مراجعة ليلة الامتحان..وبنك أسئلة لن يخلو منه الامتحان

أقصر طريق للحصول على الدرجة النهائية

أسرة الرياضيات «لغات»

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[1] Choose the correct answer from those given:

[1] The points $(-3 , 0)$, $(0 , 3)$ and $(3 , 0)$ are the vertices of ……..

(a) a scalene triangle.

(b) an equilateral triangle.

(c) an obtuse-angled triangle.

(d) a right-angled triangle and isosceles.

[2] The equation of the straight line whose slope is 1 and passes through the origin point is ……..

(a) $x = 1$ (b) $y = 1$

(c) $y = x$ (d) $y = -x$

[3] If sin 30° = cos θ, where θ is an acute angle, then m (∠ θ) = ……°

(a) 10 (b) 30 (c) 45 (d) 60

[4] In Δ ABC, if m (∠ B) = 90°, then sin A + cos C = ……

(a) 2 sin A (b) 2 sin C

(c) 2 sin b (d) 2 cos A

[5] The slope of the straight line which is parallel to x-axis is ….

(a) − 1 (b) 0 (c) 1 (d) undefined

[6] If the origin point is a centre of a circle of radius 3 unit length, then the point ……… belongs to it.

(a) (1 , 2) (b) $(-2 , \sqrt{5})$

(c) $(\sqrt{3} , 1)$ (d) $(\sqrt{2} , 1)$

[7] The straight line which passes through the two points $(1 , y)$, $(3 , 4)$ and its slope is tan 45°, then y = …..

(a) − 1 (b) 1 (c) 2 (d) 4

[8] For any acute angle A, tan A = …

(a) $\frac{\cos A}{\sin A}$ (b) sin A cos A

(c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

[9] tan 45° sin 30° = ……

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

[10] If $\frac{-2}{3}$ and $\frac{k}{2}$ are the slope of two parallel straight lines, then k = ……

(a) $-\frac{4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{1}{3}$ (d) 3

[11] If C (2 , 1) is the midpoint of $\overline{AB}$ where B (3 , 0), then A is ……

(a) (1 , 2) (b) (2 , 1)

(c) (5 , 1) (d) (1 , 5)

[12] For any acute angles A and B if sin A = cos B, then m (∠ A) + m (∠ B) = …

(a) 30° (b) 60° (c) 90° (d) 180°

[13] If the two straight lines $3x - 4y - 3 = 0$ and $ky + 4x - 8 = 0$ are perpendicular, then k = …..

(a) – 4 (b) – 3 (c) 3 (d) 4

[14] If $\overleftrightarrow{LM} \perp \overleftrightarrow{EO}$, E(−1 , 2) and O (0 , 0), then the slope of $\overleftrightarrow{LM}$ = ….

(a) −2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

[15] If Δ XYZ is right-angled at Z, XY = 25 cm, YZ = 7 cm and XZ = 24 cm, then sin X + sin Y = ……

(a) $\frac{31}{25}$ (b) $\frac{17}{25}$ (c) 2 (d) 1

[16] 2 tan 45 – $\frac{1}{\cos 60°}$ = …..

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

[17] If A (x_1 , y_1) and B (x_2 , y_2), then AB = ……

(a) $x_1 x_2 + y_1 y_2$

(b) $\sqrt{x_1 x_2 + y_1 y_2}$

(c) $(x_2 - x_1, y_2 - y_1)$

(d) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

[18] If tan $3x = \sqrt{3}$, where $3x$ is the measure of the acute angle, then m (∠ x) = ……..°

(a) 10 (b) 20 (c) 30 (d) 60

[19] If sin $(x + 5°) = \frac{1}{2}$ where $(x + 5°)$ is the measure of an acute angle, then tan $(x + 20°)$ = ….

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

[20] If $\overline{AB}$ is a diameter of a circle, where A (3 , −5) and B (5 , 1), then the centre of the circle is ……

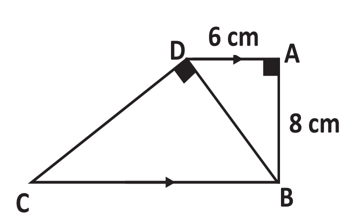
(a) (4 , −2) (b) (4 , 2)

(c) (2 , 2) (d) (8 , −2)

Answers

[1] (d)	[2] (c)	[3] (d)	[4] (a)
[5] (b)	[6] (b)	[7] (c)	[8] (c)
[9] (b)	[10] (a)	[11] (a)	[12] (c)
[13] (c)	[14] (c)	[15] (a)	[16] (a)
[17] (d)	[18] (b)	[19] (d)	[20] (a)

[2] In the figure below:



ABCD is a quadrilateral in which:

m (∠ A) = m (∠ BDC) = 90°, $\overline{AD}$ // $\overline{BC}$,

AD = 6 cm and AB = 8 cm.

Find the length of $\overline{DC}$.

Answer

In Δ ABD: ∵ m (∠ A) = 90°

∴ $(DB)^2 = (AB)^2 + (AD)^2$

= 64 + 36 = 100 cm^2

∴ DB = 10 cm

∵ $\overline{AD}$ // $\overline{BC}$ and $\overleftrightarrow{BD}$ is a transversal.

∴ m (∠ ADB) = m (∠ DBC)

"Alternate angles"

∴ tan (∠ ADB) = tan (∠ DBC)

∴ $\frac{AB}{AD} = \frac{DC}{BD}$

∴ $\frac{8}{6} = \frac{DC}{10}$ ∴ DC = $\frac{10 \times 8}{6} = 13\frac{1}{3}$ cm.

[3] Find the value of:

sin 30° cos 60° + cos² 30° + 5 tan 45° − 10 cos² 45°

Answer

The expression =

$\frac{1}{2} \times \frac{1}{2} + (\frac{\sqrt{3}}{2})^2 + 5 \times 1 - 10 \times (\frac{1}{\sqrt{2}})^2 =$

$\frac{1}{4} + \frac{3}{4} + 5 - \frac{10}{2} = 1 + 5 - 5 = 1$

[4] Find the value of x which satisfies:

[a] 2 sin x = tan² 60° − 2 tan 45° (where x is a measure of an acute angle).

[b] x sin 30° cos² 45° = cos² 30°

Answers

[a] ∵ 2 sin x = tan² 60° − 2 tan 45°

∴ 2 sin $x = (\sqrt{3})^2 - 2 \times 1 = 3 - 2 = 1$

∴ sin $x = \frac{1}{2}$ ∴ x = 30°

[b] ∵ x sin 30° cos² 45° = cos² 30°

∴ $x \times \frac{1}{2} \times (\frac{1}{\sqrt{2}})^2 = (\frac{\sqrt{3}}{2})^2$

∴ $\frac{1}{4} x = \frac{3}{4}$ ∴ $x = 3$

[5] Without using the calculator, prove each of the following:

[a] tan 60° = $\frac{2 \tan 30°}{1 - \tan^2 30°}$

[b] sin² 60° + sin² 45° + sin² 30° = cos² 30° + $\frac{1}{3}$ tan² 60° – cos² 60°

[c] cos² 60° = 5 sin² 30° – tan² 45°

[d] cos 60° = 2 cos² 30° – 1

Answers

[a] The left side = tan 60° = $\sqrt{3}$

The right side = $\frac{2 \tan 30°}{1 - \tan^2 30°}$

$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - (\frac{1}{\sqrt{3}})^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$

∴ The two sides are equal.

[b] The left side =

$(\frac{\sqrt{3}}{2})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{2})^2 = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$

The right side =

$(\frac{\sqrt{3}}{2})^2 + \frac{1}{3}(\sqrt{3})^2 - (\frac{1}{2})^2 = \frac{3}{4} + 1 - \frac{1}{4}$

$= \frac{3}{2}$

∴ The two sides are equal.

[c] The left side = cos² 60° = $(\frac{1}{2})^2 = \frac{1}{4}$

The right side = 5 sin² 30° – tan² 45°

$= 5(\frac{1}{2})^2 - 1^2 = 5 \times \frac{1}{4} - 1 = \frac{1}{4}$

∴ The two sides are equal.

[d] The left side = cos 60° = $\frac{1}{2}$

The right side = 2 cos² 30° − 1

$= 2(\frac{\sqrt{3}}{2})^2 - 1 = 2 \times \frac{3}{4} - 1 = \frac{1}{2}$

∴ The two sides are equal.

[6] If ABC is a triangle where A(0 , 0), B(3 , 4) and C(−4 , 3), find the perimeter of Δ ABC.

Answer

AB = $\sqrt{(3-0)^2 + (4-0)^2}$

$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$

= 5 length unit.

BC = $\sqrt{(-4-3)^2 + (3-4)^2}$

$= \sqrt{(-7)^2 + (-1)^2} = \sqrt{49 + 11} = \sqrt{50}$

= $5\sqrt{2}$ length unit.

CA = $\sqrt{(-4-0)^2 + (3-0)^2}$

$= \sqrt{(-4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25}$

= 5 length unit.

∴ The perimeter of Δ ABC

$= 5 + 5\sqrt{2} + 5$

= $(10 + 5\sqrt{2})$ length unit.

[7] If the points A(3 , 2), B(4 , − 3), C(− 1 , − 2) and D(− 2 , 3) are vertices of the rhombus, find:

(1) the coordinates of the point intersection of the two diagonals.

(2) the area of the rhombus ABCD.

Answer

(1) Let M be the point of intersection of the two diagonals.

∴ The coordinates of M = $(\frac{3-1}{2} , \frac{2-2}{2})$

= (1 , 0)

(2) AC = $\sqrt{(-1-3)^2 + (-2-2)^2}$

$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$ length unit.

BD = $\sqrt{(-2-4)^2 + (3+3)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$ length unit.

∴ The area of the rhombus ABCD

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ square unit.

[8] If A(5 , −6), B(3 , 7) and C(1 , −3), find the equation of the straight line which passes through the point A and the midpoint of $\overline{BC}$.

Answer

Let D be the midpoint of $\overline{BC}$.

∴ The coordinates of D = $(\frac{3+1}{2} , \frac{7-3}{2})$

= (2 , 2)

∴ The slope of $\overleftrightarrow{AD} = \frac{-6-2}{5-2} = -\frac{8}{3}$

∴ The equation of $\overleftrightarrow{AD}$ is $y = -\frac{8}{3}x + c$

∵ D ∈ $\overleftrightarrow{AD}$

∴ (2 , 2) satisfies its equation.

∴ $2 = \frac{8}{3} \times 2 + c$ ∴ $c = \frac{-10}{3}$

∴ The equation of $\overleftrightarrow{AD}$ is $y = -\frac{8}{3}x - \frac{10}{3}$

[9] If the straight line $\overleftrightarrow{AB}$ // the y-axis, where A(x , 7) and B(3 , 5), then find the value of x.

Answer

∵ $m = \frac{7-5}{x-3}$ ∵ m is undefined.

∴ $x - 3 = 0$ ∴ $x = 3$

[10] Prove that: the straight line which passes through the two points (2 , 3) and (−1 , 6) is parallel to the straight line which makes with the positive direction of x-axis a positive angle of measure 135°.

Answer

The slope of the 1st straight line m_1 = $\frac{6-3}{-1-2} = \frac{3}{-3} = -1$

The slope of the 2nd straight line m_2 = tan 135° = −1

∴ $m_1 = m_2$

∴ The two straight lines are parallel.

[11] In the Cartesian coordinates plane, if the points A(1 , 7), B(2 , 4) and C(5 , y) represent the vertices of a right-angled triangle at B, find the value of y.

Answer

∵ The slope of $\overleftrightarrow{AB} = \frac{4-7}{2-1} = -3$ and the slope of $\overleftrightarrow{BC} = \frac{y-4}{5-2} = \frac{y-4}{3}$

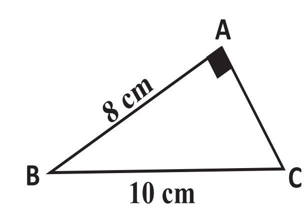
∵ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

∴ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC}$ = −1

∴ $-3 \times \frac{y-4}{3} = -1$ ∴ $y - 4 = 1$

∴ $y = 5$

[12] In the figure below:



ABC is right-angled triangle at A where AB = 8cm and BC = 10 cm.

Find the value of: sin B cos C + cos B sin C.

Answer

∵ m (∠ A) = 90°

∴ $(AC)^2 = (10)^2 - (8)^2 = 36$

∴ AC = 6 cm.

∴ sin B cos C + cos B sin C =

$\frac{6}{10} \times \frac{6}{10} + \frac{8}{10} \times \frac{8}{10} = \frac{36}{100} + \frac{64}{100} = 1$

[13] If the length between the point $(x , 5)$ and the point $(6 , 1)$ equals $2\sqrt{5}$ length units.

Find the value of x.

Answer

∵ $\sqrt{(x-6)^2 + (5-1)^2} = 2\sqrt{5}$ (By squaring both sides)

∴ $(x-6)^2 + 16 = 20$

∴ $x^2 - 12x + 36 + 16 - 20 = 0$

∴ $x^2 - 12x + 32 = 0$

∴ $(x-8)(x-4) = 0$

∴ $x = 8$ or $x = 4$

[14] Find the equation of the straight line passing through point (−3 , 2) and parallel to the straight line $3y = x - 1$

Answer

∵ The slope of the given straight line $= \frac{1}{3}$

∴ The slope of the required straight line $= \frac{1}{3}$

∴ The equation of the required straight line is $y = \frac{1}{3}x + c$

∵ (−3 , 2) satisfies the equation:

∴ $2 = \frac{1}{3} \times (-3) + c$ ∴ c = 3

∴ The equation of the straight line is

$y = \frac{1}{3}x + 3$