* chapter 9 :

→ Momentum : الزخم

→ $m \rightarrow \vec{v} \rightarrow \vec{P} = m\vec{v}$

• $P_i = m\vec{v}_i$ • $P_f = m\vec{v}_f$

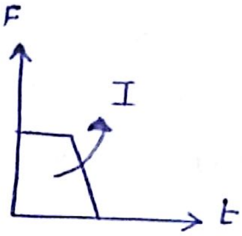
* $\vec{I} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i = m\vec{v}_f - m\vec{v}_i = F_{ext}\,\Delta t = \int_{t_1}^{t_2} F\,dt$

impulse
الدفع

له القوة الخارجية
المؤثرة على الجسم .

يستخدمه طالما يعطيني أرقام .

اقتران



P → اقتران
? F → نشتق
F → اقتران
? P → تكامل

$$\boxed{F_{ext} = \frac{dP}{dt}}$$

*Ex, :- → If $F = 4t^2 - 2t + 1$, what is the impulse from $t = 0$ to $t = 2$.

$$I = \int_0^2 (4t^2 - 2t + 1)\,dt = \frac{4}{3}t^3 - t^2 + t \Big]_0^2$$

$$= \frac{32}{3} - 4 + 2 = \left(\frac{32}{3} - 6\right) N.s .$$

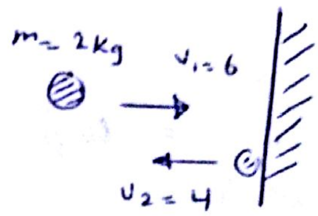
* **Ex_2:** If $p = 6t^2 - 3t + 1$ what is the force at $t = 2$.

$$F = \frac{dp}{dt} = 12t - 3$$

$$F\Big|_{t=2} = 12 \times 2 - 3 = 21 \text{ Newten.}$$

* **Ex_3:** ⓐ Find Impulse

ⓑ If the time of impact is 0.2 second what is exerted force from the wall on the ball.



ـله عبارة عن $\underline{\underline{-4\hat{i}}}$

$\boxed{a}$ $I = mv_f - mv_i$

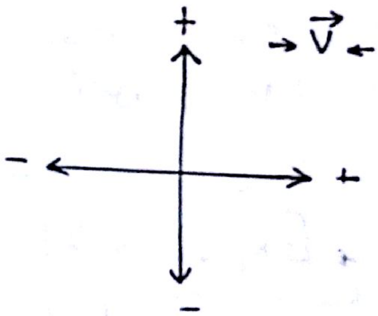
$= 2 \times 4 - 2 \times 6$

$= 8 - 12 = -4$ → X

$= 2 \times -4 - 2 \times 6$

$= -8 - 12 = -20$ Newten → ✓

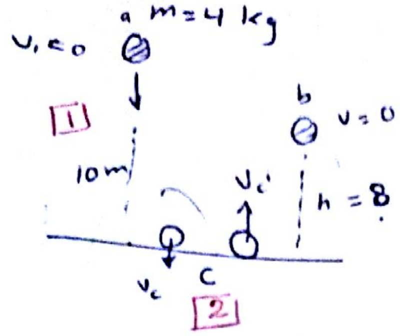
ـل دفع باتجاه اليسار .



$\boxed{b}$ $I = F \Delta t$

$$F = \frac{I}{\Delta t} = \frac{-20}{0.2} = -100 \text{ Newten.}$$

Ex₄:



$\boxed{1}$ I

$\boxed{2}$ F if $t = 0.3$

$\boxed{1}$

① $E_a = E_c$

$mgh_a = \frac{1}{2} m v_c^2$

$\cancel{4} \times 10 \times 10 = \frac{1}{2} \times \cancel{4} \times v_c^2$

$200 = v_c^2$

$v_c = 14.14 \ m/s \ (-\hat{j})$

← السرعة قبل التصادم .

② $C \rightarrow C$

قبل التصادم　　بعد التصادم

$I = m v_f - m v_i$

$= 4 v_f - 4 * (-14.14)$

$= 4 * 12.7 + 4 * 14.14$

$= 107.36 \ N.s.$

③ $c \rightarrow b$

$E_c = E_b$

$\frac{1}{2} v_{c'}^2 \cancel{m} = \cancel{m} g h_b$

$\frac{1}{2} v_{c'}^2 = 10 * 8$

$v_{c'}^2 = 160$

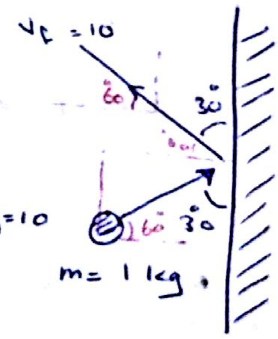
$v_c' = 12.7 \ m/s \ (+\hat{j})$

$\boxed{2}$

$I = f \Delta t$

$F = \frac{107.36}{0.3} = 357.9$ Newton $(\hat{j})$.

Ex₅: $\boxed{1}$ I　　$\boxed{2}$ If $\Delta t = 0.1$ sec what is the F.



① $\vec{I} = m (v_f - v_i)$

$= 1 \ (-5\hat{i} + \cancel{8.6\hat{j}} - 5\hat{i} - \cancel{8.6\hat{j}})$

$= -10\hat{i} \ N.S.$

$|\vec{I}| = 10 \ N.s.$

② $\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{-10\hat{i}}{0.1} = -100\hat{i}$ Newton.

$\vec{v}_i = 10 \cos 60 \ \hat{i} + 10 \sin 60$

$\vec{v}_i = 5\hat{i} + 8.6\hat{j}$

$\vec{v}_f = -10 \cos 60 \ \hat{j} + 10 \sin 60 \ \hat{j}$

$= -5\hat{i} + 8.6\hat{j}$

← تصادم جسمين معاً ←

* $F_{ext} = 0 \Rightarrow \frac{dP}{dt} = 0 \Rightarrow p = constant.$

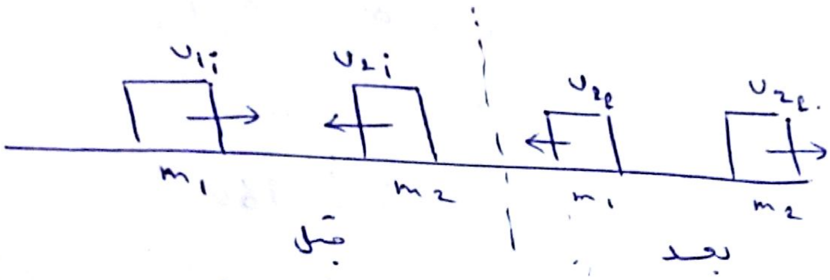
قبل التصادم $P_i = P_f$ بعد التصادم

← قانون حفظ الزخم ←

① Elastic collision تصادم مرن :- "تام المرونة"

مرن ← طاقة قبل = طاقة بعد ←

غير مرن ← طاقة قبل ≠ طاقة بعد ←



$(\vec{P}_i = \vec{P}_f)$ (نراعي الاتجاهات)

$\Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} = m_2 v_{2f}$ ①

$(K_i = K_f)$

$\Rightarrow \cancel{\frac{1}{2}} m_1 v_{1i}^2 + \cancel{\frac{1}{2}} m_2 v_{2i}^2 = \cancel{\frac{1}{2}} m_1 v_{1f}^2 + \cancel{\frac{1}{2}} m_2 v_{2f}^2$

* Lost energy throw the collision $= \Delta K = K_f - K_i = 0\,J$

← لأنه تصادم مرن

مبسط $\rightarrow v_{1i} + v_{1f} = v_{2i} + v_{2f}$ ②

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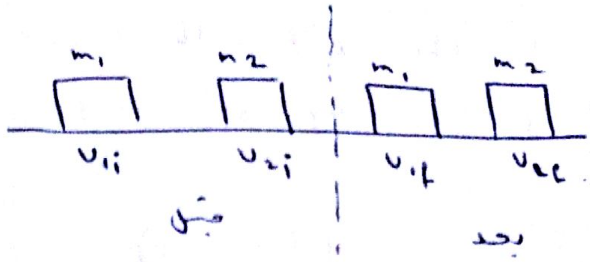
2. Non-Elastic collision : . تصادم غير-مرن

← هناك فقدان في الطاقة →



• Lost energy $= \Delta K$.

$= K_f - K_i$

$= \left(\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2\right) - \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2\right)$

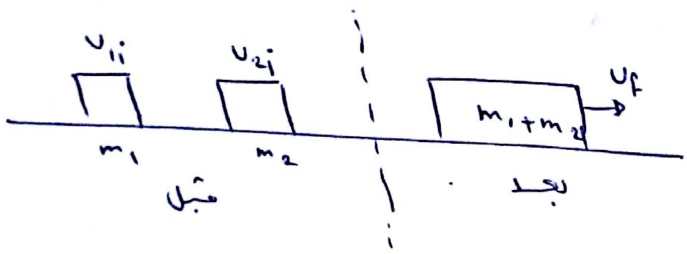
→ القوانين ← :

$(P_i = P_f)$

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

3. Completely In-elastic collision تصادم عديم المرونة :-

← طاقة غير محفوظة ← فقدان طاقة ← ΔK .



→ $\Delta K = K_f - K_i$

$= \frac{1}{2}(m_1 + m_2) v_f^2 - \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2\right)$

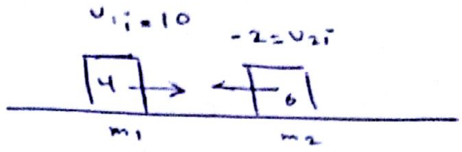
$(P_i = P_f)$

→ $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$ ←

Ex 6: Two objects of mass 4kg, 6kg, m_1 is moving. To the right with speed 10 m/s while m_2 is moving to the left with speed 2 m/s. They collides elastically find velocity of each one of them after collision.

Sol.:

$$P_i = P_f \quad \text{--- (1)}$$



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$4 \times 10 + 6 \times -2 = 4 v_{1f} + 6 v_{2f}$$

$$28 = 4 v_{1f} + 6 v_{2f}$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f} \quad \text{--- (2)}$$

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

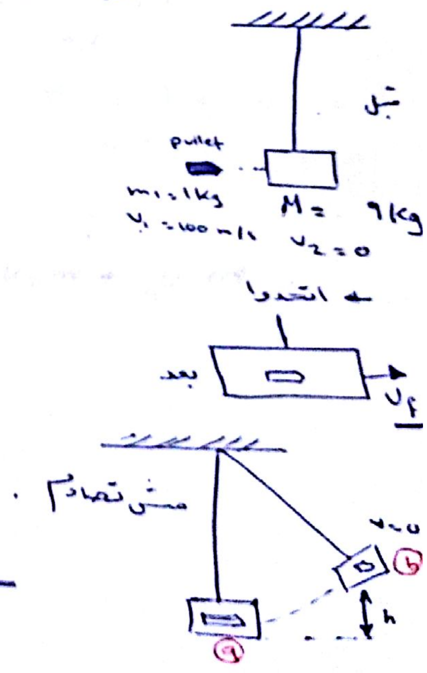
$$10 + 2 = v_{2f} - v_{1f}$$

$$12 = v_{2f} - v_{1f}$$

$\Rightarrow$ $v_{1f} = -4.4$ m/s $\qquad v_{2f} = 7.6$ m/s

→ Lost energy = 0 J ←

Ex 7: (1) find v_f



$$P_i = P_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$\cancel{9 \times 0} + 1 \times 100 = (1 + 9) v_f$$

$$\frac{100}{10} = v_f \rightarrow 10 \text{ m/s}$$

(2) find (h):

$$E_a = E_b$$

$$\cancel{mgh_a} + \frac{1}{2} m v_a^2 = mgh_b + \cancel{\frac{1}{2} m v_b^2}$$

$$\frac{1}{2} \times 10 \times \cancel{100} = \cancel{10 \times 10} \times h$$

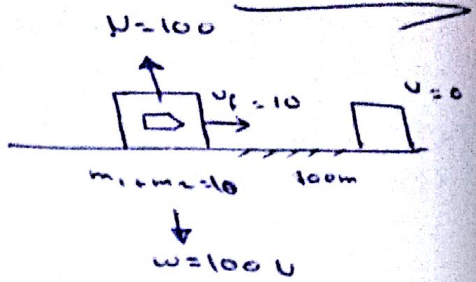
$$h = 5 \text{ m}$$

يتبع

: Ex 7 :

① → find lost energ due to friction

② → Coffeicient of kinatic friction



Sol.

① $E_a = E_b$

$w_{fk} + \cancel{mgh} + \frac{1}{2}mv_a^2 = \cancel{mgh} + \cancel{\frac{1}{2}mv_b^2}$

$w_{fk} + \frac{1}{2} \times 10 \times 10^2 = 0$

$w_{fk} = -500J$

②

$w_{fk} = -f_k d$

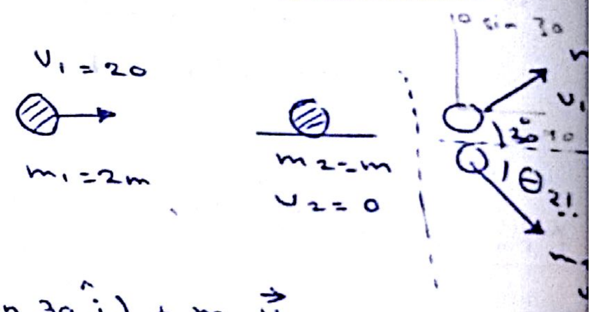
$-500 = -f_k\ 100$

$f_k = 5$ Naw

$f_k = \mu_k N$

$5 = \mu_k\ 100 \Rightarrow \mu_k = 0.05$

Ex 8 : - what is : v_{2f} , θ



Sol: $\vec{P}_i = \vec{P}_f$

$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

$2m \times 20\,\underline{\underline{\hat{i}}} + 1m \times 0 = \frac{2}{m}(10\cos 30\hat{i} + 10\sin 30\hat{j}) + m\,\vec{v}_{2f}$

$40\,\cancel{m}\,\hat{i} = 20\,\cancel{m}\cos 30\,\hat{i} + 20\,\cancel{m}\sin 30\,\hat{j} + \cancel{m}\,\vec{v}_{2f}$

$\vec{v}_{2f} = 40\hat{i} - 17.3\hat{i} - 10\hat{j}$

$= 22.7\hat{i} - 10\hat{j}$

$\tan\theta = \frac{10}{22.7} \rightarrow \theta = \tan^{-1}\left(\frac{10}{22.7}\right) = 23.8$

$\theta = -23.8°$

أو $\theta = 336.2°$

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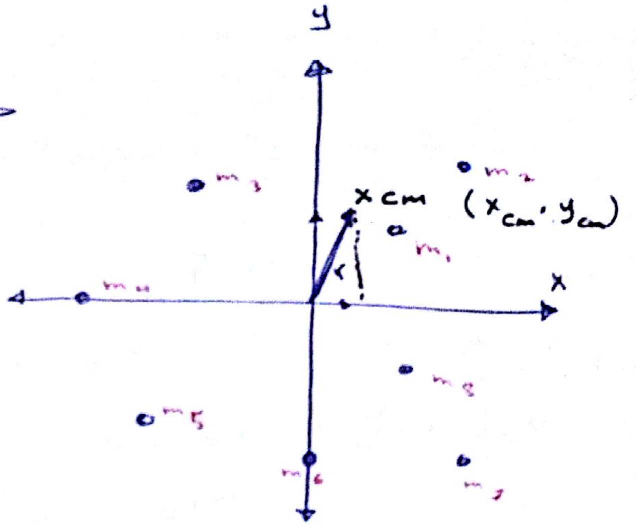
* Center of mass :-



→ $X_{cm} = \dfrac{m_1 x_1 + m_2 X_2 + \dots}{\Sigma m}$

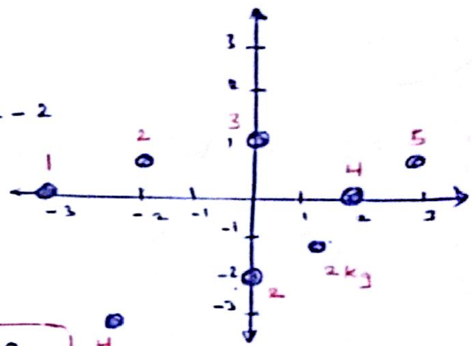
→ $y_{cm} = \dfrac{m_1 y_1 + m_2 y_1 + \dots}{\Sigma m}$

→ $\vec{r}_{cm} = X_{cm}\hat{i} + y_{cm}\hat{j}$ $\quad (x_{cm}, y_{cm})$

→ $|\vec{r}| = \sqrt{x_{cm}^2 + y_{cm}^2}$ $\quad$ → $\tan\theta = \dfrac{y_{cm}}{x_{cm}}$

* Exq :

→ Find X_{cm} , y_{cm} :-



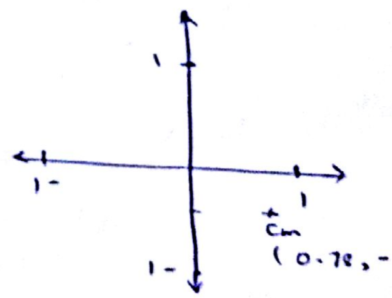
$$X_{cm} = \frac{4\times 2 + 5*3 + \cancel{3*0} + 2*-2 + 1*-3 + 4*-2 + \cancel{2*0} + 2*1}{4+5+3+2+1+4+2+2}$$

$$X_{cm} = \frac{8 + 15 - 4 - 3 - 8 + 2}{23} = \boxed{\frac{18}{23}\,m} = 0.78\,m$$

$$y_{cm} = \frac{\cancel{4*0} + 5*1 + 3*1 + 2*1 + \cancel{1*0} + 4*-3 + 2*-2 + 2*-1}{23}$$

$$y_{cm} = \frac{5 + 3 + 2 - 12 - 4 - 2}{23}$$

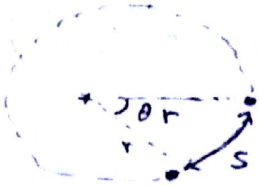
$$= \boxed{\frac{-8}{23}}\,m = -0.35$$

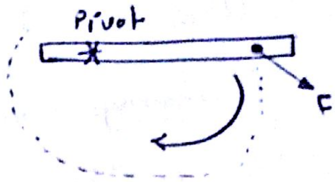


$$\vec{r} = 0.78\hat{i} - 0.35\hat{j}$$

(1 :) Done ch:9 =)

* Chapter 10 :- " Rotational motion "





* $S \longrightarrow \Delta\theta$ الإزاحة الزاوية
* $v \longrightarrow w$ السرعة الزاوية
* $a \longrightarrow \alpha$ التسارع الزاوي

θ - angular position.
$\Delta\theta$ - angular displasment.
w - angular velosity.
α - angular acceleration.

→ $\Delta x = S = \Delta\theta \, \underline{r}$

- $v = \underline{r} \, w$

→ $a = \underline{r} \, \alpha$

- $\theta \rightarrow$ rad
- $w \rightarrow$ rad / sec
- $a \rightarrow$ rad / sec^2

→ $X = f(t)$

$\rightarrow v \rightarrow v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$

$\rightarrow v_{ins} = \frac{dx}{dp}$

$\rightarrow a \rightarrow a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$\rightarrow a_{ins} = \frac{dv}{dt}$

← الحركة الانتقالية →

→ $\theta = f(t)$

$\rightarrow w \rightarrow w_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$

$\rightarrow w_{ins} = \frac{d\theta}{dt}$

$\rightarrow \alpha \rightarrow \alpha_{av} = \frac{\Delta w}{\Delta t} = \frac{w_2 - w_1}{t_2 - t_1}$

$\rightarrow \alpha_{ins} = \frac{dw}{dt}$

← الحركة الدورانية →

Ex.: $\theta = t^2 - 5t - 6$, find :

1. angular displasment from $t = 0$ - $t = 2$.
2. average angular vilosity " "
3. instantanious angular velocity . at $t = 1$ sec .
4. avarege angular accelaration between $t = 2 - 5$
5. inst. angular accekeration at $t = 4.5$ sec .

Solution:

- $\theta = t^2 - 5t - 6$

$\omega_{ins} = 2t - 5$

$\alpha_{ins} = 2$

1. $\Delta\theta = 4 - 10 - 6 - 0$

$\Delta\theta = -12$ rad

2. $\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$

$\omega = \dfrac{-12}{2} = -6$ rad/sec

4. $\alpha = \dfrac{\Delta\omega}{\Delta t}$

$\alpha = \dfrac{5+1}{3} = \dfrac{6}{3} =$

3. $\omega\big|_{t=1} = 2 - 5 = -3$ rad/sec

5. $\alpha = 2$ rad/sec^2

← إذا طلب الموقع الزاوي عند نقطة (angular position) نعوض بمعادلة

θ . أما إذا طلب (S أو Δx) نستخدم $\Delta x = r\,\Delta\theta$ →

(الارادز هو R

← إذا طلب السرعة (average velo.) نستخدم $v = r\,\omega$ وهكذا ←

→ motion with constant angular acceleration ←

• إذا دار الجسم بسرعة زاوية ثابتة فليس له تسارع زاوي $(\alpha = 0)$ $(\omega = \text{constant})$

$$\Delta\theta = \omega t$$

• إذا زادت أو قلت (تغيرت) السرعة الزاوية (ω) بانتظام يكون للجسم تسارع زاوي ثابت ويحسب كما يلي :-

1. $\omega_2 = \omega_1 + \alpha t$
2. $\omega_2^2 = \omega_1^2 + 2\alpha\,\Delta\theta$
3. $\Delta\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
4. $\Delta\theta = \Big(\underbrace{\dfrac{\omega_1 + \omega_2}{2}}_{\bar{\omega}}\Big) t$

• $a_t = \alpha r$

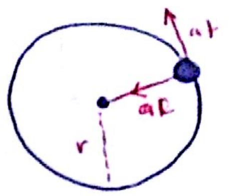
• $a_c = r\omega^2$

• $a_{total} = r\sqrt{\alpha^2 + \omega^4}$

→ $a_{tot} = \sqrt{a_c^2 + a_t^2}$

→ $a_t = \frac{dv}{dt}$

$v = r\omega$ (قوة) → $a_c = \frac{v^2}{r}$



- circular motion -

→ F → τ : Tourque.
لعزم الدوران

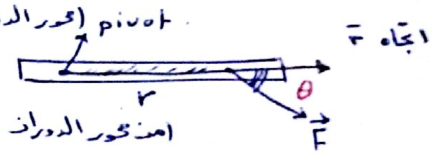
. مايقابلها في نيوتن .

→ m → I : moment of inertia.

* Torque - . عزم القوة *

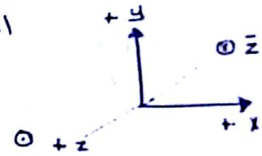
→ $\vec{\tau} = \vec{r} \times \vec{F}$

→ $|\vec{\tau}| = r F \sin\theta_{rF}$

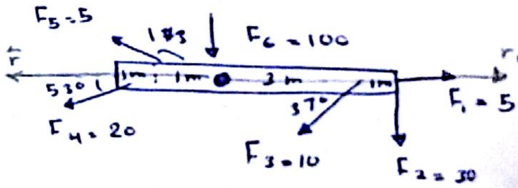


(من محور الدوران باتجاه القوة) .

اتجاه τ ($\bar{z}$, $\overset{+}{z}$) .
لـ مع عقارب الساعة .
لـ عكس عقارب الساعة .



* Ex : → Find the net Torque acting on the object.



→ Sol. $\tau_1 = r_1 F_1 \sin\theta_1$

$= 4 * 50 * \sin 0 = 0$

$\tau_2 = r_2 F_2 \sin\theta_2$

$= 4 * 30 \sin 90$

$= ⊖ 120$ N.m

$\tau_3 = r_3 F_3 \sin\theta_3$

$= 3 * 10 \sin 143°$

$= ⊖ 18$ N.m

$\tau_4 = r_4 F_4 \sin\theta_4$

$= 2 * 20 \sin 53°$

$= ⊕ 32$ N.m

$\tau_5 = r_5 F_5 \sin\theta_5$

$= 5 * 1 * \sin 37°$

$= ⊖ 3$ N.m

$\tau_6 = r_6 F_6 \sin\theta_6$

$= 0 * F$

$= 0$

$\tau_{net} = -120 - 18 + 32 - 3$

$= ⊖ 109$ N.m

الدوران الكلي مع عقارب الساعة (clock wise)

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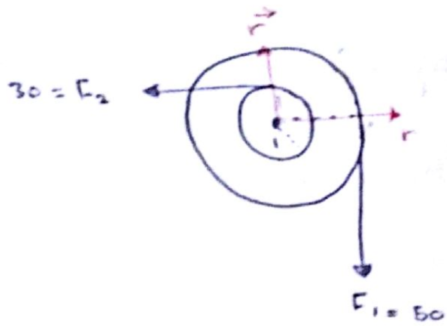
→ Ex: → Find net Torque:



$\tau_1 = r_1 F_1 \sin\theta_1$

$= 3 * 50 * \sin 90$

$= \ominus 150$ N.m

$\tau_2 = r_2 F_2 \sin\theta_2$

$= 1 * 30 \sin 90$

$= \oplus 30$ N.m

$\tau_{net} = -150 + 30 = -120$ N.m (clock wise).

→ Ex: $\vec{r} = 4\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{F} = 10\hat{j} + 2\hat{j} + 3\hat{k}$:

→ Find: ① The Torque.

② The magnitude of torque.

③ The angle between $\vec{r}$ and $\vec{F}$

Sol.: ① → $\vec{\tau} = \vec{r} \times \vec{F}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 5 \\ 10 & 2 & 3 \end{vmatrix}$$

$= \hat{i}(-9 - 10) - \hat{j}(12 - 50) + \hat{k}(8 + 30)$

$\vec{\tau} = -19\hat{i} + 38\hat{j} + 38\hat{k}$

② → $|\vec{\tau}| = \sqrt{19^2 + 38^2 + 38^2}$.

$|\vec{r}| = \sqrt{3^2 + 5^2 + 4^2} = \sqrt{50}$

$|\vec{F}| = \sqrt{10^2 + 2^2 + 3^2} = \sqrt{113}$

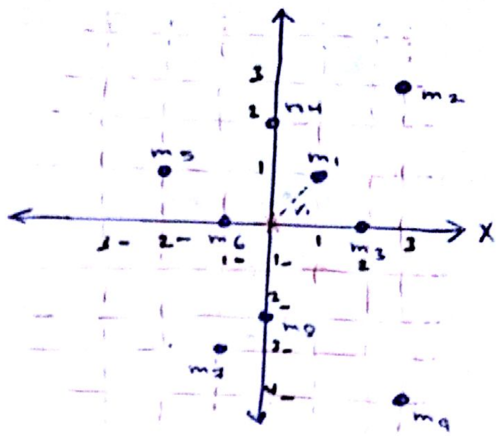
③ $|\vec{\tau}| = |\vec{r}|\,|\vec{F}| \sin\theta$

$\sqrt{19^2 + 38^2 + 38^2} = \sqrt{50}\,\sqrt{113}\,\sin\theta$

$$\sin\theta = \frac{\sqrt{19^2 + 38^2 + 38^2}}{\sqrt{50}\,\sqrt{113}}$$

→ moment of inertia :

$$I = \sum_i m_i r_i^2$$



$$* \; I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \; \ldots$$

$$= 1*2 + 2*18 + 3*4 + 4*4 + 5*5$$

$$+ 6*1 + 7*10 + 8*4 + 9*25$$

$$= (2 + 36 + 16 + 12 + 25 + 6 + 70 +$$

$$32 + 225) \; kg.m^2$$

$$r_1 = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$r_1^2 = 2$$

" حسابات تفصيلية "

[1]

$$I = \frac{1}{12} m L^2$$

(rod)

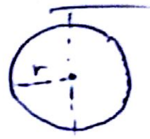
[2]

$$I = \frac{2}{5} m r^2$$

(solid sphere).

. مصمتة

[3]



$$I = \frac{2}{3} m r^2$$

(spherical shell)

. مفرغة

[4]

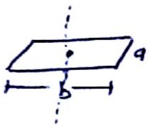
$$I = \frac{1}{2} m r^2$$

" أسطوانة صلبة "

(solid cylinder).

(disk).

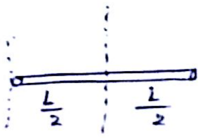
[5]



$$I = \frac{1}{12} \times m \times (a^2 + b^2)$$

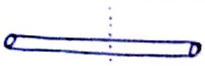
(rectangular plate)

⇒ Example :



$I_{\parallel}$

⇐



$$I_{C.M} = \frac{1}{12} m L^2$$

$$I_{\parallel} = I_{C.M} + M d^2$$

بعد المحور الجديد
عن C.M

$$\Rightarrow \left(d = \frac{L}{2}\right)$$

$$I_{\parallel} = \frac{1}{12} M L^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{12} M L^2 + M \frac{L^2}{4}$$

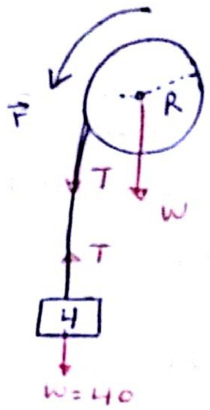
$$= \frac{1}{3} M L^2$$

. parallel axis Theorem ← نظرية المحور الموازي .

* $\Sigma F = ma \Rightarrow \Sigma \tau = I\alpha$

$$\begin{cases} m \rightarrow I \\ F \rightarrow \tau \\ \theta \rightarrow x \\ \omega \rightarrow v \\ \alpha \rightarrow a_t \end{cases}$$

+ Ex1: Find the:
1) a
2) T
3) α



$M = 0.5$ kg

$R = 0.4$ m

⇒ mass:

① $\Sigma F = ma$

$40 - T = 4 * a$

للبكرة → $\tau_T = rF\sin\theta$

$\tau = RT \quad (+)$

$I = \frac{1}{2} m r^2$ (الأسطوانة)

$(\alpha = \frac{a}{r})$

② pully :-

$\Sigma \tau = I\alpha$

$\cancel{R}T = (\frac{1}{2} m R^{\cancel{2}})\alpha$

$T = \frac{1}{2} m R \alpha$

$T = \frac{1}{2} m \cancel{R} \frac{a}{\cancel{R}}$

$T = \frac{1}{2} M a$

$$\Rightarrow \quad \begin{aligned} 40 - T &= 4a \\ T &= \frac{1}{2} M a \rightarrow \frac{1}{2} * 0.5 * a \\ \hline 40 &= 4.25\, a \end{aligned}$$

$a = \frac{40}{4.25}$

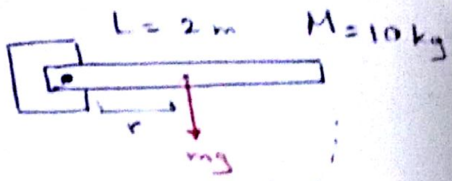
$\Rightarrow 40 - T = 4 * \frac{40}{4.25}$

$T = -\frac{4 * 40}{4.25} + 40$

$\Rightarrow \alpha = \frac{a}{R} = \frac{\frac{40}{4.25}}{0.4}$ (rad / s²)

⇒ **Ex2 :** • Find : 1) α

2) a (end of the rod).



• **Sol. :-**

$$\sum \tau = I\alpha$$

$$r F \sin\theta^{90^\circ} = I\alpha$$

$$1 * 100 * 1 = \frac{40}{3}\alpha$$

$$\alpha = \frac{30}{4} = 7.5 \ rad/s^2$$

$I = \frac{1}{3} m L^2$ ← لأن محور الدوران على الطرف

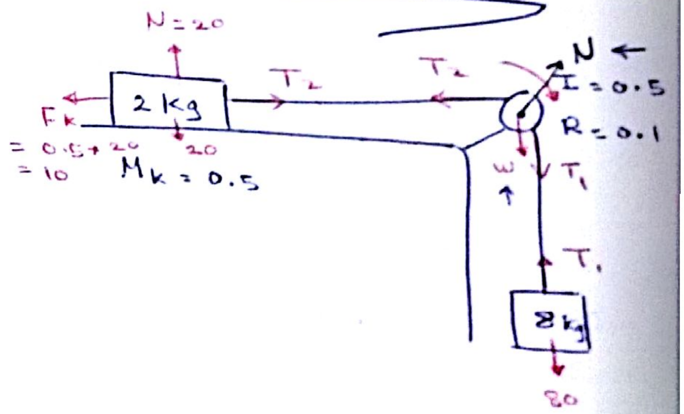
$= \frac{1}{3} * 10 * 4$

$= \frac{40}{3}$

بعد النقطة عن محور الدوران ← • $a = r\alpha$

$$a = 2 * 7.5 = 15 \ m/s^2$$

• **Ex3 :** Find T_1, T_2, a, α :



→ $\sum F_y = m_1 a$

$80 - \cancel{T_1} = 8 * a$ (1)

→ $\sum F_x = m_2 a$

$\cancel{T_2} - 10 = 2 * a$ (2)

pulley → $\sum \tau = I\alpha$

$10 * (0.1 (T_1 - T_2)) = 0.5(.10 a))$

$\cancel{T_1} - \cancel{T_2} = 5a$ (3)

$70 = 15a$ → (2) $\boxed{T_2 = 20}$

$a = \frac{70}{15} \approx 5$

$\alpha = 10 * a$
$= 50$

→ (1) $80 - T_1 = 40$

$\boxed{T_1 = 40}$

العزم لـ N و w ←
= 0 لأن r = 0
بعدهم عن محور الدوران .

$* \tau_1 = r F \sin\theta^{90^\circ}$

$\tau_1 = (0.1 \ T_1) +$

$* \tau_2 = (0.1 \ T_2) -$

$\sum \tau = 0.1 (T_1 - T_2)$

$* a = r\alpha$

$(a = 0.1\alpha) * 10$

$\alpha = 10 a$

* The work and energy :

* $K = \frac{1}{2} m v^2 \longleftrightarrow K_w = \frac{1}{2} I w^2$,

$$\Delta K = \frac{1}{2} I w_2^2 - \frac{1}{2} I w_1^2$$

→ $W = \int F \, dr \longleftrightarrow W = \int \tau \, d\theta$

العادية . الدورانية

* Ex1: $\tau = 5\theta^2 - 3\theta + 1$, what is the rotational work from $\theta = 10°$ to $\theta = 30°$

$$W = \int \tau \, d\theta = \int_{\pi/18}^{\pi/6} (5\theta^2 - 3\theta + 1) \, d\theta$$

$$= \frac{5}{3}\theta^3 - \frac{3}{2}\theta^2 + \theta \Big|_{\pi/18}^{\pi/6}$$

→ $\pi \longleftrightarrow 180°$

$X_1 \longleftrightarrow 10$

$$X_1 = \frac{10\pi}{180} = \frac{\pi}{18}$$

→ $\pi \longleftrightarrow 180°$

$X_2 \longleftrightarrow 30$

$$X_2 = \frac{30\pi}{180} = \frac{\pi}{6}$$

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$\rightarrow W_{total} = \Delta K$

$$W_{tot} = \frac{1}{2} I (\omega_2^2 - \omega_1^2)$$

$$\rightarrow E = mgh + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 + \underline{\frac{1}{2} k x^2}$$

إذا وجد سبرنغ .

$$\rightarrow E_i = E_f \leftarrow$$

$$mgh_i + \frac{1}{2} k x_i^2 + \frac{1}{2} m v_i^2 + \frac{1}{2} I \omega_i^2 = mgh_f + \frac{1}{2} k x_f^2 + \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2$$

Lenian $\leftarrow$ $\cdot P = m v$

$\boxed{m} \rightarrow v$

$\cdot L = r \times \vec{p} = \vec{r} \times m\vec{v}$

angular momentum

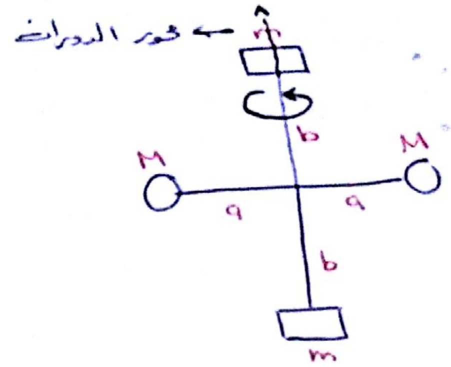
$$|\vec{L}| = m r v \sin\theta$$

ذكر تطبيقات $\leftarrow$

١: ٢:

٢٠٢٢/٢/١٥

• Ex 1 : a) I b) K_w



$\boxed{1}$

$1 \rightarrow I_i = \sum m_i r_i$

$= Ma^2 + Ma^2$

$= 2Ma^2$

$2 \rightarrow k = \frac{1}{2} I \omega^2$

$= \frac{1}{2}(2Ma^2)\omega^2$

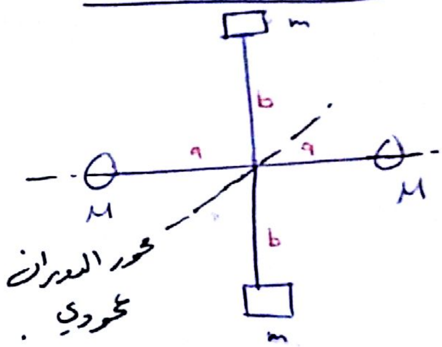
$= Ma^2\omega^2$

مكانهم $(m - m)$ ← منطبقات على المحور بعدهم = صفر .

مش مكانهم $(M - M)$:p

$\boxed{2}$ a) $\circ I = Ma^2 + Ma^2 + mb^2 + mb^2$

$= 2(Ma^2 + mb^2)$



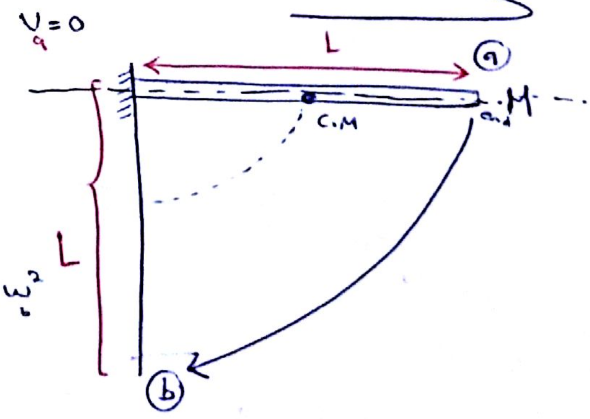
b) $\circ k = \frac{1}{2} I \omega^2$

$= \frac{1}{2}(2(Ma^2 + mb^2))\ \omega^2$

$= Ma^2 + mb^2 + \omega^2$

• Ex 2 : ① ω at ⓑ   ③ V_{end}

② $V_{c.m}$



$E_a = E_b$

$\cancel{mgh} + \cancel{\frac{1}{2}mv_a^2} + \cancel{\frac{1}{2}I\omega_a^2} = mg\overset{C.M}{h} + \cancel{\frac{1}{2}mv_b^2} + \frac{1}{2}I\omega_b^2$

لايوجد ابتدائيه

$0 = Mg(-\frac{L}{2}) + \frac{1}{2}I\ \omega^2$

$\frac{\cancel{M}g\cancel{L}}{\cancel{2}} = \cancel{\frac{1}{2}}\frac{1}{3}\cancel{M}L^{\cancel{2}}\ \omega^2$

$g = \frac{L}{3}\omega^2$

$\omega = \sqrt{\frac{3g}{L}}$

→ following :)

• $V_{C.M.} = r_{c.m} \, w$

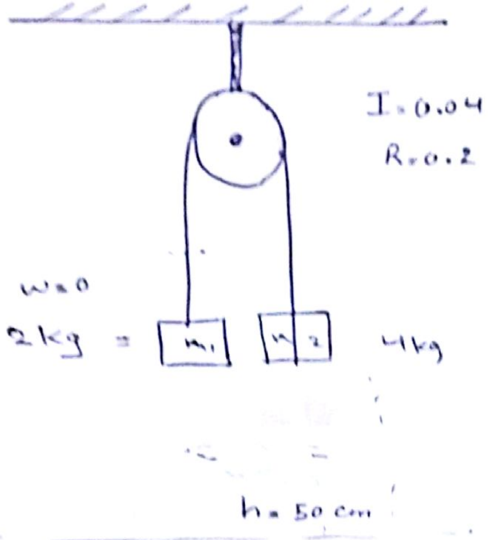
$= \frac{L}{2} \sqrt{\frac{3g}{L}} = \sqrt{\frac{3gL}{4}}$

• $V_{end} = r_{end} \, w$

$= L \sqrt{\frac{3g}{L}} = \sqrt{3gL}$

• **Ex3:**

→ Find w :
and V



*** Solu.**

$$E_a = E_b$$

$$m_1 g h + \frac{1}{2} m_1 v_a^2 =$$

* Chapter 11: → Angular Momentum ←

* مراجعة (الضرب التقاطعي):

* $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
* $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

- $\vec{A} \times \vec{B} \longrightarrow \vec{C}$

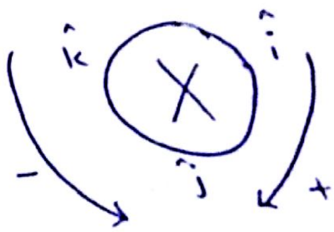
$\rightarrow |\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta_{AB}$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} +\hat{i} & -\hat{j} & +\hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\underset{(\vec{A} \times \vec{B})}{\vec{C}} = \hat{i}\underbrace{\left((A_y \times B_z) - (A_z \times B_y)\right)}_{C_x} - \hat{j}\underbrace{\left((A_x * B_z) - (A_z * B_x)\right)}_{C_y} + \hat{k}\underbrace{\left((A_x * B_y) - (A_y * B_x)\right)}_{C_z}$$

(كمقدار) $|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$$\Rightarrow \begin{cases} \hat{i} \times \hat{i} = 0 \\ \hat{j} \times \hat{j} = 0 \\ \hat{k} \times \hat{k} = 0 \end{cases} \qquad \begin{cases} \hat{i} \times \hat{j} = \hat{k} \ + \\ \hat{j} \times \hat{k} = \hat{i} \ + \\ \hat{k} \times \hat{i} = \hat{j} \ + \end{cases}$$



$$\begin{cases} \hat{j} \times \hat{i} = -\hat{k} \\ \hat{k} \times \hat{j} = -\hat{i} \\ \hat{i} \times \hat{k} = -\hat{j} \end{cases}$$

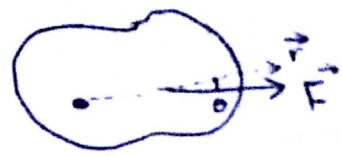
← الترتيب مهم →

ه

من التطبيقات:

- $\vec{\tau} = \vec{r} \times \vec{F}$

- $|\vec{\tau}| = r F \sin\theta$



* $\vec{\tau}_{net} = \sum_{i=1}^{n} \vec{\tau}_i = \sum_{i=1}^{n} \vec{r}_i \times F_i$

الإبهام باتجاه R
، الأصابع باتجاه F

- $\sum \vec{\tau} = I\alpha$ - $I = \sum_i m_i r_i^2$

- $P = mv \Rightarrow \frac{dp}{dt} = \frac{d(\overset{\text{ثابت}}{m} v)}{dt} = m\frac{dv}{dt} = ma = \sum F$

$$\sum F = \frac{dp}{dt}$$

- $\sum \tau = \sum r \times F$

- $\sum \tau = \sum r \times \frac{dp}{dt} = \frac{d}{dt} \sum \vec{r} \times \vec{p}$

- $\sum \tau = \sum \frac{dL}{dt}$ L : Angular momentum

Leniar $\leftarrow \vec{P} = m\vec{v} \longleftrightarrow \vec{L} = \vec{r} \times \vec{p}$

$$\sum \tau = \sum \frac{dL}{dt}$$

$$|\vec{L}| = r \times p \times \sin\theta_{rp}$$

$$= r\, m v \sin\theta_{rv}$$

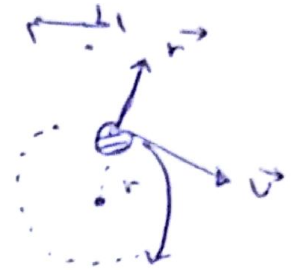
→ $\Delta p = p_f - p_i = mv_f - mv_i = F_{ext}\,\Delta t = \int F\,dt$

→ $\Delta \vec{L} = L_f - L_i = I\omega_f - I\omega_i = \tau\,\Delta t = \int \tau\,dt$.

* m $\rightarrow \vec{v}$

$\vec{P} = m\vec{v}$

بالدائري دائماً ($\vec{v} \perp r$)



$\vec{L} = \vec{r} \times \vec{p}$

$L = r\,m\,v\,\sin\theta_{rv}$

الجسم نفسه ←

$(\theta = 90^\circ \Rightarrow L = r\,m\,v)$

وجود مؤثر.

* $F = \dfrac{dp}{dt}$

$\Delta p = m(v_f - v_i)$

$= F_{ext}\,\Delta t$

$p = \int F\,dt$

* $\vec{\tau} = \dfrac{dL}{dt}$

$\Delta L = I(\omega_f - \omega_i)$

$= \tau\,\Delta t$

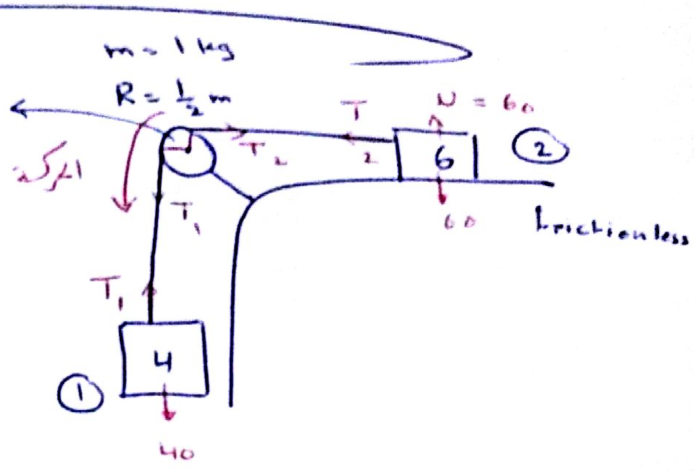
$\vec{L} = \int_{t_1}^{t_2} \tau_{ext}\,dt$

$$\vec{L}_{total} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ldots = \Sigma \vec{L}_i$$

(system of many particles)

Ex. 1: Find a:

mr^2 حلقة



$\Sigma F_{1y} = m_1 a$

$$\boxed{40 - T = 4a}$$

$\Sigma F_{2x} = m_2 a$

$$\boxed{T_2 = 6a}$$

$\Sigma \tau = I\alpha$

$(0.5T_1 - 0.5T_2 = 0.25(2a)) \times 2$

$$\boxed{T_1 - T_2 = a}$$

$40 = 11a$

$a = \frac{40}{11}\ m/s^2$

$\tau_1 = r f \sin\theta$

$= 0.5 T_1 \sin 90$

$= 0.5 T_1$ (+)

$\tau_2 = 0.5 T_2 \sin 90$

$= 0.5 T_2$ (−)

$0.5 T_1 - T_2\ 0.5$

$I = mr^2$

$= 1(0.5)^2$

$= 0.25$

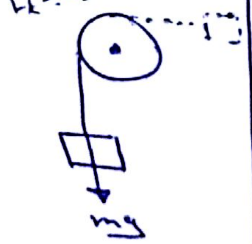
$\alpha = \frac{a}{r} = 2a$

---

① ②

$\rightarrow \Sigma \tau_{ext} = \frac{dL}{dt}$ $\rightarrow \Sigma F_{ext} = \frac{dp}{dt}$: Angular بطريقة

لا يوجد F_t



① $\rightarrow \tau_{ext} = \overset{F}{(mg)} \times r \times \sin\theta$ 90

$= 40 \times 0.5$

$= 20$ new.m

$\rightarrow L_1 = r_1 m_1 v_1 \sin\theta = 0.5 \times 4 (v_1)$

$= 2v_1$

$v_1 = v_2$

$\rightarrow L_2 = 0.5 \times 6 (v_2) \sin 90° = 3v_2$

$\rightarrow L_3 = R\, M\, v \sin 90° = 0.5 \times 1\ v$

$= 0.5v$

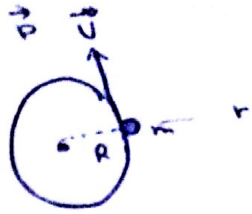
$\Sigma \tau = \frac{dL}{dt}$

$20 = (5.5v)\frac{d}{dt}$

$20 = 5.5 \frac{dv}{dt}$

$\frac{20}{5.5} = a$

* Angular Momentum of a rigid body :-



- $\vec{P} = m\vec{v}$
- $\vec{L} = r \times \vec{p}$

$= rp = rmv$

→ $\boxed{L = I\omega}$ → عام

تطبيق على المثال السابق.

استخدم رموز [illegible]

* $L_3 = I\omega$

مجوفة

$= MR^2 \dfrac{v}{R} = MRv$

بدون مؤثرات

$L_i = L_f$ → Isolated system ←

no external Torque.

$I_1\omega_1 + I_2\omega_2 = I_1\omega_1' + I_2\omega_2'$

$\sum \tau_{ext} = \dfrac{dL}{dt} \Rightarrow L = \text{constant}$

$(L_i = L_f)$

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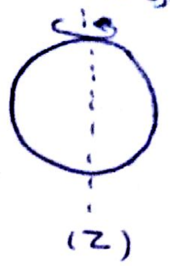
كرة مصمتة ←



* **Ex₁:** $L = ?$ $\quad v = 10$ rev/s $\quad m = 7$ kg

$= 10\,(2\pi R)$ m/s $\quad R = 12$ cm $= 0.12$ m

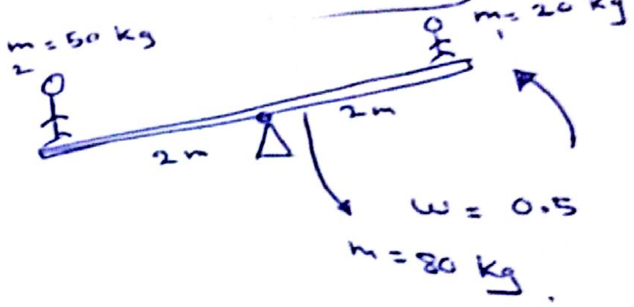
$$w = \frac{v}{R} = \frac{20\pi \cancel{R}}{\cancel{R}} = 20\pi \text{ (rad/s)}$$

$$I = \frac{2}{5} m R^2 = \frac{2}{5}\, 7 * (0.12)^2 = 0.04 \text{ kg.m}^2$$

$$L_z = I w = 0.04 * 20\pi = 2.53 \text{ kg.m}^2/\text{s}$$

---

* **Ex₂:** Find L :-



$* \; L = I w$

$$I = I_{m_1} + I_{m_2} + I_{rod}$$

$$= m_1 r^2 + m_2 r^2 + \frac{1}{12} m L^2$$

$$= 20*4 + 50*4 + \frac{1}{12}*80*16$$

$$= 80 + 200 + \frac{320}{3}$$

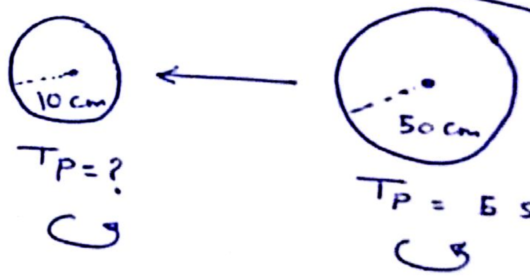
$$= 386.7 \text{ kg.m}^2$$

$* \; L = 386.7 * 0.5$

$= 193.3$ kg.m²/s

---

* **Ex₃:-** $L_i = L_f$



$$I_1 w_1 = I_2 w_2$$

$$\cancel{\frac{2}{5} m} R_1^2 w_1 = \cancel{\frac{2}{5} M} R_2^2 w_2$$

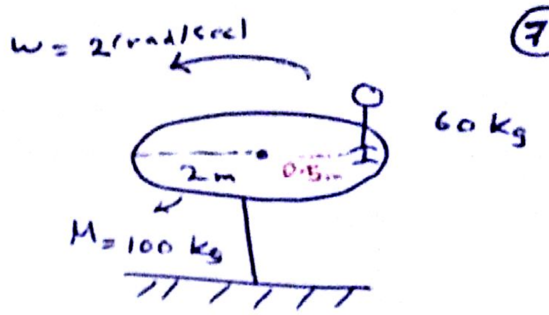
$$(0.5)^2 \frac{\cancel{2\pi}}{T_{p_1}} = (0.1)^2 \frac{\cancel{2\pi}}{T_{p_2}}$$

$$\frac{0.25}{T_{p_1}} = \frac{0.01}{T_{p_2}}$$

$$\frac{0.25}{5} = \frac{0.01}{T_{p_2}} \Rightarrow T_{p_2} = \frac{0.05}{0.25} = 0.2 \text{ sec}$$

- $w = \frac{2\pi}{T_p}$
- $v = \frac{2\pi R}{T_p}$
- $w = \frac{v}{R}$

* **Ex4:** • Find ω_{new} $(r = 0.5)_m$



$$L_i = L_f$$

$$I_p \omega_p + I_g \omega_g = I_p \omega_p' + I_g \omega_g'$$

$$200 * 2 + 240 * 2 = 200 \omega' + 15 * \omega'$$

$$400 + 480 = 215 \omega'$$

$$\omega' = \frac{880}{215} \text{ rad/sec}$$

$$= 4.1$$

$$I_p' = I_p = \frac{MR^2}{2} = \frac{100 * 4}{2} = 200 \text{ kg.m}^2$$

$$I_g = mr^2 = 60 * 4 = 240 \text{ kg.m}^2$$

$$I_g' = mr'^2 = 60 * (0.5)^2 = 15 \text{ kg.m}$$

← هل تتاج بذل شغل : (حسب k)

→ $K_i = \frac{1}{2} I \omega^2$ $(I_p + I_g)$

$$= \frac{1}{2}(440)(4) = 880 \text{ Jol}$$

للبنت والطبق

$$K_f = \frac{1}{2} I' \omega_g'^2$$

$$= \frac{1}{2}(215)(4.1)^2 = 1807 \text{ Jol}$$

$K_f > K_i \Rightarrow w = K_f - K_i = 1807 - 880$

$$= 927 \text{ Jol}$$

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