 N force along its length, what is the stress?
- (2) A pipe has an inner radius of 0.02 m and an outer radius of 0.23 m. If it is subjected to a tension stress of $5x10^7$ N/m^2, how large is the applied force?
- (3) A 0.4 m pip under compressional stress changes length by 0.005 m. What is the strain in the pipe?

- (4) The largest tension strain that can occur before fracture in aluminum is 0.003. What is the maximum change in length of 1 m aluminum pipe?
- (5) A rubber rod of length 0.5 m and radius 10^{-3} m stretches 0.1 m when a 140 N force is applied. How large a force is needed to stretch a rubber rod 0.1 m if its length is 0.5 m and its radius $2\text{x}10^{-3}$?
- (6) A steel wire 10 m long has a radius of 1 mm. Its linear limit is $2.5\text{x}10^{8}$ N/m^2, and its ultimate tension strength is $5\text{x}10^{8}$ N/m^2. The wire is attached at one end and hangs vertically with a weight at its lower end. (a) If the wire is just at its linear limit, how much large is the weight? (b) What is the largest load the wire can support?

- (7) A rod with a radius of 0.005 m and length of 2 m stretches 0.002 m when subjected to a tension force of 10,000 N. What is Young's modulus for this rod?
- (8) An aluminum wire is 20 m long and has a radius of $2\text{x}10^{-3}$ m. The linear limit for aluminum is $0.6\text{x}10^{8}$ N/m^2 (a) How large a tension force must be applied to stretch the wire to its linear limit? (b) How much will the wire stretch when this force is applied?
- (9) What is the spring constant of human femur under compression of average cross-sectional area 10^{-3} m^2 and length 0.4 m?

- (10) A 200-kg load is hung on a wire with a length of 4 m, a cross-sectional area of $0.2x10^{-4}$ m and a Young's modulus of $8x10^{10}$ N/m^2. What is its increase in length?

- (**11**) If the elastic limit of copper is $1.5x10^{8}$ N/m^2, determine the minimum diameter a copper wire can have under a load of 10.0 kg if its elastic limit is not to be exceeded.

- (12) When water freezes, it expands by about 9%. What would be the pressure increase inside your automobile's engine block if the water in it froze? (The bulk modulus of ice is $2x10^{9}$ N/m^2.)

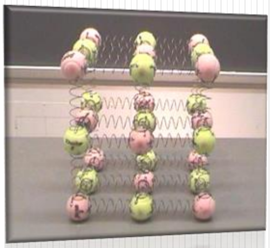
- (13) Assume that Young's modulus for bone is $1.5x10^{10}$ N/m^2 and that a bone will fracture if more than $1.5x10^{8}$ N/m^2 is exerted. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If a force of this magnitude is applied compressively, by how much does the 25.0-cm-long bone shorten?

- (14) The average cross-sectional area of a woman femur is $10^{-3}m^2$ and it is 0.4 m long. The woman weights 750 N (a) what is the length change of this bone when it supports half of the weight of the woman? (b) Assuming the stress-strain relationship is linear until fracture, what is the change in length just prior to fracture?

$$\Delta l == \frac{\sigma_B l_o}{Y}$$

- (15) A man leg can be thought of as a shaft of bone 1.2 m long. If the strain is $1.3x10^{-4}$ when the leg supports his weight, by how much is his leg shortened?
- (16) What is the spring constant of a human femur under compression of average cross-sectional area $10^{-3}m^2$ and length 0.4 m?
- (17) (a) Find the minimum diameter of a steel wire 18 m long that elongates no more than 9 mm when a load of 380 kg is hung on its lower end. (b) If the elastic limit for this steel is $3x10^8$ N/m^2, does permanent deformation occur with this load?

Selected Topics in Physics for Medical Sciences Students



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Unit 3
Elastic Properties of Materials
Lecture 11
Problem Solution

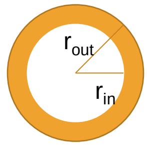
Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Problem Solution

- (1) A 2 m long bar has a rectangular cross section. 0.02 m by 0.04 m. If it is subjected to a 10,000 N force along its length, what is the stress?

$$\sigma = \frac{F}{A} = \frac{10000N}{0.02 \times 0.04} = 12.5 \times 10^{6} N/m^{2}$$

- (2) A pipe has an inner radius of 0.02 m and an outer radius of 0.23 m. If it is subjected to a tension stress of 5x10^7 N/m^2, how large is the applied force?



$$A = \pi r_{out}^2 - \pi r_{in}^2$$

$$A = \pi\left(0.23^2 - 0.02^2\right) = 0.16m^2$$

لدينا الاجهاد على هذه الانبوبة ونريد ان نحسب القوة

$$\mathbf{F} = \boldsymbol{\sigma}\mathbf{A} = 5\times10^7\, N/m^2 \times 0.16m^2 = 8\times10^6\, N$$

- (3) A 0.4 m pip under compressional stress changes length by 0.005 m. What is the strain in the pipe?

$$\epsilon = \frac{\Delta l}{l} = \frac{0.005\ m}{0.4} = 0.0125$$

- (4) The largest tension strain that can occur before fracture in aluminum is 0.003. What is the maximum change in length of 1 m aluminum pipe?

$$\varepsilon = \frac{\Delta l}{l_o}$$

$$\Delta l = \varepsilon l_o = 0.003 \times 1 = 0.003m$$

- (5) A rubber rod of length 0.5 m and radius 10^{-3} m stretches 0.1 m when a 140 N force is applied. How large a force is needed to stretch a rubber rod 0.1 m if its length is 0.5 m and its radius $2x10^{-3}$?
- من معطيات الشق الأول نحسب معامل ينج ونعوض عنه في معطيات المطلوب الثاني

$$\frac{F}{A} = Y\frac{\Delta l}{l_o}$$

$$Y = \frac{F\,l_o}{A\Delta l} = \frac{140 \times 0.5}{\pi \times (10^{-3})^2 \times 0.1} = 2.2 \times 10^8\,N/m^2$$

How large a force

$$F = Y\frac{\Delta l}{l_o}A$$

$$F = 560N$$

- (6) A steel wire 10 m long has a radius of 1 mm. Its linear limit is 2.5x10^8 N/m^2, and its ultimate tension strength is 5x10^8 N/m^2. The wire is attached at one end and hangs vertically with a weight at its lower end. (a) If the wire is just at its linear limit, how much large is the weight? (b) What is the largest load the wire can support?

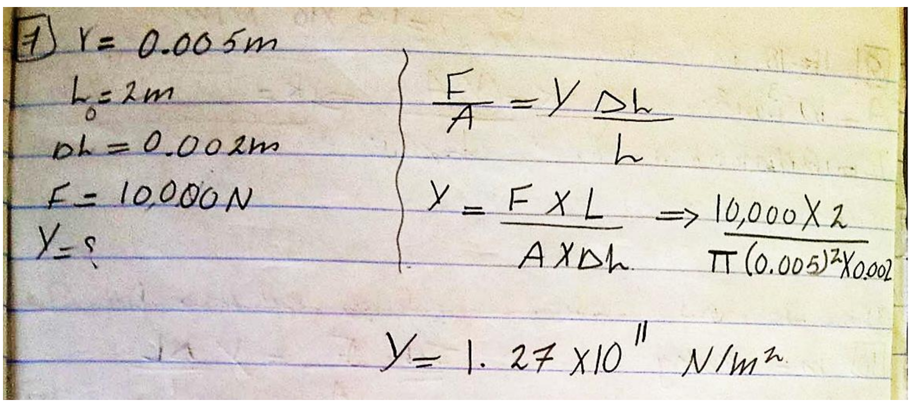
(a) If the wire is just at its linear limit, how much large is the weight?

$$F_{\text{linear limit}} = \sigma_{\text{linear limit}} A = 2.5\times10^{8}\,N/m^{2}\times\pi(1\times10^{-3})^{2}m^{2} = 785N$$

(b) What is the largest load the wire can support?

$$F_{\text{ultimate tension}} = \sigma_{\text{ultimate tension}} A = 5\times10^{8}\,N/m^{2}\times\pi(1\times10^{-3})^{2}m^{2} = 1570N$$

- (7) A rod with a radius of 0.005 m and length of 2 m stretches 0.002 m when subjected to a tension force of 10,000 N. What is Young's modulus for this rod?



- (8) An aluminum wire is 20 m long and has a radius of $2x10^{-3}$ m. The linear limit for aluminum is $0.6x10^{8}$ N/m^2 (a) How large a tension force must be applied to stretch the wire to its linear limit? (b) How much will the wire stretch when this force is applied? ($Y_{Al}=7x10^{10}$ N/m^2)

(a)

$$\sigma_{L.L.} = \frac{F_{L.L.}}{A}$$

$$\therefore F_{LL} = \sigma_{LL} . A$$

$$= 0.6\times10^{8} \times \pi (2\times10^{-3})^2$$

$$= 753 N$$

(b)

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$\therefore \Delta L = \frac{FL}{AY}$$

$$= \frac{753\times20}{\pi(2\times10^{-3})^2\times7\times10^{10}} = 0.017 m$$

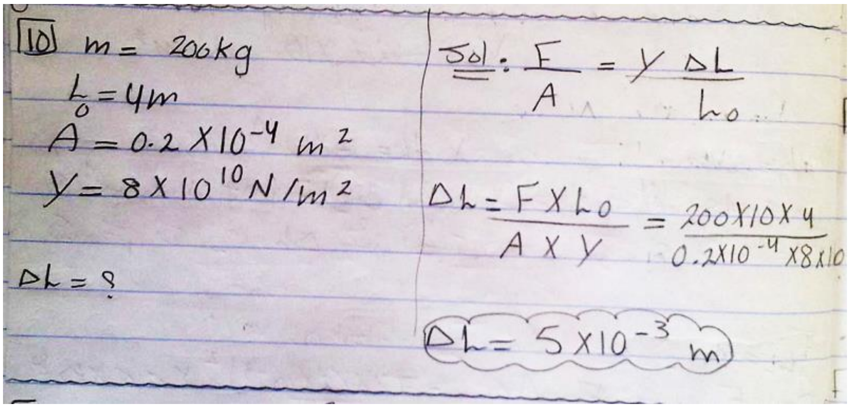
- (9) What is the spring constant of human femur under compression of average cross-sectional area 10^{-3} m^2 and length 0.4 m?

Young's modulus for the bone is 1.5×10^{10} N/m^2

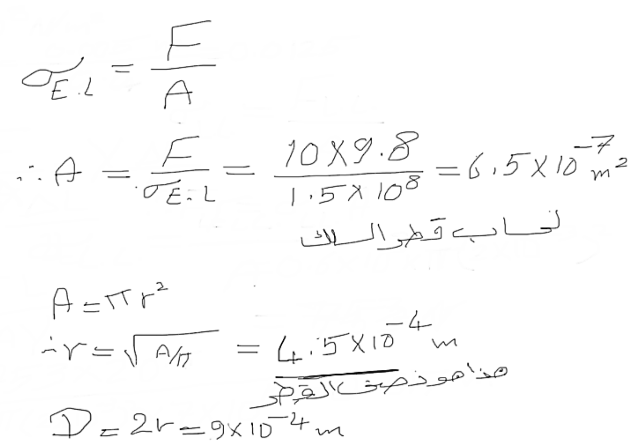
$$k = \frac{YA}{l_o}$$

$$k = \frac{YA}{l_o} = 3.75\times10^{6} N/m$$

- (10) A 200-kg load is hung on a wire with a length of 4 m, a cross-sectional area of 0.2×10^{-4} m and a Young's modulus of 8×10^{10} N/m^2. What is its increase in length?



- (11) If the elastic limit of copper is 1.5×10^8 N/m^2, determine the minimum diameter a copper wire can have under a load of 10.0 kg if its elastic limit is not to be exceeded.



- (12) When water freezes, it expands by about 9%. What would be the pressure increase inside your automobile's engine block if the water in it froze? (The bulk modulus of ice is $2x10^9$ N/m^2.)

(12) $Y = \frac{-\Delta P}{\Delta V / V_i}$ $\frac{\Delta V}{V_i} = \frac{9}{100} = 0.09$

$-\Delta P = Y \times \frac{\Delta V}{V_i}$

$\Delta P = -2 \times 10^9 \times 0.09 = -180 \times 10^6 \ N/m^2$

- (13) Assume that Young's modulus for bone is $1.5x10^{10}$ N/m^2 and that a bone will fracture if more than $1.5x10^8$ N/m^2 is exerted. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If a force of this magnitude is applied compressively, by how much does the 25.0-cm-long bone shorten?

$F_B = \sigma_B A$

$F_B = 1.5 \times 10^8 \times \pi \times (1.25 \times 10^{-2})^2$

$F_B = 73.5 \times 10^3 N$

$\Delta L = \frac{F L_0}{Y A} \Rightarrow \Delta L = \frac{73.5 \times 10^3 \times 25 \times 10^{-2}}{1.5 \times 10^{10} \times \pi \times (1.25 \times 10^{-2})^2}$

$\Delta L = 2.5 \times 10^{-3} \ m$

- (14) The average cross-sectional area of a woman femur is $10^{-3}m^2$ and it is 0.4 m long. The woman weights 750 N (a) what is the length change of this bone when it supports half of the weight of the woman? (b) Assuming the stress-strain relationship is linear until fracture, what is the change in length just prior to fracture?

$F = \frac{750}{2} = 375 N$

$A = 10^{-3} m^2$

$L = 0.4 m$

(a) $\frac{F}{A} = Y \frac{\Delta L}{L}$

$\therefore \Delta L = \frac{FL}{AY} = \frac{375 \times 0.4}{10^{-3} \times 1.5 \times 10^{10}} = 1 \times 10^{-5} m$

(b) $\Delta l = \frac{\sigma_B l_o}{Y}$

$= \frac{15 \times 10^{8} \times 0.4}{1.5 \times 10^{10}} = 4 \times 10^{-3} m$

- (15) A man leg can be thought of as a shaft of bone 1.2 m long. If the strain is 1.3×10^{-4} when the leg supports his weight, by how much is his leg shortened?

(15) $L = 1.2 m$

$\epsilon = 1.3 \times 10^{-4}$

$\epsilon = \frac{\Delta L}{L} \Rightarrow \Delta L = \epsilon L$

$= 1.3 \times 10^{-4} \times 1.2$

$\Delta L = 1.56 \times 10^{-4} m$

- (16) What is the spring constant of a human femur under compression of average cross-sectional area 10^{-3}m^2 and length 0.4 m?

$$k = \frac{YA}{l_o}$$

$$K = \frac{1.5\times10^{10}\times10^{-3}}{0.4}$$

$$K = 3.75\times10^{7}\ N/m$$

- (17) (a) Find the minimum diameter of a steel wire 18 m long that elongates no more than 9 mm when a load of 380 kg is hung on its lower end. (b) If the elastic limit for this steel is 3x10^8 N/m^2, does permanent deformation occur with this load?

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

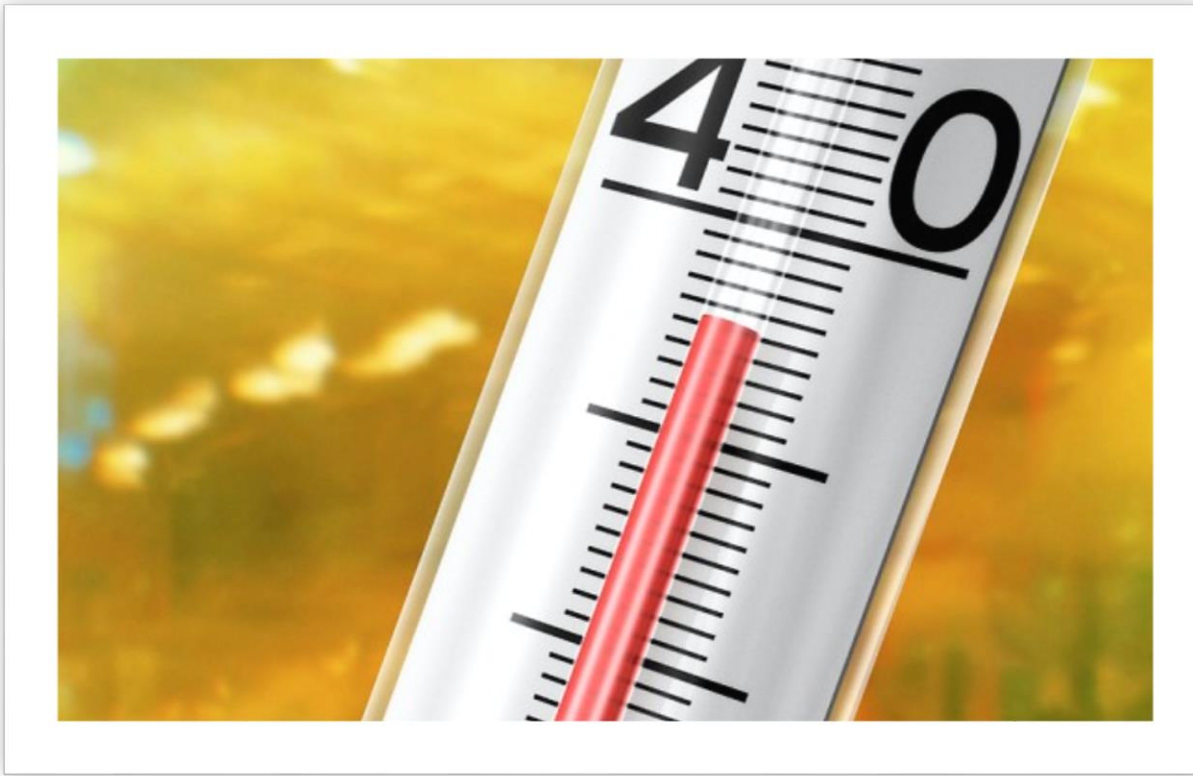
$$A = \frac{FL}{Y\Delta L} = \frac{3800\times18}{20\times10^{10}\times9\times10^{-3}} = 3\times10^{-9}\ m^2$$

$$r = \sqrt{A/\pi} = 3\times10^{-5}\ m \Longrightarrow D = 2r = 6\times10^{-5}\ m$$

$$\sigma = \frac{F}{A}$$

$$\sigma = \frac{3800}{3\times10^{-9}} = 1\times10^{12}\ N/m^2$$

لان قيمة الاجهاد هنا اكبر من
حد المرونة المعطى في السؤال
فان تشوه دائم سيحدث



الوحدة الرابعة: الحرارة ودرجة الحرارة

Heat and Temperature

Dr. Hazem Falah Sakeek | www.hazemsakeek.net | 2014

Selected Topics in Physics for Medical Sciences Students

Unit 4
Temperature and Heat
Lecture 12

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

- Temperature
- Thermal expansion of solids and Liquids
- Ideal Gas Equation
- Heat and the first law of thermodynamics

درجة الحرارة والغاز المثالي
Temperature and Ideal Gases

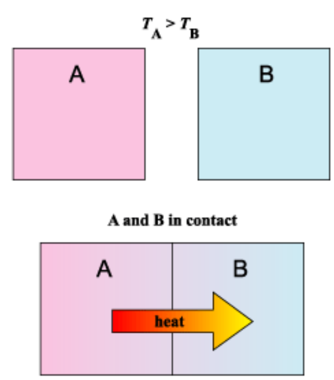
- تعتبر الحرارة احد مصادر الطاقة الرئيسية التي بدأ علماء الفيزياء في دراسة وفهم قوانينها لاهميتها ولتطبيقاتها الواسعة على حياتنا، فلو نظرنا من حولنا لوجدنا أن الحرارة هي اساس الطاقة في كل شيء فعلى سبيل المثال الثلاجة المنزلية ومكيفات الهواء ما هي الا تطبيقات على الفيزياء الحرارية وكذلك المحركات البخارية والمحركات الحديثة تعتمد على تحويل الطاقة الحرارية إلى طاقة ميكانيكيا حيث أن حرق الوقود يؤدي إلى ارتفاع في درجة حرارة الغاز الذي يضغط على مكبس المحرك الذي يؤدي حركة ميكانيكية اساسها ارتفاع في درجة الحرارة وهذه المحركات هي اساس فكرة عمل السيارات والطائرات بمختلف انواعها، كذلك مثالاً آخر وهو الكهرباء فهي تصلنا من محطات التوليد التي تقوم بحرق الفحم أو الوقود الذي يحرك التوربينات التي تولد الطاقة الكهربية وهناك الامثلة الكثيرة الأخرى.

- سنقوم في هذه المحاضرة بدراسة علم الفيزياء الحرارية والذي يسمى علم الثيرموديناميكا thermodynamics وهذا العلم هو علم تجريبي يهتم بدراسة الظواهر المتعلقة بتبادل الطاقة الحرارية بين الأجسام عند درجات حرارة مختلفة.
- عند دراسة علم الميكانيكا ركزنا على دراسة الكميات الفيزيائية مثل الكتلة mass والقوة force والطاقة energy حيث كانت تلك الكميات الفيزيائية هي الاساسات الرئيسية لذلك العلم، ولكن في دراستنا للحرارة فإننا نحتاج إلى مفاهيم أخرى هي درجة الحرارة temperature والتبادل الحراري heat والطاقة الداخلية internal energy
- لذلك سنتناول خلال هذه المحاضرة دراسة تلك المفاهيم وتوضيحها وشرح كل الأمور العلمية المتعلقة بها وسيشمل ذلك دراسة تأثير كلاً من درجة الحرارة temperature والضغط pressure والحجم volume على الغاز المثالي ideal gas

القانون الصفري للديناميكا الحرارية
The zeroth law of thermodynamics

- عند الحديث عن درجة حرارة جسم ما فإننا نقصد بذلك كم هي درجة سخونة أو برودة ذلك الجسم عند لمسه باليد، حاسة اللمس هي إحدى النعم التي انعم الله بها علينا وبناء عليها يمكن ان نقدر درجة حرارة الجسم تقدير كيفي وليس كمي، وفي بعض الأحيان نشعر ببرودة جسم ما أكثر من جسم أخر بالرغم من انهما عند نفس درجة الحرارة لأن هناك عامل مهم وهو سرعة توصيل الحرارة فالمعادن مثلاً أسرع في توصيل الحرارة منها إلى اليد من قطعة من البلاستيك، لذلك توجب ان يكون هناك مقياس دقيق لدرجة الحرارة نعتمد عليه في تحديد درجة حرارة الأجسام.
-
- قبل ان نبدأ في الحديث عن المقاييس المستخدمة لقياس درجات الحرارة دعنا نشرح بعض المفاهيم الرئيسية التي اعتمدت عليها أجهزة قياس درجات الحرارة.

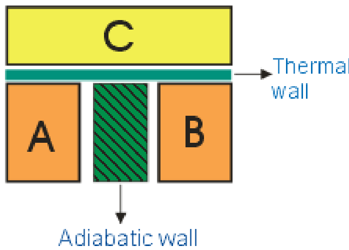
- **مفهوم الاتصال الحراري thermal contact**
- الاتصال الحراري يكون بين جسمين إذا كان من الممكن أن يتبادلا الطاقة الحرارية بدون بذل شغل.
-
- **مفهوم الاتزان الحراري thermal equilibrium**
- الاتزان الحراري بين جسمين يحدث إذا كان بينهما اتصال حراري وكذلك يكون صافي التبادل الحراري بينهما يساوي صفر.



يوضح الشكل اعلاه انتقال الحرارة من الجسم الاكثر ارتفاعاً في درجة الحرارة إلى الجسم الأقل درجة حرارة الى ان تتساوي درجات الحرارة ويصل إلى حالة الاتزان الحراري.

يوضح السهم في الشكل اتجاه انسياب الحرارة

مبدأ عمل مقياس درجة الحرارة (الثيرموميتر)



- افترض ان هناك جسمان A و B ينهما مادة عازلة أي أنهما غير متصلين حرارياً وقمنا باحضار جسم ثالث C وهذا الذي يمثل الثيرموميتر) ليكون الأداة المستخدمة لتحديد ما إذا كان الجسمان A و B في حالة اتزان حراري thermal equilibrium أم لا؟ فإننا سنستخدم الجسم C ونضعه على اتصال حراري مع الجسم A حتي نصل إلى حالة الاتزان الحراري بين A و C ونحدد درجة حرارة الاتزان الحراري بواسطة C، نقوم بتكرار الخطوة السابقة مع الجسم B. فإذا كانت درجة الحرارة للجسم A المقاسة بواسطة Cتساوي درجة الحرارة للجسم B فإننا نستطيع ان نجزم ان كلاً من A و B في حالة اتزان حراري.

معنى ذلك أنه إذا وجد جسمين معزولين وكلاً منهما في حالة اتزان حراري مع جسم ثالث فإن ذلك يؤدي إلى أن الجسمين أيضا في حالة اتزان حراري مع بعضهما البعض. وسمي هذا بالقانون الصفري للديناميكا الحرارية zeroth law of thermodynamics وسمي بالصفري لأنه من المسلمات البديهية ويعتبر هذا القانون الأساس العملي لفكرة الثيرمومتر المستخدم لقياس درجات الحرارة.

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الثيرمومتر ومقياس درجات الحرارة
Thermometer and temperature scale

- الثيرموميتر thermometer هو أداة تستخدم لقياس درجات الحرارة، والثيرمومتر يعمل من خلال تغير في أحد الخصائص الفيزيائية بتغير درجة الحرارة، مثل خاصية تمدد الأجسام مع زيادة درجة الحرارة وتغير الضغط أو مقاومة السلك الكهربي بتغير درجات الحرارة. وفيما يلي نذكر الأنواع المختلفة للثرمومتر.

Type of thermometer نوع الثيرمومتر	**Material** المادة	**Physical property** الكمية الفيزيائية
(1) Liquid thermometer	Mercury or Alcohol	Change in length
(2) Gas Thermometer	Hydrogen	Change in pressure
(3) Resistance thermometer	Platinum	Change in resistance
(4) Thermocouple thermometer	Chromel and Alumel	Change in electric potential
(5) Radiation Thermometer	Pyrometer	Change in radiation colour
(6) Magnetic thermometer		Change in susceptibility

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المقياس المئوي Celsius scale

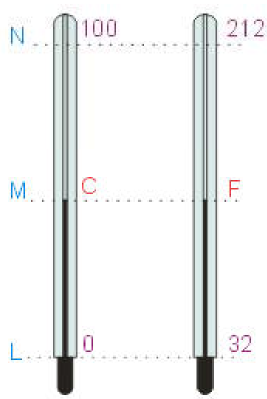
•تعتمد فكرة المقياس المئوي على وجود نقطتين لا تتغير فيهما درجة الحرارة مع تزويد المادة بحرارة وعلى هذا الاساس اعتمد العالم Celsius في ابتكاره للتدريج المئوي حيث انه من الملاحظ عملياً ثبوت درجة حرارة الماء عند نقطة الغليان أي عندما يتحول من الحالة السائلة إلى الحالة الغزية أو العكس وكذلك تثبت فيها درجة حرارة الماء عند تحوله إلى ثلج وهي درجة الانصهار أي من الحالة السائلة إلى الحالة الصلبة أو العكس، فاطلق سيليزس على درجة الانصهار بالقيمة صفرا وعلى نقطة الغليان القيمة 100 وتم تقسيم التدريج إلى 100 كل جزء يساوي درجة، ولذلك سمي بالتدريج المئوي ويسمى ايضا بتدريج سيليزس. وتبلغ درجة حرارة الانسان على هذا التدريج $37^{o}C$.

شكل يوضح ثيرمومتر زئبقي، نتيجة للتمدد الطولي يتغير مستوى ارتفاع الزئبق من 0 عند درجة التجمد إلى 100 عند درجة الغليان

المقياس الفهرنهايتي Fahrenheit scale

•يعتمد هذا التدريج لقياس درجة الحرارة على نفس المبدأ السابق للتدريج المئوي أي على نقطة تحول الماء إلى الحالة الغازية او الصلبة، ولكن اعتبر فهرنهايت درجة الانصهار هي درجة 32 بدلاً من الصفر، ودرجة الغليان للماء وهي درجة 212 بدلاً من 100.

• ولتوضيح العلاقة بين التدريج المئوي والتدريج الفهرنهايتي استعن بالشكل التالي:



$$\frac{ML}{NL} = \frac{C-0}{100-0} = \frac{F-32}{212-32}$$

$$\therefore \frac{C}{100} = \frac{F-32}{180}$$

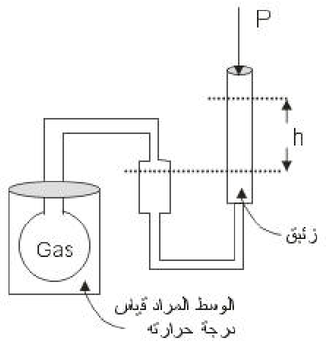
$$F = \frac{9}{5}C + 32$$

إذا للتحويل من درجة حرارة بمقياس فهرنهايت إلى مقدارها بالمقياس المئوي أو العكس نستخدم المعادلتين التاليتين:

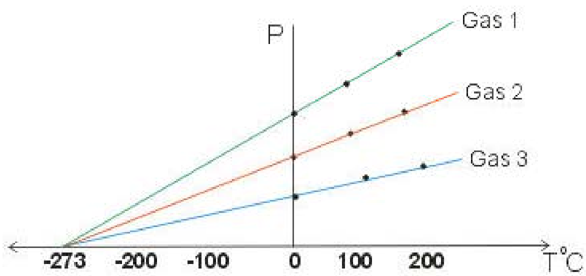
$$T(\text{in }^{\circ}\text{F}) = 32 + \frac{9}{5}T(\text{in }^{\circ}\text{C})$$

$$T(\text{in }^{\circ}\text{C}) = \frac{5}{9}[T(\text{in }^{\circ}\text{F}) - 32]$$

Kelvin scale المقياس المطلق



- مما سبق نجد أن كلا التدريجين اعتمدا على نوع مادة السائل وهو الماء حيث تم اعتبار نقطة الانصهار ونقطة الغليان كأساس للتدريج، وحيث أن هاتين النقطتين تعتمدان على الضغط وعدد من العوامل الأخرى، لذا فإننا بحاجة إلى تدريج مطلق لا يعتمد على طبيعة المادة وهذا ما قام به العالم كلفن Kelvin في تحديد تدريج مطلق لدرجة الحرارة.



قام العالم كلفن باستخدام الثيرمومتر المعتمد على التغير في الضغط Gas thermometer ودرس العلاقة بين الضغط ودرجة الحرارة، وذلك لأكثر من غاز ووجد أن جميع الغازات يقل ضغطها بنقصان درجة الحرارة وأن الضغط يصبح صفر نظرياً (أي عند مد المنحنيات كما في الشكل على استقامتها) عند درجة حرارة وقدرها -273. وقد تم اعتبار هذه الدرجة هي الصفر المطلق وأنها لا تتغير بتغير نوع الغاز وعليه تم معايرة باقي التدريجات الأخرى بالنسبة للصفر المطلق.

- إعتبر العالم كلفن نقطة تلاشي الضغط للغازات عند -273.15 درجة مئوية بأنها نقطة مرجعية لتدريج جديد لا يعتمد على نوع المادة المستخدمة (مثل الماء) في تصميم التدريج واعتبرت هذه النقطة هي الصفر المطلق والتي تساوي بتدريج سيليزس (التدريج المئوي) -273.15 وسمي هذا التدريج بالتدريج المطلق absolute scale.
- وعليه فإن العلاقة بين التدريج المئوي والتدريج المطلق هي:

$$T(\text{in } {}^\circ C) = T(\text{in } {}^\circ K) - 273.15$$

Example

An object has a temperature of 50°F. What is its temperature in degrees Celsius and in kelvins?

Solution Substituting $T_F = 50°F$ into Equation 19.5, we get

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(50 - 32) = \boxed{10°C}$$

From Equation 19.4, we find that

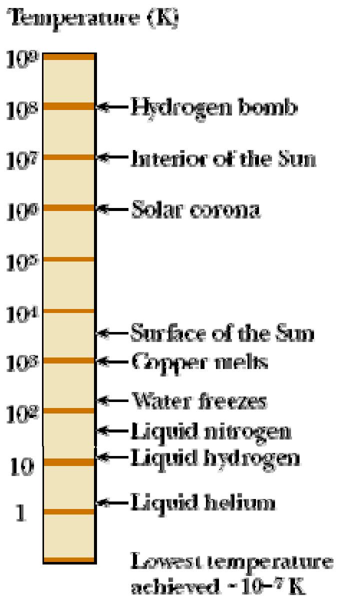
$$T = T_C + 273.15 = \boxed{283.15\ K}$$

Example

A pan of water is heated from 25°C to 80°C. What is the change in its temperature on the Kelvin scale and on the Fahrenheit scale?

$$\Delta T = \Delta T_C = 80°C - 25°C = 55°C = \boxed{55\ K}$$

$$\Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5}(55°C) = \boxed{99°F}$$

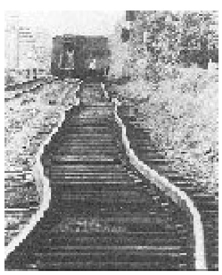


درجات الحرارة

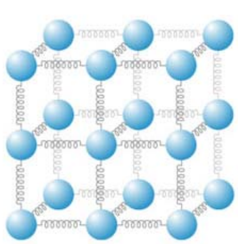
تعرف درجة حرارة جسم ما على انها مقياس للطاقة الداخلية للمادة عند تلك الدرجة.

تتراوح درجات الحرارة في هذا الكون الفسيح بين الاف الملايين كلفن إلى ما يقارب الصفر كلفن، علماً بأن اقل درجة حرارة وصل الانسان اكثر قليلا من الصفر المطلق (كما في الشكل المقابل) ولاحدود لأعلى درجة سجلت حتى الان فتصل درجة حرارة الانشطار النووي للهيلوم إلى 100 مليون كلفن .

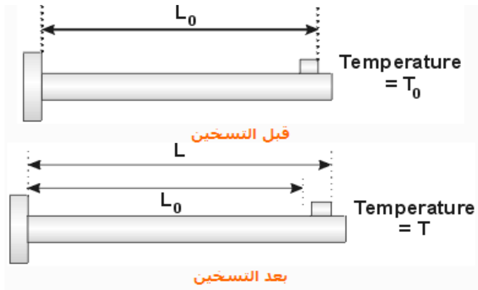
Thermal Expansion



- معظم الأجسام تتمدد expand عندما تزداد درجة حرارتها. هذه الظاهرة تلعب دوراً رئيسيا في العديد من التطبيقات الهندسية، فعلى سبيل المثال يتم ترك مسافات بين الوصلات الحديدية في المباني والجسور والسكك الحديدية والطرق السريعة لتعطي المجال للتمدد والانكماش وإذا لم يتم فعل ذلك يمكن ان يتصدع المبنى أو تنهار الجسور وتلتوي السكك الحديدية بفعل التمدد الحراري للمواد المصنوعة منه.
- إن التمدد الحراري thermal expansion للأجسام هو نتيجة عن للتغير الذي يحدث للمسافات الفاصلة بين جزيئات وذرات المادة. ولفهم أدق لما ذكرناه لننظر إلى الشكل الموضح حيث يعبر عن التركيب البلوري لمادة في الحالة الصلبة والتي تحتوي على مصفوفة مرتبة من الذرات المترابطة مع بعضها البعض بفعل القوى الكهربية (الزنبرك في الشكل يمثل القوى الكهربية).



Thermal Expansion



يحدث التمدد على كافة ابعاد الجسم كالطول والعرض والسمك وتكون نسبة الزيادة حسب الأبعاد الهندسية للمادة ومقدار الزيادة يتناسب طردياً مع الطول الأصلي لذا تكون الزيادة في الطول اكثر منها في العرض أو السمك.

$$\Delta L = \alpha \, L \, \Delta T$$

التغير في الطول (ΔL)

معامل التمدد الطولي (α)

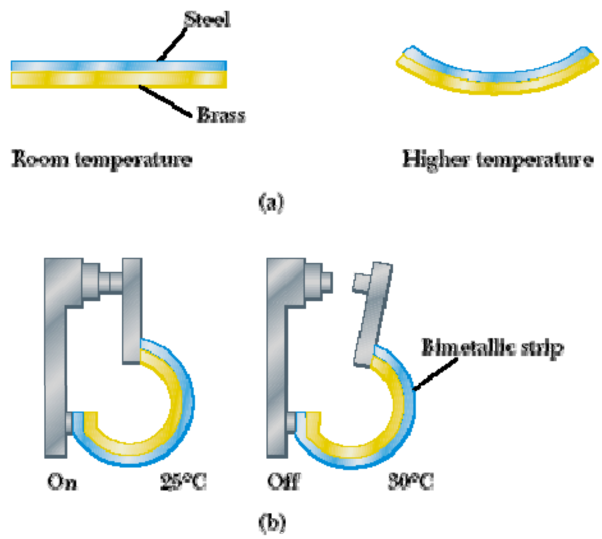
الطول الأصلي (L)

التغير في درجة الحرارة (ΔT)

coefficient of linear expansion $$\alpha = \frac{1}{L}\frac{\Delta L}{\Delta T}$$

Coefficients of Thermal Expansion at 20 °C

Substance	Linear Coefficient	Volumetric Coeff.
	α (1/ °C)	$\beta = 3\alpha$ (1/ °C)
Aluminum	24×10^{-6}	72×10^{-6}
Brass	19×10^{-6}	57×10^{-6}
Copper	17×10^{-6}	51×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Glass (Pyrex)	3×10^{-6}	9×10^{-6}
Iron/Steel	12×10^{-6}	36×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Ice	51×10^{-6}	153×10^{-6}
Gasoline		950×10^{-6}
Mercury		180×10^{-6}
Water		210×10^{-6}



Example

A segment of steel railroad track has a length of 30.000 m when the temperature is 0.0°C.

(A) What is its length when the temperature is 40.0°C?

(B) Suppose that the ends of the rail are rigidly clamped at 0.0°C so that expansion is prevented. What is the thermal stress set up in the rail if its temperature is raised to 40.0°C?

$$\Delta L = \alpha L_i \Delta T = [11 \times 10^{-6} (°C)^{-1}](30.000\ m)(40.0°C)$$
$$= 0.013\ m$$

If the track is 30.000 m long at 0.0°C, its length at 40.0°C is 30.013 m.

(B) Suppose that the ends of the rail are rigidly clamped at 0.0°C so that expansion is prevented. What is the thermal stress set up in the rail if its temperature is raised to 40.0°C?

Solution The thermal stress will be the same as that in the situation in which we allow the rail to expand freely and then compress it with a mechanical force F back to its original length. From the definition of Young's modulus for a solid (see Eq. 12.6), we have

$$\text{stress} = \frac{F}{A} = Y\frac{\Delta L}{L_i}$$

Because Y for steel is $20 \times 10^{10}\ \text{N/m}^2$ (see Table 12.1), we have

$$\frac{F}{A} = (20 \times 10^{10}\ \text{N/m}^2)\left(\frac{0.013\ \text{m}}{30.000\ \text{m}}\right) = 8.7 \times 10^{7}\ \text{N/m}^2$$

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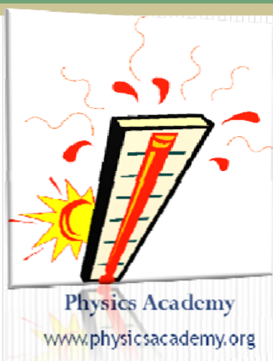


Unit 4

Temperature and Heat

Lecture 13

Ideal Gas Equation

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

- Temperature
- Thermal expansion of solids and Liquids
- Ideal Gas Equation
- Heat and the first law of thermodynamics

Ideal Gas Equation

•**الغاز المثالي هو الغاز (بغض النظر عن نوعه) والذي تنطبق عليه الشروط التالية:**

(1)حجم جزيئات الغاز مهملة بالنسبة للوعاء الذي يحتويه أي تحت ضغط منخفض.
(2) التصادمات بين جزيئات الغاز تصادمات مرنة.
(3) حركة جزيئات الغاز حركة عشوائية دون مؤثرات خارجية.

•لهذا فإن الغازات الموجودة عند درجة حرارة الغرفة وتحت ضغط يساوي الضغط الجوي تعتبر غازات تتصرف كغاز مثالي.

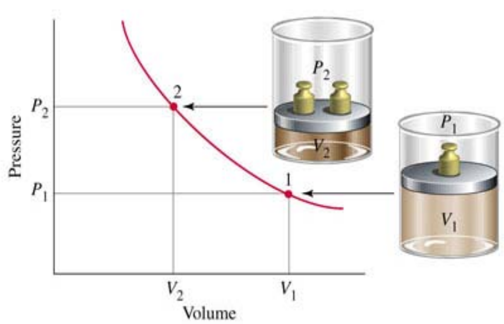
•لا شك أن الغاز المثالي لا وجود له في الطبيعة ولكن في علم الفيزياء يتم وضع مثل هذه الفروض لتسهيل دراسة تأثير المتغيرات الفيزيائية في حالة ظروف مثالية لتسهيل المعادلات الرياضية والوصول إلى علاقات رياضية تحكم تصرف الغاز المثالي ثم يتم مقارنتها مع الغاز الحقيقي. والمتغيرات الفيزيائية هنا هي درجة الحرارة والحجم والضغط، ولدراسة العلاقة بين هذه المتغيرات على الغاز المثالي سنقوم بتثبيت متغير واحد ودراسة العلاقة بين المتغيرين الآخرين، وهذا ما قام به العالمان بويل Boyle وتشارل Charle

Boyle's Law

- When gas is kept at constant temperature its pressure is inversely proportional to the volume.

$P \alpha 1/v$

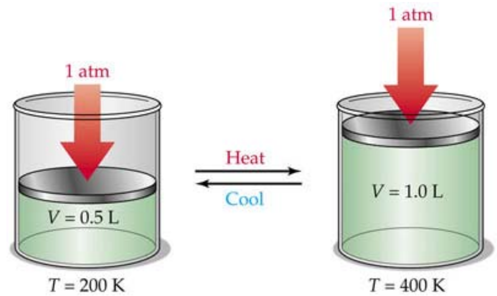
at constant temperature



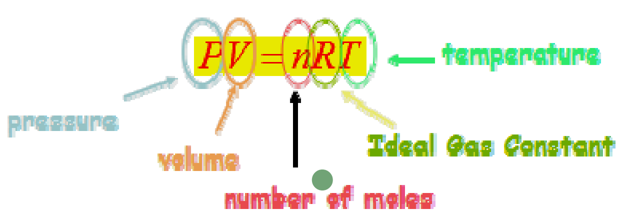
Charle's Law

- When the pressure of the gas kept constant the volume directly proportional to the temperature.

$$V \,\alpha\, T \quad \text{at constant pressure}$$



- These result can be summarized in one equation called the equation of state for an ideal gas



- Where **n** is the number of moles, **R** is a constant for a specific gas, which can be determined experimentally, and **T** is the absolute temperature in Kelvin

- When the pressure goes to zero then the quantity **PV/nT** become the same value of R for all gasses, therefore R called the universal gas constant (الثابت العام للغازات)

R = 8.31 J/mole.K

- The ideal gas law can be expressed in terms of the total number of molecules **N** where **$N = nN_A$**
- where N_A is the Avogadro's number = 6.022×10^{23} molecules/mole

$$\therefore PV = nRT = \frac{N}{N_A}RT$$

$$PV = Nk_BT$$

Boltzmann's constant
$k = 1.38\times10^{-23}$ N.m/°K
$= R/6.023\times10^{23}$

number of molecules

- where **K** is called Boltzmann's constant, which has the value R/N_A
- **$K = R/N_A = 1.38\times10^{-23}$ J/K**

- One **mole** of substance is that mass of the substance that contains Avogadro's number of molecules

Conversion between the different pressure unit systems

	Torr, mmHg	Atm	Bar	Pascal
Pascal	0.0075	9.87×10^{-6}	10^{-5}	1
Bar	750	0.987	1	10^{5}
atm	760	1	1.013	1.013×10^{5}
Torr , mmHg	1	1.32×10^{-3}	1.33×10^{-3}	133.3

- **Example (1)**
  An ideal gas occupies a volume of $100cm^3$ at 20^oC and a pressure of 100Pa. Determine the number of moles of gas in the container.

- **Solution**

$$PV = nRT$$

- What is the number of molecules in the container?

- **Example (2)**
  Pure helium gas is admitted into a tank containing a movable piston. The initial volume, pressure and temperature of the gas are $15x10^{-3}m^3$, 200kPa and 300K respectively. If the volume is decreased to $12x10^{-3}m^3$ and the pressure is increased to 350KPa, find the final temperature of the gas.
- **Solution**

  Since the gas can not escape from the tank then the number of moles is constant, therefore, PV = nRT at the initial and final points of the process

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$T_2 = (\frac{p_2 V_2}{p_1 V_1}) T_1 = \frac{3.5\, atm . 12\, liters}{2\, atm . 15\, liters}(300K) = 420\, K$$

Example (2)

- One mole of oxygen gas is at a pressure of 6 atm and a temperature of 7°C. (a) If the gas is heated at constant volume until the pressure triples, what is the final temperature? (b) If the gas is heated until both the pressure and the volume are doubled, what is the final temperature?

Solutions

- (a) T_1 = 273 + 7 = 280 K,

$$p_2 = 3p_1 \Rightarrow \frac{p_1}{T_1} = \frac{3p_1}{T_2} \Rightarrow \frac{1}{T_1} = \frac{3}{T_2}$$

$$So\ T_2 = 3T_1 = 280 \times 3 = 840\ K$$

- (b) $p_2 = 2\,p_1 , V_2 = 2V_1$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow \frac{p_1 V_1}{T_1} = \frac{4 p_1 V_1}{T_2}$$

$$T_2 = 4T_1 = 4 \times 280 = 1120\ K$$

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Unit 4

Temperature and Heat

Lecture 14

Heat and the first law of thermodynamics

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

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- Temperature
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- Ideal Gas Equation
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Example

A 0.1 mole of O_2 gas at standard conditions.

- What will the pressure be if the volume is changed to 1 Liter at constant temperature,
- and find the density of the gas after change?

Solutions

The standard condition means that the temperature of the gas T= 273K, and its pressure p=1$a$tm=1.013x10^5 Pa. The gas is expanded at constant temperature, so that

$$P_1V_1 = nRT = P_2V_2$$

$$P_2 = \frac{nRT}{V_2} = \frac{0.1(8.314)273}{1\times10^{-3}} = 2.27\times10^5\,Pa = 2.24atm$$

- To find the density of the O_2 gas, we can write the ideal gas law as

$$P_2V_2 = nRT \Rightarrow P_2\frac{m}{\rho} = \frac{m}{M_w}RT$$

$$\rho = \frac{P_2M_w}{RT}$$

$$\rho = \frac{2.27\times10^5(32\times10^{-3})}{8.314\times273} = 3.2kg/m^3$$

one mole of a gas is the quantity that contains N_A = 6.02×10^{23} molecules/mole (Avogadro's number) and equals the molecular weight of the substance expressed in grams.

$$n = \frac{N}{N_A} = \frac{m}{M_w}$$

Heat as a form of energy

• نعلم من المحاضرة السابقة أنه عند وضع جسمين عند درجات حرارة مختلفة وكان بينهما اتصال حراري thermal contact فإن الحرارة تنتقل من الجسم الأعلى درجة حرارة إلى الجسم الأقل درجة حرارة، ويسمى هذا تدفق حراري heat flow ويستمر حتى يصل الجسمين إلى نفس درجة الحرارة وعندها يكونا في حالة اتزان حراري Thermal Equilibrium

• حاول العلماء في القرن السابع عشر تفسير ظاهرة التدفق الحراري بافتراض جسيمات غير مرئية تدعى الكلوريك Caloric، تعمل على نقل الحرارة بين الأجسام. ولكن كان هذا الافتراض غير صحيح حيث لا يمكن تفسير العديد من الظواهر الحرارية مثل عدم تغير درجة الحرارة عند حالة التحول من الحالة السائلة إلى الحالة الغازية مثل غليان الماء، ولكن العالم جول Joule اثبت بالتجربة العملية أن التدفق الحراري ما هو إلا انتقال للطاقة وأن الحرارة صورة من صور الطاقة.

• تعريف الحرارة Heat

• كلمة حرارة هي اختصار للانسياب الحراري أو التدفق الحراري لأنه لا يوجد حرارة إلا إذا انتقلت من مكان إلى آخر ولتعريف الانسياب الحراري نقول أنه هو انتقال الطاقة من مكان لآخر نتيجة لاختلاف درجات الحرارة بينهما.

• The word of ''heat flow'' is an energy transfer that take place as a consequence of temperature difference only.

← أي أن التدفق الحراري أو الانسياب الحراري هو انتقال للطاقة.

• تعريف درجة الحرارة Temperature

• يجب ألا نخلط بين الحرارة ودرجة الحرارة فكما ذكرنا قبل قليل إن حرارة تعني انتقال للطاقة أما **درجة الحرارة فهي مقياس للطاقة الداخلية للمادة**، وكلما زادت درجة الحرارة زادت الطاقة الداخلية أي زادت الطاقة الحركية لجزيئاته.

وحدة الحرارة Unit of Heat

- قبل ان يتمكن العلماء من معرفة أن الحرارة هي طاقة أي قبل تجربة العالم جول الشهيرة التي اثبت فيها أن الحرارة هي صورة من صور الطاقة والتي سنشرحها بعد قليل فقد اعطى العلماء وحدة الكلوري calorie للحرارة وعرفت

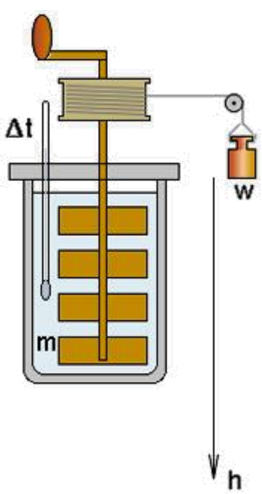
- تعرف الكلوري على إنها مقدار الحرارة اللازمة (الطاقة) لرفع درجة حرارة جرام واحد من الماء من 14.5^oC إلى 15.5^oC

- The unit of heat is "calorie" which is defined as the amount of heat (energy) required to raise the temperature of 1g of water from 14.5^oC to 15.5^oC.

- وبعد أن اثبت العالم جول أن الحرارة هي طاقة فيمكن التعبير عن وحدة الحرارة بالجول وقد اثبت عملياً أن:

- $1cal = 4.186J$ or $1J = 0.2389cal$

المكافئ الميكانيكي للحرارة The mechanical equivalent of heat



- قام العالم جول بتصميم التجربة الموضحة في الشكل المقابل والتي اثبت فيها أن الطاقة الميكانيكية تتحول إلى حرارة وان الحرارة ما هي إلا صورة من صور الطاقة ويمكن تحويلها من صورة إلى أخرى. تعتمد التجربة على قياس التغير في طاقة الوضع للأثقال التي تحرك المروحة داخل الإناء المعزول، نتيجة للاحتكاك بين المروحة والماء ينتج عنها ارتفاع في درجة حرارة الماء، وبدراسة التغير في طاقة الوضع mgh مع ارتفاع درجة حرارة الماء في الإناء وجد العالم جول ان العلاقة بين طاقة الوضع والارتفاع في درجة الحرارة هي علاقة طردية. وأن ثابت التناسب يساوي $4.186J/Kg.C^o$ وهذا يعني أن مقدار 4.18 جول من الطاقة تلزم لرفع درجة حرارة جرام واحد من الماء درجة مئوية واحدة من 14.5^oC إلى 15.5^oC وهذا هو نفسه تعريف الكالوري إذا فإن

- $1cal = 4.186J$

Example

- A student eats a dinner rated at 2000 (food) Calories. He whishes to do an equivalent amount of work in the gymnasium by lifting 50Kg mass. How many times must he raise the weight to expend this much energy? Assume that he raises the weight a distance of 2m each time and no work is done when the weight is dropped to the floor.

- **Solution**

1 (food) Calories = 1000 cal

- **then the work required is $2x10^6$cal.**
- **Converting this to joule, then the work required is**

- **$W = 2x10^6 cal \times 4.186 J/cal = 8.37x10^6 J$**

- الشغل المبذول لرفع الاثقال لمسافة h يساوي mgh والشغل الكلي لرفع الأثقال عدة مرات يعطي بالعلاقة nmgh وعليه فإن عدد مرات رفع الاثقال هو:

- **$W = nmgh = 8.37x10^6 J$**

- **Since m = 50 Kg, and h = 2m**

- **$n = 8.54x10^3$ times**

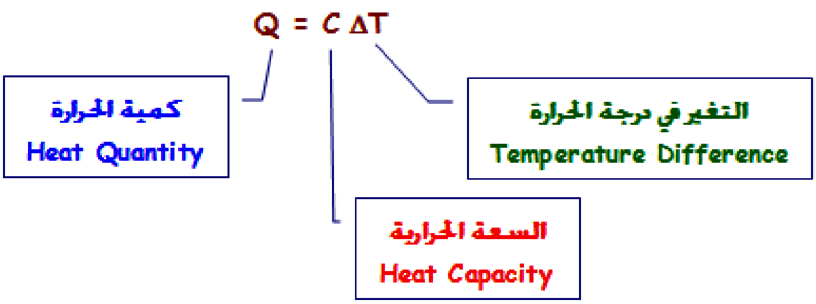
- اي انه يلزم الطالب رفع الثقل ما يقارب 8500 مرة لحرق السعرات الحرارية المطلوبة ولو كان يؤدي كل رفعة في زمن مقداره 5 ثواني فهذا يعني انه يلزمه 12 ساعة لانجاز المهمة!!

Heat capacity and specific heat

- سنستخدم الآن تعبير كمية الحرارة للدلالة على مقدار الطاقة الحرارية التي يكتسبها الجسم، وللعلم يمكن تزويد الجسم بكميات من الحرارة من مصادر مختلفة فالمصدر المعتاد هو اللهب الناتج من النار او الحرارة المكتسبة من أشعة الشمس اضافة إلى الحرارة التي يكتسبها الجسم نتيجة الاحتكاك مثل احتكاك اطارات السيارة عند التوقف المفاجئ وحك اليدين بسرعة لتدفئتها، لذا سنطلق على كل هذه الصور المختلفة لمصادر الحرارة بكمية الحرارة heat quantity.

- وجد بالتجربة العملية أن كمية الحرارة اللازمة لرفع درجة حرارة المادة تختلف حسب طبيعة المادة، فعلى سبيل المثال كمية الحرارة اللازمة لرفع درجة حرارة Kg1 من الماء درجة مئوية واحدة تساوي J4186 ولكن لرفع درجة حرارة Kg1 من النحاس درجة مئوية واحدة يلزم J387. ولهذا فإننا نحتاج إلى تعريف كمية فيزيائية جديدة تأخذ في الحسبان طبيعة المادة المكتسبة او الفاقدة للحرارة وهذه الكمية هي السعة الحرارية heat capacity. وتعرف **السعة الحرارية** بأنها مقدار الطاقة الحرارية اللازمة لرفع درجة حرارة المادة درجة مئوية واحدة

- The **heat capacity** is defined as the amount of heat energy needed to raise the temperature of a sample by 1 degree Celsius.
- ومن تعريف السعة الحرارية نستنتج أن كمية الحرارة التي تضاف للمادة تساوي التغير في درجات الحرارة مضروبة في السعة الحرارية.



ووحدة السعة الحرارية هي J/C°

- من المؤكد بأن السعة الحرارية تتناسب طرديا مع كتلة المادة ولذلك سنقوم بتقسيم السعة الحرارية على الكتلة حتى نحصل على كمية فيزيائية جديدة لا تعتمد على الكتلة وهي السعة الحرارية النوعية specific heat capacity والتي تعتمد فقط على نوع المادة.

$$c = \frac{C}{m}$$

- where c is called the specific heat capacity or specific heat.
- ووحدة السعة الحرارية النوعية $J/kg.C^o$، فمثلا السعة الحرارية النوعية للماء تساوي $4186 J/kg.C^o$ وهذا يعني اننا نحتاج إلى 4186 جول من الطاقة تلزم لرفع واحد كيلو جرام من الماء درجة مئوية واحدة. ونلاحظ ان الماء هو اكثر العناصر سعة حرارية في الطبيعة وذلك لان اجسامنا تحتوي على 70% من الماء وهذا يجعل درجة حرارة الجسم ثابتة طوال اليوم والا ارتفعت درجة الحرارة في النهار وانخفضت في الليل، كما ان مياه المحيطات والبحار لا تتغير درجة حرارتهم بسرعة حفاظاً على الكائنات الحية التي فيها وهذا من حكمة الله عز وجل بأن يكون للماء اكبر سعة حرارية أي الاقل تأثراً بتغير درجات الحرارة

- يوضح الجدول التالي السعة الحرارية لبعض المواد عند درجة حرارة 25 درجة مئوية) درجة حرارة الغرفة (وعند الضغط الجوي .

Al	$900 J/kg.C^o$	wood	$1700 J/kg.C^o$
Cu	$387 J/kg.C^o$	glass	$837 J/kg.C^o$
Ag	$129 J/kg.C^o$	water	$4186 J/kg.C^o$
Pb	$128 J/kg.C^o$	ice	$2090 J/kg.C^o$

نلاحظ أن الماء له اكبر سعة حرارية نوعية بحيث أنه يلزم 4186 جول لرفع درجة حرارة 1 كيلوجرام من الماء درجة مئوية واحدة بينما يلزم 900 جول من الحرارة للألمنيوم

- يمكننا الآن ان نعبر عن كمية الحرارة Q التي تنتقل إلى مادة كتلتها m وتتغير درجة حرارتها من درجة حرارة ابتدائية T_i إلى درجة حرارة نهائية T_f على النحو التالي:

$$Q = m\,c\,\Delta T$$

- من هذه المعادلة يتضح أنه عندما **تكتسب** المادة حرارة فإن كلا من كمية الحرارة والتغير في درجة الحرارة يكون **موجباً**، وعندما **تفقد** المادة حرارة فإن التغير في درجة الحرارة يكون بالسالب وتكون كمية الحرارة **سالبة**.

Example

- A 0.05kg of metal is heated to 200°C and then dropped into a beaker containing 0.4kg of water initially at 20°C. If the final equilibrium temperature of the mixed system is 22.4°C find the specific heat of the metal. What is the total heat transferred to water in cooling the metal?

Solution

- يعتبر هذا المثال بمثابة تجربة عملية لتعيين السعة الحرارية للمواد، وهنا قمنا بتسخين المعدن إلى درجة حرارة 200 درجة مئوية وأسقطت في كمية من الماء ذات كتلة محددة عند درجة حرارة 20 درجة مئوية ولتصبح درجة حرارة المعدن والماء 22.4 درجة مئوية. وبالتالي فإن الحرارة المفقودة بواسطة المعدن تساوي الحرارة المكتسبة بواسطة الماء.

Heat lost by the metal = heat gained by water

$$m_x\ c_x\ (T_i - T_f) = m_w\ c_w\ (T_f - T_i)$$

$$(0.05\text{Kg})\ c_x\ (200°C - 22.4°C) = (0.4\text{kg})(4186\text{J/kg.C°})(22.4°C - 20°C)$$

$$c_x = 453\text{J/kg.C°}$$

(b) $Q = m\ c\ (T_i - T_f) = 0.05\ ´\ 453\ ´\ (200-22.4) = 4020\text{J}$

Example

- A man fires a silver bullet of mass 2g with a velocity of 200m/sec into a wall. What is the temperature change of the bullet?
- **Solution**
- تكتسب الرصاصة طاقة حركة تتحول إلى حرارة عند اصطدامها بالجدار وبهذا يتضح أن الحرارة صورة من صور الطاقة.
- The kinetic energy of the bullet $E_k = 1/2\ m\ v^2 = 40\text{J}$
  - $Q = m\ c\ \Delta T$
- where c for silver is 234Jkg.C°
  - $\Delta T = Q/mc = 85.5C°$

Example

A quantity of hot water at 91°C and another cold one at 12°C. How much kilogram of each one is needed to make an 800 liter of water bath at temperature of 35°C.

- **Solution**

Assume the mass of hot water m_H and cold one is m_C,

800 liter of water is equivalent to 800 kg,

$$so\ m_H + m_C = 800,$$

From the conservation of energy

$$m_H C_w (T_H - T_f) = m_C C_w (T_f - T_C)$$

$$T_H = 92^oC,\ \ T_C = 12^oC,\ \ T_f = 35^oC,$$

$$56\ m_H = 23\ m_C,$$

- so

$$m_C = 2.43\ m_H$$

- So by substitution

$$3.43\ m_H = 800,$$

$$m_H = 233\ kg,\ \text{and}\ m_C = 567\ kg$$

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Unit 4

Temperature and Heat

Lecture 15

Heat Transfer

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Contents

- Temperature
- Thermal expansion of solids and Liquids
- Ideal Gas Equation
- Heat and the first law of thermodynamics
- Heat Transfer

Heat Transfer

- The heat is a transfer of the energy from a high temperature object to a lower temperature one.
- Heat transfer changes the internal energy of both systems
- **Heat can be transferred by three ways:**
  1. **conduction,**
  2. **convection**
  3. **radiation.**

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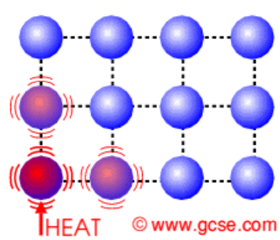
Heat conduction

- Conduction is heat transfer by means of molecular agitation within a material without any motion of the material as a whole.

Conduction is the transfer of heat within a substance, molecule by molecule.

If you put one end of a metal rod over a fire, that end will absorb the energy from the flame. The molecules at this end of the rod will gain energy and begin to vibrate faster. As they do their temperature increases and they begin to bump into the molecules next to them. The heat is being transfered from the warm end to the cold end.



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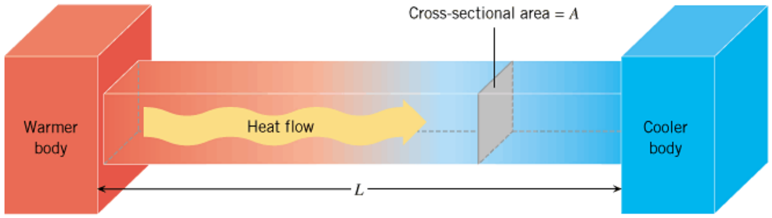
Heat conduction

- Conduction – when two objects are in physical contact.



$$H = \frac{Q}{t} = kA\left(\frac{\Delta T}{L}\right)$$

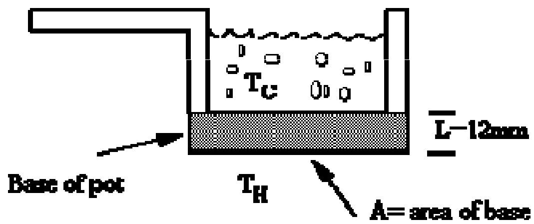
H = rate of conduction heat transfer (Watt)
k = thermal conductivity (W/m/K)
Q = heat transferred
A = cross sectional area
t = duration of heat transfer
L = length
ΔT = temperature difference between two ends

The thermal conductivity coefficient

Substance	(W m^{-1}K^{-1})	Substance	(W m^{-1}K^{-1})
Silver	427	Ice	2
Copper	397	Water	0.6
Aluminum	238	Wood	0.08
Gold	314	Air	0.023
Concrete	0.8	Hydrogen	0.1
Glass	0.8	Helium	0.138

Example

- An aluminum pot contains water that is kept steadily boiling (100 °C). The bottom surface of the pot, which is 12 mm thick and $1.5\text{x}10^4$ mm^2 in area, is maintained at a temperature of 102°C by an electric heating unit. Find the rate at which heat is transferred through the bottom surface. Compare this with a copper based pot.



Solution

$$H = kA\left(\frac{\Delta T}{L}\right)$$

- For the aluminum base: $T_H = 102$ °C, $T_C = 100$ °C, $L=12$ mm $= 0.012$ m, $K_{Al} = 238\,Wm^{-1}K^{-1}$, Base area $A = 1.5\text{x}10^4$ mm^2 = 0.015 m^2.

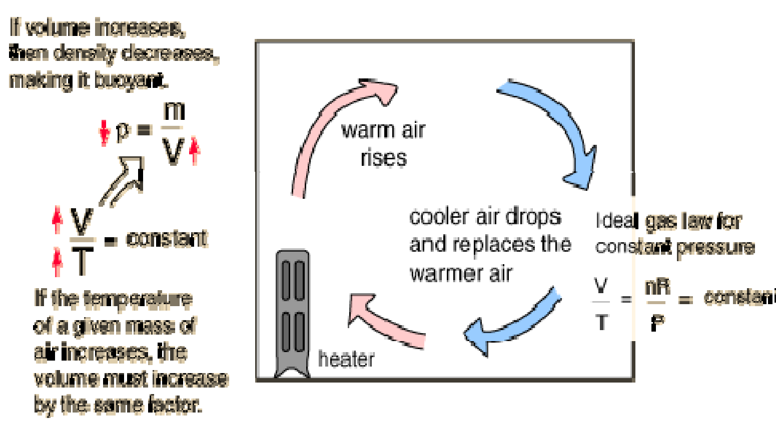
$$H_{Al} = 238\,(0.015)\,\frac{(102-100)}{0.012} = 588W$$

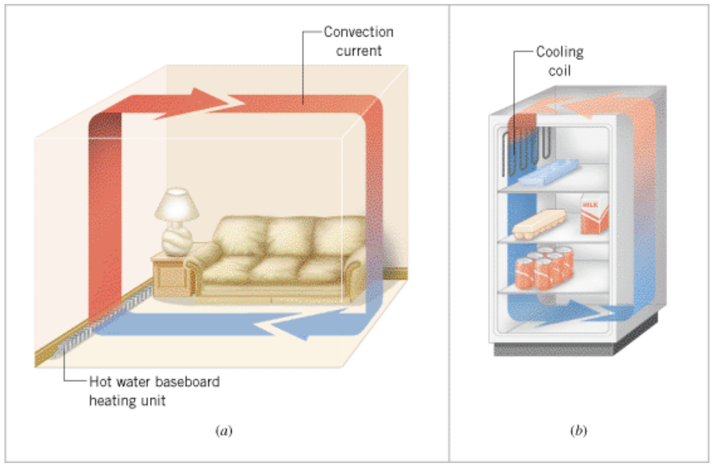
- For the copper base $K_{Cu} = 397\,Wm^{-1}K^{-1}$.

$$H_{Cu} = 397\,(0.015)\,\frac{(102-100)}{0.012} = 1003W$$

Heat Convection

- Convection is heat transfer by mass motion of a fluid such as air or water when the heated fluid is caused to move away from the source of heat, carrying energy with it.
- Convection above a hot surface occurs because hot air expands, becomes less dense, and rises. Hot water is likewise less dense than cold water and rises, causing convection currents which transport energy.





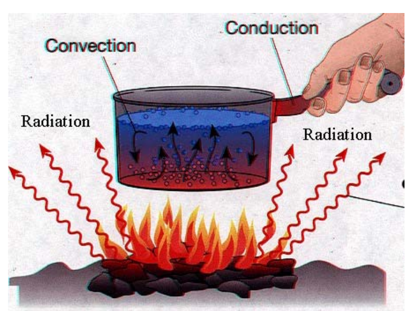
Q: In the living room, the heating unit is placed in the floor but the the refrigerator has a top-mounted cooling coil. Why?

A: Air warmed by the baseboard heating unit is pushed to the top of the room by the cooler and denser air. Air cooled by the cooling coil sinks to the bottom of the refrigerator.

Heat Radiation

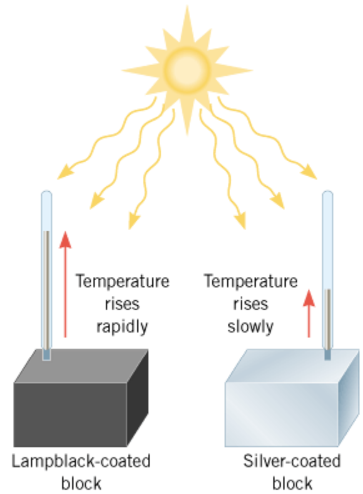
- Energy is transferred by electromagnetic radiation. All of the earth's energy is transferred from the Sun by radiation.

Our bodies radiate electromagnetic waves in a part of the spectrum that we can't see called the infra-red. However, there are some cameras that can actually see this radiation.



Heat Radiation

- The color and texture of different surfaces determines how well they absorb the radiation.
- (1) Black objects absorb more radiation than white objects.
- (2) Matt and rough surfaces absorb more than shiny and smooth surfaces.



Heat Radiation

- The relationship governing radiation from hot objects is called the Stefan-Boltzmann Law:

$$P = e\,\sigma\,A\,(T^4 - T_S^4)$$

P is the net radiated power measured in Watt,
e is the emissivity (=1 for ideal radiator),
A is the radiation area in m^2,
T is the temperature of the radiator in Kelvin,
Ts is the temperature of the surroundings in Kelvin,
$\sigma = 5.67x10^{-8}\, Watt/m^2\, K^4$ is a constant called Stefan-Boltzmann constant.

Example

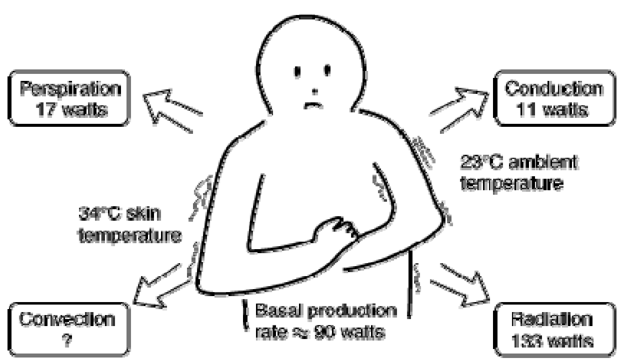
- A student tries to decide what to wear is staying in a room that is at 20°C. If the skin temperature is 37°C, how much heat is lost from the body in 10 minutes? Assume that the emissivity of the body is 0.9 and the surface area of the student is 1.5 m^2.
- **Solution**
- Using the Stefan-Boltzmann's law

$$P_{net} = e\,\sigma\,A\,(T^4 - T_s^4) = (5.67\times10^{-8})(0.9)(1.5)(310^4 - 293^4) = 143\,watt.$$

- The total energy lost during 10 min is

$$Q = P_{net}\,\Delta t = 143\times600 = 85.8\,kJ$$

Cooling of the Human Body



unclothed person at rest in a room temperature of 23 Celsius would be uncomfortably cool. The skin temperature of 34 C is a typical skin temperature taken from physiology texts, compared to the normal core body temperature of 37 C.

PROBLEMS UNIT 4: THERMAL PROPERTIES OF MATTER

1. Liquid nitrogen has a boiling point of -195.81°C at atmospheric pressure. Express this temperature in (a) degrees Fahrenheit and (b) Kelvin's.
2. The temperature difference between the inside and the outside of an automobile engine is 450°C. Express this temperature difference on the (a) Fahrenheit scale and (b) Kelvin scale.
3. 2.50 g of XeF4 gas is placed into an evacuated 3.00 liter container at 80°C. What is the pressure in the container?
4. A hydrogen gas thermometer is found to have a volume of 100.0 cm^3 when placed in an ice-water bath at 0°C. When the same thermometer is immersed in boiling liquid chlorine, the volume of hydrogen at the same pressure is found to be 87.2 cm^3. What is the temperature of the boiling point of chlorine?
5. (a) Show that the density of an ideal gas occupying a volume V is given by $\rho = p\,M / R\,T$ where M is the molar mass. (b) Determine the density of oxygen gas at atmospheric pressure and 20.0°C.
6. An ideal gas with volume of $3\ L$ is compressed at fixed temperature by increasing its pressure to $250\ kPa$. Find the bulk modulus of this gas.
7. A copper telephone wire has essentially no sag (لا يوجد ارتخاء) between poles 35.0 m apart on a winter day when the temperature is -20.0°C. How much longer is the wire on a summer day when T_C = 35.0°C?
8. A steel rod 4.00 cm in diameter is heated so that its temperature increases by 70.0°C. It is then fastened between two rigid supports. The rod is allowed to cool to its original temperature. Assuming that Young's modulus for the steel is 20.6 x 10^{10} N/m^2 and that its average coefficient of linear expansion is 11x 10^{-6} $(°C)^{-1}$, calculate the tension in the rod.
9. A steel ball bearing is 4cm in diameter at 20.0°C. A bronze plate has a hole in it that is 3.9 cm in diameter at 20.0°C. What common temperature must they have so that the ball just squeezes through the hole?
10. A concrete slab has a length of 12 m at -5 ºC on a winter's day. What is the change in length from winter to summer, when the temperature is 35 ºC ? The linear expansion coefficient of concrete is 1 X 10^{-5} $ºC^{-1}$.

11. A steel rod is initially at 20 °C and has a length of 2 m and a cross sectional area of 10 cm^2.

    a) If it is heated to 120 °C, by how much does its length increase. b) How large a force must be applied to its end to restore the original length? (Young's Modulus for steel = $2x10^{11}$ N/m^2, the coefficient of thermal expansion for steel = $1.72x10^{-5}$ K^{-1}.

12. A thermometer has a mercury-filled glass bulb with a volume of $2x10^{-7}$ m^3 attached to a thin glass capillary tube with an inner radius of $5x10^{-5}$ m. If the temperature increases by 100°C, how far will the mercury rise in the tube? (volume thermal expansion coefficient of mercury= $1.82x10^{-4}$ K^{-1}).
13. If 46.6 kJ is required to heat 0.15 kg of helium gas from 20° to 80°C at constant pressure, find the specific heat of helium?
14. sphere of iron is heated to 900K and then cooled by placing it in a container filled of 5 kg water of 300K, what is the mass of the sphere if the final temperature of the mixture is 340K, the specific heat of iron is 490 J/kgK and that of water is 4200 J/kgK. Find the heat loss by the iron in Calorie.
15. A box with a total surface area of 1.20 m^2 and a wall thickness of 4.00 cm is made of an insulating material. A 10.0-W electric heater inside the box maintains the inside temperature at 15.0°C above the outside temperature. Find the thermal conductivity k of the insulating material.
16. The surface of the Sun has a temperature of about 5800 K. The radius of the Sun is $6.96x10^8$ m. Calculate the total energy radiated by the Sun each second. (Assume that e = 0.96)
17. A person walking at a modest speed generates heat at rate of 280 W. If the surface area of the body is 1.5 m2 and if the heat is assumed to be generated 0.03 m below the skin, what temperature difference between the skin and interior of body would exist if the heat were conducted to the surface? Assume that the thermal conductivity coefficient is0.2 Wm-1K-1.
18. A 2 m length of copper pipe of 10 cm diameter containing hot water at 80^{o}C. If the surroundings are at 20^{o}C, at what rate does the pipe lose thermal energy due to radiation? (take e = 1)

الوحدة الخامسة: ميكانيكا الموائع

Fluid Mechanics

Dr. Hazem Falah Sakeek | www.hazemsakeek.net | 2014

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Unit 5

Fluid Mechanics

Lecture 16

Fluid Characteristics

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

We will study the properties of the fluids and their behavior. Along with the flow of the fluids for nonviscous and viscous fluids.

Fluid Mechanics

- **Fluid Mechanics** is concerned with the behavior of fluids at rest and in motion.
- **Distinction between solids and fluids:** According to our experience: A solid is "hard" and not easily deformed. A fluid is "soft" and deforms easily.
- **Fluid** is a substance that alters its shape in response to any force however small, that tends to flow or to conform to the outline of its container, and that includes gases and liquids.

A fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude.

- **Fluids** are playing a very important role in many fields of sciences and medical sciences.
- **Fluid** movement for Solute transport in soft connective tissue is a fundamental process, involving many physiological phenomena, such as nutrient supply, removal of metabolic waste product and movement of newly-synthesized molecules.
- **A *gas* is a fluid that is easily compressed. It fills any vessel in which it is contained.**
- **A *liquid* is a fluid which is hard to compress. A given mass of liquid will occupy a fixed volume, irrespective of the size of the container.**

Fluid Characteristics: Pressure

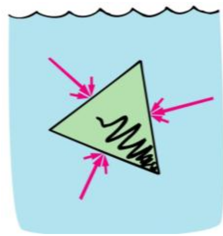
- A measure of the amount of force exerted on a surface area

$$pressure = \frac{force}{area}$$

$$p = \frac{F}{A}$$

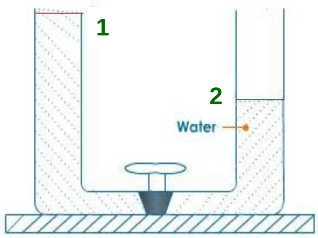
Nm^{-2} (Pa) — p; N — F; m^2 — A

•The pressure is just the weight of all the fluid above you, Atmospheric pressure is just the weight of all the air above on area on the surface of the earth.
•In a swimming pool the pressure on your body surface is just the weight of the water above you (plus the air pressure above the water).
•The deeper you go, the more weight above you and the more pressure. Go to a mountaintop and the air pressure is lower



Pressure acts perpendicular to the surface and increases at greater depth.

- **Pressure difference** cause the flowing of the fluid from point to another (from A to B in the figure).



Fluid flow through a tube of cross sectional area A from a point 1 to 2, then the **force exerted on the fluid is given by:**

$$force = (P_1 - P_2)\,A = \Delta P\,A$$

Pressure in a fluid is a measure of **energy per unit volume!**

$$pressure = \frac{force}{area} = \frac{F}{A} = \frac{F.\Delta x}{A.\Delta x} = \frac{W}{\Delta V} = \frac{energy}{volume}$$

Where ΔV is the unit volume, and the unit mass $\Delta m = \rho\,\Delta V$

Kinetic energy density

- **Kinetic energy density** is the kinetic energy per unit volume
- For a unit mass Δm of a fluid moving with average speed v, its kinetic energy is given by

$\Delta m = \rho \Delta V$

$$K.E = \frac{1}{2}\Delta m\bar{v}^2 = \frac{1}{2}\rho\Delta V\bar{v}^2$$

- kinetic energy density is written as

$$K.E\ density = \frac{kinetic\ energy}{volume} = \frac{\frac{1}{2}\rho\ \Delta V\bar{v}^2}{\Delta V} = \frac{1}{2}\rho\bar{v}^2$$

Potential energy density

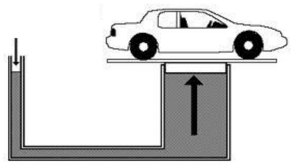
- **Potential energy density** is the potential energy per unit volume
- for a certain mass of fluid at distance h above the ground; then the potential energy density is

$\Delta m = \rho \Delta V$

$$P.E\ density = \frac{potential\ energy}{volume} = \frac{\Delta mgh}{\Delta V} = \frac{\rho\Delta Vgh}{\Delta V} = \rho gh$$

Compressibility and Incompressibility

- If the density of the fluid is constant (ρ = constant) everywhere through the flow, it is called incompressible and it is called compressible if the density is not fixed.



Viscous and Nonviscous

- The viscosity is the frictional forces originated inside the fluids and which is considered as the resistance of flow.

- Nonviscous fluid when the frictional forces are neglected.

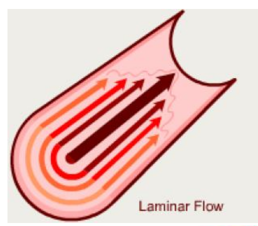
Types of flow

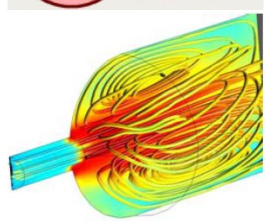
•Consider that the fluid consists of laminar or layers, so we can represent each layer by line called a streamline.

•Streamline describes the flow of the fluid and its direction

Laminar flow
- the speed of flow is low and the streamlines of flow are parallel

Turbulent flow
- the speed of flow is high and the streamlines intersect each other

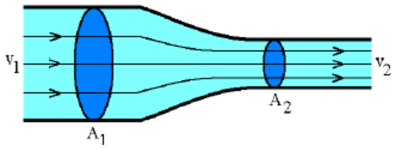




Turbulent flow

Fluid Flow and the Continuity Equation

- Consider a hose with a decreasing diameter along its length, as shown in the figure



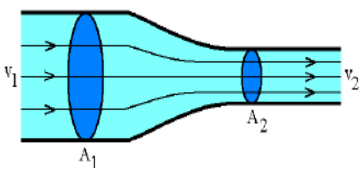
- **Continuity Equation** is a direct consequence of the fact that what goes into the hose must come out

$$\mathbf{Q}_{in} = \mathbf{Q}_{out}$$

- The volume of water flowing through the hose per unit time or the flow rate (Q) at the left must be equal to the flow rate at the right, or in fact anywhere along the hose.

$$Q = \frac{\Delta V}{\Delta t} = const.$$

- **The flow rate** is measured in the units of volume per unit time, m^3/s.
- The flow rate at any point in the hose is equal to the area of the hose at that point times the speed with which the fluid is moving.
- Consider a fluid is flowing in a tube as shown in figure, where the radius of the tube is decreasing, thus at a certain point at the tube the fluid flow rate is



$$Q = \frac{\Delta V}{\Delta t} = \frac{A\Delta x}{\Delta t} = A\bar{v} = const.$$

$$A_1\bar{v}_1 = A_2\bar{v}_2$$

- Take $A = \pi r^2$, so the continuity equation can be rewritten as

$$r_1^2\bar{v}_1 = r_2^2\bar{v}_2$$

- This means that the speed of flow increases by decrease of the diameter of the tube.

Example 5.1

- **A water pipe with radius of 1 cm leading up to a hose. Water leaves the hose at a rate of 3 liter per minute.**
  - (a) Find the velocity of water in the pipe.
  - (b) The hose has a radius of 0.5 cm. What is the velocity of water in the hose?

Solution

- The velocity can be found by using the flow rate and the area of the pipe or the hose. The flow rate is

$$Q = \frac{\Delta V}{\Delta t} = \frac{0.003}{60} = 5 \times 10^{-5}\, m^3/s$$

- So the velocity is given by:

$$v = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{5\times 10^{-5}\, m^3 s^{-1}}{3.14\times(0.01m)^2} = 0.159\, ms^{-1}$$

- The flow rate is constant, so

$$r_1^2 \overline{v_1} = r_2^2 \overline{v_2}\,, \Rightarrow \frac{v_2}{v_1} = \frac{r_1^2}{r_2^2}$$

- And so the velocity in the hose v_2 can be calculated as:

$$v_2 = v_1 \frac{r_1^2}{r_2^2} = 0.159\, \frac{1}{0.25} = 0.636\, ms^{-1}$$

- The water flow faster in the narrower channel.

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Unit 5

Fluid Mechanics

Lecture 17

Bernoulli's Equation

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

- Fluid Characteristics
- Fluid Flow and the Continuity Equation
- Bernoulli's Equation
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- Effect of acceleration on Blood pressure
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We will study the properties of the fluids and their behavior. Along with the flow of the fluids for nonviscous and viscous fluids.

Bernoulli's Equation

- Bernoulli derived an important equation to describe the flow of fluids. This equation is stated that ***the work done on a fluid as it flows from one place to another is equal to the change in its mechanical energy.***

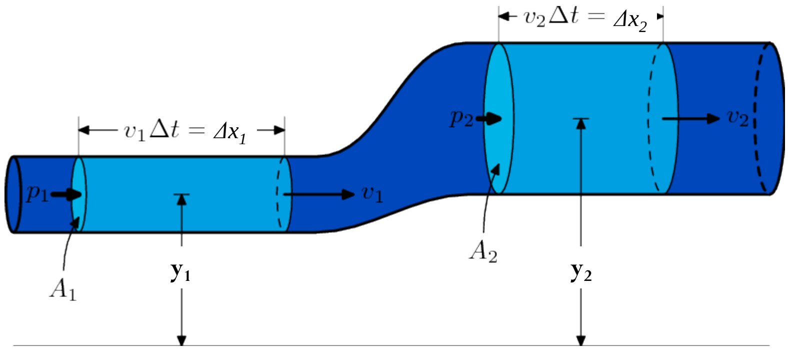
This equation is applicable for incompressible fluids, nonviscous fluids (where no energy loss), laminar flow, and for steady state flow (when the flow speed at any point is constant with time).

Daniel Bernoulli
Swiss physicist
(1700–1782)

Daniel Bernoulli made important discoveries in fluid dynamics. Born into a family of mathematicians, he was the only member of the family to make a mark in physics.

Derivation of Bernoulli's Equation

- **To derive Bernoulli's equation,** we consider the flow of a fluid in a tube of cross sectional area A from section 1 to section 2 as shown in the figure.



The flow of a liquid from point 1 to point 2 via a pressure difference $p_1 - p_2$

- The pressure, speed of flow, and height of the fluid at cross section 1 is denoted as p_1, v_1, y_1 respectively.
- For the same at section 2, we have p_2, v_2, y_2.

$$(p_1 - p_2)\,A = \Delta p\, A$$

- If the fluid in the section moves a short distance Δx, so that the work done on the fluid is given by

$$W = F.\Delta x = (p_1 - p_2)\,A\,\Delta x$$

Since the product A Δx is the volume ΔV of the fluid leaving the section, thus the equation above becomes:

$$W = (p_1 - p_2)\,\Delta V$$

- the **change in the kinetic energy** of a volume ΔV of the fluid flowing from section 1 to 2 is

$$\frac{1}{2}\rho\Delta V\bar{v}_2^2 - \frac{1}{2}\rho\Delta V\bar{v}_1^2$$

- for the **change in the potential energy**, which is

$$\rho\Delta Vgy_2 - \rho\Delta Vgy_1$$

- *Since the energy has to be conserved, the work in equation*

$$W = (p_1 - p_2)\,\Delta V$$

must equal the change in the kinetic energy and the change in the potential energy, so that we can write:

$$(p_1 - p_2)\Delta V = \frac{1}{2}\rho\Delta V\bar{v}_2^2 - \frac{1}{2}\rho\Delta V\bar{v}_1^2 + \rho\Delta Vgy_2 - \rho\Delta Vgy_1$$

- By eliminating ΔV from both sides and rearrange the similar terms in one side, we get:

$$p_1 + \frac{1}{2}\rho\bar{v}_1^2 + \rho gy_1 = p_2 + \frac{1}{2}\rho\bar{v}_2^2 + \rho gy_2$$

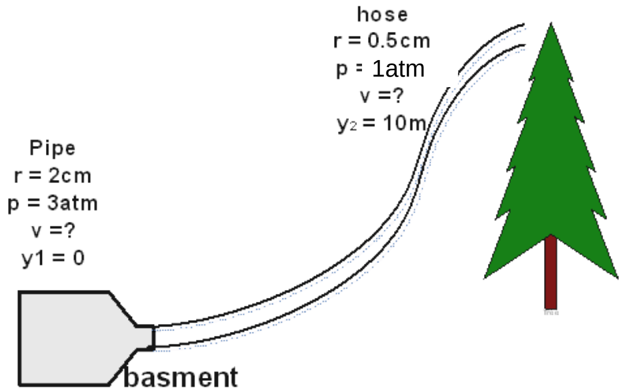
- This can be generalized for any two points through the flow of the fluid, so **Bernoulli's equation can be written as**

$$p + \frac{1}{2}\rho\bar{v}^2 + \rho gy = \text{constant}$$

Thus Bernoulli's equation can be stated as the pressure of the fluid plus its mechanical energy density (kinetic energy density+ potential energy density) is the same everywhere in the flow.

Example 5.2

Water enters the basement through a pipe 2 cm in radius at an absolute pressure of 3 atm. A hose with a 0.5 cm radius is used to water plants 10 m above the basement. Find the speed of water as it leaves the hose?



Solution

At Point 1 in the pipe at the basement, where

$p1 = 3atm,\ y_1 = 0m,\ v_1 = ?.$

At point 2 is in the hose just at the moment the water leaving the hose for planting the tree, where

$p_2 = 1atm,\ y_2 = 10m,\ v_2 = ?$

By applying Bernoulli's equation, we have

$$p_1 + \frac{1}{2}\rho \bar{v}_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho \bar{v}_2^2 + \rho g y_2$$

Rearrange the equation, we get

$$p_1 - p_2 = \frac{1}{2}\rho(\bar{v}_2^2 - \bar{v}_1^2) + \rho g(y_2 - y_1)$$

We have two unknowns v_1 and v_2 , so we can reduce them to only one unknown by using the continuity equation, where

$$v_1 = \frac{r_2^2}{r_1^2} v_2$$

So $v_1 = \frac{1}{16} v_2$ and $\overline{v_2^2} - \overline{v_1^2} = \frac{255}{256} v_2^2$ then we substitute in the last equation to get:

[redacted]

$$(3-1) \times 1.013 \times 10^5 = \frac{1}{2}(1000)\,\frac{255}{256}\,v_2^2 + 1000(10)\,(10-0)$$

Solve for v_2, *we get*

$$v_2 = 14.35\ ms^{-1}.$$

Example 5.3

Water is flowing from a hole of 1 cm radius at the bottom of a closed cylindrical container of 2m diameter. If the height of the water in the container is 2 m and the pressure over the surface of water is 3 atm, calculate how much time it took until the container became empty?

Solution

The container is cylindrical in shape with radius R = 1 m, and the height of water in it is H = 2m, so that we can calculate the amount of water in the container

$$\Delta V = \pi R^2 H = 3.14 \times 1^2 \times 2 = 6.28\ m^3$$

- If we could calculate the flow rate $Q = \pi R^2 \bar{v}$ from the hole and which is equal $\Delta V/\Delta t$, we can compute the time needed to empty the container, so we should calculate $\bar{v}$.

- To apply Bernoulli's equation we assume two points, one at the surface of water with p_s= 3atm, h_s= 2 m, v_s=? and the other point at the exit of the hole at the bottom of the container with p_h = 1atm, h_h = 0 m, v_h =?. Writing Bernoulli's equation

$$p_s + \frac{1}{2}\rho\bar{v}_s^2 + \rho g y_s = p_h + \frac{1}{2}\rho\bar{v}_h^2 + \rho g y_h$$

Rearrange the equation, we get

$$p_s - p_h = \frac{1}{2}\rho(\bar{v}_h^2 - \bar{v}_s^2) + \rho g(y_h - y_s)$$

By comparing the radius of the container to the radius of the hole

$$\frac{R}{r} = \frac{1}{1\times10^{-2}} = 100$$

so that the speed of water at the surface related with that at the hole

$$\frac{v_s}{v_h} = \frac{r^2}{R^2} = \frac{1}{10000}$$

This indicates that $v_s \ll v_h$, which leads that v_s^2 is much smaller than v_h^2.
We can consider $\overline{v_h^2} - \overline{v_s^2} \approx \overline{v_h^2}$, and $y_h - y_s = y_s = 2m$.
Substitute in Bernoulli's equation, we get:

$$p_s - p_h = \frac{1}{2}\rho\bar{v}_h^2 + \rho g y_s$$

- By solving for v_h, we get:

$$\overline{v_h^2} = \frac{2\,(p_s - p_h) - 2\rho\, g y_s}{\rho} = \frac{2 \times 2 \times 1.013 \times 10^5 - 2x10^3 \times 10 \times 2}{10^3}$$

$$\overline{v_h^2} = 365 \quad , so\ v_h = \sqrt{365} = 19.1\ ms^{-1}$$

- So the time required to empty the container

$$\Delta t = \frac{\Delta V}{Q} = \frac{\Delta V}{\pi r^2 v_h} = \frac{6.28}{3.14 \times 10^{-4} \times 19.1} = 1019\ s \sim 17\ min.$$

Selected Topics in Physics for Medical Sciences Students

Unit 5

Fluid Mechanics

Lecture 18

Applications of Bernoulli's Equation

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

- Fluid Characteristics
- Fluid Flow and the Continuity Equation
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We will study the properties of the fluids and their behavior. Along with the flow of the fluids for nonviscous and viscous fluids.

Applications of Bernoulli's equation

(1) *Static Consequence*

- In static consequence we have zero speed of flow, where the fluid is settle in its container.
- In this situation the kinetic energy density term leads to zero, and Bernoulli's equation becomes

$$p + \rho g y = constant$$

or

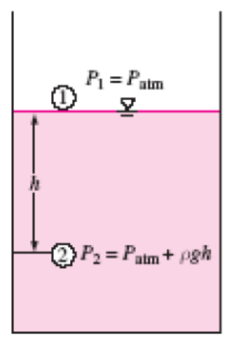
$$p_1 + \rho g y_1 = p_2 + \rho g y_2$$

This means that the sum of the pressure and the potential energy density is constant everywhere inside a fluid in static.

- **For example,**

If we have a liquid in an open air container as show in the figure and it is required to measure the pressure at a point at depth *h* inside the container.

Take two points: point 1 at the surface of the liquid, where the pressure is the atmospheric pressure and point 2 at depth $h = (y_1 - y_2)$ inside the container, also $v_1 = v_2 = 0$, and then by applying Bernoulli's equation,



- we have:

$$p_a + \rho g y_1 = p_2 + \rho g y_2$$

- so

$$p_2 = p_a + \rho g (y_1 - y_2) = p_a + \rho g h$$

- So the pressure inside any container is equal the pressure at the surface plus the potential energy density at that point.

Note:
The difference between the absolute pressure at any point and the atmospheric pressure $(p - p_a)$ is called the **gauge pressure**.

Example 5.4

What is the pressure on a swimmer 5 m below the surface of a lake?

- **Solution**

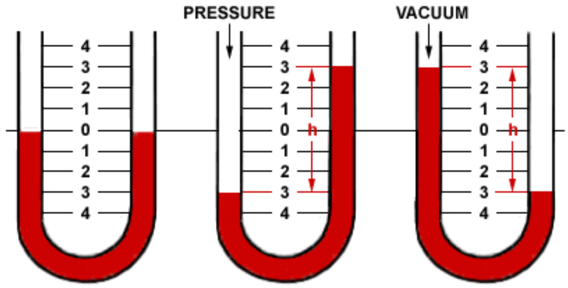
Using the depth of the swimmer is $h = 5\ mm$,
the density for water is $\rho = 1000\ kgm^{-3}$, *and*
the atmospheric pressure is $1.013\times10^5\ Pa$.
So using equation $p = p_a + \rho g h$ to *calculate the pressure on the swimmer to be:*

$$p = p_a + \rho g h = 1.013\times10^5 + (1000)(10)\,5 = 1.5\times10^5\ Pa$$

Applications of Bernoulli's equation

(2) *Manometer*

- A manometer consists of a *U* shaped tube of glass filled with some liquid.
- Typically the liquid is mercury <u>because</u> of its high density.
- With both ends of the tube open, the liquid is at the same height in each leg.
- When positive pressure is applied to one leg, the liquid is forced down in that leg and up in the other.



A U shaped tube called Manometer is used to measure the pressure of unknown gases

- The difference in height, "h," which is the sum of the readings above and below zero, indicates the gauge pressure ($p = \rho g h$).
- *When a vacuum (low pressure) is applied to one leg, the liquid rises in that leg and falls in the other.*
- *The difference in height, "h," which is the sum of the readings above and below zero, indicates the amount of vacuum.*

The manometer is a part of a device called a sphygmomanometer

Typical sphygmomanometer used to measure the blood pressure

Applications of Bernoulli's equation

(3) *Horizontal flow consequence*

- When the flow of a fluid is horizontally. In this case, the potential energy density term will vanish since all points on the flow line have the same height, so that the dynamic energy term will be considered and the equation will be given as:

$$p + \frac{1}{2}\rho \bar{v}^2 = \text{constant}$$

- Or we can write for two points on the same plane of flow, as:

$$p_1 + \frac{1}{2}\rho \bar{v}_1^2 = p_2 \frac{1}{2}\rho \bar{v}_2^2$$

$$p_1 + \frac{1}{2}\rho \bar{v}_1^2 = p_2 \frac{1}{2}\rho \bar{v}_2^2$$

- **This equation has many applications. The simple one is by blowing air between two half sheets of paper.**
- We can apply Bernoulli's equation by taking two points at the same plane. The first point is outside the sheet, where the pressure and the speed of flow are noted as p_{out}, v_{out}, and the other point is between the sheets of paper, where the pressure and the speed of flow are noted as p_{in}, v_{in}, so that we have

$$p_{out} + \frac{1}{2}\rho \bar{v}_{out}^2 = p_{in} \frac{1}{2}\rho \bar{v}_{in}^2$$

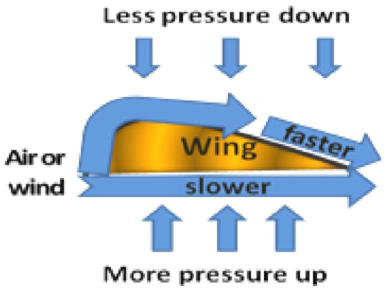
- Rearrange the equation leads to the following

$$p_{out} - p_{in} = \frac{1}{2}\rho\left(\bar{v}_{in}^2 - \bar{v}_{out}^2\right)$$

- When a person blows between the two sheets, so that the speed of air flowing inside will be larger than that out side ($v_{in} > v_{out}$): this means that the left hand side is positive and therefore $p_{out} > p_{in}$. This pressure difference results in the sheets moving closer toward one another. Thus, the pressure drops when the velocity of the flow increases for a fluid moving at a constant height.

- This pressure drop associated with increasing fluid velocities has many everyday implications.

For example, concerning flight, Bernoulli's Principle has to do with the shape of an airplane's wing as shown in the Figure. The bottom is flat, while the top is curved. Air travels across the top and bottom at the same time, so air travels slower on the bottom (creating more pressure) and faster on top (creating less pressure).



Bernoulli's principle can explain the clogging of arteries when the blood flows through an artery section of smaller cross sectional area. According to Bernoulli the pressure of blood within this section will drop inside the arterial wall, and on the other hand the pressure on the outside arterial wall will be larger than inside causing the clogging of the blood vessel.

Example 5.5

The diameter of a horizontal blood vessel is reduced from 12 to 4 mm. What is the flow rate of blood in the vessel, if the pressure at the wide part is 8 kPa and 4 kPa at the narrow one. (Take the density of blood to be 1060 kgm^{-3}.)

Solution

- By applying Bernoulli's equation for horizontal flow and by taking one point in the wider section and the other at the narrower one, we get:

$$p_{wid} - p_{narr} = \frac{1}{2}\rho\left(\overline{v}_{narr}^2 - \overline{v}_{wid}^2\right)$$

Using the **continuity equation**,

$$v_{wid} = \frac{r_{narr}^2}{r_{wid}^2} v_{narr} = \left(\frac{4}{12}\right)^2 v_{narr} = \frac{1}{9} v_{narr}$$

Then substitute and solve for v_{narr} to get

$$4 \times 10^3 = \frac{1}{2}(1060)\ \overline{v_{narr}^2}\left(1 - \frac{1}{81}\right), \text{ so then } v_{narr}^2 = \frac{81 \times 2 \times 4 \times 10^3}{80 \times 1060} = 7.64$$

- The flow rate is constant everywhere and can be calculated from the relation $Q = \pi r_{narr}^2 v_{narr} = 3.14 \times (4 \times 10^{-3})^2\ 2.76$

$$Q = 1.387 \times 10^{-4}\ m^3 s^{-1} = 138.7\ mL/s$$

Applications of Bernoulli's equation

(4) *The Role of Gravity on blood circulation*

- From Bernoulli's principle, the pressure of the fluid change according to its kinetic energy density and as well as it potential energy density.
- Because of that, the blood pressure in human organs is affected by its location from earth.
- During the blood circulation, the venous system is used to return the blood from the lower extremities to the heart. It is expected to have a problem of lifting blood long distances to the heart against the force of gravity.
- If we have a person in the reclining (laying down) position, the measurement of blood pressure in the large arteries are almost the same everywhere.

- The small drop in pressure between the **heart** and the **feet** or the **brain** is due to the viscous forces. According to Bernoulli's equation

$$p + \frac{1}{2}\rho \bar{v}^2 + \rho g y = \text{constant}$$

We can analyze the situation in the reclining position.

- The velocities in the three main arteries (Brain, heart, and feet) are small, so that the term $\frac{1}{2}\rho \bar{v}^2$ can be ignored.
- Furthermore, in this position also the height of the brain, heart and feet are almost equal, so that the term ρ g y can be ignored from the formula.
- This results in equal blood pressure in the three parts

$$p_B = p_H = p_F.$$

- Note that ***B***,***H*** and ***F*** refer to the brain, heart and feet.

- In the **standing position**, the situation is different, where only the term $\frac{1}{2}\rho\bar{v}^2$ can be ignored and the term $\rho\, gh$ has a significant effect.
- Hence the gauge pressures at the brain p_B, at the heart p_H and at the foot p_F are related by:

$$p_F = p_H + \rho g h_H = p_B + \rho g h_B$$

- Note that $h_F = 0$ in the standing position.

- Typical values for adults standing upward h_H=1.3 m and h_B = 1.7 m. Typical value of the blood pressure at the heart is p_H =13.3 kPa, and take the blood density to be 1060 kgm^{-3}, we find:

$$p_F = p_H + \rho g h_H = 13.3 \times 10^3 + (1060)(10)(1.3) \sim 27.1\, kPa$$

- In a similar way, we find that:

$$p_B = p_H + \rho g(h_H - h_B) = 13.3 \times 10^3 + (1060)(10)(-0.4) = 9.06\, kPa$$

- This explains why the pressures in the lower and upper parts of the body are very different when the person is standing, although they are about equal in the reclining.
- The high blood pressure at the foot explain the possibility of lifting blood uphill to the heart, and in addition the muscles surrounding the veins contract and cause constriction.

Example 5.6

- When a 1.7 m tall man stands, his brain is 0.5 m above his heart. If he bends so that his brain is 0.4 m below his heart, by how much does the blood pressure in his brain changes?

Solution

- We know that the blood pressure of the organ change by changing its position from the earth. The blood pressure at the brain in the standing case is given by:

$$p_{B\,stand.} = p_H + \rho g(h_H - h_B)_{stand}$$

- where

$$(h_H - h_B)_{stand} = -0.5\,m$$

So $p_{B\,stand.} = 13.3 \times 10^3 + 1060 \times 10 \times (-0.5) = 8\,kPa$

The blood pressure at the brain in the bending position is given by:

$$p_{B\,bending.} = p_H + \rho g(h_H - h_B)_{bending}$$

Where $(h_H - h_B)_{bending} = 0.4$ m, this results in

$$p_{B\,bending} = 13.3 \times 10^3 + 1060 \times 10 \times (0.4) = 17.54\,kPa$$

So the blood pressure at the brain will increase by bending, so the change in blood pressure is

$$\Delta p = p_{B\,bending} - p_{B\,stand.} = 17.54 - 8 = 9.54\,kPa$$

Selected Topics in Physics for Medical Sciences Students

Unit 5

Fluid Mechanics

Lecture 19

Effect of acceleration on Blood pressure

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

- Fluid Characteristics
- Fluid Flow and the Continuity Equation
- Bernoulli's Equation
- Applications of Bernoulli's equation
- The Role of Gravity on blood circulation
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- Turbulent Flow

We will study the properties of the fluids and their behavior. Along with the flow of the fluids for nonviscous and viscous fluids.

Effect of acceleration on Blood pressure

- It is a common symptom for some people having hypotension (هبوط ضغط الدم) to feel dizziness (دوخة) when they exist in an elevator of upward acceleration.

Is the blood pressure at the organs affected when man under upward or downward acceleration?

When a person experiences an upward or downward acceleration, his weight will be different.

Upward acceleration: *If a man experience upward acceleration* $\boldsymbol{a}$*, then his effective weight becomes* $\boldsymbol{m}\,(\boldsymbol{g}+\boldsymbol{a})$.

- Applying Bernoulli's equation to the foot, brain and heart with **g** replaced by **g+a**, so we have:

$$p_B = p_H + \rho\,(g + a)(h_H - h_B)$$

- Or

$$p_B = p_H - \rho\,(g + a)(h_B - h_H)$$

for standing person the term ($h_B - h_H$) is positive and also the same for ($g+a$),

the blood pressure at the brain will be reduced even farther by increasing the upward acceleration **a**.

At certain value of **a**, the human will loose consciousness (يفقد الوعي) because the collapse of the arteries in the brain when the blood pressure at the brain equal zero.

- Put $p_B = 0$ in the above equation, we get:

$$0 = p_H - \rho\,(g+a)(\,h_B - h_H)$$

- This can results in

$$(g+a) = \frac{p_H}{\rho\,(\,h_B - h_H)}$$

- Take $(\,h_B - h_H) = 0.4\ m$, $\boldsymbol{p_H} = \mathbf{13.3}$ *KPa, and* $\rho = 1060\ kgm^{-3}$, *we get:*

$$(g+a) = \frac{13.3 \times 10^3}{1060\ (0.4)} = 31.4\ ms^{-2} = 3.2\ g$$

- So the value of the upward acceleration causing consciousness *is 2.2 g*

This factor should limit the speed with which a pilot can pull out of dive. A related experience is the feeling of light headache that sometimes occurs when one suddenly stands up.

- We can also show the change of the blood pressure at the foot by the upward acceleration situation, by putting $g+a$ instead of

$$p_F = p_H + \rho\,(g+a)\,h_H$$

- This relation shows that the blood pressure at the foot will increase by increasing the upward acceleration.

Downward acceleration

- If a man in an erect position experience downward acceleration, then his effective weight becomes $m\,(g-a)$.
- Applying Bernoulli's equation to the foot, brain and heart with g replaced by $g-a$, so we have:

$$p_B = p_H + \rho\,(g-a)(h_H - h_B)$$

- Or

$$p_B = p_H - \rho\,(g-a)(h_B - h_H)$$

Thus the blood pressure at the brain will increase even farther by increasing the downward acceleration **a**, which opposite to what occurs by the upward acceleration.

- This increase should be controlled and observed, where at certain value of **a** the blood pressure at the brain may cause an explosion of the arteries in the brain, which is so dangerous.
- The same calculation for the blood pressure at the foot results in a decrease of the blood pressure by increasing the downward acceleration.

Example

A 1.8 m tall man stand in an elevator accelerating upward at 12 ms^{-2}, what is the blood pressure in the brain and foot. Take the height difference between the heart and the brain to be 0.35 m?

Solution

- The elevator accelerating upward, so $(g+a)= 10 + 12 = 22\ ms^{-2}$, so substitute in the related formula to get

$$p_B = p_H - \rho\,(g+a)(\,h_H - h_B)$$
$$= 13300 - 1060\times 22\times 0.35 = 5.14\ \text{Kpa}$$

- The pressure at the brain decrease

$$p_F = p_H + \rho\,(g+a)\,h_H$$
$$=13300+1060\times 10\times 1.45 = 28.67\ \text{Kpa}$$

- An increase of the blood pressure at the foot is observed.

Selected Topics in Physics for Medical Sciences Students


Physics Academy
www.physicsacademy.org

Unit 5

Fluid Mechanics

Lecture 20

Viscous Fluid Flow

Dr. Hazem Falah Sakeek
Al-Azhar University - Gaza

Contents

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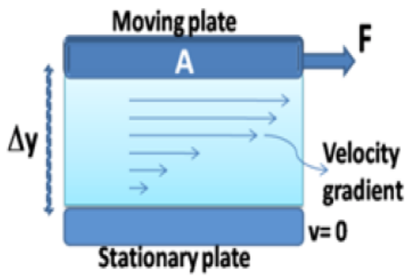
We will study the properties of the fluids and their behavior. Along with the flow of the fluids for nonviscous and viscous fluids.

Viscous Fluid Flow

- The viscosity in fluids is originated from the **frictional forces** between the **fluids laminas** and their **container**.
- The viscosity can be considered as the resistance of flow of fluids, like current resistance.
- **Viscosity in gases** is originated from the successive collision between the gas molecules, and the viscosity of the gas is a temperature dependent. By increasing the gas temperature, an increase of their kinetic energy and therefore the probability of collisions will increase.
- **Viscosity in liquids**, decrease by increasing the temperature.

- Put a small amount of liquid between two plates of glass separated by a distance Δy as shown in the figure. **The upper plate is free to move and the lower plate is fixed.** If the upper plate was forced to move with a velocity, Δv, away horizontally, there will be a resistance for this motion.



Two parallel glass plates separated by a thin layer of liquid

There will be a lamina or layer of the liquid which moves with the upper plate, and other lamina which is stationary.
There is a gradient of velocity as you move from the stationary plate to the moving one and the liquid tends to move in parallel layers, which is called **laminar flow**.

- the force **F** required to move the upper plate at constant average speed is proportional directly
  - proportional directly with the **speed gradient**
  - proportional directly the **surface area** of the plate
  - inversely proportional with the **separation distance** between the plates,
- which means that:

$$F \propto \Delta v\,,\ F \propto A \text{ and } F \propto \frac{1}{\Delta y}\,, \text{so } F \propto \frac{A\,\Delta v}{\Delta y}$$

- The constant of proportion is called the viscosity coefficient, which is represented by the symbol **η**

$$F = \eta \frac{A\,\Delta v}{\Delta y}$$

- **The viscosity can be defines** as the ratio between the **shearing stress** (**F/A**) to the rate of shearing strain or the **gradient of velocity**.

- The dimension of the viscosity coefficient can be deduced as:

$$[\eta] = \left[\frac{F/A}{\Delta v/\Delta y}\right] = \left[\frac{MLT^{-2}/L^2}{LT^{-1}/L}\right] = ML^{-1}T^{-1}$$

- The S.I. unit of the viscosity coefficient is Pascal second, where **$kgm^{-1}s^{-1}$=1 Pa.s.**

- The **Pascal second** is rarely (نادرا ما تستخدم) used, The common used unit is called poise (P), where

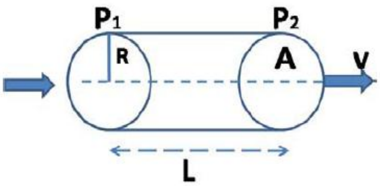
$$1\,poise = \frac{dyn\,e.s}{cm^2} = 10^{-1}\,Pa.s$$

Typical values of viscosity coefficient for some fluids in units of Pa.s.

Temperature C	Castor Oil	Water	Air	Normal blood	Blood Plasma
0	5.3	$1.792x10^{-3}$	$1.71x10^{-5}$		
20	0.986	$1.005x10^{-3}$	$1.81x10^{-5}$	$3.015x10^{-3}$	$1.81x10^{-3}$
37	-	$0.695x10^{-3}$	$1.87x10^{-5}$	$2.08x10^{-3}$	$1.257x10^{-3}$
40	0.231	$0.656x10^{-3}$	$1.9x10^{-5}$		
60	0.08	$0.469x10^{-3}$	$2.00x10^{-5}$		
80	0.03	$0.357x10^{-3}$	$2.09x10^{-5}$		
100	0.017	$0.284x10^{-3}$	$2.18x10^{-5}$		

Laminar Flow in a Tube

- Consider a fluid moving through a tube of length **L** and cross sectional area, $A = \pi R^2$.
- The pressure difference across the segment of the tube is $\Delta p = p_2 - p_1$ as shown in the figure.
- Because of the viscosity inside the fluid, the layer of the fluid adjacent to the cylindrical wall moves very slowly and the inward successive layers move at increasing velocities.
-



A laminar flow in a tube for viscous liquid

- The maximum velocity will be for the fluid at the central axis of the tube, v_{max} and the minimum velocity will be for the layer adjacent to the wall, $v_{min} = 0$.
- So the average velocity, $\bar{v}$, is half the maximum velocity at the center of the tube is

$$\bar{v} = \frac{1}{2} v_{\max}$$

- From the continuity equation, the flow rate then

$$Q = A\bar{v} = \frac{1}{2} A v_{\max}$$

- The pressure drop $\Delta p = p_2 - p_1$ along the tube of length L is directly proportional to the average velocity of flow and to the length L of the tube. **Then the average velocity of flow and the flow rate of the fluid are proportional to the pressure gradient,** $\Delta P / L$.

- Another factor affect the average velocity of flow is the radius of the tube and the coefficient of viscosity of the moving fluid. In general we can write such proportionalities as:

$$\bar{v} \propto \frac{\Delta p}{L} R^m \eta^n$$

- Use the dimensional analysis method to find the values of the unknown powers **m** and **n**.
- So,

$$[\bar{v}] = \left[\frac{\Delta p}{L} R^m \eta^n\right]$$

$$L\,T^{-1} = \frac{ML^{-1}T^{-2}}{L} L^m \left(M\,L^{-1}T^{-1}\right)^n = M^{n+1}L^{-2+m-n}T^{-2-n}$$

- it is found $n=-1$ and $m=2$. Thus the formula for the average velocity becomes:

$$\bar{v} \propto \frac{\Delta p}{L} R^2 \eta^{-1} = constant. \frac{\Delta p R^2}{\eta L}$$

- From the mathematical calculation, we have the same formula, where the value of the constant is $\mathbf{1/8}$

- the average velocity and the flow rate of laminar flow of a fluid through a tube is given by:

$$\bar{v} = \frac{\Delta p R^2}{8 \eta L}$$

$$Q = A\bar{v} = \pi R^2 \bar{v} = \frac{\pi \Delta p R^4}{8 \eta L}$$

Poiseuille law

Poiseuille law

- It indicates that **high viscosity** leads to **low flow rate** and **low speed of flow**, It also shows that the flow rate is proportional to the 4th power of **R**.
- This indicates that for blood vessel, any small change in the radius of the vessel results in a considerable change of the flow rate.

- **For example**, if the radius of an artery is halved, so the flow rate will be reduced to

$$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

Example

- **What** is the pressure drop in the blood as it passes through a capillary 5 mm long and $3\mu m$ in radius if the speed of the blood at the center of the capillary is 0.6 ms^{-1}. Take the viscosity of blood to be 2.08×10^{-3} Pa.s. If the radius of the capillary is reduced by 20 %, **find** the change in the flow rate?

Solution

- The speed of blood at the center of the capillary is the maximum speed of flow,

$$v_{max} = 2\bar{v} = 0.6\, m/s$$

So that

$$\bar{\mathbf{v}} = 3\times10^{-1} = \frac{\Delta p\, R^2}{8\,\eta\, L},$$

We have $R = 3\times10^{-6}\, m$, $L=5\times10^{-3}\, m$ and $\eta = 2.08\times10^{-3}$ Pa.s,

Substitute to get

$$\Delta p = \frac{8\bar{v}\eta L}{R^2} = \frac{8\times3\times10^{-1}\times2.08\times10^{-3}\times5\times10^{-3}}{(3\times10^{-6})^2} = 2.77\times10^{6}\, Pa$$

The flow rate is proportional R^4, when $\mathbf{R_2 = 0.8\, R_1}$

So

$$\mathbf{Q_2 = (0.8)^4\, Q_1 = 0.407\, Q_1}$$

Selected Topics in Physics for Medical Sciences Students

Unit 5

Fluid Mechanics

Lecture 21

Power dissipation & Flow Resistance & Turbulent Flow

Dr. Hazem Falah Sakeek
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Contents

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We will study the properties of the fluids and their behavior. Along with the flow of the fluids for nonviscous and viscous fluids.

Power dissipation

- The power dissipated during the flow of a fluid is the rate of energy required to maintain the flow.
- the power is defined as the net force **F** times the average speed,

$$\mathbb{p} = F\,\bar{v}.$$

- But the force on a segment is the pressure drop times the cross sectional area,

$$F = \Delta p A$$

- Thus the power is given as:

$$\mathbb{p} = \Delta \boldsymbol{p}\, A\, \bar{v} = \Delta \boldsymbol{p} Q$$

Where Q is the flow rate measured in $\boldsymbol{m^3 s^{-1}}$ and $\Delta p = (p_2 - p_1)$ is the pressure difference measured in N/m^2, so the unit of the power is

$$\frac{N}{m^2}\frac{m^3}{s} = \frac{N.m}{s} = \frac{J}{s} = Watt$$

Example

Determine the power dissipated to maintain the flow of blood in the capillary as described in the last example?

Solution

- From the data given in the last example, the power is given as:

$$\mathbb{p} = \Delta p A\,\bar{v} = \pi R^2 \bar{v}\,\Delta p$$

$$= 3.14\left(3\times10^{-6}\right)^2(0.3)\left(1.15\times10^{6}\right)$$

$$= 9.75\times10^{-6} W$$

Flow Resistance

- The viscosity of fluids is defined as the flow resistance, which is originated from the frictional forces inside the fluid.
- The flow resistance can be defined in general as the ratio of the pressure drop through a segment and the flow rate.

(notice the analogy to electric resistance, the pressure difference like the potential difference, and the flow rate like the electric current)

$$\mathcal{R}_f = \frac{\Delta p}{Q}$$

When the flow is Laminar, and from Poiseuille's equation, we get:

$$\mathcal{R}_f = \frac{\Delta p}{\pi \Delta p R^4/8\eta L} = \frac{8\eta L}{\pi R^4}$$

The unit of $\mathcal{R}_f$ is, $\frac{Pa}{m^3/s} = Pa.s/m^3$ which is the unit of the viscosity per volume

- We can define the flow resistance as the viscosity density.
- It is observed that the flow resistance is directly proportional with the coefficient of viscosity, and inversely proportional with the 4th power of the radius of the tube. This means that most of flow resistances and pressure drops occur in smaller arteries and vascular beds of the body.

Example

Compare the flow resistance in a capillary of 5 μm in radius and that in an artery of 5 cm in radius?

solution

Since the flow resistance is inversely proportional to the 4th power of the radius, so that:

$$\frac{\mathcal{R}_{f capillary}}{\mathcal{R}_{f artery}} = \frac{R^4_{artery}}{R^4_{capillary}} = \frac{(5\times10^{-2})^4}{(5\times10^{-6})^4} = 10^{16}$$

The flow resistance in small capillaries is much larger than that of the arteries.

From the definition of the flow resistance, the power dissipation can be expressed in terms of $\mathcal{R}_f$ as:

$$\mathbb{p} = \Delta p Q = \mathcal{R}_f Q^2 = \frac{\Delta p^2}{\mathcal{R}_f}$$

Example

- A large artery has an inner radius of 4 mm. Blood flows through the artery at the rate of 1 $cm^3 s^{-1}$.

Find

a) The average and maximum speed of the blood in the artery
b) The pressure drop in a 10 cm long segment of the artery
c) The flow resistance of blood over the 10 cm segment
d) The power dissipated through the flow

Solution

The artery has a radius $R = 4\times10^{-3}\ m$. The flow rate of blood is $\boldsymbol{Q} = 1\times10^{-6} m^3/s$

a) the average velocity can be found from continuity equation as:

$$\bar{v} = \frac{Q}{\pi R^2} = \frac{1 \times 10^{-6}}{3.14\,(4 \times 10^{-3})^2} = 0.02\; ms^{-1}$$

So the maximum speed, $v_{max} = 2\,\bar{v} = 0.04\ ms^{-1}$

b) For a segment of the artery $L = 0.1\ m$, and take viscosity coefficient of blood to be

$$\eta = 2.08 \times 10^{-3} Pa.s$$

Using the equation

$$\bar{v} = \frac{\Delta p\, R^2}{8\, \eta\, L}$$

We can write

$$\Delta p = \frac{8\eta L\bar{v}}{R^2} = \frac{8\,(2.08 \times 10^{-3})(0.1)(0.02)}{(4 \times 10^{-3})^2} = 2.08\; Pa$$

c) The flow resistance is given by:

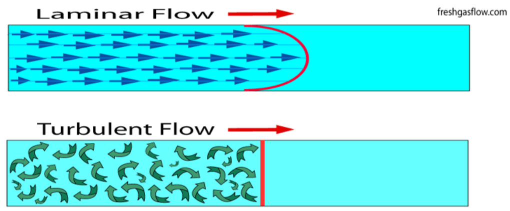
$$\mathcal{R}_f = \frac{\Delta p}{Q} = \frac{2.08}{1 \times 10^{-6}} = 2.08 \times 10^{6}\; Pa.s/m^3$$

d) The power dissipation through the flow is

$$\mathbb{p} = \Delta p Q = 2.08 \times 1 \times 10^{-6} = 2.08\; \mu W$$

Turbulent Flow

- We learned that when flow speed becomes high the streamlines of the flow start to intersect each other; and this flow is called turbulent flow.



- In turbulent flow, the mechanical energy dissipated is much larger than that in the laminar flow, so that it is often desirable to ensure that the flow does not become turbulent.

- Poiseuille's law is applicable for laminar flow, so it is necessary to determine whether the flow is laminar or turbulent. There is a dimensionless quantity called Reynolds Number ($\mathbf{N_R}$) used to distinguish the type of the flow.
- Consider a fluid of density $\boldsymbol{\rho}$ and viscosity coefficient $\boldsymbol{\eta}$ flows with an average velocity $\bar{v}$ through a tube of radius $\mathbf{R}$, hence the Reynolds number is defined by

$$N_R = \frac{2\rho\bar{v}R}{\eta}$$

It is found experimentally that if

$N_R < 2000$	*flow is laminar*
$N_R > 3000$	*flow **is turbulent***
$\mathbf{2000 < N_R < 3000}$	***flow is unstable***

Example

The flow rate of blood in a blood vessel of 2 cm in diameter is 1 Liter per minute, Determine whether the flow is laminar or turbulent, if the density of blood is 1060 kgm^{-3} and the coefficient of viscosity of the blood is 2.1x10^{-3} Pa.s

Solution

The flow rate of the blood

$$Q = \pi R^2 \bar{v} = \frac{1\times10^{-3}}{60} = 1.66 \times 10^{-5}\ m^3/s$$

so that

$$\bar{v} = \frac{Q}{\pi R^2} = \frac{1.66 \times 10^{-5}}{3.14\ (1 \times 10^{-2})^2} = 0.053\ ms^{-1}$$

The Reynolds number

$$N_R = \frac{2\rho\bar{v}R}{\eta} = \frac{2\ (1060)\ 0.053(1 \times 10^{-2})}{2.1 \times 10^{-3}} = 535$$

N_R<2000, *so the flow is laminar*

Example

A small human capillary of 100 µm radius has a length of 2 cm, **calculate**

a) The blood flow resistance across this capillary
b) If the pressure drop across the capillary is 2.3 kPa, what is the flow rate
c) What is the maximum speed of blood through the capillary
d) The power dissipated across the capillary
e) Reynolds's number and then determine the type of flow.

Solution

a) Take the viscosity of blood to be η = 2.1x10-3 Pa.s, the length of the capillary L =2x10^{-2}m and its radius R = 10^{-4}m

$$R_f = \frac{8\,\eta L}{\pi R^4} = \frac{8 \times 2.1 \times 10^{-3} \times 0.02}{3.14 \times 10^{-16}} = 1.07 \times 10^{12}\ Pa.sm^{-3}$$

b) If $\Delta p = 2.3\ kPa$, so $Q = \frac{\Delta p}{R_f} = \frac{2.3\times10^3}{1.07\times10^{12}} = 2.15 \times 10^{-9}\,m^3/s$

c) The flow rate is defined as $Q = \pi R^2 \bar{v} = \frac{1}{2}\pi R^2 v_{max}$

so the maximum speed

$$v_{max} = \frac{2\,Q}{\pi R^2} = \frac{2\,(2.15 \times 10^{-9})}{3.14 \times 10^{-8}} = 0.136\ m/s$$

d) The power dissipation is given as:

$$\mathbb{P} = Q\Delta p = 2.15 \times 10^{-9} \times 2.3 \times 10^3 = 4.95\ \mu W$$

e) Reynolds's number

$$N_R = \frac{2\rho\bar{v}R}{\eta} = \frac{2\,(1060)\,0.136/2 \times (100 \times 10^{-6})}{2.1 \times 10^{-3}} = 50.5 < 2000$$

So, the flow is laminar

Chapter 5 Fluid Mechanics: Problems

1. Calculate the flow rate in an aorta with a cross sectional area of 2 cm^2. if the flow speed is 40 cm/s.

2. A needle of radius 0.3 mm and length 3 cm is used to give a patient a blood transfusion. Assume the pressure differential across the needle is achieved by elevating the blood 1 m above the patient´s arm. a) What is the rate of flow of blood through the needle? b) At this rate of flow, how long it take to inject 500cm^3 of blood into patient?

3. If an elevator accelerates upward at 10 ms^{-2}, what is the average blood pressure in the brain? What is the average blood pressure in the feet? If the elevator accelerates downward with the same acceleration, what is the average blood pressure in the brain and feet? *take g =10 ms^{-2}.

4. The aorta in humans has a diameter of about 2 cm and at certain times, the blood speed through it is about 55 cm /s. What is the type of flow. Find also the flow resistance over 30 cm long segment and the power for dissipating the blood through this segment.

5. A standing 1.8 m tall person in an elevator accelerates downward with 1.5 g. The height difference between the brain and heart is 40 cm. Find the pressure at the brain and foot if the pressure at the heart is 13.3 kPa. If this person is suddenly bending so his brain is 30 cm below his heart, calculate the change of the pressure at the brain.

6. An artery has an inner radius of 2 mm. If the flow is laminar and the average flow velocity is 0.03 ms^{-1}, What is the (a)maximum velocity (b) the flow rate, (c) and the pressure drop in 0.05 m, if the artery is horizontal?

7. The pressure drop along a length of a horizontal artery is 100 Pa. The radius of the artery is 10 cm, and the flow is laminar. (a) what is the net force on blood in this portion of the artery? (b) If the average speed of blood is 0.015 ms^{-1}, , find the power expended in maintaining the flow?

8. The radius of an artery is increased by a factor of 1.5 (a) The pressure drop remains the same, what happens to the flow rate? (b) If the flow rate stays the same, what happens to the pressure drop? (Assume laminar flow).

9. A small artery has a length of 0.11 cm and a radius of 25 µm. (a) calculate its resistance.(b) If the pressure drop across the artery is 1.3 kPa, what is the flow rate?

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Selected Topics in Physics for Medical Sciences Students

Unit 6

Bioelectricity "Signals in the Body"

Lecture 22

RC circuit

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Bioelectricity

- Bioelectricity refers to electrical potentials and currents occurring within or produced by living organisms. It results from the conversion of chemical energy into electrical energy.
- Bioelectric potentials are generated by a number of different biological processes, and are used by cells to govern metabolism, to conduct impulses along nerve fibers, and to regulate muscular contraction. In most organisms bioelectric potentials vary in strength from one to several hundred millivolts.

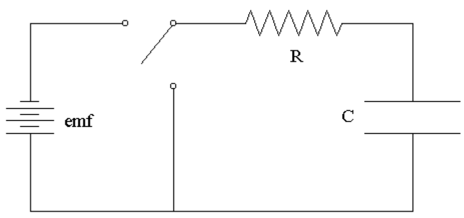
The most important difference between bioelectric currents in living organisms and the type of electric current used to produce light, heat, or power is that a bioelectrical current is a flow of ions (atoms or molecules carrying an electric charge), while standard electricity is a movement of electrons.

Electrostatics

- Electric Force
- Electric field
- Electrical Potential
- Electric Current
- Electric Resistance
- Electric Power
- Electromotive Force
- The Capacitor

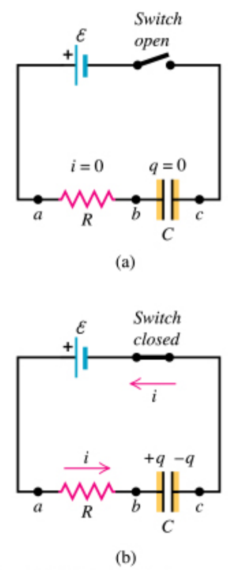
RC circuit

- In circuits containing capacitors, the current may vary in time.
- A circuit containing a series combination of a resistor and a capacitor is called an RC circuit.



Charging a Capacitor

- Assume the capacitor in is initially uncharged.
- There is no current while switch S is open.
- If the switch is closed, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge.
- During charging, charges do not jump across the capacitor plates because the gap between the plates, until the capacitor is fully charged.
- As the plates become charged, the potential difference across the capacitor is increased. The value of the maximum charge depends on the voltage of the battery.



- Once the maximum charge is reached, the current in the circuit is zero because the potential difference across the capacitor matches that supplied by the battery.
- During the charging process the battery voltage is divided between the Capacitor C and the resistor R. The voltages across the resistor and the Capacitor change with time, so we can write,

$$\mathcal{E} = V_C(t) + V_R(t)$$

- the voltages is time dependent, which can be rewritten as

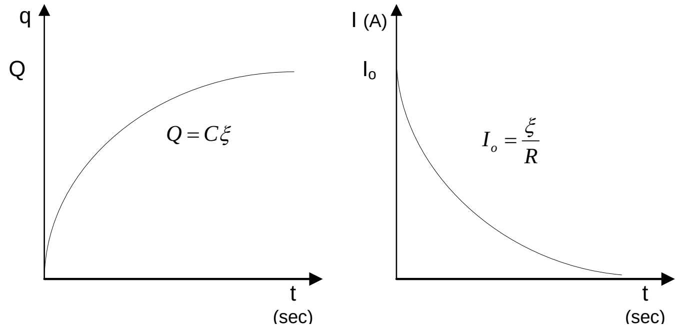
$$\varepsilon = \frac{q(t)}{c} + I(t)R$$

- Solving this equation for $q\ (t)$,

$$q(t) = Q(1 - e^{-t/RC}) \qquad Q = C\mathcal{E}$$

- The current I is

$$I = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$



Plots of the charge Q and the current I as a function of time in the ***charging process***

- Note that the quantity *RC* in the equation has a unit of time (sec). Therefore it is called the *time constant* of the circuit.
- Unit of *RC* is *Ohm . Farad=Sec*

$$Ohm.Farad = Ohm.\frac{Coulomb}{Volt} = Ohm.\frac{Amp.Sec}{Volt} = \frac{Volt.Sec}{Volt} = Sec$$

Example

- An uncharged capacitor and a resistor are connected in series to a battery. If $\mathcal{E}$=12.0volt,C=5.0μm, and R=0.8 $M\Omega$, find the

  time constant of the circuit,

  the maximum charge on the capacitor,

  the maximum current in the circuit,

  the charge and current as functions of time

Solution

- The time constant

$$\tau = RC = 0.8 \times 10^{6} \times 5.0 \times 10^{-6} = 4sec$$

- The maximum charge

$$Q = C\mathcal{E} = 5.0 \times 10^{-6} \times 12.0 = 60\mu C$$

- The maximum charging current

$$I_0 = \frac{\mathcal{E}}{R} = \frac{12}{0.8 \times 10^{6}} = 15\mu A$$

- Charge and current as function of time

$$q(t) = Q\left(1 - e^{-t/RC}\right) \quad and \;\; I(t) = I_0 e^{-t/RC}$$

$$q(t) = 60\mu C\left(1 - e^{-t/4}\right) \; and \;\; I(t) = 15\mu A e^{-t/4}$$

Example

- An uncharged capacitor and a resistor are connected in series to a battery. If $\mathcal{E}$=12.0volt, C=3.0μm, and R=1 $M\Omega$. Find how long it will take before the capacitor receives 99% of its final charge.

Solution

- The time constant is

$$\tau = RC = 1.0 \times 10^{6} \times 3.0 \times 10^{-6} = 3sec$$

$$q(t) = Q\left(1 - e^{-t/RC}\right)$$

$$\frac{q(t)}{Q} = 0.99 = \left(1 - e^{-t/3}\right)$$

- Now take the natural logarithm for both side and solve for t, we will find $t = 13.8\ sec$

Example

- A resistance R and a capacitor are connected in series across a 200 Volt source. Across the capacitor is a neon lamp that strikes (glows) at 120 volt. Calculate the value of R to make the lamp glows after 3.0 seconds after the switch has been closed.

Solution

- This means that the voltage across the capacitor has to rise to 120 Volt is 5 seconds,

$$V_c(t) = \mathcal{E}(1 - e^{-t/RC})$$

$$120 = 200(1 - e^{-5/RC})$$

- Now solve for RC, we find

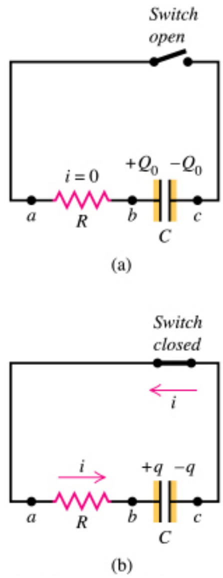
$$RC = 5.464\ sec \longrightarrow R = 5.464 sec/10\mu F$$

$$R = 1.366 M\Omega$$

Discharging a Capacitor

- When the switch is open, a potential difference Q/C exists across the capacitor and there is zero potential difference across the resistor because I=0A.
- If the switch is closed at t=0 the capacitor begins to discharge through the resistor.
- At some time t during the discharge, the current in the circuit is I and the charge on the capacitor is q.
- The loop equation for the circuit

ε=0, no battery → $$0 = \frac{q(t)}{c} + I(t)R$$



- the current through the resistance R

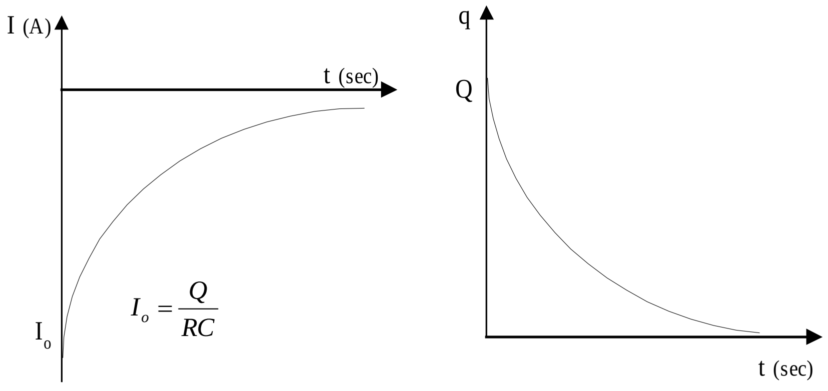
$$q(t) = Qe^{-t/RC}$$

- The current I during the discharging process is

$$I = \frac{dq(t)}{dt} = -\frac{Q}{RC}e^{-t/RC} = -I_o\frac{Q}{RC}e^{-t/RC}$$

- $I_0 = Q/RC$ is the initial current

- The negative sign indicates that discharging current direction is opposite to that of charging.



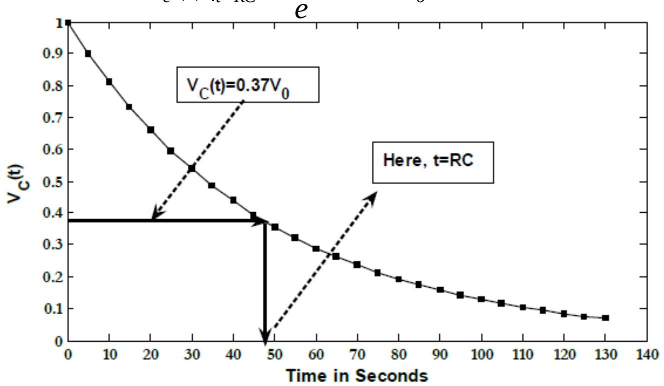
Plots of the charge Q and the current I as a function of time in the ***discharging process***

- the voltage across a capacitor with time during discharging:

$$V_c(t) = V_o e^{-t/RC}$$

- At $t = RC$ the potential difference across the capacitor equals to

$$V_c(t)|_{t=RC} = \frac{V_o}{e} = 0.37V_o$$



The Voltage across a discharging Capacitor with timeTime

Example

- Consider a capacitor of capacitance C that is being discharged through a resistor of resistance R. After how many time constants is the charge on the capacitor one-fourth its initial value?

Solution

- We need to find the time required for the capacitor's charge to drop to its one fourth, that is

$$q\,(t) = Q/4$$

$$q(t) = Qe^{-t/\tau} \rightarrow Q/4 = Qe^{-t/\tau} \rightarrow 1/4 = e^{-t/\tau}$$

Now solve for t in terms of $\tau = RC$ (the time constant),

$$t = 1.39\,\tau$$

Example

- A 20μF capacitor initially charged to potential difference of 500 Volt is discharged through an unknown resistance. After 60 seconds, the potential difference at the capacitor terminal is 185 volt. What is the magnitude of the resistance?

Solution

- The voltage across the capacitor as a function of time is given by,

$$V_c(t) = V_o e^{-t/RC}$$

- Since the ratio 185/500=0.37. This means the Voltage drops after a time equals one time constant, that is τ=60 s
- Thus

$$R \times 20\mu F = 60 \rightarrow R = \frac{60sec}{20\mu F} = 3M\Omega$$

Example

- A 0.1μF capacitor initially charged to potential difference of 100 Volt is discharged through 1$M\Omega$ resistance. Find the following: Initial value of the discharged current; its value 0.1 second later; charge of the capacitor plates after 0.1 second.

Solution

- The initial discharge current, at t=0, is given by

$$I_0 = \frac{V}{R} = \frac{100}{1M\Omega} = 100\mu A$$

- Check the RC circuit time constant

$$\tau = RC = 0.1\mu F \times 1M\Omega = 0.1 \text{ sec}$$

- That means the current, charge, and voltage will drop to 0.37 of its initial value.
- The current after 0.1 sec is equal to

$$I = 0.37I_0 = 0.37 \times 100\mu A = 37\mu A$$

- The charge after 0.1 sec is also 0.37 of its initial value, that is

$$Q = C\mathcal{E} = 01.\mu F \times 100Volt = 10\mu C$$

$$q = 0.37Q = 0.37 \times 10\mu C = 3.7\mu C$$

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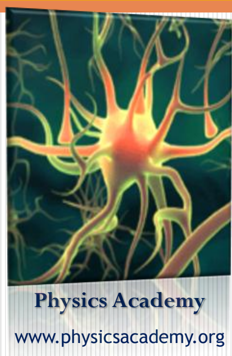

Unit 6

Bioelectricity "Signals in the Body"

Lecture 23

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Electrostatics

- Electric Force
- Electric field
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Bioelectricity

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The most important difference between bioelectric currents in living organisms and the type of electric current used to produce light, heat, or power is that a bioelectrical current is a flow of ions (atoms or molecules carrying an electric charge), while standard electricity is a movement of electrons.

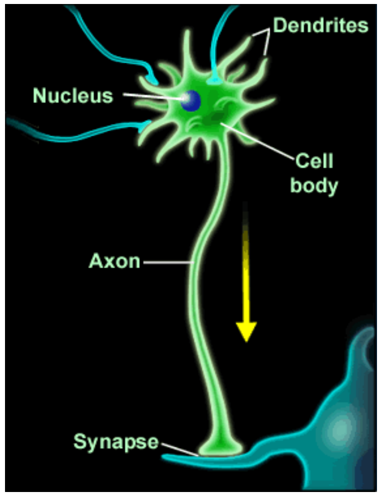
Nerve Conduction

- The human nervous system contains roughly 100 billion neurons, connected in networks that transmit information from one location in the body to another.
- The human nervous system consisting of the **brain** and **spinal cord**, the central nervous system interprets sensory input, initiates muscle contraction, and carries out all other cognitive tasks. The nerves that communicate messages between the central nervous system and the rest of the body compose the peripheral nervous system.
- The neurons send information to one another **via electrical signals**, we can treat them like classical electrical circuits.

In this topic we will review basic concepts in neurobiology and then describe the circuit model.

Neurobiology Review

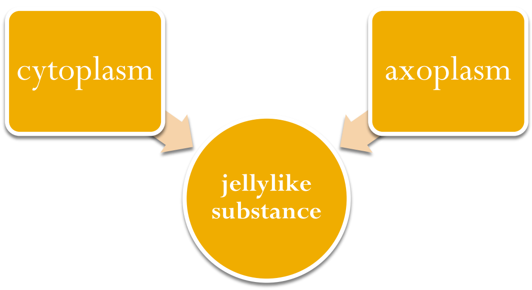
- Neurons can be divided into three main parts: the soma, or cell body, which contains the nucleus and other organelles, and two types of fiber-like extensions called dendrites and axons.
- Dendrites receive inputs from other cells and conduct signals towards the cell body.
- Axons conduct signals away from the cell body towards their tips, where they are then passed on to other neurons or to muscle cells.
- A neuron may have many dendrites but usually has only one axon, which can be as long as 1 m.
- The junction between the axon of one neuron and the dendrite or cell body of another is called the synapse.



Dendrites: receive messages,
Axon: sends messages

Resting and Acting Potential

- Enclosed in the membrane of any cell is a jellylike substance that contains both inorganic and organic matter.
- In the cell body, this substance is called cytoplasm, but in the axon it is called axoplasm.



- For an inactive neuron, the axoplasm has an overall negative charge.

- The resting potential is originated from the differences in concentration of ions inside and outside the membrane.
- The resting potential depends also on the difference in permeability of the cell wall to the different ions.
- Two types of positively-charged ions, potassium (K+) and sodium (Na+), can cross the cell membrane through selective ion channels. Normally there are more potassium ions inside the cell than outside, whereas there are more sodium ions outside the cell than inside.
- Due to these ionic effects, the resting potential of the axoplasm is about –90 mV relative to the extracellular fluid.
- Because the resting potential is negative, the electrical field is directed from outside to inside.

- Therefore, the electric field drives additional ions through the membrane into the cell.
- The migration of the ions cause a potential difference, V, leading to the formation of a potential hill against the movement of extra ions till equilibrium takes place.
- **When the thermal energy of the ions equal the height of the potential hill, then the potential difference across the membrane is given by Nernst equation:**

$$V = \pm 2.3 \frac{k_B T}{e} \log \left(\frac{C_o}{C_i} \right)$$

- Where $k_B T$ is the thermal energy, e is the electronic charge, C_o and C_i is the concentration of the ions outside and inside the axon respectively.

- For example T = 37°C = 310K, so

$$\frac{k_B T}{e} = \frac{(1.38\times10^{-23} J/K)(310\,K)}{1.6\times10^{-19}\,C} = 0.0267\,V = 26.7\,mV$$

- Then equation can be written as:

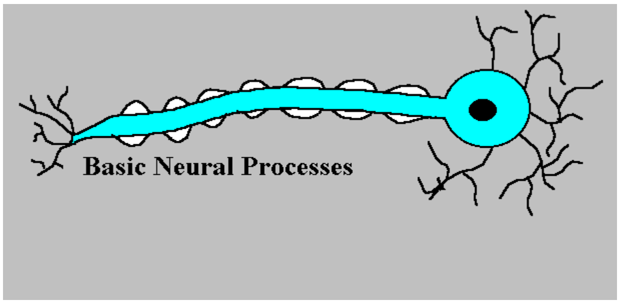
$$V = \pm 61.4\,mV \log\left(\frac{C_o}{C_i}\right)$$

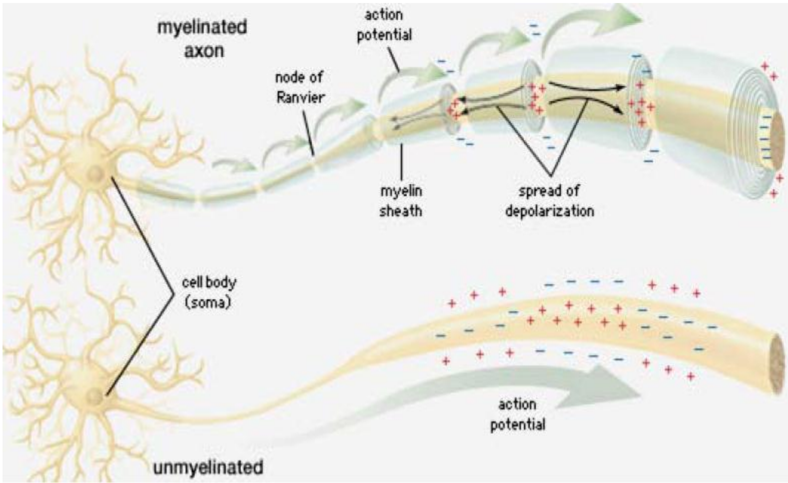
- For a nerve cell, the intercellular has K+ concentration, C_i = 0.141 mol/liter, whereas the extracellular fluid has a K+ concentration, C_o = 0.005 mol/liter, which give the Nernst potential to be:

$$V = +61.4\,mV \log\left(\frac{0.005}{0.141}\right) = -89.2\,mV$$

- In practice, the actual resting potential measured by the electrophysiological units is about -85 mV.

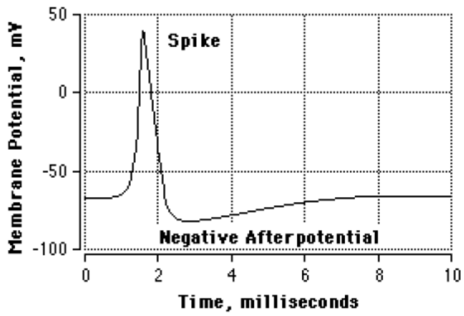
- Excitable cells have special Na and K channels with gates that open and close in response to the membrane voltage (voltage-gated channels).
- Between Schwann cells are small regions of exposed axon called nodes of Ranvier.
- These nodes contain the voltage-gated ion channels that allow action potentials to propagate down the axon, so that the signal jumps from node to node as shown in the figure





Structure of a neuron with myelinated axon and unmyelinated axon

- When a nerve cell receives a stimulating action such as electric signal, thermal mechanical or chemical, **its membrane suddenly become permeable to the Na+.**
- This will permit the Na+ to diffuse through the cell membrane to the interior and combine with the Cl-. Thus membrane potential will depolarize (becomes more positive) producing spike as shown in the figure. The rate of flow through the cellular membrane increases rapidly to 100 times that of K+ after short time ~ 2ms.

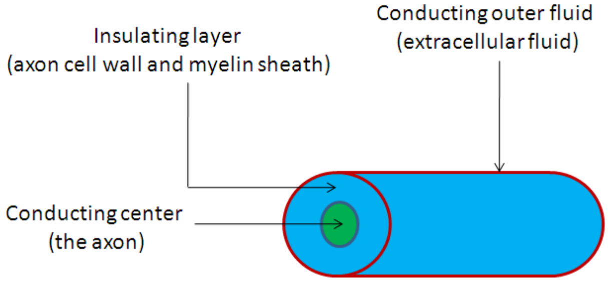


The depolarization and repolarization process resulted from external stimulation of the nerve

- During this period the interior potential changes from – 90 mV to 35 mV.
- After this period, the sodium gate is closed, where the membrane become impermeable to sodium ions and in the same time K+ gate is opened diffusing it out of the cell.
- This makes the membrane repolarized (becomes more negative) till it returns to its normal state.
- This change of the cell potential is called the action potential.

Electrical Properties of Neurons

- **Axon acts** as cables that transmit bioelectric impulses from one nerve cell to another **cells** or the **central nervous system**.
- **A nerve fiber** consists of a thin hollow tube containing positive and negative ions.
- **The fiber** is immersed in an extracellular fluid which contains the same ionic composition.



Axon as an insulated wire

- **The wall** of the axon tube is semi-permeable membrane which, although a dielectric, allows ions to migrate into and out of the fiber.

- Examination of the axons of various neurons with an electron microscopy indicates that there are two different types of nerve fibers.
  - **The membrane of some axons** are covered with a fatty insulating layer called myelin (النخاع) and they are called myelinated axons (MA).
  - **The membrane of other axons** has no myelin sheath and they are called unmyelinated axons (UA).

Resistance

- In the neuron, there are two substances that exhibit electrical resistance:
  - the axoplasm
  - the cell membrane plus myelin sheath, if present.

Axoplasm Resistance

- **The electrical resistance R along the length of the axon follows the same principles as a wire:**

$$R = \rho_a \frac{l}{\pi\, r^2}$$

- For both myelinated and unmyelinated neurons, the resistivity ρ_a of the axoplasm is 2.0 Ω.m.

- If the average neuron has an axon 1 mm long and a 5μm radius, we can find that the resistance of the axoplasm $R_{axoplasm}$ = 2.5x10^7 Ω. This huge value indicates that axons are actually poor electrical conductors.

Cell Membrane Resistance

- The cell membrane is also permeable to charge; its resistance is not infinite, even when myelinated. the resistance through the membrane (R_m) depends on the surface area of the axonal membrane 2πrl:

$$R_m = \frac{\rho_m}{2\pi\, r\, l}$$

Where ρ_m is the membrane resistivity measured in $\Omega.m^2$.

- For an unmyelinated axon (UA), ρ_{mUA}= 0.2 $\Omega.m^2$. So, again for an average axon 1 mm long with radius 5 μm, so R= 6.4x10^6 Ω.
- For Myelinated axons (MA) have a much higher resistivity, ρ_{mMA}= 40 $\Omega.m^2$, so R= 1.3x10^7 Ω.

The membrane resistance R_m represents the resistance of the leakage current.

- There is an operator called the space parameter λ, which represent how can the impulse signal travel along the axon before the leakage is complete.
- This occurs when the axon resistance R is equal to the membrane resistance R. So that at $R = R_m$, then put $l = \lambda$, which gives us:

$$\frac{\rho_a \lambda}{\pi r^2} = \frac{\rho_m}{2\pi r \lambda}$$

- So we can solve for λ, which gives us

$$\lambda = \sqrt{\frac{\rho_m r}{2\rho_a}}$$

The space parameter ranges from 0.05 to 0.7 cm for unmyelinated to myelinated axons

Example

A myelinated axon has a space parameter of 1cm, find its radius? ρ_{mMA}= 40 $\Omega.m^2$ and ρ_a= 2 $\Omega.m$.

Solution

From equation,

$$\lambda = \sqrt{\frac{\rho_m r}{2\rho_a}}$$

we can write r in terms of the space parameter as:

$$r = \frac{2\rho_a \lambda^2}{\rho_m} = \frac{2\,(2\Omega.m).10^{-4}m^2}{40\ \Omega.m^2} = 10^{-5}m = 10\mu m$$

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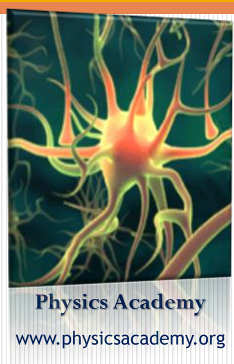

Unit 6
Bioelectricity
"Signals in the Body"

Lecture 24

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Capacitance

- The capacitor is an electrical device that stores electric charge.
- It consists of two conductors side by side, separated by some insulating substance called the dielectric.
- The Capacitance of parallel plate capacitor is given by

$$C = \frac{\kappa \varepsilon_o A}{d} = \frac{\varepsilon A}{d}$$

where $\varepsilon = \kappa \varepsilon_o$

The larger the surface area between the plates the more charge can be stored. Furthermore, the smaller plate separation the greater the attraction between the charges, which also increases the capacity for charge storage.

- For a lipid bilayer, $\varepsilon = 5\text{x}10^{-11}\ F/m$ and $d = 50\ \text{Å} = 5\text{x}10^{-9}\ mm$. Thus, the capacitance per unit area for an unmyelinated axon of 5 nm thickness is:

$$C_m = \frac{C}{A} = \frac{\epsilon}{d} = \frac{5\times10^{-11}}{5\times10^{-9}} = 10^{-2} F/m^2 \text{(Unmyelinated axon)}$$

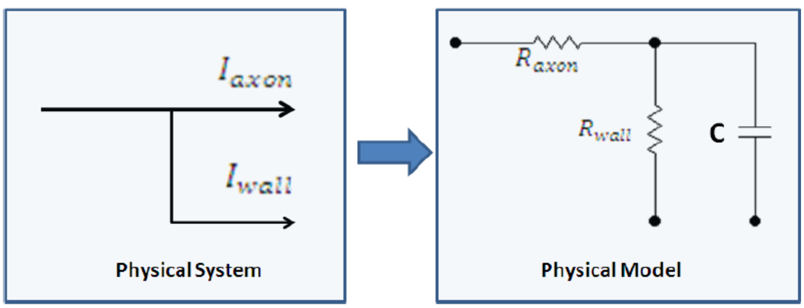
- For myelinated axons, the myelin sheath contains a membrane that wraps around the axon a couple of hundred times. This multilayer arrangement effectively increases the thickness of the lipid bilayer by a factor of 200 (1 μm total thickness),

so capacitance per unit area for a myelinated axon is:

$$C_m = \frac{C}{A} = \frac{\epsilon}{d} = \frac{5\times10^{-11}}{1\times10^{-6}} = 5\times10^{-5} F/m^2 \text{(Myelinated axon)}$$

Neuron's Equivalent Circuit

- The electrical properties of neurons are summarized by the diagram,



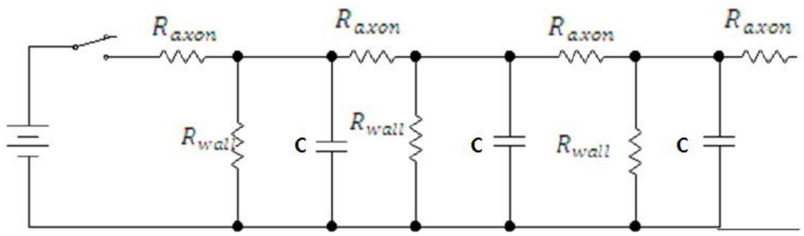
The physical model shows wires, two resistors, and a capacitor that approximate the physical flow of charge through real axons

Useful Constants

	Unmyelinated axons (UA)	Myelinated axons (MA)
Axoplasm resistivity	$\rho_{axoplasm} = 2.0\ \Omega.m$	$\rho_{axoplasm} = 2.0\ \Omega.m$
Wall resistivity	$\rho_{UA} = 0.20\ \Omega.m^2$	$\rho_{MA} = 40.0\ \Omega.m^2$
Wall capacitance/area	$C/A = 10^{-2}\ F/m^2$	$C/A = 5x10^{-5}\ F/m^2$

Impulse Propagation

- Neurons are connected so that action potentials travel between them in only one direction.
- The impulse propagation through the nerve cells can be modeled as an array of resistors and capacitors.

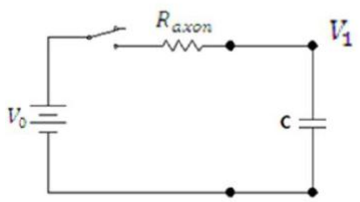


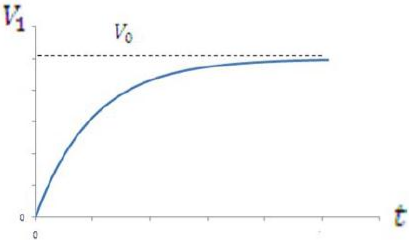
Physical model of neural connections

Propagation Speed

- For simplicity, we will at first ignore the wall resistance. For one circuit unit (i.e. one neuron), the voltage changes over time according to the equation

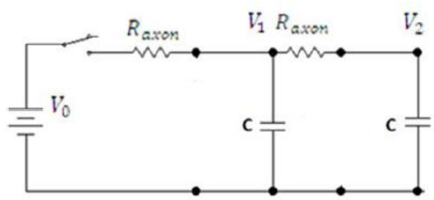
$$V(t) = V_o\left(1 - e^{-t/R_{axoplasm}C}\right)$$ where V_0 is the resting potential

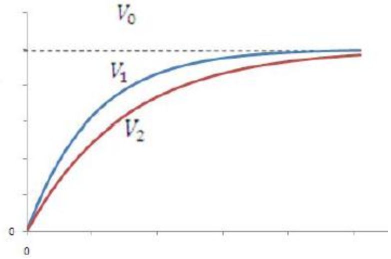




Voltage V of a single neuron as a function of time

- The charging time is $\tau = R_{axoplasm}C$.
- When we add another unit, the first unit charges up before the second unit begins to charge. With every additional unit, there is an additional delay of $\tau = R_{axoplasm}C$





The circuit with two neurons charges twice as slow as the circuit with one

- Since a unit must charge completely before it can discharge to the next unit, there is a time delay equal to τ in the propagation of the electrical signal between two units.
- If the length of each unit is x, then the speed of propagation is given by.

$$v = \frac{x}{t} = \frac{x}{R_{axon}\,C}$$

- Substituting the expressions for the resistance of the axoplasm as

$$R_a = \frac{\rho_a x}{\pi r^2}$$

- Substituting the capacitance/area of the membrane $C=C_m 2\pi rx$ into the expression for the velocity we obtain

$$v = \frac{x}{\left(\frac{\rho_a x}{\pi r^2}\right)(C_m\, 2\pi r x)} = \frac{r}{2C_m \rho_a\, x}$$

- The wider the axon, the faster the axonal speed of propagation.
- For myelinated neurons, the myelin sheath covers the axon in 1 mm long sections. Thus, within each myelinated section, one would predict that the speed of propagation is faster.
- For unmyelinated neurons,

$v=5\times10^{-6}m/(2\times2.0\Omega m\times5\times10^{-5}F/m^2\times1\times10^{-3}m) \approx 20m/sec$

the speed of propagation is 200 times slower

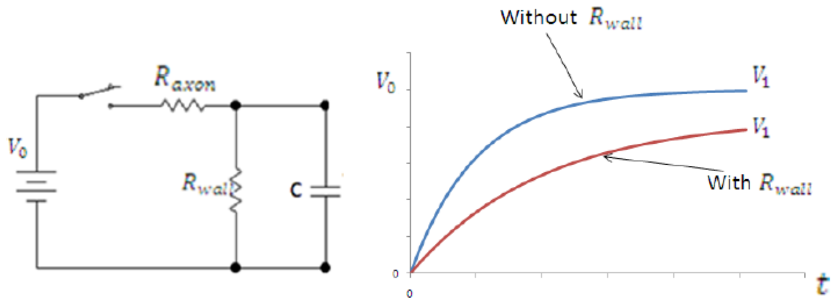
While most neurons share the same basic structure, they vary greatly in length and speed of signal propagation.

In the brain where axons are 0.01 mm, signals travel 0.5-2.0 m/s.

In the limbs axons can be up to to 1.0 m in length and carry signals at 100 m/s.

The resistance of the cell membrane

- Recall that our calculations do not take into account the resistance of the cell membrane and myelin sheath. Including it leads to leakage of the electrical signal through the wall, called signal attenuation.



The voltage is much lower if we consider the wall resistance

- the voltage decreases geometrically with the number of units traveled

$$V = V_o e^{-x/\lambda}$$

where x is the distance traveled down the circuit.

- The value of λ is 0.05 cm for unmyelinated neurons and 0.7 cm for myelinated neurons.

- Myelinated nerve fibers carry nerve impulses farther

Example

- A myelinated nerve of 10 μm thick with a space parameter of 0.5 cm. **Calculate** the speed of the pulse propagation along 1 mm. **Find** the dielectric constant of the membrane material?

Solution

$$v = \frac{r}{2C_m \rho_a\, x}$$

where for myelinated axon ρ_a=2$\Omega.m$, ρ_m =40 $\Omega.m^2$ and C_m= 5×10^{-5} F/m^2.

$$r = \frac{2\rho_a \lambda^2}{\rho_m}$$

$$v = \frac{\lambda^2}{C_m \rho_m x} = \frac{(5\times10^{-3})^2 m^2}{\left(5\times\frac{10^{-5}\,F}{m}\right)(40\,\Omega.m^2)(1\times10^{-3}m)} = 12.5\ m/s$$