

(Collisions)



1-7

()

(Impulse & Momentum)

$$\Delta t = t_2 - t_1$$

F

m

:

(1-7)
$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

: dt

p

$$d\mathbf{p} = \mathbf{F} dt$$

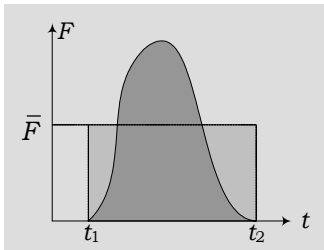
: t_2 t_1

(2-7)

$$\mathbf{J} = \mathbf{p}_2 - \mathbf{p}_1 = \int_{t_1}^{t_2} \mathbf{F} dt$$

.kg.m/s N.s

$$\mathbf{J} = \mathbf{p}_2 - \mathbf{p}_1$$



(1-7)

\bar{F}

(1-7)

$$\bar{F}\Delta t \quad (2-7)$$

Δt

(3-7)

$$J = \bar{F}\Delta t$$

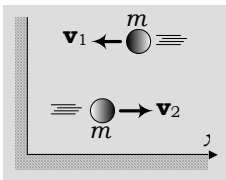
1-7

20 m/s

30 m/s

0.4 kg

0.1 s



(2-7)

(2-7)

$$\mathbf{p}_1 = m\mathbf{v}_1 = (0.4 \text{ kg})(-30 \text{ m/s})\mathbf{i} = (-12 \text{ kg}\cdot\text{m/s})\mathbf{i}$$

$$\mathbf{p}_2 = m\mathbf{v}_2 = (0.4 \text{ kg})(20 \text{ m/s})\mathbf{i} = (8 \text{ kg}\cdot\text{m/s})\mathbf{i}$$

$$\mathbf{J} = \mathbf{p}_2 - \mathbf{p}_1 = (20 \text{ kg}\cdot\text{m/s})\mathbf{i}$$

(3-7)

$$J = \bar{F}\Delta t \Rightarrow \bar{F} = \frac{J}{\Delta t} = \frac{20 \text{ kg}\cdot\text{m/s}}{0.1 \text{ s}} = 200 \text{ N}$$

J

\bar{F}

$$\bar{\mathbf{F}}_{21} \quad \mathbf{v}_2 \quad \mathbf{v}_1 \quad m_2 \quad m_1$$

$$\bar{\mathbf{F}}_{12}$$

$$\mathbf{v}'_2 \quad \mathbf{v}'_1$$

$$\bar{\mathbf{F}}_{12} = -\bar{\mathbf{F}}_{21}$$

$$\bar{\mathbf{F}}_{21} = \frac{\Delta \mathbf{P}_1}{\Delta t}$$

$$\bar{\mathbf{F}}_{12} = \frac{\Delta \mathbf{P}_2}{\Delta t}$$

$$\Delta \mathbf{P}_1 = -\Delta \mathbf{P}_2$$

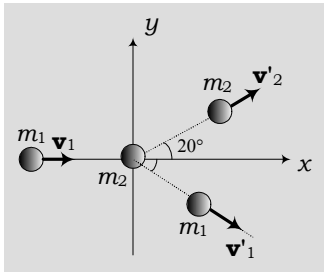
$$\Delta \mathbf{P}_1 + \Delta \mathbf{P}_2 = 0 \Rightarrow \Delta(\mathbf{P}_1 + \mathbf{P}_2) = 0$$

(4-7) $\mathbf{P}_T(\quad) = \mathbf{P}_T(\quad)$

(5-7) $m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$

10 m/s

4 kg



20°

(3-7)

(5-7)

(3-7)

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

$$\mathbf{v}'_1 = \frac{1}{m_1} (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 - m_2 \mathbf{v}'_2)$$

$$v'_{1x} = (m_1 v_{1x} + m_2 v_{2x} - m_2 v'_{2x}) / m_1$$

$$v'_{1y} = (m_1 v_{1y} + m_2 v_{2y} - m_2 v'_{2y}) / m_1$$

$$v_{1x} = 10 \text{ m/s}, \quad v_{1y} = 0$$

$$v_{2x} = v_{2y} = 0, \quad v'_{2x} = 5 \cos 20^\circ = 4.70 \text{ m/s}, \quad v'_{2y} = 5 \sin 20^\circ = 1.71 \text{ m/s}$$

$$m_1 = m_2$$

$$v'_{1x} = v_{1x} + v_{2x} - v'_{2x} = 10 + 0 - 4.70 = 5.30 \text{ m/s}$$

$$v'_{1y} = v_{1y} + v_{2y} - v'_{2y} = 0 + 0 - 1.71 = -1.71 \text{ m/s}$$

$$v' = \sqrt{v'^2_{1x} + v'^2_{1y}} = \sqrt{(5.30)^2 + (-1.71)^2} = 5.57 \text{ m/s}$$

$$\tan \theta = \frac{v'_{1y}}{v'_{1x}} = \frac{-1.71}{5.30} = -0.32 \Rightarrow \theta = -17.9^\circ$$

18°

(elastic & inelastic collisions)

4-7

(elastic collisions)

(6-7)

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

(inelastic collisions)

(7-7)

$$[\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2] - [\frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2] = Q$$

Q

Q > 0

Q < 0

(partially inelastic collisions)

-1

() (3H)

(⁴He)

(totally inelastic collisions)

-2

(fusion)

(fission)

()

5-7

v_1

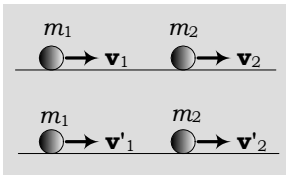
$m_2 \quad m_1$

$v'_2 \quad v'_1$

v_2

:(5-7)

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$



(4-7)

(4-7)

:(6-7)

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$v'_2 \quad v'_1$

:

(8-7)

$$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \left(\frac{2m_2}{m_1 + m_2} \right) v_2$$

$$v'_2 = \left(\frac{2m_1}{m_1 + m_2} \right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2$$

$$m_{Au}=197 \quad m_{\alpha}=4$$

α

α

:

:

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2$$

:

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 \Rightarrow m_1 v_1^2 = m_1 v'^2_1 + m_2 v'^2_2$$

:

$$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 = \left(\frac{4 - 197}{4 + 197} \right) v_1 = -0.96 v_1$$

$$v'_2 = \left(\frac{2m_2}{m_1 + m_2} \right) v_1 = \left(\frac{8}{4 + 197} \right) v_1 = -0.04 v_1$$

: α

$$K' = \frac{1}{2} m_1 v'^2_1 = \frac{1}{2} m_1 (-0.96 v_1)^2 = (0.92) \left(\frac{1}{2} m_1 v_1^2 \right) = 0.92 K_1$$

0.92

0.08

6-7

(relative speed of approach and separation and coefficient of restitution)

$$\mathbf{v}_2 \quad \mathbf{v}_1 \qquad m_2 \quad m_1$$

$$|v_2 - v_1|$$

:

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2$$

:

$$m_1(v_1^2 - v_1'^2) = m_2(v_2^2 - v_2'^2)$$

(9-7) $m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2 + v_2')(v_2 - v_2')$

:

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

(10-7) $m_1(v_1 - v_1') = m_2(v_2 - v_2')$

: (10-7) (9-7)

$$(v_1 + v_1') = (v_2 + v_2')$$

:

(11-7) $(v_1 - v_2) = (v_1' - v_2')$

$$(v_1' - v_2')$$

$$(v_1 - v_2)$$

(coefficient of restitution)

:

(12-7)
$$e = \frac{|v_2' - v_1'|}{|v_2 - v_1|}$$

$$e=1$$

7-7

$(m_2 \gg m_1)$

-1

: (8-7)

$m_2 \gg m_1$

$$v_1' = \left(\frac{m_1/m_2 - 1}{m_1/m_2 + 1} \right) v_1 + \left(\frac{2}{m_1/m_2 + 1} \right) v_2$$
$$v_2' = \left(\frac{2m_1/m_2}{m_1/m_2 + 1} \right) v_1 + \left(\frac{1 - m_1/m_2}{m_1/m_2 + 1} \right) v_2$$

$$m_1/m_2 \ll 1$$

(13-7)
$$\begin{cases} v'_1 = -v_1 + 2v_2 \\ v'_2 = v_2 \end{cases}$$

m_2

(14-7)
$$\begin{cases} v'_1 = -v_1 \\ v'_2 = 0 \end{cases}$$
 $v_2=0$

$(m_1 \gg m_2)$

-2

$(8-7)$ $m_1 \gg m_2$

$$v'_1 = \left(\frac{1 - m_2/m_1}{m_2/m_1 + 1} \right) v_1 + \left(\frac{2m_2/m_1}{m_2/m_1 + 1} \right) v_2$$

$$v'_2 = \left(\frac{2}{m_2/m_1 + 1} \right) v_1 + \left(\frac{m_2/m_1 - 1}{m_2/m_1 + 1} \right) v_2$$

$$m_2/m_1 \ll 1$$

(15-7)
$$\begin{cases} v'_1 = v_1 \\ v'_2 = 2v_1 \end{cases}$$

m_2

m_1

!

:

$$(m_1 = m_2)$$

-3

$$(8-7)$$

$$m_1 = m_2$$

(16-7)

$$\begin{cases} v'_1 = v_2 \\ v'_2 = v_1 \end{cases}$$

!

4-7

500 g

150 m/s

10 g

1 m

()

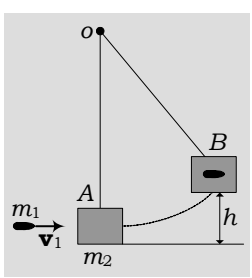
() (5-7)

h

h

:

()



(5-7)

$$m_1 v_1 = (m_1 + m_2) v'$$

$$v' = 2,9 \text{ m/s}$$

:

$$K_1 = \frac{1}{2} m_1 v_1^2 = 112,5 \text{ J}$$

:

$$K' = \frac{1}{2} (m_1 + m_2) v'^2 = 2,2 \text{ J}$$

:

$$Q = K_1 - K' = 110,3 \text{ J}$$

()

:

$$E_A = E_B$$

8-7

$$E_A = K' = 2.2 \text{ J}$$

$$E_B = (m_1 + m_2)gh \approx 5h$$

$$h = 0.4 \text{ m}$$

:

:

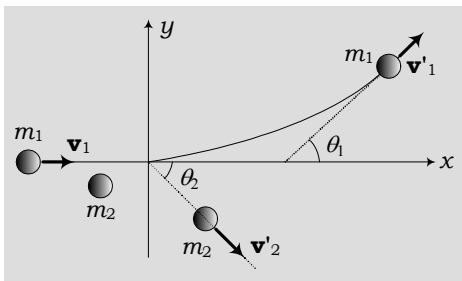
8-7

()

)

(Ernst Rutherford 1837-1937)

.(



(6-7)

()

α

(glancing collisions)

(head-on collisions)

$$m_2 \mathbf{v}_1 + m_1 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

(6-7)

$$v'_1 \quad v'_2$$

:

$$m_1 \mathbf{v}_1 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

: oy ox

(17-7)

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2$$

:

(18-7) $0 = m_1 v'_1 \sin \theta_1 - m_2 v'_2 \sin \theta_2$

:

(19-7) $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$

5-7

30° 2.0×10^7 m/s α
50

α

(6-7) α :

:(18-7) (17-7)

$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2 \Rightarrow 2 \times 10^7 = v'_1 \cos 30^\circ + 50 v'_2 \cos \theta_2$

$0 = m_1 v'_1 \sin \theta_1 - m_2 v'_2 \sin \theta_2 \Rightarrow 0 = v'_1 \sin 30^\circ - 50 v'_2 \sin \theta_2$

:

$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \Rightarrow 4 \times 10^{14} = v_1'^2 + 50 v_2'^2$

: v'_2 v'_1 θ_2

$v'_2 \approx 0.02 \times 10^7$ m/s $v'_1 \approx 1.9 \times 10^7$ m/s $\theta_2 \approx 72^\circ$

6-7

m

:

:

8-7

$$m_1 \mathbf{v}_1 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

:

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$\mathbf{v}'_1 = -\frac{1}{3} \mathbf{v}_1$$

:

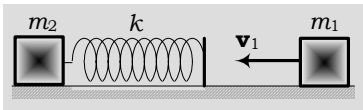
$$\mathbf{v}'_2 = \frac{4m_1}{3m_2} \mathbf{v}_1$$

$$v_2'^2 = \frac{8m_1}{9m_2} v_1^2$$

:

$$m_2 = 2m_1$$

7-7



(7-7)

$$m_1 = 3.5 \text{ kg}$$

$$m_2 = 6 \text{ kg}$$

$$8 \text{ m/s}$$

$$750 \text{ N/m}$$

(7-7)

$$m_1$$

:

:

$$m_1 \mathbf{v}_1 = (m_1 + m_2) \mathbf{v}'$$

:

$$v' = \frac{m_1 v_1}{m_1 + m_2} = 2.9 \text{ m/s}$$

:

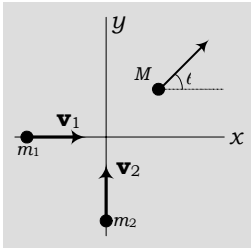
$$E_1 - E_2 = \frac{1}{2} kx^2$$

$$E_1 = \frac{1}{2} m_1 v_1^2 = 112 \text{ J}$$

$$E_2 = \frac{1}{2} (m_1 + m_2) v'^2 = 41.3 \text{ J}$$

$$x = 43.4 \text{ m}$$

7-7



(8-7)

$m_2 = 50 \text{ kg}$ 6 km/h $m_1 = 80 \text{ kg}$
 8 km/h

(8-7)

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2) \mathbf{v}'$$

(8-7) oy ox

$$m_1 v_1 = (m_1 + m_2) v' \cos \theta$$

$$m_2 v_2 = (m_1 + m_2) v' \sin \theta$$

$$\tan \theta = \frac{m_2 v_2}{m_1 v_1} \approx 0.8$$

$$v = \frac{m_1 v_1}{(m_1 + m_2) \cos \theta} \approx 4.8 \text{ km/h}$$

$$K_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (80 \text{ kg})(6 \text{ km/h})^2 + \frac{1}{2} (50 \text{ kg})(8 \text{ km/h})^2 = 235 \text{ J}$$

$$K_2 = \frac{1}{2} (m_1 + m_2) v'^2 = \frac{1}{2} (130 \text{ kg})(4.8 \text{ km/h})^2 = 116 \text{ J}$$

$$\Delta K = K_2 - K_1 = -119 \text{ J}$$

$$\mathbf{J} = \Delta \mathbf{P} = \int \mathbf{F} \Delta t$$

$$\bar{\mathbf{F}} = \mathbf{J} / \Delta t$$

$$\mathbf{F} = 0 \Rightarrow \mathbf{P} =$$

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

:

:

$$e = |v'_2 - v'_1| / |v_2 - v_1|$$

- 65 m/s 60 g **1-7**
- . 0.03 s
- .50 m/s 30 m/s 0.2 kg **2-7**
- 400 m/s 50 g **3-7**
- () . 10 cm
- () () ()
- 45° v m **4-7**
- 150 kg **5-7**
- () 2200 kg () .25 m/s
- () 0.4 s
- 4 m 0.5 kg **6-7**
- () () .
- 2 ms

.5 m/s 300 cm³/s **7-7**

$F = 480 - 1.6 \times 10^5 t$ m **8-7**

() .3 ms ()
() .() () .

.320 m/s

.12 cm 2 kg 10 g **9-7**

9 m/s 6 kg **10-7**

30° 6 m/s .12 kg **11-7**
300 g

10 ms

0.6 kg **12-7**

3 m/s 2 m/s **13-7**

2 kg **14-7**

2 m/s 10,000 kg **15-7**

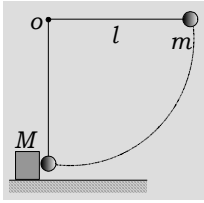
() () . 20,000 kg

()

1.8 kg 4.5 g **16-7**

1.8 m 0.2

v m **17-7**



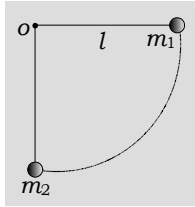
70 cm 0.5 kg

(9-7)

2.5 kg

(9-7)

18-7



(10-7)

l

(10-7)

m_1

19-7

m_2

$.h$ $.h$

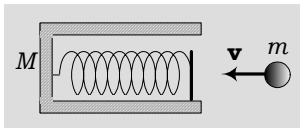
20 m/s

1 kg

20-7

(11-7)

100 kg

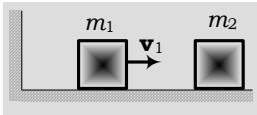


(11-7)

m_1

21-7

m_2



(12-7)

(12-7)

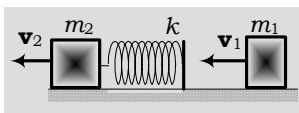
m_2

m_2/m_1

10 m/s

$m_1=2$ kg

22-7



(13-

3 m/s

m_2

1120 N/m

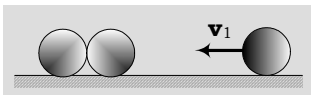
$m_2=5$ kg

(13-7)

1 m/s

23-7

(14-7)



(14-7)

M

24-7

$m > M$

$m \leq M$

			:
	1400 kg	2200 kg	25-7
			18 m
		0.95	
	1400 kg	540 kg	26-7
80 km/h			()
	()		()
		60 cm	
			27-7
	$(m_p=1860m_e)$		
		$m_1=2m_2$	28-7
		.60 J	
		2 m/s	29-7
		() .	20 m
			()
12 g	5 m/s	0.5 kg	30-7
			.
	40 km/h	3×10^7 kg	31-7
		$.8 \times 10^8$ kg	
	m_2	m_1	32-7
	:	e	v_2 v_1
		$v_1' = \frac{1}{M}[(m_1 - em_2)v_1 + (1 + e)m_2v_2]$	
		$v_2' = \frac{1}{M}[(1 + e)m_1v_1 + (m_2 - em_1)v_2]$	
	$\frac{1}{2} \mu v^2(1 - e^2)$		33-7
e	$ v_2 - v_1 $	v $\mu = m_1 m_2 / (m_1 + m_2)$	

34-7

θ_1 v_0 **35-7**

$\theta_1 + \theta_2 = 90^\circ$ θ_2

$\theta_1 = 30^\circ$ **36-7**

500 m/s **37-7**

() () $.60^\circ$

2.2 m/s **38-7**

() 60° 1.1 m/s

()

39-7

() 8×10^6 m/s 6×10^6 m/s

() ($u = 10^{-27}$ kg) $12u$ $8u$ $17u$

30° 30 m/s **40-7**

$.45^\circ$

1200 N 600 N 800 N **41-7**

5 m/s

30° 9 m/s 60°

12.5 m/s 0.4 kg **42-7**

10 m/s 0.6 kg

() $.ox$ 37°

37° 3 kg 60 kg **43-7**

$.20$ m/s

0.1 kg 400 m/s 2.5 g **44-7**

$.300$ m/s