من إصدارات

أكاديمية الفيزياء للتعليم الإلكتروني

Magnetism and Alternating Current

سلسلة محاضرات
المغناطيسية والتيار المرتدد

الدكتور حازم فلاح سكيك

استاذ الفيزياء المشارك

جامعة الأزهر-غزة

أكاديمية الفيزياء

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سلسلة محاضرات مقرر
المغناطيسية والتيار المتردد

Magnetism and Alternating Current

إعداد
الدكتور حازم فلاح سكيك
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المحتويات

مقدمة

يعتبر المغناطيس من أهم المواد الموجودة في عصرنا الحالي وهو من أهم الاختراعات التي ظهرت في مجال الفيزياء فمنذ بداية اكتشاف حجر المغناطيس في مدينة مغنيسيا في تركيا والفكر البشري يحاول أن يجد استخدامات واستعمالات لهذا الحجر الجاذب للمعادن فمع بداية التطور صنع واخترع المغناطيس الصناعي كما نراه اليوم وتم الاستفادة منه بشكل مذهل ومدهش خاصة في مجال المواصلات) القطارات الكهربائية السريعة والحافلات الكهربائية (وتخزين المعلومات في الحاسوب وأجهزة تسجيل الصورة وتسجيل لصوت مثل القرص الصلب والكاسيت وغيرها في الأجهزة التي يستخدمها الفيزيائيون في تجاربهم مثل معجلات الجسيمات، مثل مصادم الهدرونات الكبير وفيرميلاب.

تختص هذه السلسلة من المحاضرات بدراسة علم المغناطيسية والتيار المتردد وقد اعتمدت في اعدادها على كتاب الوحدة الرابعة من كتاب الفيزياء للعلوم والهندسة مع الفيزياء الحديثة Physics for Scientists and Engineers with Modern Physics للمؤلف ريموند سيروي Raymond A. Serway. هذه المحاضرات موجهة لطلبة الفيزياء والهندسة وكل الدارسين والمهتمين في مجال المغناطيسية والتيار المتردد، ولقد راعيت ان يكون أسلوب الشرح واضح وميسر مدعما بالشروحات والامثلة المحلولة بخطوات واضحة ومتسلسلة.

تغطي المحاضرات خمسة فصول دراسية يختص كل فصل بموضوع محدد حيث ان الفصل الأول يتناول خواص المجال المغناطيسي والقوى المغناطيسية، وكذلك دارسة تأثير المجال المغناطيسي على شحنة كهربية متحركة مع دراسة للعديد من التطبيقات المعتمدة على المجال المغناطيسي مثل مرشح السرعة ومطياف الكتلة ومعجل السيكلترون وفكرة عملهم كما سوف نتطرق إلى دراسة تأثير المجال المغناطيسي على موصل يمر فيه تيار كهربي وتأثير عزم الازدواج على حلقة سلك يمر فيها تيار كهربي. في نهاية الفصل الأول سوف ندرس ظاهرة هول المستخدمة في معرفة نوع حاملات الشحنة في المواد المختلفة وعلاقتها بقياس المجال المغناطيسي. اما الفصل الثاني فانه يتناول شرح كيف نقوم بحساب المجالات المغناطيسية المختلفة باستخدام قانون بيوت-سافارت وقانون امبير، ودراسة القوة المغناطيسية المتبادلة بين موصلين متوازيين. كما يستعرض هذا الفصل قانون جاوس في المغناطيسية والذي يتعامل مع الفيض المغناطيسي ونختم هذا الفصل بدراسة مختصرة لتصنيف المواد حسب خواصها المغناطيسية مع دراسة التأثيرات المغناطيسية في المادة المعتمدة على العزم المغناطيسي الذري، والذي ينشأ

من الحركة المدارية والمغزلية للالكترونات. اما الفصل الثالث فانه يركز على تأثير جديد وهام للمجال المغناطيسي المتغير مع الزمن والذي اكتشف هذا التأثير كلا من العالمين هنري وفارادي واطلق عليه قانون فارادي للحث الذي ربط المجال المغناطيسي مع المجال الكهربي، الذي فتح المجال لتطبيقات هامة سندرسها مثل المولدات والمواتير ونختم هذا الفصل بدراسة التيارات الدوامية. يركز الفصل الرابع على الحث الذاتي لملف والحث المتبادل، هذا بالإضافة إلى دراسة الدوائر الكهربية التي تحتوي على مقاومة وملف حثي ومكثف، بالإضافة إلى دراسة التذبذبات المتولدة في دوائر المكثف والملف الحثي. نبدأ في الفصل الخامس بدراسة دوائر التيار المتردد وندرس سلوك التيار الكهربي والجهد في دوائر كهربية تحتوي على مقاومة او ملف حثي او مكثف تمهيدا لدراسة دوائر المكثف والملف الحثي والمقاومة المتصلين على التوالي، ونختم هذا الفصل بدراسة مجموعة من التطبيقات العلمية مثل حساب القدرة الكهربية في دوائر التيار المتردد، ودائرة الرنين والمحول الكهربي ومقومات ومرشحات التيار المتردد.

للاستفادة من هذه المحاضرات يفضل مشاهدة تسجيل المحاضرات على موقع اكاديمية الفيزياء للتعليم الالكتروني. والعمل على حل المسائل والتمارين في نهاية كل محاضرة

أتمنى ان تكون هذه السلسلة من المحاضرات مفيدة وممتعة ومكملة لما هو مشروح وموضح على موقع اكاديمية الفيزياء.

مع خالص تحياتي

د. حازم فلاح سكيك
جامعة الأزهر – غزة

غزة في 25-1-2015

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نبذة عن المحاضر

د. حازم فلاح سكيك

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- مؤسس قناة الفيزياء التعليمي على اليوتيوب
- مؤسس ورئيس تحرير مجلة الفيزياء العصرية

لمزيد من المعلومات يرجى زيارة

المؤسسة الإعلامية لشبكة الفيزياء التعليمية

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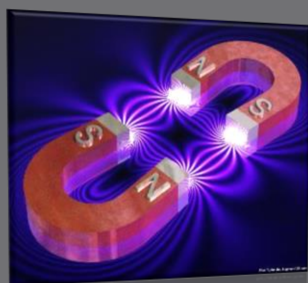
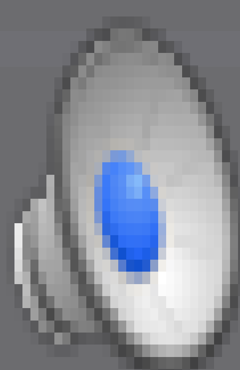
Magnetism and Alternating Current



Lecture 0: Introduction

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza



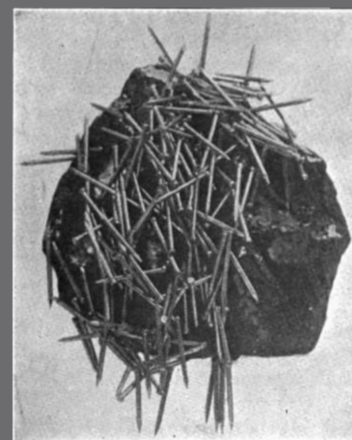
History

The compass, which uses a magnetic needle, was used in China as early as the 13th century BC, its invention being of Arabic or Indian origin.

The early Greeks knew about magnetism as early as 800 BC.

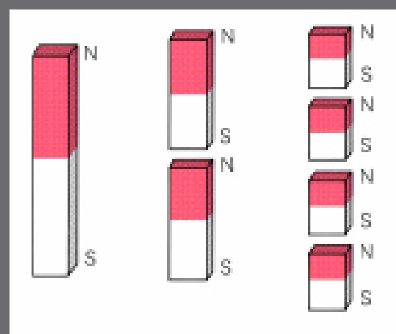
They discovered that the stone magnetite (Fe_3O_4) attracts pieces of iron.

Legend ascribes the name magnetite to the shepherd Magnes, the nails of whose shoes stuck to chunks of magnetite.

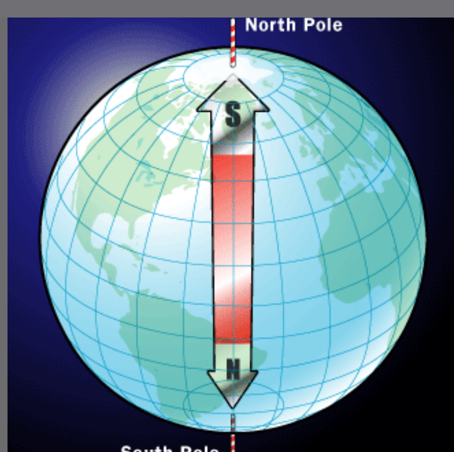


In 1269, Pierre de Maricourt of France found that the directions of a needle near a spherical natural magnet formed lines that encircled the sphere and passed through two points diametrically opposite each other, which he called the poles of the magnet.

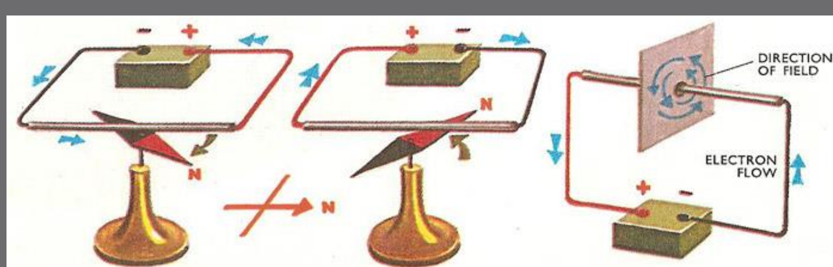
Subsequent experiments showed that every magnet, regardless of its shape, has two poles, called north (N) and south (S) poles, that exert forces on other magnetic poles similar to the way electric charges exert forces on one another. That is, like poles (N–N or S–S) repel each other, and opposite poles (N–S) attract each other.



In 1600, William Gilbert extended de Maricourt's experiments to a variety of materials. He knew that a compass needle orients in preferred directions, so he suggested that the Earth itself is a large, permanent magnet.



The relationship between magnetism and electricity was discovered in 1819 when, during a lecture demonstration, Hans Christian Oersted found that an electric current in a wire deflected a nearby compass needle.



In the 1820s, further connections between electricity and magnetism were demonstrated independently by Faraday and Joseph Henry (1797–1878).

They showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field.

Years later, theoretical work by Maxwell showed that the reverse is also true: a changing electric field creates a magnetic field.

Unit 1: Magnetic Fields

1.1 Magnetic Fields and Forces.

1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

1.3 Applications Involving Charged Particles Moving in a Magnetic Field.

1.4 Magnetic Force Acting on a Current-Carrying Conductor.

1.5 Torque on a Current Loop in a Uniform Magnetic Field.

1.6 The Hall Effect.

Unit 2: Sources of the Magnetic Field

2.1 The Biot–Savart Law

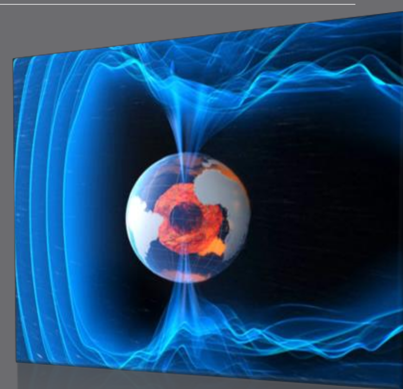
2.2 The Magnetic Force Between Two Parallel Conductors

2.3 Ampère's Law

2.4 The Magnetic Field of a Solenoid

2.5 Gauss's Law in Magnetism

2.6 Magnetism in Matter



Unit 3: Faraday's Law

3.1 Faraday's Law of Induction

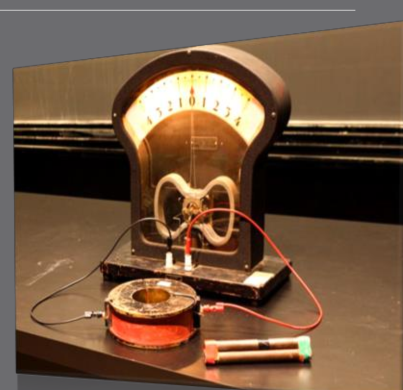
3.2 Motional emf

3.3 Lenz's Law

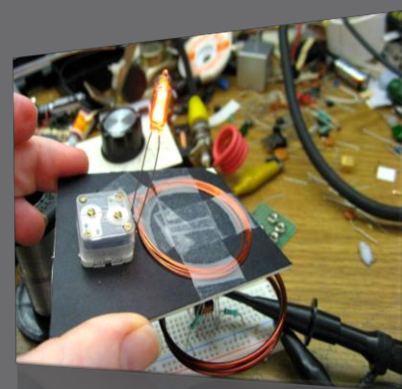
3.4 Induced emf and Electric Fields

3.5 Generators and Motors

3.6 Eddy Currents



Unit 4: Inductance

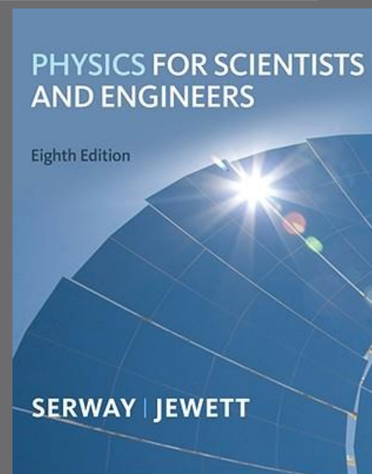


Unit 5: Alternating-Current Circuits

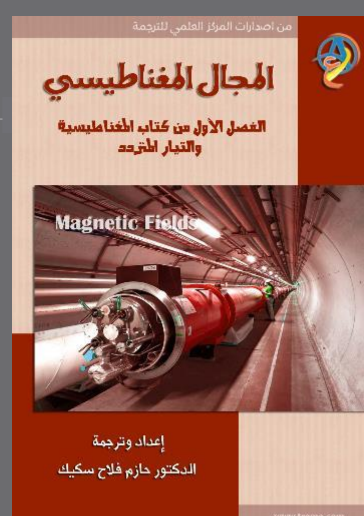
Physics for Scientists and Engineers by Serway & Jewett

Unit 4: Electricity and Magnetism

Chapters: 29,
30,
31,
32,
33.



ترجمة باللغة العربية للفصل الأول



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إلى اللقاء مع المحاضرة (١) بعنوان المجال المغناطيسي

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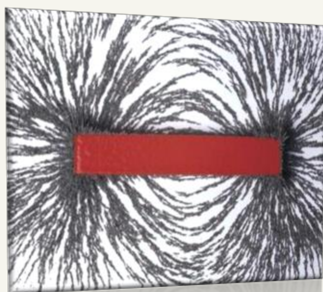
Magnetism and Alternating Current



Unit 1: Magnetic Fields

Lecture 1: Magnetic Fields and Forces

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 1: Magnetic Fields

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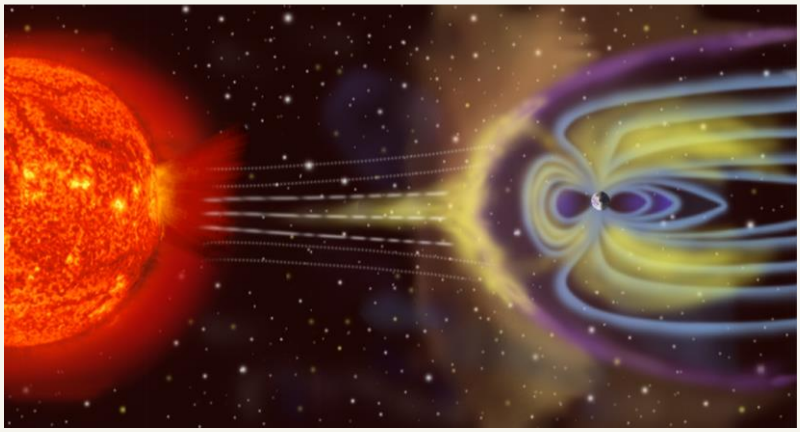
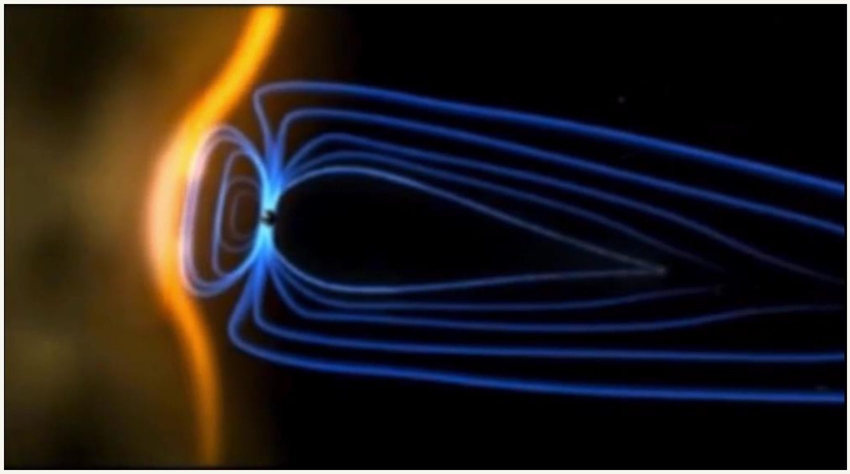
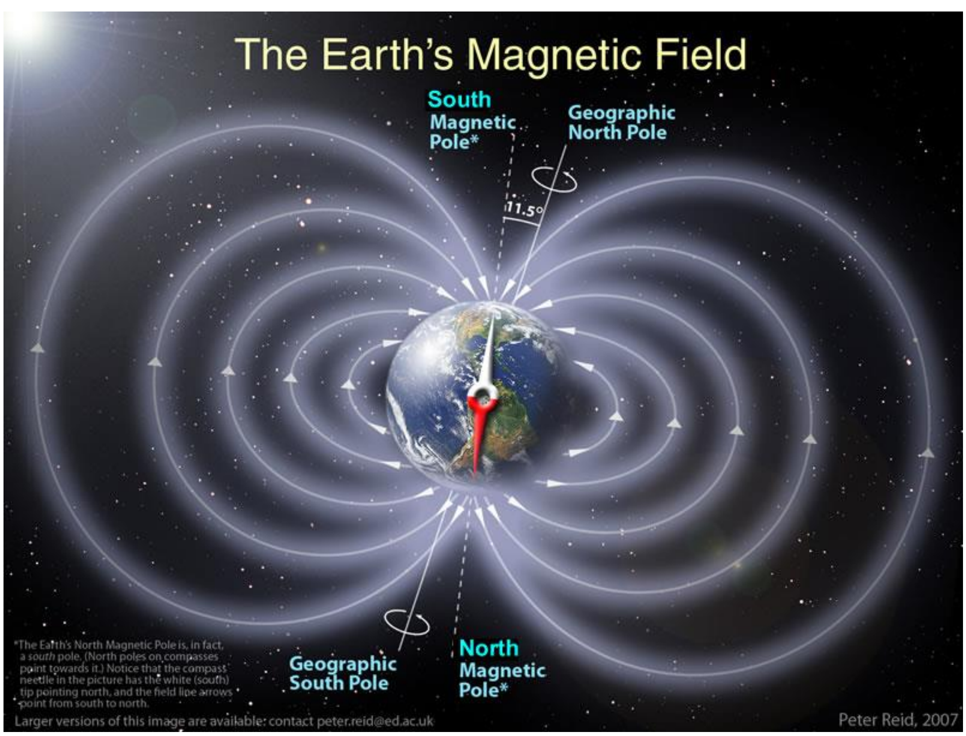
Earth's magnetic field serves to deflect most of the solar wind, whose charged particles would otherwise strip away the ozone layer that protects the Earth from harmful ultraviolet radiation.

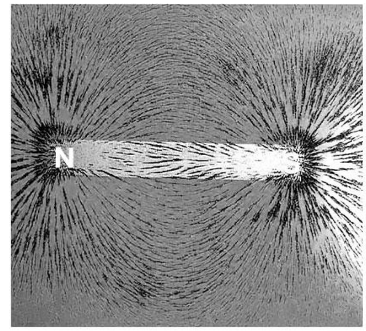


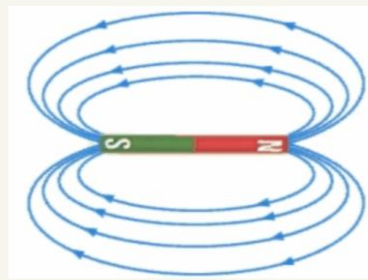
Aurora Polaris




The Earth's Magnetic Field
South Magnetic Pole*
Geographic North Pole
11.5°
Geographic South Pole
North Magnetic Pole*
*The Earth's North Magnetic Pole is, in fact, a south pole. (North poles on compasses point towards it.) Notice that the compass needle in the picture has the white (south) tip pointing north, and the field line arrows point from south to north.
Larger versions of this image are available: contact peter.reid@ed.ac.uk
Peter Reid, 2007

The magnetic field



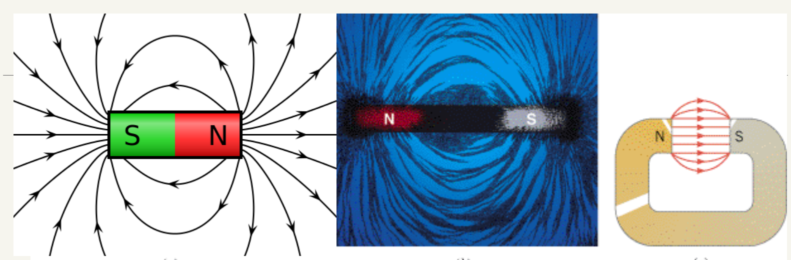


A bar magnet has a magnetic field around it. This field is 3D in nature and often represented by lines LEAVING north and ENTERING south

The magnetic field is a vector that has both magnitude and direction.

The direction of the magnetic field at any point in space is the direction indicated by the north pole of a small compass needle placed at that point.

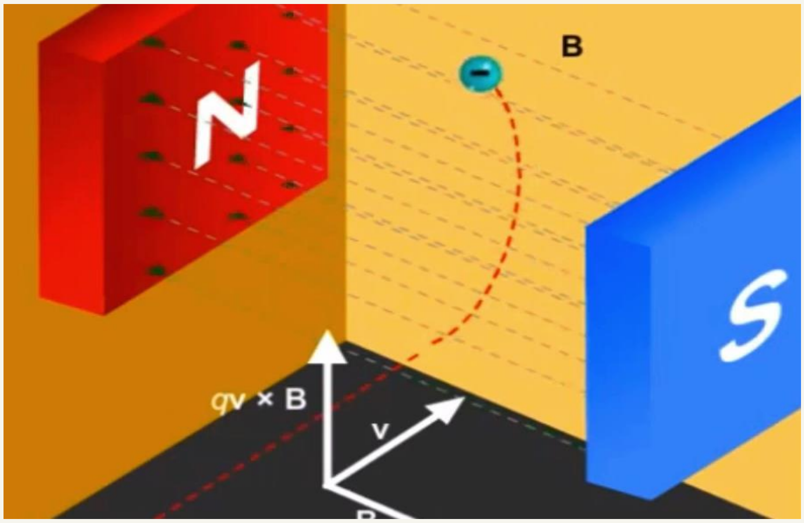
The properties of magnetic field line



(a) (b) (c)

1. The lines originate from the north pole and end on the south pole; they do not start or stop in mid-space.
2. The magnetic field at any point is tangent to the magnetic field line at that point.
3. The strength of the field is proportional to the number of lines per unit area that passes through a surface oriented perpendicular to the lines.
4. The magnetic field lines will never come to cross each other.

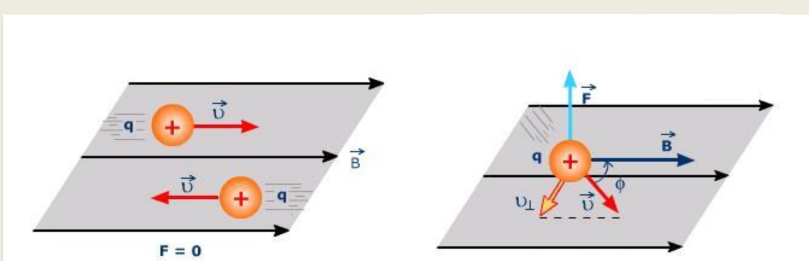
Magnetic force on moving charge



Magnetic force on moving charge

When a charge is placed in a magnetic field, it experiences a magnetic force if two conditions are met:

1. The charge must be moving. No magnetic force acts on a stationary charge.
2. The velocity of the moving charge must have a component that is perpendicular to the direction of the field.

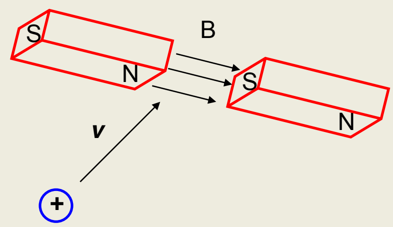


Properties of the magnetic force on a charged particle moving in a magnetic field

We can define a magnetic field **B** at some point in space in terms of the magnetic force F_B the field exerts on a charged particle moving with a velocity **v**, which we call the test object.

Experiments on various charged particles moving in a magnetic field give the following results:

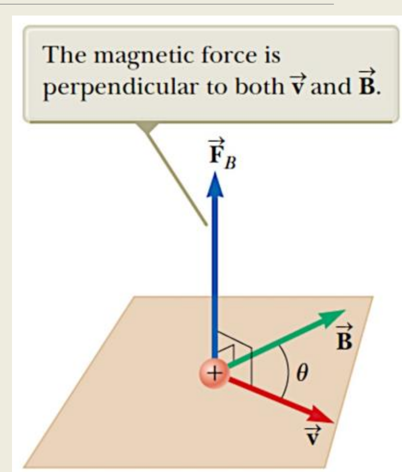
(1) The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.



Properties of the magnetic force on a charged particle moving in a magnetic field

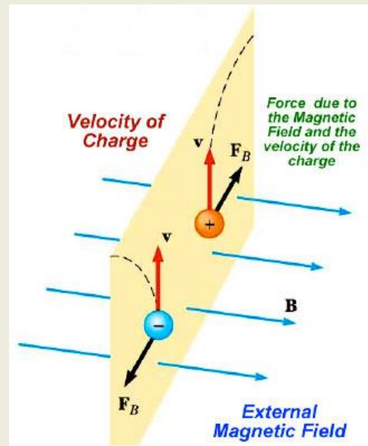
(2) When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.

(3) When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and ***B***; that is, F_B is perpendicular to the plane formed by **v** and **B.**



Properties of the magnetic force on a charged particle moving in a magnetic field

(4) The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.



Properties of the magnetic force on a charged particle moving in a magnetic field

(5) The magnitude of the magnetic force exerted on the moving particle is proportional to sinθ, where θ is the angle the particle's velocity vector makes with the direction of **B**.

$$F_B = qvB\sin$$

Vector expression for the magnetic force on a charged particle moving in a magnetic field

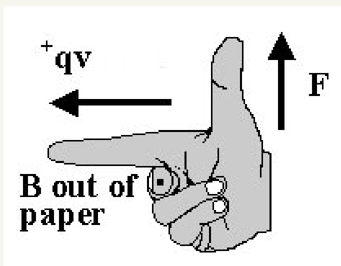
Magnetic Force — Magnetic Field

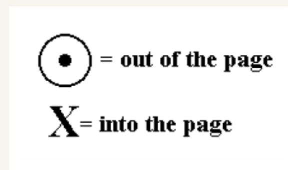
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

velocity of charge

Direction of the magnetic force? Right Hand Rule

To determine the DIRECTION of the force on a **POSITIVE** charge we use a special technique that helps us understand the 3D perpendicular nature of magnetic fields.





Unit of Magnetic Field

SI unit of magnetic field is the newton per coulomb-meter per second, which is called the tesla (T):

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{C.m/s}}$$

Because a coulomb per second is defined to be an ampere,

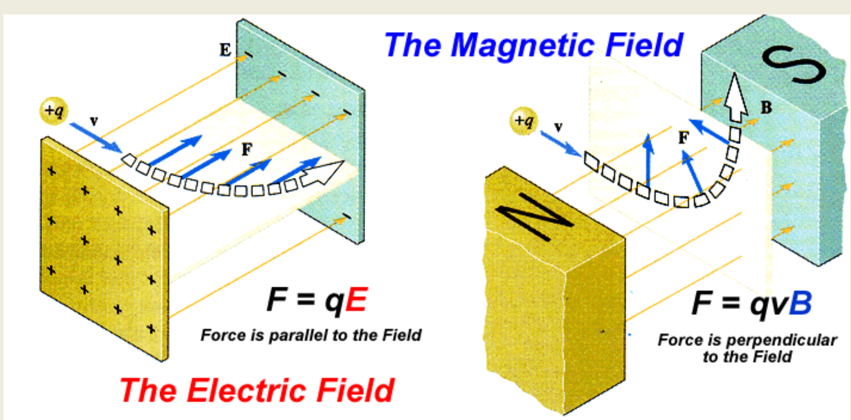
$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A.m}}$$

A non-SI magnetic-field unit in common use, called the gauss (G), is related to the tesla through the conversion 1 T = 10^4 G.

Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	**30**
Strong conventional laboratory magnet	**2**
Medical MRI unit	**1.5**
Magnetic Bar	**10^{-2}**
Surface of the Sun	**10^{-2}**
Surface of the Earth	**0.5×10^{-4}**
Inside human brain due to nerve impulses	**10^{-13}**

Motion of charge particle in
- Electric field
- Magnetic field



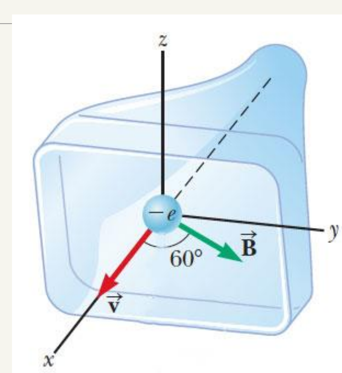
Differences between Electric and Magnetic Forces

1. The electric force vector is along the direction of the electric field, whereas the magnetic force vector is perpendicular to the magnetic field.
2. The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
3. The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement of its point of application.

The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. The field can alter the direction of the velocity vector, but it cannot change the speed or kinetic energy of the particle.

Example 1

An electron in an old-style television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the *x* axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the *x* axis and lying in the *xy* plane.



Calculate the magnetic force on the electron.

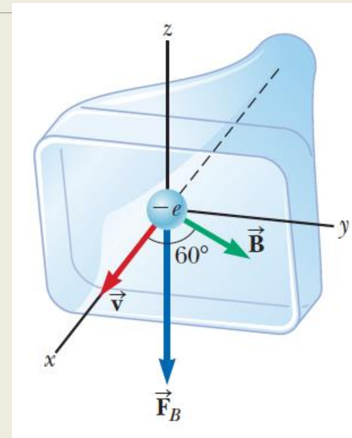
Solution

Use one of the right-hand rules to determine the direction of the force on the electron

$$F_B = qvB\sin$$

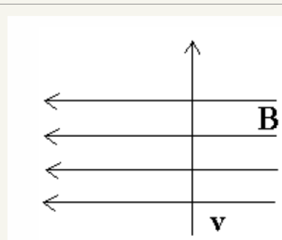
= (1.6 × 10^{-19} C)(8.0 × 10^{6} m/s)(0.025 T) (sin 60°)

= 2.8 × 10^{-14} N

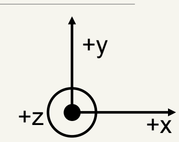


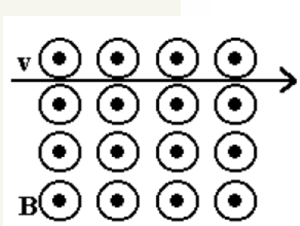
Example 2

Determine the direction of the unknown variable for a proton moving in the field using the coordinate axis given

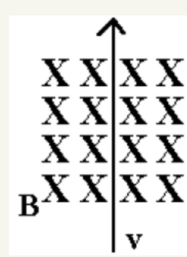


B = -x
v = +y
F = **+z**





B =+Z
v = +x
F = **-y**



B = -z
v = +y
F = **-x**

Example 3

A spatially uniform magnetic field cannot exert a magnetic force on a particle in which of the following circumstances? There may be more than one correct statement.

- (a) The particle is charged.
- (b) The particle moves perpendicular to the magnetic field.
- (c) The particle moves parallel to the magnetic field.
- (d) The magnitude of the magnetic field changes with time.
- (e) The particle is at rest.

Example 4

A particle with electric charge is fired into a region of space where the electric field is zero. It moves in a straight line. Can you conclude that the magnetic field in that region is zero?

- (a) Yes, you can.
- (b) No; the field might be perpendicular to the particle's velocity.
- (c) No; the field might be parallel to the particle's velocity.
- (d) No; the particle might need to have charge of the opposite sign
- to have a force exerted on it.
- (e) No; an observation of an object with electric charge gives no information about a magnetic field.

Example 5

Classify each of the following statements as a characteristic (a) of electric forces only, (b) of magnetic forces only, (c) of both electric and magnetic forces, or (d) of neither electric nor magnetic forces.

(1) The force is proportional to the magnitude of the field exerting it.

(2) The force is proportional to the magnitude of the charge of the object on which the force is exerted.

(3) The force exerted on a negatively charged object is opposite in direction to the force on a positive charge.

(4) The force exerted on a stationary charged object is nonzero.

(5) The force exerted on a moving charged object is zero.

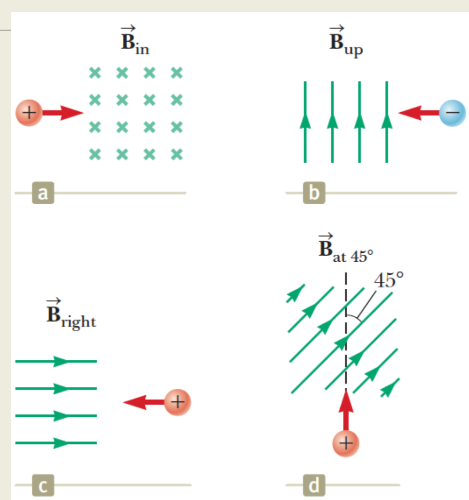
(6) The force exerted on a charged object is proportional to its speed.

(7) The force exerted on a charged object cannot alter the object's speed.

(8) The magnitude of the force depends on the charged object's direction of motion.

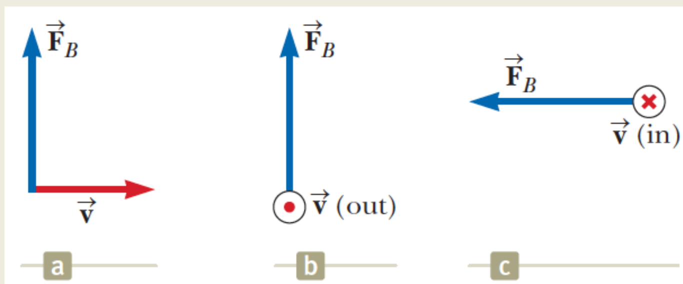
Example 6

Determine the initial direction of the deflection of charged particles as they enter the magnetic fields shown in the Figure



Example 7

Find the direction of the magnetic field acting on a positively charged particle moving in the various situations shown in the Figure if the direction of the magnetic force acting on it is as indicated.



Solve by Your self

- Two charged particles are projected in the same direction into a magnetic field perpendicular to their velocities. If the particles are deflected in opposite directions, what can you say about them?
- How can the motion of a moving charged particle be used to distinguish between a magnetic field and an electric field?
- Can a constant magnetic field set into motion an electron initially at rest? Explain your answer.

Solve by Your self

- A proton travels with a speed of 5.02×10^6 m/s in a direction that makes an angle of 60.0^o with the direction of a magnetic field of magnitude 0.180 T in the positive x direction. What are the magnitudes of (a) the magnetic force on the proton and (b) the proton's acceleration?

- A proton moves perpendicular to a uniform magnetic field **B** at a speed of 1.00×10^7 m/s and experiences an acceleration of 2.00×10^{13} m/s^2 in the positive x direction when its velocity is in the positive z direction. Determine the magnitude and direction of the field.

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Magnetism and Alternating Current

Unit 1: Magnetic Fields

Lecture 2: Motion of a charged particle in a uniform magnetic field

Dr. Hazem Falah Sakeek
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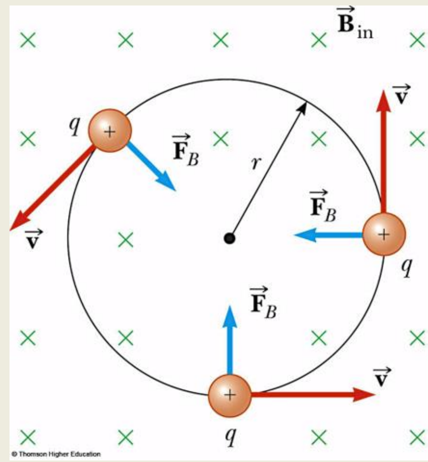
Unit 1: Magnetic Fields

Charged Particle in a Magnetic Field

Consider a +ve charged particle moving in an external magnetic field with its velocity perpendicular to the field.

The magnetic force is always directed toward the center of the circular path.

The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle.



We use the particle under a net force model to write Newton's second law for the particle:

$$F = F_B = ma$$

Because the particle moves in a circle, we also model it as a particle in uniform circular motion and we replace the acceleration with centripetal acceleration:

$$F_B = qvB = \frac{mv^2}{r}$$

This expression leads to the following equation for the radius of the circular path:

$$r = \frac{mv}{qB}$$ **Radius of the circular path**

The **radius** of the path is proportional to the linear momentum *mv* of the particle and inversely proportional to the magnitude of the charge *q* on the particle and to the magnitude of the magnetic field B.

The angular speed of the particle

$$r = \frac{mv}{qB}$$

$$= \frac{v}{r} = \frac{qB}{m}$$

angular speed

The period of the motion (the time interval the particle requires to complete one revolution) is equal to the circumference of the circle divided by the speed of the particle:

$$T = \frac{2\ \ r}{v} = \frac{2}{\ } = \frac{2\ \ m}{qB}$$

period of the motion

These results show that the angular speed of the particle and the period of the circular motion do not depend on the speed of the particle or on the radius of the orbit.

The angular speed ω is often referred to as the cyclotron frequency because charged particles circulate at this angular frequency in the type of accelerator called a cyclotron.

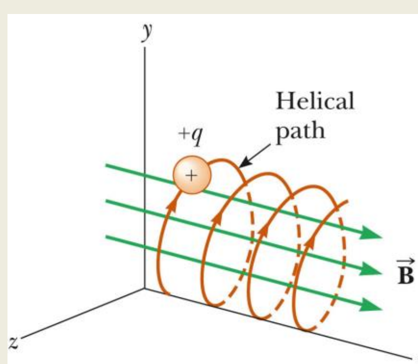
General Case

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to **B**, its path is a helix.

Same equations apply,

with

$$V = \sqrt{V_y^2 + V_z^2}$$



Example 1

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. **Find the speed of the proton.**

Solution

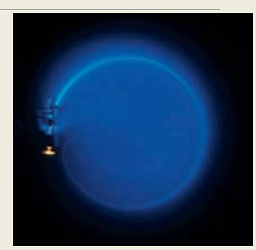
$$= \frac{v}{r} = \frac{qB}{m} \quad \Rightarrow \quad v = \frac{qBr}{m}$$

$$v = \frac{(1.60 \times 10^{-19}\ \mathrm{C})(0.35\ \mathrm{T})(0.14\ \mathrm{m})}{1.67 \times 10^{-27}\ \mathrm{kg}}$$

$$= 4.7 \times 10^{6}\ \mathrm{m/s}$$

Example 2

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm.



(A) What is the magnitude of the magnetic field?

(B) What is the angular speed of the electrons?

Solution (A) the magnitude of the magnetic field

$$\Delta K + \Delta U = 0$$

$$(\tfrac{1}{2}m_e v^2 - 0) + (q\,\Delta V) = 0$$

$$v = \sqrt{\frac{-2q\,\Delta V}{m_e}}$$

$$v = \sqrt{\frac{-2(-1.60 \times 10^{-19}\text{ C})(350\text{ V})}{9.11 \times 10^{-31}\text{ kg}}} = 1.11 \times 10^{7}\text{ m/s}$$

$$r = \frac{mv}{qB}$$

$$B = \frac{m_e v}{er}$$

$$B = \frac{m_e v}{er}$$

$$B = \frac{(9.11 \times 10^{-31}\text{ kg})(1.11 \times 10^{7}\text{ m/s})}{(1.60 \times 10^{-19}\text{ C})(0.075\text{ m})} = 8.4 \times 10^{-4}\text{ T}$$

Solution B the angular speed of the electrons

$$\omega = \frac{v}{r} = \frac{1.11 \times 10^{7}\text{ m/s}}{0.075\text{ m}} = 1.5 \times 10^{8}\text{ rad/s}$$

$$\omega = (1.5 \times 10^{8}\text{ rad/s})(1\text{ rev}/2\pi\text{ rad}) = 2.4 \times 10^{7}\text{ rev/s}$$

Applications

- Velocity Selector
- Mass Spectrometer
- The Cyclotron

Applications involving charged particles moving in a magnetic field

In many applications, charged particles will move in the presence of both magnetic and electric fields.

In that case, the total force is the sum of the forces due to the individual fields.

In general (The Lorentz force):

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Electric Force   Magnetic Force

Velocity Selector

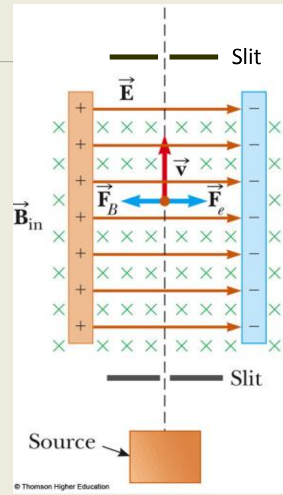
A uniform electric field is perpendicular to a uniform magnetic field.

When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line.

$$qE = qvB$$

This selects particles with velocities of the value

$$v = \frac{E}{B}$$



Mass Spectrometer

A mass spectrometer separates ions according to their mass-to-charge ratio.

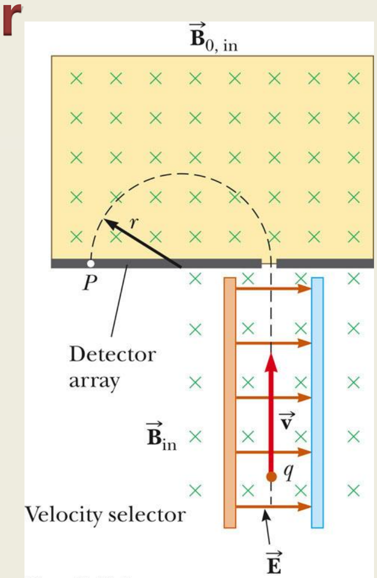
A beam of ions passes through a velocity selector and then enters a second magnetic field where the ions move in a semicircle of radius *r* before striking a detector at P.

From the equation

$$r = \frac{mv}{qB_o}$$

The ratio of m/q

$$\frac{m}{q} = \frac{rB_o}{v}$$



Mass Spectrometer

$$\frac{m}{q} = \frac{rB_o}{v}$$

The velocity is given by the velocity selector of the first part as

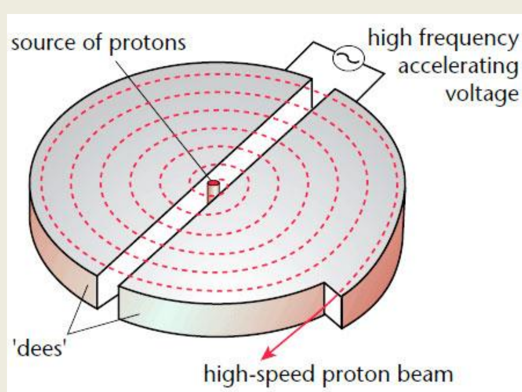
$$v = \frac{E}{B}$$

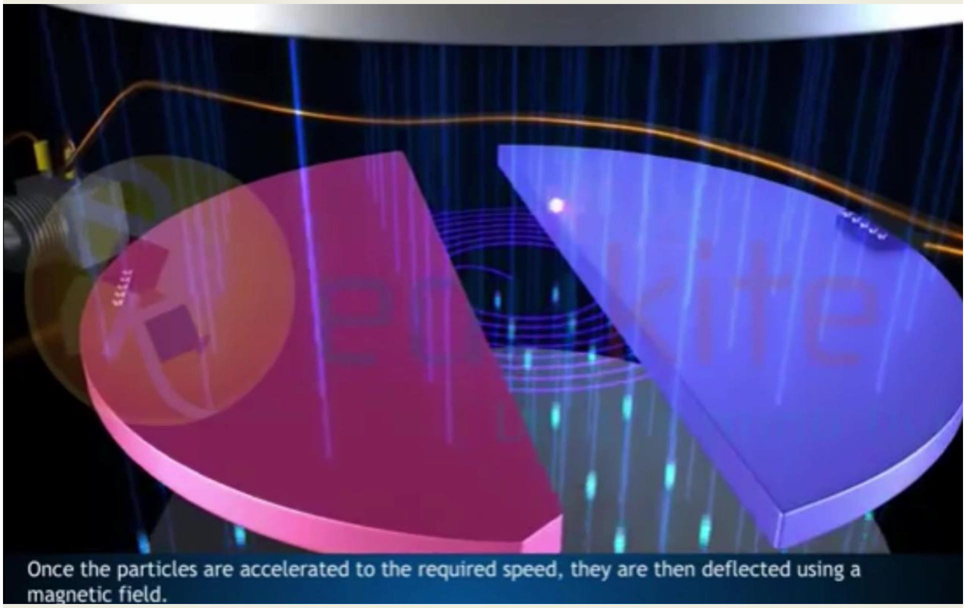
$$\frac{m}{q} = \frac{rB_o B}{E}$$

we can determine **m /q** by measuring the radius of curvature and knowing the field magnitudes **B, B_0** , and **E.**

The Cyclotron

A cyclotron is a device that can accelerate charged particles to very high speeds.





The Cyclotron

We can obtain an expression for the kinetic energy of the ion when it exits the cyclotron in terms of the radius R of the dees.

we know that

$$\frac{m}{q} = \frac{RB_o}{v} \quad \Rightarrow \quad v = \frac{qBR}{m}$$

the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

Solve by your self

1. An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 2.00 mT. If the speed of the electron is 1.50×10^7 m/s, determine (a) the radius of the circular path and (b) the time interval required to complete one revolution.
2. An electron moves in a circular path perpendicular to a constant magnetic field of magnitude 1.00 mT. The angular momentum of the electron about the center of the circle is 4.00×10^{-25} kg.m^2/s. Determine (a) the radius of the circular path and (b) the speed of the electron.
3. Consider the mass spectrometer. The magnitude of the electric field between the plates of the velocity selector is 2.50×10^3 V/m, and the magnetic field in both the velocity selector and the deflection chamber has a magnitude of 0.035T. Calculate the radius of the path for a singly charged ion having a mass $m = 2.18 \times 10^{-26}$ kg.

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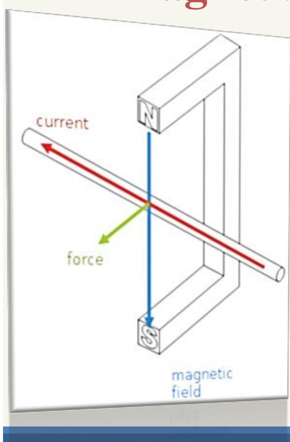
Magnetism and Alternating Current



Unit 1: Magnetic Fields

Lecture 3: Magnetic force acting a current-carrying conductor

Dr. Hazem Falah Sakeek
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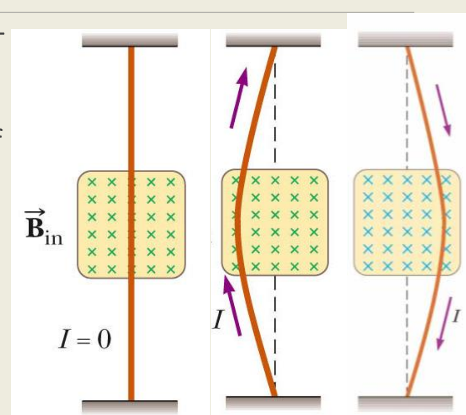
Unit 1: Magnetic Fields

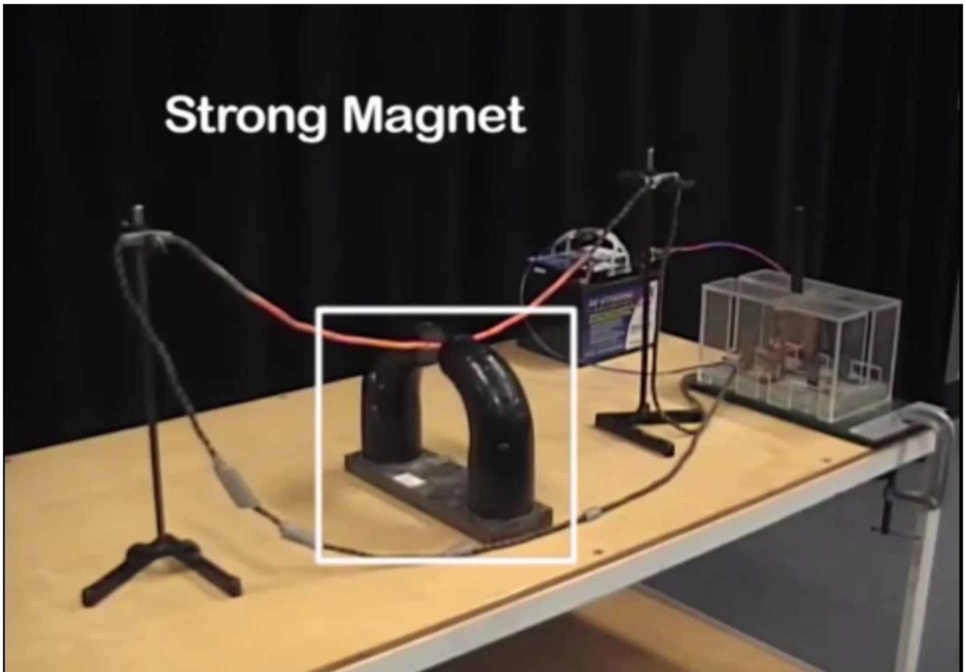
Magnetic Force on a Current Carrying Conductor, a wire

A force is exerted on a current-carrying wire placed in a magnetic field.

- The current is a collection of many charged particles in motion.

The direction of the force is given by the right-hand rule





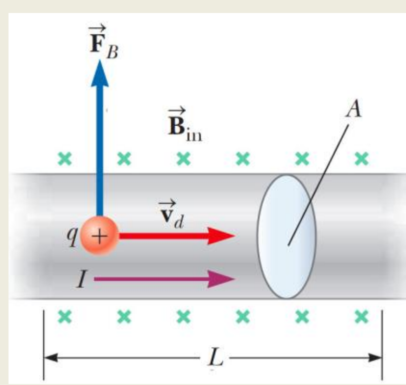
Force on a Wire, the equation

Consider a straight segment of wire of length L and cross-sectional area A carrying a current I in a uniform magnetic field **B.**

The magnetic force exerted on a charge q moving with a drift velocity $\mathbf{v}_d$.

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$$

To find the total force acting on the wire, we multiply the $q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$ force exerted on one charge by the number of charges in the segment.



Force on a Wire, the equation, continue

The number of charges in the segment is nAL, where n is the number of charges per unit volume. Hence, the total magnetic force on the segment of wire of length L is

$$\vec{\mathbf{F}} = \left(q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}\right) nAL$$

the current in the wire is $I = nqv_dA$. Therefore,

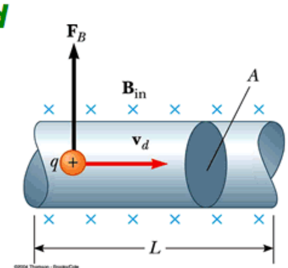
$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

where **L** is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment. This expression applies only to a straight segment of wire in a uniform magnetic field.

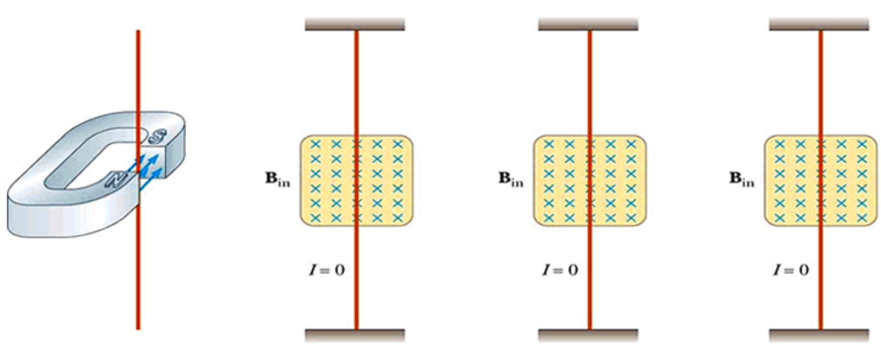
Magnetic Force **Magnetic Field**

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

current in wire



The moving electrons in the wire is immersed in an external B-Field and feels a magnetic force given by the right hand rule as shown.

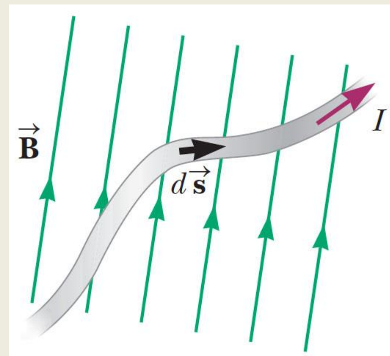


General Equation

Now consider an arbitrarily shaped wire segment of uniform cross section in a magnetic field as shown in the Figure.

The magnetic force exerted on a small segment of vector length **ds** in the presence of a field **B** is,

$$d\vec{\mathbf{F}}_B = I d\vec{s} \times \vec{\mathbf{B}}$$

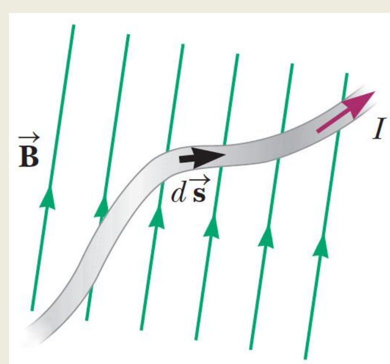


General Equation, continue

To calculate the total force $\mathbf{F_B}$ acting on the wire shown in the Figure, we integrate Equation over the length of the wire:

$$\vec{\mathbf{F}}_B = I \int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

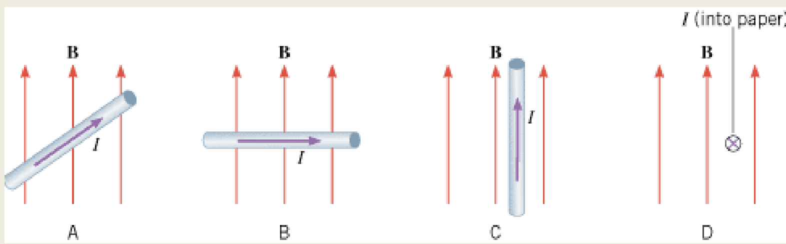
where a and b represent the endpoints of the wire.



Example 1

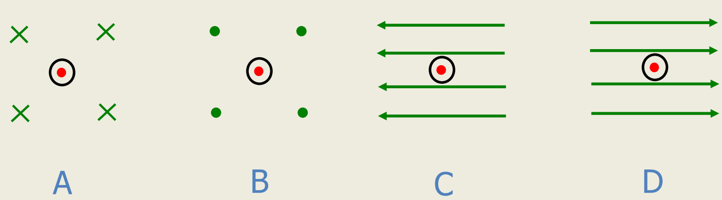
The same current-carrying wire is placed in the same magnetic field **B** in four different orientations.

Rank the orientations according to the magnitude of the magnetic force exerted on the wire, largest to smallest.



Example 2

A straight, horizontal length of copper wire is immersed in a uniform magnetic field. The current through the wire is out of page. Which magnetic field can possibly suspend this wire to balance the gravity?

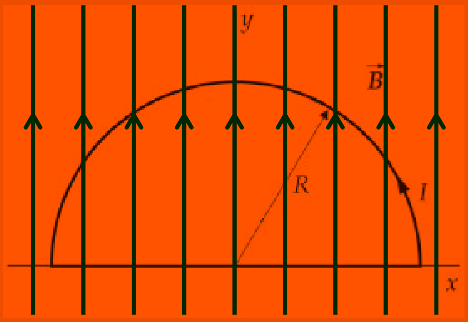


See problem 3 in problems to solve by your self..

Example 3

A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis as in the Figure.

Find (A) the magnitude and direction of the magnetic force acting on the straight portion of the wire and (B) on the curved portion.



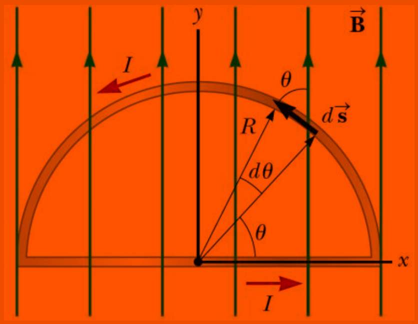
Solution

The force F_1 on the straight portion of the wire is out of the page.

$$\vec{\mathbf{F}}_1 = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

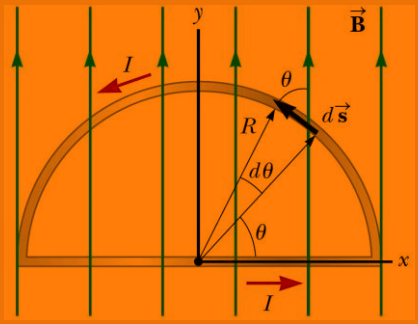
$$= I\int_{-R}^{R} B\, dx\, \hat{\mathbf{k}}$$

$$= 2IRB\,\hat{\mathbf{k}}$$



Solution, continue

The force F_2 on the curved portion is into the page.



$$d\vec{\mathbf{F}}_2 = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$= -IB \sin\theta \; ds \,\hat{\mathbf{k}}$$

$$ds = R \, d\theta$$

$$\vec{\mathbf{F}}_2 = -\int_0^{\pi} IRB \sin\theta \; d\theta \,\hat{\mathbf{k}}$$

$$= -IRB \int_0^{\pi} \sin\theta \; d\theta \,\hat{\mathbf{k}}$$

$$= -IRB[-\cos\theta]_0^{\pi} \,\hat{\mathbf{k}}$$

$$= IRB(\cos\pi - \cos 0)\hat{\mathbf{k}}$$

$$= IRB(-1 - 1)\hat{\mathbf{k}} = -2IRB\,\hat{\mathbf{k}}$$

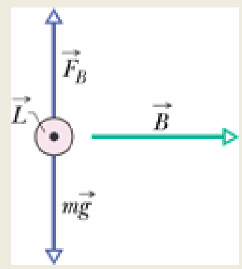
The force on the curved portion is the same in magnitude as the force on a straight wire between the same two points.

The net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$$

Solve by your self

1. A conductor carrying a current I=15.0 A is directed along the positive x axis and perpendicular to a uniform magnetic field. A magnetic force per unit length of 0.120 N/m acts on the conductor in the negative y direction. Determine (a) the magnitude and (b) the direction of the magnetic field in the region through which the current passes.
2. A wire carries a steady current of 2.40 A. A straight section of the wire is 0.750 m long and lies along the x axis within a uniform magnetic field, B = 1.60k T. If the current is in the positive x direction, what is the magnetic force on the section of wire?
3. A straight, horizontal length of copper wire has a current i=28 A through it. What are the magnitude and direction of the minimum magnetic field needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.



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Magnetism and Alternating Current

Unit 1: Magnetic Fields

Lecture 4: Examples

Dr. Hazem Falah Sakeek
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Unit 1: Magnetic Fields

1.1 Magnetic Fields and Forces.

1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

1.3 Applications Involving Charged Particles Moving in a Magnetic Field.

1.4 Magnetic Force Acting on a Current-Carrying Conductor.

1.5 Torque on a Current Loop in a Uniform Magnetic Field.

1.6 The Hall Effect.

Example 1

A proton travels with a speed of 3.00 x 10^6 m/s at an angle of 37.0° with the direction of a magnetic field of 0.300 T in the *y* direction. What are (a) the magnitude of the **magnetic force** on the proton and (b) its **acceleration**?

Solution (a)

$$F_B = qvB \sin\theta$$

$$= (1.60 \times 10^{-19}\ \text{C})(3.00 \times 10^6\ \text{m/s})(3.00 \times 10^{-1}\ \text{T}) \sin 37.0°$$

$$F_B = \boxed{8.67 \times 10^{-14}\ \text{N}}$$

Solution (b)

$$a = \frac{F}{m} = \frac{8.67 \times 10^{-14}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= \boxed{5.19 \times 10^{13}\ \text{m/s}^2}$$

Example 2

An electron is accelerated through 2400 V from rest and then enters a uniform 1.70-T magnetic field. What are (a) the maximum and (b) the minimum values of the magnetic force this charge can experience?

Solution

We first find the speed of the electron as follow

$$\Delta K = \frac{1}{2} mv^2 = e(\Delta V)$$

$$v = \sqrt{\frac{2e(\Delta V)}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})(2400\ \text{J/C})}{(9.11 \times 10^{-31}\ \text{kg})}} = 2.90 \times 10^7\ \text{m/s}$$

(a) $F_{B,\max} = qvB = \boxed{7.90 \times 10^{-12}\ \text{N}}$ When θ = 90°

(b) $F_{B,\min} = \boxed{0}$ When θ = 0 or 180°

Example 3

A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

Solution

$$F_B = ILB \sin\theta$$

$$mg = ILB \sin\theta$$

$$\frac{m}{L} g = IB \sin\theta$$

$$\frac{m}{L} = (0.500 \text{ g / cm})\left(\frac{100 \text{ cm / m}}{1000 \text{ g / kg}}\right)$$

$$= 5.00 \times 10^{-2} \text{ kg / m}$$

$$(5.00 \times 10^{-2})(9.80) = (2.00)B \sin 90.0°$$

$$B = \boxed{0.245 \text{ Tesla}}$$

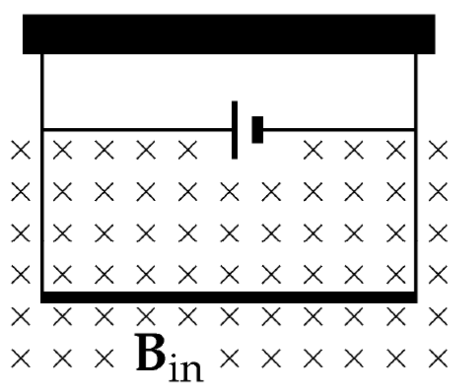
To the east

Example 4

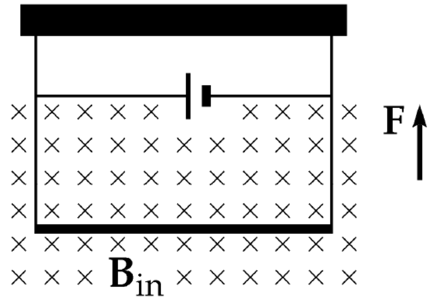
A conductor suspended by two flexible wires as shown in the Figure, has a mass per unit length of 0.040 kg/m.

What current must exist in the conductor in order for the tension in the supporting wires to be zero when the magnetic field is 3.60 T into the page?

What is the required direction for the current?



Solution 4



$$\frac{|\mathbf{F}_B|}{L} = \frac{mg}{L} = \frac{I|\mathbf{L}\times\mathbf{B}|}{L}$$

$$I = \frac{mg}{BL}$$

$$= \frac{(0.0400\ \text{kg/m})(9.80\ \text{m/s}^2)}{3.60\ \text{T}}$$

$$= \boxed{0.109\ \text{A}}$$

The direction of I in the bar is to the right

Example 5

A singly charged positive ion has a mass of $3.20\text{x}10^{-26}$ kg. After being accelerated from rest through a potential difference of 833 V, the ion enters a magnetic field of 0.920T along a direction perpendicular to the direction of the field. Calculate the radius of the path of the ion in the field.

Solution

$$\frac{1}{2}mv^2 = q(\Delta V)$$

$$\frac{1}{2}(3.20\times10^{-26}\ \text{kg})\,v^2 = (1.60\times10^{-19}\ \text{C})(833\ \text{V})$$

$$v = 91.3\ \text{km/s}$$

Solution 5, continue

The magnetic force provides the centripetal force:

$$qvB\sin\theta = \frac{mv^2}{r}$$

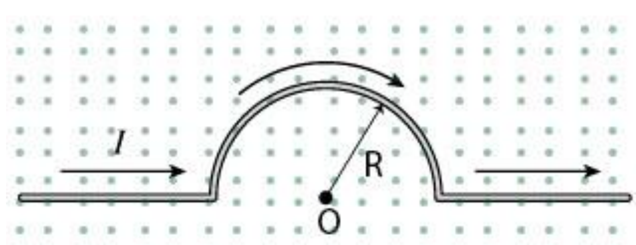
$$r = \frac{mv}{qB\sin 90.0°}$$

$$= \frac{(3.20\times10^{-26}\ \text{kg})(9.13\times10^{4}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.920\ \text{N}\cdot\text{s/C}\cdot\text{m})}$$

$$= \boxed{1.98\ \text{cm}}$$

Homework

A wire bent as shown in the figure carries a current I and is placed in a uniform magnetic field B that emerges from the plane of the figure. Calculate the force acting on the wire.



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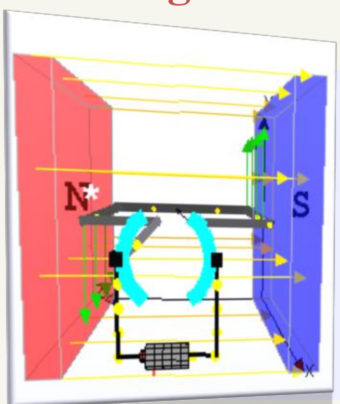
Magnetism and Alternating Current



Unit 1: Magnetic Fields

Lecture 5: Torque on a Current loop in a Uniform Magnetic Field

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 1: Magnetic Fields

1.1 Magnetic Fields and Forces.

1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

1.3 Applications Involving Charged Particles Moving in a Magnetic Field.

1.4 Magnetic Force Acting on a Current-Carrying Conductor.

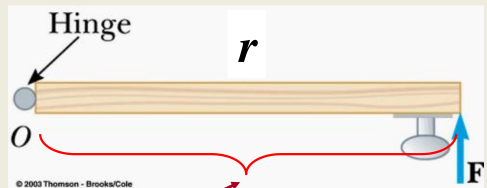
1.5 Torque on a Current Loop in a Uniform Magnetic Field.

1.6 The Hall Effect.

What is Torque

Torque, τ, is the force to rotate an object about some axis

$$\tau = Fr$$



SI unit: [N m]

- τ is the torque
- r is the *lever arm* (or moment arm)
- F is the force

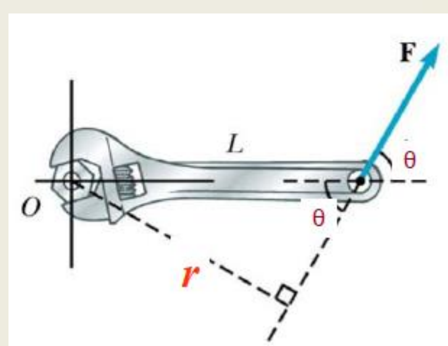
The lever arm is the distance between the axis of rotation and the "line of action".

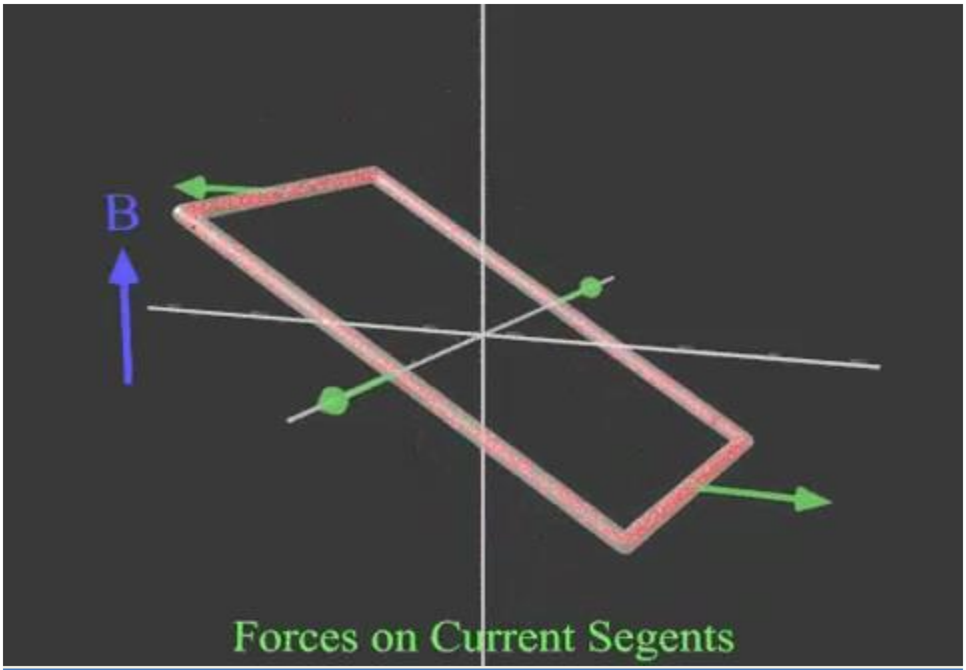
The lever arm, r, is the shortest *(perpendicular)* distance from the axis of rotation to a line drawn along the direction of the force

r = L sin θ

$$\tau = rF\sin\theta$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

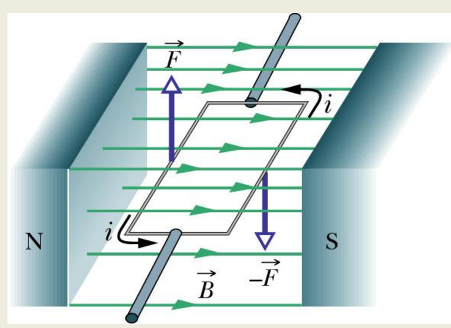




Torque on a Current Carrying Coil

With the knowledge that a force is exerted on a current-carrying conductor when the conductor is placed in an external magnetic field, this force can produce a torque on a current loop placed in the magnetic field.

Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field in the plane of the loop.



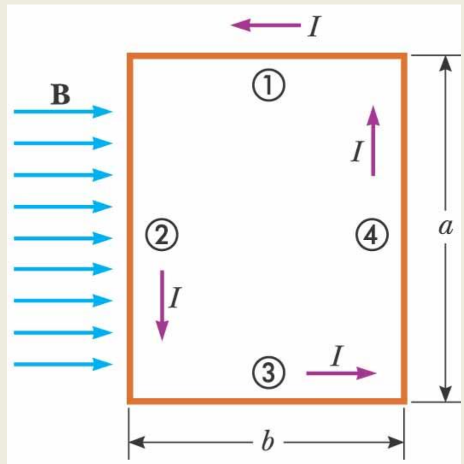
Torque on a Current Carrying Coil

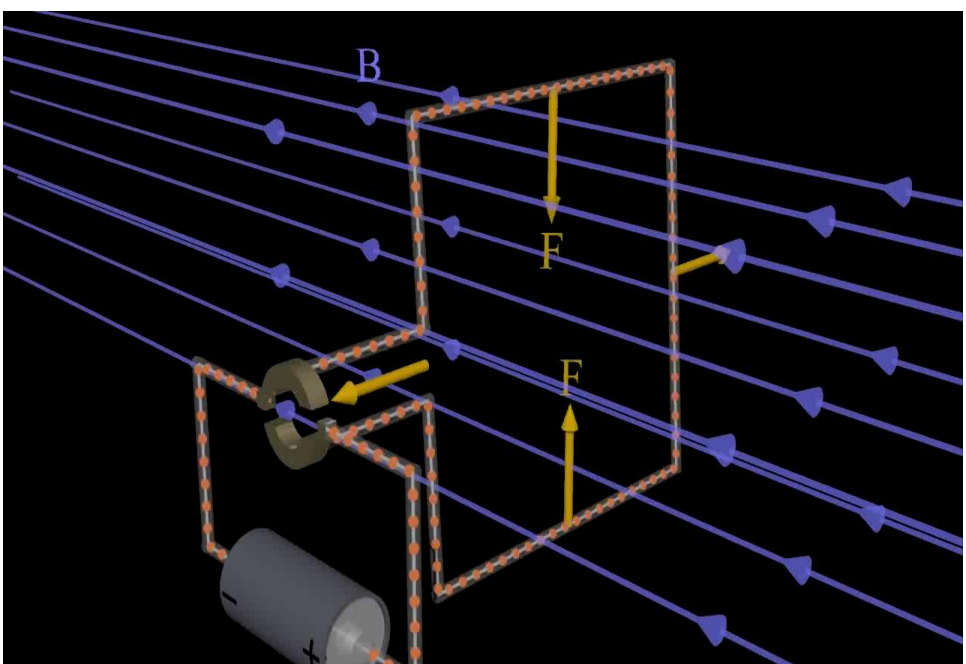
No magnetic forces act on sides (1) and (3) because these wires are parallel to the field; hence,

$$F_1 = F_3 = 0$$

Magnetic forces act on sides (2) and (4) because these sides are oriented perpendicular to the field.

$$F_2 = F_4 = IaB$$





If viewing the loop from the end, and we assume that the loop is pivoted so that it can rotate about point O, the two forces produce a torque about O that rotates the loop clockwise.

The magnitude of the torque, which is maximum in this position, is:

$$\tau_{max} = F_2 \cdot \frac{b}{2} + F_4 \cdot \frac{b}{2}$$

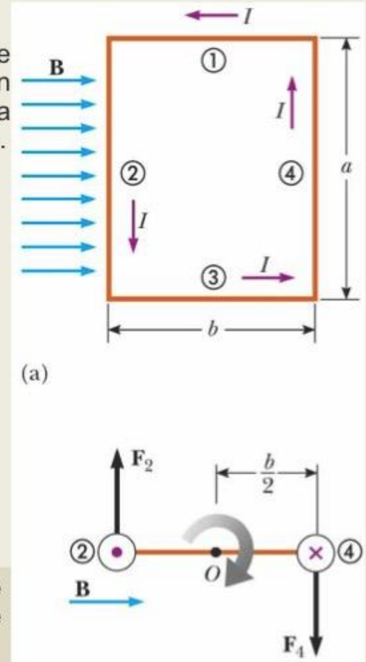
$$\tau_{max} = I \cdot a \cdot B \cdot \frac{b}{2} + I \cdot a \cdot B \cdot \frac{b}{2}$$

$$\tau_{max} = I \cdot a \cdot b \cdot B$$

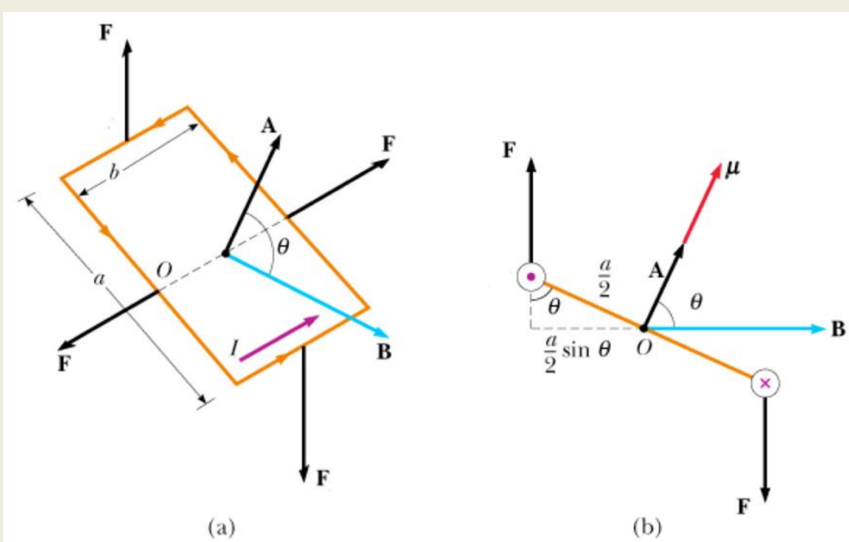
The area of the loop   A = a·b;

$$\tau_{max} = I \cdot A \cdot B$$

If the current direction were reversed, the forces would reverse their directions and the rotational tendency would be counterclockwise.



Suppose the magnetic field makes an angle θ with respect to a line perpendicular to the plane of the loop (the dashed line).

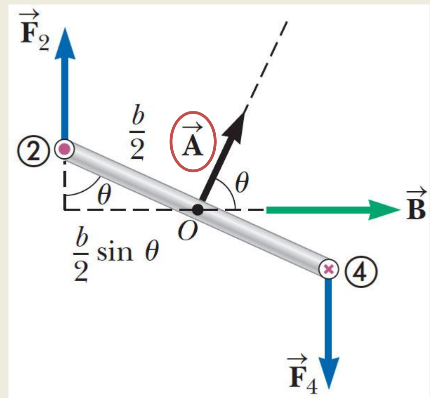


Torque on a Current Carrying Coil, General Case

The vector **A** which is perpendicular to the plane of the loop and its magnitude is the area A of the loop.

Only the forces F_2 and F_4 contribute to the torque about the axis of rotation O.

The other two forces on the loop would not produce a rotation as these forces would be equal in magnitude and opposite in direction and would also pass through the axis of rotation O, making the torque arm 0 m.



Torque on a Current Carrying Coil, General Case

The magnitude of the net torque about O is

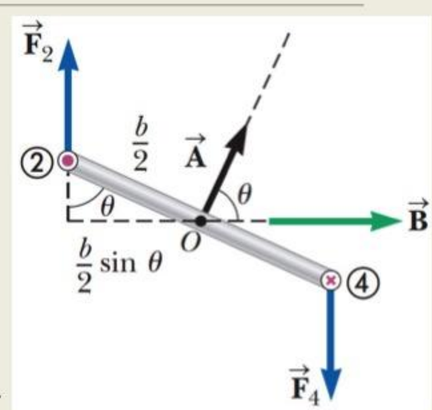
$$\tau = F_2 \cdot \frac{b}{2}\sin\theta + F_4 \cdot \frac{b}{2}\sin\theta$$

$$\tau = IaB\left(\frac{b}{2}\sin\theta\right) + IaB\left(\frac{b}{2}\sin\theta\right)$$

$$\tau = IabB\sin\theta$$

where A = ab is the area of the loop.

$$\tau = IAB\sin\theta$$

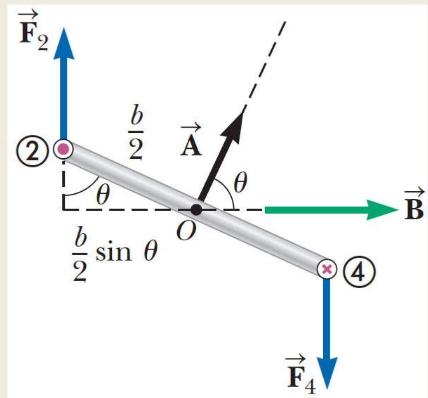


The torque has a maximum value IAB when the magnetic field is parallel to the plane of the loop (angle θ between **A** and **B** = 90°).

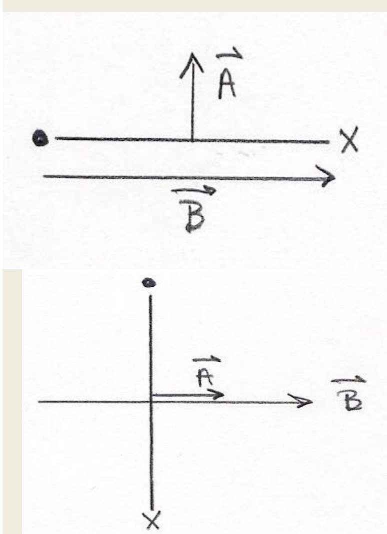
The torque is 0 N·m when the magnetic field is perpendicular to the plane of the loop (angle θ between **A** and **B** = 0° = 180°).

Torque on a current loop in a magnetic field

$$\vec{\tau} = I\vec{A} \times \vec{B}$$



Important notice



Maximum torque occurs when the plane of the loop is parallel to the magnetic field B.

- Angle between plane of loop and B is 0°.
- Angle between area vector A and B is 90°.

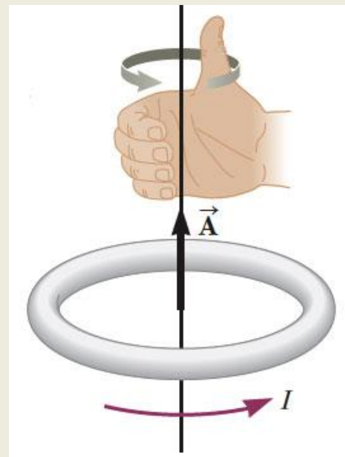
Zero torque occurs when the plane of the loop is perpendicular to the magnetic field B.

- Angle between plane of loop and B is 90°.
- Angle between area vector A and B is 0°.

Direction of area vector A

The direction of the area vector **A** is determined by the right hand rule:

Rotate the fingers of the right hand in the direction of the current in the loop, the thumb points in the direction of the area vector **A**.



Magnetic dipole moment

The product I A is defined as the magnetic moment μ of the loop;

$$\vec{\mu} = I\vec{A}$$

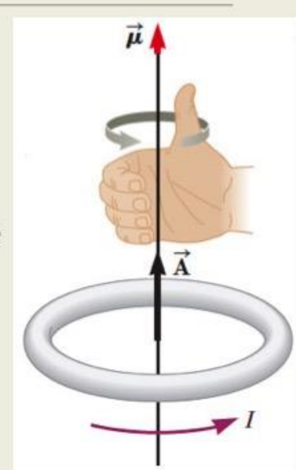
The SI unit of μ is the (A m²).

If a coil of wire contains N loops of the same area, the magnetic moment of the coil is

$$\vec{\mu} = NI\vec{A}$$

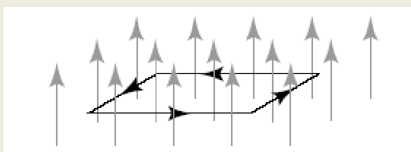
$$\because \tau = IAB\sin\theta \Rightarrow \tau = \mu B\sin\theta$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



Question

A rectangular loop is placed in a uniform magnetic field with the plane of the loop perpendicular to the direction of the field.
If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



1. a net force.
2. a net torque.
3. a net force and a net torque.
4. neither a net force nor a net torque.

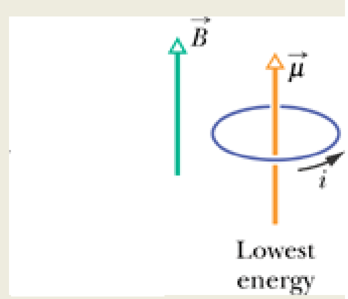
Potential energy of a magnetic dipole in a magnetic field

The potential energy of a system of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field and is given by

$$U = -\vec{\mu} \bullet \vec{B} = -\mu B \cos\theta$$

The system has its lowest U when μ points in the direction of B; the angle between μ and B = 0°.

The system has its highest U when μ points in the opposite direction of B; the angle between μ and B = 180°.



Derive U = -μ.B

Work = Torque x angular displacement θ

For a displacement from θ_i to θ_f:

$$\int_{\theta_i}^{\theta_f} dW = \int_{\theta_i}^{\theta_f} \tau \, d\theta = \int_{\theta_i}^{\theta_f} NIAB\sin\theta \, d\theta$$

$$W = U_f - U_i = NIAB\int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$$

$$U_f - U_i = NIAB(-\cos\theta)\Big|_{\theta_i}^{\theta_f}$$

$$U_f - U_i = NIAB(-\cos\theta_f - (-\cos\theta_i))$$

$$U_f - U_i = NIAB(-\cos\theta_f + \cos\theta_i)$$

$$U_f - U_i = NIAB(-\cos\theta_f + \cos\theta_i)$$

The term that contains $\cos\theta_i$ is a constant that depends on the initial orientation of the dipole.

It is convenient to choose a reference angle of $\theta_i = 90°$ so that $\cos\theta_i = \cos 90° = 0$.

Furthermore, let's choose $U_i = 0$ at $\theta_i = 90°$ as our reference value of potential energy. Hence, we can express a general value of $U = U_f$ as

$$U = -\mu B\cos\theta$$

$$U = -\vec{\mu} \bullet \vec{B}$$

Example 1

A rectangular coil of dimensions 5.40 cm 3 8.50 cm consists of 25 turns of wire and carries a current of 15.0 mA. A 0.350-T magnetic field is applied parallel to the plane of the coil.

(A) Calculate the magnitude of the magnetic dipole moment of the coil.

(B) What is the magnitude of the torque acting on the loop?

Solution

(A) the magnetic dipole moment of the coil.

$$\mu_{\text{coil}} = NIA = (25)(15.0 \times 10^{-3}\text{ A})(0.054\,0\text{ m})(0.085\,0\text{ m})$$

$$= 1.72 \times 10^{-3}\text{ A}\cdot\text{m}^2$$

(B) the magnitude of the torque acting on the loop

$$\tau = \mu_{\text{coil}}B = (1.72 \times 10^{-3}\text{ A}\cdot\text{m}^2)(0.350\text{ T})$$

$$= 6.02 \times 10^{-4}\text{ N}\cdot\text{m}$$

Solve by your self

1. A current of 17.0 mA is maintained in a single circular loop of 2.00 m circumference. A magnetic field of 0.800 T is directed parallel to the plane of the loop. (a) Calculate the magnetic moment of the loop. (b) What is the magnitude of the torque exerted by the magnetic field on the loop?
2. A 50.0-turn circular coil of radius 5.00 cm can be oriented in any direction in a uniform magnetic field having a magnitude of 0.500 T. If the coil carries a current of 25.0 mA, find the magnitude of the maximum possible torque exerted on the coil.
3. A wire is formed into a circle having a diameter of 10.0 cm and is placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A. Find (a) the maximum torque on the wire and (b) the range of potential energies of the wire–field system for different orientations of the circle.

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Magnetism and Alternating Current

Unit 1: Magnetic Fields

Lecture 6: The Hall Effect

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 1: Magnetic Fields

1.1 Magnetic Fields and Forces.

1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

1.3 Applications Involving Charged Particles Moving in a Magnetic Field.

1.4 Magnetic Force Acting on a Current-Carrying Conductor.

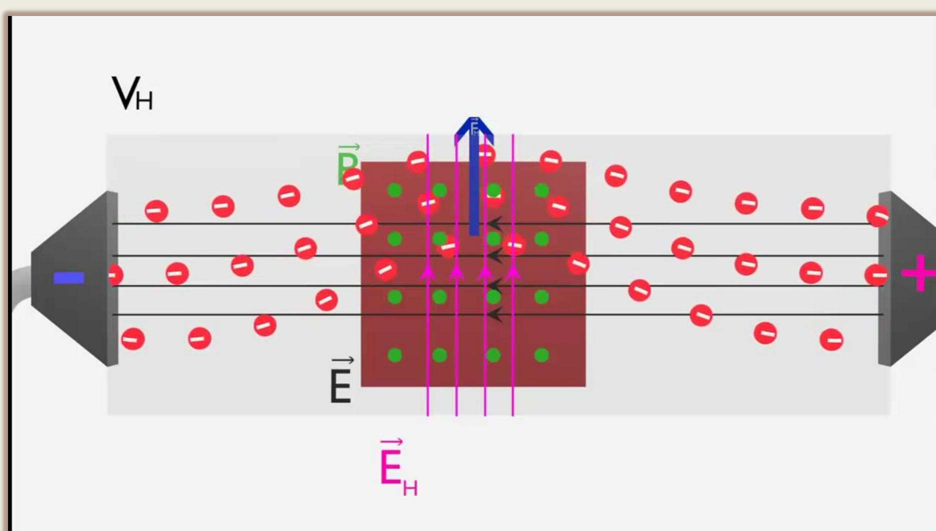
1.5 Torque on a Current Loop in a Uniform Magnetic Field.

1.6 The Hall Effect.

Hall effect

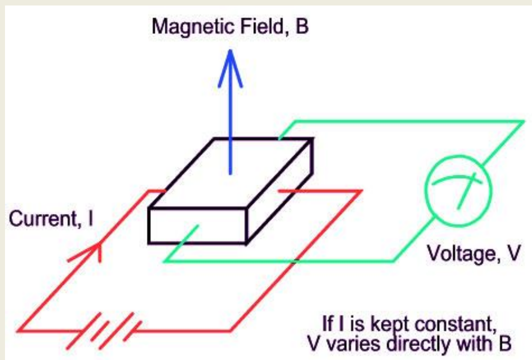
The Hall effect is the production of a voltage difference (the Hall voltage) across a current carrying conductor (in presence of magnetic field), perpendicular to both current and the magnetic field.

The Hall effect was discovered in 1879 by Edwin Hall while working on his doctoral degree at the Johns Hopkins University in Baltimore, Maryland, USA.



Observing the Hall effect

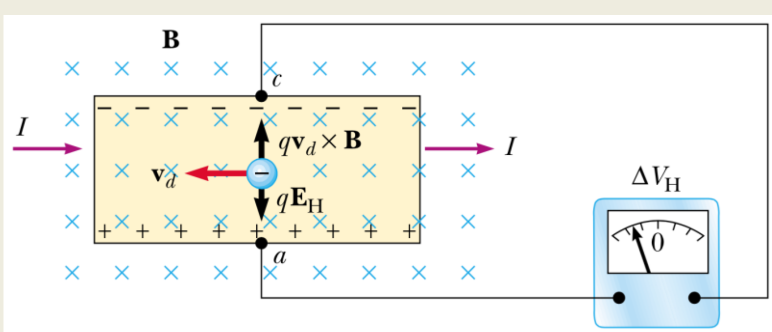
The arrangement for observing the Hall effect consists of a flat conductor carrying a current I in the x direction. A uniform magnetic field B is applied in the y direction.



If the charge carriers are electrons moving in the negative x direction with a drift velocity $\mathbf{v}_d$, they experience an upward magnetic force

$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

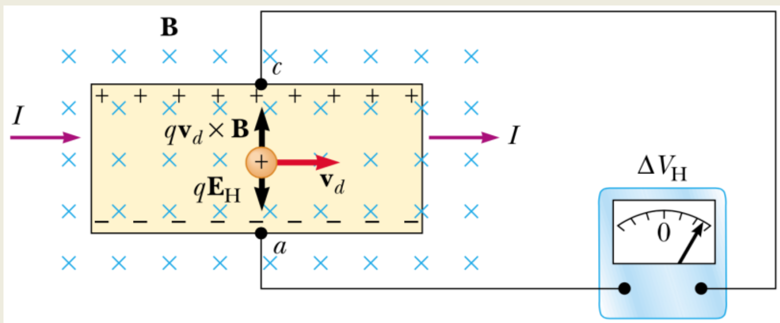
are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge.



This accumulation of charge at the edges establishes an electric field in the conductor and increases until the electric force balances the magnetic force acting on the carriers.

When this equilibrium condition is reached, deflection stops.

A sensitive voltmeter connected across the sample as shown in the Figure can measure the potential difference, known as the Hall voltage **Δ**$\mathbf{V_H}$, generated across the conductor.



Deriving an expression for the Hall voltage

The magnetic force exerted on the carriers has magnitude qv_dB. In equilibrium, this force is balanced by the electric force qE_H , (E_H is the Hall field).

$$qv_dB = qE_H$$

$$E_H = v_dB$$

If d is the width of the conductor, the **Hall voltage** is

$$V_H = E_H d = v_d Bd$$

Therefore, the measured Hall voltage gives a value for the drift speed of the charge carriers if d and B are known

The charge-carrier density n

We can obtain the charge-carrier density n by measuring the current in the sample. From Equation of the drift velocity and the current, we can express the drift speed as

$$v_d = \frac{I}{nqA}$$

where A is the cross-sectional area of the conductor.

$$V_H = v_d B d$$

$$V_H = \frac{IBd}{nqA}$$

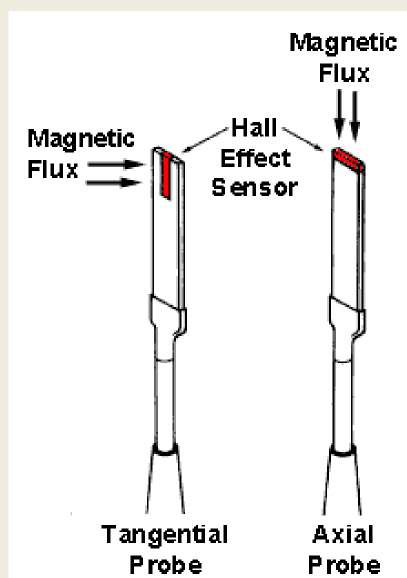
$$V_H = \frac{IBd}{nqA}$$

Because A = td, where t is the thickness of the conductor, we can also express

$$V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

where R_H = 1/**nq** is called the Hall coefficient.

This relationship shows that a properly calibrated conductor can be used to measure the magnitude of an unknown magnetic field.



Hall Effect Measurement Experiment

Example 1

$$V_H = \frac{IB}{nqt}$$

A rectangular copper strip 1.5 cm wide and 0.10 cm thick carries a current of 5.0 A. Find the Hall voltage for a 1.2-T magnetic field applied in a direction perpendicular to the strip.

Solution

Assuming one electron per atom is available for conduction, find the charge-carrier density in terms of the molar mass M and density ρ of copper:

$$n = \frac{N_A}{Volume} = \frac{N_A}{M}$$

$$n = \frac{N_A}{M}$$

$$V_H = \frac{IB}{nqt} = \frac{MIB}{N_A \ qt}$$

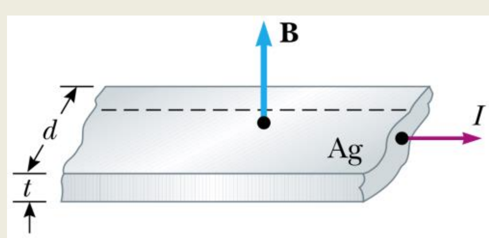
$$\Delta V_H = \frac{(0.063\,5 \text{ kg/mol})(5.0 \text{ A})(1.2 \text{ T})}{(6.02 \times 10^{23} \text{ mol}^{-1})(8\,920 \text{ kg/m}^3)(1.60 \times 10^{-19} \text{ C})(0.001\,0 \text{ m})}$$

$$= 0.44 \ \mu\text{V}$$

Such an extremely small Hall voltage is expected in good conductors. What if the strip has the same dimensions but is made of a semiconductor? Will the Hall voltage be smaller or larger?

Example 2

A flat ribbon of silver having a thickness t = 0.200 mm is used in a Hall-effect measurement of a uniform magnetic field perpendicular to the ribbon, as shown in the Figure. The Hall coefficient for silver is R_H = 0.840 x 10^{10} m^3/C. **(a)** What is the density of charge carriers in silver? **(b)** If a current I = 20.0 A produces a Hall voltage V_H = 15.0 V, what is the magnitude of the applied magnetic field?



Solution (A)

(A) charge carriers in silver

$R_H = 1/nq$

$$n = \frac{1}{qR_H}$$

$$= \frac{1}{(1.60\times10^{-19}\ \mathrm{C})(0.840\times10^{-10}\ \mathrm{m^3/C})}$$

$$= \boxed{7.44\times10^{28}\ \mathrm{m^{-3}}}$$

Solution (B)

(B) magnitude of the applied magnetic field

$$V_H = \frac{IB}{nqt} \quad \longrightarrow \quad B = \frac{nqt(\Delta V_H)}{I}$$

$$= \frac{(7.44\times10^{28}\ \mathrm{m^{-3}})(1.60\times10^{-19}\ \mathrm{C})(0.200\times10^{-3}\ \mathrm{m})(15.0\times10^{-6}\ \mathrm{V})}{20.0\ \mathrm{A}}$$

$$= \boxed{1.79\ \mathrm{T}}$$

Solve by your self

1. A Hall-effect probe operates with a 120-mA current. When the probe is placed in a uniform magnetic field of magnitude 0.0800T, it produces a Hall voltage of 0.700V. (a) When it is measuring an unknown magnetic field, the Hall voltage is 0.330 μV. What is the magnitude of the unknown field? (b) The thickness of the probe in the direction of B is 2.00 mm. Find the density of the charge carriers, each of which has charge of magnitude *e*.

2. In an experiment that is designed to measure the Earth's magnetic field using the Hall effect, a copper bar 0.500 cm thick is positioned along an east–west direction. If a current of 8.00 A in the conductor results in a Hall voltage of 5.10×10^{12} V, what is the magnitude of the Earth's magnetic field? (Assume that $n = 8.49 \times 10^{28}$ electrons/m^3 and that the plane of the bar is rotated to be perpendicular to the direction of **B**.)

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Magnetism and Alternating Current

Unit 1: Magnetic Fields

Lecture 7: Examples 2

Dr. Hazem Falah Sakeek
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1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

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1.6 The Hall Effect.

Example 6

A velocity selector consists of electric and magnetic fields described by the expressions **E** = *E* **k** and **B** = *B* **j**, with *B*= 15.0 mT. Find the value of *E* such that a 750-eV electron moving along the positive *x* axis is undeflected.

$$F_B = F_e$$

$$qvB = qE$$

$$v = \sqrt{2K/m}$$

$$E = vB = \sqrt{\frac{2K}{m}}B$$

$$= \left(\frac{2(750)\left(1.60\times10^{-19}\right)}{9.11\times10^{-31}}\right)^{1/2}(0.0150) \quad = \boxed{244\ \text{kV/m}}$$

Example 7

Consider the mass spectrometer. The magnitude of the electric field between the plates of the velocity selector is 2500 V/m, and the magnetic field in both the velocity selector and the deflection chamber has a magnitude of 0.035 T. Calculate the radius of the path for a singly charged ion having a mass *m* = 2.18 x 10^{26} kg.

Solution

In the velocity selector:

$$v = \frac{E}{B} = \frac{2500\ \text{V/m}}{0.0350\ \text{T}} = 7.14\times10^4\ \text{m/s}$$

In the deflection chamber:

$$r = \frac{mv}{qB}$$

$$= \frac{\left(2.18\times10^{-26}\ \text{kg}\right)\left(7.14\times10^4\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)(0.0350\ \text{T})} \quad = \boxed{0.278\ \text{m}}$$

Example 8

A small bar magnet is suspended in a uniform 0.250 T magnetic field. The maximum torque experienced by the bar magnet is 4.60×10^{-3} Nm. Calculate the magnetic moment of the bar magnet.

Solution:

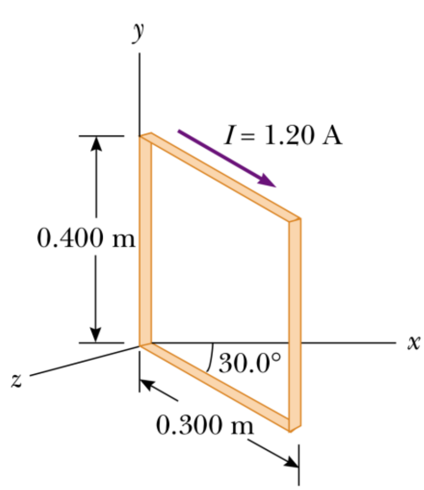
$$\tau = \mu B \sin\theta$$

$$4.60\times10^{-3}\ \mathrm{N\cdot m} = \mu(0.250)\sin 90.0°$$

$$\mu = 1.84\times10^{-2}\ \mathrm{A\cdot m^2}$$

Example 9

A rectangular coil consists of N=100 closely wrapped turns and has dimensions a=0.400 m and b=0.300 m. The coil is hinged along the y axis, and its plane makes an angle $\phi = 30.0°$ with the x axis. What is the magnitude of the torque exerted on the coil by a uniform magnetic field $\vec{B}$ =0.800 T directed along the x axis when the current is I =1.20 A in the direction shown? What is the expected direction of rotation of the coil?



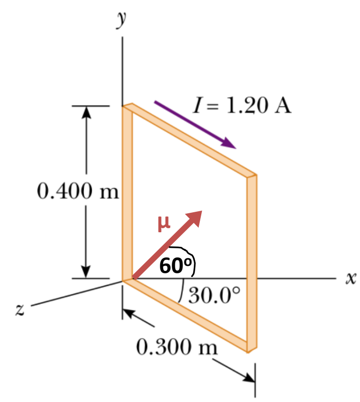
Solution 9

Note that ϕ=60º is the angle between the magnetic moment and the B field.

$$\tau = NBAI \sin\phi$$

$$\tau = 100(0.800\ \text{T})\left(0.400 \times 0.300\ \text{m}^2\right)(1.20\ \text{A})\sin 60°$$

$$\tau = \boxed{9.98\ \text{N}\cdot\text{m}}$$



Direction of rotation of the coil is in clockwise in order the *u* align with B

Example 10

A Hall-effect probe operates with a 120-mA current. When the probe is placed in a uniform magnetic field of magnitude 0.0800T, it produces a Hall voltage of 0.700μV.

(a) When it is measuring an unknown magnetic field, the Hall voltage is 0.330 μV. What is the magnitude of the unknown field?

(b) The thickness of the probe in the direction of B is 2.00 mm. Find the density of the charge carriers, each of which has charge of magnitude *e*.

Solution 10

(a) To find the unknown magnetic field B_2

B_1 = 0.0800 T produces a V_{H1} = 0.700 μV

B_2 = ???? produces a V_{H2} = 0.330 μV

Therefore B_2 = 0.0377 T

(b) The density of the charge carriers

$$\Delta V_H = \frac{IB}{nqt} \rightarrow \frac{nqt}{I} = \frac{B}{\Delta V_H} = \frac{0.0800\ \text{T}}{0.700 \times 10^{-6}\ \text{V}} = 1.14 \times 10^5\ \text{T/V}$$

$$n = \left(1.14 \times 10^5\ \text{T/V}\right)\frac{I}{qt}$$

$$n = \left(1.14 \times 10^5\ \text{T/V}\right)\frac{0.120\ \text{A}}{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{-3}\ \text{m}\right)}$$

$$= \boxed{4.29 \times 10^{25}\ \text{m}^{-3}}$$

Homework

1. A cyclotron designed to accelerate protons has a magnetic field of magnitude 0.450 T over a region of radius 1.20 m. What are (a) the cyclotron frequency and (b) the maximum speed acquired by the protons?

2. A wire is formed into a circle having a diameter of 10.0 cm and placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A. Find (a) the maximum torque on the wire and (b) the range of potential energies of the wire–field system for different orientations of the circle.

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Magnetism and Alternating Current

Unit 2: Sources of the Magnetic Field

Lecture 8: The Biot–Savart Law

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Unit 2: Sources of the Magnetic Field

2.1 The Biot–Savart Law

2.2 The Magnetic Force Between Two Parallel Conductors

2.3 Ampère's Law

2.4 The Magnetic Field of a Solenoid

2.5 Gauss's Law in Magnetism

2.6 Magnetism in Matter

About Unit 2 Sources of the Magnetic Field

- Explores the origin of the magnetic field.
- How to use the Biot-Savart law to calculate the magnetic field produced at some point in space by a various current distributions.
- Determine the force between two current-carrying conductors.
- Calculating the magnetic field of a highly symmetric configuration carrying a steady current using Ampère's law.

Magnetic Fields Produced by Currents

When studying magnetic forces so far, we examined how a magnetic field, presumably produced by a permanent magnet, affects moving charges and currents in a wire.

Now we consider the phenomenon in which a current carrying wire produces a magnetic field.

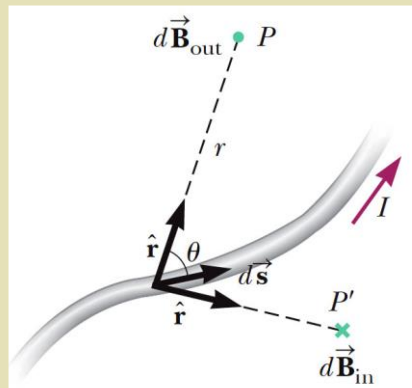
Hans Christian Oersted first discovered this effect in 1820 when he observed that a current carrying wire influenced the orientation of a compass needle.

Biot-Savart experimental results

Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field.

(1) The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\mathbf{r}$ directed from dS toward P.

(2) The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.



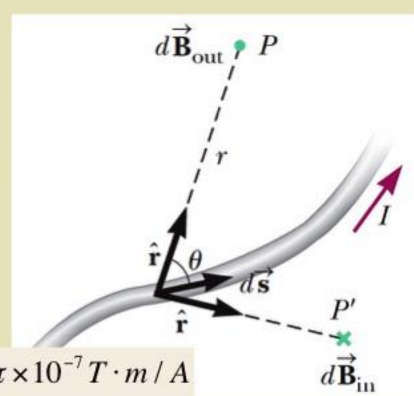
Biot-Savart experimental results

(3) The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude ds.

(4) The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vector dS and $\mathbf{r}$.

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$$

μ_o = permeability of free space = $4\pi\times10^{-7}\,T\cdot m/A$



Biot-Savart Law

Total magnetic field **B** created at some point by a current of finite size, we must sum up contributions from all current elements *I* d**s** that make up the current.

$$d\vec{B} = \frac{\mu_o}{4\pi}\int \frac{I\,d\vec{s}\times\hat{r}}{r^2}$$

Using Biot-Savart Law, determine the magnetic field at a distance a away from a current carrying wire lying along the x axis.

Similarities and differences between the magnetic field due to a current element and for the electric field due to a point charge.

$$d\vec{B} = \frac{\mu_o}{4\pi}\int \frac{I\,d\vec{s}\times\hat{r}}{r^2} \qquad E = \frac{1}{4\ \ _o}\frac{q}{r^2}$$

The magnitude of the magnetic field varies as the inverse square of the distance from the source, as does the electric field due to a point charge.

The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element *d***s** and the unit vector **r.**

An electric field is established by an isolated electric charge. The Biot–Savart law gives the magnetic field of an isolated current element at some point, such an isolated current element cannot exist. Hence we must integrate over the current distribution.

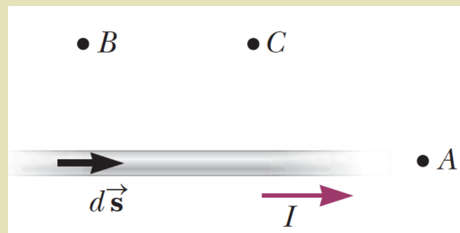
Exercise

Rank the points *A, B,* and *C* in terms of magnitude of the magnetic field that is due to the current in just the length element *d***s** shown from greatest to least.

(1) C > B > A

(2) B > C > A

(3) A > C > B

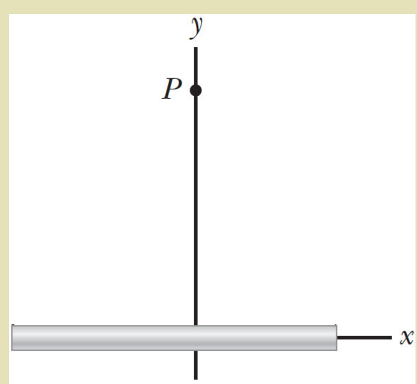


Example 1

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Consider a thin, straight wire carrying a constant current *I* and placed along the *x* axis as shown in the Figure.

Determine the magnitude and direction of the magnetic field at point *P* due to this current.

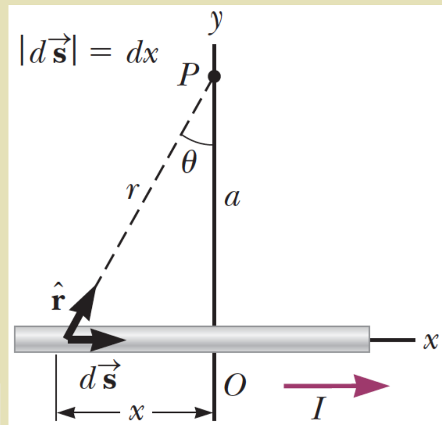


Solution

$$d\vec{B} = \frac{\mu_o}{4\pi}\int \frac{I\, d\vec{s}\times\hat{r}}{r^2}$$

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|\hat{\mathbf{k}}$$
$$= \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}}$$
$$= (dx \cos\theta)\hat{\mathbf{k}}$$

$$d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}}$$
$$= \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$



$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

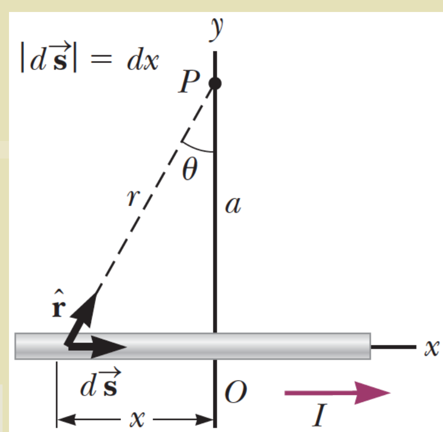
$$r = \frac{a}{\cos\theta}$$

$$x = -a\tan\theta$$

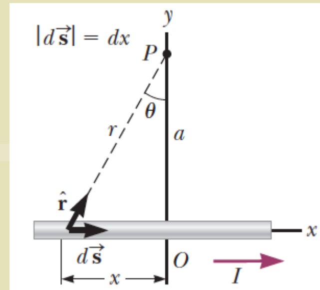
The negative sign because ds is located at a negative value of x.

$$dx = -a\sec^2\theta\, d\theta = -\frac{a\, d\theta}{\cos^2\theta}$$

$$dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\, d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta$$



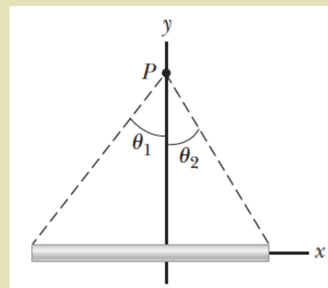
$$dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta$$



Integrate over all length elements on the wire, where the subtending angles range from θ_1 to θ_2

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta$$

$$= \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

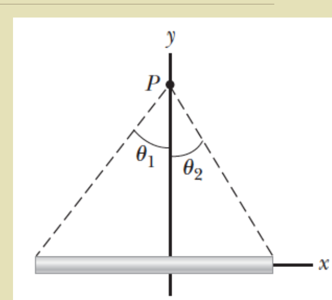


The direction of the magnetic field at point P is out of the page

Special case infinitely long, straight wire

Consider the special case of an infinitely long, straight wire.

If the wire in becomes infinitely long, $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$ for length elements ranging between positions $x = -\infty$ and $x = +\infty$.



Because

$(\sin\theta_1 - \sin\theta_2) = [\sin\pi/2 - \sin(-\pi/2)] = 2$

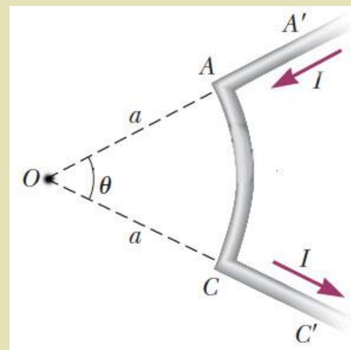
$$B = \frac{\mu_0 I}{2\pi a}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Example 2

Calculate the magnetic field at point *O* for the current-carrying wire segment shown in the Figure.

The wire consists of two straight portions and a circular arc of radius *a,* which subtends an angle θ.



Solution

The magnetic field at *O* due to the current in the straight segment *AA'* and *CC'* is zero, because *d***s** is parallel to **r** along these paths.

$$d\vec{B} = \frac{\mu_o}{4\pi}\int \frac{I\,d\vec{s}\times\hat{r}}{r^2}$$

$$|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$$

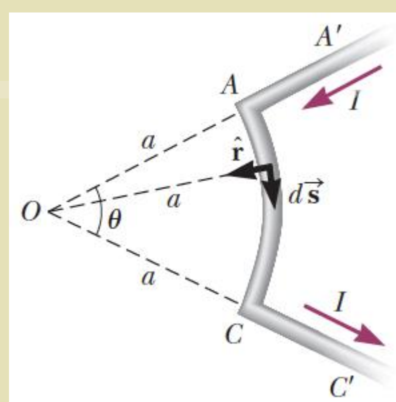
$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds}{a^2}$$

$$B = \frac{\mu_0 I}{4\pi a^2}\int ds = \frac{\mu_0 I}{4\pi a^2}s$$

$$s = a\theta$$

$$B = \frac{\mu_0 I}{4\pi a^2}(a\theta)$$

$$B = \frac{\mu_0 I}{4\pi a}\theta$$

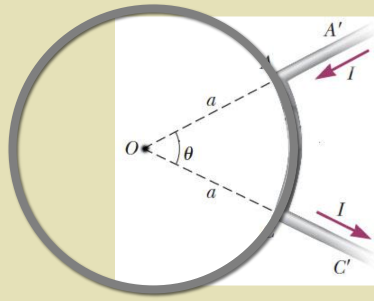


The direction of B is into the page at O

Special case, circular wire loop

What if you were asked to find the magnetic field at the center of a circular wire loop of radius *R* that carries a current *I*?

As the angle θ increases, the curved segment becomes a full circle when θ = 2π. Therefore, you can find the magnetic field at the center of a wire loop by letting θ = 2π

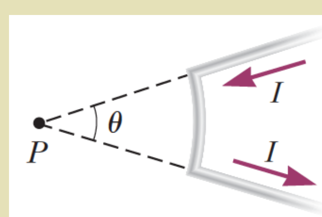


$$B = \frac{\mu_0 I}{4\pi a}\theta \rightarrow B = \frac{\mu_0 I}{4\pi a}2\pi \rightarrow B = \frac{\mu_0 I}{2a}$$

Solve by your self

1. Calculate the magnitude of the magnetic field at a point 25.0 cm from a long, thin conductor carrying a current of 2.00 A.

2. A current path shaped as shown in the Figure produces a magnetic field at P, the center of the arc. If the arc subtends an angle of θ = 30.0º and the radius of the arc is 0.600 m, what are the magnitude and direction of the field produced at P if the current is 3.00 A?



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Magnetism and Alternating Current

Unit 2: Sources of the Magnetic Field

Lecture 9: More Examples on using Biot – Savart Law

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Unit 2: Sources of the Magnetic Field

2.1 The Biot–Savart Law

2.2 The Magnetic Force Between Two Parallel Conductors

2.3 Ampère's Law

2.4 The Magnetic Field of a Solenoid

2.5 Gauss's Law in Magnetism

2.6 Magnetism in Matter

Example 1

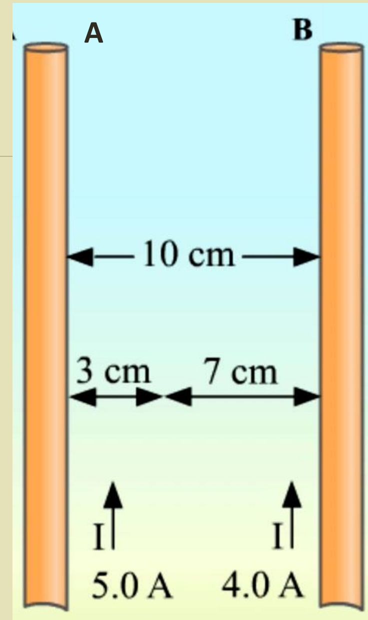
Assume you have two parallel wires 10 cm apart. The wire on the left A, Carries 5.0 A and the wire on the right B, Carries 4.0A.

What is the strength of the magnetic field at 3 cm from the left wire and 7 cm from the right wire?

a) 4.4×10^{-5} T

b) 3.3×10^{-5} T

c) 2.2×10^{-5} T



Solution

The direction of B_A at point p is directed into the page and the direction of B_B at point p is directed out of the page

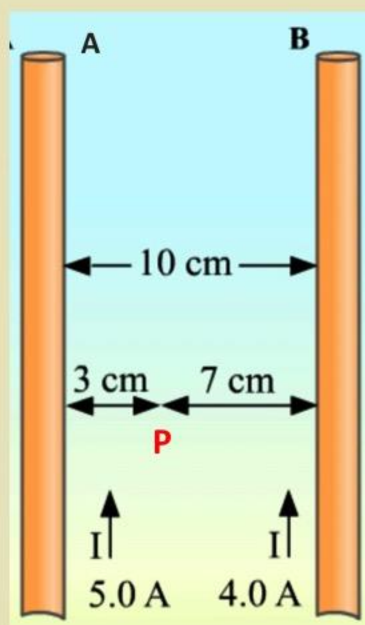
$B_{net} = B_A - B_B$

$$B = \frac{\mu_o I}{2\pi a} \qquad \frac{\mu_o}{2\pi} = 2\times10^{-7} T.m/A$$

$$B_A = \frac{\mu_o I_2}{2\pi a} = 3.3\times10^{-5} T$$

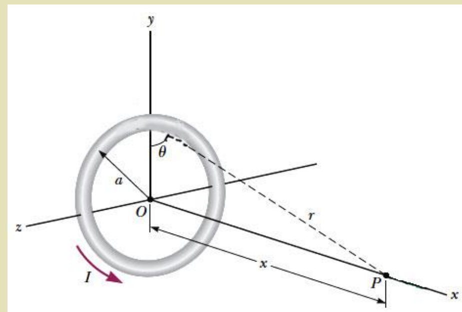
$$B_B = \frac{\mu_o I_2}{2\pi a} = 1.1\times10^{-5} T$$

$B_{net} = B_A - B_B = 2.2 \times 10^{-5}$ T

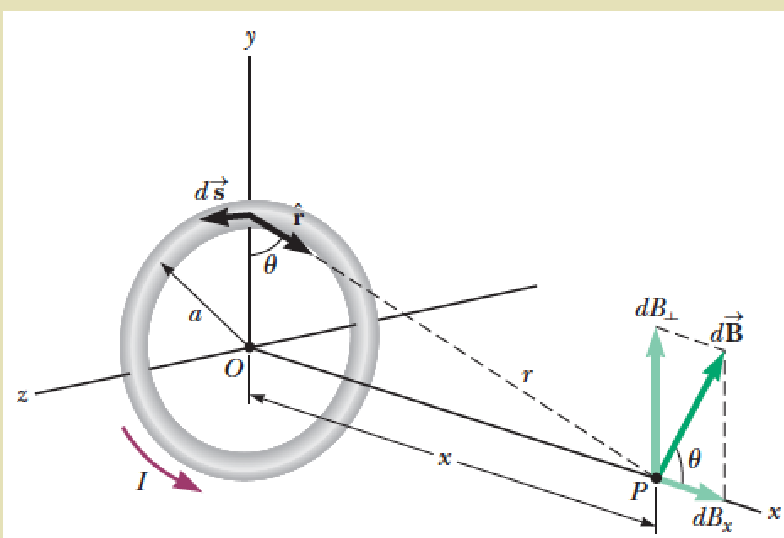


Example 3

Consider a circular wire loop of radius *R* located in the *yz* plane and carrying a steady current *I*, as in the Figure. **Calculate the magnetic field at an axial point *P* a distance *x* from the center of the loop.**



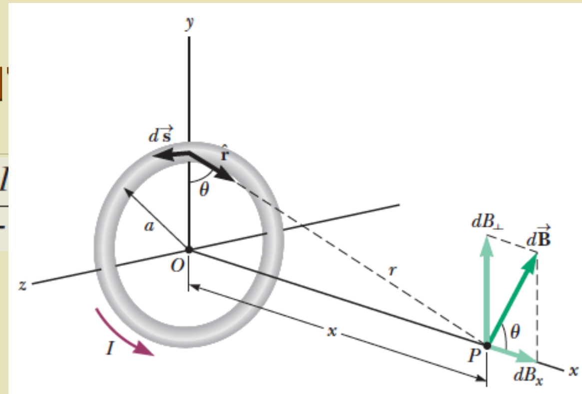
Solution



Solu

$dB = \frac{\mu_0 I}{4\pi}$

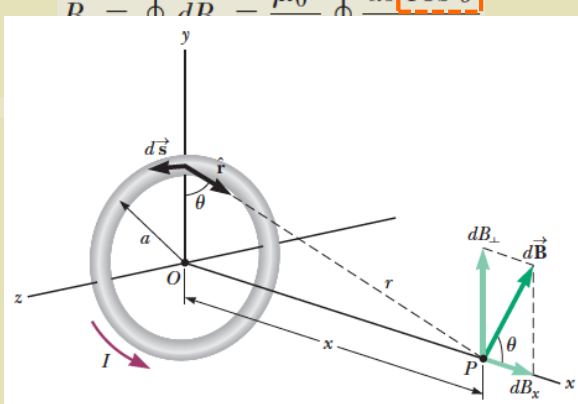
$)^\circ = ds$



$$dB_x = \frac{\mu_0 I}{4\pi} \frac{ds}{(a^2 + x^2)} \cos\theta$$

$$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi} \oint \frac{ds \cos\theta}{a^2 + x^2}$$

$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi} \oint \frac{ds \cos\theta}{}$



$B_x = \frac{}{4\pi} \frac{}{(a^2 + x^2)^{3/2}} (2\pi a)$

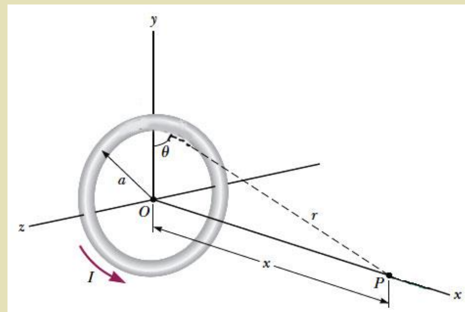
$$B_x = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

To find the magnetic field at the center of the loop, set x = 0

$$B_x = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{2a}$$

What if we consider points on the x axis very far from the loop? How does the magnetic field behave at these distant points?

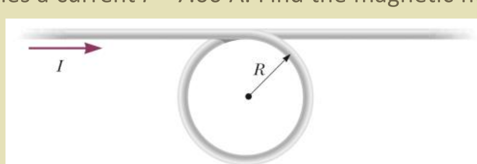


$$B \approx \frac{\mu_0 I a^2}{2x^3} \quad (\text{for } x >> a)$$

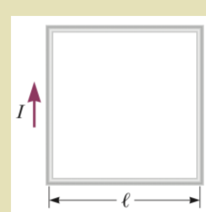
$$\mu = I(\pi a^2) \longrightarrow B \approx \frac{\mu_0}{2\pi}\frac{\mu}{x^3}$$

Solve by your self

(1) A conductor consists of a circular loop of radius R = 15.0 cm and two long, straight sections as shown in the Figure. The wire lies in the plane of the paper and carries a current I = 7.00 A. Find the magnetic field at the center of the loop.



(2) (a) A conducting loop in the shape of a square of edge length l=0.400m carries a current I=10.0A as shown in the Figure. Calculate the magnitude and direction of the magnetic field at the center of the square. (b) What If? If this conductor is reshaped to form a circular loop and carries the same current, what is the value of the magnetic field at the center?



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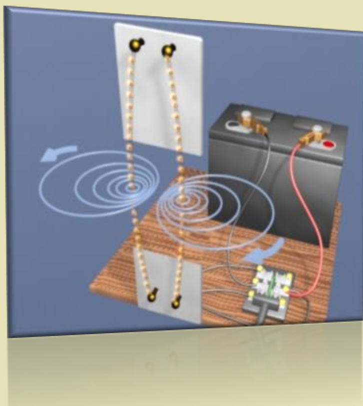
Magnetism and Alternating Current



Unit 2: Sources of the Magnetic Field

Lecture 10: The Magnetic Force Between two Parallel Conductors

Dr. Hazem Falah Sakeek
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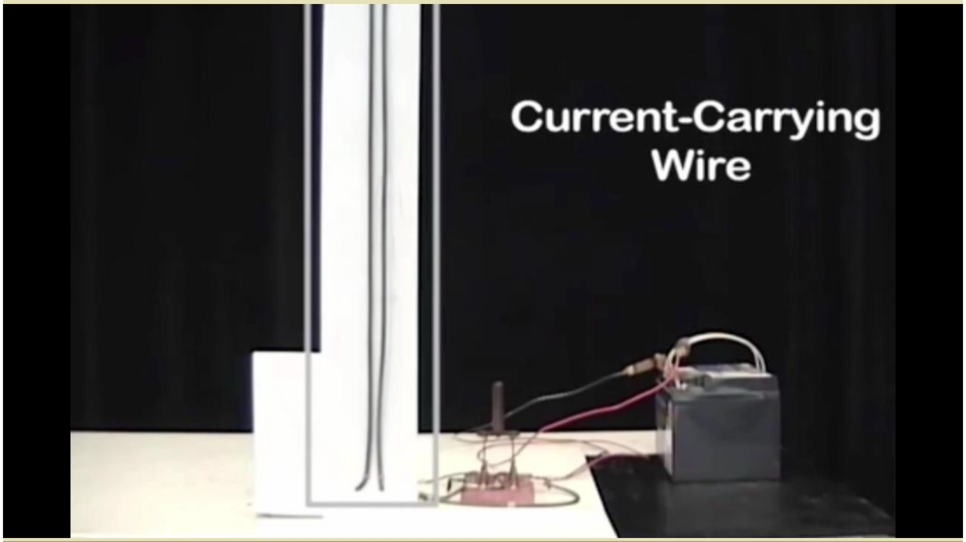
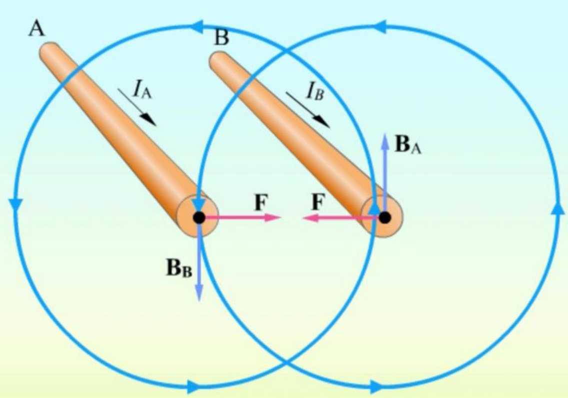
Unit 2: Sources of the Magnetic Field

The Magnetic Force Between Two Parallel Conductors





Magnetic Force Between Two Parallel Conductors

Two parallel wires each carry steady currents.

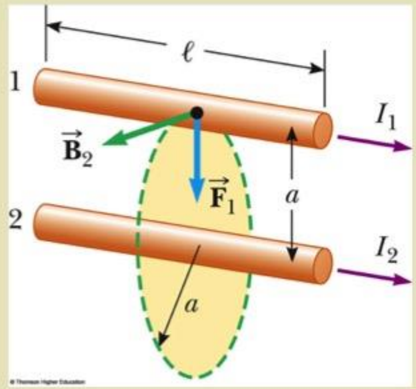
The field B_2 due to the current in wire 2 exerts a force on wire 1 of

$$F_1 = I_1 \ell B_2$$

$$B_2 = \frac{\mu_o I_2}{2\pi a}$$

Substituting the equation for B_2 gives

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a} l$$

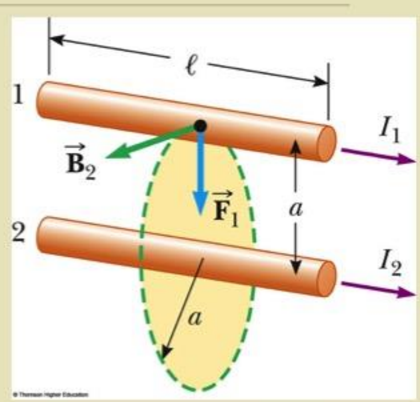


Magnetic Force Between Two Parallel Conductors

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a} l$$

Check with right-hand rule:

- same direction currents attract each other
- opposite directions currents repel each other



The force per unit length on the wire is

$$\frac{F_B}{l} = \frac{\mu_o I_1 I_2}{2\pi a}$$

And this formula defines the current unit Ampere.

Definition of the Ampere and Coulomb

$$\frac{F_B}{l} = \frac{\mu_o I_1 I_2}{2\pi a}$$

Ampere

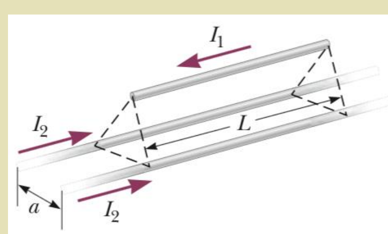
When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

Coulomb

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

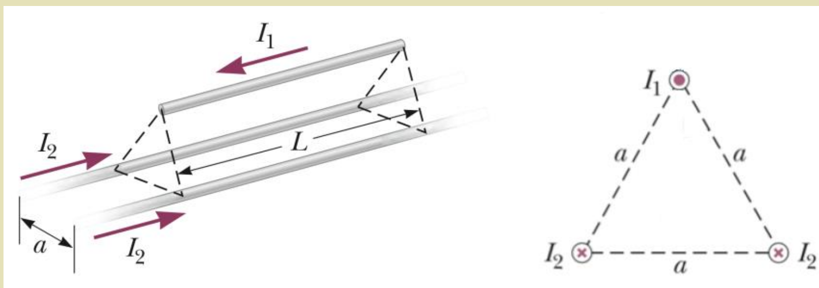
Example 1

Two infinitely long, parallel wires are lying on the ground a distance a=1.00 cm apart as shown in the Figure. A third wire, of length L=10.0 m and mass 400 g, carries a current of I_1=100 A and is levitated above the first two wires, at a horizontal position midway between them. The infinitely long wires carry equal currents I_2 in the same direction, but in the direction opposite that in the levitated wire. **What current must the infinitely long wires carry so that the three wires form an equilateral triangle?**



Solution

The horizontal components of the magnetic forces on the levitated wire cancel. The vertical components are both positive and add together. Choose the *z* axis to be upward through the top wire and in the plane of the page.



Solution

$$\vec{\mathbf{F}}_B = 2\left(\frac{\mu_0 I_1 I_2}{2\pi a}\ell\right)\cos\theta\,\hat{\mathbf{k}} = \frac{\mu_0 I_1 I_2}{\pi a}\ell\cos\theta\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{F}}_g = -mg\hat{\mathbf{k}}$$

$$\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_B + \vec{\mathbf{F}}_g = \frac{\mu_0 I_1 I_2}{\pi a}\ell\cos\theta\,\hat{\mathbf{k}} - mg\hat{\mathbf{k}} = 0$$

$$I_2 = \frac{mg\pi a}{\mu_0 I_1 \ell\cos\theta}$$

$$I_2 = \frac{(0.400\text{ kg})(9.80\text{ m/s}^2)\pi(0.010\,0\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(100\text{ A})(10.0\text{ m})\cos 30.0^\circ}$$

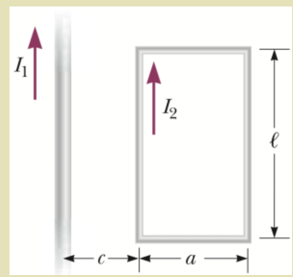
$$= \boxed{113\text{ A}}$$

this situation would be difficult to establish in practice!

Solve by your self

(1) Two long, parallel conductors, separated by 10.0 cm, carry currents in the same direction. The first wire carries a current I_1 = 5.00 A, and the second carries I_2 = 8.00 A. **(a)** What is the magnitude of the magnetic field created by I_1 at the location of I_2? **(b)** What is the force per unit length exerted by I_1 on I_2? **(c)** What is the magnitude of the magnetic field created by I_2 at the location of I_1? **(d)** What is the force per length exerted by I_2 on I_1?

(2) In the Figure, the current in the long, straight wire is I_1 = 5.00 A and the wire lies in the plane of the rectangular loop, which carries a current I_2 = 10.0 A. The dimensions in the figure are c = 0.100 m, a = 0.150 m, and l = 0.450 m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.



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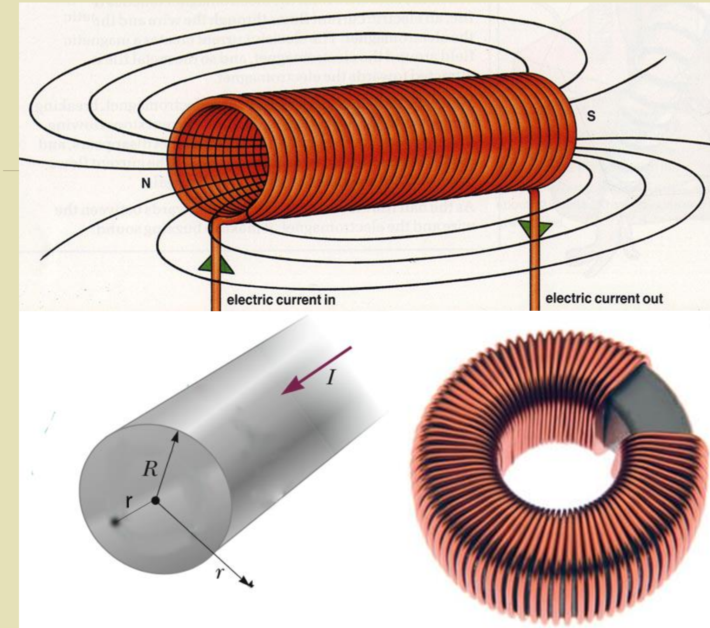
Magnetism and Alternating Current

Unit 2: Sources of the Magnetic Field

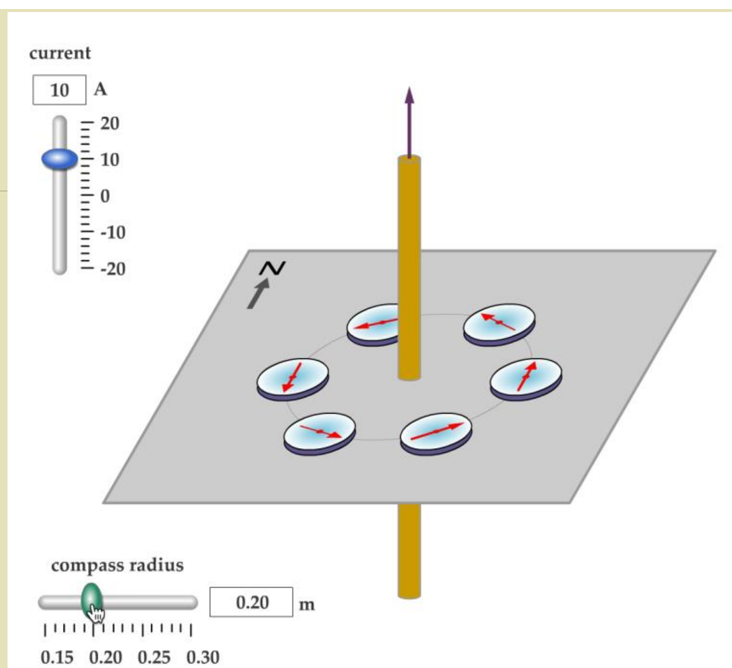
Lecture 11: Ampère's Law

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 2: Sources of the Magnetic Field



S
N
electric current in
electric current out
I
R
r
r



current
10 A
20
10
0
-10
-20
N
compass radius
0.20 m
0.15 0.20 0.25 0.30

About Ampere's Law

A useful law that relates the net magnetic field along a closed loop (*amperian loop*) to the electric current passing through the loop.

First discovered by André-Marie Ampère in 1826.

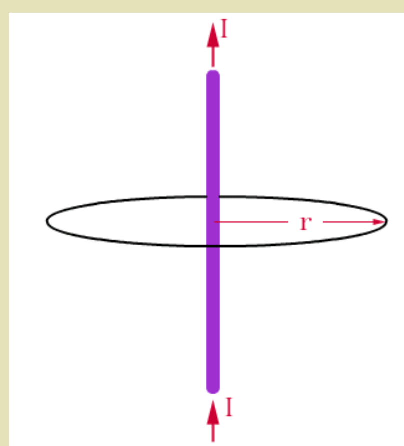
Definition of Ampere's Law

The line integral of $\oint \vec{B}.d\vec{s}$ around any closed path equals $\mu_0 I$,

where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{B}.d\vec{s} = \mu_o I$$

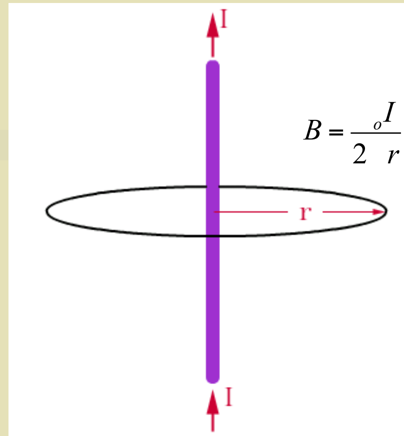
Ampere's Law



the product $\vec{B}.\vec{ds}$ for a small length element *d*S on the circular path defined by the compass needles and sum of the products for all elements over the closed circular path.

Along this path, the vectors ***d*s** and **B** are parallel at each point.

$$\vec{B}.\vec{ds} = B\,ds$$



$$\oint \vec{B}.\vec{ds} = B\oint ds = \frac{\mu_o I}{2\pi r}(2\pi r) = \mu_o I$$

Remarks

$$\oint \vec{B}.\vec{ds} = \mu_o I$$

In order to apply Ampère's Law all currents have to be steady (i.e. do not change with time)

- Only currents crossing the area inside the path are taken into account and have some contribution to the magnetic field.
- Currents have to be taken with their algebraic signs (those going "out" of the surface are positive, those going "in" are negative)- use right hand's rule to determine directions and signs.

Remarks

$$\oint \vec{B}.d\vec{s} = \mu_o I$$

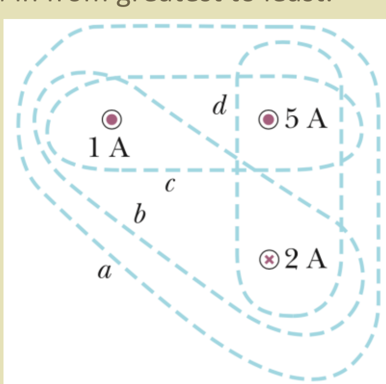
The total magnetic is zero only in the following cases:

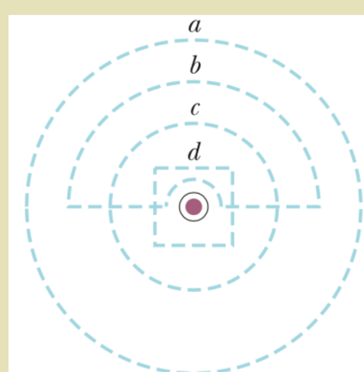
- -the enclosed net current is zero.
- -the magnetic field is normal to the selected path at any point.
- -the magnetic field is zero.

•Ampère's Law can be useful when calculating magnetic fields of current distributions with a high degree of symmetry (similar to symmetrical charge distributions in the case of Gauss' Law).

Exercise

Rank the magnitudes of $\oint \vec{B}.d\vec{s}$ for the closed paths *a* through *d* in from greatest to least.

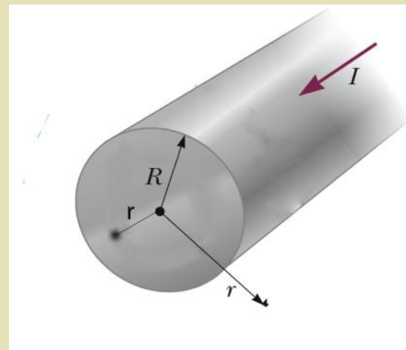




Example 1

A long, straight wire of radius *R* carries a steady current *I* that is uniformly distributed through the cross section of the wire.

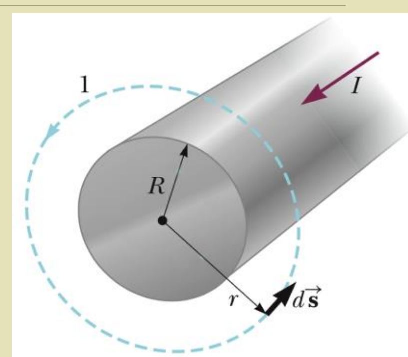
Calculate the magnetic field a distance *r* from the center of the wire in the regions $r \geq R$ and $r < R$.



Solution

For $r \geq R$

From symmetry, **B** must be constant in magnitude and parallel to *d***s** at every point on this circle.



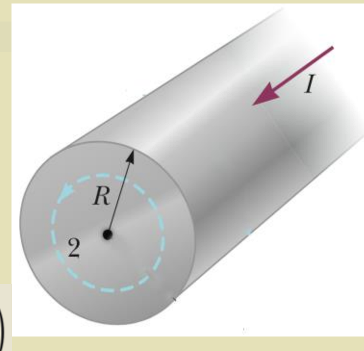
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (\text{for } r \geq R)$$

Note that the total current passing through the plane of the circle is I

For r < R

Here the current *I'* passing through the plane of circle 2 is less than the total current *I.*



$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$$

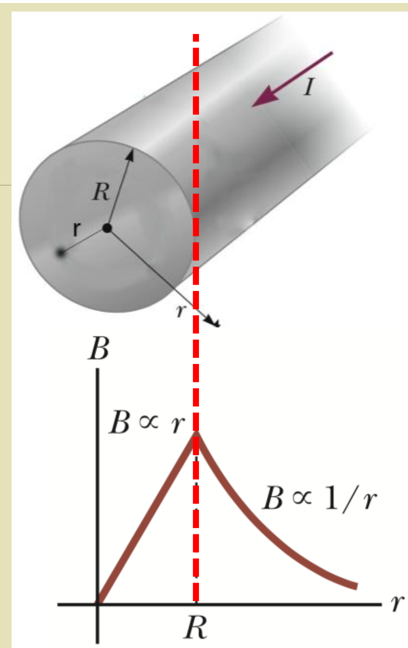
$$I' = \frac{r^2}{R^2} I$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I' = \mu_0\left(\frac{r^2}{R^2} I\right)$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r \qquad (\text{for } r < R)$$

Remarks

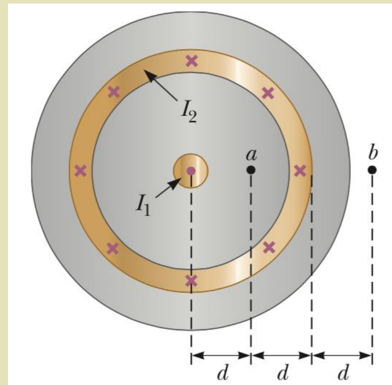
The magnetic field exterior to the wire is identical in form to the one we obtained from Biot-Savart Law.



The magnetic field interior to the wire is similar in form to the expression for the electric field inside a uniformly charged sphere

Solve by your self

(1) A cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, an outer conductor, and another rubber layer. In a particular application, the current in the inner conductor is I_1=1.00 A out of the page and the current in the outer conductor is I_2=3.00 A into the page. Assuming the distance d=1.00 mm, determine the magnitude and direction of the magnetic field at (a) point *a* and (b) point *b.*



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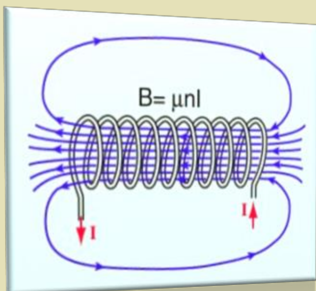
Magnetism and Alternating Current



Unit 2: Sources of the Magnetic Field

Lecture 12: The Magnetic Field of a Solenoid

Gauss's Law in Magnetism

Dr. Hazem Falah Sakeek
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Unit 2: Sources of the Magnetic Field

2.1 The Biot–Savart Law

2.2 The Magnetic Force Between Two Parallel Conductors

2.3 Ampère's Law

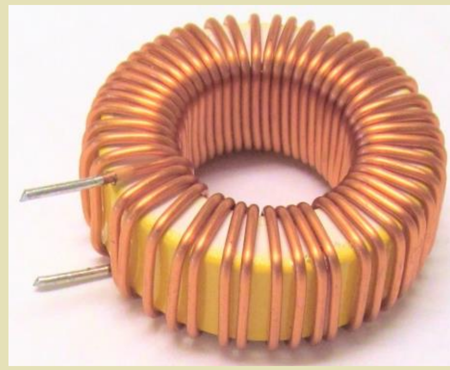
2.4 The Magnetic Field of a Solenoid

2.5 Gauss's Law in Magnetism

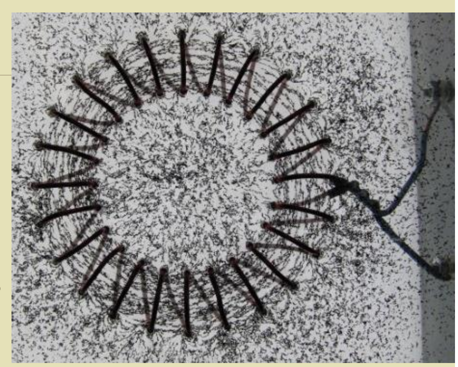
2.6 Magnetism in Matter

Example 1

A device called a *toroid* is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material. For a toroid having *N* closely spaced turns of wire, **calculate** the magnetic field in the region occupied by the torus, a distance *r* from the center.



Solution



For loop of radius *r*.

By symmetry, the magnitude of the field is constant on this circle and tangent to it, so **B**.***ds*** = *B ds*.

Furthermore, the wire passes through the loop *N* times, so the total current through the loop is *NI*.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

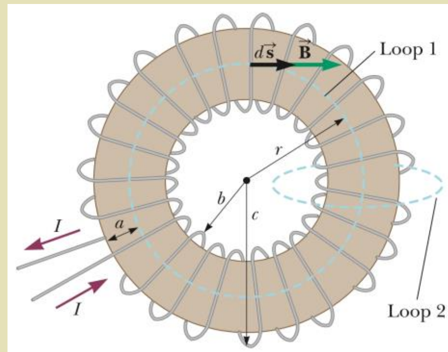
Magnetic Field of a toroid

Remarks

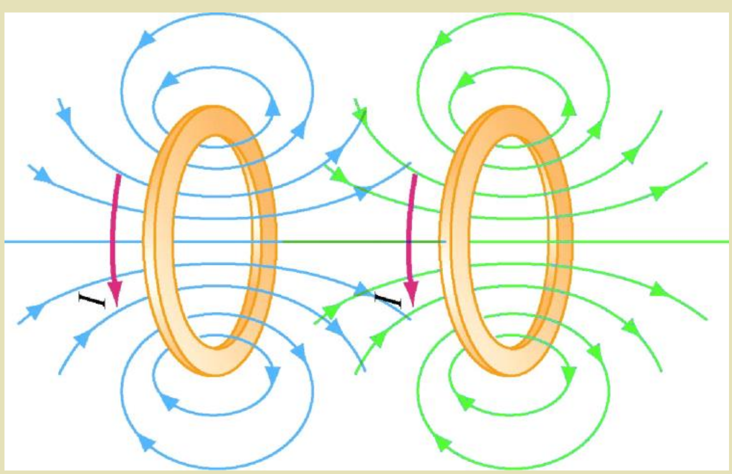
$$B = \frac{\mu_0 NI}{2\pi r}$$

This result shows that B varies as $1/r$ and hence is *nonuniform* in the region occupied by the torus.

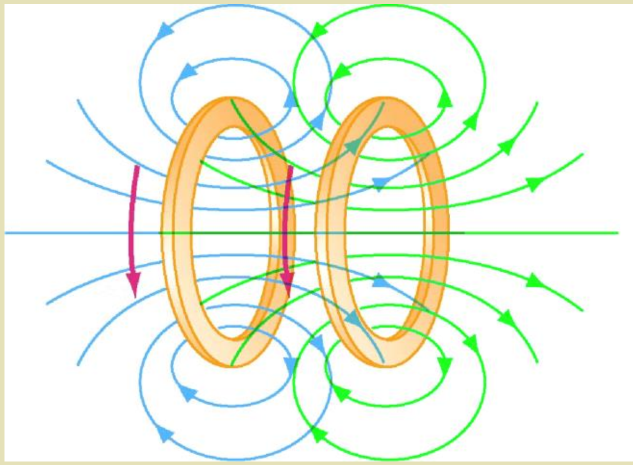
If r is very large compared with the cross-sectional radius a of the torus, the field is approximately uniform inside the torus.



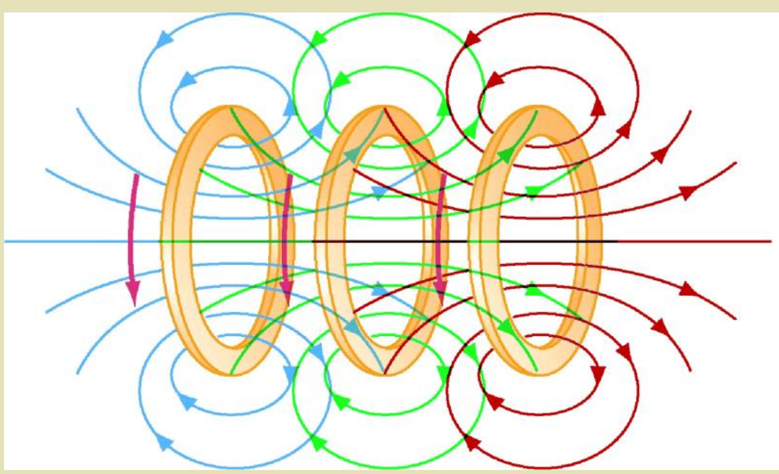
Two Loops



Two Loops Moved Closer Together



Multiple Wire Loops

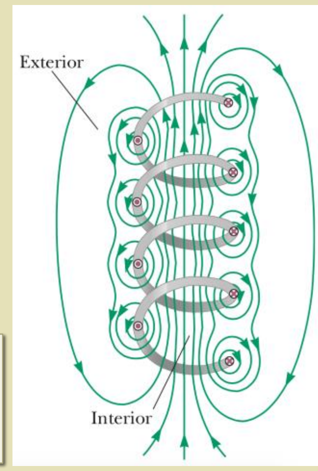


The Magnetic Field of a Solenoid

A **solenoid** is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire (*interior* of the solenoid).

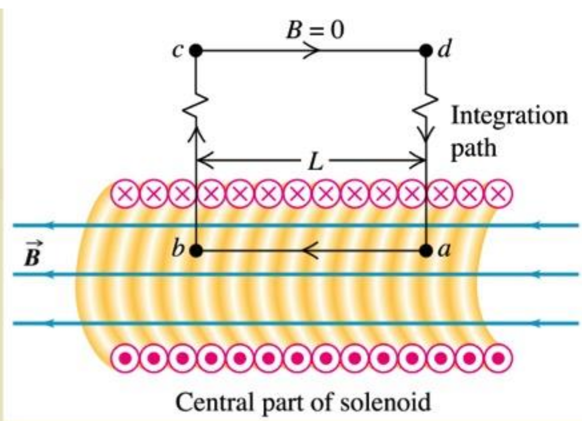
When the turns are closely spaced, each can be approximated as a circular loop; the net magnetic field is the vector sum of the fields resulting from all the turns.

An *ideal solenoid* is approached when the turns are closely spaced and the length is much greater than the radius of the turns.



For an ideal solenoid

$$\oint \vec{B}.d\vec{s} = \mu_o I$$



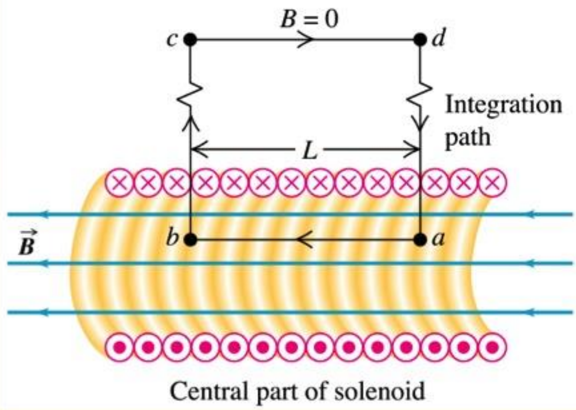
$$\oint \vec{B}.d\vec{s} = \int_{ab} B\cos 0\, ds + \int_{bc} B\cos 90\, ds + \int_{cd} 0\cos 180\, ds + \int_{da} B\cos 90\, ds$$

$$\oint \vec{B}.d\vec{s} = \int_{ab} B\cos 0\, ds = BL$$

$$\oint \vec{B}.d\vec{s} = \mu_o I$$

$$\mu_o I = \mu_o NI$$

$$\oint \vec{B}.d\vec{s} = BL = \mu_o NI$$



the total current through the rectangular path equals the current through each turn multiplied by the number of turns. If *N* is the number of turns in the length *l* the total current through the rectangle is *NI.*

$$B = \mu_o \frac{N}{L} I \qquad B = \mu_o nI$$

Magnetic Field of a Solenoid

where $n = N/L$, is the number of turns per unit length.

Gauss's Law in Magnetism

Consider an element of area *dA* on an arbitrarily shaped surface as shown in the Figure.

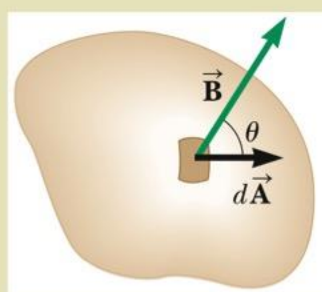
If the magnetic field at this element is **B**, the magnetic flux through the element is **B** . *d***A**,

where *d***A** is a vector that is perpendicular to the surface and has a magnitude equal to the area *dA*.

Therefore, the total magnetic flux Φ_B through the surface is

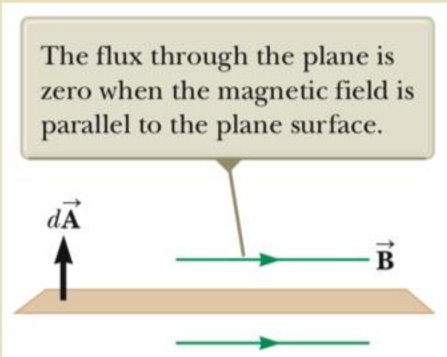
$$\phi_B = \int \vec{B}.d\vec{A}$$

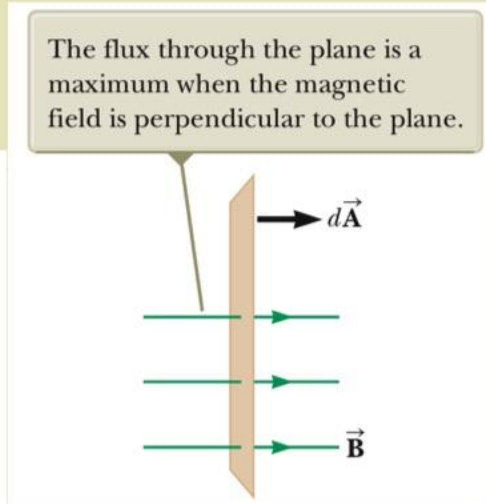
$$\phi_B = BA\cos\theta$$



The unit of magnetic flux is T . m^2 , which is defined as a weber (Wb);
1 Wb = 1 T . m^2.

$$\phi_B = BA\cos\theta$$





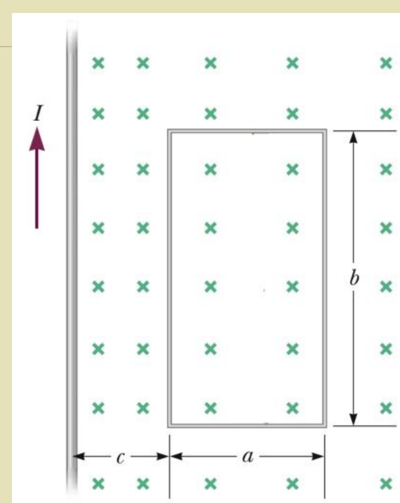
$$\phi_B = 0 \quad (\theta = 90)$$

$$\phi_B = BA \quad (\theta = 0)$$

Example 2

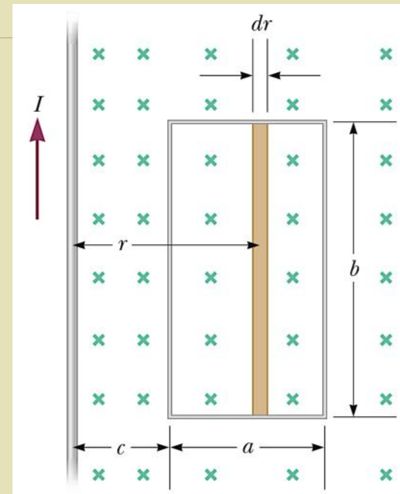
A rectangular loop of width *a* and length *b* is located near a long wire carrying a current *I*. The distance between the wire and the closest side of the loop is *c*. The wire is parallel to the long side of the loop.

Find the total magnetic flux through the loop due to the cur- rent in the wire.



Solution

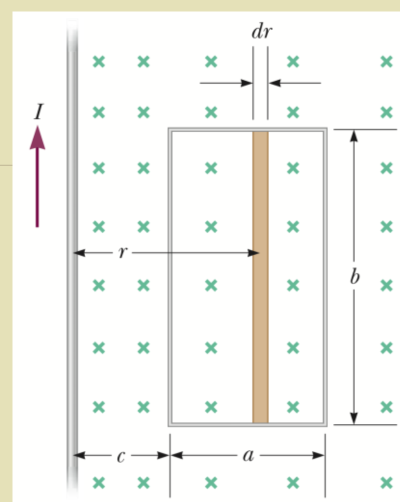
$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\, dA = \int \frac{\mu_0 I}{2\pi r}\, dA$$



dA = b dr

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r}\, b\, dr = \frac{\mu_0 I b}{2\pi} \int \frac{dr}{r}$$

Solution



$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r}$$

$$= \frac{\mu_0 I b}{2\pi} \ln r \Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

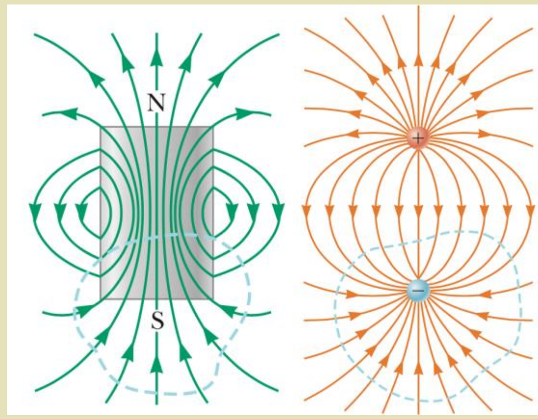
$$= \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right)$$

Notice how the flux depends on the size of the loop. Increasing either *a* or *b* increases the flux as expected.

Remark

The net magnetic flux through any closed surface is always zero:

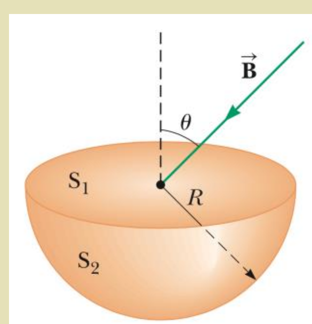
$$\oint \vec{B}.d\vec{A} = 0$$



Solve by your self

(1) The magnetic coils of a tokamak fusion reactor are in the shape of a toroid having an inner radius of 0.700 m and an outer radius of 1.30 m. The toroid has 900 turns of large- diameter wire, each of which carries a current of 14.0 kA. Find the magnitude of the magnetic field inside the toroid along (a) the inner radius and (b) the outer radius.

(2) Consider the hemispherical closed surface in the Figure. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the magnetic flux through (a) the flat surface S_1 and (b) the hemispheri- cal surface S_2.



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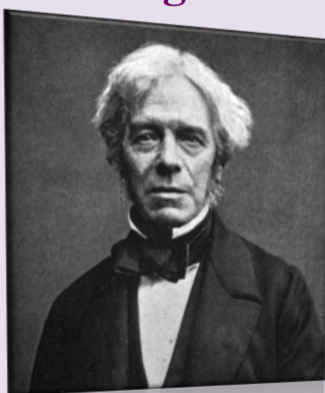
Magnetism and Alternating Current

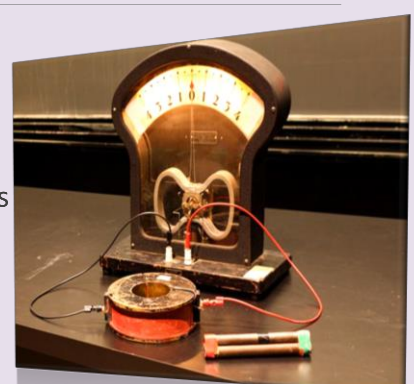


Unit 3: Faraday's Law

Lecture 13: Faraday's Law of Induction

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 3: Faraday's Law



Objectives of this unit

From the previous lectures we have focused on the electric fields produced by stationary charges and the magnetic fields produced by moving charges.

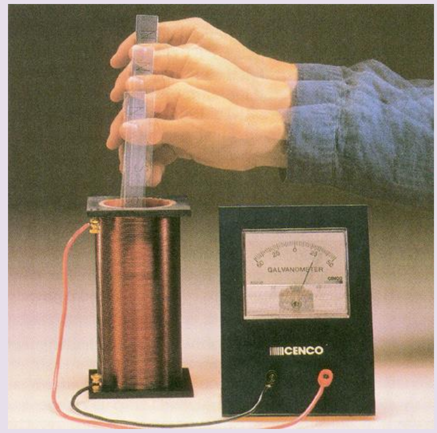
This chapter explores the effects produced by magnetic fields that vary in time.

Current can be induced in various processes that involve a change in a magnetic flux.

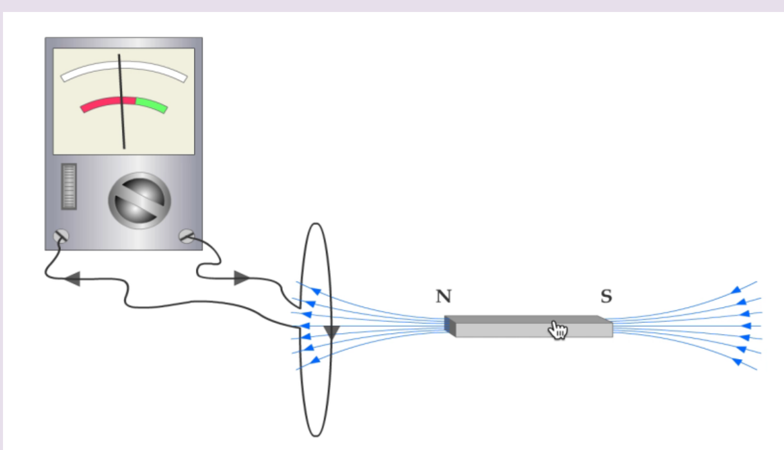
We will study Generator and Motors.

Faraday's Law of Induction

Two simple experiments demonstrate that a current can be produced by a changing magnetic field.



Faraday First Experiment: consider a loop of wire connected to a galvanometer.

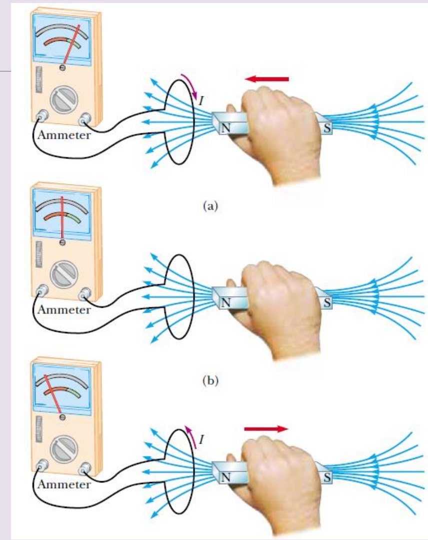


Observations

If a magnet is moved toward the loop, the galvanometer needle will deflect in one direction.

If a magnet is moved away from the loop, the galvanometer needle will deflect in the opposite direction.

If the magnet is held stationary relative to the loop, no galvanometer needle deflection is observed.

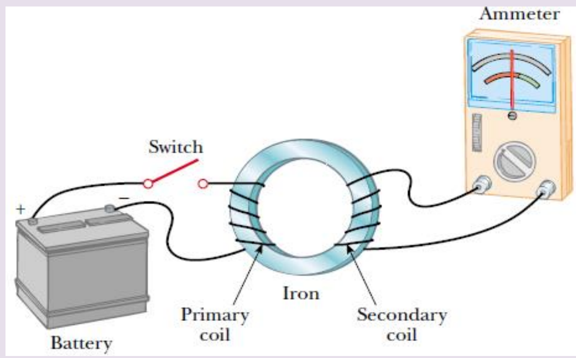


Remarks

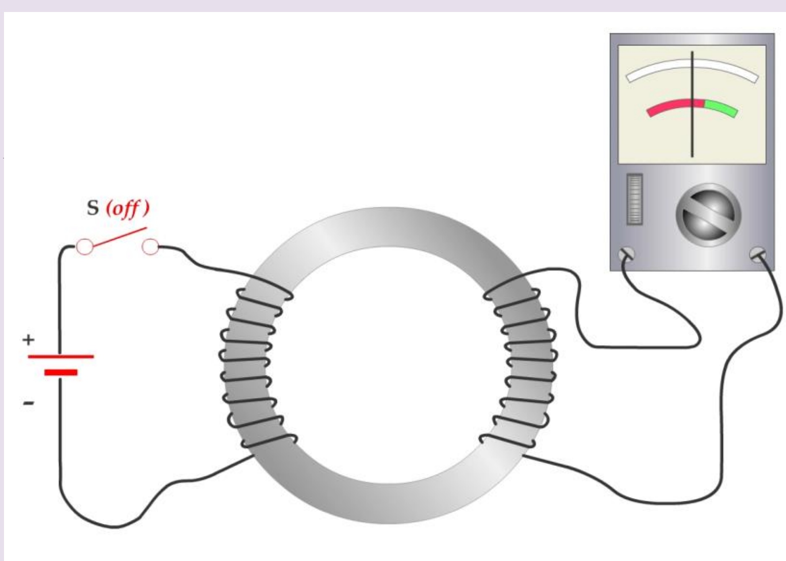
From these observations, you can conclude that:

1. Current is set up in the circuit as long as there is relative motion between the magnet and the coil.
2. This current is set up in the circuit even though there are no batteries in the circuit.
3. The current is said to be an induced current, which is produced by an induced EMF.

Faraday Second Experiment: **A coil is connected to a switch and a battery.**



The only purpose of this circuit is to detect any current that might be produced by a change in the magnetic field.



Observations

1. When the switch in the primary circuit is closed, the galvanometer in the secondary circuit deflects in one direction and then returns to zero.
2. When the switch is opened, the galvanometer deflects in the opposite direction and again returns to zero.
3. The galvanometer reads zero when there is a steady current in the primary circuit.

Remarks

Faraday concluded that

1. An electric current can be produced by a changing magnetic field.
2. A current cannot be produced by a steady magnetic field.
3. The current that is produced in the secondary circuit occurs for only an instant while the magnetic field through the secondary coil is changing.
4. In effect, the secondary circuit behaves as though there were a source of EMF connected to it for a short instant.
5. An induced EMF is produced in the secondary circuit by the changing magnetic field.

Faraday's Law of Induction

In both experiments, an EMF is induced in a circuit when the magnetic flux through the circuit changes with time.

Faraday's Law of Induction: The EMF induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit.

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- where Φ_m is the magnetic flux threading the circuit.
- Magnetic flux Φ_m :

$$\Phi_B = \oint \vec{B}.d\vec{A}$$

The integral of the magnetic flux is taken over the area bounded by the circuit.

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The negative sign is a consequence of Lenz's law and is discussed later (the induced EMF opposes the change in the magnetic flux in the circuit).

If the circuit is a coil consisting of N loops all of the same area and if the flux threads all loops, the induced EMF is:

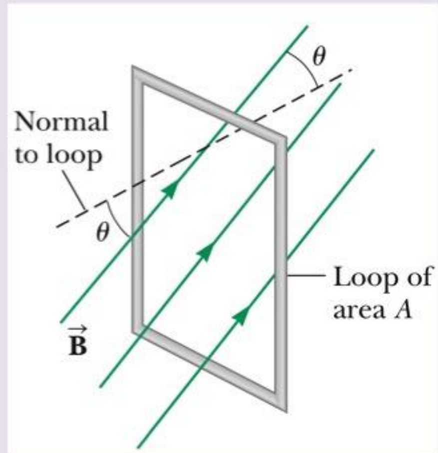
$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Special Case

Suppose the magnetic field is uniform over a loop of area A lying in a plane as shown in the figure below.

The flux through the loop is equal to B·A·cos θ; and the induced EMF is:

$$\varepsilon = -\frac{d}{dt}\left(BA\cos\theta\right)$$



Induced EMF

$$\varepsilon = -\frac{d}{dt}\left(BA\cos\theta\right)$$

An EMF can be induced in the circuit in several ways:

1. The magnitude of **B** can vary with time;
2. The area of the circuit can change with time;
3. The angle θ between **B** and the normal to the plane can change with time; and
4. Any combination of these can occur.

Applications of Faraday's Law

1. The ground fault circuit interrupter (GFCI).
2. production of sound in an electric guitar.
3. The Magnetic Playback Head of a Tape Deck.
4. Tape / Hard Drive etc.
1. Credit Card Reader.

Example 1

A coil consists of 200 turns of wire. Each turn is a square of side d = 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s,

what is the magnitude of the induced emf in the coil while the field is changing?

Solution

$$|\varepsilon| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t}$$

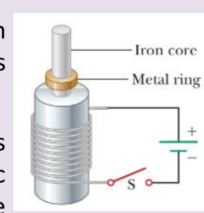
$$= Nd^2\frac{B_f - B_i}{\Delta t}$$

$$|\varepsilon| = (200)(0.18\text{ m})^2\frac{(0.50\text{ T} - 0)}{0.80\text{ s}} = 4.0\text{ V}$$

Solve by your self

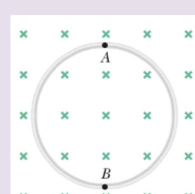
(1) What is the difference between magnetic flux and magnetic field?

(2) When the switch in Figure is closed, a current is set up in the coil and the metal ring springs upward. Explain this behavior.



(3) A 25-turn circular coil of wire has diameter 1.00 m. It is placed with its axis along the direction of the Earth's mag- netic field of 50.0 µT and then in 0.200 s is flipped 180º. An average emf of what magnitude is generated in the coil?

(4) A circular loop of wire of radius 12.0 cm is placed in a magnetic field directed perpendicular to the plane of the loop as in the Figure. If the field decreases at the rate of 0.050 T/s in some time interval, find the magnitude of the emf induced in the loop during this interval.



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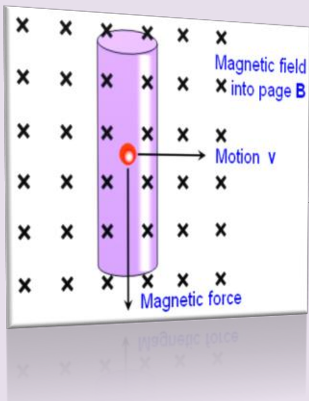
Magnetism and Alternating Current



Unit 3: Faraday's Law

Lecture 14: Motional emf and Lenz's Law

Dr. Hazem Falah Sakeek
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Unit 3: Faraday's Law

3.1 Faraday's Law of Induction

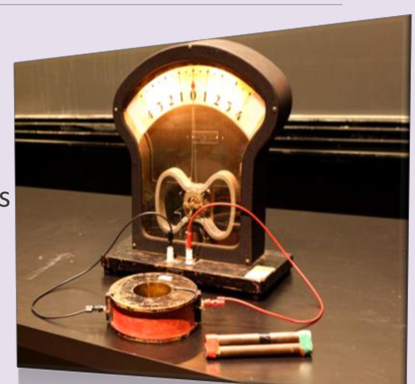
3.2 Motional emf

3.3 Lenz's Law

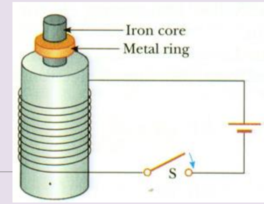
3.4 Induced emf and Electric Fields

3.5 Generators and Motors

3.6 Eddy Currents



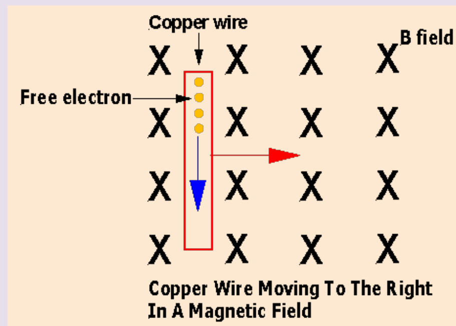
When the switch in Figure is closed, a current is set up in the coil and the metal ring springs upward. Explain this behavior.



Objective

We considered in the previous lecture, an **emf** induced in a stationary circuit placed in a magnetic field when the field changes with time.

In this section, we describe **motional emf**, the emf induced in a **conductor moving through a constant magnetic field**.



Conductor moving through a constant magnetic field

A wire passes through a uniform magnetic field.

The length of the wire, the magnetic field, and the velocity of the wire are all perpendicular to one another:

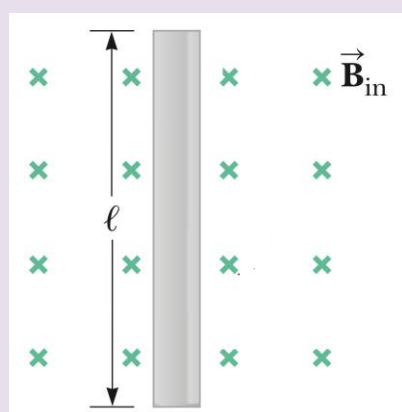
A positive charge in the wire experiences a magnetic force, directed upward:

$$F_B = qvB\sin 90° = qvB$$

A negative charge in the wire experiences the same magnetic force, but directed downward:

These forces tend to separate the charges.

$$F_B = qvB$$



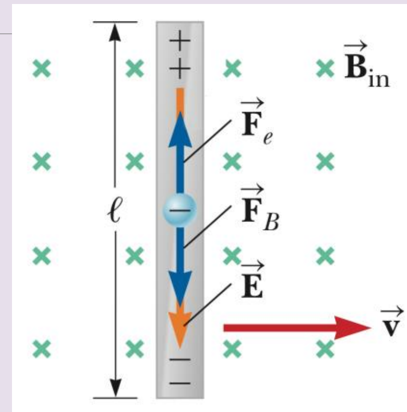
Conductor moving through a constant magnetic field

The separation of the charges produces an electric field, E. It exerts an attractive force on the charges:

$$F_e = Eq$$

In the steady state (at equilibrium), the magnitudes of the magnetic force – separating the charges – and the Coulomb force – attracting them – are equal.

$$qvB = Eq$$



Motional EMF

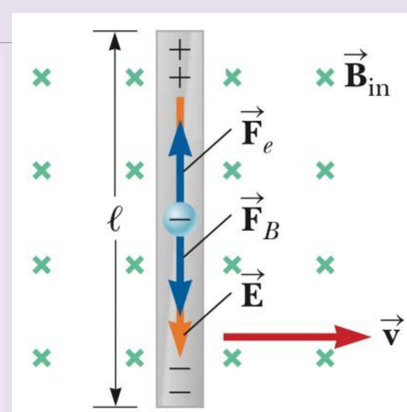
Rewrite the electric field as a potential gradient:

$$E = \frac{\Delta V}{L} = \frac{EMF}{L} = \frac{\varepsilon}{L}$$

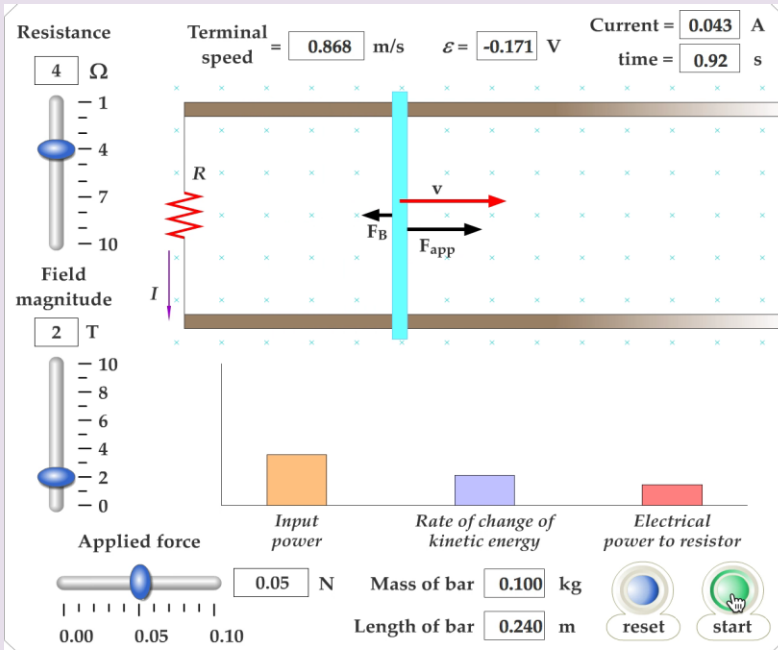
Substitute this result back into our earlier equation:

$$Eq = qvB \rightarrow \frac{\varepsilon}{L} q = qvB$$

$$\varepsilon = vLB$$

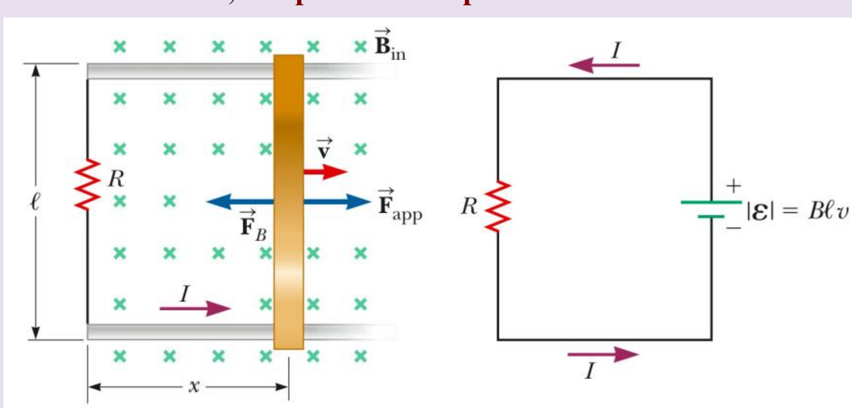


This is called ***motional EMF***. It results from the constant velocity of the wire through the magnetic field, B.



Motional EMF (Induced Current)

Now, our moving wire slides over two other wires, forming a circuit. A current will flow, and power is dissipated in the resistive load:



The **area** enclosed by the circuit at any instant is lx, the **magnetic flux** through that area is

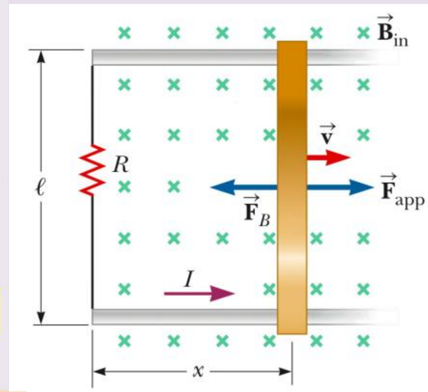
$$\Phi_B = Blx$$

Using Faraday's law

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt}$$

$$\varepsilon = -Blv$$ *Motional emf*

$$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$$ *Induced current*



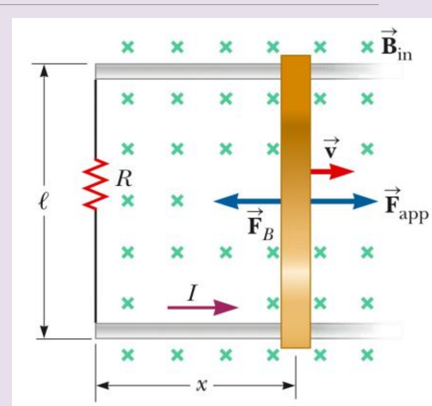
Motional EMF (Power Dissipated)

$$\varepsilon = V = vBL$$

$$I = \frac{V}{R} = \frac{vBL}{R}$$

$$P = VI = (vBL)\left(\frac{vBL}{R}\right)$$

$$P = \frac{(vBL)^2}{R} = \frac{\varepsilon^2}{R}$$



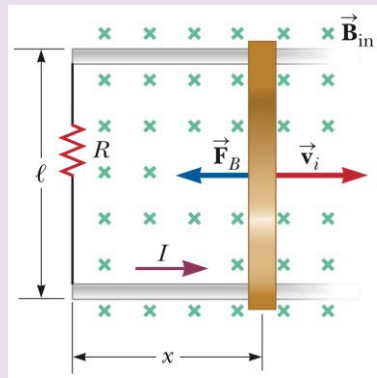
Example 1

The conducting bar illustrated in the Figure moves on two frictionless, parallel rails in the presence of a uniform magnetic field directed into the page.

The bar has mass *m*, and its length is l.

The bar is given an initial velocity $\mathbf{v}_i$ to the right and is released at $t = 0$.

Using Newton's laws, find the velocity of the bar as a function of time.



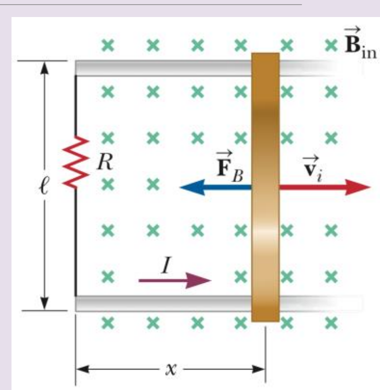
Solution

The magnetic force is $F_B = -IlB$, where the negative sign indicates that the force is to the left. The magnetic force is the *only* horizontal force acting on the bar.

$$F_x = ma = m\frac{dv}{dt} = -I\ell B$$

$I = B\ell v/R$ Induced current

$$m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v$$



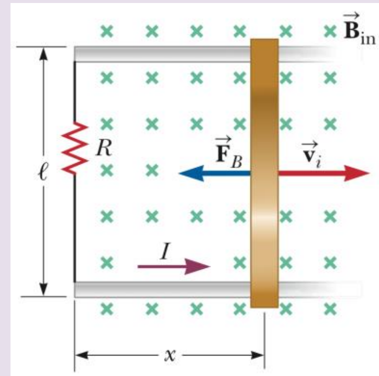
$$m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v$$

Rearrange the equation

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right)dt$$

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\int_0^t dt$$

$$\ln\left(\frac{v}{v_i}\right) = -\left(\frac{B^2\ell^2}{mR}\right)t$$

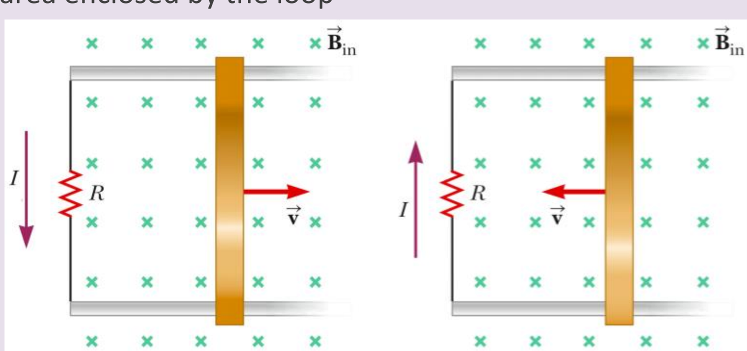


$$v = v_i e^{-t/\tau}$$
$$\tau = mR/B^2\ell^2$$

This expression for v indicates that the velocity of the bar decreases with time under the action of the magnetic force as expected

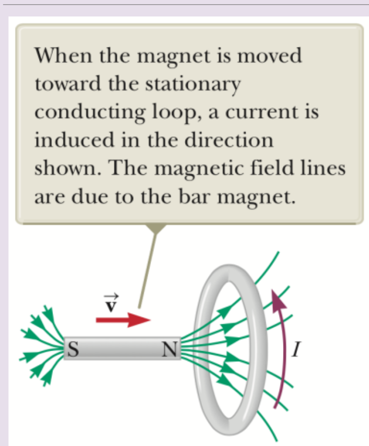
Lenz's Law

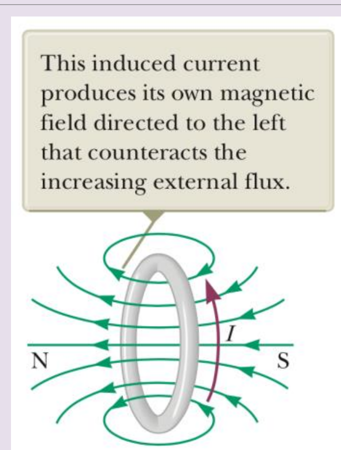
The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop



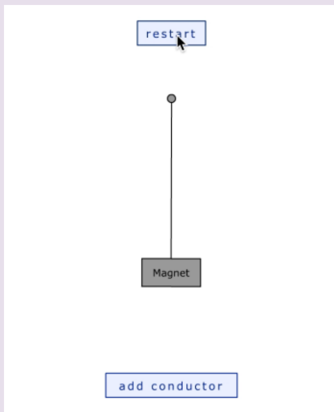

Iron Ball

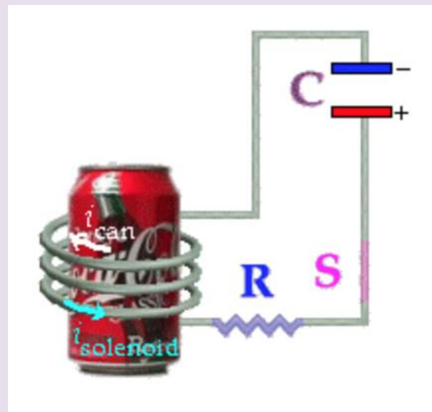
Lenz's Law


When the magnet is moved toward the stationary conducting loop, a current is induced in the direction shown. The magnetic field lines are due to the bar magnet.
$\vec{v}$
S
N
I


This induced current produces its own magnetic field directed to the left that counteracts the increasing external flux.
N
S
I

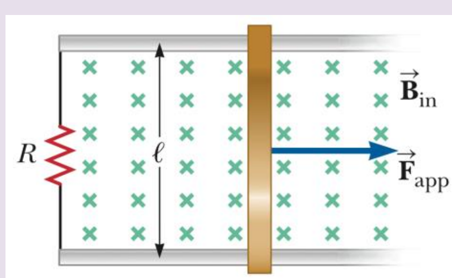
Lenz's Law





Solve by your self

(1) The resistor is R = 6.00 Ω, and a 2.50-T magnetic field is directed perpendicularly downward, into the paper. Let , l= 1.20 m. (a) Calculate the applied force required to move the bar to the right at a constant speed of 2.00 m/s. (b) At what rate is energy delivered to the resistor?



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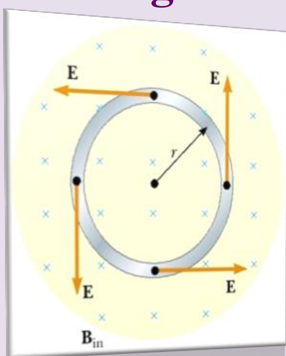
Magnetism and Alternating Current



Unit 3: Faraday's Law

Lecture 15: Induced emf and Electric Fields

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 3: Faraday's Law

3.1 Faraday's Law of Induction

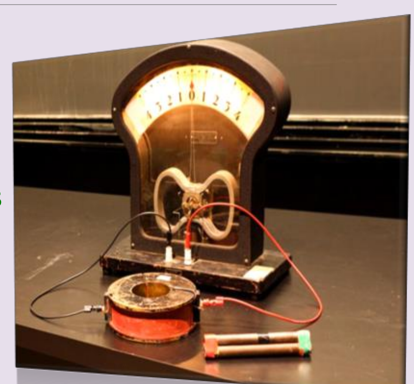
3.2 Motional emf

3.3 Lenz's Law

3.4 Induced emf and Electric Fields

3.5 Generators and Motors

3.6 Eddy Currents

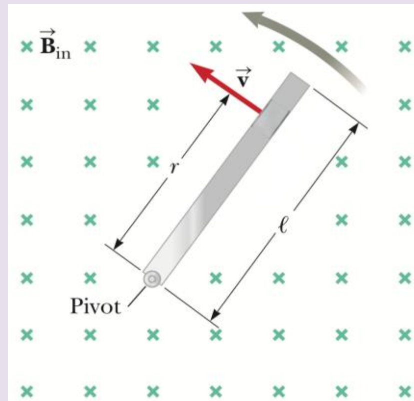


Example (1)
Motional emf Induced in a Rotating Bar

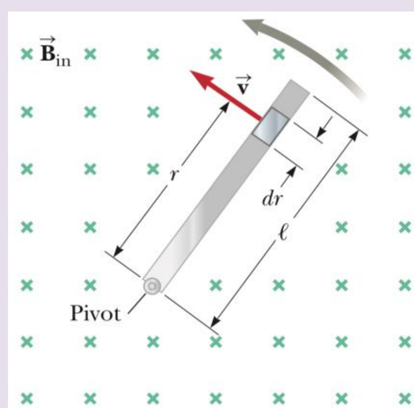
A conducting bar of length *l* , rotates with a constant angular speed *v* about a pivot at one end.

A uniform magnetic field **B** is directed perpendicular to the plane of rotation as shown in the Figure.

Find the motional emf induced between the ends of the bar.



Solution



Consider a small segment of the bar, all segments are in series and the emfs add.

$$d\mathcal{E} = Bv\,dr$$

$$\mathcal{E} = \int Bv\,dr$$

$$v = r\omega$$

$$\mathcal{E} = B\int v\,dr = B\omega\int_0^{\ell} r\,dr = \tfrac{1}{2}B\omega\ell^2$$

For the rotating rod, there is an advantage to increasing of the length of the rod to raise the emf because *l*, is squared. Doubling the length gives four times the emf, whereas doubling the angular speed only doubles the emf.

Notes to remember

- Changing magnetic flux induces an emf and a current in a conducting loop.
- Current is related to an electric field that applies electric forces on charged particles.
- Induced current in a conducting loop is related to an electric field by claiming that an electric field is created in the conductor as a result of the changing magnetic flux.
- The existence of an electric field is independent of the presence of any test charges. **This independence suggests that even in the absence of a conducting loop, a changing magnetic field generates an electric field in empty space.**

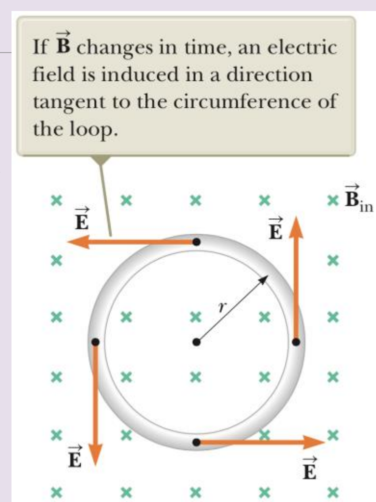
Induced emf and Electric Fields

Consider a conducting loop of radius *r* in a uniform magnetic field that is perpendicular to the plane of the loop.

If the magnetic field changes with time, an emf induced in the loop

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The induction of a current in the loop implies the presence of an induced electric field **E**, which must be tangent to the loop because that is the direction in which the charges in the wire move in response to the electric force.



Induced emf and Electric Fields

The work done by the electric field in moving a test charge q once around the loop

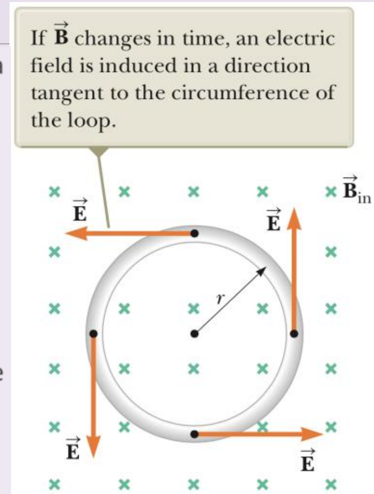
$$W = q\varepsilon$$

Since the electric force acting on the charge

$$F_e = qE$$

The work done by the electric field in moving the charge once around the loop is

$$W = qE(2\pi r)$$



Induced emf and Electric Fields

Therefore,

$$q\varepsilon = qE(2\pi r) \quad \Rightarrow \quad E = \frac{\varepsilon}{2\pi r}$$

Induced electric field

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{1}{2\pi r}\frac{dBA}{dt} = -\frac{1}{2\pi r}\frac{dB(\pi r^2)}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

Induced electric field can be calculated from the time variation of the magnetic field.

Faraday's law of induction

The emf for any closed path can be expressed as

$$\varepsilon = \oint \vec{E}.d\vec{s}$$

E may not be constant and the path may not be a circle.

$$\oint \vec{E}.d\vec{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law of induction

The induced electric field E in Equation above is a nonconservative field that is generated by a changing magnetic field.
The field E that satisfies that Equation cannot possibly be an electrostatic field because were the field electrostatic and hence conservative, the line integral of **E**.d**s** over a closed loop would be zero.

Example

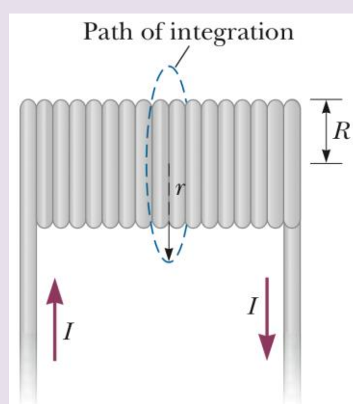
A long solenoid of radius *R* has *n* turns of wire per unit length and carries a time- varying current that varies sinusoidally as

$$I = I_{max} \cos \omega t,$$

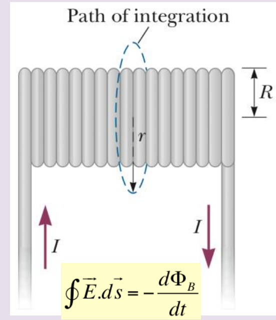
where I_{max} is the maximum current and ω is the angular frequency of the alternating current source.

(A) Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis.

(B) What is the magnitude of the induced electric field inside the solenoid, a distance *r* from its axis?



Solution (A)



$$\oint \vec{E}.d\vec{s} = -\frac{d\Phi_B}{dt}$$

Consider an external point and take the path for the line integral to be a circle of radius *r* centered on the solenoid

$$(1) \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$$

$$(2) \quad B = \mu_0 n I = \mu_0 n I_{\max} \cos \omega t$$

$$(3) \quad -\frac{d\Phi_B}{dt} = -\pi R^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$(4) \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E(2\pi r) \qquad E(2\pi r) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \frac{\mu_0 n I_{\max} \omega R^2}{2r} \sin \omega t \qquad (\text{for } r > R)$$

This result shows that the amplitude of the electric field outside the solenoid falls off as 1/r and varies sinusoidally with time.

Solution (B)

$$\oint \vec{E}.d\vec{s} = -\frac{d\Phi_B}{dt}$$

For an interior point ($r < R$), the magnetic flux through an integration loop is given by $\Phi_B = B\pi r^2$.

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi r^2) = -\pi r^2 \frac{dB}{dt}$$

$$-\frac{d\Phi_B}{dt} = -\pi r^2 \mu_0 \, n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

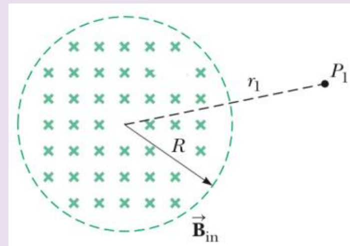
$$E(2\pi r) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \frac{\mu_0 n I_{\max} \omega}{2} r \sin \omega t \qquad (\text{for } r < R)$$

The amplitude of the electric field induced inside the solenoid by the changing magnetic flux through the solenoid increases linearly with r and varies sinusoidally with time.

Solve by your self

(1) Within the green dashed circle shown in the Figure, the magnetic field changes with time according to the expression $B = 2.00t^3 - 4.00t^2 + 0.800$, where B is in teslas, t is in seconds, and R = 2.50 cm. When t = 2.00 s, calculate (a) the magnitude and (b) the direction of the force exerted on an electron located at point P_1, which is at a distance r_1 = 5.00 cm from the center of the circular field region. (c) At what instant is this force equal to zero?



(2) A long solenoid with 1.00×10^3 turns per meter and radius 2.00 cm carries an oscillating current $I = 5.00 \sin 100\pi t$, where I is in amperes and t is in seconds. (a) What is the electric field induced at a radius r = 1.00 cm from the axis of the solenoid? (b) What is the direction of this electric field when the current is increasing counterclockwise in the solenoid?

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Magnetism and Alternating Current

Unit 3: Faraday's Law

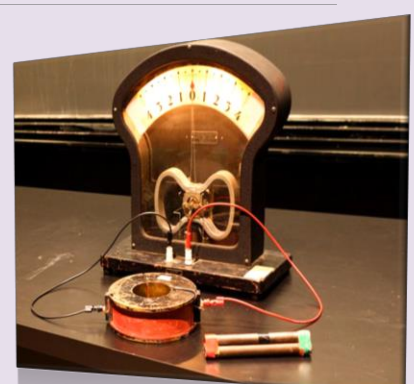
Lecture 16: Generators and Motors

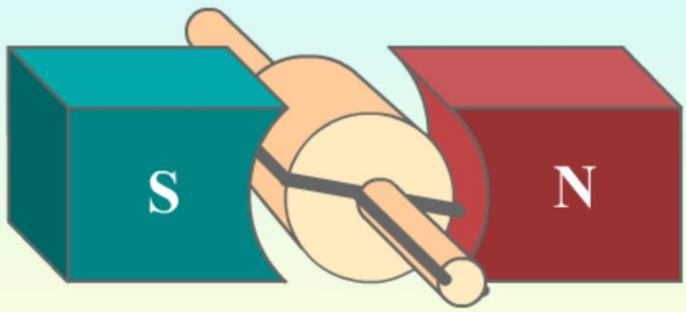
Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Unit 3: Faraday's Law

- 3.1 Faraday's Law of Induction
- 3.2 Motional emf
- 3.3 Lenz's Law
- 3.4 Induced emf and Electric Fields
- **3.5 Generators and Motors**
- 3.6 Eddy Currents



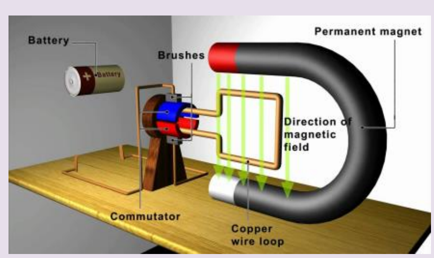


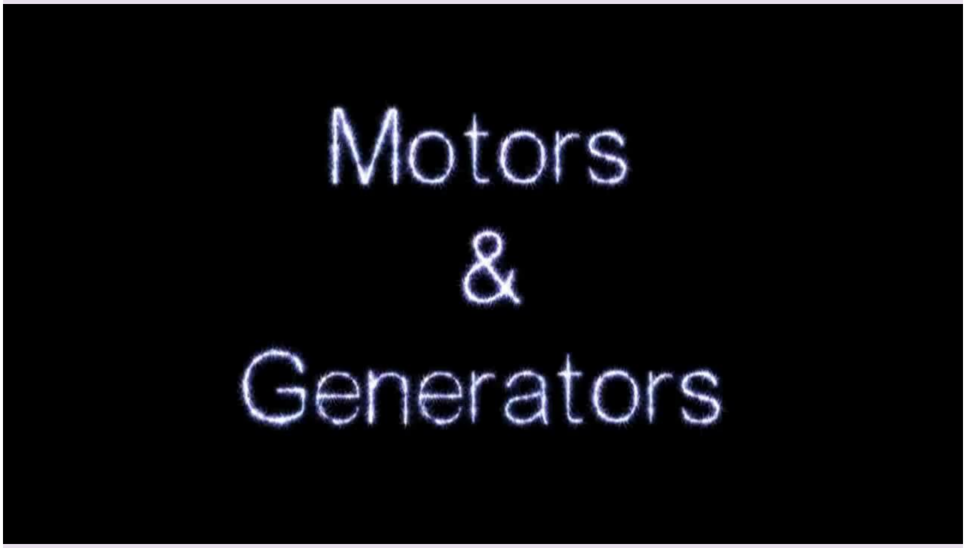
Generators and Motors

Electric generators take in energy by work and transfer it out by electrical transmission.

A **motor** is a device into which energy is transferred by electrical transmission while energy is transferred out by work.

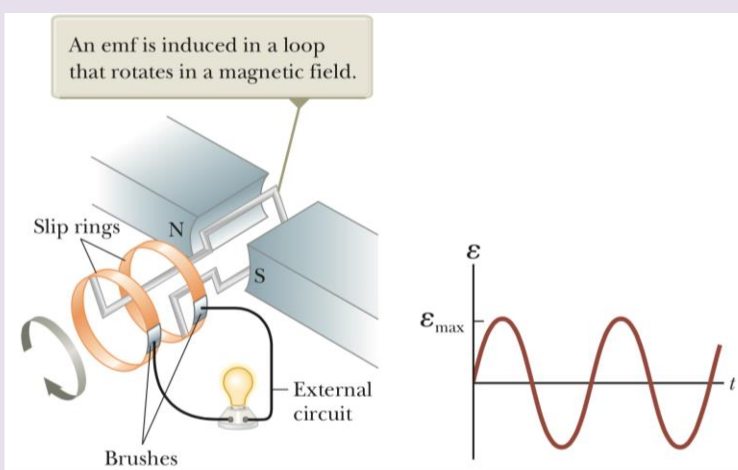
A motor is essentially a generator operating in reverse. Instead of generating a current by rotating a coil, a current is supplied to the coil by a battery, and the torque acting on the current-carrying coil causes it to rotate.





Generators

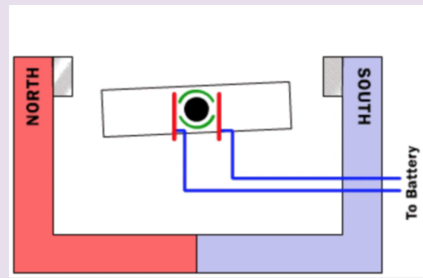
Consider the alternating-current (AC) generator. In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field.



Generators

As a loop rotates in a magnetic field, the magnetic flux through the area enclosed by the loop changes with time, and this change induces an emf and a current in the loop according to Faraday's law.

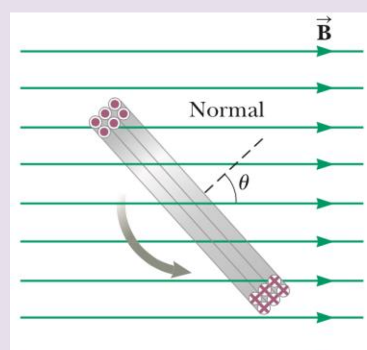
The ends of the loop are connected to slip rings that rotate with the loop.



Connections from these slip rings, which act as output terminals of the generator, to the external circuit are made by stationary metallic brushes in contact with the slip rings.

Instead of a single turn, suppose a coil with *N* turns, with the same area *A*, rotates in a magnetic field with a constant angular speed v.

If θ is the angle between the magnetic field and the normal to the plane of the coil as in the Figure, the magnetic flux through the coil at any time *t* is



$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

The induced emf in the coil is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t) = NAB\omega\sin\omega t$$

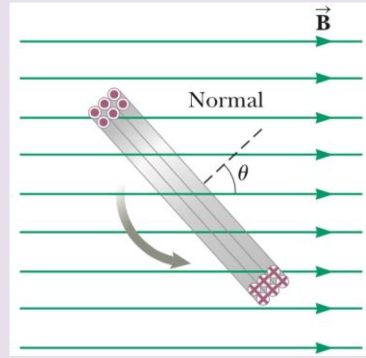
This result shows that the emf varies sinusoidally with time.

The maximum emf has the value is

$$\mathcal{E}_{max} = NAB\omega$$

which occurs when ωt = 90° or 270°.

The emf is zero when ωt = 0 or 180°.



Example 1

The coil in an AC generator consists of 8 turns of wire, each of area A = 0.090 m^2, and the total resistance of the wire is 12.0 V. The coil rotates in a 0.500-T magnetic field at a constant frequency of 60.0 Hz.

(A) Find the maximum induced emf in the coil.

$$\mathcal{E}_{max} = NAB\omega = NAB(2\pi f)$$

$$\mathcal{E}_{max} = 8(0.090\,0\ \mathrm{m^2})(0.500\ \mathrm{T})(2\pi)(60.0\ \mathrm{Hz}) = 136\ \mathrm{V}$$

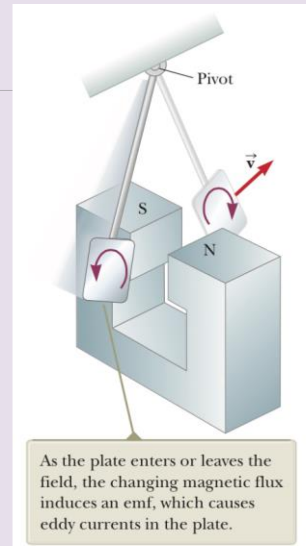
(B) What is the maximum induced current in the coil when the output terminals are connected to a low-resistance conductor?

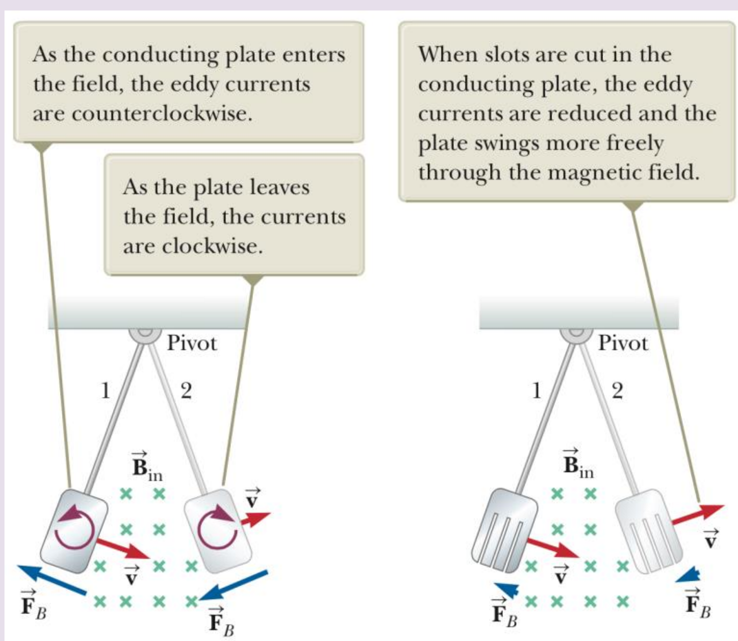
$$I_{max} = \frac{\mathcal{E}_{max}}{R} = \frac{136\ \mathrm{V}}{12.0\ \Omega} = 11.3\ \mathrm{A}$$

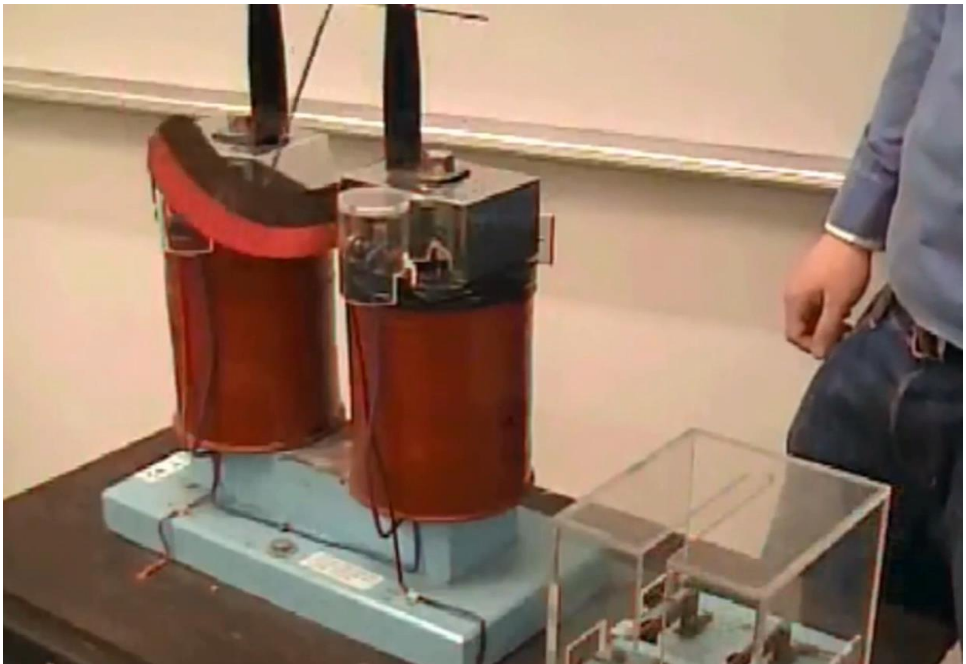
Eddy Currents

An emf and a current are induced in a circuit by a changing mag- netic flux. In the same manner, circulating currents called **eddy currents** are induced in bulk pieces of metal moving through a magnetic field.

According to Lenz's law, the direction of the eddy currents is such that they create magnetic fields that oppose the change that causes the currents. For this reason, the eddy currents must produce effective magnetic poles on the plate, which are repelled by the poles of the magnet; this situation gives rise to a repulsive force that opposes the motion of the plate.



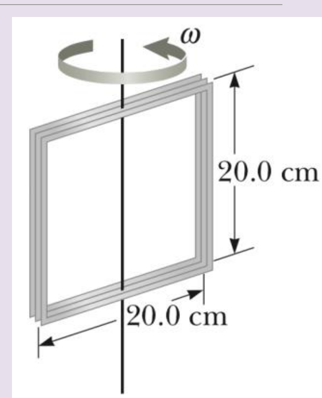




Solve by your self

A 100-turn square coil of side 20.0 cm rotates about a vertical axis at 1.50 x 10^3 rev/min as indicated in the Figure. The horizontal component of the Earth's magnetic field at the coil's location is equal to 2.00 x 10^{-5} T.

(a) Calculate the maximum emf induced in the coil by this field.
(b) (b) What is the orientation of the coil with respect to the magnetic field when the maximum emf occurs?



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Magnetism and Alternating Current

Unit 4: Inductance

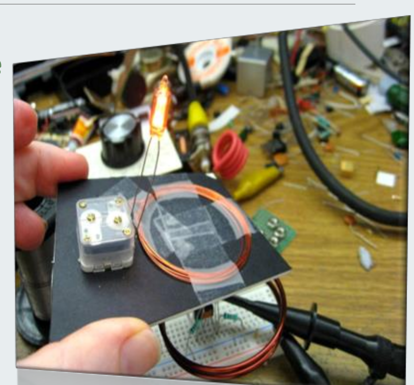
Lecture 17: Self-Induction and Inductance

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

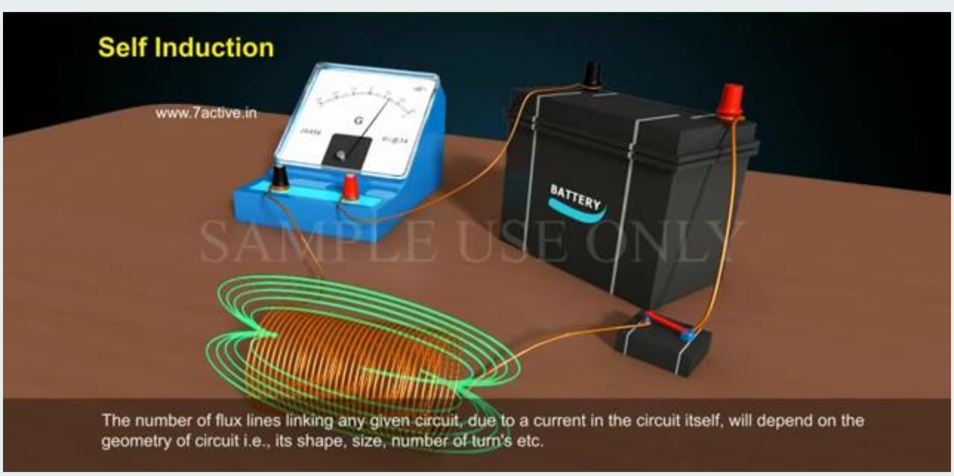
Unit 4: Inductance

- 4.1 Self-Induction and Inductance
- 4.2 RL Circuits
- 4.3 Energy in a Magnetic Field
- 4.4 Mutual Inductance
- 4.5 Oscillations in an LC Circuit
- 4.6 The RLC Circuit



Objective of unit 4

- *Self-induction,* in which a time-varying current in a circuit produces an induced emf opposing the emf that initially set up the time-varying current.
- The energy stored in the magnetic field of an inductor and the energy density associated with the magnetic field.
- Study how an emf is induced in a coil as a result of a changing magnetic flux produced by a second coil, which is the basic principle of *mutual induction.*
- Study the characteristics of circuits that contain inductors, resistors, and capacitors in various combinations.



Resistance, Capacitance & Inductance

Ohm's law defines resistance as: $R = \frac{\Delta V}{I}$

Resistors do not store energy; they transform electrical energy into thermal energy at a rate of: $P = I^2 \cdot R = \frac{V^2}{R}$

Capacitance is the ability to hold charge: $C = \frac{Q}{\Delta V}$

Capacitors store electric energy in the electric field between the plates when fully charged: $U_E = 1/2 \times C \times V^2$

Inductance can be described as the ability to "hold" current.

Inductors store energy in the magnetic field inside the inductor once the current flows through it.

Terminology

EMF and *current* are associated with batteries or other primary voltage sources.

Induced EMF and *induced current* are associated with changing magnetic flux.

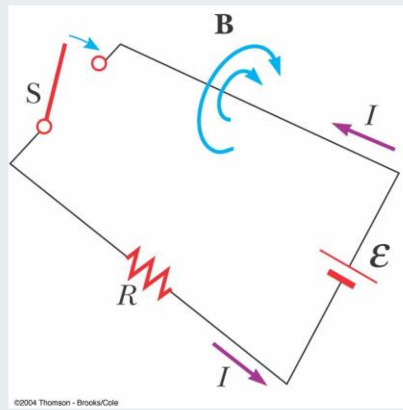
Self-Inductance

When the switch is closed, the current does not immediately reach its maximum value.

Faraday's law can be used to describe the effect.

As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time.

This increasing flux creates an induced emf in the circuit.



Self-Inductance

The direction of the induced emf is such that it would cause an induced current in the loop which would establish a magnetic field opposing the change in the original magnetic field.

The direction of the induced emf is opposite the direction of the emf of the battery.

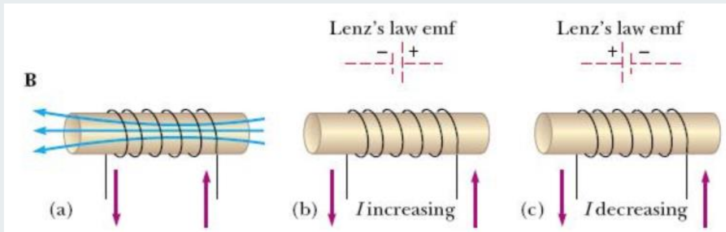
This results in a gradual increase in the current to its final equilibrium value.

This effect is called self-inductance

- Because the changing flux through the circuit and the resultant induced emf arise from the circuit itself.

The emf ε_L is called a self-induced emf

Self-induced emf

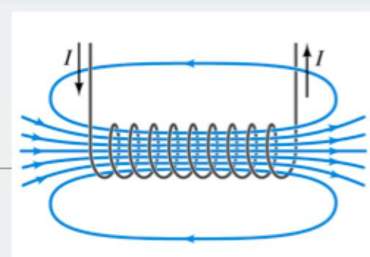


A current in the coil produces a magnetic field directed toward the left (a)

If the current increases, the increasing flux creates an induced emf of the polarity shown (b)

The polarity of the induced emf reverses if the current decreases (c)

Self Inductance



$$B = \mu_o n I$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d(\mu_o nIA)}{dt} = -N\mu_o nA\frac{dI}{dt} = -\frac{NBA}{I}\frac{dI}{dt}$$

$$\varepsilon = -\frac{N\Phi_B}{I}\frac{dI}{dt} = -L\frac{dI}{dt}$$

Define: Self Inductance $L = \frac{N\Phi_B}{I}$

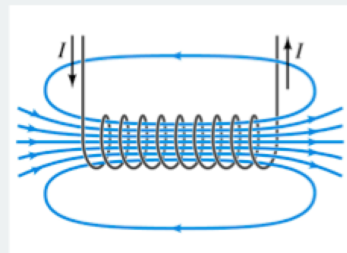
Inductance of a Solenoid

The magnetic flux through each turn is

$$\Phi_B = BA = \left(\mu_o \frac{N}{l} I\right) A$$

Therefore, the inductance is

$$L = \frac{N\Phi_B}{I} \rightarrow L = \frac{\mu_o N^2 A}{l}$$

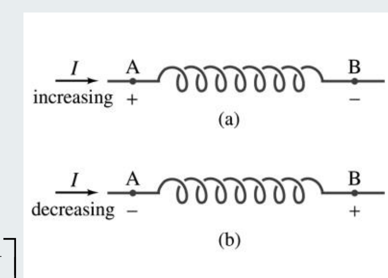


This shows that *L* depends on the geometry of the object

Inductance Units

$$\varepsilon = -L\frac{dI}{dt} \qquad L = \frac{N\Phi_B}{I}$$

$$L = \left[\frac{V}{A/s}\right] = [\ \ .s] = [\text{Henry}] = [H]$$



If the rate of change of **current** in a circuit is one ampere per second and the resulting **electromotive force** is one volt, then the **inductance** of the circuit is one henry.

Example (1)

Consider a uniformly wound solenoid having *N* turns and length L. Assume L is much longer than the radius of the windings and the core of the solenoid is air.

(A) Find the inductance of the solenoid.

(B) Calculate the inductance of the solenoid if it contains 300 turns, its length is 25.0 cm, and its cross-sectional area is 4.00 cm^2.

(C) Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50.0 A/s.

Solution (A)

The magnetic flux through each turn of area *A* in the solenoid

$$\Phi_B = BA = \mu_0 n I A = \mu_0 \frac{N}{\ell} I A$$

The inductance of the solenoid

$$L = \frac{N\Phi_B}{I} = \mu_0 \frac{N^2}{\ell} A$$

L depends on geometry and is proportional to the square of the number of turns.

Solution (B)

$$L = \frac{N\Phi_B}{I} = \mu_0 \frac{N^2}{\ell} A$$

$$L = (4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}) \frac{300^2}{25.0 \times 10^{-2}\ \text{m}} (4.00 \times 10^{-4}\ \text{m}^2)$$

$$= 1.81 \times 10^{-4}\ \text{T} \cdot \text{m}^2/\text{A} = 0.181\ \text{mH}$$

Solution (C)

$$dI/dt = -50.0\ \text{A/s}$$

$$\varepsilon_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \text{H})(-50.0\ \text{A/s})$$

$$= 9.05\ \text{mV}$$

Solve by your self

(1) The current in a coil changes from 3.50 A to 2.00 A in the same direction in 0.500 s. If the average emf induced in the coil is 12.0 mV, what is the inductance of the coil?

(2) A solenoid of radius 2.50 cm has 400 turns and a length of 20.0 cm. Find (a) its inductance and (b) the rate at which current must change through it to produce an emf of 75.0 μV.

(3) An inductor in the form of a solenoid contains 420 turns and is 16.0 cm in length. A uniform rate of decrease of current through the inductor of 0.421 A/s induces an emf of 175 μV. What is the radius of the solenoid?

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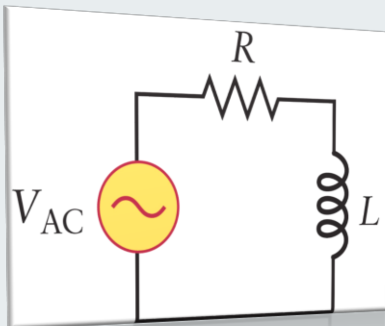
Magnetism and Alternating Current



Unit 4: Inductance

Lecture 18: RL Circuits

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 4: Inductance

4.1 Self-Induction and Inductance

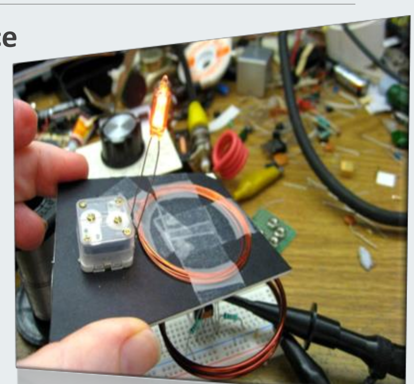
4.2 RL Circuits

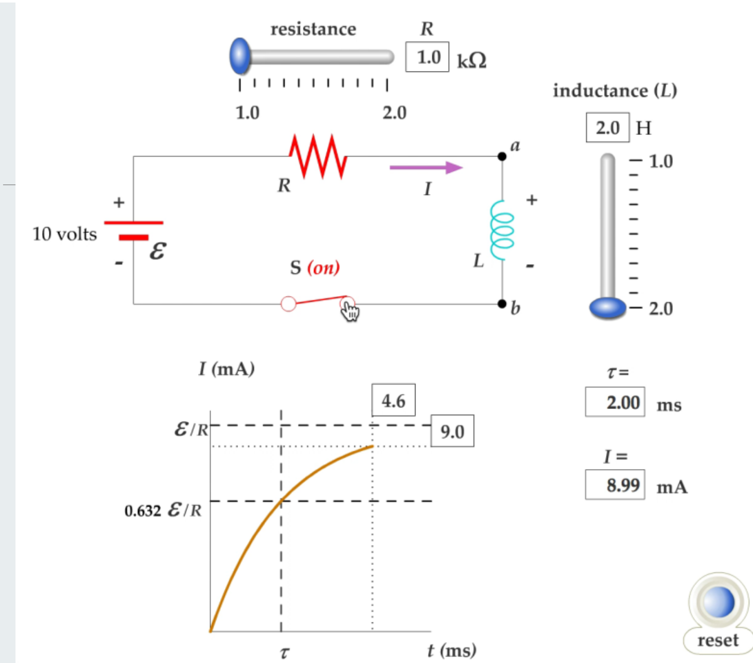
4.3 Energy in a Magnetic Field

4.4 Mutual Inductance

4.5 Oscillations in an LC Circuit

4.6 The RLC Circuit





RL Circuit, Introduction

A circuit element that has a large self-inductance is called an **inductor.**

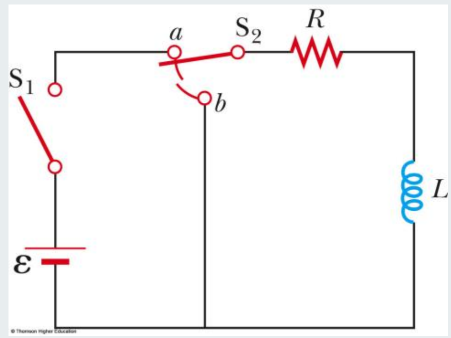
The circuit symbol is 

We assume the self-inductance of the rest of the circuit is negligible compared to the inductor.

- However, even without a coil, a circuit will have some self-inductance.

RL Circuit, Analysis

- An *RL* circuit contains an inductor and a resistor
- Assume S_2 is connected to *(a)*
- When switch S_1 is closed (at time t = 0), the current begins to increase.
- At the same time, a back emf is induced in the inductor that opposes the original increasing current.

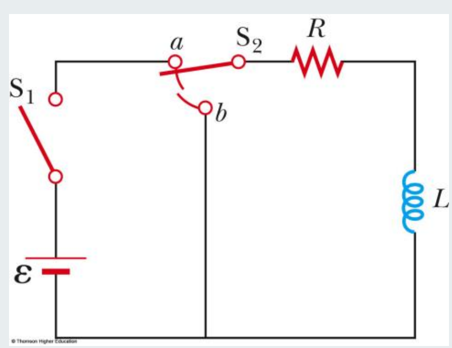


RL Circuit, Analysis, cont.

Applying Kirchhoff's loop rule to the previous circuit in the clockwise direction gives

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

Looking at the current as a function of time, we find



$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

To find this solution, we change variables for convenience, lets assume

$$x = (\mathcal{E}/R) - I \quad \longrightarrow \quad dx = -dI$$

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

Rearranging and integrating this last expression gives

$$\int_{x_0}^{x} \frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

where x_0 is the value of x at time t = 0.

$$\int_{x_0}^{x} \frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

$$\ln\frac{x}{x_0} = -\frac{R}{L}t$$

Taking the antilogarithm of this result gives

$$x = x_0 e^{-Rt/L} \qquad x = (\mathcal{E}/R) - I$$

Because I = 0 at t = 0, note from the definition of x that $x_0 = \varepsilon/R$. Hence, this last expression is equivalent to

$$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R}e^{-Rt/L}$$

$$I = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

Conclusion

$$I = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$$

- The inductor affects the current exponentially.
- The current does not instantly increase to its final equilibrium value.
- If there is no inductor, the exponential term goes to zero and the current would instantaneously reach its maximum value as expected.

RL Circuit, Time Constant

The expression for the current can also be expressed in terms of the time constant, τ, of the circuit

$$I = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$$

- where $\tau = L / R$

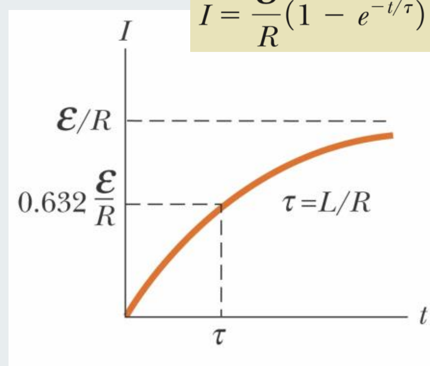
Physically, time constant t is the time required for the current to reach $(1 - e^{-1}) = 0.632 = 63.2\%$ of its maximum value ε/R.

The time constant is a useful parameter for comparing the time responses of various circuits.

RL Circuit, Current-Time Graph

- The equilibrium value of the current is $\mathcal{E}/R$ and is reached as *t* approaches infinity.
- The current initially increases very rapidly.
- The current then gradually approaches the equilibrium value.

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$$



RL Circuit, Current-Time Graph

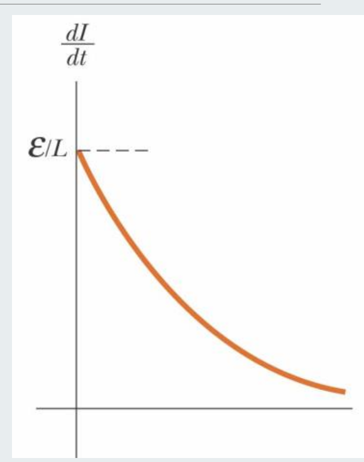
The time rate of change of the current by taking the first time derivative of

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$$

We get

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-t/\tau}$$

This result shows that the time rate of change of the current is a maximum (equal to ε/L) at t = 0 and falls off exponentially to zero as t approaches infinity.



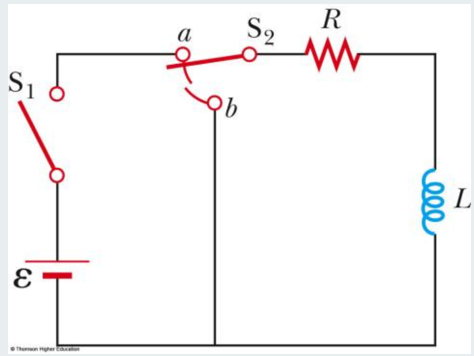
RL Circuit Without A Battery

Now set S_2 to position (b).

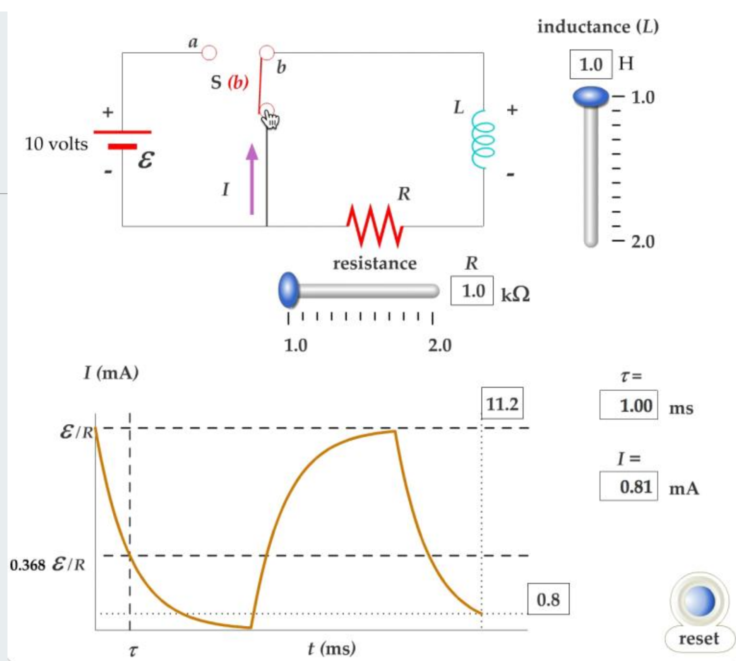
The circuit now contains just the right hand loop.

The battery has been eliminated.

The expression for the current becomes.



$$IR + L\frac{dI}{dt} = 0 \rightarrow I = \frac{\varepsilon}{R}e^{-t/\tau} = I_i e^{-t/\tau}$$



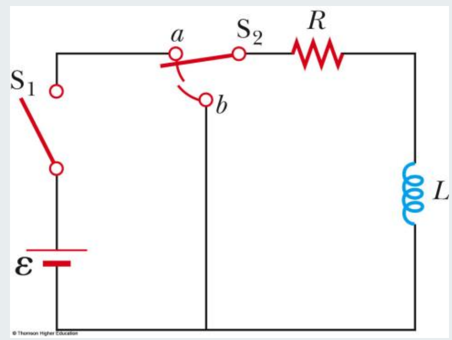
Example

Consider the circuit in the Figure. Suppose the circuit elements have the following values: ε = 12.0 V, *R* = 6.00 Ω, and *L* = 30.0 mH.

(A) Find the time constant of the circuit.

(B) Switch S_2 is at position *a,* and switch S_1 is thrown closed at *t* = 0. Calculate the current in the circuit at *t* = 2.00 ms.

(C) Compare the potential difference across the resistor with that across the inductor.



Solution (A) & (B)

(A) the time constant

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \text{H}}{6.00\ \Omega} = 5.00\ \text{ms}$$

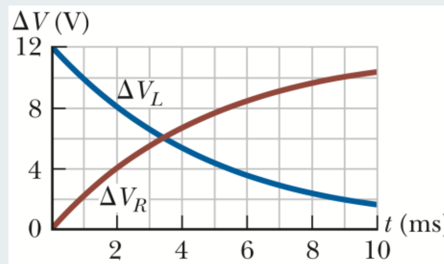
(B) The current at *t*=2.00ms

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = \frac{12.0\ \text{V}}{6.00\ \Omega}(1 - e^{-2.00\ \text{ms}/5.00\ \text{ms}}) = 2.00\ \text{A}\,(1 - e^{-0.400})$$

$$= 0.659\ \text{A}$$

Solution (C)

At the instant the switch is closed, there is no current and therefore no potential difference across the resistor.



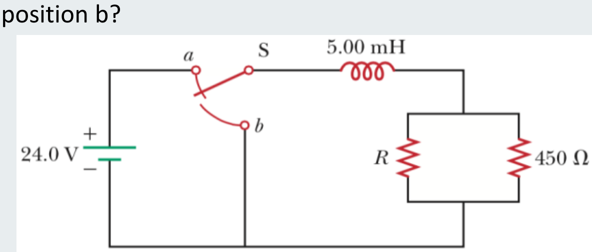
At this instant, the battery voltage appears entirely across the inductor in the form of a back emf of 12.0 V as the inductor tries to maintain the zero-current condition. (The top end of the inductor in the Figure is at a higher electric potential than the bottom end.)

As time passes, the emf across the inductor decreases and the current in the resistor (and hence the voltage across it) increases as shown in the Figure. The sum of the two voltages at all times is 12.0 V.

Solve by your self

(1) A 510-turn solenoid has a radius of 8.00 mm and an over- all length of 14.0 cm. (a) What is its inductance? (b) If the solenoid is connected in series with a 2.50-V resistor and a battery, what is the time constant of the circuit?

(2) Consider the circuit shown in the Figure (a) When the switch is in position a, for what value of R will the circuit have a time constant of 15.0 ms? (b) What is the current in the inductor at the instant the switch is thrown to position b?



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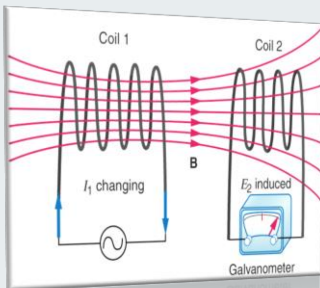
Magnetism and Alternating Current



Unit 4: Inductance

Lecture 19: Energy in a Magnetic Field and Mutual Inductance

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 4: Inductance

4.1 Self-Induction and Inductance

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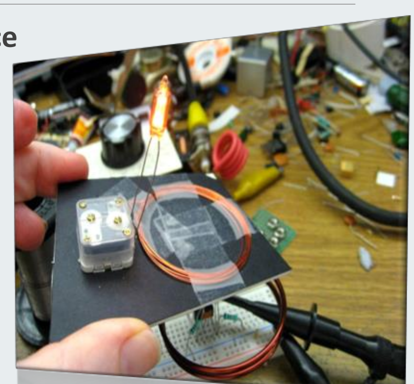
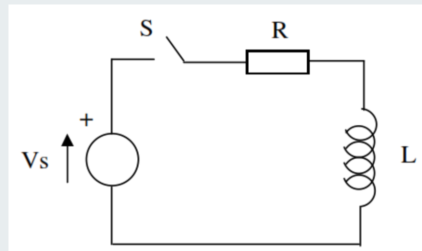


Energy in a Magnetic Field

- In a circuit with an inductor, the battery must supply more energy than in a circuit without an inductor.
- Part of the energy supplied by the battery appears as internal energy in the resistor.
- The remaining energy is stored in the magnetic field of the inductor.



$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

Multiplying each term in the Equation by the current (I) and rearranging the expression gives

Energy in a Magnetic Field

Looking at this energy (in terms of rate)

$$I\varepsilon = I^2R + LI\frac{dI}{dt}$$

- **$I\varepsilon$ is the rate at which energy is being supplied by the battery.**
- **I^2R is the rate at which the energy is being delivered to the resistor.**
- **Therefore, LI (dI/dt) must be the rate at which the energy is being stored in the magnetic field.**

Energy in a Magnetic Field

Let U denote the energy stored in the inductor at any time

The rate at which the energy is stored is

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

To find the **total energy U**, we integrate the equation above

$$U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$U = \frac{1}{2}LI^2$$

Energy Density of a Magnetic Field

Given $U = 1/2LI^2$ and assume (for simplicity) a solenoid with the inductance

$$L = \frac{N\Phi_B}{I} \rightarrow L = \frac{\mu_o N^2 A}{l} \rightarrow L = \frac{\mu_o (nl)^2 A}{l}$$

$$L = \mu_o n^2 V$$

The current

$$B = \mu_o nI \rightarrow I = \frac{B}{\mu_o n}$$

Energy Density of a Magnetic Field

$$U = \frac{1}{2}LI^2 \qquad L = \mu_o n^2 V \qquad I = \frac{B}{\mu_o n}$$

$$U = \frac{1}{2}\mu_0 n^2 V\left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}V$$

Since V is the volume of the solenoid, **the magnetic energy density, u_B is**

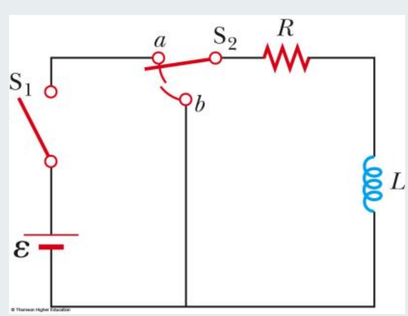
$$u_B = \frac{U}{V} = \frac{B^2}{2\mu_0}$$

Magnetic energy density

This applies to any region in which a magnetic field exists (not just the solenoid)

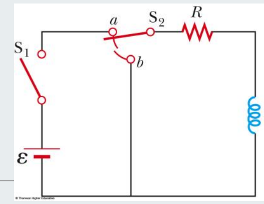
Example

Consider the *RL* circuit shown in the Figure, with switch S_2 at position *a* and the current having reached its steady-state value. When S_2 is thrown to position *b*, the current in the right-hand loop decays exponentially with time according to the expression $I = I_i e^{-t/\tau}$, where $I_i = \varepsilon/R$ is the initial current in the circuit and $\tau = L/R$ is the time constant.



Show that all the energy initially stored in the magnetic field of the inductor appears as internal energy in the resistor as the current decays to zero.

Solution



Before S_2 is thrown to *b,* energy is being delivered at a constant rate to the resistor from the battery and energy is stored in the magnetic field of the inductor. After t = 0, when S_2 is thrown to *b,* the battery can no longer provide energy and energy is delivered to the resistor only from the inductor.

The energy in the magnetic field of the inductor at any time is *U.* The rate *dU/dt* at which energy leaves the inductor and is delivered to the resistor is equal to I^2R, where I is the instantaneous current.

$$\frac{dU}{dt} = I^2R \qquad I = \frac{\mathcal{E}}{R}e^{-t/\tau} = I_i e^{-t/\tau}$$

$$\frac{dU}{dt} = I^2R = (I_i e^{-Rt/L})^2 R = I_i^{\,2} R e^{-2Rt/L}$$

Solve for *dU* and integrate this expression over the limits t=0 to $t \rightarrow \infty$

$$U = \int_0^\infty I_i^{\,2} R e^{-2Rt/L}\,dt = I_i^{\,2} R \int_0^\infty e^{-2Rt/L}\,dt$$

$$\int_0^\infty e^{-2Rt/L}dt = -\frac{L}{2R}e^{-2Rt/L}\Big|_0^\infty = -\frac{L}{2R}\left(e^{-\infty} - e^0\right) = \frac{L}{2R}(0-1) = \boxed{\frac{L}{2R}}$$

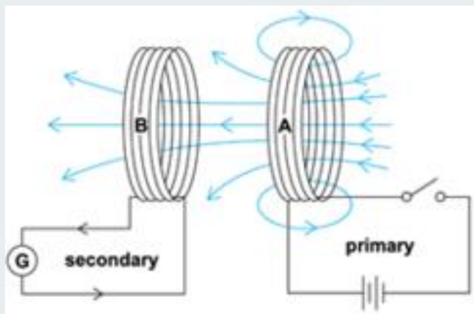
$$U = I_i^{\,2} R\left(\frac{L}{2R}\right) = \tfrac{1}{2}LI_i^{\,2}$$

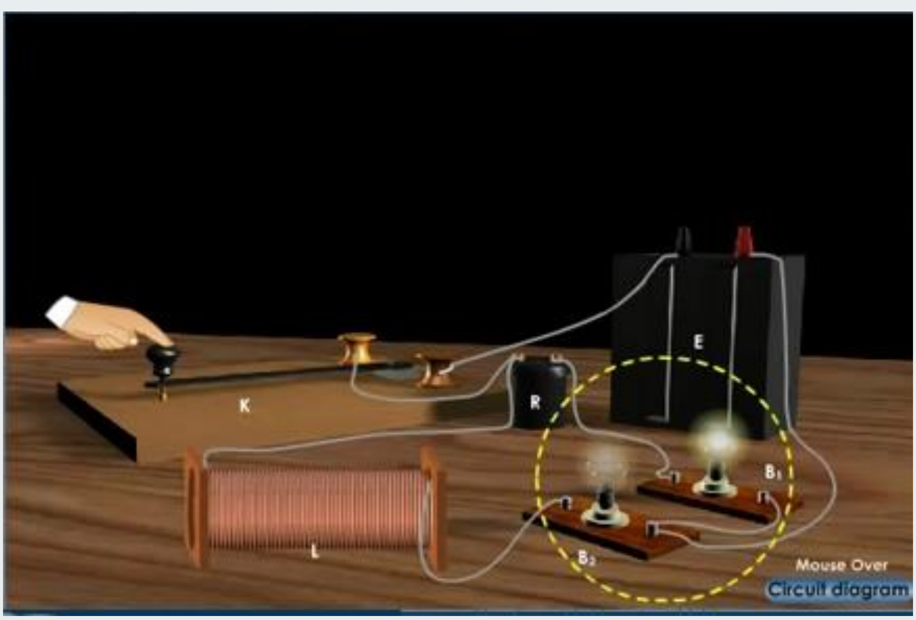
The initial energy stored in the magnetic field of the inductor when the current is I.

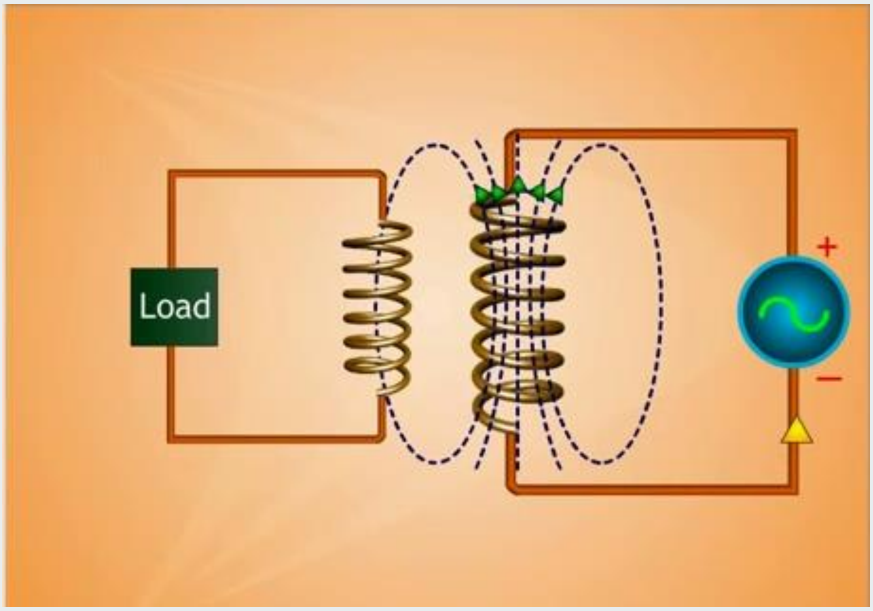
Mutual Inductance

As we have seen previously, changes in the magnetic flux due to one circuit can effect what goes on in other circuits.

The changing magnetic flux induces an emf in the second circuit.







Mutual Inductance

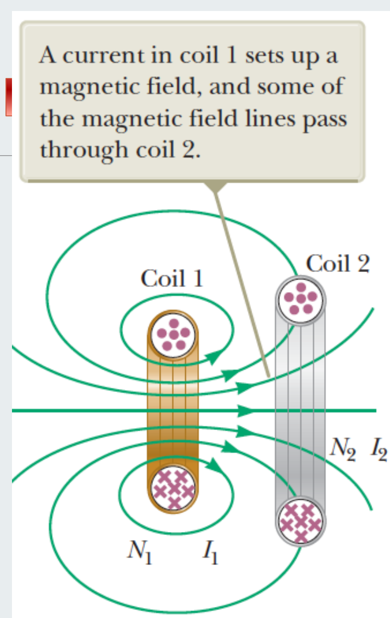
Suppose that we have two coils,

Coil 1 with N_1 turns and Coil 2 with N_2 turns.

Coil 1 has a current I_1 which produces a magnetic flux, Φ_{12} , going through one turn of Coil 2

we can identify the **mutual inductance** M_{12} of coil 2 with respect to coil 1:

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$



If the current I_1 varies with time, then the flux changes and an emf is induced induced by coil 1 in **Coil 2** which is given by

$$\varepsilon_2 = -N_2 \frac{d\Phi_{12}}{dt} \Rightarrow \varepsilon_2 = -N_2 \frac{d}{dt}\left(\frac{M_{12} I_1}{N_2}\right) \qquad M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

$$\varepsilon_2 = -M_{12} \frac{dI_1}{dt}$$

Imagine the current I_2 in coil 2. If the current I_2 varies with time, the emf induced by coil 2 in **coil 1** is

$$\varepsilon_1 = -M_{21} \frac{dI_2}{dt}$$

In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing.

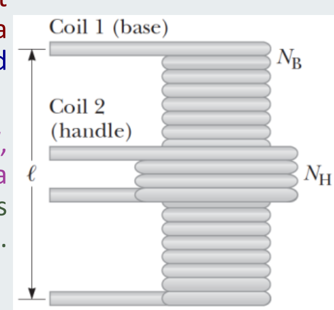
$$\varepsilon_2 = -M \frac{dI_1}{dt} \qquad \varepsilon_1 = -M \frac{dI_2}{dt}$$

The unit of mutual inductance is the henry.

Example

An electric toothbrush has a base designed to hold the toothbrush handle when not in use. The handle has a cylindrical hole that fits loosely over a matching cylinder on the base. When the handle is placed on the base, a changing current in a solenoid inside the base cylinder induces a current in a coil inside the handle. This induced current charges the battery in the handle.

We can model the base as a solenoid of length l, with N_B turns, carrying a current I, and having a cross-sectional area A. The handle coil contains N_H turns and completely surrounds the base coil. **Find the mutual inductance of the system.**



Solution

The magnetic field in the interior of the **base solenoid:**

$$B = \mu_0 \frac{N_B}{\ell} I$$

The mutual inductance, noting that the magnetic flux Φ_{BH} through the handle's coil caused by the magnetic field of the base coil is BA:

$$M = \frac{N_H \Phi_{BH}}{I} = \frac{N_H BA}{I} = \mu_0 \frac{N_B N_H}{\ell} A$$

Wireless charging is used in a number of other "cordless" devices. One significant example is the inductive charging used by some manufacturers of electric cars that avoids direct metal-to-metal contact between the car and the charging apparatus.

Solve by your self

(1) Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a magnetic flux of 3.70 x 10^{-4} T . m^2 in each turn.

(2) A 10.0-V battery, a 5.00-Ω resistor, and a 10.0-H inductor are connected in series. After the current in the circuit has reached its maximum value, calculate (a) the power being supplied by the battery, (b) the power being delivered to the resistor, (c) the power being delivered to the inductor, and (d) the energy stored in the magnetic field of the inductor.

(3) An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. What is the mutual inductance of the two coils?

(4) Two coils are close to each other. The first coil carries a current given by $I(t) = 5\, e^{-0.025t} \sin 120\pi t$, where I is in amperes and t is in seconds. At $t = 0.8$ s, the emf measured across the second coil is 23.20 V. What is the mutual inductance of the coils?

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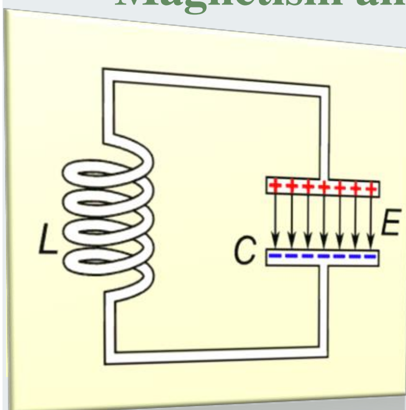
Magnetism and Alternating Current



Unit 4: Inductance

Lecture 20: Oscillations in an LC and RLC Circuit

Dr. Hazem Falah Sakeek

Al-Azhar University of Gaza

Unit 4: Inductance

4.1 Self-Induction and Inductance

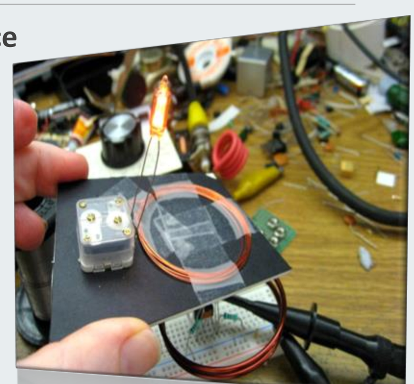
4.2 RL Circuits

4.3 Energy in a Magnetic Field

4.4 Mutual Inductance

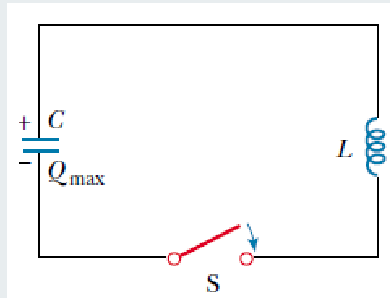
4.5 Oscillations in an LC Circuit

4.6 The RLC Circuit



Oscillations in an LC Circuit

- When a charged **capacitor** is connected to an **inductor** as shown in the figure and the switch is then closed, **oscillations** will occur in the **current and charge on the capacitor.**
- If the resistance of the circuit is zero, no energy is dissipated as joule heat and the oscillations will persist.
- The resistance of the circuit will be ignored.
- Assume that the capacitor has an initial charge Q and that the switch is closed at t = 0 s.

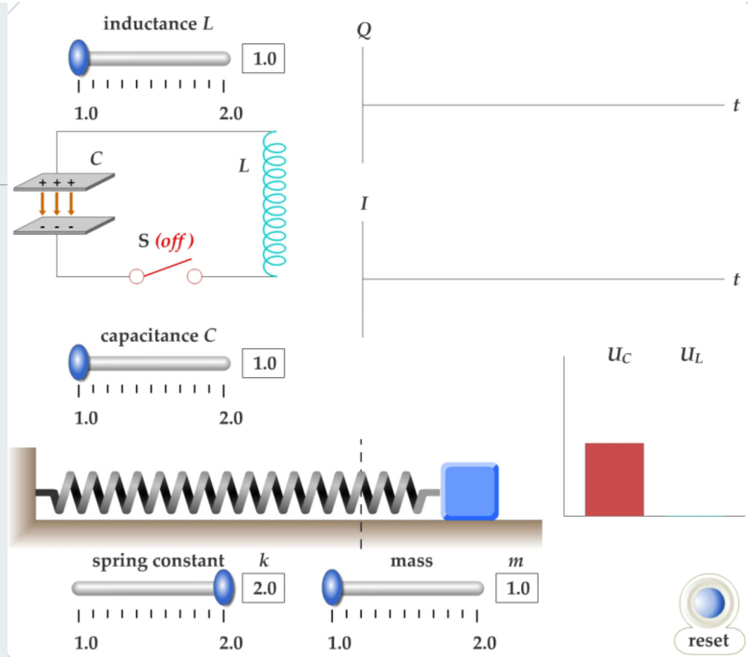


Oscillations in an LC Circuit

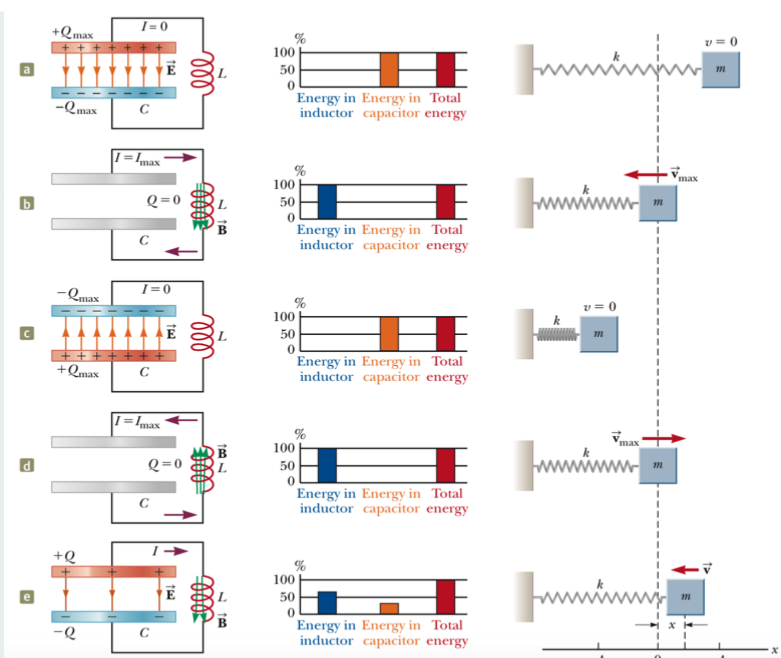
- When the capacitor is fully charged, the total energy **U** in the circuit is stored in the **electric field** of the capacitor and is equal to

$$U = \frac{Q_{max}^2}{2C}$$

- At this time, the current is zero and there is no energy stored in the inductor.
- As the capacitor begins to discharge, the energy stored in its electric field decreases.
- The circuit behavior is analogous to an oscillating mass-spring system.



inductance L
1.0
1.0
2.0
C
L
S (off)
Q
t
I
t
capacitance C
1.0
1.0
2.0
U_C
U_L
spring constant k
2.0
mass m
1.0
1.0
2.0
1.0
2.0
reset



a
+Q_max
I = 0
E
L
−Q_max
C
%
100
50
0
Energy in inductor
Energy in capacitor
Total energy
k
v = 0
m
b
I = I_max
Q = 0
L
B
C
v_max
c
−Q_max
I = 0
+Q_max
d
I = I_max
e
+Q
I
−Q
v
x
−A
0

Energy Consideration of LC Circuit

At any time, the sum of the two energies must equal the total initial energy *U* stored in the fully charged capacitor at *t* = 0:

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$

Total energy stored in an LC circuit

We have assumed the circuit resistance to be zero and we ignore electromagnetic radiation. Therefore,

$$\frac{dU}{dt} = 0$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{1}{2}LI^2\right) = 0 \quad \Rightarrow \quad \frac{dU}{dt} = \frac{1}{2C}\frac{dQ^2}{dt} + \frac{1}{2}L\frac{dI^2}{dt} = 0$$

$$\frac{dU}{dt} = \frac{1}{2C}2Q\frac{dQ}{dt} + \frac{1}{2}L2I\frac{dI}{dt} = 0$$

$$I = \frac{dQ}{dt}$$

$$\frac{dI}{dt} = \frac{d^2Q}{dt^2}$$

$$\frac{dU}{dt} = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{dU}{dt} = \frac{Q}{C}I + LI\frac{d^2Q}{dt^2} = 0$$

$$\frac{Q}{C} = L\frac{d^2Q}{dt^2} \quad \Rightarrow \quad \frac{d^2Q}{dt^2} = \frac{1}{LC}Q \quad \Rightarrow \quad \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

We can solve for the function Q by noting that the equation is of the same form as that of the mass-spring system (simple harmonic oscillator):

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

- where k is the spring constant, m is the mass, and $\omega = \sqrt{k/m}$

The solution for the equation has the general form

$$x = A\cos(\omega t + \phi)$$

- where ω is the angular frequency of the simple harmonic motion, A is the amplitude of the motion (the maximum value of x), and φ is the phase constant; the values of A and φ depend on the initial conditions.

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \qquad \frac{d^2x}{dt^2} = -\omega^2 x$$

$$Q = Q_{max}\cos(\omega t + \phi)$$

Charge as a function of time for an ideal LC circuit

where Q_{max} is the maximum charge of the capacitor and the angular frequency ω is given by:

$$\omega = \frac{1}{\sqrt{LC}}$$

Angular frequency of oscillation in an LC circuit

The angular frequency of the oscillation depends on the inductance and capacitance of the circuit.

Since Q varies periodically, the current also varies periodically.

$$I = \frac{dQ}{dt} = -\omega Q_{\max} \sin(\omega t + \phi)$$

Current as a function of time for an ideal LC current

To determine the value of the phase angle φ, let's examine the initial conditions, which in our situation require that at t = 0, I = 0, and Q = Q_{max}. Setting I = 0 at t = 0 in Equation above we get:

$$0 = -\omega Q_{\max} \sin\phi$$

which shows that φ = 0.

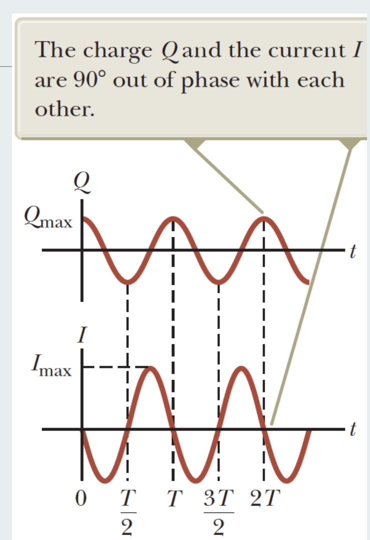
$$Q = Q_{\max} \cos\omega t$$

$$I = -\omega Q_{\max} \sin\omega t = -I_{\max} \sin\omega t$$

Graphs of *Q* versus *t* and *I* versus *t*

the current is 90º out of phase with the charge.

That is, when the charge is a maximum, the current is zero, and when the charge is zero, the current has its maximum value.



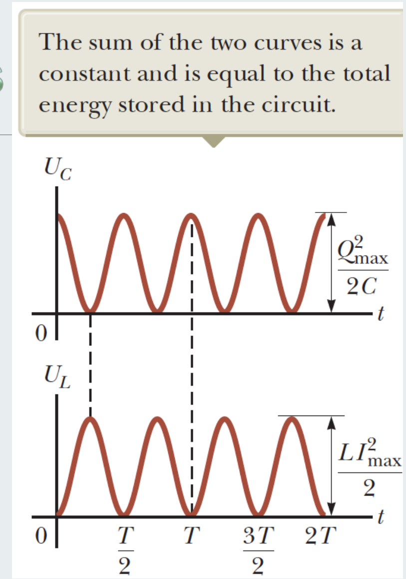
Energy Oscillations

$$U = U_C + U_L$$

$$U = \frac{Q_{max}^2}{2C}\cos^2 \quad t + \frac{1}{2}LI_{max}^2 \sin^2 \quad t$$

The sum U_C + U_L is a constant and is equal to the total energy

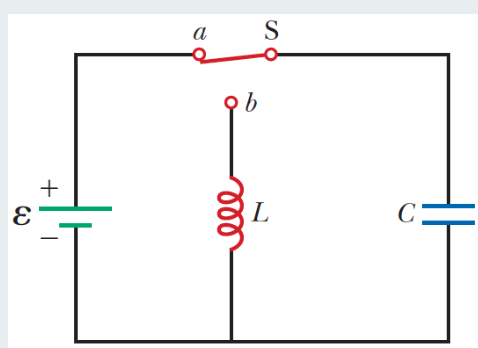
$$U = \frac{Q_{max}^2}{2C} = \frac{1}{2}LI_{max}^2$$



Example

The battery has an emf of 12.0 V, the inductance is 2.81 mH, and the capacitance is 9.00 pF. The switch has been set to position *a* for a long time so that the capacitor is charged. The switch is then thrown to position *b*, removing the battery from the circuit and connecting the capacitor directly across the inductor.

(A) Find the frequency of oscillation of the circuit.
(B) What are the maximum values of charge on the capacitor and current in the circuit?



Solution (A)

The frequency:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi[(2.81 \times 10^{-3}\,\mathrm{H})(9.00 \times 10^{-12}\,\mathrm{F})]^{1/2}}$$

$$= 1.00 \times 10^{6}\,\mathrm{Hz}$$

Solution (B)

The initial charge on the capacitor, which equals the maximum charge:

$$Q_{\max} = C\,\Delta V$$

$$= (9.00 \times 10^{-12}\,\mathrm{F})(12.0\,\mathrm{V})$$

$$= 1.08 \times 10^{-10}\,\mathrm{C}$$

The maximum current from the maximum charge

$$I_{\max} = \omega Q_{\max} = 2\pi f Q_{\max}$$

$$= (2\pi \times 10^{6}\,\mathrm{s}^{-1})(1.08 \times 10^{-10}\,\mathrm{C})$$

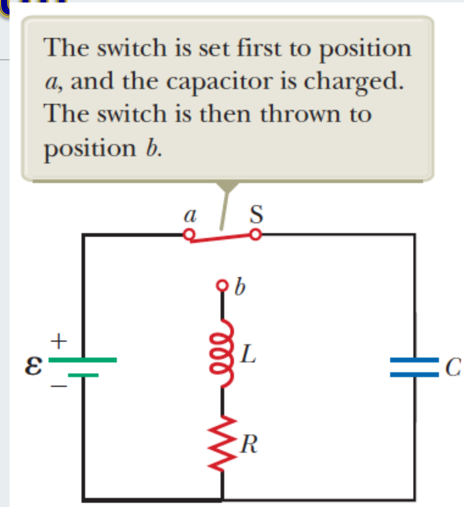
$$= 6.79 \times 10^{-4}\,\mathrm{A}$$

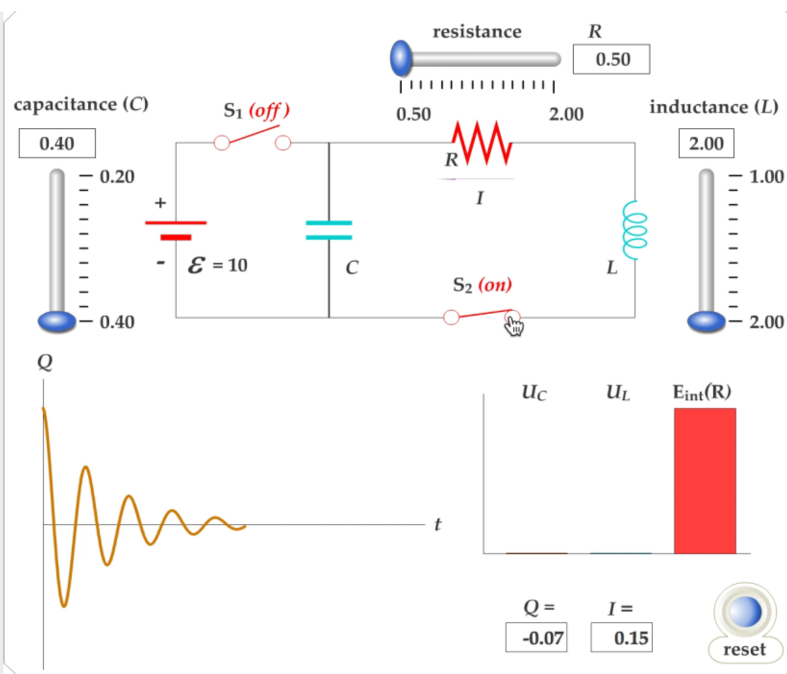
The *RLC* Circuit

Consider a more realistic circuit consisting of a resistor, an inductor, and a capacitor connected in series.

Suppose the switch is at position *a* so that the capacitor has an initial charge Q_{max}.

The total energy, however, is no longer constant as it was in the *LC* circuit because the resistor causes transformation to internal energy.



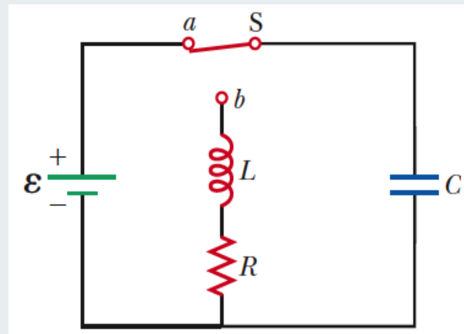


The *RLC* Circuit Oscillation

Because the rate of energy transformation to internal energy within a resistor is I^2R,

$$\frac{dU}{dt} = I^2R$$

the negative sign signifies that the energy U of the circuit is decreasing in time.



$$\frac{dU}{dt} = LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = I^2R$$

$$\frac{dU}{dt} = LI\frac{d^2Q}{dt^2} + I^2R + \frac{Q}{C}I = 0$$

$$I = \frac{dQ}{dt}$$

$$\frac{dI}{dt} = \frac{d^2Q}{dt^2}$$

Now divide through by *I*

$$L\frac{d^2Q}{dt^2} + IR + \frac{Q}{C} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

The *RLC* circuit is analogous to the damped harmonic oscillator

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

The analytical solution of Equation is too long so we will give only a qualitative description of the circuit behavior.

When R = 0, Equation above reduces to that of a simple LC circuit as expected, and the charge and the current oscillate sinusoidally in time.

$$Q = Q_{\max}\cos\omega t$$

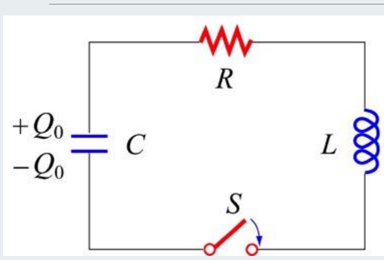
When R is small, a situation that is analogous to light damping in the mechanical oscillator,

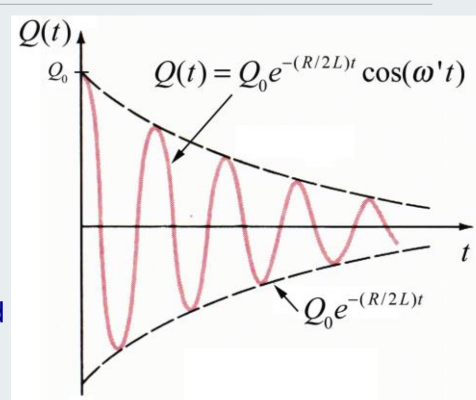
$$Q = Q_{\max}e^{-Rt/2L}\cos\omega_d t$$

where ω_d, the angular frequency at which the circuit oscillates, is given by

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

Damped LC or RLC Oscillations





Resistor dissipates energy and system rings down over time. Also, frequency decreases.

Solve by your self

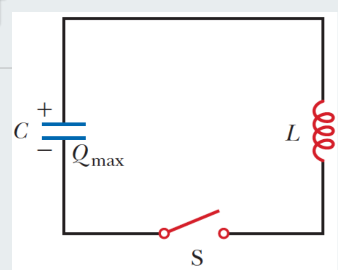
(1) A 1.05-mH inductor is connected in series with a variable capacitor in the tuning section of a short wave radio set. What capacitance tunes the circuit to the signal from a transmitter broadcasting at 6.30 MHz?

(2) Calculate the inductance of an *LC* circuit that oscillates at 120 Hz when the capacitance is 8.00 mF.

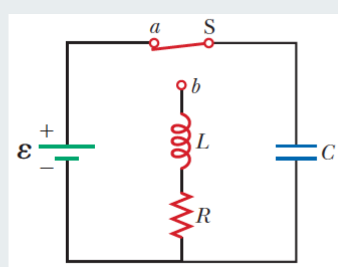
(3) A 1.00-mF capacitor is charged by a 40.0-V power supply. The fully charged capacitor is then discharged through a 10.0-mH inductor. Find the maximum current in the resulting oscillations.

Solve by your self

(4) An *LC* circuit like the one in the Figure contains an 82.0-mH inductor and a 17.0-mF capacitor that initially carries a 180-mC charge. The switch is open for $t < 0$ and is then thrown closed at $t = 0$. (a) Find the frequency (in hertz) of the resulting oscillations. At $t = 1.00$ ms, find (b) the charge on the capacitor and (c) the current in the circuit.



(5) In the Figure, let $R = 7.60\ \Omega$, $L = 2.20$ mH, and $C = 1.80$ mF. (a) Calculate the frequency of the damped oscillation of the circuit when the switch is thrown to position *b.* (b) What is the critical resistance for damped oscillations?



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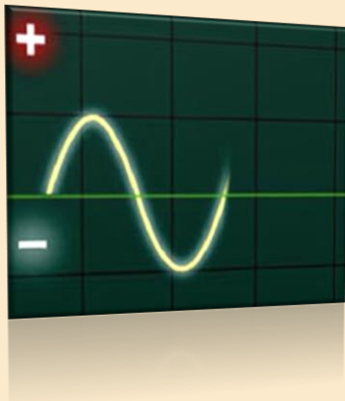
Magnetism and Alternating Current



Unit 5: Alternating-Current Circuits

Lecture 21: Resistors, Inductors and Capacitors in AC Circuit

Dr. Hazem Falah Sakeek
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Unit 4: Alternating-Current Circuits

- 5.1 AC Sources
- 5.2 Resistors in an AC Circuit
- 5.3 Inductors in an AC Circuit
- 5.4 Capacitors in an AC Circuit
- 5.5 The RLC Series Circuit
- 5.6 Power in an AC Circuit
- 5.7 Resonance in a Series RLC Circuit
- 5.8 The Transformer and Power Transmission
- 5.9 Rectifiers and Filters

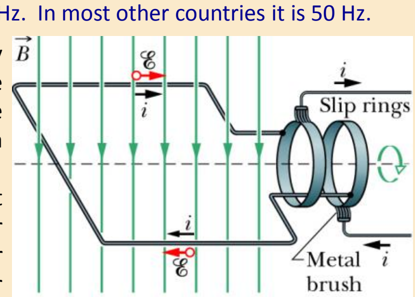
About this unit 5

- We describe alternating-current (AC) circuits.
- Investigating the characteristics of simple series circuits that contain resistors, inductors, and capacitors and that are driven by a sinusoidal voltage.
- If an AC source applies an alternating voltage to a series circuit containing resistors, inductors, and capacitors, we want to know the amplitude and time characteristics of the alternating current.
- Learning about transformers, power transmission, and electrical filters.

Alternating Current

The electric power out of a home or office power socket is in the form of alternating current (AC), as opposed to the direct current (DC) of a battery.
Alternating current is used because it is easier to transport, and easier to "transform" from one voltage to another using a transformer.
In the U.S., the frequency of oscillation of AC is 60 Hz. In most other countries it is 50 Hz.

- One way to make an alternating current by rotating a coil of wire in a magnetic field. The slip rings and brushes allow the coil to rotate without twisting the connecting wires. Such a device is called a *generator*.
- It takes power to rotate the coil, but that power can come from moving water (a water turbine), or air (windmill), or a gasoline motor (as in your car), or steam (as in a nuclear power plant).



$= {}_m \sin \ t$ $i = I \sin(\ t \ )$

AC Sources Characteristics

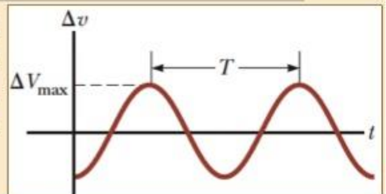
An AC circuit consists of circuit elements and a power source that provides an alternating voltage Δv. This time-varying voltage from the source is described by

$$\Delta v = \Delta V_{max} \sin \omega t$$

The angular frequency of the AC voltage is

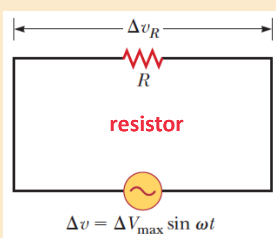
$$\omega = 2\pi f = \frac{2\pi}{T}$$

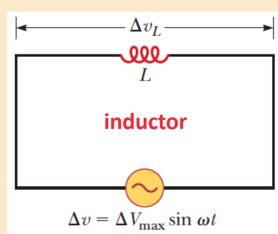
where f is the frequency of the source and T is the period.

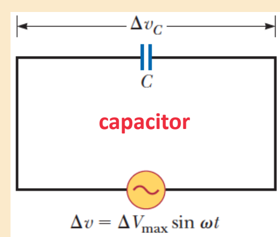


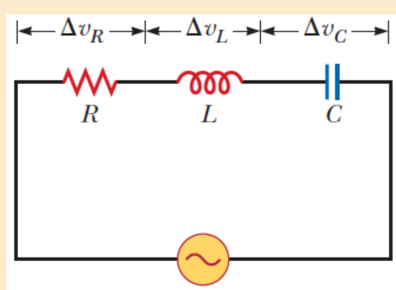
The voltage supplied by an AC source is sinusoidal with a period T.

ΔV_{max} is the maximum output voltage of the source, or the voltage amplitude.









- Lower case symbols will indicate instantaneous values.
- Capital letters will indicate fixed values.

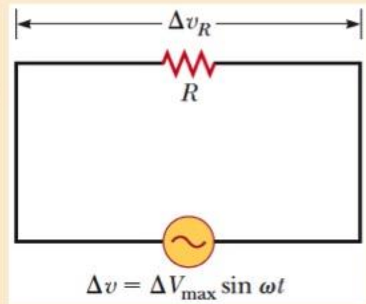
Resistors in an AC Circuit

At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule).

$$\Delta v + \Delta v_R = 0$$

$$\Delta v - i_R R = 0$$

$$\because \Delta v = \Delta V_{\max} \sin \omega t$$



$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{\max}}{R} \sin \omega t = I_{\max} \sin \omega t \Rightarrow I_{\max} = \frac{\Delta V_{\max}}{R}$$

Maximum current in a resistor

The instantaneous current in the resistor and the instantaneous voltage across the resistor

$$i_R = I_{\max} \sin \omega t \qquad \Delta v_R = i_R R = I_{\max} R \sin \omega t$$

The graph shows the current through and the voltage across the resistor.

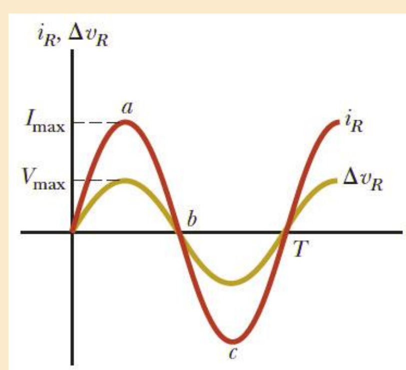
The current and the voltage reach their maximum values at the same time.

The current and the voltage are said to be *in phase.*

For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.

The direction of the current has no effect on the behavior of the resistor.

Resistors behave essentially the same way in both DC and AC circuits.



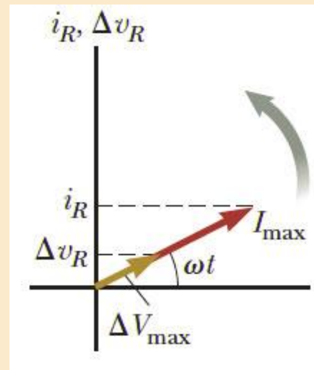
Phasor Diagram

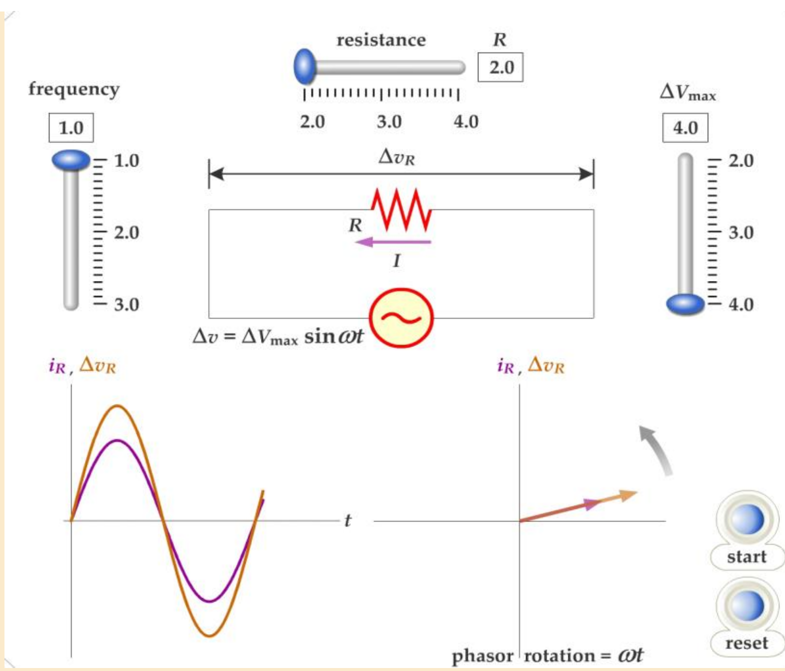
To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used.

A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents.

The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.

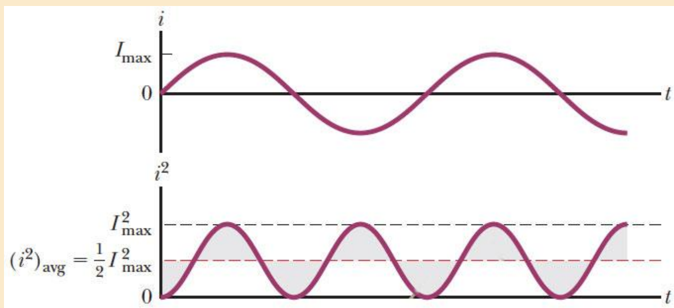
The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.





rms Current and Voltage

- The average current in one cycle is zero.
- Resistors experience a temperature increase which depends on the magnitude of the current, but not the direction of the current.
- The power is related to the square of the current.



The rate at which energy is delivered to a resistor is the power $P = i^2R$, where i is the instantaneous current in the resistor.

The temperature increase produced by an alternating current having a maximum value I_{max}, however, is not the same as that produced by a direct current equal to I_{max} because the alternating current has this maximum value for only an instant during each cycle.

What is of importance in an AC circuit is an average value of current, referred to as the rms (root-mean-square) current.

$$I_{rms} = \sqrt{(i^2)_{avg}}$$

i^2 varies with $\sin^2\omega t$

$$(i^2)_{avg} = \frac{1}{2} I^2_{max}$$

The rms current is the average of importance in an AC circuit.

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$$

rms current

Alternating current whose maximum value is 2.00 A delivers to a resistor the same power as a direct current that has a value of (0.707)x(2.00 A) = 1.41 A.

Power

The rate at which electrical energy is delivered to a resistor in the circuit is given by

$$P = i^2 R$$

- *i* is the *instantaneous current.*
- The heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value.
- The maximum current occurs for a small amount of time.

The **average power** delivered to a resistor that carries an alternating current is

$$P_{av} = I_{rms}^2 R$$

AC ammeters and voltmeters are designed to read rms values.

Example 1

- The voltage output of an AC source is given by the expression Δ*v* = 200 sin ω*t*, where Δ*v* is in volts. Find the rms current in the circuit when this source is connected to a 100-Ω resistor.

Solution

- **rms voltage**

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{200\ \text{V}}{\sqrt{2}} = 141\ \text{V}$$

- **rms current**

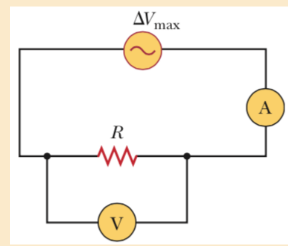
$$I_{rms} = \frac{\Delta V_{rms}}{R} = \frac{141\ \text{V}}{100\ \Omega} = 1.41\ \text{A}$$

Solve by your self

(1) When an AC source is connected across a 12.0-V resistor, the rms current in the resistor is 8.00 A. Find (a) the rms voltage across the resistor, (b) the peak voltage of the source, (c) the maximum current in the resistor, and (d) the average power delivered to the resistor.

(2) An AC source has an output rms voltage of 78.0 V at a frequency of 80.0 Hz. If the source is connected across a 25.0-mH inductor, what are (a) the inductive reactance of the circuit, (b) the rms current in the circuit, and (c) the maximum current in the circuit?

(3) An AC power supply produces a maximum voltage ΔV_{max} = 100 V. This power supply is connected to a resistor R = 24.0 Ω, and the current and resistor voltage are measured with an ideal AC ammeter and voltmeter as shown in the Figure. An ideal ammeter has zero resistance, and an ideal voltmeter has infinite resistance. What is the reading on (a) the ammeter and (b) the voltmeter?



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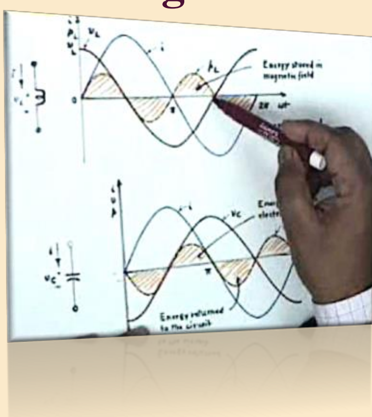
Magnetism and Alternating Current



Unit 5: Alternating-Current Circuits

Lecture 22: Resistors, Inductors and Capacitors in AC Circuit

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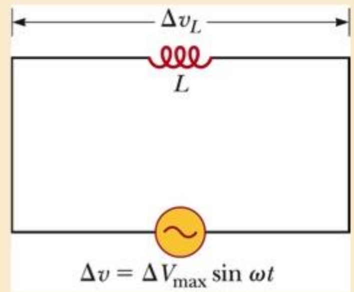
Unit 4: Alternating-Current Circuits

Inductors in an AC Circuit

- Kirchhoff's loop rule can be applied and gives:



$$\Delta v + \Delta v_L = 0$$

$$\Delta v - L\frac{di_L}{dt} = 0$$

$$\Delta v_L = L\frac{di_L}{dt} = \Delta V_{max} \sin \omega t$$

$$di_L = \frac{\Delta V_{max}}{L} \sin \omega t \, dt$$

$$i_L = \frac{\Delta V_{max}}{L} \int \sin \omega t \, dt \quad \Rightarrow \quad i_L = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

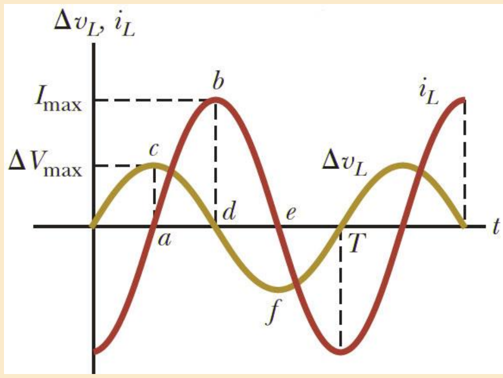
$$i_L = -\frac{\Delta V_{max}}{\omega L} \cos \omega t \qquad \cos \omega t = -\sin(\omega t - \pi/2)$$

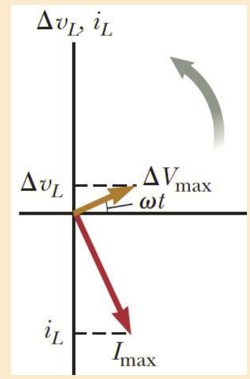
$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\Delta v_L = \Delta V_{max} \sin \omega t$$

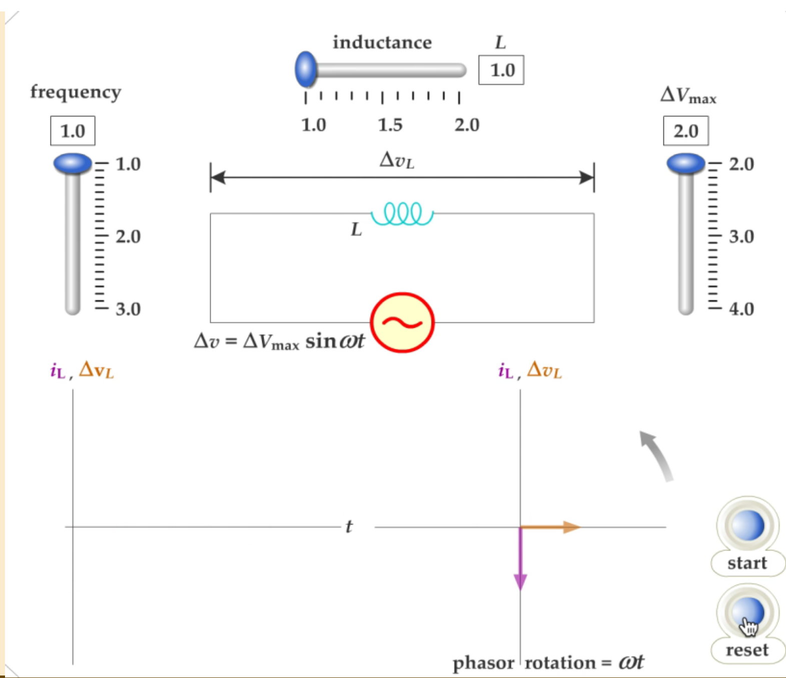
This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by ($\pi/2$).

Phase Relationship of Inductors in an AC Circuit





The current in an inductor always lags behind the voltage across the inductor by 90°



Inductive Reactance

$$i_L = \frac{\Delta V_{max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance.

Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.

The factor is the **inductive reactance** and is given by:

$$X_L = \omega L$$

Inductive Reactance, cont.

Maximum Current can be expressed in terms of the inductive reactance:

$$I_{max} = \frac{\Delta V_{max}}{X_L} \quad \text{or} \quad I_{rms} = \frac{\Delta V_{rms}}{X_L}$$

As the frequency increases, the inductive reactance increases

- This is consistent with Faraday's Law:
  - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current.

The instantaneous voltage across the inductor is

$$\Delta v_L = -L\frac{di}{dt} = -\Delta V_{max}\sin\omega t = -I_{max}\,X_L\sin\omega t$$

Example 2

- In a purely inductive AC circuit, L = 25.0 mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

Solution

- the inductive reactance

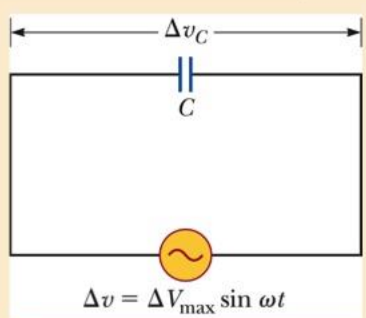
$$X_L = \omega L = 2\pi f L = 2\pi(60.0\ \text{Hz})(25.0 \times 10^{-3}\ \text{H})$$
$$= 9.42\ \Omega$$

- the rms current

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{150\ \text{V}}{9.42\ \Omega} = 15.9\ \text{A}$$

Capacitors in an AC Circuit

The circuit contains a capacitor and an AC source. Kirchhoff's loop rule gives:



$$\Delta v + \Delta v_C = 0$$

Δv_c is the instantaneous voltage across the capacitor.

$$\Delta v - \frac{q}{C} = 0$$

$$\Delta v = \Delta V_{\text{max}} \sin \omega t$$

$$q = C\Delta V_{\text{max}} \sin \omega t$$

where q is the instantaneous charge on the capacitor.

- The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t$$

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

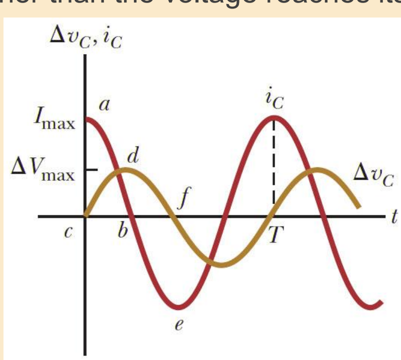
Current in a capacitor

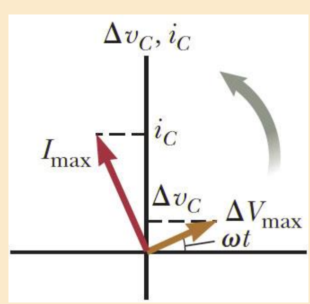
$$\Delta v = \Delta V_{\max} \sin \omega t$$

This shows that the instantaneous current i_C in the capacitor and the instantaneous voltage Δv_C across the capacitor are out of phase by ($\pi/2$).

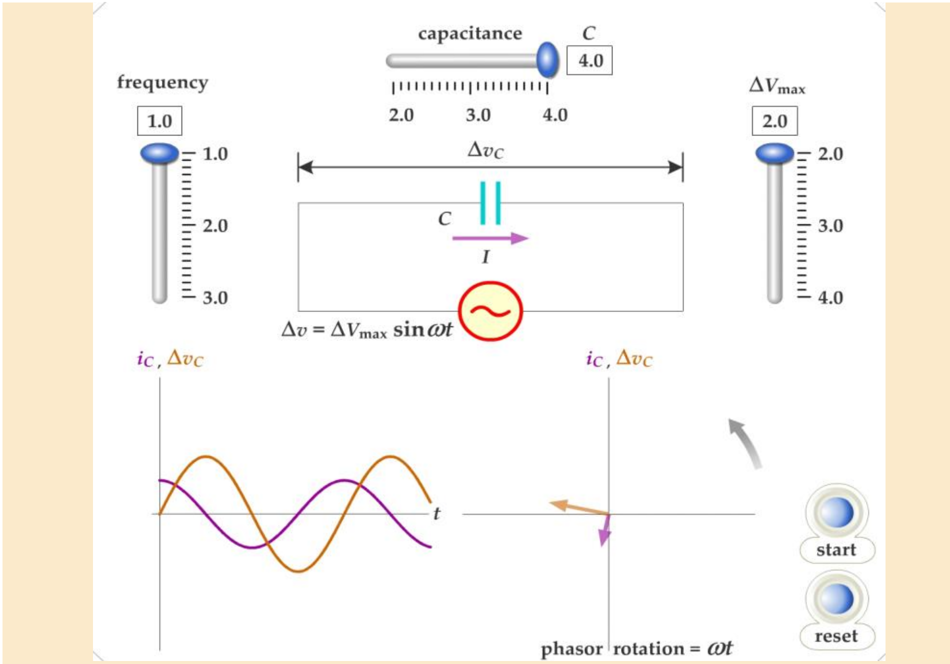
Phase Relationship of Capacitor in an AC Circuit

The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.





The current leads the voltage by 90°.



Capacitive Reactance

The maximum current in the circuit occurs at cos ωt = ±1 which gives

$$i_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{(1/\omega C)}$$

The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C = \frac{1}{\omega C}$$

Capacitive reactance

$$I_{max} = \frac{\Delta V_{max}}{X_C}$$

Maximum current in a capacitor

Voltage Across a Capacitor

The instantaneous Voltage across a capacitor

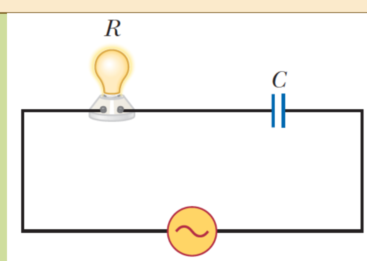
$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t$$

As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.

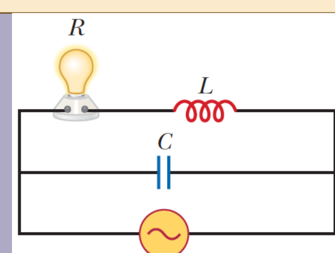
As the frequency approaches zero, X_C approaches infinity and the current approaches zero.

- This would act like a DC voltage and the capacitor would act as an open circuit.

Quiz



The frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest? **(a)** It glows brightest at high frequencies. **(b)** It glows brightest at low frequencies. **(c)** The brightness is the same at all frequencies.



The frequency of the AC source is adjusted while its voltage amplitude is held constant. When does the lightbulb glow the brightest? **(a)** It glows brightest at high frequencies. **(b)** It glows brightest at low frequencies. **(c)** The brightness is the same at all frequencies.

Example 3

- An 8.00-μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

Solution

- the capacitive reactance:

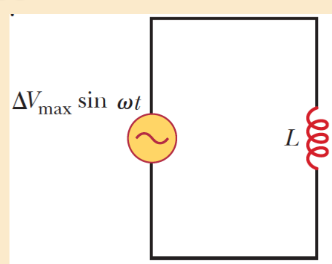
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\ \text{Hz})(8.00 \times 10^{-6}\ \text{F})} = 332\ \Omega$$

- the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\ \text{V}}{332\ \Omega} = 0.452\ \text{A}$$

Solve by your self

(1) In a purely inductive AC circuit as shown in the Figure, ΔV_{max} = 100 V. (a) The maximum current is 7.50 A at 50.0 Hz. Calculate the inductance L. (b) What If? At what angular frequency ω is the maximum current 2.50 A?



(2) An inductor has a 54.0-Ω reactance when connected to a 60.0–Hz source. The inductor is removed and then connected to a 50.0-Hz source that produces a 100-V rms voltage. What is the maximum current in the inductor?

Solve by your self

(3) What is the maximum current in a 2.20-μF capacitor when it is connected across (a) a North American electrical outlet having ΔV_{rms} = 120 V and f = 60.0 Hz and (b) a European electrical outlet having ΔV_{rms} = 240 V and f = 50.0 Hz?

(4) A source delivers an AC voltage of the form Δv = 98.0 sin 80πt, where Δv is in volts and t is in seconds, to a capacitor. The maximum current in the circuit is 0.500 A. Find (a) the rms voltage of the source, (b) the frequency of the source, and (c) the value of the capacitance.

(5) What maximum current is delivered by an AC source with ΔV_{max} = 48.0 V and f = 90.0 Hz when connected across a 3.70-μF capacitor?

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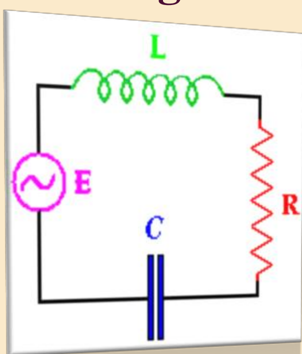
Magnetism and Alternating Current



Unit 5: Alternating-Current Circuits

Lecture 23: The RLC Series Circuit

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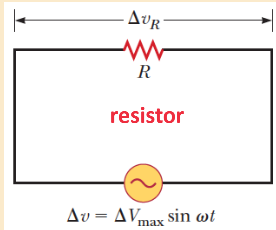
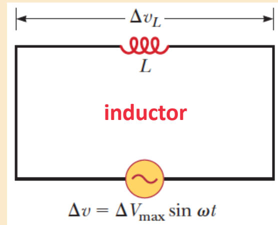
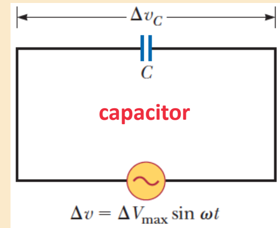
Unit 4: Alternating-Current Circuits







Circuit Element	Symbol	Resistance or Reactance	Phase of Current	Phase Constant	Amplitude Relation
Resistor	R	R	In phase with v_R	0º (0 rad)	$V_R = I_R R$
Capacitor	C	$X_C=1/w_dC$	Leads v_R by 90º	-90º (-p/2)	$V_C = I_C X_C$
Inductor	L	$X_L=w_dL$	Lags v_R by 90º	+90º (p/2)	$V_L = I_L X_L$

The *RLC* Series Circuit

The resistor, inductor, and capacitor can be combined in a circuit.

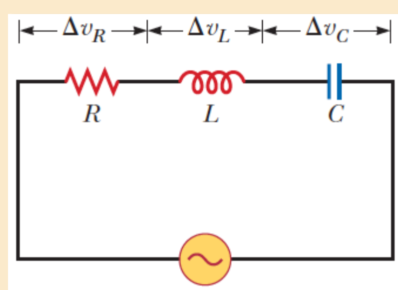
The current and the voltage in the circuit vary sinusoidally with time.

The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$.

The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$.

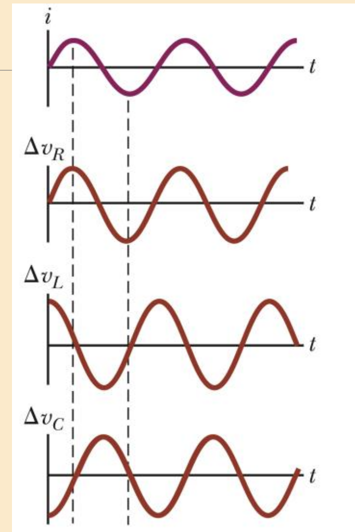
- φ is the **phase angle** between the current and the applied voltage.

Since the elements are in series, the current at all points in the circuit has the same amplitude and phase.



A series circuit consisting of a resistor, an inductor, and a capacitor connected to an AC source.

i and *v* Phase Relationships Graphical View



- The instantaneous voltage across the resistor is in phase with the current.
- The instantaneous voltage across the inductor leads the current by 90°.
- The instantaneous voltage across the capacitor lags the current by 90°.

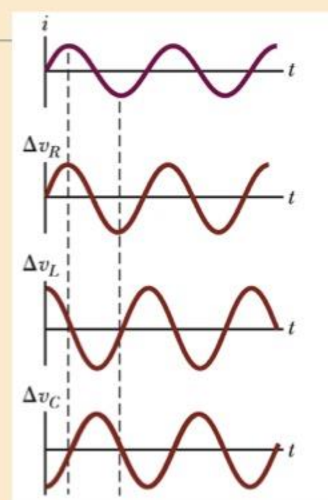
i and *v* Phase Relationships Equations

- The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = \mathrm{I}_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = \mathrm{I}_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$



More About Voltage in RLC Circuits

ΔV_R is the maximum voltage across the **resistor** and $\Delta V_R = I_{max}R$.

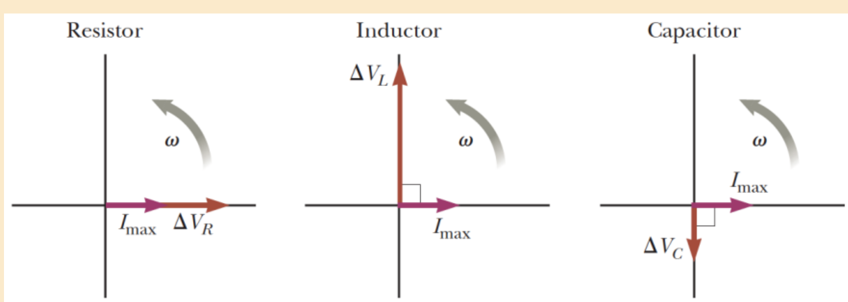
ΔV_L is the maximum voltage across the **inductor** and $\Delta V_L = I_{max}X_L$.

ΔV_C is the maximum voltage across the **capacitor** and $\Delta V_C = I_{max}X_C$.

The sum of these voltages must equal the voltage from the AC source.

Note: Because of the different phase relationships with the current, they cannot be added directly.

Phasor Diagrams



To account for the different phases of the voltage drops, vector techniques are used.

Remember the phasors are rotating vectors.

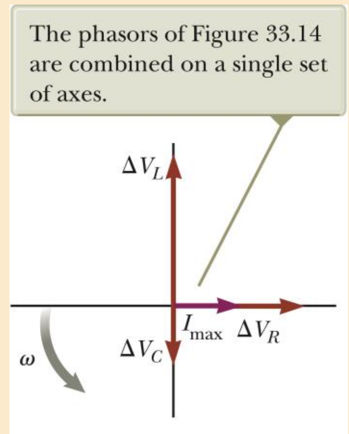
The phasors for the individual elements are shown.

Resulting Phasor Diagram

The individual phasor diagrams can be combined.

Here a single phasor I_{max} is used to represent the current in each element.

- In series, the current is the same in each element.



Vector Addition of the Phasor Diagram

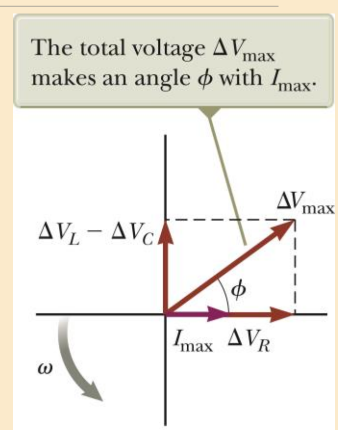
Vector addition is used to combine the voltage phasors.

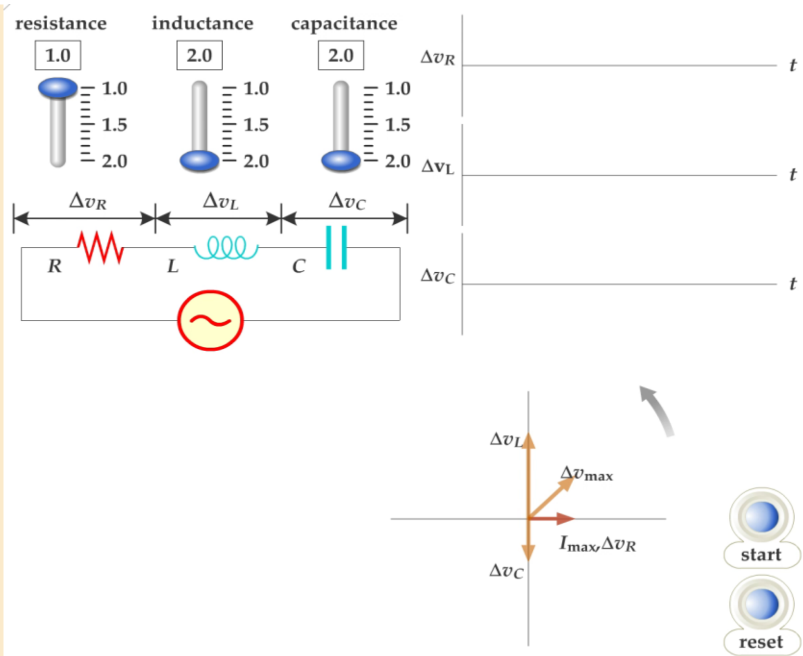
ΔV_L and ΔV_C are in opposite directions, so they can be combined.

Their resultant is perpendicular to $\Delta V_{R.}$

The resultant of all the individual voltages across the individual elements is $\Delta v_{max.}$

- This resultant makes an angle of φ with the current phasor $I_{max.}$





Total Voltage in RLC Circuits

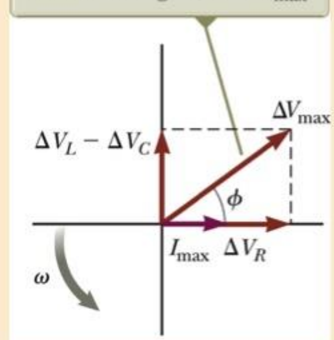
From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{max} R)^2 + \left(I_{max} X_L - I_{max} X_C\right)^2}$$

$$\Delta V_{max} = I_{max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

The total voltage ΔV_{max} makes an angle ϕ with I_{max}.



Impedance

The current in an *RLC* circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance has units of **ohms**

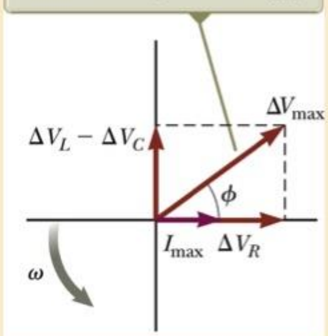
Phase Angle

The right triangle in the phasor diagram can be used to find the phase angle, φ.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The phase angle can be positive or negative and determines the nature of the circuit.

The total voltage ΔV_{max} makes an angle ϕ with I_{max}.



Determining the Nature of the Circuit

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

If ϕ is positive

- $X_L > X_C$ (which occurs at high frequencies)
- The current lags the applied voltage.
- The circuit is *more inductive than capacitive*.

If ϕ is negative

- $X_L < X_C$ (which occurs at low frequencies)
- The current leads the applied voltage.
- The circuit is *more capacitive than inductive*.

If ϕ is zero

- $X_L = X_C$
- The circuit is *purely resistive*.

Example

A series *RLC* circuit has *R* = 425 Ω, *L* = 1.25 H, and *C* = 3.50 μF. It is connected to an AC source with *f* = 60.0 Hz and ΔV_{max} = 150 V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

(B) Find the maximum current in the circuit.

(C) Find the phase angle between the current and voltage.

(D) Find the maximum voltage across each element.

(E) What replacement value of *L* should an engineer analyzing the circuit choose such that the current leads the applied voltage by 30.0° rather than 34.0°? All other values in the circuit stay the same.

Solution (A)

- The angular frequency:

$$\omega = 2\pi f = 2\pi(60.0\ \text{Hz}) = 377\ \text{s}^{-1}$$

- the inductive reactance:

$$X_L = \omega L = (377\ \text{s}^{-1})(1.25\ \text{H}) = \boxed{471\ \Omega}$$

- the capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{s}^{-1})(3.50 \times 10^{-6}\ \text{F})} = \boxed{758\ \Omega}$$

- Impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = \boxed{513\ \Omega}$$

Solution (B)

- **the maximum current:**

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{150\ \text{V}}{513\ \Omega} = \boxed{0.293\ \text{A}}$$

Solution (C)

- **the phase angle between the current and voltage.**

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$= \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right) = \boxed{-34.0^\circ}$$

Solution (D)

- the maximum voltage across each element.

$$\Delta V_R = I_{max} R = (0.293\ \text{A})(425\ \Omega) = 124\ \text{V}$$

$$\Delta V_L = I_{max} X_L = (0.293\ \text{A})(471\ \Omega) = 138\ \text{V}$$

$$\Delta V_C = I_{max} X_C = (0.293\ \text{A})(758\ \Omega) = 222\ \text{V}$$

Solution (E)

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$X_L = X_C + R\tan\phi$$

$$\omega L = \frac{1}{\omega C} + R\tan\phi$$

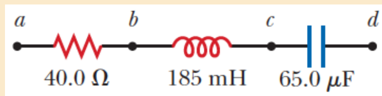
$$L = \frac{1}{\omega}\left(\frac{1}{\omega C} + R\tan\phi\right)$$

$$L = \frac{1}{(377\ \text{s}^{-1})}\left[\frac{1}{(377\ \text{s}^{-1})(3.50 \times 10^{-6}\ \text{F})} + (425\ \Omega)\tan(-30.0°)\right]$$

$$L = 1.36\ \text{H}$$

Solve by your self

(1) An AC source with ΔV =150V and f = 50.0Hz is connected between points a and d in the Figure. Calculate the maximum voltages between (a) points a and b, (b) points b and c, (c) points c and d, and (d) points b and d.



(2) At what frequency does the inductive reactance of a 57.0-μH inductor equal the capacitive reactance of a 57.0-μF capacitor?

(3) An RLC circuit consists of a 150-Ω resistor, a 21.0-μF capacitor, and a 460-mH inductor connected in series with a 120-V, 60.0-Hz power supply. (a) What is the phase angle between the current and the applied voltage? (b) Which reaches its maximum earlier, the current or the voltage?

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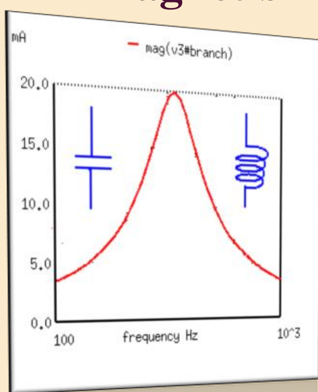
Magnetism and Alternating Current



Unit 5: **Alternating-Current Circuits**

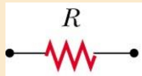
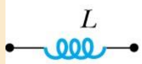
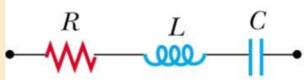
Lecture 24: **Power in an AC Circuit and Resonance in a Series RLC Circuit**

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 4: Alternating-Current Circuits

- **5.1** AC Sources
- **5.2** Resistors in an AC Circuit
- **5.3** Inductors in an AC Circuit
- **5.4** Capacitors in an AC Circuit
- **5.5** The RLC Series Circuit
- **5.6 Power in an AC Circuit**
- **5.7 Resonance in a Series RLC Circuit**
- **5.8** The Transformer and Power Transmission
- **5.9** Rectifiers and Filters

Summary of Circuit Elements, Impedance and Phase Angles

	R	0°
	X_C	–90°
	X_L	+90°
	$\sqrt{R^2 + X_C^2}$	Negative, between –90° and 0°
	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Power in an AC Circuit

The average power delivered by the AC source is converted to internal energy in the resistor.

$$P_{ave} = \frac{1}{2} I_{max} \Delta V_{max} \cos\phi$$

$$I_{max} = \sqrt{2} I_{rms}$$
$$\Delta V_{max} = \sqrt{2} \Delta V_{rms}$$

$$P_{ave} = I_{rms} \Delta V_{rms} \cos\phi$$

Average power delivered to an RLC circuit

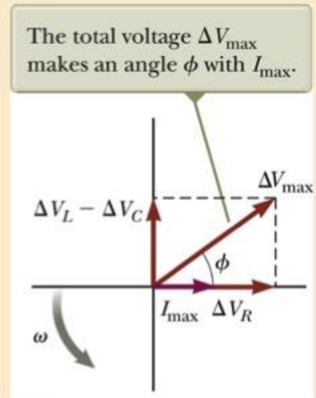
cos ϕ is called the power factor of the circuit

We can also find the average power in terms of R.

$$\Delta V_R = \Delta V_{\max} \cos\phi$$

$$\Delta V_R = I_{\max} R$$

$$\cos\phi = \frac{I_{\max} R}{\Delta V_{\max}} = \frac{R}{Z}$$



$$P_{ave} = I_{rms} \Delta V_{rms} \cos\phi$$

$$P_{ave} = I_{rms} \Delta V_{rms} \left(\frac{R}{Z}\right) \rightarrow P_{ave} = I_{rms} \left(\frac{\Delta V_{rms}}{Z}\right) R \rightarrow P_{ave} = I_{rms}^2 R$$

When the load is purely resistive, $\phi = 0$ and $\cos\phi = 1$

$$P_{ave} = I_{rms} \Delta V_{rms}$$

Power in an AC Circuit, cont.

The average power delivered by the source is converted to internal energy in the resistor.

No power losses are associated with pure capacitors and pure inductors in an AC circuit.

- **In a capacitor,** during one-half of a cycle, energy is stored and during the other half the energy is returned to the circuit and no power losses occur in the capacitor.
- **In an inductor,** the source does work against the back emf of the inductor and energy is stored in the inductor, but when the current begins to decrease in the circuit, the energy is returned to the circuit.

The power delivered by an AC circuit depends on the phase.

Example 1

A series *RLC* circuit has $R = 425\ \Omega$, $L = 1.25$ H, and $C = 3.50\ \mu$F. It is connected to an AC source with $f = 60.0$ Hz and $\Delta v_{max} = 150$ V. Calculate the average power delivered to the series *RLC* circuit

Solution

the rms voltage:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{150\ \text{V}}{\sqrt{2}} = 106\ \text{V}$$

the rms current in the circuit:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{0.293\ \text{A}}{\sqrt{2}} = 0.207\ \text{A}$$

the power delivered by the source:

$$P_{avg} = I_{rms} V_{rms} \cos \phi$$

$$= (0.207\ \text{A})(106\ \text{V}) \cos (-34.0°)$$

$$= 18.2\ \text{W}$$

Resonance in an AC Circuit

Resonance occurs at the frequency ω_o where the current has its maximum value.

- To achieve maximum current, the impedance must have a minimum value.
- This occurs when $X_L = X_C$

$$I_{rms} = \frac{\Delta V_{rms}}{Z}$$

$$I_{rms} = \frac{\Delta V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Resonance in an AC Circuit

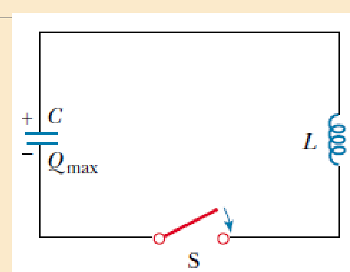
Because the impedance depends on the frequency of the source, the current in the RLC circuit also depends on the frequency.

The angular frequency ω_0 at which $X_L - X_C = 0$ is called the resonance frequency of the circuit. To find ω_0, we set $X_L = X_C$, which gives $\omega_0 L = 1/\omega_0 C$, or

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Remarks

- The resonance frequency also corresponds to the natural frequency of oscillation of an LC circuit.



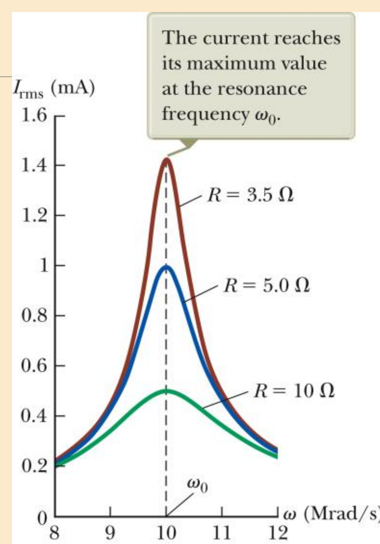
- The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.
- At the resonance frequency, the current is in phase with the applied voltage.

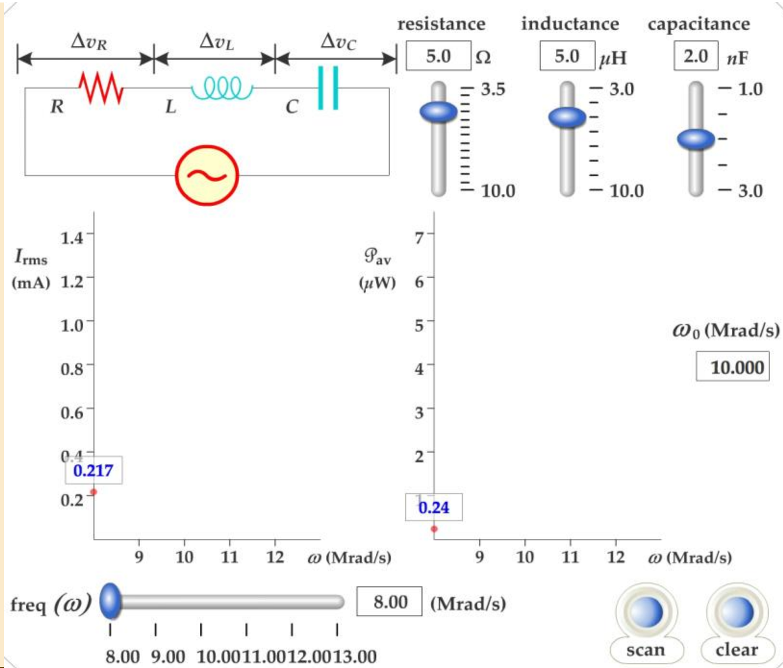
Plot of I_{rms} versus ω for a series *RLC* circuit

Resonance occurs at the same frequency regardless of the value of *R.*

As *R* decreases, the curve becomes narrower and taller.

Theoretically, if *R* = 0 the current would be infinite at resonance.





Power as a Function of Frequency

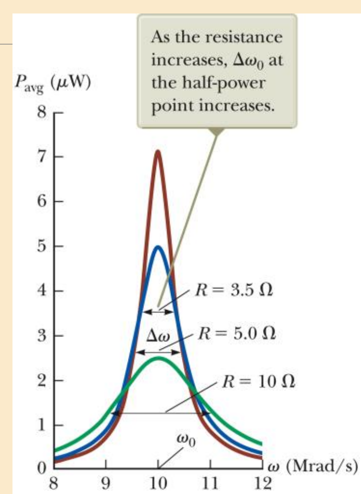
Power can be expressed as a function of frequency in an *RLC* circuit.

$$P_{avg} = \frac{(\Delta V_{rms})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^{\ 2})^2}$$

This shows that at resonance, the average power is a maximum.

When $\omega = \omega_0$

$$P_{ave} = \frac{V_{rms}^2}{R}$$



Quality Factor

The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor, Q.

$$Q = \frac{\ {}_o}{\ }$$

Quality factor

Δω is the width of the curve, measured between the two values of ω for which P_{avg} has half its maximum value. These points are called the *half-power points.*

$$= \frac{R}{L}$$

$$Q = \frac{{}_o L}{R}$$

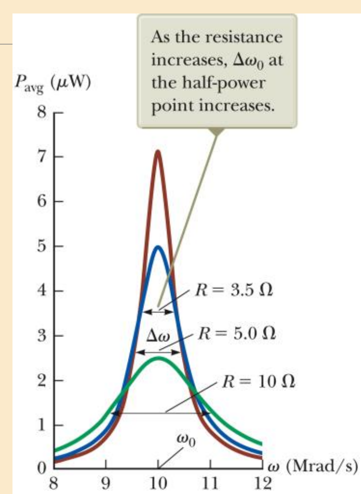
Quality factor

Quality Factor

A high-*Q* circuit responds only to a narrow range of frequencies. Narrow peak

A low-*Q* circuit can detect a much broader range of frequencies.

A radio's receiving circuit is an important application of a resonant circuit.



Example 2

Consider a series *RLC* circuit for which R = 150 Ω, L = 20.0 mH, ΔV_{rms} = 20.0 V, and ω = 5000 s^{-1}. Determine the value of the capacitance for which the current is a maximum.

Solution

The current in the circuit has its maximum value at the resonance frequency ω_0.

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_0{}^2 L}$$

$$C = \frac{1}{(5.00 \times 10^3\ \mathrm{s}^{-1})^2(20.0 \times 10^{-3}\ \mathrm{H})} = 2.00\ \mu\mathrm{F}$$

Solve by your self

(1) An AC voltage of the form Δv = 90.0 sin350t, where Δv is in volts and t is in seconds, is applied to a series *RLC* circuit. If R=50.0Ω, C=25.0μF,and L=0.20H, find (a) the impedance of the circuit, (b) the rms current in the circuit, and (c) the average power delivered to the circuit.

(2) The *LC* circuit of a radar transmitter oscillates at 9.00 GHz. (a) What inductance is required for the circuit to resonate at this frequency if its capacitance is 2.00 pF? (b) What is the inductive reactance of the circuit at this frequency?

(3) An *RLC* circuit is used in a radio to tune into an FM station broadcasting at f = 99.7 MHz. The resistance in the circuit is R = 12.0 Ω, and the inductance is L = 1.40 μH. What capacitance should be used?

(4) A series *RLC* circuit has components with the following values: L=20.0mH, C=100nF, R=20.0Ω, and ΔV_{max}= 100 V, with $\Delta v = \Delta V_{max}$ sin ωt. Find (a) the resonant frequency of the circuit, (b) the amplitude of the current at the resonant frequency, (c) the Q of the circuit, and (d) the amplitude of the voltage across the inductor at resonance.

51.

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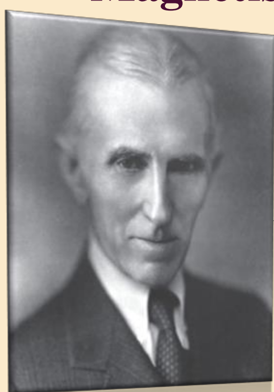
Magnetism and Alternating Current



Unit 5: Alternating-Current Circuits

Lecture 25: The Transformer and Rectifiers and Filters

Dr. Hazem Falah Sakeek
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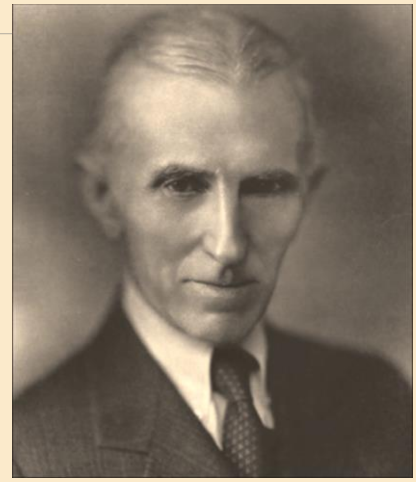
Unit 4: Alternating-Current Circuits

Nikola Tesla

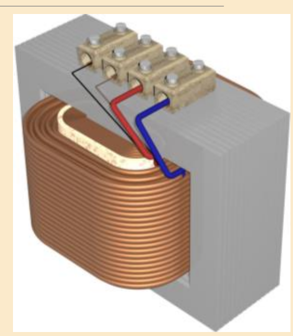


1856 – 1943

American physicist / inventor

Key figure in development of
- Alternating-current electricity
- High-voltage transformers
- Transport of electric power using AC transmission lines

Why Do we need Transformers?



It is economical to use a high voltage and a low current to minimize the I^2R loss in transmission lines when electric power is transmitted over great distances.

High voltage lines (350-kV) are common, and in many areas, even higher-voltage (765-kV) are used.

The voltage is decreased to approximately 20000 V at a distributing station, then to 4000 V for delivery to residential areas, and finally to 120 V and 240 V at the customer's site.

Therefore, a device is needed that can change the alternating voltage and current without causing appreciable changes in the power delivered. The **AC transformer** is that device.

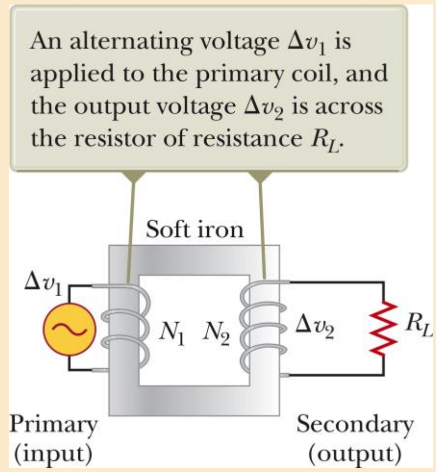
Transformers

An **AC transformer** consists of two coils of wire wound around a core of iron.

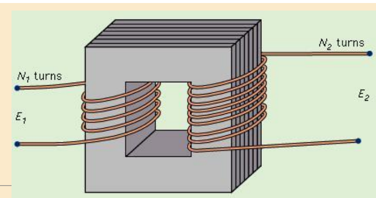
The side connected to the input AC voltage source is called the *primary* and has N_1 turns.

The other side, called the secondary, is connected to a resistor and has N_2 turns.

The core is used to increase the magnetic flux and to provide a medium for the flux to pass from one coil to the other.



Transformers



Eddy-current losses are minimized by using a laminated core.

Assume an ideal transformer

- One in which the energy losses in the windings and the core are zero.
  - Typical transformers have power efficiencies of 90% to 99%.

The voltage across the primary coil is,

$$v_1 = N_1 \frac{d \quad _B}{dt}$$

The rate of change of the flux is the same for both coils.

The voltage across the secondary coil is

$$v_2 = N_2 \frac{d \quad _B}{dt}$$

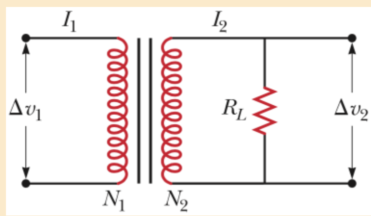
Transformers – Step-up and Step-down

The voltages are related by

$$v_2 = \frac{N_2}{N_1} v_1$$

When $N_2 > N_1$, the transformer is referred to as a step-up transformer.

When $N_2 < N_1$, the transformer is referred to as a step-down transformer.



Example

An electricity-generating station needs to deliver energy at a rate of 20 MW to a city 1.0 km away. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

(A) If the resistance of the wires is 2.0 Ω and the energy costs are about 0.11$/kWh, estimate the cost of the energy converted to internal energy in the wires during one day.

(B) Repeat the calculation for the situation in which the power plant delivers the energy at its original voltage of 22 kV.

Solution (A)

The I_{rms} in the wires

$$I_{rms} = \frac{P_{avg}}{\Delta V_{rms}} = \frac{20 \times 10^6 \text{ W}}{230 \times 10^3 \text{ V}} = 87 \text{ A}$$

The rate at which energy is delivered to the resistance in the wires

$$P_{wires} = I_{rms}^2 R = (87 \text{ A})^2 (2.0 \ \Omega) = 15 \text{ kW}$$

The energy T_{ET} delivered to the wires over the day

$$T_{ET} = P_{wires} \Delta t = (15 \text{ kW})(24 \text{ h}) = 363 \text{ kWh}$$

The cost of this energy

$$\text{Cost} = (363 \text{ kWh})(\$0.11/\text{kWh}) = \$40$$

Solution (B)

The I_{rms} in the wires

$$I_{rms} = \frac{P_{avg}}{\Delta V_{rms}} = \frac{20 \times 10^6 \text{ W}}{22 \times 10^3 \text{ V}} = 909 \text{ A}$$

The rate at which energy is delivered to the resistance in the wires

$$P_{wires} = I_{rms}^2 R = (909 \text{ A})^2 (2.0 \ \Omega) = 1.7 \times 10^3 \text{ kW}$$

The energy T_{ET} delivered to the wires over the day

$$T_{ET} = P_{wires} \Delta t = (1.7 \times 10^3 \text{ kW})(24 \text{ h}) = 4.0 \times 10^4 \text{ kWh}$$

The cost of this energy

$$\text{Cost} = (4.0 \times 10^4 \text{ kWh})(\$0.11/\text{kWh}) = \$4.4 \times 10^3$$

Notice the tremendous savings that are possible through the use of transformers and high-voltage transmission lines. Such savings in combination with the efficiency of using alternating current to operate motors led to the universal adoption of alternating current instead of direct current for commercial power grids.

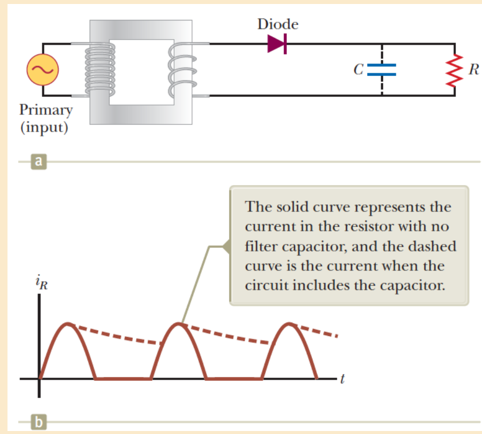
Rectifier

The process of converting alternating current to direct current is called **rectification.**

A **rectifier** is the converting device.

The most important element in a rectifier circuit is the diode.

- A diode is a circuit element that conducts current in one direction but not the other.



Rectifier Circuit

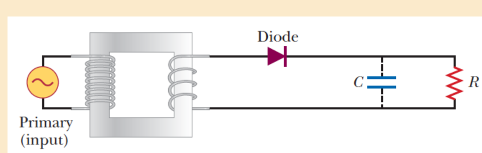
The arrow on the diode () indicates the direction of the current in the diode.

- The diode has low resistance to current flow in this direction.
- It has high resistance to current flow in the opposite direction.

Because of the diode, the alternating current in the load resistor is reduced to the positive portion of the cycle.

The transformer reduces the 120 V AC to the voltage needed by the device.

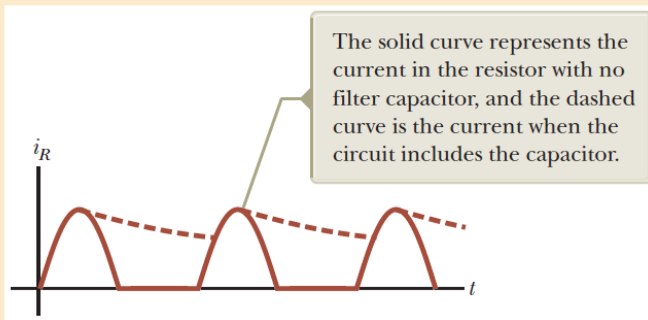
- Typically 6 V or 9 V



Half-Wave Rectifier

The solid line in the graph is the result through the resistor.

It is called a *half-wave rectifier* because current is present in the circuit during only half of each cycle.



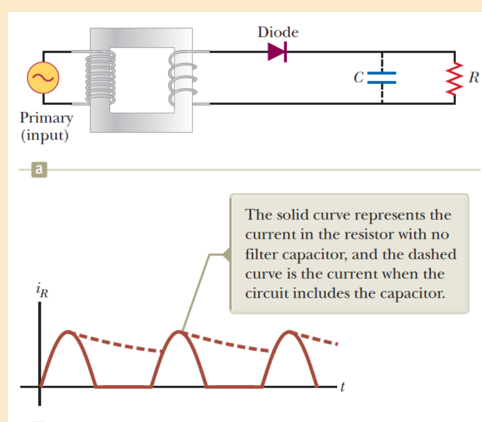
Half-Wave Rectifier, Modification

A capacitor can be added to the circuit.

The circuit is now a simple DC power supply.

The time variation in the circuit is close to zero.

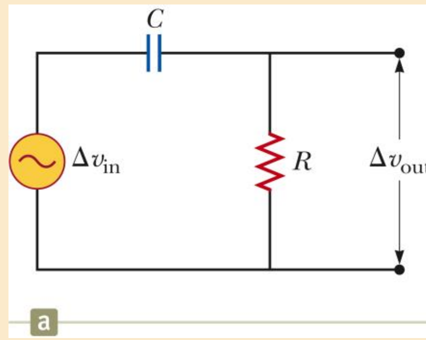
This is represented by the dotted lines in the graph shown in the figure.



High-Pass Filter

The circuit shown is one example of a **high-pass filter.**

A high-pass filter is designed to preferentially pass signals of higher frequency and block lower frequency signals.



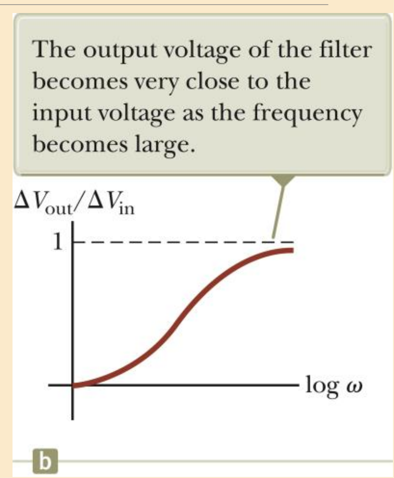
High-Pass Filter, cont

At low frequencies, ΔV_{out} is much smaller than Δv_{in}.

At low frequencies, the capacitor has **high reactance** and much of the applied voltage appears across the capacitor.

At high frequencies, the two voltages are equal.

At high frequencies, the capacitive reactance is small and the voltage appears across the resistor.

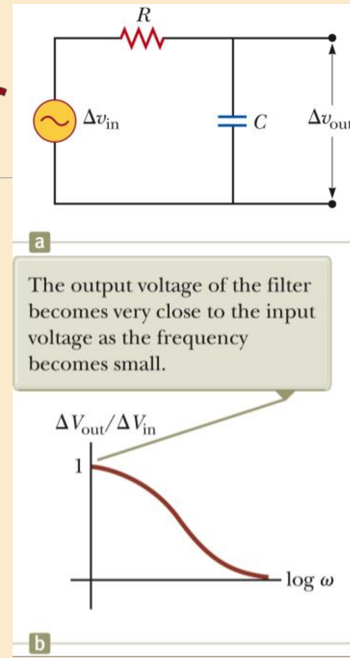


Low-Pass Filter

At **low frequencies**, the reactance and voltage across the capacitor are high.

As the frequency increases, the reactance and voltage decrease.

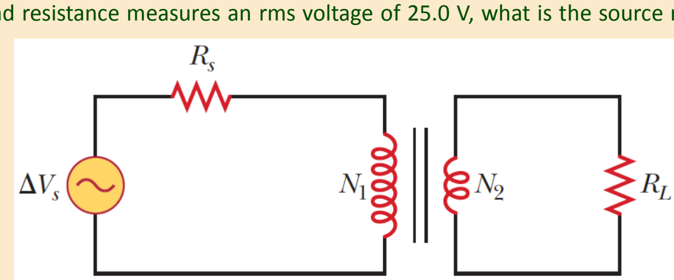
This is an example of a low-pass filter.



Solve by your self

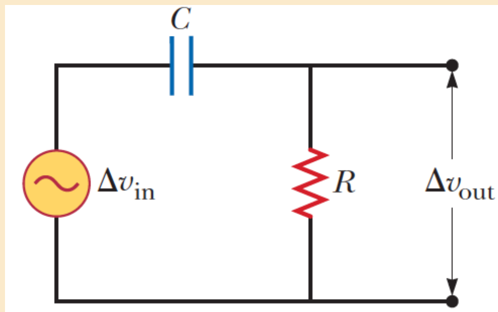
(1) The primary coil of a transformer has N_1 = 350 turns, and the secondary coil has N_2 = 2 000 turns. If the input voltage across the primary coil is $\Delta v = 170 \cos \omega t$, where Δv is in volts and t is in seconds, what rms voltage is developed across the secondary coil?

(2) In the transformer shown in the Figure, the load resistance R_L is 50.0 Ω. The turns ratio N_1/N_2 is 2.50, and the rms source voltage is ΔV_s = 80.0 V. If a voltmeter across the load resistance measures an rms voltage of 25.0 V, what is the source resistance R_s?



Solve by your self

(3) The *RC* high-pass filter shown in the Figure has a resistance $R = 0.500\ \Omega$ and a capacitance $C = 613\ \mu F$. What is the ratio of the amplitude of the output voltage to that of the input voltage for this filter for a source frequency of 600 Hz?



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