

كل ما يحتاجه الطالب في جميع الصفوف من أوراق عمل واختبارات ومذكرات، يجده هنا في الروابط التالية لأفضل مواقع تعليمي إماراتي 100 %

<u>تطبيق المناهج الإماراتية</u>	<u>الاجتماعيات</u>	<u>الرياضيات</u>
<u>الصفحة الرسمية على التلغرام</u>	<u>الاسلامية</u>	<u>العلوم</u>
<u>الصفحة الرسمية على الفيسبوك</u>	<u>الانجليزية</u>	
<u>التربية الاخلاقية لجميع الصفوف</u>	<u>اللغة العربية</u>	
<u>التربية الرياضية</u>		
مجموعات التلغرام.	مجموعات الفيسبوك	قنوات تلغرام
<u>الصف الأول</u>	<u>الصف الأول</u>	<u>الصف الأول</u>
<u>الصف الثاني</u>	<u>الصف الثاني</u>	<u>الصف الثاني</u>
<u>الصف الثالث</u>	<u>الصف الثالث</u>	<u>الصف الثالث</u>
<u>الصف الرابع</u>	<u>الصف الرابع</u>	<u>الصف الرابع</u>
<u>الصف الخامس</u>	<u>الصف الخامس</u>	<u>الصف الخامس</u>
<u>الصف السادس</u>	<u>الصف السادس</u>	<u>الصف السادس</u>
<u>الصف السابع</u>	<u>الصف السابع</u>	<u>الصف السابع</u>
<u>الصف الثامن</u>	<u>الصف الثامن</u>	<u>الصف الثامن</u>
<u>الصف التاسع عام</u>	<u>الصف التاسع عام</u>	<u>الصف التاسع عام</u>
<u>الصف التاسع متقدم</u>	<u>الصف التاسع متقدم</u>	<u>الصف التاسع متقدم</u>
<u>الصف العاشر عام</u>	<u>الصف العاشر عام</u>	<u>الصف العاشر عام</u>
<u>الصف العاشر متقدم</u>	<u>الصف العاشر متقدم</u>	<u>الصف العاشر متقدم</u>
<u>الحادي عشر عام</u>	<u>الحادي عشر عام</u>	<u>الحادي عشر عام</u>
<u>الحادي عشر متقدم</u>	<u>الحادي عشر متقدم</u>	<u>الحادي عشر متقدم</u>
<u>ثاني عشر عام</u>	<u>الثاني عشر عام</u>	<u>الثاني عشر عام</u>
<u>ثاني عشر متقدم</u>	<u>الثاني عشر متقدم</u>	<u>الثاني عشر متقدم</u>

Chapter 29: Electromagnetic Induction

الوحدة 9: الحث الكهرومغناطيسي

Concept Checks

29.1. c 29.2. a 29.3. c 29.4. c 29.5. a 29.6. a 29.7. a 29.8. e

Multiple-Choice Questions

29.1. d 29.2. c 29.3. a 29.4. a 29.5. a 29.6. c 29.7. d 29.8. b 29.9. a 29.10. d 29.11. c 29.12. d 29.13. e 29.14. a

Conceptual Questions

- 29.15. A refrigerator's electrical circuit contains a motor with a large number of winding coils, making it highly inductive. The electromagnetic induction due to the coil can create a large voltage, on the order of kV between the prongs. This voltage is great enough to ionize the air and the process of ionization produces light, creating a visible spark.
- 29.16. Large machinery and motors often convert electrical energy to mechanical energy or vice-versa to complete a task. The conversion from electrical energy to mechanical energy requires the creation of magnetic fluxes. Changes in the magnetic flux reaching a pacemaker, due to movements of the machine or the person, will create currents in the circuitry of the pacemaker, changing its behavior; this can be dangerous.
- 29.17. As the metal moves through the non-uniform magnetic field, it experiences a changing magnetic flux. The flux induces an emf in the metal, if it is a conductor, and produces eddy currents. Lenz's law states that the induced currents create a force to oppose the movement of the metal through the field. This action is analogous to the drag force or force of friction used to create the damping of a harmonic oscillator.
- 29.18. Lenz's law requires that as the magnet moves down the cylinder, a current is produced in the aluminum cylinder, which in turn creates a magnetic field that opposes the magnet's motion. The force of the currents on the magnet is proportional to the velocity of the magnet. Thus, the magnet will continue to accelerate until it reaches a terminal speed that creates a force equal and opposite to the force of gravity.
- 29.19. (a) The currents produced in the aluminum, by induction, create a force that opposes the motion of the magnet. The magnet falling in the glass tube does not create a current since glass is an insulator. Thus, the magnet in the glass tube falls faster since there is no magnetic field produced to oppose the force of gravity. (b) Because the glass has nearly infinite resistance, no eddy currents are created as the magnet passes through it. The aluminum being a good conductor does produce eddy currents as the magnet falls through it. Thus, the aluminum tube has a larger eddy current.
- 29.20. (a) The B field inside the solenoid is uniform and equal to $B_i = \mu_0 ni$. Outside the solenoid, the field is zero, $B_o = 0$. The B field through the ring is only that of the field inside the solenoid of radius, a . The flux is then $\Phi = BA = \mu_0 n i \pi a^2 = \mu_0 n \pi a^2 C t^2$. Thus, the emf is $|\Delta V_{\text{ind}}| = \frac{d\Phi}{dt} = 2\mu_0 n \pi a^2 C t$.
- (b) The magnitude of the electric field is then $2\pi r E = \Delta V = 2\mu_0 n \pi a^2 C t$ or $E = \frac{\mu_0 n a^2 C t}{r}$.
- (c) The ring is not necessary for the induced electric field to exist. The solenoid will produce a magnetic field from the current being passed through the wire inducing an electric field on each concurrent loop of wire.
- 29.21. Lenz's law requires that the induced current opposes the change in the magnetic field. Therefore, the B field created by the induced current is downward. To produce a magnetic field in this direction, the current must flow clockwise as seen from above.

29.22. The area of the loop perpendicular to the field is given by $A = L^2 \cos(\omega t)$. The potential difference is:

$$\Delta V_{\text{ind}} = -\frac{d\Phi}{dt} = -\frac{d(AB)}{dt} = -B\frac{dA}{dt} = -B\frac{d}{dt}(L^2 \cos(\omega t)) = -BL^2(-\omega \sin(\omega t)) = BL^2\omega \sin(\omega t).$$

29.23. The emf produced by a loop is given by $\Delta V_{\text{ind}} = vBL$, where L is the length of the moving conductor. By taking a differentially small element of the disk, we convert L into the differential, dr , and integrate from the center of the disk to the edge for the emf of the disk: $\Delta V_{\text{ind}} = \int_0^R vBdr$. The velocity of an element, dr , is given by $v = r\omega$. The emf is then:

$$\Delta V_{\text{ind}} = \int_0^R r\omega Bdr = \frac{1}{2}\omega R^2 B.$$

29.24. Separation of charge due to the magnetic force, $q\vec{v} \times \vec{B}$, engenders a compensating electric field of magnitude $E = \vec{v} \times \vec{B} = vB$. The corresponding potential difference across height, l , is:
 $V = lE = lvB = (1.80 \text{ m})(2.00 \text{ m/s})(25.0 \text{ T}) = 90.0 \text{ V}$. In equilibrium this drives no current. However, such a large magnetic field offers further hazards due to any metal objects about the man's body and to stress on blood vessels, which are carrying conducting fluids in motion like iron.

29.25. The flux through the inside copper cylinder is constant during the process, so:

$$\Phi_i = \Phi_f \Rightarrow B_i A_i = B_f A_f \Rightarrow B_i \pi r_i^2 = B_f \pi r_f^2.$$

The final magnetic field is given by:

$$B_f = \left(\frac{r_i}{r_f}\right)^2 B_i.$$

If the initial B field is 1.0 T and the radius compresses by a factor of 14, then final field is given by:

$$B_f = \left(\frac{r_i}{r_i/14}\right)^2 B_i = (14)^2 B_i = (14)^2 (1.0 \text{ T}) = 2.0 \cdot 10^2 \text{ T}.$$

Experimental magnetic fields are typically lower than 10 T. This is a huge magnetic field.

29.26. Lenz's law requires that the induced current opposes the change in the magnetic field. Therefore, the B field created by the induced current is downward. To produce a magnetic field in this direction, the current must flow clockwise as seen from above.

29.27. The inductance of a solenoid is given by $L = \mu_0 n^2 lA$. Let d denote the length of the wire. The number of turns in each case is $N = d / 2\pi r$. The inductance is then:

$$L = \mu_0 n^2 lA = \mu_0 n(nl)A = \mu_0 nNA = \mu_0 n \left(\frac{d}{2\pi r}\right) \pi r^2 = \frac{1}{2} \mu_0 ndr.$$

For both solenoids, the number of turns per unit length is equal, and the distance of the wire is the same. Therefore, the ratio of the inductances is:

$$\frac{L_1}{L_2} = \frac{\mu_0 ndr / 2}{\mu_0 nd2r / 2} = \frac{1}{2}.$$

Thus, the inductance of the second solenoid is twice that of the first solenoid.

Exercises

29.28. The magnetic flux through the coil is given by:

$$\Phi = NBA \cos \theta = 20(5.00 \text{ T})\pi(0.400 \text{ m})^2 \cos(90^\circ - 25.8^\circ) = 21.9 \text{ T m}^2$$

29.29. The potential difference around the loop is:

$$V_{\text{emf}} = -\frac{d\Phi}{dt} \approx -\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta(AB)}{\Delta t} = -A \frac{\Delta B}{\Delta t} = -\pi r^2 \frac{\Delta B}{\Delta t} = -\pi(0.0100 \text{ m})^2 \left(\frac{0 \text{ T} - 1.20 \text{ T}}{20.0 \text{ s}} \right) = 1.89 \cdot 10^{-5} \text{ V}.$$

Note that the area of the ring is perpendicular to the field. Thus, the normal of the area is parallel to the field and $\cos\theta = 1$.

29.30. If the angle between the B -field and the plane of the loop is 40° , then the angle between the B -field and the normal to the loop is $90^\circ - 40^\circ = 50^\circ$, and so the voltage across the loop is given by:

$$V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(NAB \cos\theta) = -NA \cos\theta \frac{dB}{dt} = -NA \cos\theta \frac{d(1.50t^3)}{dt} = -NL^2 (\cos 50.0^\circ) 4.50t^2.$$

The current induced if the loop has a resistance of $R = 3.00 \Omega$ is:

$$i = \frac{|V_{\text{ind}}|}{R} = \frac{NL^2 (\cos 50.0^\circ) 4.50t^2}{R} = \frac{(8)(0.200 \text{ m})^2 (\cos 50.0^\circ) 4.50(2.00 \text{ s})^2}{(3.00 \Omega)} = 1.23 \text{ A}.$$

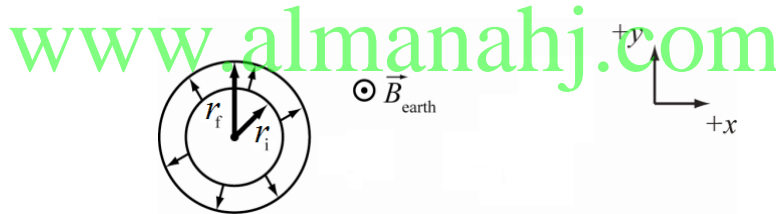
29.31. Because the magnetic field is perpendicular to the normal of the loop, there is no flux through the loop:

$$\Phi = AB \cos 90^\circ = 0.$$

Since there is no flux through the loop, there is no induced voltage: $V = -\frac{d\Phi}{dt} = -\frac{d(0)}{dt} = 0$.

29.32. **THINK:** The change in the area of the loop creates a change in the magnetic flux through the loop. The change in flux produces a current. The loop has a resistance of $R = 30.0 \Omega$ and a radius which changes from $r_i = 20.0 \text{ cm}$ to $r_f = 25.0 \text{ cm}$ in 1.00 s . The magnetic field of the Earth is about $4.26 \cdot 10^{-5} \text{ T}$.

SKETCH:



RESEARCH: The flux through the loop is $\Phi_B = AB \cos\theta$ or $\Phi_B = AB$, since the B field is perpendicular to the surface of the loop. The induced potential difference is given by $V_{\text{ind}} = -d\Phi_B / dt$. This potential must also satisfy $V = iR$.

SIMPLIFY: The induced current in the loop is:

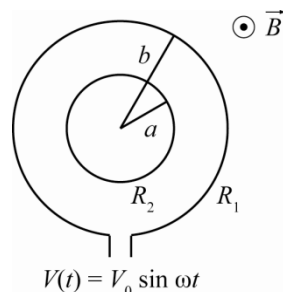
$$i = \frac{V_{\text{ind}}}{R} = \frac{1}{R} \left(-\frac{d\Phi_B}{dt} \right) = -\frac{1}{R} \left(\frac{dAB}{dt} \right) = -\frac{B}{R} \left(\frac{dA}{dt} \right) = -\frac{B}{R} \frac{d\pi r^2}{dt} = -\frac{B\pi}{R} \left(\frac{dr^2}{dt} \right) \approx -\frac{B\pi}{R} \left(\frac{r_f^2 - r_i^2}{\Delta t} \right).$$

$$\text{CALCULATE: } i = -\frac{(4.26 \cdot 10^{-5} \text{ T})\pi}{30.0 \Omega} \left(\frac{(0.250 \text{ m})^2 - (0.200 \text{ m})^2}{1.00 \text{ s}} \right) = -1.00374 \cdot 10^{-7} \text{ A}$$

ROUND: The induced current in the loop is $i = -1.00 \cdot 10^{-7} \text{ A}$.

DOUBLE-CHECK: This current is very small, as one would expect. The negative sign indicates that the direction of the induced current is such that the magnetic field due to the induced current opposes the change in magnetic flux that induces the current.

29.33. **THINK:** The current in the outer loop generates a magnetic field. Because the magnitude of the current in the outer loop changes with time, the magnetic field it generates also changes. The changing magnetic field, in turn, induces a potential difference and thus a current in the inner loop. Let I be the current in the outer loop and i be the induced current in the inner loop.

SKETCH:


RESEARCH: The current through the large loop is $I = \frac{V_0 \sin \omega t}{R_1}$. This creates a magnetic field at the center of the loop of:

$$B_1 = \frac{\mu_0 I}{2b}$$

which is derived from the Biot-Savart Law. Since the radius of the inner loop is much smaller than the radius of the outer loop, the magnetic field through the inner loop is $B_1 = \mu_0 V_0 \sin \omega t / 2bR_1$. This magnetic field creates a flux of:

$$\Phi_B = B_1 A = B_1 \pi a^2 = \frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t.$$

The induced potential across the inner loop is then:

$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right).$$

This voltage corresponds to a current in the inner loop of:

$$i = \frac{\Delta V_{\text{ind}}}{R_2} = -\frac{1}{R_2} \frac{d}{dt} \left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right).$$

SIMPLIFY: The potential difference induced in the inner loop is:

$$\Delta V_{\text{ind}} = -\frac{d}{dt} \left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right) = -\frac{\mu_0 \pi a^2 V_0}{2bR_1} \frac{d}{dt} (\sin \omega t) = -\frac{\mu_0 \pi a^2 V_0 \omega}{2bR_1} \cos \omega t,$$

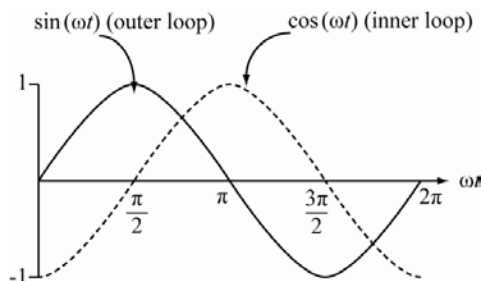
and the induced current in the inner loop is:

$$i = \frac{\Delta V_{\text{ind}}}{R_2} = -\frac{1}{R_2} \frac{d}{dt} \left(\frac{\mu_0 \pi a^2 V_0}{2bR_1} \sin \omega t \right) = -\frac{\mu_0 \pi a^2 V_0}{2bR_1 R_2} \frac{d}{dt} (\sin \omega t) = -\frac{\mu_0 \pi a^2 V_0 \omega}{2bR_1 R_2} \cos \omega t.$$

CALCULATE: Not applicable.

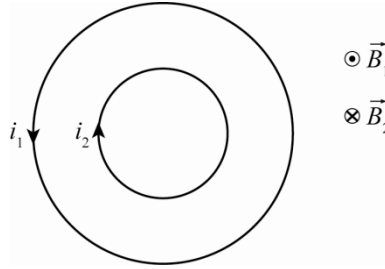
ROUND: Not applicable.

DOUBLE CHECK: The time dependence on the current for the outer loop and inner loop is shown in the plot below. For example, for $\omega t < \pi/2$ (taking positive values to be the counterclockwise direction) if the current in the outer loop is moving counterclockwise and increasing then the current in the inner loop is increasing in the clockwise direction. This is consistent with Lenz's Law.



- 29.34. **THINK:** The varying current, i_1 , through the outer solenoid creates a varying magnetic field, B_1 , within the coil. This varying B field creates a flux in the inner solenoid, which in turn creates an induced emf .

SKETCH:



RESEARCH: The magnetic field generated by the outer solenoid is given by $B_1 = \mu_0 n i_1 = \mu_0 n i_0 \cos \omega t$. The flux generated in the inner solenoid is given by $\Phi = A_2 B_1$. The induced emf in the inner solenoid is given

$$\text{by } \Delta V_{\text{ind}} = -\frac{d\Phi}{dt} = -A_2 \frac{dB_1}{dt}.$$

SIMPLIFY: $\Delta V_{\text{ind}} = -A_2 \frac{dB_1}{dt} = -A_2 \frac{d(\mu_0 n i_0 \cos \omega t)}{dt} = -A_2 \mu_0 n i_0 \frac{d(\cos \omega t)}{dt} = A_2 \mu_0 n i_0 \omega \sin \omega t$. This

corresponds to a current of $i_2 = \frac{\Delta V}{R} = \frac{A_2 \mu_0 n i_0 \omega \sin(\omega t)}{R}$, in the inner solenoid. The current of the inner solenoid induces a B field of:

$$B_2 = \mu_0 n i_2 = \frac{\mu_0 n (A_2 \mu_0 n i_0 \omega \sin \omega t)}{R} = \frac{\mu_0^2 n^2 A_2 i_0 \omega \sin \omega t}{R}.$$

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE CHECK: The induced magnetic field of the inner solenoid must oppose the change in flux of the outer solenoid. It can be seen from the expressions for B_2 and B_1 the two fields will always have opposite directions, satisfying this requirement.

- 29.35. (a) The decreasing B field creates a changing flux through the loop, confined to the area of the dotted circle of radius, $r = 3.00$ cm. The varying flux creates an emf of:

$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d(AB)}{dt} = -\frac{d(\pi r^2 B)}{dt} = -\pi r^2 \frac{dB}{dt} \approx -\pi r^2 \frac{\Delta B}{\Delta t} = -\pi r^2 \left(\frac{B_f - B_i}{\Delta t} \right).$$

This corresponds to a current of:

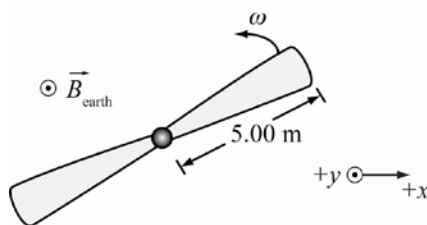
$$i = \frac{V}{R} = -\frac{\pi r^2 (B_f - B_i)}{R \Delta t} = -\frac{\pi (0.0300 \text{ m})^2 (1.00 \text{ T} - 2.00 \text{ T})}{0.200 \Omega (2.00 \text{ s})} = 0.00707 \text{ A} = 7.07 \text{ mA}$$

(b) The B field points into the page, thus a decrease in the B field will induce a current corresponding to a B field which points into the page. By the right-hand rule, the induced current flows clockwise.

- 29.36. The airplane's wings are approximated by a straight wire. The voltage across a wire moving in a B field is:

$$V = vLB = 3v_{\text{mach}} LB = 3(340. \text{ m/s})(10.0 \text{ m})(0.500 \cdot 10^{-4} \text{ T}) = 0.510 \text{ V}.$$

- 29.37. **THINK:** As a conductor travels through a magnetic field, perpendicular to the ground, of intensity $B = 0.426$ G, it creates a voltage difference between its ends. The length of metal of interest is $L = 5.00$ m and rotates at $1.00 \cdot 10^4$ rpm.

SKETCH:

RESEARCH: The potential difference across a wire moving in a magnetic field is $\Delta V_{\text{ind}} = vLB$. Each element of the blade travels at a different speed, $v = r\omega$. To calculate the potential difference, the length must be divided into pieces of length, dl , which travel at $v = l\omega$. The value should be integrated over the total length, from 0 to L .

SIMPLIFY:
$$\int \Delta V = \int_0^L vBdl = \int_0^L l\omega Bdl = \frac{1}{2}\omega BL^2$$

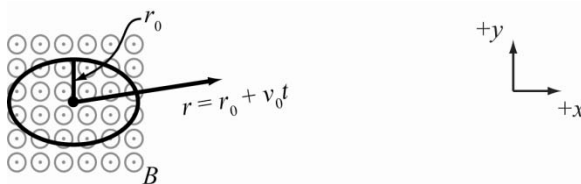
In terms of the blade's rpm, the potential difference is
$$V = \frac{1}{2} \left(\frac{2\pi(\text{rpm})}{60 \text{ s}} \right) BL^2.$$

CALCULATE:
$$V = \left(\frac{\pi(1.00 \cdot 10^4 \text{ rpm})}{60.0 \text{ s/rpm}} \right) (0.426 \cdot 10^{-4} \text{ T})(5.00 \text{ m})^2 = 0.557633 \text{ V} \approx 0.558 \text{ V}$$

ROUND: The potential difference from the hub of the helicopter's blade to its far end is $\Delta V_{\text{ind}} = 0.558 \text{ V}$.

DOUBLE-CHECK: We can double-check this result by assuming that the blade moves with a constant speed equal to the speed of the middle of the blade, $v = \left(\frac{L}{2}\right)\omega = \left(\frac{L}{2}\right)2\pi f = 2\pi Lf$. The induced potential difference would be $\Delta V = vB\frac{L}{2} = (2\pi Lf)B\frac{L}{2} = \pi fBL^2$, which is the same answer we got by integrating over the length of the blade.

- 29.38. THINK:** The expanding loop creates a changing flux through the loop. Lenz's law implies that the changing flux induces a current in the loop. This is similar to increasing the magnetic field within the loop. To counteract the increase in flux, the current must create a magnetic field opposite to the B field. By the right-hand rule, the current must flow clockwise. The radius of the loop expands by $r = r_0 + vt$, where $r_0 = 0.100 \text{ m}$ and $v = 0.0150 \text{ m/s}$. The resistance of the wire is $R = 12.0 \Omega$. The B field has a uniform value of $B_0 = 0.750 \text{ T}$ upward. The problem asks for the induced current at the time, $t = 5.00 \text{ s}$.

SKETCH:

RESEARCH: The flux through the loop is $\Phi_B = AB = \pi r^2 B$. The induced current of the loop is $i = V/R$, where the voltage is given by $V = -d\Phi_B/dt$.

SIMPLIFY: The induced current in the wire is:

$$i = \frac{V}{R} = -\frac{1}{R} \frac{d\Phi_B}{dt} = -\frac{1}{R} \frac{d}{dt}(\pi r^2) = -\left(\frac{\pi B}{R}\right) \frac{d}{dt}(r_0 + vt)^2 = -\frac{\pi B 2(r_0 + vt)v}{R} = -\frac{2\pi B}{R} v(r_0 + vt).$$

CALCULATE: The magnitude of the induced current at $t = 5.00 \text{ s}$ is:

$$i = -\frac{2\pi(0.750 \text{ T})}{12.0 \Omega} (0.0150 \text{ m/s}) [0.100 \text{ m} + (0.0150 \text{ m/s})(5.00 \text{ s})] = 0.0010308 \text{ A}.$$

ROUND: $i = 1.03 \text{ mA}$ at 5.00 s , travelling clockwise through the loop.

DOUBLE-CHECK: $[i] = \frac{[\text{T}]}{[\Omega]} [\text{m/s}] ([\text{m}] + [\text{m/s}][\text{s}]) = \frac{[\text{T}][\text{m}^2]}{[\Omega][\text{s}]} = \frac{[\text{V}][\text{s}][\text{m}^2][\text{A}]}{[\text{m}^2][\text{V}][\text{s}]} = [\text{A}]$

29.39. THINK: Terminal velocity will be reached when the force due to the changing magnetic flux cancels the weight of the bar.

SKETCH: A sketch is not necessary.

RESEARCH: $\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -Bw\frac{dy}{dt} = Bwv_{\text{term}}$

$i = \frac{\Delta V_{\text{ind}}}{R}$, $F_B = iLB = iwB$, $F_B = F_{\text{gravity}} = mg$.

SIMPLIFY: $iBw = mg \Rightarrow \frac{\Delta V_{\text{ind}}}{R} Bw = mg \Rightarrow \frac{Bwv_{\text{term}}}{R} Bw = mg \Rightarrow v_{\text{term}} = \frac{mgR}{w^2 B^2}$.

CALCULATE: No calculations are necessary.

ROUND: Rounding is not necessary.

DOUBLE-CHECK: It makes sense the larger m is, the higher v_{term} has to be to compensate for the greater gravitational force.

29.40. THINK:

- (a) The change in area causes an induced voltage.
- (b) After finding the induced voltage, the induced current can be determined.
- (c) The induced current will cause a force opposite to the direction of motion (from Lenz's law) which requires F_{ext} compensating for it.
- (d) Determine W_{ext} and P_{ext} from F_{ext} .

SKETCH: Provided with the question.

RESEARCH:

(a) $|\Delta V_{\text{ind}}| = \left| -\frac{d\Phi_B}{dt} \right| = B\frac{dA}{dt} = BvL$

(b) $i_{\text{ind}} = \frac{\Delta V}{R}$, in the clockwise direction.

(c) $F_B = i_{\text{ind}}LB = F_{\text{ext}}$

(d) $W_{\text{ext}} = F_{\text{ext}}\Delta y$, $P_{\text{ext}} = Fv$

(e) $P_{\text{ext}} = P_R = i_{\text{ind}}^2 R$

SIMPLIFY:

(a) $|\Delta V_{\text{ind}}| = BvL$

(b) $i_{\text{ind}} = \frac{BvL}{R}$

(c) $|F_B| = |F_{\text{ext}}| = \frac{L^2 B^2 v}{R}$

(d) $W_{\text{ext}} = \frac{L^2 B^2 v}{R} \Delta y$, $P_{\text{ext}} = \frac{L^2 B^2 v^2}{R}$

(e) $P_R = \frac{L^2 B^2 v^2}{R}$

CALCULATE: Not necessary.

ROUND: Not necessary.

DOUBLE-CHECK:

(e) This is due to the law of conservation of energy. The work done has to go somewhere, and in this case is dissipated by the resistor as heat.

29.41. THINK: The current in the wire will cause a magnetic field. The changing current will cause a changing flux through the loop, inducing a potential.

SKETCH: Provided with question.

RESEARCH: For a wire: $B = \frac{\mu_0}{4\pi} \left(\frac{2i}{r} \right)$. $\Delta V_{\text{ind}} = \frac{d\Phi_B}{dt}$, $i = 2.00 \text{ A} + (0.300 \text{ A/s})t$, $A = 7.00 \text{ m by } 5.00 \text{ m}$,

$$\Phi_B = \iint \vec{B} \cdot d\vec{A}.$$

SIMPLIFY: $\Phi_B = (5.00 \text{ m}) \int_{1 \text{ m}}^{8 \text{ m}} \frac{\mu_0}{4\pi} \left(\frac{2i}{r} \right) dr = (5.00 \text{ m}) \left(\frac{\mu_0 i}{2\pi} \right) \ln \left(\frac{8.00 \text{ m}}{1.00 \text{ m}} \right) = (5.00 \text{ m}) \left(\frac{\mu_0 i}{2\pi} \right) \ln 8.00$

$$\Delta V_{\text{ind}} = \frac{d\Phi_B}{dt} = (5.00 \text{ m}) \left(\frac{\mu_0}{2\pi} \right) (\ln 8.00) \left(\frac{di}{dt} \right) = (5.00 \text{ m}) \left(\frac{\mu_0}{2\pi} \right) (\ln 8.00) (0.300 \text{ A/s})$$

CALCULATE: $\Delta V_{\text{ind}} = (5.00 \text{ m}) \left(\frac{4\pi \cdot 10^{-7} \text{ H/m}}{2\pi} \right) (\ln 8.00) (0.300 \text{ A/s}) = 6.238 \cdot 10^{-7} \text{ V}$

ROUND: $\Delta V_{\text{ind}} = 6.24 \cdot 10^{-7} \text{ V}$

DOUBLE-CHECK: It makes sense that the larger the rate of change of the current, the larger the induced voltage.

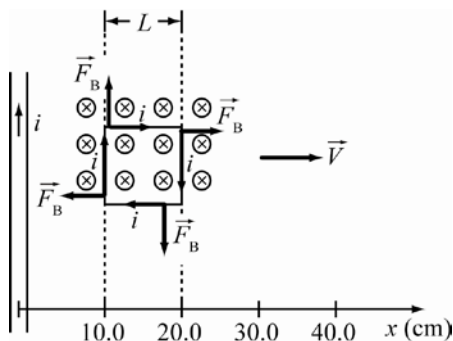
29.42. THINK:

(a) By the right-hand rule, the flux is into the page. Since the square is moving away from the wire, the flux is decreasing. Lenz's law states that the current is moving clockwise.

(c) The top and bottom parts have the same contributions and cancel each other.

SKETCH:

(b)



RESEARCH: Use $x_2 = 20.0 \text{ cm}$ and $x_1 = 10.0 \text{ cm}$ as the end points. $B = \frac{\mu_0}{4\pi} \left(\frac{2i}{r} \right)$, $\Phi_B = \iint \vec{B} \cdot d\vec{A}$,

$$\Delta V_{\text{ind}} = -\frac{d\Phi}{dt}, \quad i_{\text{ind}} = \frac{\Delta V_{\text{ind}}}{R}, \quad r = 10.0 \text{ cm}, \quad i = 1.00 \text{ A}, \quad v = 10.0 \text{ cm/s}, \quad R = 0.0200 \Omega, \quad L = 10.0 \text{ cm},$$

$$F_{\text{left}} = i_{\text{ind}} L B x_1, \quad F_{\text{right}} = i_{\text{ind}} L B x_2, \quad \text{and} \quad F_{\text{net}} = F_{\text{right}} - F_{\text{left}}.$$

SIMPLIFY: $\Phi_B = L \int_{x_1+vt}^{x_2+vt} \frac{\mu_0}{4\pi} \frac{2i}{r} dr = L \frac{\mu_0}{2\pi} i \left[\ln(x_2 + vt) - \ln(x_1 + vt) \right]$

$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{Li\mu_0}{2\pi} \left(\frac{v}{x_2 + vt} - \frac{v}{x_1 + vt} \right)$$

$$\begin{aligned}
 F_{\text{net}} &= i_{\text{ind}}LBx_2 - i_{\text{ind}}LBx_1 = i_{\text{ind}}LB(x_2 - x_1) = \frac{\Delta V_{\text{ind}}}{R}LB(x_2 - x_1) \\
 &= \left(-\frac{Li\mu_0}{2\pi R}v \left(\frac{1}{x_2 + vt} - \frac{1}{x_1 + vt} \right) \right) L \left(\frac{\mu_0}{4\pi} \left(\frac{2i}{r} \right) \right) (x_2 - x_1) \\
 &= -\frac{L^2i^2\mu_0v}{2\pi R} \left(\frac{1}{x_2 + vt} - \frac{1}{x_1 + vt} \right) \left(\frac{\mu_0}{2\pi} \right) \left(\frac{x_2 - x_1}{r} \right) \\
 &= -\frac{L^2i^2\mu_0^2v}{(2\pi)^2 R} \left(\frac{1}{x_2 + vt} - \frac{1}{x_1 + vt} \right) \left(\frac{x_2 - x_1}{r} \right)
 \end{aligned}$$

CALCULATE: At time $t = 0$:

$$\begin{aligned}
 F_{\text{net}} &= -\frac{(0.100 \text{ m})^2 (1.00 \text{ A})^2 (4\pi \cdot 10^{-7} \text{ H/m})^2 (0.100 \text{ m/s})}{(2\pi)^2 0.0200 \Omega} \left(\frac{1}{20.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} \right) \left(\frac{20.0 \text{ cm} - 10.0 \text{ cm}}{10.0 \text{ cm}} \right) \\
 &= 1.00 \cdot 10^{-16} \text{ N}
 \end{aligned}$$

ROUND: $F_{\text{net}} = 1.00 \cdot 10^{-16} \text{ N}$

DOUBLE-CHECK: It makes sense that for larger velocities and currents through the wire, the induced force is larger. This is in some ways analogous to how a car traveling faster than another has a larger drag force.

29.43. $\Phi(t) = BA \cos(2\pi ft)$, $\Delta V_{\text{ind}} = \frac{d\Phi}{dt} = -2\pi fBA \sin(2\pi ft)$. The maximum occurs when $|\sin(2\pi ft)| = 1$.

$$\Delta V_{\text{ind,max}} = 2\pi fBA = 110 \text{ V}. \text{ Substitute the values to obtain: } f = \frac{110 \text{ V}}{2\pi BA} = \frac{110 \text{ V}}{2\pi (1.00 \text{ T})(1.00 \text{ m}^2)} = 17.5 \text{ Hz}.$$

29.44. THINK: First relate the magnetic flux to the angular speed and then determine the maximum angular speed. Use the values $B = 0.87 \text{ T}$, $A = 0.0300 \text{ m}^2$.

SKETCH: A sketch is not necessary.

RESEARCH: For a single loop: $\Phi(t) = BA \cos(\omega t)$. $\Delta V_{\text{ind}} = -\frac{d\Phi}{dt} = \omega BA \sin(\omega t)$

$$\Delta V_{\text{ind,max}} = 170 \text{ V} = \omega BA, \text{ since the maximum occurs when } |\sin(\omega t)| = 1.$$

SIMPLIFY: $\omega = \frac{\Delta V_{\text{ind,max}}}{BA}$

CALCULATE: $\omega = \frac{170 \text{ V}}{0.87 \text{ T}(0.0300 \text{ m}^2)} = 6513 \text{ Hz}$

ROUND: $\omega = 6500 \text{ Hz}$

DOUBLE-CHECK: It is reasonable that the higher the applied voltage, the higher the angular speed.

29.45. THINK: First determine an expression for the magnetic flux, and then use Faraday's law to determine the induced voltage.

SKETCH: A sketch is not necessary.

RESEARCH: $B_{\text{Earth}} = 0.300 \text{ G} = 0.300 \cdot 10^{-4} \text{ T}$, $\Phi_B = NBA \cos(\omega t)$, $A = \pi r^2$, $r = 0.250 \text{ m}$, $N = 1.00 \cdot 10^5$,

$$\omega = 2\pi(150 \text{ Hz}), \quad i_{\text{ind}} = \frac{\Delta V_{\text{ind}}}{R} = -\left(\frac{1}{R} \right) \frac{d\Phi_B}{dt}, \quad i_{\text{ind,peak}} = -\left(\frac{1}{R} \right) \frac{d\Phi_B}{dt}_{\text{peak}}, \quad R = 1500 \Omega$$

SIMPLIFY:

(a) $i_{\text{ind}} = -\left(\frac{1}{R} \right) (-NBA\omega \sin(\omega t)) = \frac{NBA\omega}{R} \sin(\omega t)$; The peak occurs at $|\sin(\omega t)| = 1$: $i_{\text{ind,peak}} = \frac{NBA\omega}{R}$.

$$(b) i_{\text{avg}} = 0.7071(i_{\text{ind,peak}}), P_{\text{avg}} = i_{\text{avg}}^2 R$$

CALCULATE:

$$(a) i_{\text{ind,peak}} = \frac{(1.00 \cdot 10^5)(0.300 \cdot 10^{-4} \text{ T})(0.250 \text{ m})^2 2\pi^2 (150. \text{ Hz})}{(1500. \Omega)} = 0.3701 \text{ A}$$

$$(b) i_{\text{avg}} = 0.7071(0.3701 \text{ A}) = 0.2617 \text{ A}, P_{\text{avg}} = (0.2617 \text{ A})^2 (1500. \Omega) = 102.7 \text{ W}$$

ROUND:

$$(a) i_{\text{ind,peak}} = 0.370 \text{ A}$$

$$(b) i_{\text{avg}} = 0.262 \text{ A}, P_{\text{avg}} = 103 \text{ W}$$

DOUBLE-CHECK: The answer seems reasonable since there are a very large number of turns for the generator turning at a very fast rate.

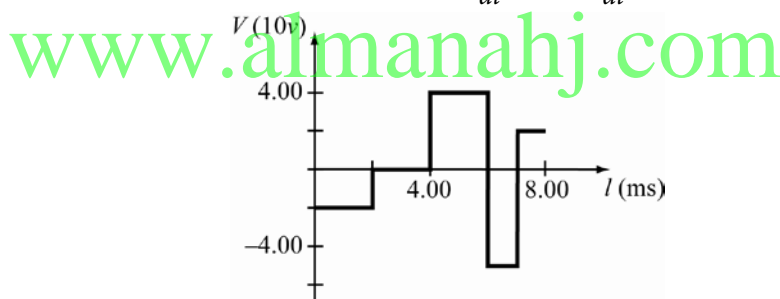
29.46. First solve for n : $B = \mu_0 n i \Rightarrow n = \frac{B}{\mu_0 i} = \frac{0.025 \text{ T}}{(1.2566 \cdot 10^{-6} \text{ m kg s}^{-2} \text{ A}^{-2})(0.60 \text{ A})} = 33158.$

$$M = N_1 \pi \mu_0 n_2 r_1^2 = 200\pi (1.2566 \cdot 10^{-6} \text{ m kg s}^{-2} \text{ A}^{-2})(33158)(0.034 \text{ m})^2 = 0.0302 \text{ H}, i(t) = i_0 (1 + (2.4 \text{ s}^{-2})t^2)$$

$$V = -M \frac{di}{dt} = -(0.0302 \text{ H})(2)(0.60 \text{ A})(2.4 \text{ s}^{-2})(2.0 \text{ s}) = -0.17 \text{ V}$$

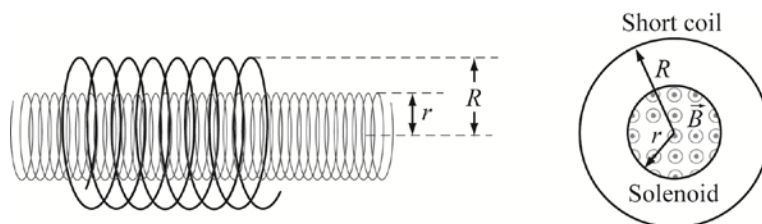
The results match those of the example.

29.47. The potential across an inductor is given by: $\Delta V_{\text{ind}} = -L \frac{di}{dt}$, where $\frac{di}{dt}$ is the slope.



29.48. THINK: The potential difference induced in the solenoid is due to the changing current in the coil. Using the mutual inductance of the solenoid due to the coil, the potential difference induced in the solenoid can be calculated. Assume the magnetic field of the short coil is uniform. This is not strictly accurate, but necessary to answer the question and will give a reasonable approximation.

SKETCH:



RESEARCH: The mutual inductance between the coil and the solenoid is

$$M = \frac{N_s \Phi_{c \rightarrow s}}{i_c},$$

where N_s is the number of turns in the solenoid, $\Phi_{c \rightarrow s}$ is the flux in the solenoid resulting from the magnetic field through the coil, and i_c is the current in the coil. The flux is given by

$$\Phi_{c \rightarrow s} = in\pi\mu_0 r^2.$$

$$\Delta V_{\text{ind}} = M \frac{di}{dt}, \quad N_s = 30, \quad n = 60/\text{cm} = 6000/\text{m}, \quad r = 0.0800 \text{ m}, \quad \frac{di}{dt} = \frac{2.00 \text{ A}}{12.0 \text{ s}}.$$

$$\text{SIMPLIFY: } \Delta V_{\text{ind}} = N_s n \pi \mu_0 r^2 \frac{di}{dt}$$

$$\text{CALCULATE: } \Delta V_{\text{ind}} = (30)(6000/\text{m})\pi(4\pi \cdot 10^{-7} \text{ H/m})(0.0800 \text{ m})^2 \left(\frac{2.00 \text{ A}}{12.0 \text{ s}} \right) = 7.57986 \cdot 10^{-4} \text{ V}$$

$$\text{ROUND: } \Delta V_{\text{ind}} = 7.58 \cdot 10^{-4} \text{ V}$$

DOUBLE-CHECK: It makes sense that for larger changes in current, larger potential differences are induced.

29.49. (a) $\tau_L = \frac{L}{R} = \frac{1.00 \text{ H}}{1.00 \text{ M}\Omega} = 1.00 \mu\text{s}$

(b) $i(t) = \frac{V_{\text{emf}}}{R} (1 - e^{-t/\tau_L})$. At $t = 0$, $i(t) = 0$. At $t = 2.00 \mu\text{s}$, $i(t) = \frac{10.0 \text{ V}}{1.00 \text{ M}\Omega} (1 - e^{-(2.00 \mu\text{s})/(1.00 \mu\text{s})}) = 8.65$

At steady state. $t \rightarrow \infty$: $i(\infty) = \frac{V_{\text{emf}}}{R} = 10.0 \mu\text{A}$.

29.50. For an RL circuit: $i(t) = \frac{V_{\text{emf}}}{R} (1 - e^{-t/\tau})$, where $\tau = \frac{L}{R} = 0.0250 \text{ s}$.

$$\frac{i(t)R}{V_{\text{emf}}} = 1 - e^{-t/\tau} \Rightarrow -\tau \ln \left(1 - \frac{i(t)R}{V_{\text{emf}}} \right) = t \Rightarrow t = -(0.0250 \text{ s}) \ln \left(1 - \frac{(0.300 \text{ A})(120. \Omega)}{40.0 \text{ V}} \right) = 0.0576 \text{ s}$$

29.51. The potential drop is the sum of the potential drop across the resistor and the inductor:

$$\Delta V = iR + L \frac{di}{dt} = (3.0 \text{ A})(3.25 \Omega) + (0.440)(3.6 \text{ A/s}) = 11 \approx 11.3 \text{ V}.$$

29.52. **THINK:** In a circuit containing only a resistor, the current would be established almost instantaneously. However, with the RL circuit, the current must increase exponentially from zero to the steady state. $V_{\text{emf}} = 18 \text{ V}$, $R_1 = R_2 = 6.0 \Omega$, $L = 5.0 \text{ H}$.

SKETCH: Provided with question.

RESEARCH:

(a) The inductor functions as an open-circuit, so $i = V_{\text{emf}} / R_2 = 18 \text{ V} / 6.0 \Omega = 3.0 \text{ A}$.

(b) The inductor acts as an open-circuit, so there is no current across it and hence no current across R_1 .

(c) The current across R_2 is given by Ohm's Law, $i = V_{\text{emf}} / R$.

(d) The potential difference across a resistor is also given by Ohm's Law, $\Delta V = iR$.

(e) Same as (d).

(f) The sum of the voltages around any loop is zero.

(g) The rate of current change across R_1 is the same as that of L .

SIMPLIFY:

(a) $i = V / R_2$

(b) Not applicable.

(c) $i_{R_2} = V_{\text{emf}} / R_2$

(d) $\Delta V_{R_1} = i_{R_1} R_1$

(e) $\Delta V_{R_2} = i_{R_2} R_2$

(f) $V_{\text{emf}} - V_L - V_{R_1} = 0 \Rightarrow V_L = V_{\text{emf}} - V_{R_1}$

$$(g) \quad V_L = L \frac{di}{dt} \Rightarrow \frac{di}{dt} = \frac{V_L}{L}$$

CALCULATE:

$$(a) \quad i = 18 \text{ V} / 6.0 \, \Omega = 3.0 \text{ A}$$

$$(b) \quad i_{R_1} = 0$$

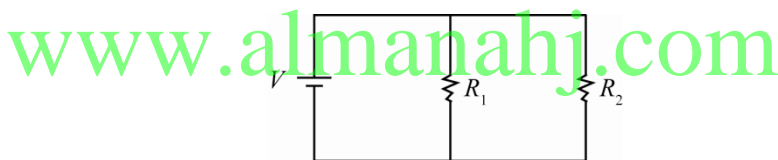
$$(c) \quad i_{R_2} = 18 \text{ V} / 6.0 \, \Omega = 3.0 \text{ A}$$

$$(d) \quad \Delta V_{R_1} = (0 \text{ A})(6.0 \, \Omega) = 0$$

$$(e) \quad \Delta V_{R_2} = (3.0 \text{ A})(6.0 \, \Omega) = 18 \text{ V}$$

$$(f) \quad V_L = 18 \text{ V} - 0 = 18 \text{ V}$$

$$(g) \quad \frac{di}{dt} = \frac{18 \text{ V}}{5.0 \text{ H}} = 3.6 \text{ A/s}$$

ROUND: Not necessary. The values are already to the correct number of significant figures.**DOUBLE CHECK:** The branch of the circuit which contains only a resistor and a source of emf behaves as a simple resistor circuit, with the current being established almost instantaneously. For the branch ofthe circuit which contains a resistor and an inductor, equation 29.29 states $i(t) = \frac{V_{\text{emf}}}{R} [1 - e^{-t/(L/R)}]$.When $t = 0$, $i(t) = 0$, as found above.**29.53. THINK:** After a long time, the inductor acts like a short-circuit. The circuit is in steady state, so the current is no longer changing. $V_{\text{emf}} = 18 \text{ V}$, $R_1 = R_2 = 6.0 \, \Omega$, $L = 5.0 \text{ H}$.**SKETCH:** An equivalent sketch when the circuit is in steady-state is as follows.**RESEARCH:** The current from the battery is given by $i_{\text{tot}} = \frac{V_{\text{emf}}}{R_{\text{net}}}$, where $R_{\text{net}} = \left(\frac{1}{R_2} + \frac{1}{R_1} \right)^{-1}$. The current through each resistor is given by Ohm's Law, $i = V / R$. The sum of the potentials around any loop must be zero: $V_{\text{emf}} + V_{R_1} = 0$, $V_{\text{emf}} + V_{R_2} + V_L = 0$.**SIMPLIFY:**

$$(a) \quad i_{\text{tot}} = \frac{V_{\text{emf}}}{R_1 R_2} (R_1 + R_2)$$

$$(b) \quad i_{R_1} = \frac{V_{R_1}}{R_1}$$

$$(c) \quad i_{R_2} = \frac{V_{R_2}}{R_2}$$

$$(d) \quad V_{\text{emf}} + V_{R_1} = 0 \Rightarrow V_{R_1} = -V_{\text{emf}}$$

$$(e) \quad V_{\text{emf}} + V_{R_2} + V_L = 0 \Rightarrow V_{R_2} = -V_{\text{emf}} - V_L = -V_{\text{emf}} - L \frac{di}{dt}$$

$$(f) \quad V_L = L \frac{di}{dt}$$

$$(g) \quad \frac{di_{R_1}}{dt} = \frac{di_L}{dt} = \frac{V_L}{L}$$

CALCULATE:

$$(a) i_{\text{tot}} = \frac{18 \text{ V}}{(6.0 \Omega)(6.0 \Omega)}(6.0 \Omega + 6.0 \Omega) = 6.0 \text{ A}$$

$$(b) i_{R_1} = \frac{18 \text{ V}}{6.0 \Omega} = 3.0 \text{ A}$$

$$(c) i_{R_2} = \frac{18 \text{ V}}{6.0 \Omega} = 3.0 \text{ A}$$

$$(d) V_{R_1} = -18 \text{ V}$$

$$(e) V_{R_2} = -18 \text{ V} - (5.0 \text{ H})(0) = -18 \text{ V}$$

$$(f) V_L = 5.0 \text{ H}(0) = 0$$

$$(g) \frac{di_{R_1}}{dt} = \frac{0}{5.0 \text{ H}} = 0$$

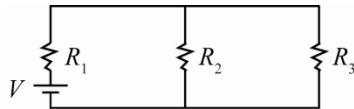
ROUND: Not necessary.

DOUBLE CHECK: Evaluating the loop containing the inductor using equation 29.29 shows that after a long time, $i_2(t) = \frac{V_{\text{emf}}}{R_2}(1 - e^{-t/(L/R)}) = \frac{V_{\text{emf}}}{R_2}$, as found above. Kirchoff's rules can be used to show that $i_{R_1} = i_{R_2}$, also as found above.

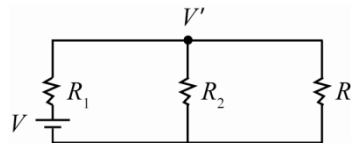
- 29.54. THINK:** As the current begins to flow through the circuit, the self induced potential difference in the inductor opposes the change in current. As the change in current decreases, the self induced potential difference also decreases until the current reaches the steady state given by Ohm's Law, $i = V_{\text{emf}} / R$. When the switch is opened, the current will continue to flow, at a decreasing rate, through the loop composed of R_3 , L , and R_2 until the energy which has been stored in the inductor is dissipated.

SKETCH:

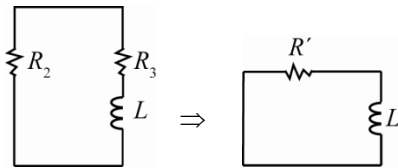
(a)



(b)



(c)


RESEARCH:

(a) Immediately after the switch is closed, the inductor is like an open-circuit. Clearly, $i_{R_3} = 0$, and

$$V + i_{R_1} R_1 + i_{R_2} R_2 + i_{R_3} R_3 = 0. \text{ so } i_{R_2} = i_{R_1} = \frac{V}{R_1 + R_2}.$$

(b) After a long time, the inductor acts like a short-circuit. $R_{\text{tot}} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$, $V' = V - R_1 \left(\frac{V}{R_{\text{tot}}} \right)$

(c) When the switch is opened, $i_L = i_{R_2} = i_{R_3}$. In fact, the equivalent resistance of this circuit is $R' = R_2 + R_3$ and the circuit can be redrawn accordingly. Since the current in an inductor cannot change

instantaneously, from part (b): $i_{\text{initial}} = \frac{V'}{R_3} = \frac{V - R_1(V/R_{\text{tot}})}{R_3}$, $R_{\text{tot}} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$ and $\tau = \frac{L}{R'}$. The current for an RL circuit is $i(t) = i_{\text{initial}}(e^{-t/\tau})$. Immediately after opening the switch, $t \approx 0$ and $i(t_0) = i_{\text{initial}}(e^0) = i_{\text{initial}}$.

SIMPLIFY:

$$(a) V + i_{R_1} R_1 + i_{R_2} R_2 + 0 = 0 \Rightarrow i_{R_2} = i_{R_1} = \frac{V}{R_1 + R_2}, \quad i_{R_3} = 0$$

$$(b) i_{R_1} = \frac{V}{R_{\text{tot}}} = \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$i_{R_2} = \frac{V'}{R_2} = \frac{V}{R_2} - \frac{R_1}{R_2} \left(\frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} \right) = \frac{V(R_1 R_2 + R_1 R_3 + R_2 R_3) - V(R_1 R_2 + R_1 R_3)}{R_2(R_1 R_2 + R_1 R_3 + R_2 R_3)} = \frac{V(R_3)}{(R_1 R_2 + R_1 R_3 + R_2 R_3)}$$

$$i_{R_3} = \frac{V'}{R_3} = \frac{V}{R_3} - \frac{R_1}{R_3} \left(\frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} \right) = \frac{V(R_1 R_2 + R_1 R_3 + R_2 R_3) - V(R_1 R_2 + R_1 R_3)}{R_3(R_1 R_2 + R_1 R_3 + R_2 R_3)} = \frac{V(R_2)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$(c) i(t) = i_{\text{initial}}(e^{-t/\tau}), \text{ where } i_{R_1} = 0, \quad i_{R_3} = -i_{R_2} = i(t), \quad i_{\text{initial}} = \frac{V - R_1(V/R_{\text{tot}})}{R_3} = \frac{V(R_2)}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE CHECK: When the switch is closed and the current is increasing according to

$$i(t) = \frac{V_{\text{emf}}}{R} (1 - e^{-t/\tau}), \text{ as } t \rightarrow \infty, \quad i(t) = \frac{V_{\text{emf}}}{R}, \text{ which agrees with the result.}$$

- 29.55. The energy density is given by $u_B = \frac{B^2}{2\mu_0}$. Determine the volume that gives $Vu_B = 1 \text{ J}$:

$$V = \frac{2\mu_0(1 \text{ J})}{B^2} = \frac{2(4\pi \cdot 10^{-7} \text{ H} \cdot \text{m}^{-1})(1 \text{ J})}{(5.0 \cdot 10^{-5} \text{ T})^2} = 1.01 \cdot 10^3 \text{ m}^3.$$

This volume is equivalent to a 10 m by 10 m by 10 m cube. This is a fraction of the size of a house.

- 29.56. (a) The magnetic energy density is given by: $u_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2(4\pi \cdot 10^{-7} \text{ H} \cdot \text{m}^{-1})} (3.00 \text{ T})^2 = 3.58 \cdot 10^6 \text{ J/m}^3$.

(b) The total energy is given by $U_B = Vu_B$. $V = \pi R^2 L$, $R = 0.500 \text{ m}$, $L = 1.50 \text{ m}$.

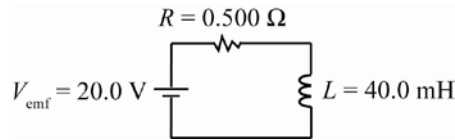
$$\Rightarrow U_B = \pi(0.500 \text{ m})^2(1.50 \text{ m})(3.58 \cdot 10^6 \text{ J/m}^3) = 4.22 \cdot 10^6 \text{ J}$$

- 29.57. (a) $u_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2(4\pi \cdot 10^{-7} \text{ H} \cdot \text{m}^{-1})} (4.00 \cdot 10^{10} \text{ T})^2 = 6.366 \cdot 10^{26} \text{ J/m}^3$

(b) The associated mass density is then: $\frac{u_B}{c^2} = \rho_{\text{rest}} = \frac{6.366 \cdot 10^{26} \text{ J/m}^3}{(3.00 \cdot 10^8 \text{ m/s}^2)^2} = 7.07 \cdot 10^9 \text{ kg/m}^3$

- 29.58. **THINK:** The emf potential and the resistance can be used to find the maximum current. Then the energy stored in the magnetic field of the inductor at one fourth of this current can be found. The equation for the rise in current as a function of time can be used to find the time for the circuit to reach a current of one fourth of its maximum value. The inductance of the inductor is $L = 40.0 \text{ mH}$, the resistance of the resistor is $R = 0.500 \Omega$, and the emf potential is $V_{\text{emf}} = 20.0 \text{ V}$.

SKETCH:



RESEARCH:

(a) At steady-state,

$$i_{\max} = \frac{V_{\text{emf}}}{R}.$$

The time of interest is when $i = i_{\max} / 4$. Use the equation $U_B = \frac{1}{2} Li^2$.

$$(b) i(t) = i_{\max} (1 - e^{-t/\tau_{\text{RL}}}) = \frac{1}{4} i_{\max}, \quad \tau_{\text{RL}} = \frac{L}{R}$$

SIMPLIFY:

$$(a) U_B = \frac{1}{2} L \left(\frac{1}{4} \frac{V_{\text{emf}}}{R} \right)^2 = \left(\frac{1}{32} \right) \frac{L V_{\text{emf}}^2}{R^2}$$

$$(b) \frac{1}{4} i_{\max} = i_{\max} (1 - e^{-t/\tau_{\text{RL}}}) \Rightarrow \ln\left(\frac{3}{4}\right) = -\frac{t}{\tau_{\text{RL}}} \Rightarrow t = -\tau_{\text{RL}} \ln\left(\frac{3}{4}\right) = -\frac{L}{R} \ln\left(\frac{3}{4}\right)$$

CALCULATE:

$$(a) U = \left(\frac{1}{32} \right) \frac{(0.0400 \text{ H})(20.0 \text{ V})^2}{(0.500 \Omega)^2} = 2.00 \text{ J}$$

$$(b) t = -\left(\frac{0.0400 \text{ H}}{0.500 \Omega} \right) \ln\left(\frac{3}{4}\right) = 0.0230 \text{ s}$$

ROUND:

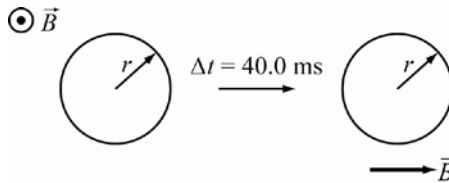
$$(a) U = 2.00 \text{ J}$$

$$(b) t = 0.0230 \text{ s}$$

DOUBLE-CHECK: It makes sense that the time it takes to reach one fourth of the maximum value is comparable to the time constant, τ_{RL} .

- 29.59. THINK:** The equation for the rate of energy production due to a potential across a resistance can be used to determine the heat generated. The induced potential can be found by using Faraday's Law. Then the rise in temperature due to this heat can be found for the ring of mass $m = 0.0150 \text{ kg}$ and specific heat capacity $c = 129 \text{ J/kg} \cdot ^\circ\text{C}$. The strength of the magnetic field is $B = 0.0800 \text{ T}$, the radius of the ring is $r = 0.00750 \text{ m}$, the time change between maximum and zero magnetic flux is $\Delta t = 0.0400 \text{ s}$, and the resistance of the ring is $R = 61.9 \cdot 10^{-6} \Omega$.

SKETCH:



RESEARCH: The induced potential in the ring is given by: $\Delta V_{\text{ind}} = -\frac{d\Phi}{dt} = -\frac{BA}{\Delta t}$. The rate of energy production as heat is given by

$$P = \frac{(\Delta V_{\text{ind}})^2}{R}.$$

The power produced multiplied by the time difference is equal to the heat generated:

$$P\Delta t = Q = mc\Delta T.$$

SIMPLIFY: The temperature rise is

$$\Delta T = \frac{(\Delta V_{\text{ind}})^2 \Delta t}{mcR} = \frac{\Delta t}{mcR} \left(-\frac{BA}{\Delta t} \right)^2 = \frac{(\pi Br^2)^2}{mcR\Delta t}$$

CALCULATE:
$$\Delta T = \frac{(\pi(0.0800 \text{ T})(0.00750 \text{ m})^2)^2}{(0.0150 \text{ kg})(129 \text{ J/kg } ^\circ\text{C})(61.9 \cdot 10^{-6} \Omega)(0.0400 \text{ s})} = 4.1715 \cdot 10^{-5} \text{ } ^\circ\text{C}$$

ROUND: To three significant figures, the temperature rise is $\Delta T = 4.17 \cdot 10^{-5} \text{ } ^\circ\text{C}$.

DOUBLE-CHECK: It makes sense that for larger fields, ΔT is larger, and for larger masses, ΔT is smaller since it would take more work to heat up the ring. As expected, the temperature increase is quite small.

29.60. THINK: Consider the energy of the dipole before and after the flip and relate this to the work done.

SKETCH: A sketch is not necessary.

RESEARCH: When the dipole is in alignment: $U = -NiAB$. When the dipole is anti-parallel to the field: $U = NiAB$.

SIMPLIFY: The work done must therefore be $W = \Delta U = 2NiAB$.

CALCULATE: No calculations are necessary.

ROUND: Rounding is not necessary.

DOUBLE-CHECK: Larger fluxes (larger NAB) yield more work for the power supply.

29.61. THINK: Determine the energy density of the electric field and the magnetic field separately.

SKETCH: A sketch is not necessary.

RESEARCH: $u_B = \frac{1}{2\mu_0} |B^2|$, $u_E = \frac{1}{2}\epsilon_0 |E^2|$, $|\vec{B}_0| = \left| \frac{k \times \vec{E}_0}{\omega} \right|$, $\omega = \frac{|\vec{k}|}{\sqrt{\mu_0 \epsilon_0}}$, $\vec{E}(\vec{x}, t) = \vec{E}_0 \cos(\vec{k} \cdot \vec{x} - \omega t)$,

$$\vec{B}(\vec{x}, t) = \vec{B}_0 \cos(\vec{k} \cdot \vec{x} - \omega t).$$

SIMPLIFY:

$$\frac{u_B}{u_E} = \frac{|B^2|}{2\mu_0} \left(\frac{1}{2}\epsilon_0 |E^2| \right)^{-1} = \frac{1}{\mu_0 \epsilon_0} \frac{|B^2|}{|E^2|} = \frac{1}{\mu_0 \epsilon_0} \frac{|\vec{B}_0|^2 \cos^2(\vec{k} \cdot \vec{x} - \omega t)}{|\vec{E}_0|^2 \cos^2(\vec{k} \cdot \vec{x} - \omega t)} = \frac{1}{\mu_0 \epsilon_0} \left(\frac{1}{\omega^2} \right) \frac{|k \times \vec{E}_0|^2}{|\vec{E}_0|^2} = \frac{|k \times \vec{E}_0|^2}{|\vec{k}|^2 |\vec{E}_0|^2}$$

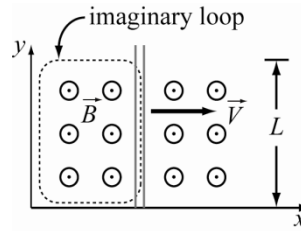
Note that \vec{k} is perpendicular to \vec{E}_0 so $|\vec{k} \times \vec{E}_0|^2 = |\vec{k}|^2 |\vec{E}_0|^2$, so the above expression becomes $\frac{u_B}{u_E} = 1$.

CALCULATE: No calculations are necessary.

ROUND: Rounding is not necessary.

DOUBLE-CHECK: This result shows that the energy in this type of wave is partitioned equally between the electric and magnetic fields.

29.62.



The induced voltage is given by:

$$|\Delta V_{\text{ind}}| = \left| -\frac{d\Phi}{dt} \right| = vLB = 2.00 \text{ V} \Rightarrow v = \frac{\Delta V}{BL} = \frac{2.00 \text{ V}}{(0.100 \text{ m})(1.00 \text{ T})} = 20.0 \text{ m/s}.$$

29.63. The potential difference is given by Faraday's law:

$$|\Delta V_{\text{ind}}| = \left| -\frac{d\Phi}{dt} \right| = A \frac{dB}{dt} = \pi R^2 \frac{dB}{dt} = \pi (0.0400 \text{ m})^2 (1.50 \text{ T/s}) = 7.54 \cdot 10^{-3} \text{ V}$$

Note that the radius of the coil is irrelevant.

29.64. The inductor cannot have the current jump instantaneously. From Kirchoff's loop law:

$$V_{\text{emf}} - L \frac{di}{dt} = 0 \Rightarrow \frac{V_{\text{emf}}}{L} = \frac{di}{dt}$$

$$\frac{V_{\text{emf}}}{L} = \frac{di}{dt} \Rightarrow di = \frac{V_{\text{emf}}}{L} dt \Rightarrow \int_0^i di = \int_0^t \frac{V_{\text{emf}}}{L} dt \Rightarrow i(t) = \frac{V_{\text{emf}}}{L} t + C$$

Since $i(0) = 0$, $C = 0$. The expression is then $i(t) = \frac{V_{\text{emf}}}{L} t$.

 29.65. The energy stored in a solenoid is given by $U_B = Li^2/2$. The energy is dependent only on the magnitude, not the direction of the current. Therefore the energy stored in the magnetic field does not change.

 29.66. Use the formulas: $u_B = \frac{1}{2\mu_0} B^2$, $u_E = \frac{1}{2} \epsilon_0 E^2$ and $\frac{u_B}{u_E} = \frac{1}{\mu_0 \epsilon_0} \frac{B^2}{E^2}$. In particular, the values of the energy densities are:

$$u_B = \frac{1}{2 \left(1.257 \cdot 10^{-6} \frac{\text{m kg}}{\text{s}^2 \text{ A}^2} \right)} (50.0 \mu\text{T})^2 = 9.94 \cdot 10^{-4} \text{ J/m}^3,$$

and

$$u_E = \frac{1}{2} \left(8.842 \cdot 10^{-12} \frac{\text{s}^4 \text{ A}^2}{\text{m}^3 \text{ kg}} \right) (150. \text{ N/C})^2 = 9.95 \cdot 10^{-8} \text{ J/m}^3.$$

To compute the ratios, it is useful to remember that $1/\mu_0 \epsilon_0 = c^2$. This is a result from light being an electromagnetic wave where c is the speed of light in a vacuum.

$$\frac{u_B}{u_E} = c^2 \frac{B^2}{E^2} = (3.00 \cdot 10^8 \text{ m/s})^2 \left(\frac{50.0 \cdot 10^{-6} \text{ T}}{150. \text{ N/C}} \right)^2 = 1.00 \cdot 10^5$$

The energy density of the magnetic field is much larger than that of the electric field.

 29.67. The current of an RL circuit is given by: $i(t) = \frac{V_{\text{emf}}}{R} (1 - e^{-t/\tau})$, where $\tau = L/R$. For $t = 20.0 \mu\text{s}$:

$$\frac{1}{2} \left(\frac{V_{\text{emf}}}{R} \right) = \frac{V_{\text{emf}}}{R} (1 - e^{-t/\tau}) \Rightarrow 1 - \frac{1}{2} = e^{-t/\tau} \Rightarrow \tau \ln\left(\frac{1}{2}\right) = -t \Rightarrow \frac{L}{R} \ln\left(\frac{1}{2}\right) = -t \Rightarrow L = -\frac{Rt}{\ln(1/2)}$$

$$\Rightarrow L = -\frac{(3.00 \cdot 10^3 \Omega)(20.0 \cdot 10^{-6} \text{ s})}{\ln(1/2)} = 0.0866 \text{ H.}$$

29.68. The current of an RL circuit is given by $i(t) = i_{\max}(1 - e^{-t/\tau})$, where $\tau = L/R$.

$$0.995i_{\max} = i_{\max}(1 - e^{-t/\tau}) \Rightarrow e^{-t/\tau} = 0.00500 \Rightarrow$$

$$t = -\frac{L}{R} \ln(0.00500) = -\frac{0.200 \cdot 10^{-6} \text{ H}}{500. \Omega} \ln(0.00500) = 2.12 \text{ ns}$$

It is interesting to note that the voltage of the battery is irrelevant to the result of the problem.

29.69. For a single loop of wire ($N = 1$), the induced potential difference is:

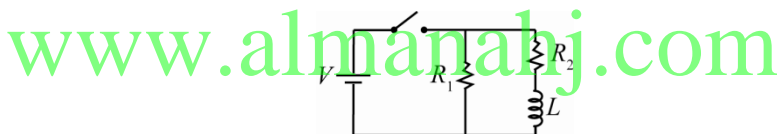
$$\Delta V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA \cos\theta).$$

Since the normal vector of the loop and the magnetic field is parallel, $\cos\theta = 1$. The negative sign can be dropped and ΔV_{ind} becomes:

$$\Delta V_{\text{ind}} = A \frac{dB}{dt} = A \frac{d}{dt}(3.00 \text{ T} + 2.00t \text{ T/s}) = A(2.00 \text{ T/s}) = (5.00 \text{ m}^2)(2.00 \text{ T/s}) = 10.0 \text{ V.}$$

Note the magnetic field, \vec{B} , is increasing, and it is directed into the page. By Lenz's law, the induced magnetic field, \vec{B}_i , opposes the change in magnetic flux, Φ_B . In this case, \vec{B}_i is directed out of the page to oppose the increasing field, \vec{B} , directed into the page. The induced current is therefore counterclockwise.

29.70. The following circuit has values: $V = 9.00 \text{ V}$, $R_1 = R_2 = 100. \Omega$, and $L = 3.00 \text{ H}$.



(a) When the switch is closed at $t = 0 \text{ s}$, the current through R_1 is: $i_1 = \frac{V}{R_1} = \frac{9.00 \text{ V}}{100. \Omega} = 0.0900 \text{ A}$. The

current through R_2 is $i_2(t) = \frac{V}{R_2} [1 - e^{-t/(L/R)}] \Rightarrow i_2(0) = 0$.

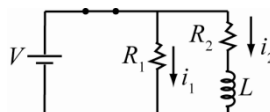
(b) At $t = 50.0 \text{ ms} = 0.0500 \text{ s}$, i_1 is still 0.0900 A , while i_2 is:

$$i_2(0.0500 \text{ s}) = \frac{9.00 \text{ V}}{100. \Omega} \left[1 - \exp\left\{(-0.0500 \text{ s})\left(\frac{100. \Omega}{3.00 \text{ H}}\right)\right\}\right] = 0.0900(1 - 0.189) \text{ A} = 0.0730 \text{ A}$$

(c) At $t = 500. \text{ ms} = 0.500 \text{ s}$, i_1 is still 0.0900 A , and i_2 is:

$$i_2(0.500 \text{ s}) = \frac{9.00 \text{ V}}{100. \Omega} \left[1 - \exp\left\{(-0.500 \text{ s})\left(\frac{100. \Omega}{3.00 \text{ H}}\right)\right\}\right] = 0.0900(1 - 5.78 \cdot 10^{-8}) \text{ A} = 0.0900 \text{ A.}$$

(d) After 10.0 s , the equilibrium current of 0.0900 A has long since been reached. Before the switch is opened, the currents i_1 and i_2 oppose each other in the right-most loop, as shown below.



When the switch is opened (after achieving an equilibrium current in the circuit), $i_1 = -i_2$. After opening the switch, Kirchhoff's loop rule becomes $L di/dt + iR_1 + iR_2 = 0$. With $R_1 = R_2 = R$, this expression

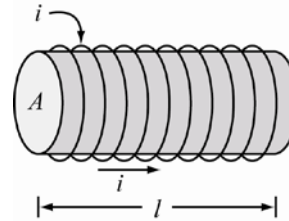
becomes $L di/dt + i(2R) = 0$. Solving for i yields: $i(t) = i_0 e^{-t/\tau_L}$, $\tau_L = L/2R$, and i_0 is the achieved equilibrium current, $i = 0.0900$ A. At $t = 0$ s, $-i_1(0) = i_2(0) = i_0 e^{-0/\tau_L} = i_0 = 0.0900$ A.

(e) At $t = 50.0$ ms = 0.0500 s, $i_1(0.0500 \text{ s}) = -i_2(0.0500 \text{ s}) = (0.0900 \text{ A}) e^{-2(100. \Omega)(0.0500 \text{ s})/3.00 \text{ H}} = 0.00321$ A.

(f) At $t = 500.$ ms = 0.500 s, $i_1(0.500 \text{ s}) = -i_2(0.500 \text{ s}) = (0.0900 \text{ A}) e^{-2(100. \Omega)(0.500 \text{ s})/3.00 \text{ H}} \approx 0$ A.

29.71. THINK: A solenoid of length, $l = 3.0$ m, and $n = 290$ turns/m has a current of $i = 3.0$ A, and stores an energy of $U_B = 2.8$ J. Find the cross-sectional area, A , of the solenoid.

SKETCH:



RESEARCH: The energy stored in the magnetic field of an ideal solenoid is $U_B = \mu_0 n^2 l A i^2 / 2$.

SIMPLIFY: Solving for A yields: $A = \frac{2U_B}{\mu_0 n^2 l i^2}$.

CALCULATE:

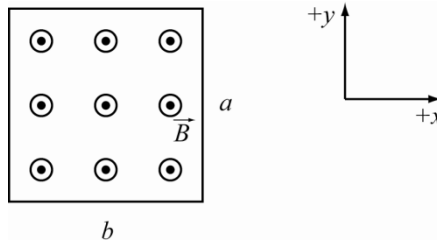
$$A = \frac{2(2.80 \text{ J})}{(4\pi \cdot 10^{-7} \text{ T m/A})(290 \text{ m}^{-1})^2 (3.00 \text{ m})(3.00 \text{ A})^2} = 1.9625 \text{ J/T A} = 1.9625 \frac{\text{N m}}{(\text{V s/m}^2)(\text{J/V s})} = 1.9625 \text{ m}^2$$

ROUND: Rounding to three significant figures, $A = 1.96 \text{ m}^2$.

DOUBLE-CHECK: Considering the length, l , of the solenoid, this is a reasonable cross-sectional area. The units of the result are also correct.

29.72. THINK: The rectangular loop has dimensions a by b and resistance R . It is placed on the xy -plane. The magnetic field direction points out of the page and varies in time according to $B = B_0(l + c_1 t^3)$. Determine the direction of the current induced in the loop, i_{ind} , and its value at $t = 1$ s (in terms of a , b , R , B_0 and c_1).

SKETCH:



RESEARCH: Since the magnetic field is increasing as it comes out of the page, the induced magnetic field, B_i , points into the page according to Lenz's law. The induced current flows clockwise. The current is found from $V_{\text{ind}} = i_{\text{ind}} R$, where $V_{\text{ind}} = -d\Phi_B / dt = -dBA \cos\theta / dt$.

SIMPLIFY: With $\cos\theta = \cos(0^\circ) = 1$, and A constant:

$$V_{\text{ind}} = -A \frac{dB}{dt} = -A \frac{d}{dt} [B_0 (l + c_1 t^3)] = -3AB_0 c_1 t^2. \text{ Then, } i_{\text{ind}} = \frac{|V_{\text{ind}}|}{R} = \frac{3AB_0 c_1 t^2}{R} = \frac{3abB_0 c_1 t^2}{R}.$$

CALCULATE: At $t = 1$ s, $i_{\text{ind}} = \frac{3abB_0c_1}{R}$, clockwise

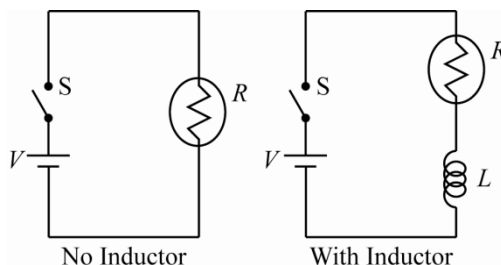
ROUND: Not applicable.

DOUBLE-CHECK: By dimensional analysis, the result has units of current:

$$\left[\frac{abB_0c_1 t^2}{R} \right] = \left[\frac{\text{m}^2 \text{T s}^2}{\text{s}^3 \Omega} \right] = \left[\frac{\text{m}^2 \text{V s/m}^2 \text{s}^2}{\text{s}^3 \text{V/A}} \right] = [\text{A}].$$

- 29.73. THINK:** The battery with $V_{\text{emf}} = 12.0$ V, is connected in series with a switch and a light-bulb. When the light-bulb draws a current of $i = 0.100$ A, its glow becomes visible. This bulb draws $P = 2.00$ W when it has been connected and when the switch has been closed for a long time. When an inductor is put in series with the bulb and the rest of the circuit, the light-bulb begins to glow $t = 3.50$ ms after the switch is closed. Find the size of the inductor, L .

SKETCH:



RESEARCH: The resistance of the light-bulb can be determined from $P = V^2 / R$. When the inductor is attached, the current is given by $i(t) = i_0 (1 - e^{-tR/L}) = \frac{V_{\text{emf}}}{R} (1 - e^{-tR/L})$.

SIMPLIFY: $R = \frac{V^2}{P} = \frac{V_{\text{emf}}^2}{P}$. Substitute this expression into the equation for the current to get:

$$i(t) = \frac{V_{\text{emf}}}{R} (1 - e^{-tR/L}) = \frac{V_{\text{emf}}}{V_{\text{emf}}^2 / P} (1 - e^{-tR/L}) = \frac{P}{V_{\text{emf}}} (1 - e^{-tR/L}) \Rightarrow e^{-tR/L} = 1 - \frac{V_{\text{emf}} i(t)}{P}$$

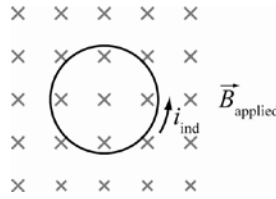
$$\Rightarrow -tR/L = \ln \left[1 - \frac{V_{\text{emf}} i(t)}{P} \right] \Rightarrow L = -\frac{tR}{\ln \left[1 - \frac{V_{\text{emf}} i(t)}{P} \right]}$$

CALCULATE: $R = \frac{(12.0 \text{ V})^2}{2.00 \text{ W}} = 72.0 \Omega$, $L = -\frac{(0.00350 \text{ s})(72.0 \Omega)}{\ln \left\{ 1 - \left[(12.0 \text{ V})(0.100 \text{ A}) / 2.00 \text{ W} \right] \right\}} = 0.27502 \text{ H}$

ROUND: $L = 0.275 \text{ H}$.

DOUBLE-CHECK: An inductor of this capacity in this circuit is capable of storing energy $U_B = \frac{1}{2} Li^2 = \frac{1}{2} (0.300 \text{ H})(0.100 \text{ A})^2 = 1.50 \text{ mJ}$. This is sufficient energy to power a 2.00 W light bulb for 0.750 ms. This is a reasonable value for L in this light-bulb circuit.

- 29.74. THINK:** A circular loop of cross-section A is placed perpendicular to a time-varying magnetic field of $B(t) = B_0 + at + bt^2$, where B_0 , a , and b are constants. For purposes of making a sketch, view the loop so that the field points into the plane of the page. Determine (a) the magnetic flux, Φ_B , through the loop at $t = 0$, (b) an equation for the induced potential difference, V_{ind} , in the loop as a function of time, and (c) the magnitude and direction of the induced current if the resistance of the loop is R .

SKETCH:

RESEARCH:

(a) Since the loop is perpendicular to the field, the magnetic flux is given by $\Phi_B = BA$.

(b) From Faraday's law, $V_{\text{ind}} = -d\Phi_B / dt$. Since A is constant while B varies with time, this expression becomes $V_{\text{ind}} = -A(dB / dt)$.

(c) The magnitude of the induced current is found from $V = iR$. With the applied magnetic field directed into the page and increasing in time, the induced magnetic field will point out of the page to oppose the change in magnetic flux. The induced current flows counterclockwise.

SIMPLIFY:

(a) $\Phi_B(t) = BA = (B_0 + at + bt^2)A$. At $t = 0$, $\Phi_B(0) = B_0A$.

(b) $V_{\text{ind}}(t) = -A \frac{d}{dt}(B_0 + at + bt^2) = -A(a + 2bt)$

(c) The magnitude of i_{ind} is given by: $i_{\text{ind}} = \left| \frac{V_{\text{ind}}}{R} \right| = \frac{A(a + 2bt)}{R}$, counterclockwise.

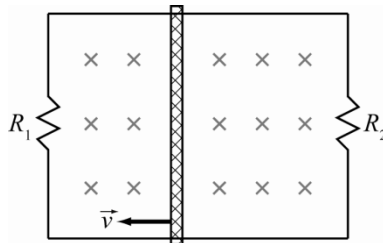
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: By dimensional analysis, the units are correct:

$$[\Phi] = [BA] \Rightarrow \text{Wb} = \text{T m}^2; [V] = [AB/t] \Rightarrow \text{V} = \text{m}^2 \text{T/s}; [i] = [V/R] \Rightarrow [A] = [V/ \Omega]$$

- 29.75. **THINK:** A conducting rod of length, $L = 0.500$ m, slides over a frame of two metal bars placed in a magnetic field of strength, $B = 1000$ gauss = 0.1000 T. The ends of the rods are connected by two resistors, $R_1 = 100. \Omega$ and $R_2 = 200. \Omega$. The conducting rod moves with a constant velocity of $v = 8.00$ m/s. Determine (a) the current flowing through the two resistors, i_1 and i_2 , (b) the power, P , delivered to the resistors, and (c) the force, F , needed for the motion of the rod with constant velocity.

SKETCH:

RESEARCH:

(a) The induced potential difference across the resistors is $V_{\text{ind}} = -d\Phi_B / dt$. Since B is constant while A varies in time at a velocity of v , this expression becomes $V_{\text{ind}} = -B(dA / dt) = -BLv$. The current in each resistor can be determined from $V_{\text{ind}} = i_{\text{ind}}R$.

(b) The power delivered to the resistors is $P = i_1^2 R_1 + i_2^2 R_2$.

(c) The force needed to move the rod with a constant velocity is obtained by calculating the total force acting on the rod. The magnetic force on the rod, F_{mag} , is given by $F_{\text{mag}} = BiL = B(V / R_{\text{eq}})L$, where R_{eq} is the equivalent resistance.

Note for n resistors in parallel, the equivalent resistance is:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

SIMPLIFY:

$$(a) V_{\text{ind}} = -BLv, \quad i_1 = \frac{|V_{\text{ind}}|}{R_1}, \quad i_2 = \frac{|V_{\text{ind}}|}{R_2}$$

$$(b) P = i_1^2 R_1 + i_2^2 R_2$$

$$(c) F_{\text{mag}} = BV_{\text{ind}}L \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = B^2 L^2 v \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

CALCULATE:

$$(a) V_{\text{ind}} = -(0.100 \text{ T})(0.500 \text{ m})(8.00 \text{ m/s}) = -0.400 \text{ V}, \quad i_1 = \frac{|-0.400 \text{ V}|}{100. \Omega} = 0.00400 \text{ A},$$

$$i_2 = \frac{|-0.400 \text{ V}|}{200. \Omega} = 0.00200 \text{ A}$$

$$(b) P = (0.00400 \text{ A})^2 (100. \Omega) + (0.00200 \text{ A})^2 (200. \Omega) = 0.00240 \text{ W}$$

$$(c) F_{\text{mag}} = (0.100 \text{ T})^2 (0.500 \text{ m})^2 (8.00 \text{ m/s}) \left(\frac{1}{100. \Omega} + \frac{1}{200. \Omega} \right) = 0.000300 \text{ N}$$

ROUND:

$$(a) i_1 = 4.00 \text{ mA}, \quad i_2 = 2.00 \text{ mA}$$

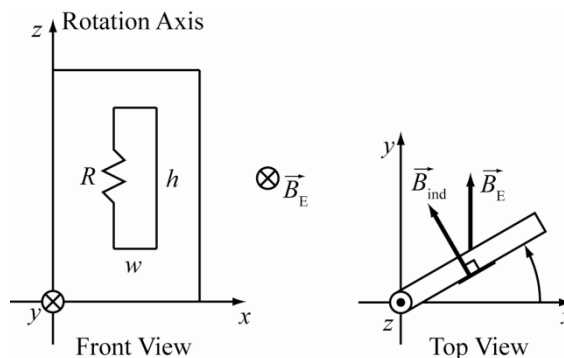
$$(b) P = 2.40 \text{ mW}$$

$$(c) F_{\text{mag}} = 0.300 \text{ mN}$$

DOUBLE-CHECK: The calculated values are consistent with the given values. Dimensional analysis confirms all the units are correct.

- 29.76. THINK:** The loop on the door has dimensions $h = 0.150 \text{ m}$, $w = 0.0800 \text{ m}$ and resistance, $R = 5.00 \Omega$. When the door is closed, it is perpendicular ($\theta = 0^\circ$) to the Earth's uniform magnetic field, $B_E = 2.6 \cdot 10^{-5} \text{ T}$. At time, $t = 0 \text{ s}$, the door is opened (right edge moving into the page in the figure below) at a constant rate, with an opening angle of $\theta(t) = \omega t$, where $\omega = 3.5 \text{ rad/s}$. Determine the direction and magnitude of the induced current, $i(t)$, at $t = 0.200 \text{ s}$.

SKETCH:



RESEARCH: The induced current, i , is found from $i = V_{\text{ind}} / R$, where V_{ind} is given by $V_{\text{ind}} = -d\Phi_B / dt$, and $\Phi_B = BA \cos\theta$. As the door opens, the B field through the loop decreases; by Lenz's law the induced B field points into the page, at an angle of $\theta(t)$ from the plane of the page. The induced current flows clockwise.

SIMPLIFY: $\Phi_B = BA \cos(\theta(t)) = whB_E \cos \omega t$, $V_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(whB_E \cos \omega t) = whB_E \omega \sin \omega t$

The magnitude of i is $i(t) = \frac{|V_{\text{ind}}|}{R} = \frac{whB_E \omega \sin \omega t}{R}$.

CALCULATE: At $t = 0.200$ s:

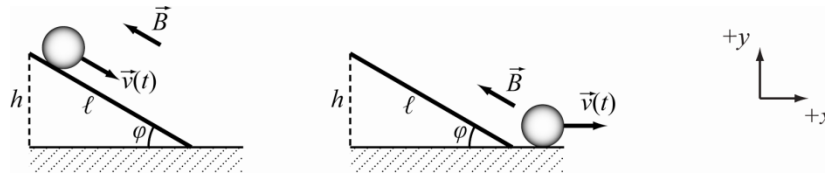
$$i(0.200 \text{ s}) = \frac{(0.0800 \text{ m})(0.150 \text{ m})(2.6 \cdot 10^{-5} \text{ T})(3.5 \text{ rad/s}) \sin[(3.5 \text{ rad/s})(0.200 \text{ s})]}{5.00 \Omega} = 1.407 \cdot 10^{-7} \text{ A.}$$

ROUND: Rounding to two significant figures, $i(0.200 \text{ s}) = 140 \text{ nA}$ clockwise.

DOUBLE-CHECK: This induced current is reasonable for a loop with such a small cross-sectional area in the Earth's magnetic field.

- 29.77. **THINK:** The steel cylinder has radius $r = 2.5 \text{ cm} = 0.025 \text{ m}$ and length $L = 10.0 \text{ cm} = 0.100 \text{ m}$. The ramp is inclined at $\phi = 15.0^\circ$ and has a length $l = 3.00 \text{ m}$. Determine the induced potential difference, V_{ind} between the ends at the bottom of the ramp if the ramp points in the direction of magnetic North. Use $0.426 \cdot 10^{-4} \text{ T}$ as the magnetic field of the Earth.

SKETCH:



RESEARCH: The magnetic field of the Earth lies at an angle to the surface of the Earth (i.e. it is usually not parallel or perpendicular to the Earth's surface). Generally, as a charge q moves with velocity through a magnetic field, the magnetic force acting on the electrons in the conductor is $F_{\text{mag}} = q|\vec{v} \times \vec{B}| = qvB \sin \theta$,

where θ is the angle between the velocity \vec{v} of the charge and the magnetic field vector \vec{B} . When the electric force, $F_E = qE$, and the magnetic force on the electrons are in equilibrium, then $E = vB \sin \theta$. This means that the induced potential difference, V_{ind} , between the ends of the conductor is given by $V_{\text{ind}} = EL = vBL \sin \theta$. As the cylinder rolls down the ramp (in the direction of the Earth's magnetic field, B), the angle between the cylinder's velocity vector and the Earth's magnetic field vector is zero, so the induced voltage between the ends of the cylinder is zero. At the bottom of the ramp, the cylinder changes direction, and the induced potential difference between the ends is $V_{\text{ind}} = vBL \sin \theta$. To determine the speed, v , of the cylinder, recall that the cylinder rolls without slipping so the change in potential energy for the cylinder is equal in magnitude to the change in its kinetic energy: $\Delta K = -\Delta U \Rightarrow mv^2/2 + I\omega^2/2 = mgh$. Here I is the moment of inertia for the cylinder: $I = mr^2/2$, and $\omega = v/r$.

SIMPLIFY: Determine v :

$$\begin{aligned} \Delta K = -\Delta U &\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{v^2}{r^2} \\ &\Rightarrow \frac{1}{2}v^2 + \frac{1}{4}v^2 = gh \Rightarrow v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}gl \sin \phi} \end{aligned}$$

At the bottom of the ramp, the angle between \vec{B} and \vec{v} is $\theta = \phi$.

$$V_{\text{ind}} = vBL \sin \theta = \sqrt{\frac{4}{3}gl \sin \phi} [BL \sin(\phi)]$$

CALCULATE: $V_{\text{ind}} = \sqrt{\frac{4}{3}(9.81 \text{ m/s}^2)(3.00 \text{ m}) \sin 15.0^\circ} [(0.426 \cdot 10^{-4} \text{ T})(0.100 \text{ m}) \sin(15^\circ)] = 3.514 \cdot 10^{-6} \text{ V}$

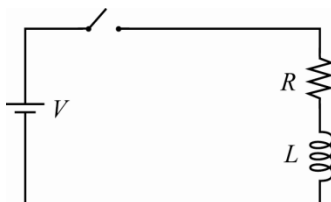
ROUND: $V_{\text{ind}} = 3.51 \mu\text{V}$.

DOUBLE-CHECK: Considering the given values for this problem, this result is reasonable and also has

the correct units: $V_{\text{ind}} = \sqrt{[\text{m/s}^2][\text{m}][\text{T}][\text{m}]} \Rightarrow \sqrt{\frac{[\text{m}][\text{m}]}{[\text{s}^2]}} \cdot \frac{[\text{V}][\text{s}][\text{m}]}{[\text{m}^2]} \Rightarrow [\text{V}]$.

- 29.78. THINK:** The battery is connected to a resistor and an inductor in series. Determine (a) the current, $i(t)$, across the circuit after the switch is closed, (b) the total energy, U , provided by the battery from time, $t = 0$ to $t = L/R$, (c) the total energy, U_R , dissipated from the resistor, R , for the same time period, and (d) discuss the conservation of energy in this circuit.

SKETCH:



RESEARCH:

(a) In this RL circuit, current at any given time, t , is given by equation 29.29 in the text, namely $i = i_0(1 - e^{-t/\tau})$, where $i_0 = V/R$, and the time constant is $\tau = L/R$.

(b) The power provided by the battery is $P = Vi$. In the given time period, the total energy provided by the battery is $U = \int Vidt$.

(c) The power dissipated in the resistor is $P = i^2R$. In the given time period, the total energy dissipated in the resistor is $U_R = \int i^2Rdt$.

(d) Any discrepancy between the energy provided by the battery and the energy dissipated in the resistor is due to the fact that there is energy stored in the inductor, $U_L = Li^2/2$.

SIMPLIFY:

$$(a) \quad i(t) = \frac{V}{R}(1 - e^{-tR/L})$$

$$(b) \quad U = \int_{t=0}^{t=\tau} Vi(t)dt = \int_0^\tau \frac{V^2}{R}(1 - e^{-tR/L})dt = \frac{V^2}{R} [t]_0^\tau + \frac{V^2}{R} \left(\frac{L}{R}\right) [e^{-tR/L}]_0^\tau = \frac{V^2}{R} \tau + \frac{V^2L}{R^2} e^{-\tau R/L} - \frac{V^2L}{R^2} = \frac{V^2L}{R^2} e^{-1} = \frac{V^2L}{R^2} (0.368)$$

$$(c) \quad U_R = \int_{t=0}^{t=\tau} i^2Rdt = \int_0^\tau \frac{V^2}{R}(1 - e^{-tR/L})^2 dt = -\frac{V^2L}{2R^2} [e^{-tR/L}]_0^\tau + \left[\frac{2V^2Le^{-tR/L}}{R^2} \right]_0^\tau - \frac{V^2L}{R^2} [\ln(e^{-tR/L})]_0^\tau = \frac{V^2L}{R^2} \left(-\frac{e^{-2}}{2} + \frac{1}{2} + 2e^{-1} - 2 + 1 + 0 \right) = 0.168 \frac{V^2L}{R^2}$$

$$(d) \quad \text{At time, } t = \tau = L/R: \quad U_L = \frac{1}{2} Li^2 \tau = \frac{LV^2}{2R^2} (1 - e^{-1})^2 = (0.200) \frac{V^2L}{R^2}.$$

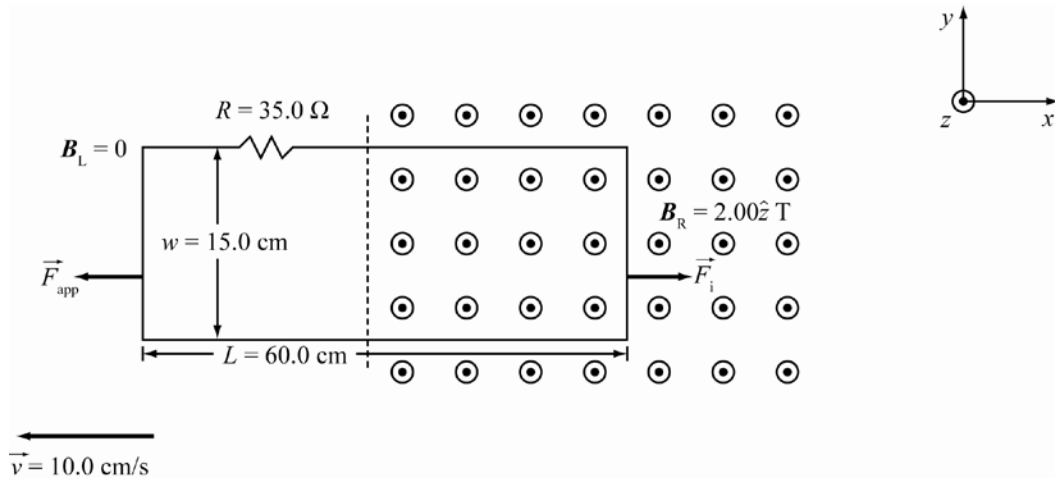
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: The total energy of the battery is the sum of the energy dissipated in the resistor and the energy stored in the inductor; energy is conserved.

- 29.79. **THINK:** The rectangular circuit loop has length, $L = 0.600$ m, and width, $w = 0.150$ m, with resistance, $R = 35.0 \Omega$. It is held parallel to the xy -plane with one end inside a uniform magnetic field as shown in the figure. The magnetic field is $\vec{B}_R = 2.00\hat{z}$ T along the positive z -axis to the right of the dotted line; $\vec{B}_L = 0$ T to the left of the dotted line. Determine the magnitude of the force, F_{app} , required to move the loop to the left at a constant speed of $v = 0.100$ m/s, while the right end of the loop is still in the magnetic field. Determine the power, P , used by an agent to pull the loop out of the magnetic field at this speed, and the power, P_R , dissipated by the resistor.

SKETCH:



RESEARCH: The magnitude of the force required to move the loop will be equal to the magnitude of the force, F_i , on the current induced in the segment of the loop that lies along the y -axis in the magnetic field. That is, $F_{\text{app}} = F_2 = iwB\sin\theta$. Since the angle, θ , between the loop segment of length, w , and the magnetic field is 90° : $\sin\theta = 1$. The induced current, i , is $i = V_{\text{ind}} / R$, where $V_{\text{ind}} = vwB$ (see equation 29.15). The power, P , used by an agent to pull the loop out of the magnetic field is given by $P = F_{\text{app}}v$. The power dissipated by the resistor is given by $P_R = i^2R$.

SIMPLIFY: The current is $i = \frac{V_{\text{ind}}}{R} = \frac{vwB}{R}$.

(a) $F_{\text{app}} = iwB$

(b) $P = F_{\text{app}}v$

(c) $P_R = i^2R$

CALCULATE:

(a) $i = \frac{(0.100 \text{ m/s})(0.150 \text{ m})(2.00 \text{ T})}{35.0 \Omega} = 0.85714 \text{ mA}$

$F_{\text{app}} = (0.85714 \text{ mA})(0.150 \text{ m})(2.00 \text{ T}) = 0.25714 \text{ mN}$

(b) $P = (0.25714 \text{ mN})(0.100 \text{ m/s}) = 25.714 \mu\text{W}$

(c) $P_R = (0.85714 \text{ mA})^2 (35.0 \Omega) = 25.714 \mu\text{W}$

ROUND:

(a) $F_{\text{app}} = 0.257 \text{ mN}$

(b) $P = 25.8 \mu\text{W}$

(c) $P_R = 25.7 \mu\text{W}$

DOUBLE-CHECK: All the power used to move the loop while in the magnetic field is dissipated in the resistor: $P = P_R$.

Multi-Version Exercises

29.80. $i(t) = i_{\max}(1 - e^{-t/\tau})$, $\tau = L/R$

$$\frac{3}{4}i_{\max} = i_{\max}(1 - e^{-t/\tau}) \Rightarrow -\frac{t}{\tau} = -\frac{tR}{L} = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow R = -\frac{L}{t} \ln\left(\frac{1}{4}\right) = \frac{L}{t} \ln(4) = \frac{33.03 \cdot 10^{-3} \text{ H}}{3.35 \cdot 10^{-3} \text{ s}} \ln(4) = 13.7 \Omega$$

29.81. $i(t) = i_{\max}(1 - e^{-t/\tau})$, $\tau = L/R$

$$\frac{3}{4}i_{\max} = i_{\max}(1 - e^{-t/\tau}) \Rightarrow -\frac{t}{\tau} = -\frac{tR}{L} = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow L = -tR / \ln\left(\frac{1}{4}\right) = tR / \ln(4) = (3.45 \cdot 10^{-3} \text{ s})(17.88 \Omega) / \ln(4) = 44.5 \text{ mH}$$

29.82. $i(t) = i_{\max}(1 - e^{-t/\tau})$, $\tau = L/R$

$$\frac{3}{4}i_{\max} = i_{\max}(1 - e^{-t/\tau}) \Rightarrow -\frac{t}{\tau} = -\frac{tR}{L} = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow t = -\frac{L}{R} \ln\left(\frac{1}{4}\right) = \frac{L}{R} \ln(4) = \frac{55.93 \cdot 10^{-3} \text{ H}}{21.84 \Omega} \ln(4) = 3.55 \text{ ms}$$

29.83. $\Phi_B = NAB \cos(2\pi ft)$, which means that $V_{\text{ind}} = -d\Phi_B / dt = NAB(2\pi f) \sin(2\pi ft)$.

The maximum occurs when $\sin(2\pi ft) = 1$. $N = 1$, so

$$V_{\max} = AB(2\pi f) = (0.25\pi d^2)B(2\pi f) = 0.5\pi^2 (0.0195 \text{ m})^2 (4.77 \cdot 10^{-5} \text{ T})(13.3 \text{ s}^{-1}) = 1.19 \cdot 10^{-6} \text{ V}.$$

29.84. $\Phi_B = NAB \cos(2\pi ft)$, which means that $V_{\text{ind}} = -d\Phi_B / dt = NAB(2\pi f) \sin(2\pi ft)$.

The maximum occurs when $\sin(2\pi ft) = 1$. $N = 1$, so

$$V_{\max} = AB(2\pi f) = (0.25\pi d^2)B(2\pi f)$$

$$\Rightarrow d = \sqrt{2V_{\max} / Bf\pi^2} = \sqrt{2(1.446 \cdot 10^{-6} \text{ V}) / (4.97 \cdot 10^{-5} \text{ T})(13.5 \text{ s}^{-1})\pi^2} = 2.09 \text{ cm}.$$

29.85. $\Phi_B = NAB \cos(2\pi ft)$, which means that $V_{\text{ind}} = -d\Phi_B / dt = NAB(2\pi f) \sin(2\pi ft)$.

The maximum occurs when $\sin(2\pi ft) = 1$. $N = 1$, so

$$V_{\max} = AB(2\pi f) = (0.25\pi d^2)B(2\pi f)$$

$$\Rightarrow B = 2V_{\max} / (f\pi^2 d^2) = 2(6.556 \cdot 10^{-7} \text{ V}) / ((13.7 \text{ s}^{-1})\pi^2 (1.63 \text{ cm})^2) = 3.65 \cdot 10^{-5} \text{ T}.$$

Chapter 30: Alternating Current Circuits

الوحدة 10 : دوائر التيار المتردد

Concept Checks

30.1. b 30.2. b 30.3. a 30.4. a 30.5. d 30.6. b 30.7. d 30.8. c

Multiple-Choice Questions

30.1. d 30.2. c 30.3. b 30.4. a 30.5. d 30.6. b 30.7. d 30.8. c 30.9. a 30.10. b

Conceptual Questions

30.11. The impedance of an RLC circuit in series is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C$. Therefore, the impedance is at its minimum value of $Z = R$.

30.12. The total energy stored in a magnetic field is the magnetic energy density times the volume of the field, $U_B = u_B V$, where $u_B = B^2 / (2\mu_0)$ (Equation 29.35). For the $d = 5.00$ km thick shell above the Earth's surface, the volume is, $V = (4/3)\pi(R_E + d)^3 - (4/3)\pi R_E^3$, where $R_E = 6378$ km is the Earth's radius. The total energy stored in this field above the Earth's surface is

$$U_B = \frac{4\pi}{6} \frac{B^2}{\mu_0} \left((R_E + d)^3 - R_E^3 \right) = \frac{2\pi}{3} \left(\frac{(0.500 \cdot 10^{-4} \text{ T})^2}{(4\pi \cdot 10^{-7} \text{ T m/A})} \right) \left((6.383 \cdot 10^6 \text{ m})^3 - (6.378 \cdot 10^6 \text{ m})^3 \right) = 2.54 \cdot 10^{12} \text{ kJ}.$$

30.13. No, charges are not crossing the gap (dielectric) of the capacitor. It simply means that, because of the periodic change in the polarity of the emf source, the capacitor is being periodically charged and discharged. No charge crosses the gap of the capacitor, whether in a DC or AC circuit. (This presumes that the potential on the capacitor does not get so big that the electric field exceeds the dielectric strength.)

30.14. In an RL circuit with alternating current, the expression "the current lags behind the voltage" means that the current achieves its maximum value at a delayed time compared to the time when the applied voltage achieves its maximum value. This is due to the phenomenon of self-induction: the changing current through the coil of the inductor creates a changing magnetic flux through the coil. Faraday's law will result in an induced emf in the coil, which will oppose the externally applied emf, in compliance with Lenz's law. The net effect is that the current will always try to "catch up" with the applied voltage, but will "lag behind" because of the self induced emf of the inductor.

30.15. The voltages given in the problem are root-mean-square values. Of course, Kirchhoff's rules will be obeyed at any instant of time, but not when using root-mean-square values since the voltages are out of phase with each other. This circuit does not violate Kirchhoff's rules.

30.16. The power depends upon the voltage. For an AC circuit, the voltage oscillate about zero, so the average voltage is zero. Thus, the average power for any AC circuit would be zero regardless of amplitude, which would not be very informative as an average value. Hence, RMS power is used instead.

30.17. Each device has its own specific operating current and voltage. Each needs its own transformer with a specific ratio of primary to secondary coils to convert the normal household current and voltage into the required current and voltage in order to prevent damage to the device.

- 30.18.** When electrons arrive at one plate of the capacitor, they repel an equal number of electrons off the opposite plate. For this reason, the amount of current flowing into one plate of the capacitor is equal to the amount of current flowing out of the other plate, despite the fact that charge does not actually flow across the gap.
- 30.19.** Any surrounding high-frequency electromagnetic waves can induce unwanted signals. In parallel pairs, noise sources may affect one wire more than the other and this can be disruptive. Twisting the pairs minimizes this effect, since for each half twist the wire nearest to the noise source is exchanged.
- 30.20.** (a) In this circuit capacitive reactance can be neglected since there is no capacitor. The root-mean-square input voltage V_{rms} and the root-mean-square current I_{rms} are related by $I_{\text{rms}} = V_{\text{rms}} / Z = V_{\text{rms}} / (R^2 + \omega^2 L^2)^{1/2}$, where R is the resistance in the circuit and L is the inductance of the solenoid. The effect of the ferromagnetic core is to increase the inductance of the solenoid by the factor κ_m . If the inductive reactance ωL is substantially greater than the resistance R , inserting the core will greatly increase the total impedance. Thus, the root-mean-square current will decrease.
 (b) With a DC power source ($\omega = 0$), the current would have fluctuated as the magnet was being inserted due to induction. Once the insertion was complete the current would return to its original value since the resistance of the circuit is unaffected by the presence of the core.
- 30.21.** The response of the tuner at any frequency is related to the amplitude of the input signal and how close to resonance the input signal is. By design the response should be dominated by the signal that is at resonance. However, if the signal at some other non-resonant frequency is large and somewhat close to the resonant frequency, noticeable crosstalk can occur.
- 30.22.** A sine or cosine signal is termed “pure” or “monochromatic”; it consists of a single frequency. Any other periodic signal is a superposition of harmonics. For simplicity, consider a square wave $S(t)$ which takes the value $+1$ (“on”) for $0 \leq t < (T/2)$ and -1 (“off”) for $(T/2) \leq t < T$, repeating periodically for all time, t , in both directions. As this is an odd function, it can be written as a sum of sine functions of period T : $S(t) = \sum_{n=1}^{\infty} b_n \sin((2\pi n/T)t)$. From Fourier analysis (see below), the coefficients b_n can be determined by multiplying both sides by $\sin((2\pi k/T)t)$ and integrating from $t=0$ to $t=T$.

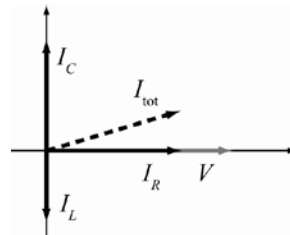
$$\begin{aligned}
 b_k &= \frac{1}{T/2} \int_0^T S(t) \sin\left(\frac{2\pi k}{T}t\right) dt \\
 &= \frac{2}{T} \left(\int_0^{T/2} (+1) \sin\left(\frac{2\pi k}{T}t\right) dt - \int_{T/2}^T (-1) \sin\left(\frac{2\pi k}{T}t\right) dt \right) \\
 &= \frac{2}{T} \left[\frac{-T}{2\pi k} \cos\left(\frac{2\pi k}{T}t\right) \right]_{t=0}^{t=T/2} - \frac{2}{T} \left[\frac{-T}{2\pi k} \cos\left(\frac{2\pi k}{T}t\right) \right]_{t=T/2}^{t=T} \\
 &= \frac{1}{\pi k} \left((-\cos(\pi k)) - (-\cos(0)) \right) + \left(\cos(2\pi k) - \cos(\pi k) \right) \\
 &= \frac{1}{\pi k} (2 - 2\cos(\pi k)) \\
 &= \begin{cases} \frac{4}{\pi k} & \text{if } k \text{ is odd;} \\ 0 & \text{if } k \text{ is even.} \end{cases}
 \end{aligned}$$

Hence, this square wave can be written as $S(t) = \frac{4}{\pi} \sum_{j=0}^{\infty} \frac{1}{2j+1} \sin\left(\frac{2\pi(2j+1)}{T}t\right)$, where the harmonic frequencies are all odd multiples of the fundamental frequency $2\pi/T$. *Fourier analysis* is a very important

mathematical tool, named for Jean Baptist Joseph Fourier (1768-1830), with a long and curious history. If such a square wave is applied as the driving voltage of an RLC circuit, the response of the circuit will be the sum of the responses to the fundamental ($j=0$) term. But if the frequency of the square wave is any odd submultiple (one-third, one-fifth, etc.) of the resonant frequency, the square wave will contain a harmonic at the resonant frequency and the circuit will resonate in response to that term. The frequency response curve of the circuit with square-wave input will contain a sequence of resonance peaks, at its resonant frequency and all odd submultiples of it.

- 30.23.** Yes, it is possible to have the voltage amplitude across the inductor exceed the voltage amplitude of the voltage supply. Since the voltage across each component is related by $V_m^2 = \sqrt{(V_R)^2 + (V_L - V_C)^2}$, the voltage across the battery is equal to the voltage across the resistor at resonance. Therefore, at resonance, the voltage across the inductor could be anything since it is countered by the same voltage across the capacitor.
- 30.24.** The transformer works on the principle of mutual inductance, and depends on the (back) emf that is generated in the set of two coils in the transformer due to changing magnetic flux within the coils. If the current is DC, then there is no change in flux, and therefore, no possibility of operating the two coils as a transformer.
- 30.25.** There can be no non-zero steady state current through the circuit. The current will vary until the capacitor is fully charged. Since the battery remains in the circuit, the current is not allowed to oscillate as it would in an LC circuit. The current will rise to a maximum and fall to zero eventually, where a steady state is achieved.
- 30.26.** **THINK:** In this RLC circuit, L , C and R are connected in parallel, as shown in the provided figure. The circuit is connected to an AC source providing V_{rms} at frequency f . A phasor diagram can be used to find an expression for I_{rms} in terms of V_{rms} , f , L , R and C .

SKETCH: Phasor diagram for current across each component in the circuit:



I_R is in phase with V , and I_C leads I_L .

RESEARCH: Since this is a parallel circuit, the voltage V_{rms} is the same across all components. The current, however, has different phases and amplitudes within each component, as shown in the above phasor diagram. By a similar analysis to determining the voltage in a series RLC circuit, the phasor diagram above shows that the current is:

$$I_{tot} = I_{rms} = \sqrt{I_R^2 + (I_L - I_C)^2}.$$

Since V_{rms} is the same in each component, $V_{rms} = I_L X_L = I_C X_C = I_R R$, where $X_L = \omega L$, $X_C = 1/\omega C$ and $\omega = 2\pi f$.

SIMPLIFY:
$$I_{rms} = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{\frac{V_{rms}^2}{R^2} + \left(\frac{V_{rms}}{X_L} - \frac{V_{rms}}{X_C}\right)^2} = V_{rms} \sqrt{\frac{1}{R^2} + \left(\frac{1}{2\pi fL} - 2\pi fC\right)^2}$$

CALCULATE: Not required.

ROUND: Not required.

DOUBLE-CHECK: By dimensional analysis of the above expression, the units are correct:

$$[I_{\text{rms}}] = V \left(\frac{1}{\Omega^2} + \left(\frac{\text{s}}{\text{H}} \frac{\text{F}}{\Omega \text{s}} \right)^2 \right)^{1/2} = V \left(\frac{1}{\Omega^2} + \left(\frac{1}{\Omega} - \frac{1}{\Omega} \right)^2 \right)^{1/2} = \frac{V}{\Omega} = \text{A}.$$

Exercises

- 30.27.** From the inductance and capacitance, $L = 32.0 \text{ mH}$ and $C = 45.0 \mu\text{F}$, the frequency of oscillation is $\omega_0 = (LC)^{-1/2}$. The total energy is constant at $U = q_0^2 / 2C$ where $q_0 = 10.0 \mu\text{C}$, and the charge varies as $q = q_0 \cos(\omega_0 t)$. Since energy remains constant, when the energy in both is the same, it is $(1/2)U$.

$$U_E = \frac{1}{2}U \Rightarrow \frac{q_0^2 \cos^2(\omega_0 t)}{2C} = \frac{1}{2} \left(\frac{q_0^2}{2C} \right) \Rightarrow \cos^2(\omega_0 t) = \frac{1}{2} \Rightarrow t = \frac{1}{\omega_0} \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) \\ \Rightarrow t = \sqrt{LC} \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = \sqrt{(32.0 \text{ mH})(45.0 \mu\text{F})} \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 0.4210 \text{ s}$$

Note that the result does not depend on the original amount of charge, q_0 . Dimensional analysis shows

that the result has the correct units: $\sqrt{\left[\frac{\text{s}^2}{\text{F}} \right]} \cdot [\text{F}] = [\text{s}]$.

- 30.28.** (a) From conservation of energy, $U_E = U_B \Rightarrow q_{\text{max}}^2 / (2C) = Li_{\text{max}}^2 / 2$, where $q_{\text{max}} = CV$, with $C = 2.00 \mu\text{F}$, $L = 0.250 \text{ H}$ and $V = 12.0 \text{ V}$. Therefore,

$$\frac{q_{\text{max}}^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2} CV^2 = \frac{Li_{\text{max}}^2}{2} \Rightarrow i_{\text{max}} = \sqrt{\frac{C}{L}} V = \sqrt{\frac{2.00 \mu\text{F}}{0.250 \text{ H}}} (12.0 \text{ V}) = 33.9 \text{ mA}.$$

- (b) The angular frequency of the current is $\omega_0 = (LC)^{-1/2}$, so that the frequency of the oscillation is $f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.250 \text{ H})(2.00 \mu\text{F})}} = 225 \text{ Hz}$.

- 30.29.** In general, the frequency of oscillation is $\omega_0 = (LC)^{-1/2}$, where $L = 1.00 \text{ mH}$. The maximum energy in the capacitor is $U_E = q_{\text{max}}^2 / (2C)$. Since the charge varies as $q = q_{\text{max}} \cos(\omega_0 t)$, and at time $t = 2.10 \text{ ms}$ the energy on capacitor is half the maximum value, $U_E = (1/2)U$. This means

$$U_E = \frac{1}{2}U \Rightarrow \frac{q_{\text{max}}^2 \cos^2(\omega_0 t)}{2C} = \frac{1}{2} \left(\frac{q_{\text{max}}^2}{2C} \right) \Rightarrow \cos^2(\omega_0 t) = \frac{1}{2} \Rightarrow \omega_0 = \left(\frac{1}{t} \right) \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) \\ \Rightarrow \frac{1}{\sqrt{LC}} = \frac{\cos^{-1}(1/\sqrt{2})}{t} \Rightarrow C = \frac{1}{L} \left[\frac{t}{\cos^{-1}(1/\sqrt{2})} \right]^2 = \frac{1}{(1.00 \cdot 10^{-3} \text{ H})} \left(\frac{2.10 \cdot 10^{-3} \text{ s}}{\cos^{-1}(1/\sqrt{2})} \right)^2 = 7.15 \text{ mF}.$$

- 30.30.** (a) Since the current is proportional to $\sin(\omega_0 t)$ where $\omega_0 = 1200. \text{ s}^{-1}$, this is at a maximum when $\omega_0 t = \pi / 2$; therefore, $t = \pi / (2\omega_0) = \pi / (2(1200. \text{ s}^{-1})) = 1.31 \text{ ms}$.

- (b) The total energy in the circuit is $U = Li_{\text{max}}^2 / 2$, where $i_{\text{max}} = 1.00 \text{ A}$. Since the angular frequency is $\omega_0 = (LC)^{-1/2}$, then $L = 1 / (\omega_0^2 C)$, where $C = 10.0 \mu\text{F}$. The total energy of the circuit is then

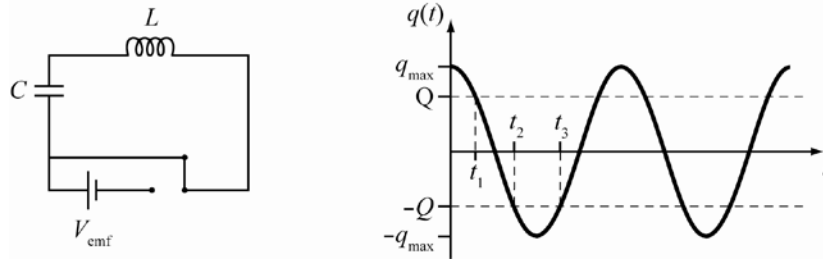
$$U = \frac{Li_{\text{max}}^2}{2} = \frac{i_{\text{max}}^2}{2\omega_0^2 C} = \frac{(1.00 \text{ A})^2}{2(1200. \text{ s}^{-1})^2 (10.0 \mu\text{F})} \approx 34.7 \text{ mJ}.$$

(c) The inductance is

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(1200. \text{ s}^{-1})^2 (10.0 \text{ } \mu\text{F})} = 69.4 \text{ mH.}$$

- 30.31. THINK:** The charge on the capacitor will oscillate with time as a cosine function with a period determined by the inductance, $L = 0.200 \text{ H}$, and capacitance, $C = 10.0 \text{ } \mu\text{F}$. The potential, $V_{\text{emf}} = 12.0 \text{ V}$, will give the initial charge on the capacitor. Ignoring the sign of the charge, the charge on the capacitor will equal $Q = 80.0 \text{ } \mu\text{C}$ periodically.

SKETCH:



RESEARCH: The initial (and maximum) charge on the capacitor is $q_{\text{max}} = CV$. The charge will oscillate as $q = q_{\text{max}} \cos(\omega_0 t)$, where $\omega_0 = 1/\sqrt{LC}$.

SIMPLIFY: The first time t_1 when the charge on the capacitor is equal to Q is

$$q = Q = q_{\text{max}} \cos(\omega_0 t_1) \Rightarrow t_1 = \left(\frac{1}{\omega_0} \right) \cos^{-1} \left(\frac{Q}{q_{\text{max}}} \right);$$

By symmetry, the second time t_2 and the third time t_3 are given by

$$\omega_0 t_2 = \pi - \omega_0 t_1 = \omega_0 \left(\frac{\pi}{\omega_0} - t_1 \right) \Rightarrow t_2 = \frac{\pi}{\omega_0} - t_1 \text{ and}$$

$$\omega_0 t_3 = \pi + \omega_0 t_1 = \omega_0 \left(\frac{\pi}{\omega_0} + t_1 \right) \Rightarrow t_3 = \frac{\pi}{\omega_0} + t_1.$$

CALCULATE: $\omega_0 = \frac{1}{\sqrt{(0.200 \text{ H})(10.0 \text{ } \mu\text{F})}} = 707.107 \text{ rad/s}$

$$q_{\text{max}} = (10.0 \text{ } \mu\text{F})(12.0 \text{ V}) = 120. \text{ } \mu\text{C}$$

$$t_1 = \left(\frac{1}{(707.107 \text{ rad/s})} \right) \cos^{-1} \left(\frac{80.0 \text{ } \mu\text{C}}{120. \text{ } \mu\text{C}} \right) = 0.0011895 \text{ s}$$

$$t_2 = \frac{\pi}{(707.107 \text{ rad/s})} - 0.0011895 \text{ s} = 0.0032534 \text{ s}$$

$$t_3 = \frac{\pi}{(707.107 \text{ rad/s})} + 0.0011895 \text{ s} = 0.0056323 \text{ s}$$

ROUND: Rounding the times to three significant figures gives: $t_1 = 1.19 \text{ ms}$, $t_2 = 3.25 \text{ ms}$, and $t_3 = 5.63 \text{ ms}$.

DOUBLE-CHECK: Given the high frequency, small times are expected, so the answers are reasonable.

- 30.32. THINK:** The current, $i_{\text{max}} = 3.00 \text{ A}$, will oscillate with time as a sine function with a period determined by the inductance, $L = 7.00 \text{ mH}$, and capacitance, $C = 4.00 \text{ mF}$. The charge will also vary with the same period as the current, but as a cosine function. The maximum charge on the capacitor is related to the energy of the system.

SKETCH: Not required.

RESEARCH: The total energy in circuit is found either when the current is at a maximum, $E = (1/2)Li_{\max}^2$, or when charge on capacitor is a maximum, $E = q_{\max}^2 / (2C)$. The charge varies as $q = q_{\max} \cos(\omega_0 t)$, where $\omega_0 = 1/\sqrt{LC}$.

SIMPLIFY:

(a) The energy in the circuit is $U = \frac{1}{2}Li_{\max}^2$.

(b) The maximum charge on the capacitors is found by $U = \frac{q_{\max}^2}{2C} \Rightarrow q_{\max} = \sqrt{2CU} = i_{\max} \sqrt{LC}$. Therefore,

the expression for charge is $q = i_{\max} \sqrt{LC} \cos\left(\frac{t}{\sqrt{LC}}\right)$.

CALCULATE:

(a) $U = \frac{1}{2}(7.00 \text{ mH})(3.00 \text{ A})^2 = 0.0315 \text{ J}$

(b) $q = (3.00 \text{ A})\sqrt{(7.00 \text{ mH})(4.00 \text{ mF})} \cos\left(\frac{t}{\sqrt{(7.00 \text{ mH})(4.00 \text{ mF})}}\right) = 0.01587 \cos(189.0t) \text{ C}$

ROUND:

(a) $U = 31.5 \text{ mJ}$

(b) It is best not to round the coefficients of the expression. Keep the answer as $q = 0.01587 \cos(189.0t)$, with the intention of rounding after particular values of t are substituted.

DOUBLE-CHECK: The equation for the charge has the proper units since \sqrt{LC} has units of s and i_{\max} has units of C/s.

- 30.33.** The angular frequency without the resistor is $\omega_0 = 1/\sqrt{LC}$, and with the resistor it is $\omega = \sqrt{\omega_0^2 - (R/2L)^2}$. The fractional change is

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega - \omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1 = \sqrt{1 - \left(\frac{R}{2L\omega_0}\right)^2} - 1 = \sqrt{1 - \frac{R^2C}{4L}} - 1.$$

Inserting the numbers, we find

$$\frac{\Delta\omega}{\omega_0} = \sqrt{1 - \frac{(1.00 \cdot 10^3 \text{ } \Omega)^2 (4.5 \cdot 10^{-9} \text{ F})}{4(4.00 \cdot 10^{-3} \text{ H})}} - 1 = -0.152.$$

This means that the fractional change is a drop of 15.2%.

- 30.34. THINK:** For an RLC circuit, the charge on the capacitor is described by equation 30.6. The capacitance is found by equating the term in the exponential to the period determined from the term in the cosine. The resistance and inductance are $R = 50.0 \text{ } \Omega$ and $L = 1.00 \text{ mH}$.

SKETCH: Not required.

RESEARCH: The general expression for an RLC circuit is

$$q = q_{\max} e^{-Rt/2L} \cos(\omega t),$$

where $\omega = \sqrt{\omega_0^2 - (R/2L)^2}$ and where $\omega_0 = 1/\sqrt{LC}$. The period of the oscillation is $T = 2\pi/\omega$. The decay rate of the exponential is $\tau = 2L/R$.

SIMPLIFY: Capacitance is found by letting $\tau = T$, which gives the result:

$$\frac{2L}{R} = \frac{2\pi}{\omega} \Rightarrow \left(\frac{R}{2L}\right)^2 = \left(\frac{\sqrt{\omega_0^2 - (R/2L)^2}}{2\pi}\right)^2 \Rightarrow \frac{R^2}{4L^2} = \frac{\omega_0^2 - \frac{R^2}{4L^2}}{4\pi^2} \Rightarrow (4\pi^2 + 1)\frac{R^2}{4L^2} = \omega_0^2 = \frac{1}{LC}.$$

Therefore, $C = \frac{4L}{R^2(4\pi^2 + 1)}$.

CALCULATE: $C = \frac{4(1.00 \cdot 10^{-3} \text{ H})}{(50.0 \Omega)^2(4\pi^2 + 1)} = 3.9527 \cdot 10^{-8} \text{ F}$

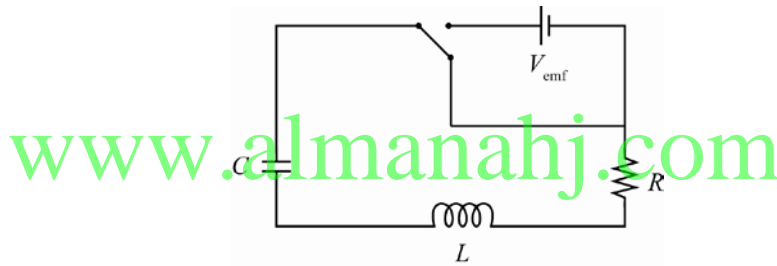
ROUND: Rounding to three significant figures, $C = 39.5 \text{ nF}$.

DOUBLE-CHECK: The time constant is $\tau = 2L/R = 4 \cdot 10^{-5} \text{ s}$. The angular frequency, ω , is

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = 1.5708 \cdot 10^5 \text{ s}^{-1},$$

and the period is $T = 2\pi/\omega = 4.00 \cdot 10^{-5} \text{ s}$. So $\tau = T$, as required.

- 30.35. THINK:** The frequency of the damped oscillation is independent of the initial charge, and hence potential. It then only depends on the inductance, $L = 0.200 \text{ H}$, the resistance, $R = 50.0 \Omega$, capacitance $C = 2.00 \mu\text{F}$.
SKETCH:



RESEARCH: In general the charge on the capacitor is

$$q = q_{\max} e^{-Rt/2L} \cos(\omega t),$$

where $\omega = \sqrt{\omega_0^2 - (R/2L)^2}$ and where $\omega_0 = 1/\sqrt{LC}$. The frequency of oscillation is $f = \omega/(2\pi)$.

SIMPLIFY: $f = \frac{\omega}{2\pi} = \frac{\sqrt{(\omega_0^2 - R/2L)^2}}{2\pi} = \frac{\sqrt{1/(LC) - R^2/(4L^2)}}{2\pi}$

CALCULATE: $f = \frac{\sqrt{1/((0.200 \text{ H})(2.00 \cdot 10^{-6} \text{ F})) - (50.0 \Omega)^2/(4(0.200 \text{ H})^2)}}{2\pi} = 250.858 \text{ Hz}$

ROUND: Rounding to three significant figures, $f = 251 \text{ Hz}$.

DOUBLE-CHECK: This frequency is typical of RLC circuits. Additionally, dimensional analysis yields:

$$\left[\frac{[\text{F}]}{[\text{s}^2]} \frac{1}{[\text{F}]} - \frac{[\Omega]^2}{[\Omega \cdot \text{s}]^2} \right]^{\frac{1}{2}} = \left[\frac{1}{[\text{s}^2]} \right]^{\frac{1}{2}} = \frac{1}{[\text{s}]}.$$

- 30.36. THINK:** The angular frequency of the RLC is 20% less than the angular frequency of the LC circuit where the inductance is $L = 4.0 \text{ mH}$ and the capacitance is $C = 2.50 \mu\text{F}$. Even though the current is actually a damped oscillation, the magnitude of the oscillation is describe simply by the exponential decay, similar to that of the charge. Therefore, the current is at half of its maximum when the exponential is at a half. The number of periods in a given time is the number of oscillations.

SKETCH: Not required.

RESEARCH: The RLC frequency is related to LC frequency by $\omega_{\text{RLC}} = 0.8\omega_{\text{LC}}$. The RLC frequency is $\omega_{\text{RLC}} = \sqrt{\omega_0^2 - (R/(2L))^2}$, where $\omega_0 = \omega_{\text{LC}} = 1/\sqrt{LC}$. The current varies as $i = i_{\text{max}} e^{-Rt/2L}$. A wave goes through n oscillations in time $t = nT$, where T is the period.

SIMPLIFY:

(a) For the oscillation frequency,

$$\begin{aligned}\omega_{\text{RLC}} = 0.8\omega_{\text{LC}} &\Rightarrow \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2} = 0.8\omega_0 \Rightarrow \omega_0^2 - \left(\frac{R}{2L}\right)^2 = 0.64\omega_0^2 \Rightarrow 0.36\omega_0^2 = \frac{R^2}{4L^2} \\ &\Rightarrow R = 1.2L\omega_0 = 1.2\left(\frac{L}{\sqrt{LC}}\right) = 1.2\sqrt{\frac{L}{C}}.\end{aligned}$$

$$(b) i = \left(\frac{1}{2}\right)i_{\text{max}} = i_{\text{max}} e^{-Rt/2L} \Rightarrow \frac{1}{2} = e^{-Rt/2L} \Rightarrow t = \frac{2L}{R} \ln 2$$

$$(c) \text{ The period is } T = \frac{2\pi}{\omega_{\text{RLC}}}. \text{ Therefore, the number of oscillations is } n = \frac{t}{T} = \frac{t\omega_{\text{RLC}}}{2\pi}.$$

CALCULATE:

$$(a) R = 1.2 \sqrt{\frac{(4.0 \cdot 10^{-3} \text{ H})}{(2.50 \cdot 10^{-6} \text{ F})}} = 48 \Omega$$

$$(b) t = \frac{2(4.0 \cdot 10^{-3} \text{ H})}{(48 \Omega)} \ln(2) = 0.000115525 \text{ s}$$

$$(c) \omega_{\text{RLC}} = \sqrt{\frac{1}{(4.0 \cdot 10^{-3} \text{ H})(2.50 \cdot 10^{-6} \text{ F})} - \left(\frac{(48 \Omega)}{2(4.0 \cdot 10^{-3} \text{ H})}\right)^2} = 8000 \text{ rad/s, and}$$

$$n = \frac{(0.000115525 \text{ s})(8000 \text{ rad/s})}{2\pi} = 0.14709 \text{ cycles.}$$

ROUND: To three significant figures,

$$(a) R = 48.0 \Omega$$

$$(b) t = 116 \mu\text{s}$$

$$(c) n = 0.147 \text{ cycles}$$

DOUBLE-CHECK: Since the resistance is large compared to the inductance, the current will die off quickly, so a complete oscillations will not occur before it is at half its maximum value.

30.37. The capacitive reactance, $X_C = 200. \Omega$, is given by $X_C = 1/(\omega C)$, where $C = 10.0 \mu\text{F}$; therefore, the frequency is

$$\omega = \frac{1}{X_C C} = \frac{1}{(200. \Omega)(10.0 \mu\text{F})} = 500. \text{ rad/s.}$$

30.38. The capacitive reactance is given by $X_C = 1/(\omega C)$, where $C = 5.00 \cdot 10^{-6} \text{ F}$ and $\omega = 2\pi f$, where $f = 100. \text{ Hz}$. Therefore, the capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(100. \text{ Hz})(5.00 \cdot 10^{-6} \text{ F})} = 318 \Omega.$$

The maximum current through the capacitor, I_C , is given by $I_C = V_C / X_C$, where $V_C = 10.0 \text{ V}$. Therefore,

$$I_C = \frac{V_C}{X_C} = \frac{(10.0 \text{ V})}{(318 \Omega)} = 31.4 \text{ mA.}$$

- 30.39. (a) The resonant angular frequency of an RLC circuit is $\omega_0 = 1/\sqrt{LC}$ where $L = 0.500$ H and $C = 0.400$ μ F. Therefore,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.400 \cdot 10^{-6} \text{ F})(0.500 \text{ H})}} = 2240 \text{ rad/s}$$

(b) At resonance, only the resistor contributes to the overall impedance, so $I = V_{\text{emf}} / R$ where $V_{\text{emf}} = 40.0$ V and $R = 100.0$ Ω . Therefore, $I = (40.0 \text{ V}) / (100.0 \text{ } \Omega) = 0.400$ A.

- 30.40. In general, the resonant frequency is determined by $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC_0}}$, when $f_0 = 5.0$ MHz and $C_0 = 15$ pF, the circuit has an inductance of L that satisfies the given relation. When $C_1 = 380$ pF, the same L will give another resonant frequency. Therefore,

$$f_0 = \frac{1}{2\pi\sqrt{LC_0}} \Rightarrow L = \frac{1}{4\pi^2 f_0^2 C_0},$$

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{\frac{C_0}{4\pi^2 f_0^2 C_1}}} = f_0 \sqrt{\frac{C_0}{C_1}} = f_1 = (5.0 \text{ MHz}) \sqrt{\frac{(15 \text{ pF})}{(380 \text{ pF})}} = 1.0 \text{ MHz}.$$

- 30.41. Given the frequency, $f = 1.00$ kHz, the angular frequency is $\omega = 2\pi f$. The phase constant for an RLC circuit is given by

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/(\omega C)}{R} \Rightarrow \phi = \tan^{-1} \left(\frac{2\pi f \cdot L - 1/(2\pi f \cdot C)}{R} \right)$$

For the values $R = 100.$ Ω , $L = 10.0$ mH and $C = 100.$ μ F, the phase constant is

$$\phi = \tan^{-1} \left(\frac{\left((2\pi)(1.00 \cdot 10^3 \text{ Hz})(10.0 \cdot 10^{-3} \text{ H}) - \left((2\pi)(1.00 \cdot 10^3 \text{ Hz})(100. \cdot 10^{-6} \text{ F}) \right)^{-1} \right)}{(100. \text{ } \Omega)} \right) = 0.549 \text{ rad}.$$

The impedance for this circuit is

$$Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2} = \sqrt{R^2 + (2\pi f \cdot L - 1/(2\pi f \cdot C))^2}$$

$$= \sqrt{(100. \text{ } \Omega)^2 + \left((2\pi)(1.00 \cdot 10^3 \text{ Hz})(10.0 \cdot 10^{-3} \text{ H}) - \left((2\pi)(1.00 \cdot 10^3 \text{ Hz})(100. \cdot 10^{-6} \text{ F}) \right)^{-1} \right)^2} = 117 \text{ } \Omega.$$

- 30.42. For an RLC circuit with $L = 5.00$ mH and $C = 4.00$ μ F, the resonant frequency, ω_0 , is $\omega_0 = 1/\sqrt{LC}$. Therefore,

$$\omega_0 = \frac{1}{\sqrt{(5.00 \cdot 10^{-3} \text{ H})(4.00 \cdot 10^{-6} \text{ F})}} = 7070 \text{ rad/s}.$$

At resonance, the resistor with $R = 1.00$ k Ω , is the only contribution to the impedance. For a peak voltage of $V_m = 10.0$ V, the maximum current is

$$I_m = \frac{V_m}{R} = \frac{(10.0 \text{ V})}{(1000. \text{ } \Omega)} = 10.0 \text{ mA}.$$

- 30.43. **THINK:** Given the equation, $V = (12.0 \text{ V})\sin(\omega t)$ the peak voltage is clearly $V_m = 12.0$ V. When the circuit is in resonance, the current is dictated solely by the resistance, $R = 10.0$ Ω . This means the inductor, $L = 2.00$ H, and the capacitor, $C = 10.0$ μ F will not influence the current. However, the current will dictate the voltage drop across each.

SKETCH: Not required.

RESEARCH: At resonance, impedance is $Z = R$. The maximum current at resonance is $I_m = V_m / R$. The resonant frequency is $\omega_0 = 1/\sqrt{LC}$. The voltage drop across the inductor is $V_L = IX_L$, where $X_L = 1/\omega_0 C$.

SIMPLIFY: The voltage drop across the inductor is,

$$V_L = IX_L = \frac{V_m}{R} \omega_0 L = \frac{V_m}{R} \frac{L}{\sqrt{LC}} = \frac{V_m}{R} \sqrt{\frac{L}{C}}.$$

CALCULATE: $V_L = \frac{(12.0 \text{ V})}{(10.0 \ \Omega)} \sqrt{\frac{(2.00 \text{ H})}{(10.0 \cdot 10^{-6} \text{ F})}} = 536.66 \text{ V}$

ROUND: To three significant figures, $V_L = 537 \text{ V}$.

DOUBLE-CHECK: Even though the answer seems large compared to V_m , the voltage drop across the capacitor is the exact same. Since the voltage across each component is related by $V_m^2 = \sqrt{(V_R)^2 + (V_L - V_C)^2}$, the voltage across the battery is equal to the voltage across the resistor at resonance. Therefore, at resonance, the voltage across the inductor could be anything since it is countered by the same voltage across the capacitor.

- 30.44. THINK:** The inductive reactance and the capacitive reactance are needed to find the impedance of the RLC circuit. The AC power source oscillates with frequency $f = 60.0 \text{ Hz}$ and has an amplitude of $V_m = 220 \text{ V}$. The resistance is $R = 50.0 \ \Omega$, the inductance is $L = 0.200 \text{ H}$ and the capacitance is $C = 0.040 \text{ mF}$.

SKETCH: Not required.

RESEARCH: The angular frequency of oscillation is $\omega = 2\pi f$. The inductive reactance is $X_L = \omega L$. The capacitive reactance is $X_C = 1/\omega C$. The impedance of circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The maximum current through the circuit is $I_m = V_m / Z$. The maximum potential drop across each component is $V_i = I_m X_i$, where i denotes either R , C or L .

SIMPLIFY:

(a) $X_L = \omega L = 2\pi fL$

(b) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

(d) $I_m = \frac{V_m}{Z}$

(e) $V_R = I_m R$, $V_C = I_m X_C$ and $V_L = I_m X_L$

CALCULATE:

(a) $X_L = 2\pi(60.0 \text{ Hz})(0.200 \text{ H}) = 75.398 \ \Omega$

(b)

(c) $Z = \sqrt{(50.0 \ \Omega)^2 + (75.398 \ \Omega - 66.315 \ \Omega)^2} = 50.818 \ \Omega$

(d)

(e) $V_R = (4.329 \text{ A})(50.0 \ \Omega) = 216.457 \text{ V}$, $V_C = (4.329 \text{ A})(66.315 \ \Omega) = 287.085 \text{ V}$ and

$V_L = (4.329 \text{ A})(75.398 \ \Omega) = 326.409 \text{ V}$.

ROUND:

(a) $X_L = 75.4 \ \Omega$

(b) $X_C = 66.3 \ \Omega$

- (c) $Z = 50.8 \Omega$
 (d) $I_m = 4.33 \text{ A}$
 (e) $V_R = 220 \text{ V}$, $V_C = 290 \text{ V}$ and $V_L = 330 \text{ V}$.

DOUBLE-CHECK: In order to satisfy Kirchoff's loop rule, the vector phasors must sum as vectors to match \vec{V}_m : $V_m^2 = V_R^2 + (V_L - V_C)^2$. Therefore,

$$V_m = \sqrt{(216.457 \text{ V})^2 + (326.409 \text{ V} - 287.085 \text{ V})^2} = 220 \text{ V},$$

as required.

- 30.45. THINK:** The maximum current occurs for when the AC voltage is at its peak, $V_m = 110 \text{ V}$. The angular frequency of the oscillation is $\omega = 377 \text{ rad/s}$. The voltage acts across the total impedance of the circuit where $R = 2.20 \Omega$, $L = 9.30 \text{ mH}$ and $C = 2.27 \text{ mF}$. The maximum current I'_m occurs for a capacitance C' that puts the RLC circuit in resonance with the supplied voltage. At resonance, the phase constant is zero and only the resistor influences the current.

SKETCH: Not required.

RESEARCH: The impedance of inductor and capacitor are $X_L = \omega L$ and $X_C = 1/\omega C$. The maximum current is $I_m = V_m / Z$, where $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The phase angle for the circuit is $\phi = \tan^{-1}((X_L - X_C)/R)$. When at resonance, $\omega = \omega_0 = 1/\sqrt{LC}$. The maximum current at resonance is $I_m = V_m / R$.

SIMPLIFY:

(a) The maximum current is $I_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_m}{\sqrt{R^2 + (\omega L - (\omega C)^{-1})^2}}$.

(b) The phase constant is $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - (\omega C)^{-1}}{R}\right)$.

(c) The required capacitance for resonance is

$$\omega = \frac{1}{\sqrt{LC'}} \Rightarrow C' = \frac{1}{\omega^2 L},$$

and maximum current at resonance is $I_m = \frac{V_m}{R}$.

CALCULATE:

(a) $I_m = \frac{(110 \text{ V})}{\sqrt{(2.20 \Omega)^2 + \left((377 \text{ rad/s})(9.30 \cdot 10^{-3} \text{ H}) - \left((377 \text{ rad/s})(2.27 \cdot 10^{-3} \text{ F})\right)^{-1}\right)^2}} = 34.2675 \text{ A}$

(b) $\phi = \tan^{-1} \frac{(377 \text{ rad/s})(9.30 \cdot 10^{-3} \text{ H}) - \left((377 \text{ rad/s})(2.27 \cdot 10^{-3} \text{ F})\right)^{-1}}{(2.20 \Omega)} = 0.8157 \text{ rad}$

(c) $C' = \frac{1}{(377 \text{ rad/s})^2 (9.30 \cdot 10^{-3} \text{ H})} = 7.565 \cdot 10^{-4} \text{ F}$ and $I'_m = \frac{(110 \text{ V})}{(2.20 \Omega)} = 50.0 \text{ A}$

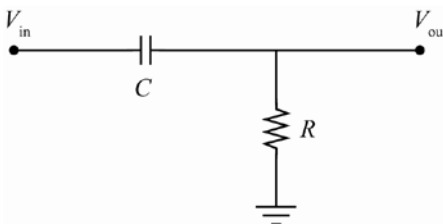
ROUND: To three significant figures:

- (a) $I_m = 34.3 \text{ A}$
 (b) $\phi = 0.816 \text{ rad}$
 (c) $C' = 757 \mu\text{F}$, $I'_m = 50.0 \text{ A}$, $\phi' = 0 \text{ rad}$

DOUBLE-CHECK: The current is at a maximum when at resonance, so having $I'_m > I_m$ is reasonable.

- 30.46. THINK:** Although not explicitly stated, since the circuit should pass a frequency of $f = 5.00$ kHz, it is natural to assume that it is a high-pass filter. The ratio of the voltages is $V_{\text{out}} / V_{\text{in}} = 1/2$.

SKETCH:



RESEARCH: The ratio of the voltages for the high-pass RC filter is given by $V_{\text{out}} / V_{\text{in}} = \left[1 + \frac{1}{\omega^2 R^2 C^2} \right]^{-\frac{1}{2}}$,

where $\omega = 2\pi f$. The phase constant for the circuit is $\phi = \tan^{-1}((X_L - X_C)/R)$, with $X_L = 0$ and $X_C = 1/(\omega C)$.

SIMPLIFY:

$$(a) \frac{V_{\text{out}}}{V_{\text{in}}} = 0.500 = \frac{1}{\sqrt{1 + 1/(\omega^2 R^2 C^2)}} \Rightarrow 1 + \frac{1}{\omega^2 R^2 C^2} = 4.00 \Rightarrow \omega^2 R^2 C^2 = \frac{1}{3.00}$$

$$\Rightarrow C = \frac{1}{\sqrt{3.00}} \frac{1}{\omega R} = \frac{1}{\sqrt{3.00}} \frac{1}{2\pi f R}$$

$$(b) \text{ The phase constant is } \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - (1/(\omega C))}{R}\right) = \tan^{-1}\left(-\frac{1}{2\pi f R C}\right).$$

CALCULATE:

$$(a) C = \frac{1}{\sqrt{3.00}} \frac{1}{2\pi(5.00 \cdot 10^3 \text{ Hz})(1.00 \cdot 10^3 \Omega)} = 1.838 \cdot 10^{-8} \text{ F}$$

$$(b) \phi = \tan^{-1}\left(-\frac{1}{2\pi(5.00 \cdot 10^3 \text{ Hz})(1.00 \cdot 10^3 \Omega)(1.838 \cdot 10^{-8} \text{ F})}\right) = -1.0472 \text{ rad}$$

ROUND:

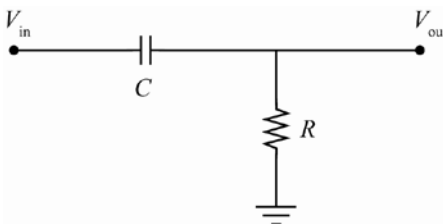
$$(a) C = 18.4 \text{ nF}$$

$$(b) \phi = -1.05 \text{ rad}$$

DOUBLE-CHECK: The negative phase constant simply means the current lags the voltage, which makes sense since the current must first reach the capacitor before it can charge it. The values for the capacitor and phase constant seem reasonable.

- 30.47. THINK:** For signals of frequency $f = 60.0$ Hz the required noise factor is $V_{\text{out}} / V_{\text{in}} = 0.00100$. The given total impedance $R = 20.0$ k Ω . Since the ratio of voltages will increase as the frequency does, the lowest frequency that has 90.0% signal strength is when $V_{\text{out}} / V_{\text{in}} = 0.900$.

SKETCH:



RESEARCH: The signal ratio is $V_{\text{out}} / V_{\text{in}} = 1 / \sqrt{1 + 1 / (\omega^2 R^2 C^2)}$, where the angular frequency is $\omega = 2\pi f$.

SIMPLIFY:

(a) When $V_{\text{out}} / V_{\text{in}} = 0.001000 = 1.00 \cdot 10^{-3}$, then

$$\frac{V_{\text{out}}}{V_{\text{in}}} = 1.00 \cdot 10^{-3} = \frac{1}{\sqrt{1 + 1 / (\omega^2 R^2 C^2)}} \Rightarrow 1 + \frac{1}{\omega^2 R^2 C^2} = 10^6 \Rightarrow \omega^2 R^2 C^2 = (10^6 - 1)^{-1}$$

$$\Rightarrow C = \frac{1}{\sqrt{(10^6 - 1)(2\pi f R)}}$$

(b) When $\frac{V_{\text{out}}}{V_{\text{in}}} = 0.900$, then

$$\frac{V_{\text{out}}}{V_{\text{in}}} = 0.900 = \frac{1}{\sqrt{1 + 1 / (\omega^2 R^2 C^2)}} \Rightarrow 1 + \frac{1}{\omega^2 R^2 C^2} = \frac{1}{0.81} \Rightarrow \omega^2 R^2 C^2 = \left(\frac{1}{0.81} - 1\right)^{-1} = \frac{81}{19}$$

$$\Rightarrow \omega = \sqrt{\frac{81}{19}} \left(\frac{1}{RC}\right) \Rightarrow f = \sqrt{\frac{81}{19}} \left(\frac{1}{2\pi RC}\right)$$

CALCULATE:

(a) $C = \frac{1}{\sqrt{(1.00 \cdot 10^6 - 1)(2\pi(60.0 \text{ Hz})(2.00 \cdot 10^3 \Omega))}} = 1.3263 \cdot 10^{-9} \text{ F}$

(b) $f = \sqrt{\frac{81}{19}} \left(\frac{1}{2\pi(2.00 \cdot 10^3 \Omega)(1.3263 \cdot 10^{-9} \text{ F})}\right) = 123884 \text{ Hz}$

ROUND: To three significant figures,

(a) $C = 1.33 \text{ nF}$

A capacitor of capacitance 1.00 nF needs to be used in this high-pass filter.

(b) $f = 124 \text{ kHz}$

Frequencies of 120 kHz and higher will be passed with at least 90% of their amplitude.

DOUBLE-CHECK: Since frequencies higher than 120 kHz will pass with 90% of their strength, this seems like a reasonable high-pass filter.

30.48. In general, $V_{\text{rms}} = V_{\text{m}} / \sqrt{2}$; therefore, $V_{\text{m}} = \sqrt{2}V_{\text{rms}}$.

(a) For $V_{\text{rms}} = 110 \text{ V}$, $V_{\text{m}} = \sqrt{2}(110 \text{ V}) = 160 \text{ V}$.

(b) For $V_{\text{rms}} = 220 \text{ V}$, $V_{\text{m}} = \sqrt{2}(220 \text{ V}) = 310 \text{ V}$.

30.49. The quality factor for an RLC circuit is defined as $Q = \omega_0 (\text{Energy stored} / \text{Power lost})$. For the RLC circuit, the resonant frequency is $\omega_0 = 1 / \sqrt{LC}$. In general, the energy stored in the circuit is $U = U_0 e^{-Rt/L}$. The power lost is defined as $P = -|dU / dt|$. Therefore,

$$Q = \frac{\omega_0 (U_0 e^{-Rt/L})}{-\frac{d}{dt}(U_0 e^{-Rt/L})} = \frac{(U_0 e^{-Rt/L})}{\sqrt{LC}(R/L)(U_0 e^{-Rt/L})} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

30.50. On any product label the voltage and power displayed are the rms voltage and average power. Therefore, take $V_{\text{rms}} = 110 \text{ V}$ and $\langle P \rangle = 1250 \text{ W}$. The peak value of current is related to rms current by $I_{\text{rms}} = I_{\text{m}} / \sqrt{2}$.

In general, $\langle P \rangle = I_{\text{rms}} V_{\text{rms}} \cos \phi$.

Assuming that the hair dryer acts like a resistor, $\cos\phi = R/Z = 1$, so

$$I_{\text{rms}} = \frac{\langle P \rangle}{V_{\text{rms}}} \Rightarrow I_{\text{m}} = \sqrt{2} \frac{\langle P \rangle}{V_{\text{rms}}} = \sqrt{2} \frac{(1250 \text{ W})}{(110 \text{ V})} = 16 \text{ A}.$$

- 30.51.** (a) The resonant frequency of the radio tuner is related to the inductance, $L = 3.00 \text{ mH}$ and capacitance $C = 25.0 \text{ nF}$, by $\omega_0 = 1/\sqrt{LC}$. Keeping in mind that $\omega_0 = 2\pi f_0$,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(3.00 \cdot 10^{-3} \text{ H})(25.0 \cdot 10^{-9} \text{ F})}} = 18.4 \text{ kHz}.$$

(b) At resonance, the total impedance is solely that from the resistance, $R = 1.00 \mu\Omega$. Given that the voltage drop across the resistor is $V_{\text{rms}} = 1.50 \text{ mV}$, the power in the circuit is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{(1.50 \cdot 10^{-3} \text{ V})^2}{(1.00 \cdot 10^{-6} \Omega)} = 2.25 \text{ W}.$$

- 30.52.** **THINK:** The current through the circuit is driven by the AC potential, $V_{\text{rms}} = 50.0 \text{ V}$ and $f = 2000. \text{ Hz}$, which has a total impedance from the resistance, $R = 100. \Omega$, capacitance, $C = 0.400 \mu\text{F}$ and inductance, $L = 0.0500 \text{ H}$. The average power lost over the circuit is determined by the rms voltage and rms current.

SKETCH: Not required.

RESEARCH: The rms current is given by

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

The rms voltage drop across each component is $V_i = I_{\text{rms}} X_i$, where i denotes R , L or C . The average power drawn from the circuit is

$$\langle P \rangle = I_{\text{rms}}^2 R.$$

SIMPLIFY:

(a) Since the angular frequency is $\omega = 2\pi f$,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}.$$

(b) The rms voltage drop across the resistor is $V_R = I_{\text{rms}} R$. The rms voltage drop across the capacitor is

$V_C = I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{2\pi fC}$. The rms voltage drop across the inductor is $V_L = I_{\text{rms}} X_L = 2\pi I_{\text{rms}} fL$.

(c) $\langle P \rangle = I_{\text{rms}}^2 R$

CALCULATE:

$$(a) \quad I_{\text{rms}} = \frac{(50.0 \text{ V})}{\sqrt{(100. \Omega)^2 + \left(2\pi(2000. \text{ Hz})(0.0500 \text{ H}) - \frac{1}{2\pi(2000. \text{ Hz})(0.400 \cdot 10^{-6} \text{ F})}\right)^2}} = 0.1134 \text{ A}$$

$$(b) \quad V_R = (0.1134 \text{ A})(100. \Omega) = 11.34 \text{ V}$$

$$V_C = \frac{(0.1134 \text{ A})}{2\pi(2000. \text{ Hz})(0.400 \cdot 10^{-6} \text{ F})} = 22.56 \text{ V}$$

$$V_L = 2\pi(0.1134 \text{ A})(2000. \text{ Hz})(0.0500 \text{ H}) = 71.26 \text{ V}$$

$$(c) \langle P \rangle = (0.1134 \text{ A})^2 (100. \Omega) = 1.286 \text{ W}$$

ROUND: To three significant figures,

$$(a) I_{\text{rms}} = 0.113 \text{ A}$$

$$(b) V_R = 11.3 \text{ V}, V_C = 22.6 \text{ V}, V_L = 71.3 \text{ V}$$

$$(c) \langle P \rangle = 1.29 \text{ W}$$

DOUBLE-CHECK: For an RLC circuit, the maximum voltage is given by $V_m^2 = V_R^2 + (V_L - V_C)^2$; therefore, as a check $V_m = \sqrt{(11.34 \text{ V})^2 + (71.26 \text{ V} - 22.56 \text{ V})^2}$, which equals 50.003 V, which is V_m within rounding errors, so the answer is reasonable.

- 30.53. THINK:** In order to receive the best signal, the radio should be tuned at resonance with the incoming frequency, $f_0 = 88.7 \text{ MHz}$. The inductance of the radio receiver is $L = 8.22 \mu\text{H}$. Signal strength is $V_m = 12.9 \mu\text{V}$. The similar signal with frequency $f = 88.5 \text{ MHz}$, is not at resonance, so its total impedance is influenced by the resistor, capacitor and inductor such that its current is half that of the current for the frequency at resonance.

SKETCH: Not required.

RESEARCH: At resonance, $\omega = \omega_0 = 1/\sqrt{LC}$, where the angular frequency is $\omega_0 = 2\pi f_0$. The impedances of the inductor and capacitor are $X_L = \omega L$ and $X_C = 1/(\omega C)$, respectively. The impedance of the RLC circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance, the current amplitude is $I_m = V_m / R$, and when not at resonance, the current amplitude is $I'_m = V_m / Z$.

SIMPLIFY:

$$(a) \text{ At resonance, } \omega_0 = \frac{1}{\sqrt{LC_0}} = 2\pi f_0 \Rightarrow C_0 = \frac{1}{4\pi^2 f_0^2 L}$$

$$(b) \text{ When } I'_m = \frac{1}{2} I_m, \text{ then}$$

$$\frac{V_m}{Z} = \frac{1}{2} \frac{V_m}{R_0} \Rightarrow 2R_0 = Z = \sqrt{R_0^2 + (X_L - X_C)^2}$$

$$4R_0^2 = R_0^2 + (X_L - X_C)^2 \Rightarrow R_0 = \left| \frac{(X_L - X_C)}{\sqrt{3}} \right| = \left| \frac{2\pi fL - (2\pi fc)^{-1}}{\sqrt{3}} \right|$$

CALCULATE:

$$(a) C = \frac{1}{4\pi^2 (88.7 \cdot 10^6 \text{ Hz})^2 (8.22 \cdot 10^{-6} \text{ H})} = 3.9167 \cdot 10^{-13} \text{ F}$$

$$(b) R_0 = \left| \frac{2\pi(88.5 \cdot 10^6 \text{ Hz})(8.22 \cdot 10^{-6} \text{ H}) - (2\pi(88.5 \cdot 10^6 \text{ Hz})(3.9167 \cdot 10^{-13} \text{ F}))^{-1}}{\sqrt{3}} \right| = 11.941 \Omega$$

ROUND: To three significant figures,

$$(a) C = 0.392 \text{ pF}$$

$$(b) R_0 = 11.9 \Omega$$

DOUBLE-CHECK: This is an RLC circuit, so the current across the circuit decays exponentially with a time constant $\tau = 2L/R = 1.38 \mu\text{s}$. Assuming that the time constant represents the delay from when the radio picks up the signal to when it transmits it as sound, it is reasonable that the value is small.

30.54. If a power station provides power P , and delivers it over potential difference V , then the current out of the station is $I = P/V$. The power loss over the lines is $P' = I^2 R = (P/V)^2 R$. Therefore, if $V \rightarrow 10V$, the power loss over the lines is now $P'' = (P/10V)^2 R = (P/V)^2 R/100 = P'/100$. Therefore, if voltage is raised by a factor of 10, the power loss is 100 times smaller.

30.55. (a) From solved problem 29.1, the coil has $N_C = 31$ turns, and the solenoid has $n = 290$ turns/cm and length of $l = 12.0$ cm. Therefore, the number of turns in the solenoid is $N_S = nl$. If the voltage across the solenoid is $V_S = 120$ V, and the coil and solenoid act as a transformer, then

$$\frac{V_C}{N_C} = \frac{V_S}{N_S} \Rightarrow V_C = \frac{V_S N_C}{N_S} = \frac{V_S N_C}{nl} = \frac{(120 \text{ V})(31)}{(290 \text{ cm}^{-1})(12.0 \text{ cm})} = 1.1 \text{ V}.$$

(b) With DC current, no magnetic flux is created within the solenoid so there is no voltage in the coil.

30.56. The primary coil has $N_p = 800$ turns, and the secondary coil has $N_s = 40$ turns.

(a) The voltages across each coil is given by $V_p/N_p = V_s/N_s$, so when $V_p = 100$. V, the voltage on the secondary coil is

$$V_s = N_s V_p / N_p = (40)(100. \text{ V}) / (800) = 5.00 \text{ V}.$$

(b) The current on the secondary coil when $I_p = 5.00$ A is

$$I_s = I_p V_p / V_s = (5.00 \text{ A})(100. \text{ V}) / (5.00 \text{ V}) = 100. \text{ A}.$$

(c) When the voltage is DC, no magnetic flux is created within the secondary coil, so there is no voltage on the secondary coil.

(d) When the voltage is DC, the voltage on the secondary coil is zero, so the secondary coil does not carry a current.

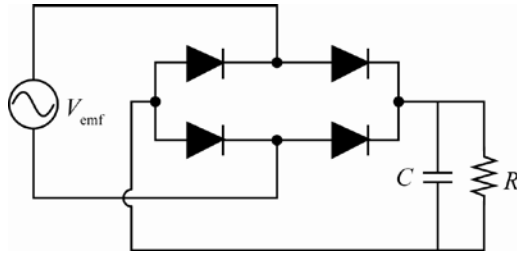
30.57. The primary coil has $N_p = 200$ turns and the secondary coil has $N_s = 120$ turns. The secondary coil drives a current I through a resistance of $R = 1.00$ k Ω . The input voltage applied across the primary coil is $V_{\text{rms}} = 75.0$ V. The voltage across the secondary coil is $V_s = V_{\text{rms}} N_s / N_p$. The power dissipated in the

$$\text{resistor is } P = \frac{(V_{\text{rms}} N_s / N_p)^2}{R} = \frac{((75.0 \text{ V})(120) / (200))^2}{(1.00 \cdot 10^3 \Omega)} = 2.03 \text{ W}.$$

30.58. The frequency of the source is $f_s = 60$. Hz. The full wave rectifier is a configuration of diodes that allows all of the current in the circuit to flow in one direction. This is illustrated in Figure 30.35 of the textbook. Comparing the plot of emf as a function of the time (Figure 30.35 (a)) to the rectified current (Figure 30.35 (d)), it can be seen that the frequency is doubled, $f = 2f_s = 120$ Hz. The capacitor minimizes the size of the ripples, but the frequency stays the same.

30.59. THINK: The voltage applied to the primary side of the transformer is $V_{\text{rms}} = V_p = 110$ V and its frequency is $f = 60$. Hz. The ratio of primary coil turns to secondary coil turns in the transformer is $N_p / N_s = 11$. The secondary coil voltage, V_s is used as the source voltage for the fullwave rectifier shown in Problem 30.56. Using the equations for transformers, the rms voltage in the secondary coil can be found. To find the DC voltage V_{DC} provided to the resistor, it is necessary to integrate over the AC voltage to obtain the time-averaged value.

SKETCH:



RESEARCH:

(a) The secondary rms voltage is given by $V_s = V_p N_s / N_p$. For any rms voltage, the maximum voltage is given $V_m = \sqrt{2} V_{\text{rms}}$.

(b) The DC voltage V_{DC} is the time-averaged value of the rectified V_{emf} . The time average is found from the equation

$$V_{\text{DC}} = \frac{2}{T} \int_0^{T/2} V_{\text{emf}}(t) dt,$$

where $V_{\text{emf}}(t) = V_m \sin(\omega t)$.

SIMPLIFY:

(a) $V_m = \sqrt{2} \frac{V_p N_s}{N_p}$

(b) Substitute $T = \frac{2\pi}{\omega}$ into the equation:

$$V_{\text{DC}} = \frac{\omega}{\pi} V_m \int_0^{\pi/\omega} \sin(\omega t) dt = -\frac{V_m}{\pi} \cos(\omega t) \Big|_0^{\pi/\omega} = -\frac{V_m}{\pi} (-1 - 1) = \frac{2}{\pi} V_m.$$

CALCULATE:

(a) $V_m = \sqrt{2} (110 \text{ V}) \left(\frac{1}{11} \right) = 14.14 \text{ V}$

(b) $V_{\text{DC}} = \frac{2}{\pi} (14.14 \text{ V}) = 9.003 \text{ V}$

ROUND: To two significant figures,

(a) $V_m = 14 \text{ V}$

(b) $V_{\text{DC}} = 9.0 \text{ V}$

DOUBLE-CHECK: The number of turns in the primary is greater than the number of turns in the secondary so it is expected that the voltage in the secondary is lower.

- 30.60.** The given quantities are: the inductance, $L = 100. \text{ mH}$; the frequency, $f = 60.0 \text{ Hz}$; and the rms voltage, $V_{\text{rms}} = 115 \text{ V}$. The average power is given by $\langle P \rangle = I_{\text{rms}} V_{\text{rms}} R / Z$, so to maximize the power output the impedance must be minimized. The impedance is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$. To minimize Z , the expression in the brackets must equal zero:

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi(60.0 \text{ Hz}))^2 (0.100 \text{ H})} = 7.04 \cdot 10^{-5} \text{ F}.$$

- 30.61.** This question deals with an LC circuit. The given quantities are the frequency, $f = 1000. \text{ kHz}$ and the inductance, $L = 10.0 \text{ mH}$. What is the capacitance, C , of the capacitor when the station is properly tuned? Equating the expressions $\omega = 2\pi f$ and $\omega = 1/\sqrt{LC}$ and solving for C : $2\pi f = 1/\sqrt{LC}$,

$$2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{(2\pi f)^2 L}$$

$$C = \frac{1}{(2\pi f)^2 L} = \frac{1}{((2\pi)(1.000 \cdot 10^6 \text{ Hz}))^2 (1.00 \cdot 10^{-2} \text{ H})} = 2.53 \cdot 10^{-12} \text{ F.}$$

- 30.62.** This question deals with an RLC circuit. The circuit is driven by a generator with $V_{\text{rms}} = 12.0 \text{ V}$ and frequency, f_0 . The inductance is $L = 7.00 \text{ mH}$, the resistance is $R = 100. \Omega$ and the capacitance is $C = 0.0500 \text{ mF}$.

(a) The angular resonant frequency of the RLC circuit is given by $\omega_0 = 1/\sqrt{LC}$, where $\omega_0 = 2\pi f_0$. Therefore, the resonant frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(7.00 \cdot 10^{-3} \text{ H})(0.0500 \cdot 10^{-3} \text{ F})}} = 269 \text{ Hz.}$$

(b) The average power dissipated in the resistor at the resonant frequency is

$$\langle P \rangle = \frac{V_{\text{rms}}^2}{R} = \frac{(12.0 \text{ V})^2}{(100. \Omega)} = 1.44 \text{ W.}$$

- 30.63.** A “60-W light bulb” dissipates power at $\langle P \rangle = 60. \text{ W}$. The question gives $V_{\text{rms}} = 110 \text{ V}$.

(a) The maximum current is $I_m = \sqrt{2}I_{\text{rms}}$. The average power is $\langle P \rangle = I_{\text{rms}} V_{\text{rms}}$, so the maximum current is

$$I_m = \frac{\sqrt{2}\langle P \rangle}{V_{\text{rms}}} = \frac{\sqrt{2}(60. \text{ W})}{(110 \text{ V})} = 0.77 \text{ A.}$$

(b) The maximum voltage is $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(110 \text{ V}) = 160 \text{ V}$.

- 30.64.** The given quantities are the frequency, $f = 360 \text{ Hz}$, the inductance $L = 25 \text{ mH}$ and the resistance $R = 0.80 \Omega$. In order for the current and voltage to be in phase, the circuit must be in resonance. This occurs when the inductive reactance and the capacitive reactance are equal (Note that this is result is independent of the resistor value):

$$X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}} = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi(360 \text{ Hz}))^2 (25 \cdot 10^{-3} \text{ H})} = 7.8$$

- 30.65.** In an RLC circuit, the inductance is $L = 65.0 \text{ mH}$ and the capacitance is $C = 1.00 \mu\text{F}$. The circuit loses electromagnetic energy at a rate of $\Delta U = -3.50\%$ per cycle. The energy stored in the electric field of the

capacitor is expressed by $\Delta U_E = q_{\text{max}}^2 e^{-Rt/L} \cos(\omega_0 t)$. The rate of energy loss is $\frac{\Delta U_E}{U_E} = \frac{U_{E, \text{final}} - U_{E, \text{initial}}}{U_{E, \text{initial}}}$,

where time t_{initial} is zero and t_{final} is the time to complete one cycle, $t_{\text{final}} = 2\pi / \omega_0$. The rate of energy loss per cycle can now be written as

$$\frac{\Delta U_E}{U_E} = -0.035 = \frac{q_{\text{max}}^2 e^{-2\pi R/\omega_0 L} \cos^2(2\pi) - q_{\text{max}}^2 e^{-0} \cos^2(0)}{q_{\text{max}}^2 e^{-0} \cos^2(0)} = e^{-2\pi R/\omega_0 L} - 1.$$

Since $\omega_0 = 1/\sqrt{LC}$,

$$1 - 0.0350 = e^{-(2\pi R\sqrt{LC})/L} \Rightarrow \ln(0.9650) = -\frac{2\pi R\sqrt{LC}}{L}$$

$$R = -\frac{\ln(0.9650)}{2\pi} \sqrt{\frac{L}{C}} = -\frac{\ln(0.9650)}{2\pi} \sqrt{\frac{(65.0 \cdot 10^{-3} \text{ H})}{(1.00 \cdot 10^{-6} \text{ F})}} = 1.45 \Omega.$$

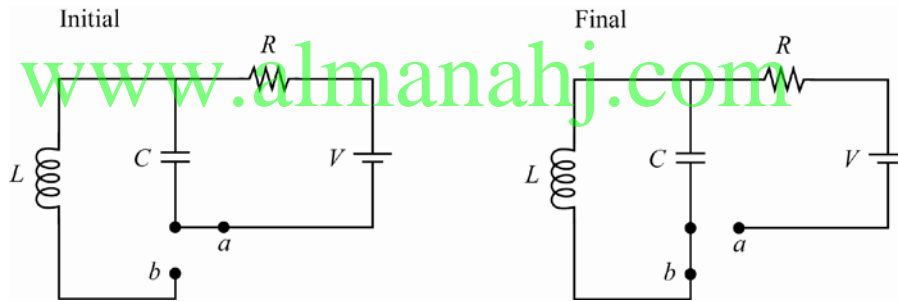
- 30.66.** The transformer has $N_p = 400$ turns on its primary coil and $N_s = 20$ turns on its secondary coil. The average power output from the secondary coil is $\langle P \rangle = 1200$ W and the maximum voltage output from the secondary coil is $V_{s,m} = 60.0$ V. The rms current in the primary coil is given by,

$$I_{p,rms} = \frac{I_{s,rms} N_s}{N_p} = \frac{\langle P \rangle N_s}{V_{s,rms} N_p}.$$

Substituting the maximum values for the rms values gives

$$\frac{I_{p,m}}{\sqrt{2}} = \frac{\langle P \rangle N_s}{\left(\frac{V_{s,m}}{\sqrt{2}}\right) N_p} \Rightarrow I_{p,m} = 2 \frac{\langle P \rangle N_s}{V_{s,m} N_p} = 2 \frac{(1200 \text{ W})(20)}{(60.0 \text{ V})(400)} = 2.00 \text{ A}.$$

- 30.67.** The given quantities are the capacitance, $C = 5.00 \mu\text{F}$, the resistance, $R = 4.00 \Omega$ and the battery voltage, $V = 9.00$ V. The capacitor is charged for a long time by closing the switch to position *a*. At time $t = 0$ the switch is closed at position *b* and the capacitor is discharged through an inductor with $L = 40.0$ mH.



- (a) The maximum current through the inductor is given by $i_{\max} = \omega_0 q_{\max}$, where q_{\max} is the maximum charge on the capacitor, and $\omega_0 = 1/\sqrt{LC}$. The fully charged capacitor has charge $q_{\max} = CV$. Substituting for ω_0 and q_{\max} gives

$$i_{\max} = \frac{CV}{\sqrt{LC}} = \sqrt{\frac{C}{L}} V = \sqrt{\frac{(5.00 \cdot 10^{-6} \text{ F})}{(40.0 \cdot 10^{-3} \text{ H})}} (9.00 \text{ V}) = 0.101 \text{ A}.$$

- (b) The current is given by $i = -i_{\max} \sin(\omega_0 t)$. For the current to be a maximum $\sin(\omega_0 t) = 1$, or when $\omega_0 t = \pi/2$. This occurs at time

$$t = \frac{\pi}{2} \sqrt{LC} = \frac{\pi}{2} \sqrt{(40.0 \cdot 10^{-3} \text{ H})(5.00 \cdot 10^{-6} \text{ F})} = 7.02 \cdot 10^{-4} \text{ s}.$$

- 30.68. THINK:** The resistance is $R = 10.0 \text{ k}\Omega$ and the capacitance is $C = 0.0470 \mu\text{F}$ in the RC high-pass filter. To find the frequency f where the ratio of the output voltage to the input voltage gives $20 \log(V_{\text{out}}/V_{\text{in}}) = -3.00$, use the voltage ratio for an RC high-pass filter.

SKETCH: A sketch of the circuit is provided in the problem.

RESEARCH: For an RC high-pass filter, the ratio $V_{\text{out}} / V_{\text{in}}$ is given by:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\sqrt{1 + \frac{1}{\omega^2 R^2 C^2}}}$$

The frequency is $f = \omega / 2\pi$. From the properties of logarithms, if $y = \log_b(x)$ then $x = b^y$.

SIMPLIFY: $20 \log\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = -3.00 \Rightarrow \log\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = -0.150$. This can be rewritten as $\frac{V_{\text{out}}}{V_{\text{in}}} = 10^{-0.150}$.

Substituting for $\frac{V_{\text{out}}}{V_{\text{in}}}$ gives:

$$\frac{1}{\sqrt{1 + \frac{1}{\omega^2 R^2 C^2}}} = 10^{-0.150} \Rightarrow \frac{1}{1 + \frac{1}{\omega^2 R^2 C^2}} = 10^{-0.300} \Rightarrow \frac{1}{\omega^2 R^2 C^2} + 1 = 10^{0.300} \Rightarrow \omega^2 R^2 C^2 = \frac{1}{10^{0.300} - 1}$$

$$\omega = \sqrt{\frac{1}{(10^{0.300} - 1) RC}} \Rightarrow f = \sqrt{\frac{1}{(10^{0.300} - 1) (2\pi RC)}}$$

CALCULATE: $f = \sqrt{\frac{1}{(10^{0.300} - 1) \left(2\pi (10.0 \cdot 10^3 \Omega) (0.0470 \cdot 10^{-6}) \right)}} = 339.43 \text{ Hz}$

ROUND: To three significant figures, $f = 339 \text{ Hz}$.

DOUBLE-CHECK: The breakpoint frequency for this RC high-pass filter is $f_B = 1/(2\pi RC) = 338 \text{ Hz}$. Since the calculated frequency is larger than this value, the answer is reasonable.

30.69. THINK: The unknown wire-wound resistor R is initially connected to a DC power supply. When there is a voltage of $V_{\text{emf}} = 10.0 \text{ V}$ across the resistor, the current is $I = 1.00 \text{ A}$. Next the resistor is connected to an AC power source with $V_{\text{rms}} = 10.0 \text{ V}$. When the AC power source is operated at frequency $f = 20.0 \text{ kHz}$, a current of $I_{\text{rms}} = 0.800 \text{ A}$ is measured. Find:

- the resistance, R ;
- the inductive reactance, X_L , of the resistor;
- the inductance, L , of the resistor; and
- the frequency, f' , of the AC power source at which $X_L = R$.

SKETCH: Not required.

RESEARCH:

- The resistance of the resistor when used with the DC source can be found using Ohm's law, $R = V / I$.
- When connected to the AC power source, the resistor can be treated as an RL series circuit. The impedance of the RL circuit is $Z = \sqrt{R^2 + X_L^2} = V_{\text{rms}} / I_{\text{rms}}$.

(c) The inductance can be found with $X_L = \omega L$ and $\omega = 2\pi f$.

(d) $\omega L = R$, $\omega = 2\pi f'$

SIMPLIFY:

(a) $R = \frac{V}{I}$

(b) $Z = \sqrt{R^2 + X_L^2} \Rightarrow X_L^2 = Z^2 - R^2 \Rightarrow$ Substituting $Z = \frac{V_{\text{rms}}}{I_{\text{rms}}}$ gives, $X_L = \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}$.

$$(c) L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f}$$

$$(d) f' = \frac{R}{2\pi L}$$

CALCULATE:

$$(a) R = \frac{(10.0 \text{ V})}{(1.00 \text{ A})} = 10.0 \Omega$$

$$(b) X_L = \sqrt{\left(\frac{10.0 \text{ V}}{0.800 \text{ A}}\right)^2 - (10.0 \Omega)^2} = 7.50 \Omega$$

$$(c) L = \frac{(7.50 \Omega)}{2\pi(20.0 \cdot 10^3 \text{ Hz})} = 5.968 \cdot 10^{-5} \text{ H}$$

$$(d) f' = \frac{10.0 \Omega}{2\pi(5.968 \cdot 10^{-5} \text{ H})} = 26667 \text{ Hz}$$

ROUND: The answers should be reported to three significant figures.

$$(a) R = 10.0 \Omega$$

$$(b) X_L = 7.50 \Omega$$

$$(c) L = 5.97 \cdot 10^{-5} \text{ H}$$

$$(d) f' = 26.7 \text{ kHz}$$

DOUBLE-CHECK: Since the current decreased when the power supply was changed from DC to AC, the resistance must have increased. This additional resistance is explained by the inductive reactance of the resistor. The units for all calculated values are correct.

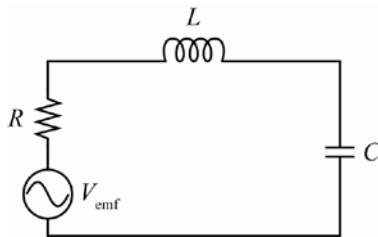
30.70. THINK: The components of the RLC circuit are connected in series, and their values are $R = 20.0 \Omega$, $L = 10.0 \text{ mH}$ and $C = 5.00 \cdot 10^{-6} \text{ F}$. They are connected to an AC source of peak voltage $V = 10.0 \text{ V}$ and frequency $f = 100. \text{ Hz}$. Find:

(a) The amplitude of the current I .

(b) The phase difference ϕ between the current and the voltage.

(c) The maximum voltage across R , L , and C .

SKETCH:



RESEARCH:

(a) The amplitude of the current in an RLC series circuit is

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}},$$

where $X_L = \omega L$ and $X_C = 1/\omega C$. Recall that $\omega = 2\pi f$ and $V_m = \sqrt{2}V_{\text{rms}}$.

(b) The phase difference between the current and the voltage is $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$.

(c) For the maximum voltage across each circuit component, $V_L = I_m X_L$, $V_C = I_m X_C$ and $V_R = I_m R$.

SIMPLIFY:

$$(a) I_m = \frac{\sqrt{2}V_{\text{rms}}}{\sqrt{R^2 + (2\pi fL - 1/(2\pi fC))^2}}$$

$$(b) \phi = \tan^{-1}\left(\frac{2\pi fL - 1/(2\pi fC)}{R}\right)$$

$$(c) V_L = 2\pi fI_m L, V_C = \frac{I_m}{2\pi fC} \text{ and } V_R = I_m R.$$

CALCULATE:

$$(a) I_m = \frac{\sqrt{2}(10.0 \text{ V})}{\sqrt{(20.0 \Omega)^2 + \left((200\pi \text{ rad/s})(0.0100 \text{ H}) - \frac{1}{(200\pi \text{ rad/s})(5.00 \cdot 10^{-6} \text{ F})} \right)^2}} = 0.04523 \text{ A}$$

$$(b) \phi = \tan^{-1}\left(\frac{(200\pi \text{ rad/s})(0.0100 \text{ H}) - 1/((200\pi \text{ rad/s})(5.00 \cdot 10^{-6} \text{ F}))}{(20.0 \Omega)}\right) = -1.507 \text{ rad}$$

$$(c) V_L = (0.04523 \text{ A})(200\pi \text{ rad/s})(0.0100 \text{ H}) = 0.2842 \text{ V}$$

$$V_C = \frac{(0.04523 \text{ A})}{(200\pi \text{ rad/s})(5.00 \cdot 10^{-6} \text{ F})} = 14.40 \text{ V}$$

$$V_R = (0.04523 \text{ A})(20.0 \Omega) = 0.9046 \text{ V}$$

ROUND: To three significant figures.

$$(a) I_m = 45.2 \text{ mA}$$

$$(b) \phi = -1.51 \text{ rad}$$

$$(c) V_L = 0.284 \text{ V}, V_C = 14.4 \text{ V}, V_R = 0.905 \text{ V}$$

DOUBLE-CHECK: It must be true that $V_m^2 = V_R^2 + (V_L - V_C)^2$. To check this, plug in the calculated values for each side of the equation:

$$V_m^2 = (\sqrt{2}(V_{\text{rms}}))^2 = 2(10.0 \text{ V})^2 = 200. \text{ V}$$

$$V_R^2 + (V_L - V_C)^2 = (0.9046 \text{ V})^2 + ((0.2842 \text{ V}) - (14.40 \text{ V}))^2 = 200. \text{ V},$$

as required.

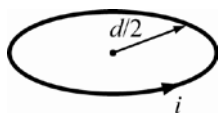
30.71. THINK:

(a) The loop of wire has a diameter of $d = 5.00 \text{ cm} = 0.0500 \text{ m}$ and it carries current $i = 2.00 \text{ A}$. Find the magnetic energy density, u_B , at the loop's center.

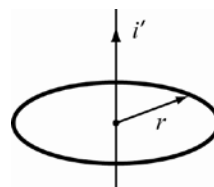
(b) Find the current, i' in a straight wire that gives the same value of u_B at a point, $r = 4.00 \text{ cm} = 0.0400 \text{ m}$ from the wire.

SKETCH:

(a)



(b)



RESEARCH:

(a) The magnetic energy density is $u_B = (1/(2\mu_0))B^2$. The magnetic field, B for the wire loop is given by

$$\text{Equation 28.8 in the textbook, } B_{\text{loop}} = \frac{\mu_0 i}{2(d/2)}.$$

(b) The magnetic field due to current traveling in a straight wire is given by Equation 28.4,

$$B_{\text{wire}} = \mu_0 i' / (2\pi r).$$

SIMPLIFY:

$$(a) \quad u_B = \left(\frac{1}{2\mu_0}\right) B_{\text{loop}}^2, \text{ substituting for } B_{\text{loop}} \text{ gives: } u_B = \left(\frac{1}{2\mu_0}\right) \left(\frac{\mu_0 i}{2(d/2)}\right)^2 = \frac{1}{2} \frac{\mu_0 i^2}{d^2}.$$

$$(b) \quad u_B = \left(\frac{1}{2\mu_0}\right) B_{\text{wire}}^2, \text{ substituting for } B_{\text{wire}} \text{ gives:}$$

$$u_B = \left(\frac{1}{2\mu_0}\right) \frac{\mu_0^2 i'^2}{4\pi^2 r^2} \Rightarrow i'^2 = \frac{8\pi^2 r^2 u_B}{\mu_0} \Rightarrow i' = \sqrt{\frac{8\pi^2 r^2 u_B}{\mu_0}}.$$

CALCULATE:

$$(a) \quad u_B = \frac{1}{2} \frac{(4\pi \cdot 10^{-7} \text{ T m/A})(2.00 \text{ A})^2}{(0.0500 \text{ m})^2} = 1.005 \cdot 10^{-3} \text{ J/m}^3$$

$$(b) \quad i' = \sqrt{\frac{8\pi^2 (0.0400 \text{ m})^2 (1.005 \cdot 10^{-3} \text{ J/m}^3)}{4\pi \cdot 10^{-7} \text{ T m/A}}} = 10.053 \text{ A}$$

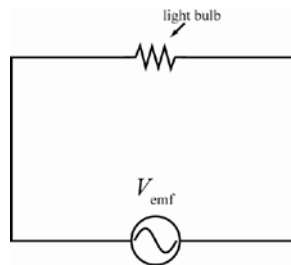
ROUND: The answers should be reported to three significant figures.

$$(a) \quad u_B = 1.01 \cdot 10^{-3} \text{ J/m}^3$$

$$(b) \quad i' = 10.1 \text{ A}$$

DOUBLE-CHECK: The calculated values have the proper units. It is expected that the current required to generate a given magnetic field would be much larger for a straight wire than for a loop.

- 30.72. THINK:** The bulb of average power $\langle P \rangle = 75000 \text{ W}$ operates at a current of $I_{\text{rms}} = 200. \text{ A}$, a voltage of $V_{\text{rms}} = 440. \text{ V}$, and a frequency of $f = 60.0 \text{ Hz}$. The inductive reactance of the bulb is not negligible so its impedance needs to be considered. The inductive reactance can be neglected, so $X_C = 0$.

SKETCH:

RESEARCH: The average power of the bulb is $\langle P \rangle = I_{\text{rms}}^2 R$. The rms voltage is $V_{\text{rms}} = I_{\text{rms}} Z$, where

$$Z = \sqrt{R^2 + X_L^2} \text{ and } X_L = \omega L.$$

SIMPLIFY: The resistance of the bulb is

$$R = \frac{\langle P \rangle}{I_{\text{rms}}^2}.$$

The inductance of the bulb is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (\omega L)^2}} \Rightarrow (R^2 + (\omega L)^2)(I_{\text{rms}}^2) = V_{\text{rms}}^2 \Rightarrow (\omega L)^2 = \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2$$

$$\Rightarrow L = \pm \frac{1}{\omega} \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2} = \frac{1}{2\pi f} \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}.$$

CALCULATE: $R = \frac{(75000 \text{ W})}{(200. \text{ A})^2} = 1.875 \Omega$

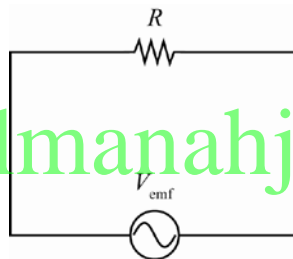
$$L = \frac{1}{2\pi(60.0 \text{ Hz})} \sqrt{\left(\frac{(440. \text{ V})}{(200. \text{ A})}\right)^2 - (1.875 \Omega)^2} = 0.003053 \text{ H}$$

ROUND: To three significant figures, $R = 1.88 \Omega$ and $L = 3.05 \text{ mH}$.

DOUBLE-CHECK: These are reasonable values. If this was a DC source Ohm's Law would give, $R = V/I = 440. \text{ V} / 200. \text{ A} = 2.20 \Omega$ which is comparable to the total calculated impedance. All values have the correct units.

- 30.73. THINK:** A resistor R is connected across an AC source which oscillates at angular frequency ω . Show that the power dissipated in R oscillates with frequency 2ω .

SKETCH:



RESEARCH: The power is $P = i_R^2 R$. For an AC power supply, $i_R = I_R \sin(\omega t)$. Also useful is the trigonometric identity $\cos(2\theta) = 1 - 2\sin^2 \theta$.

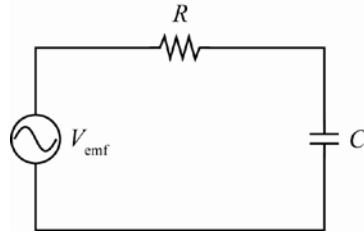
SIMPLIFY: $P = i_R^2 R = I_R^2 \sin^2(\omega t) R = I_R^2 \frac{1 - \cos(2(\omega t))}{2} R = \frac{1}{2} I_R^2 R (1 - \cos(2\omega t))$. It can be seen from the above equation that the power oscillates with a frequency twice that of the voltage.

CALCULATE: Not required.

ROUND: Not required.

DOUBLE-CHECK: Power, P , is proportional to i^2 . Since i varies proportionately with V , it must be the case that i^2 varies proportionately with V^2 . Since V varies proportionately with ω , it must be the case that V^2 varies proportionately with ω^2 . Therefore by transitivity, P is proportional to ω^2 . Therefore, there exists a constant, c , such that $P = c\omega^2$. So the change in P with respect to time, dP/dt , will be proportional to $d\omega^2/dt$, or 2ω .

- 30.74. THINK:** The resistor has a resistance of $R = 300. \Omega$ and is connected in series with a capacitor with $C = 4.00 \mu\text{F}$. The AC power supply has $V_{\text{rms}} = 40.0 \text{ V}$. Find:
- the frequency f at which $V_C = V_R$;
 - the current I_{rms} at which this occurs.

SKETCH:**RESEARCH:**(a) $V_C = iX_C$, and $X_C = 1/\omega C$, while $V_R = iR$. Recall $\omega = 2\pi f$.(b) In this circuit, $V_{\text{rms}} = I_{\text{rms}} Z$, where $Z = \sqrt{R^2 + X_C^2}$ (as there is no inductor).**SIMPLIFY:**

$$(a) V_C = V_R \Rightarrow iX_C = iR \Rightarrow \frac{1}{\omega C} = R \Rightarrow \frac{1}{2\pi f C} = R \Rightarrow f = \frac{1}{2\pi RC}$$

$$(b) I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}} \frac{V_{\text{rms}}}{R}$$

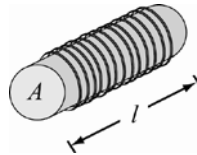
CALCULATE:

$$(a) f = \frac{1}{2\pi(300. \Omega)(4.00 \mu\text{F})} = 132.6 \text{ Hz}$$

$$(b) I_{\text{rms}} = \frac{1}{\sqrt{2}} \frac{(40.0 \text{ V})}{(300. \Omega)} = 0.09428 \text{ A}$$

ROUND: To three significant figures, $f = 133 \text{ Hz}$ and $I_{\text{rms}} = 0.0943 \text{ A}$.**DOUBLE-CHECK:** These values are reasonable given the initial conditions. Dimensional analysis confirms the units are correct.

- 30.75. **THINK:** The electromagnet has $N = 200$ loops, a length $l = 0.100 \text{ m}$, and a cross-sectional area $A = 5.00 \text{ cm}^2$. Find its resonant frequency f_0 when it is attached to the Earth.

SKETCH:**RESEARCH:** The resonant frequency is $\omega_0 = 1/\sqrt{LC}$. Recall $\omega = 2\pi f$. The radius of the Earth is $r = 6.38 \cdot 10^6 \text{ m}$ and for a spherical capacitor, $C = 4\pi\epsilon_0 r$. From Chapter 29, $L = \mu_0 n^2 l A$ for a solenoid, where $n \equiv N/l$.

$$\text{SIMPLIFY: } f_0 = \frac{1}{2\pi\sqrt{\mu_0 (N/l)^2 l A (4\pi\epsilon_0 r)}} = \frac{1}{4N} \sqrt{\frac{l}{\pi^3 \mu_0 \epsilon_0 A r}}$$

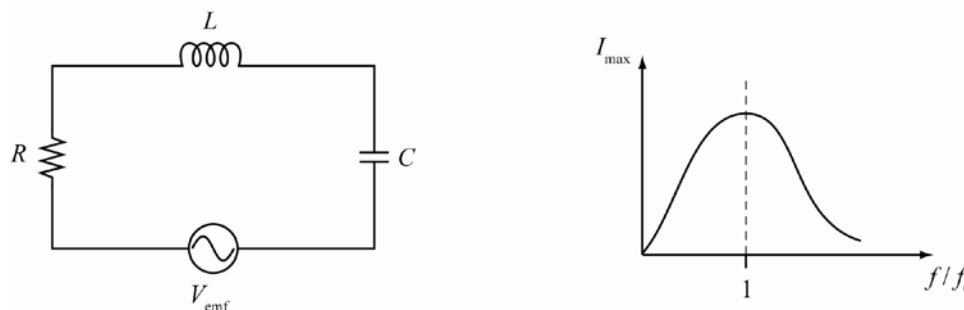
CALCULATE:

$$f_0 = \frac{1}{4(200)} \sqrt{\frac{(0.100 \text{ m})}{\pi^3 (4\pi \cdot 10^{-7} \text{ T m/A})(8.854 \cdot 10^{-12} \text{ F/m})(5.00 \cdot 10^{-4} \text{ m}^2)(6.38 \cdot 10^6 \text{ m})}} = 376.8 \text{ Hz}$$

ROUND: To three significant figures, $f_0 = 377 \text{ Hz}$.**DOUBLE-CHECK:** This is a reasonable frequency for an electromagnet. Dimensional analysis confirms the units are correct.

- 30.76. THINK:** The inductance of the inductor is $L = 1.00$ H. The resonance of the series RLC circuit is to occur at frequency $f_0 = 60.0$ Hz. The voltage across the capacitor (or inductor), V_C (or V_L) is to be 20.0 times that across the resistor, V_R . Find the capacitance C and the resistance R .

SKETCH:



RESEARCH: At resonance, the angular frequency is $\omega_0 = 1/\sqrt{LC}$, where $\omega_0 = 2\pi f_0$. At resonance, $V_R = V_{\text{emf}}$ and $V_L = -V_C$, where $V_L = IX_L$. Recall $V = IR$ and $X_L = \omega_0 L$.

SIMPLIFY: $C = \frac{1}{\omega_0^2 L} = \frac{1}{4\pi^2 f_0^2 L}$, $V_L = IX_L = \frac{V_{\text{emf}}}{R} \omega_0 L = \frac{2\pi V_R f_0 L}{R} \Rightarrow R = \frac{2\pi V_R f_0 L}{V_L}$

Since $V_L = 20.0V_R$, $R = \frac{2\pi V_R f_0 L}{(20.0V_R)} = \frac{\pi f_0 L}{10.0}$.

CALCULATE: The capacitance of the capacitor must be $C = \frac{1}{4\pi^2 (60.0 \text{ Hz})^2 (1.00 \text{ H})} = 7.0362 \cdot 10^{-6} \text{ F}$.

The resistance of the resistor must be $R = \frac{\pi (60.0 \text{ Hz})(1.00 \text{ H})}{10.0} = 18.8496 \ \Omega$.

ROUND: Rounding to three significant figures, $C = 7.04 \ \mu\text{F}$ and $R = 18.8 \ \Omega$.

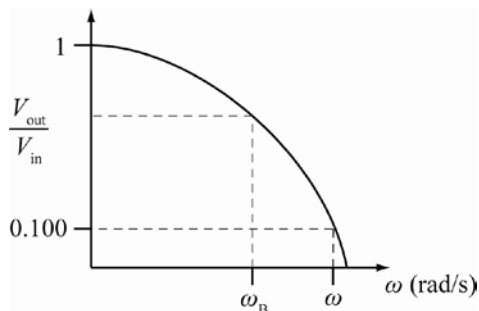
DOUBLE-CHECK: At resonance, $X_L - X_C = 0$. To check this, plug in the value found for the capacitance:

$$X_L - X_C = \omega_0 L - \frac{1}{\omega_0 C} = 2\pi f_0 L - \frac{1}{2\pi f_0 C} = 2\pi (60.0 \text{ Hz})(1.00 \text{ H}) - \frac{1}{2\pi (60.0 \text{ Hz})(7.0362 \cdot 10^{-6} \text{ F})} = 0,$$

as required.

- 30.77. THINK:** The RC low-pass filter has a breakpoint frequency of $f_B = 200$. Hz. Find the frequency at which the output voltage divided by the input voltage is $V_{\text{out}}/V_{\text{in}} = 0.100$.

SKETCH:



RESEARCH: For a RC low-pass filter, the breakpoint frequency is: $\omega_B = 1/(RC)$, where $\omega_B = 2\pi f_B$. The ratio of the input voltage to output voltage is

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}.$$

SIMPLIFY: $\omega_B = 2\pi f_B = \frac{1}{RC} \Rightarrow RC = \frac{1}{2\pi f_B}$

$$\frac{V_{\text{in}}}{V_{\text{out}}} = \sqrt{1 + \omega^2 (RC)^2} = \sqrt{1 + \omega^2 \left(\frac{1}{2\pi f_B}\right)^2} \Rightarrow \left(\frac{V_{\text{in}}}{V_{\text{out}}}\right)^2 = 1 + \left(\frac{\omega}{2\pi f_B}\right)^2 \Rightarrow \left(\frac{V_{\text{in}}}{V_{\text{out}}}\right)^2 - 1 = \frac{\omega^2}{(2\pi f_B)^2}$$

$$\omega = 2\pi f_B \sqrt{\left(\frac{V_{\text{in}}}{V_{\text{out}}}\right)^2 - 1} \Rightarrow f = f_B \sqrt{\left(\frac{V_{\text{in}}}{V_{\text{out}}}\right)^2 - 1}$$

CALCULATE: $f = (200. \text{ Hz}) \sqrt{\left(\frac{1}{0.100}\right)^2 - 1} = 1989.97 \text{ Hz}$

ROUND: To three significant figures, $f = 1990 \text{ Hz}$.

DOUBLE-CHECK: Since $V_{\text{out}}/V_{\text{in}}$ is less than $1/\sqrt{2}$ (the value associated with the breakpoint frequency), by the above sketch, the frequency f must be greater than the breakpoint frequency f_B .

Multi-Version Exercises

30.78. $X_L = 2\pi fL = 2\pi(605 \text{ Hz})(42.1 \text{ mH}) = 160. \Omega$

30.79. $I_L = V_L / X_L = V_L / (2\pi fL) = (19.9 \text{ V}) / [2\pi(669 \text{ Hz})(52.5 \text{ mH})] = 90.2 \text{ mA}$

30.80. $X_L = 2\pi fL \Rightarrow L = X_L / (2\pi f) = (81.52 \Omega) / [2\pi(733 \text{ Hz})] = 17.7 \text{ mH}$

30.81. $I_L = V_L / (2\pi fL) \Rightarrow L = V_L / (2\pi fI_L) = (21.5 \text{ V}) / [2\pi(797 \text{ Hz})(0.1528 \text{ A})] = 28.1 \text{ mH}$

www.almanahj.com

Chapter 31: Electromagnetic Waves

الوحدة 11 : الموجات الكهرومغناطيسية

Concept Checks

31.1. e 31.2. c 31.3. e 31.4. c 31.5. a 31.6. d 31.7. b

Multiple-Choice Questions

31.1. c 31.2. b, c, e 31.3. b 31.4. a 31.5. a 31.6. a 31.7. c 31.8. b 31.9. c 31.10. b

Conceptual Questions

31.11. (a) The intensity of the light passing through the first polarizer is $I_1 = I_0 / 2$. The angle between the transmission axis of the first and second polarizer as a function of time is $\theta_{1,2}(t) = 45^\circ + \omega t$, where t is in seconds and ω is in rad/s. The intensity of the light passing the second polarizer is then $I_2 = I_1 \cos^2 \theta_{1,2} = \frac{1}{2} I_0 \cos^2(45^\circ + \omega t) = I_2(t)$. The angle between the transmission axis of the second and third polarizer as a function of time is $\theta_{2,3}(t) = 45^\circ - \omega t$. The intensity of the light passing the third polarizer is:

$$I_3 = I_2 \cos^2 \theta_{2,3} = \frac{1}{2} I_0 \cos^2(45^\circ + \omega t) \cos^2(45^\circ - \omega t) = \frac{1}{2} I_0 [\cos(45^\circ + \omega t) \cos(45^\circ - \omega t)]^2 = I_3(t).$$

Now, use the trigonometric identity, $\cos u \cos v = \frac{1}{2} [\cos(u+v) + \cos(u-v)]$:

$$\begin{aligned} \cos(45^\circ + \omega t) \cos(45^\circ - \omega t) &= \frac{1}{2} [\cos(45^\circ + \omega t + 45^\circ - \omega t) + \cos(45^\circ + \omega t - 45^\circ + \omega t)] \\ &= \frac{1}{2} (\cos 90^\circ + \cos 2\omega t) = \frac{1}{2} \cos 2\omega t. \end{aligned}$$

Therefore, $I_3 = \frac{1}{2} I_0 \left[\frac{1}{2} \cos 2\omega t \right]^2 = \left(\frac{1}{2} I_0 \right) \frac{1}{4} \cos^2(2\omega t) = \frac{1}{8} I_0 \cos^2(2\omega t)$. Next, make use of the identity

$$\cos^2 u = \frac{1 + \cos 2u}{2}, \text{ which in this case means: } \cos^2(2\omega t) = \frac{1 + \cos(4\omega t)}{2}. \text{ So:}$$

$$I_3(t) = \frac{1}{8} I_0 \left(\frac{1 + \cos(4\omega t)}{2} \right) = \frac{1}{16} I_0 + \frac{1}{16} I_0 \cos(4\omega t).$$

So, the intensity of the light making it past the third polarizer will oscillate about the value of $I_0 / 16$ as a cosine function with amplitude of $I_0 / 16$ and an angular frequency that is four times the angular frequency at which the polarizer is rotating. Thus, the intensity oscillates between a minimum value of zero (when polarizer 2 is parallel to either polarizer 1 or polarizer 3) and a maximum value of $I_0 / 8$ when polarizer 2 is at 45° with polarizer 1 and 3. The result is thus consistent with the $I_0 / 8$ result of Example 31.4, where polarizer 2 was at a fixed angle of 45° .

(b) The transmission axis of a polarizer is a direction in the plane of the polarizer, not a single specific line in the plane of the polarizer. Therefore, moving the second polarizer parallel to itself in any direction will not change anything for the light passing through the second polarizer. Light that is incident on the first polarizer but that does not pass through the second polarizer will not pass through the third polarizer at all. In other words, if the light is initially incident on the total surface area of the first polarizer, the total amount of light (i.e. number of photons) that passes through the third polarizer after the second polarizer is displaced by a distance $d < R$, will be proportional to the fraction of surface area of overlap between all three polarizers.

- 31.12.** Charge moving up and down along the antenna creates an electric dipole on the antenna. This produces an electric field along $\pm\vec{z}$ at point A (parallel to the antenna). From Ampere's law, the current produces a magnetic field along $\pm\vec{x}$ at point A. Since radiation is moving along the $\pm\vec{y}$ direction away from the antenna, the possible directions for \vec{E} and \vec{B} are:
- (a) \vec{E} in positive \vec{z} -direction and \vec{B} in the positive \vec{x} -direction.
- (b) \vec{E} in the negative \vec{z} -direction and \vec{B} in the negative \vec{x} -direction.
- See Figure 31.16 as a visual aid.
- 31.13.** Assuming that the randomly polarized light source (the sun) was replaced by a polarized source, the results are still correct since for randomly polarized light, the average of E^2 is the same as the average of E^2 for a polarized light.
- 31.14.** A magnetic monopole is a magnet with only one pole. The magnetic field produced by the monopole is similar to the electric field produced by an electric charge. The magnetic field vector is directed radially outward, that is, $\vec{B} \propto \vec{r}$. If a charged particle is moving parallel to \vec{r} , its motion is not affected by the magnetic field. However, if the particle is moving perpendicular to \vec{r} , then its motion is affected and there is a perpendicular force to its velocity producing a helical motion.
- 31.15.** Both signals would arrive at the same time, since the speed of both signals is the speed of light. This is correct provided there is no medium between the Earth and the Moon. It is known that the speed of light in a medium depends on its refractive index. The refractive index of the medium depends also on the frequency of light.
- 31.16.**



Ampere's law is defined as:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 \oint_S \vec{J} \cdot d\vec{A},$$

where \vec{J} is a charge density within the surface. Since the surface is closed, the Amperian loop can be made very small, such that $\oint \vec{B} \cdot d\vec{S} = 0$. The integral $\oint \vec{J} \cdot d\vec{A}$ represents the net rate of charge transport out of the region bounded by the surface, S. In a static situation, the charge in the region is constant and the integral is $\oint \vec{J} \cdot d\vec{A} = 0$. For a dynamical situation, the integral must be equal to the negative rate of change of charge in the region, that is,

$$\oint \vec{J} \cdot d\vec{A} = -\frac{dQ_{\text{in}}}{dt} \neq 0.$$

Therefore, there is inconsistency in Ampere's law. The Maxwell-Ampere law, however, take the form:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 \oint_S \vec{J} \cdot d\vec{A} + \mu_0 \epsilon_0 \frac{d}{dt} \oint_S \vec{E} \cdot d\vec{A}.$$

Applying Gauss's law, it is found that: $-\mu_0 \frac{dQ_{\text{in}}}{dt} + \mu_0 \frac{dQ_{\text{in}}}{dt} = 0$. Therefore, the Ampere-Maxwell law is

satisfied. For Faraday's law: $\oint \vec{E} \cdot d\vec{S} = -\frac{d}{dt} \oint_S \vec{B} \cdot d\vec{A}$. Both sides give zero since Gauss' law states:

$\oint_S \vec{B} \cdot d\vec{A} = 0$. That is, there is no magnetic charge in the region enclosed by S . Therefore, there is no inconsistency.

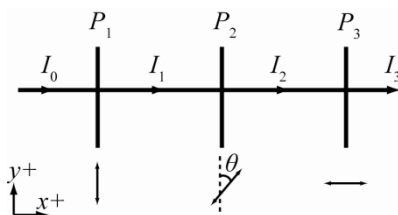
31.17. According to Maxwell's equations, the velocity of light always has a fixed value regardless of the observer's speed. This is in direct contradiction to Newton's laws of motion based on the Galilean addition law for velocities. According to the Galilean addition law, the velocity of light should not be the same in all inertial frames. Therefore, Maxwell's equations and Newton's laws of motion are mutually inconsistent.

31.18. Our vision begins with chemical reactions in the rod and cone cells of the retinas of our eyes, which release neurotransmitters in response to electromagnetic radiation in the visible range. These are resonant processes –the transfer of energy from the electromagnetic field to the nervous system is enhanced by a matching of frequencies. As in all resonances, high amplification can only be achieved in a narrow resonance peak. In order to achieve the sensitivity necessary for seeing, a narrow bandwidth is needed. Therefore, the narrow frequency band is necessary for our eyes to have sufficient sensitivity. This sensitivity allows seeing in high and low intensity situations. This is the reason that it is impossible to have a wide range of frequencies that can be seen.

31.19. (a) From energy conservation, the power per unit area or intensity of radiation from a point source must be inversely proportional to r^2 .
 (b)The radiation field falls off with distance at the same rate as the electrostatic field of a point charge which falls off according to $E = kQ / r^2$.

31.20. As discussed in Section 31.10, LCD displays use polarizing filters in one of their display components. Therefore, the light emitted by the LCD is a polarized light. Since some sunglasses also have polarizing filters for their lenses, the intensity of LCD light passing through the glass varies as the sunglasses are rotated. It can be concluded that the sunglasses must be polarizing the light.

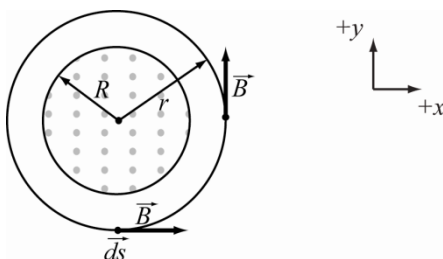
31.21.



Since the light reaching P_2 is polarized, the transmitted intensity is given by $I_2 = I_1 \cos^2 \theta$. Similarly, the intensity passing through P_3 is given by $I_3 = I_2 \cos^2 (90^\circ - \theta) = I_1 \cos^2 \theta \sin^2 \theta = \sin^2 (2\theta) / 4$. Therefore, as the intermediate filter is rotated, the intensity of light passing through the polarizers will increase to a maximum at $\theta = 45^\circ$ and then it will decrease to zero at $\theta = 90^\circ$ and it will continue in this pattern every 45° increase in the rotation of the second polarizer.

Exercises

31.22.



Applying Maxwell's law of induction along a closed loop of radius, r , gives:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}, \text{ where } \Phi_E \text{ is given by } \Phi_E = EA = E\pi R^2.$$

Substituting the expression for Φ_E into the equation for Maxwell's law gives:

$$B \oint dS = \mu_0 \epsilon_0 \pi R^2 \frac{dE}{dt} \Rightarrow B(2\pi r) = \mu_0 \epsilon_0 \pi R^2 \frac{dE}{dt}.$$

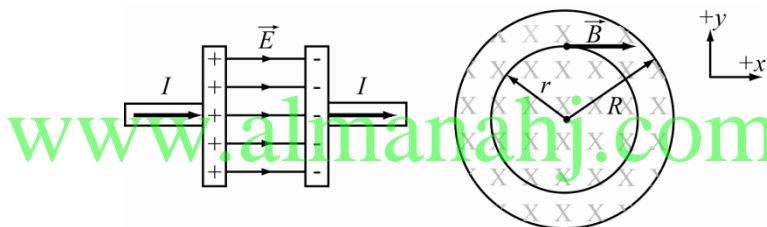
Thus, the magnetic field is: $B = \frac{1}{2} \mu_0 \epsilon_0 \frac{R^2}{r} \left(\frac{dE}{dt} \right)$. Substituting $R = 0.0600$ m, $r = 0.100$ m and $dE/dt = 10.0$ V/m s yields:

$$B = \frac{1}{2} (4\pi \cdot 10^{-7} \text{ H/m}) (8.85 \cdot 10^{-12} \text{ F/m}) \left(\frac{0.0600 \text{ m}}{0.100 \text{ m}} \right)^2 (10.0 \text{ V/m s}) = 2.00 \cdot 10^{-18} \text{ T}.$$

Because dE/dt is positive, the direction of \vec{B} is counterclockwise, as shown in the figure above.

- 31.23. THINK:** A magnetic field can be produced by a current and by induction due to a change in an electric flux. To solve this problem, use the Maxwell-Ampere law. There is no current between the plates, but there is a change in the electric flux. The wire carries a current $i = 20.0$ A. The parallel plate capacitor has radius $R = 4.00$ cm, and separation $s = 2.00$ mm. The radius of interest is $r = 1.00$ cm from the center of the parallel plates.

SKETCH:



RESEARCH: Since there is no current between the capacitor plates, the Maxwell-Ampere law becomes:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}.$$

SIMPLIFY: Applying this law along a circular Amperian loop with a radius, $r \leq R$, as shown above. Since \vec{B} is parallel to $d\vec{S}$, the left-hand side of the above equation is $\oint \vec{B} \cdot d\vec{S} = B \oint dS = B2\pi r$. Assuming the electric field, \vec{E} , is uniform between the capacitor plates and directed perpendicular to the plates, the electric flux through the loop is $\Phi_E = EA_r = E\pi r^2$. Thus, the Ampere-Maxwell law becomes:

$$B(2\pi r) = \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}.$$

Therefore, the magnetic field is: $B = \left(\frac{\mu_0 \epsilon_0 r}{2} \right) \frac{dE}{dt}$. Since the electric field of the capacitor is $E = \sigma / \epsilon_0$, the rate of change of the electric field is given by:

$$\frac{dE}{dt} = \frac{d}{dt} (\sigma / \epsilon_0) = \left(\frac{1}{\epsilon_0} \right) \frac{d\sigma}{dt} = \left(\frac{1}{\epsilon_0} \right) \frac{d}{dt} (q / A_R) = \left(\frac{1}{\epsilon_0 A_R} \right) \frac{dq}{dt}.$$

Since $i = dq/dt$, $\frac{dE}{dt} = \frac{i}{\epsilon_0 \pi R^2}$. Using this result, the magnetic field is: $B = \left(\frac{\mu_0 \epsilon_0 r}{2} \right) \frac{i}{\epsilon_0 \pi R^2} = \left(\frac{\mu_0 i}{2\pi R^2} \right) r$.

CALCULATE: $B = \frac{(4\pi \cdot 10^{-7} \text{ H/m})(20.0 \text{ A})}{2\pi (0.0400 \text{ m})^2} (0.0100 \text{ m}) = 2.50 \cdot 10^{-5} \text{ T}$

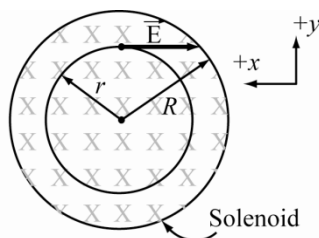
ROUND: Three significant figures are required: $B = 2.50 \cdot 10^{-5}$ T.

DOUBLE-CHECK: This is the same as calculating a magnetic field inside a wire with radius, R . Applying Ampere's law gives:

$$B = \frac{\mu_0}{2\pi r} \left(\frac{\pi r^2}{\pi R^2} \right) i = \left(\frac{\mu_0 i}{2\pi R^2} \right) r. \text{ This is the same result as above.}$$

- 31.24. THINK:** To determine the electric field, apply Faraday's law of induction. The solenoid is 20.0 cm long, 2.00 cm in radius, and has 500. turns. The current varies from 3.00 A to 1.00 A in 0.100 s.

SKETCH:



RESEARCH: The magnetic field inside a solenoid is given by $B = \mu_0 ni = \mu_0 Ni / L$. Applying Faraday's law along a loop with radius, r , gives:

$$E = - \left(\frac{1}{2\pi r} \right) \frac{d\Phi_B}{dt}.$$

SIMPLIFY: Substituting $\Phi_B = BA = \mu_0 Ni\pi r^2 / L$ into the above equation yields:

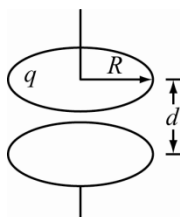
$$E = - \left(\frac{1}{2\pi r} \right) \left(\frac{\mu_0 N \pi r^2}{L} \right) \frac{di}{dt} = - \left(\frac{\mu_0 N r}{2L} \right) \frac{di}{dt} = - \left(\frac{\mu_0 N r}{2L} \right) \left(\frac{i_2 - i_1}{\Delta t} \right).$$

CALCULATE: $E = - \left(\frac{(4\pi \cdot 10^{-7} \text{ H/m})(500.)(0.0100 \text{ m})}{2(0.200 \text{ m})} \right) \left(\frac{1.00 \text{ A} - 3.00 \text{ A}}{0.100 \text{ s}} \right) = 3.142 \cdot 10^{-4} \text{ V/m}$

ROUND: Keeping three significant figures: $E = 3.14 \cdot 10^{-4}$ V/m.

DOUBLE-CHECK: The direction of the induced electric field must be such that the magnetic field induced by the current opposes the change in magnetic flux. Because the magnetic flux is decreasing, the induced magnetic field will be in the same direction as the original magnetic field. The fact that the calculated electric field is positive confirms that this requirement is satisfied.

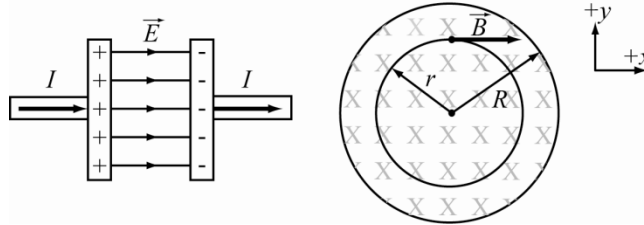
- 31.25.**



The displacement current i_d produced by a rate of change in the electric field of a parallel plate capacitor is $i_d = \epsilon_0 d\Phi_E / dt$. The flux Φ_E is given by $\Phi_E = \sigma A / \epsilon_0 = q / \epsilon_0$. Therefore, the displacement current is:

$$i_d = \epsilon_0 \frac{1}{\epsilon_0} \frac{dq}{dt} = 10.0 \mu\text{A}.$$

- 31.26. THINK** This problem is similar to problem 31.21 except that here, the rate of change of the potential difference across the capacitor is given. In order to get the induced magnetic field, apply Maxwell's law of induction. The parallel plate capacitor has radius $R = 10.0$ cm and separation $d = 5.00$ mm. The potential is increasing at a rate of 1.20 kV/s. The radius of interest is $r = 4.00$ cm from the center of the capacitor.
SKETCH:



RESEARCH: Applying Maxwell's law of induction along a circular loop with a radius, $r \leq R$, and assuming a uniform electric field yields $\oint \vec{B} \cdot d\vec{S} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$.

SIMPLIFY: $\oint \vec{B} \cdot d\vec{S} = B2\pi r = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \epsilon_0 \frac{dEA}{dt} = \mu_0 \epsilon_0 A \frac{d(V/d)}{dt} = \frac{\mu_0 \epsilon_0 A}{d} \frac{dV}{dt}$

The magnetic field is: $B = \left(\frac{\mu_0 \epsilon_0 \pi r^2}{2\pi r d} \right) \cdot \frac{d(\Delta V)}{dt} = \left(\frac{\mu_0 \epsilon_0 r}{2d} \right) \frac{d(\Delta V)}{dt}$.

CALCULATE: $B = \left(\frac{(4\pi \cdot 10^{-7} \text{ H/m})(8.85 \cdot 10^{-12} \text{ F/m})(0.0400 \text{ m})}{2(5.00 \cdot 10^{-3} \text{ m})} \right) 1.20 \cdot 10^3 \text{ V/s} = 5.338 \cdot 10^{-14} \text{ T}$

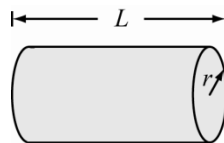
ROUND: Rounding to three significant figure gives $B = 5.34 \cdot 10^{-14} \text{ T}$.

DOUBLE-CHECK: Treating the area between the parallel plates as a solid conductor carrying a current of magnitude equal to the displacement current, $i_d = \epsilon_0 A dE/dt$, the problem becomes one of finding the magnetic field inside a current carrying wire. Applying Ampere's Law,

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2} \right) r = \left(\frac{\mu_0 \epsilon_0 A dE/dt}{2\pi R^2} \right) r = \left(\frac{\mu_0 \epsilon_0 (\pi R^2) dV/dt}{2\pi R^2 d} \right) r = \frac{\mu_0 \epsilon_0}{2d} \frac{dV}{dt} r, \text{ which is the same result as that}$$

obtained by applying Maxwell's law of induction.

- 31.27. THINK:** To determine the displacement current, the electric field inside the conductor is needed.
SKETCH:



RESEARCH: The displacement current is defined as: $i_d = \epsilon_0 d\Phi_E/dt$. The electric flux inside the conductor is: $\Phi_E = EA = (V/L)A$.

SIMPLIFY: Since $V = iR$, the electric flux becomes $\Phi = iRA/L$. Therefore, the displacement current is:

$$i_d = \epsilon_0 R \left(\frac{A}{L} \right) \frac{di}{dt}$$

Using $R = \rho L/A$ or $\rho = RA/L$, the displacement current simplifies to: $i_d = \epsilon_0 \rho \frac{di}{dt}$.

CALCULATE: Not required.

ROUND: Not required.

DOUBLE-CHECK: Since the current depends in part on the resistance of the current carrying conductor, and the resistance depends on the geometry and resistivity of the material, it makes sense that the current is some function of the resistivity.

- 31.28.** The amplitude of the B field of an electromagnetic field is related to the electric field by $B = E/c$. Therefore,

$$B = \frac{250. \text{ V/m}}{3.00 \cdot 10^8 \text{ m/s}} = 8.33 \cdot 10^{-7} \text{ T.}$$

- 31.29.** The distance traveled by light is given by:

$$x = c\Delta t = 3.00 \cdot 10^8 \text{ m/s} (1.00 \cdot 10^{-9} \text{ s}) = 0.300 \text{ m} = 0.300 \text{ m} \left(3.28 \frac{\text{ft}}{\text{m}} \right) = 0.984 \text{ ft.}$$

- 31.30.** The interval of time taken by light to travel:

(a) from the Moon to the Earth is: $\Delta t = \frac{d}{c} = \frac{3.84 \cdot 10^8 \text{ m}}{3.00 \cdot 10^8 \text{ m/s}} = 1.28 \text{ s,}$

(b) from the Sun to the Earth is: $\Delta t = \frac{1.50 \cdot 10^{11} \text{ m}}{3.00 \cdot 10^8 \text{ m/s}} = 500 \text{ s} = 8.33 \text{ min,}$

(c) Here we first calculate the time from Jupiter to the Sun:

Perihelion: $\Delta t = \frac{7.41 \cdot 10^{11} \text{ m}}{3.00 \cdot 10^8 \text{ m/s}} = 2470 \text{ s}$

Apehilion: $\Delta t = \frac{8.17 \cdot 10^{11} \text{ m}}{3.00 \cdot 10^8 \text{ m/s}} = 2723 \text{ s}$

The shortest time is then the time from Jupiter at perihelion to the Earth when it is on the same side, which is $2470 - 500 = 1970 \text{ s} = 32.8 \text{ min}$. The longest time is the time from Jupiter at aphelion to the Earth on the other side, which is $2723 + 500 = 3223 \text{ s} = 53.7 \text{ min}$.

- 31.31.** (a) The time delay from New York to Baghdad by cable is $\Delta t = \frac{d}{c} = \frac{1 \cdot 10^7 \text{ m}}{3.00 \cdot 10^8 \text{ m/s}} = 0.03 \text{ s.}$

(b) The time delay via satellite is given by $\Delta t = d/c$. The distance, d , is given by twice the distance from New York to the satellite, that is, $d = 2\sqrt{(36000 \text{ km})^2 + (5000 \text{ km}/2)^2} = 2 \cdot 36345 \text{ km} = 72691 \text{ km}$. The time delay is:

$$\Delta t = \frac{7.269 \cdot 10^7 \text{ m}}{3.00 \cdot 10^8 \text{ m/s}} = 0.24 \text{ s.}$$

When the signal travels by the cable, the time delay is very short, so it is not noticeable. However, the time delay for the signal traveling via satellite is about a quarter of a second. This means in a conversation, Alice will find that she receives a response from her fiancé after 0.5 s, which is quite noticeable.

- 31.32. THINK:** The speed of electromagnetic waves in a vacuum is different from the speed of such waves in different media. The difference depends on the dielectric constant, κ , and the relative permeability, κ_m , of the material.

SKETCH: Not required.

RESEARCH: The speed of electromagnetic waves in a material is $v = 1/\sqrt{\mu\epsilon}$ and the speed in a vacuum is $c = 1/\sqrt{\mu_0\epsilon_0}$. The permittivity is $\epsilon = \kappa\epsilon_0$, and the permeability is $\mu = \kappa_m\mu_0$.

SIMPLIFY: The ratio of the speed of electromagnetic waves in a vacuum to the speed in a material is:

$$\frac{c}{v} = \frac{1/\sqrt{\mu_0\epsilon_0}}{1/\sqrt{\mu\epsilon}} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} = \sqrt{\kappa\kappa_m}. \text{ This ratio is the index of refraction.}$$

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE CHECK: The calculated ratio is the index of refraction, which is a measure of how much the speed of light, or other electromagnetic waves, is reduced in a medium compared to the speed in a vacuum.

- 31.33.** The relation between the wavelength of light and the frequency is $\lambda f = c$. Therefore, the frequencies for the wavelengths of 400 nm and 700 nm are:

$$f_1 = \frac{3.00 \cdot 10^8 \text{ m/s}}{400 \cdot 10^{-9} \text{ m}} = 7.5 \cdot 10^{14} \text{ Hz} \quad \text{and} \quad f_2 = \frac{3.00 \cdot 10^8 \text{ m/s}}{700 \cdot 10^{-9} \text{ m}} = 4.3 \cdot 10^{14} \text{ Hz}.$$

The range of frequencies is $4 \cdot 10^{14} \text{ Hz}$ to $8 \cdot 10^{14} \text{ Hz}$.

- 31.34.** Using the relation between frequency and wavelength, the operating frequency of the signal of a cell phone is $f = c/\lambda$. Since $L = \lambda/4$, the frequency is:

$$f = \frac{c}{4L} = \frac{3.00 \cdot 10^8 \text{ m/s}}{4(0.080 \text{ m})} = 9.4 \cdot 10^8 \text{ Hz} = 940 \text{ MHz}.$$

- 31.35.** **THINK:** To solve this problem, the frequency of oscillation of an RLC circuit must be determined. The circuit has a capacitor $C = 2.0 \cdot 10^{-12} \text{ F}$, and must have a resonance frequency such that it will generate a radio wave with wavelength $\lambda = 150 \text{ m}$.

SKETCH: A sketch is not required.

RESEARCH: The angular frequency of the RLC circuit in resonance is $\omega_0 = 1/\sqrt{LC}$.

SIMPLIFY: Using $\omega_0 = 2\pi f$ and $f = c/\lambda$, the above equation becomes: $\frac{2\pi c}{\lambda} = \frac{1}{\sqrt{LC}}$. The inductance

required in the RLC circuit is $L = \frac{\lambda^2}{(2\pi c)^2 C}$.

$$\text{CALCULATE: } L = \frac{(150 \text{ m})^2}{[2\pi(3.00 \cdot 10^8 \text{ m/s})]^2 (2.0 \cdot 10^{-12} \text{ F})} = 0.00317 \text{ H}$$

ROUND: Rounding to two significant figures yields $L = 3.2 \text{ mH}$.

DOUBLE-CHECK: A wavelength of 150 m corresponds to a frequency of $2 \cdot 10^6 \text{ Hz}$. Such a large frequency necessarily requires a fairly small inductance.

- 31.36.** **THINK:** The radio frequencies given are: $f_1 = 91.1 \text{ MHz}$, $f_2 = 91.3 \text{ MHz}$, and $f_3 = 91.5 \text{ MHz}$. To determine the wavelength width of the band-pass filter used in a radio receiver, the wavelengths of the three radio frequencies are required.

SKETCH: A sketch is not required.

RESEARCH: Wavelength is related to frequency by $\lambda = c/f$. The maximum bandwidth required to distinguish between two adjacent frequencies is given by $\lambda_{12} = 2(\lambda_1 - \lambda_2)$ for f_1 and f_2 , and $\lambda_{23} = 2(\lambda_2 - \lambda_3)$ for f_2 and f_3 . Thus, the maximum allowable bandwidth to distinguish all three frequencies is $\Delta\lambda = \min(\Delta\lambda_{12}, \Delta\lambda_{23})$.

SIMPLIFY: Simplification is not necessary.

CALCULATE: The corresponding wavelengths of the three radio frequencies are given by:

$$\lambda_1 = \frac{c}{f_1} = \frac{3.00 \cdot 10^8 \text{ m/s}}{91.1 \cdot 10^6 \text{ Hz}} = 3.293 \text{ m}, \quad \lambda_2 = \frac{c}{f_2} = \frac{3.00 \cdot 10^8 \text{ m/s}}{91.3 \cdot 10^6 \text{ Hz}} = 3.286 \text{ m}, \quad \text{and} \quad \text{finally,}$$

$$\lambda_3 = \frac{c}{f_3} = \frac{3.00 \cdot 10^8 \text{ m/s}}{91.5 \cdot 10^6 \text{ Hz}} = 3.279 \text{ m}.$$

The differences of two adjacent wavelengths are:

$$\Delta\lambda_{12} = 2(3293 \text{ mm} - 3286 \text{ mm}) = 14 \text{ mm} \text{ and } \Delta\lambda_{23} = 2(3286 \text{ mm} - 3279 \text{ mm}) = 14 \text{ mm}.$$

Therefore, the maximum allowable wavelength bandwidth is $\Delta\lambda = 14 \text{ mm}$.

ROUND: Rounding is not necessary.

DOUBLE-CHECK: A larger wavelength width in the band pass filter would allow overlap between two signals, resulting in interference. This result is reasonable.

31.37. The magnitude of a Poynting vector is given by: $S = \frac{\text{Power}}{\text{Spherical Area}} = \frac{\text{Power}}{4\pi R^2}$. Therefore, the magnitudes

of the Poynting vectors are:

$$(a) S = \frac{1.5 \text{ W}}{4\pi(0.30 \text{ m})^2} = 1.3 \text{ W/m}^2,$$

$$(b) S = \frac{1.5 \text{ W}}{4\pi(0.32 \text{ m})^2} = 1.2 \text{ W/m}^2,$$

$$(c) S = \frac{1.5 \text{ W}}{4\pi(1.00 \text{ m})^2} = 0.12 \text{ W/m}^2.$$

31.38. (a) The electric field experienced by an electron is:

$$E = \frac{kq}{r^2} = \frac{(8.99 \cdot 10^9 \text{ N m}^2 / \text{C}^2)(1.602 \cdot 10^{-19} \text{ C})}{(0.050 \cdot 10^{-9} \text{ m})^2} = 5.761 \cdot 10^{11} \text{ V/m} \approx 5.8 \cdot 10^{11} \text{ V/m}.$$

(b) The intensity of a laser beam is related to the rms electric field by:

$$I = \frac{1}{\mu_0 c} E_{\text{rms}}^2 = \frac{(5.761 \cdot 10^{11} \text{ V/m})^2}{(4\pi \cdot 10^{-7} \text{ H/m})(3.00 \cdot 10^8 \text{ m/s})} = 8.8 \cdot 10^{20} \text{ W/m}^2.$$

31.39. The intensity of the laser beam is $I = P/A$. This intensity is related to the amplitude of the electric field by $I = E^2 / (2\mu_0 c)$. Therefore, the amplitude of the electric field in the beam is:

$$E = \sqrt{\frac{2\mu_0 c P}{A}} = \sqrt{\frac{2(4\pi \cdot 10^{-7} \text{ H/m})(3.00 \cdot 10^8 \text{ m/s})(3.00 \cdot 10^3 \text{ W})}{\pi(0.500 \cdot 10^{-3} \text{ m})^2}} = 1.697 \cdot 10^6 \text{ V/m} \approx 1.70 \cdot 10^6 \text{ V/m}.$$

31.40. The electric field of an electromagnetic radiation is related to its magnetic field by $E = cB$. Therefore, the maximum E in the region is $E_m = cB_m = (3.00 \cdot 10^8 \text{ m/s})(0.00100 \text{ T}) = 3.00 \cdot 10^5 \text{ V/m}$. The period of

oscillation is: $T = \frac{1}{f} = \frac{1}{1 \text{ Hz}} = 1 \text{ s}$. The magnitude of the Poynting vector is:

$$S_m = \frac{1}{\mu_0} E_m B_m = \frac{(3.00 \cdot 10^5 \text{ V/m})(0.001 \text{ T})}{4\pi \cdot 10^{-7} \text{ H/m}} = 2.3873 \cdot 10^8 \text{ W/m}^2 = 2.39 \cdot 10^8 \text{ W/m}^2.$$

31.41. The average value of the Poynting vector, S_{ave} , is:

$$S_{\text{ave}} = \frac{1}{2\mu_0 c} E_m^2 = \frac{(100. \text{ V/m})^2}{2(4\pi \cdot 10^{-7} \text{ H/m})(3.00 \cdot 10^8 \text{ m/s})} = 13.3 \text{ W/m}^2.$$

(a) The average energy density is: $u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} (8.85 \cdot 10^{-12} \text{ C}^2 / \text{N m})(100. \text{ V/m})^2 = 4.43 \cdot 10^{-8} \text{ J/m}^3$.

(b) The amplitude of the magnetic field is: $B = \frac{E}{c} = \frac{100. \text{ V/m}}{3.00 \cdot 10^8 \text{ m/s}} = 3.33 \cdot 10^{-7} \text{ T}$.

31.42. THINK: The maximum electric field of a beam of light is given as $E_m = 3.0 \cdot 10^6$ V/m.

SKETCH: A sketch is not necessary.

RESEARCH:

(a) The magnitude of a magnetic field is related to the magnitude of an electric field by $B = E / c$.

(b) The intensity of the wave is given by $I = E_m^2 / (2\mu_0 c)$.

(c) If the electric field is above this maximum value, the air will be ionized by the presence of the electric field. The energy of the wave will be dissipated in the ionized air.

SIMPLIFY: Simplification is not necessary.

CALCULATE:

$$(a) B = \frac{3.0 \cdot 10^6 \text{ V/m}}{3.00 \cdot 10^8 \text{ m/s}} = 1.0 \cdot 10^{-2} \text{ T}$$

$$(b) I = \frac{(3.0 \cdot 10^6 \text{ V/m})^2}{2(4\pi \cdot 10^{-7} \text{ H/m})(3.00 \cdot 10^8 \text{ m/s})} = 1.19 \cdot 10^{10} \text{ W/m}^2$$

ROUND: Round the results to two significant figures.

$$(a) B = 1.0 \cdot 10^{-2} \text{ T}$$

$$(b) I = 1.2 \cdot 10^{10} \text{ W/m}^2$$

DOUBLE-CHECK: This is a very large magnetic field, and a very high intensity, as expected for a field at the breakdown threshold. The results are reasonable.

31.43. THINK: A laser beam has a power of 10.0 W and a beam diameter of 1.00 mm. Assume the intensity of the beam is the same throughout the cross section of the beam.

SKETCH: A sketch is not required.

RESEARCH:

(a) The intensity of the laser beam is given by $I = P/A$. Area $A = \pi r^2$.

(b) The intensity is related to the rms electric field by $I = E_{\text{rms}}^2 / (\mu_0 c) \Rightarrow E_{\text{rms}} = \sqrt{\mu_0 c I}$.

(c) The time-averaged Poynting vector is equal the intensity of the beam, $S_{\text{ave}} = I$.

$$(d) S(x, t) = \frac{[E(x, t)]^2}{\mu_0 c} \text{ and } E(x, t) = E_m \sin(kx - \omega t + \phi).$$

(e) The rms magnetic field is $B_{\text{rms}} = E_{\text{rms}} / c$.

SIMPLIFY:

(d) Substituting the expression for $E(x, t)$ gives: $S(x, t) = \frac{1}{\mu_0 c} E_m^2 \sin^2(kx - \omega t + \phi)$. Because $S(0, 0) = 0$,

take $\phi = 0$. Therefore, $S(x, t) = 2I \sin^2(kx - \omega t)$. Note that $\omega = 2\pi f = 2\pi c / \lambda$ and $k = 2\pi / \lambda$.

CALCULATE:

$$(a) I = \frac{10.0 \text{ W}}{\pi(0.500 \cdot 10^{-3} \text{ m})^2} = 1.2732 \cdot 10^7 \text{ W/m}^2$$

$$(b) E_{\text{rms}} = \sqrt{(4\pi \cdot 10^{-7} \text{ H/m})(3.00 \cdot 10^8 \text{ m/s})(1.2732 \cdot 10^7 \text{ W/m}^2)} = 6.932809 \cdot 10^4 \text{ V/m}$$

$$(c) S_{\text{ave}} = 1.2732 \cdot 10^7 \text{ W/m}^2$$

$$(d) S(x, t) = 2(1.2732 \cdot 10^7 \text{ W/m}^2) \sin^2 \left[\left(\frac{2\pi}{514.5 \cdot 10^{-9} \text{ m}} \right) x - \left(\frac{2\pi(3.00 \cdot 10^8 \text{ m/s})}{514.5 \cdot 10^{-9} \text{ m}} \right) t \right]$$

$$= 2.5464 \cdot 10^7 \text{ W/m}^2 \sin^2 \left[(1.22122 \cdot 10^7 \text{ m}^{-1}) x - (3.66366 \cdot 10^{15} \text{ Hz}) t \right]$$

$$(e) B_{\text{rms}} = \frac{6.932809 \cdot 10^4 \text{ V/m}}{3.00 \cdot 10^8 \text{ m/s}} = 2.30936 \cdot 10^{-4} \text{ T}$$

ROUND:

(a) $I = 1.27 \cdot 10^7 \text{ W/m}^2$. This intensity is much larger than the intensity of sunlight on Earth (1400 W/m^2).

$$(b) E_{\text{rms}} = 6.93 \cdot 10^4 \text{ V/m}$$

$$(c) S_{\text{ave}} = 1.27 \cdot 10^7 \text{ W/m}^2$$

$$(d) S(x, t) = 2.5464 \cdot 10^7 \text{ W/m}^2 \sin^2 \left((1.22122 \cdot 10^7 \text{ m}^{-1})x - (3.66366 \cdot 10^{15} \text{ Hz})t \right).$$

Rounding the coefficients to three significant figures,

$$S(x, t) = 2.55 \cdot 10^7 \text{ W/m}^2 \sin^2 \left((1.22 \cdot 10^7 \text{ m}^{-1})x - (3.66 \cdot 10^{15} \text{ Hz})t \right).$$

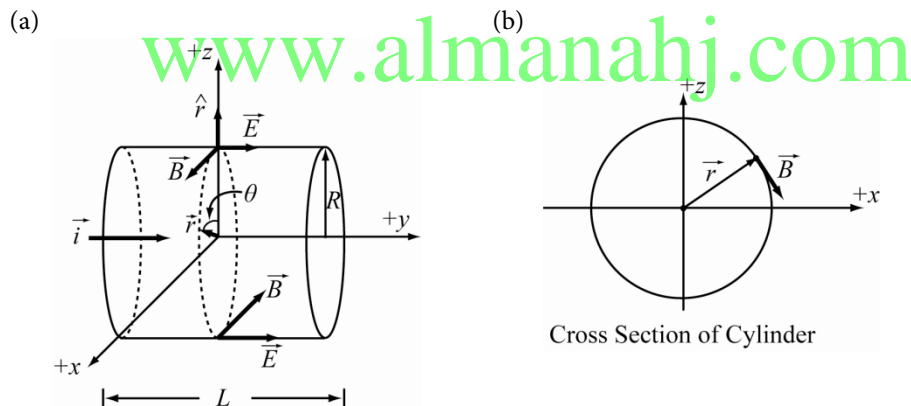
Note that for given values for x and t , it would be better to keep the unrounded coefficients and then round the calculated value of S .

$$(e) B_{\text{rms}} = 2.31 \cdot 10^{-4} \text{ T}$$

DOUBLE-CHECK: The laser has a very high power output in a very narrow beam. This is a desirable property in a laser. The results make sense.

31.44. THINK: The Poynting vector is proportional to $\vec{E} \times \vec{B}$. Assume that the electric field in a conductor is uniform. The conductor is placed along the y -axis and the current is flowing along the positive y -direction. This means the electric field is in the positive y -direction.

SKETCH:



RESEARCH: The Poynting vector is defined as $\vec{S} = \vec{E} \times \vec{B} / \mu_0$.

(a) The electric field on the surface of the conductor is $\vec{E} = (V/L)\hat{y}$. Assume a long cylindrical conductor and the magnetic field on the surface is:

$$\vec{B} = \frac{\mu_0 I}{2\pi R} \hat{\theta} = \frac{\mu_0 I}{2\pi R} (\cos\theta\hat{x} - \sin\theta\hat{z}).$$

(b) The integral of $\vec{S} \cdot d\vec{A}$ is: $\oint \vec{S} \cdot d\vec{A} = -S \oint dA$, since $d\vec{A} = dA\hat{r}$.

SIMPLIFY:

(a) The Poynting vector is the cross product:

$$\vec{S} = \frac{1}{\mu_0} \left(\frac{V}{L} \right) \left(\frac{\mu_0 I}{2\pi R} \right) \hat{y} \times (\cos\theta\hat{x} - \sin\theta\hat{z}) = \left(\frac{VI}{2\pi RL} \right) (-\cos\theta\hat{z} - \sin\theta\hat{x}) = \frac{VI}{2\pi RL} (-\hat{r}).$$

This means that the Poynting vector is directed toward the cylindrical conductor with a magnitude of $S = VI / (2\pi RL)$.

(b) Taking the integral over the surface of the cylinder:

$$\oint dA = 2\pi RL \Rightarrow \oint \vec{S} \cdot d\vec{A} = -S(2\pi RL) = -\frac{VI}{2\pi RL}(2\pi RL) = -VI = -IR_R I = -I^2 R_R.$$

Note that the subscript, R, is to distinguish that R_R means resistance.

CALCULATE: Not necessary.

ROUND: Not necessary.

DOUBLE-CHECK: Dimensional analysis shows that the units of the calculated result are W/m^2 , as required for the Poynting vector.

31.45. (a) The intensity above the Earth's atmosphere is $I = 1.40 \text{ kW/m}^2$.

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2 \Rightarrow E_{\text{rms}} = \sqrt{Sc\mu_0} = \sqrt{(1.40 \cdot 10^3 \text{ W/m}^2)(3.00 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ T m/A})}$$

$$E_{\text{rms}} = 726.49 \text{ V/m} \rightarrow E_{\text{max}} = \sqrt{2}(726.49 \text{ V/m}) = 1027.4 \text{ V/m.}$$

$$B = \frac{E}{c} = \frac{1027.4 \text{ V/m}}{3.00 \cdot 10^8 \text{ m/s}} = 3.4247 \cdot 10^{-6} \text{ T.}$$

$$E_{\text{max}} = 1030 \text{ V/m} = 1.03 \text{ kV/m}, B_{\text{max}} = 3.42 \cdot 10^{-6} \text{ T.}$$

(b) $P_r = \frac{I}{c} = \frac{1.00 \cdot 10^3 \text{ W/m}^2}{3.00 \cdot 10^8 \text{ m/s}} = 3.33333 \mu\text{Pa} \approx 3.33 \mu\text{Pa},$

$$P_r = \frac{F}{A} \Rightarrow F = P_r A = (3.33333 \cdot 10^{-6} \text{ Pa})(0.750 \text{ m}^2) = 2.50 \mu\text{N.}$$

31.46. (a) First, determine the force needed to accelerate a 10.0 ton spaceship by 1 m/s^2 . Newton's second law, $F = ma$, gives: $F = (10.0 \cdot 10^3 \text{ kg})(1.00 \text{ m/s}^2) = 1.00 \cdot 10^4 \text{ N}$. Now, the radiation pressure is given by $P_r = F/A$. The radiation pressure is related to the intensity of the radiation by $P_r = I/c$ (total absorption). Comparing the two equations for the radiation pressure gives:

$$\frac{F}{A} = \frac{I}{c} \Rightarrow A = \frac{Fc}{I} = \frac{(1.00 \cdot 10^4 \text{ N})(3.00 \cdot 10^8 \text{ m/s})}{1.40 \cdot 10^3 \text{ W/m}^2} = 2.14 \cdot 10^9 \text{ m}^2.$$

This area is large. Moreover, even if the scientists are able to get the astronauts to another planet, how do they get them back to Earth?

(b) For perfect reflection: $P_r = \frac{2I}{c} = \frac{F}{A} \Rightarrow A = \frac{Fc}{2I} = \frac{1}{2}(2.14 \cdot 10^9 \text{ m}^2) = 1.07 \cdot 10^9 \text{ m}^2.$

31.47. The net force on the sail is $F = A\Delta P$. The area, A , is $A = \pi R^2 = \pi(10.0 \cdot 10^3 \text{ m})^2 = 3.142 \cdot 10^8 \text{ m}^2$. The differential pressure, ΔP , is:

$$\Delta P = \frac{2I}{c} - \frac{I}{c} = \frac{I}{c}.$$

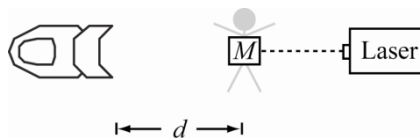
The intensity, I , is given by the Stefan-Boltzman law as:

$$I = \sigma T^4 = (5.67 \cdot 10^{-8} \text{ W/m}^2 \text{ K}^4)(2.725 \text{ K})^4 = 3.126 \cdot 10^{-6} \text{ W/m}^2.$$

$$\Rightarrow \Delta P = \frac{3.126 \cdot 10^{-6} \text{ W/m}^2}{3.00 \cdot 10^8 \text{ m/s}} = 1.042 \cdot 10^{-14} \text{ Pa}$$

$$\Rightarrow F = (3.142 \cdot 10^8 \text{ m}^2)(1.042 \cdot 10^{-14} \text{ Pa}) = 3.27 \cdot 10^{-6} \text{ N}$$

31.48. **THINK:** There will be a constant force on the astronaut due to the radiation pressure from the laser. This force can be determined from the laser power, and then the time required to reach the shuttle can be determined. $d = 20.0 \text{ m}$, $m = 100.0 \text{ kg}$ and $P = 100.0 \text{ W}$.

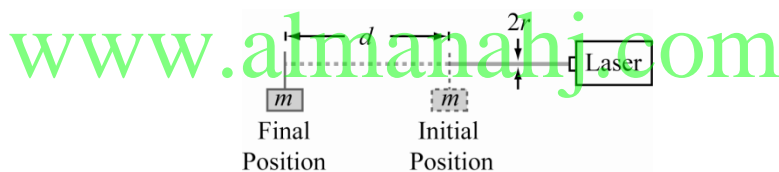
SKETCH:

RESEARCH: $P_r = \frac{2I}{c}$ (totally reflecting), $I = \frac{P}{A}$, $P_r = \frac{F}{A} = \frac{ma}{A}$, $x = \frac{1}{2}at^2$ (constant acceleration)

SIMPLIFY: $P_r = \frac{ma}{A} \Rightarrow a = \frac{P_r A}{m}$, $P_r = \frac{2I}{c} = \frac{2(P/A)}{c} \Rightarrow a = \frac{A}{m} \left(\frac{2P}{Ac} \right) = \frac{2P}{mc}$

$$x = d = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2d}{a}} = \sqrt{2d \left(\frac{mc}{2P} \right)} = \sqrt{\left(\frac{dmc}{P} \right)}$$

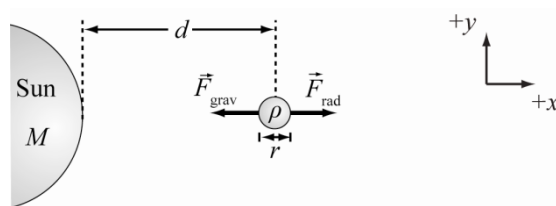
CALCULATE: $t = \sqrt{\frac{(20.0 \text{ m})(100.0 \text{ kg})(3.00 \cdot 10^8 \text{ m/s})}{100.0 \text{ W}}} = 7.746 \cdot 10^4 \text{ s}$
ROUND: $t = 7.75 \cdot 10^4 \text{ s} = 21.5 \text{ h}$
DOUBLE-CHECK: The time decreases as the laser power increases or the mass decreases. This is what would be expected.

- 31.49. THINK:** The applied force can be determined from the given data. From the force, the radiation pressure, and then the laser power used in the demonstration can be determined. $d = 2.00 \text{ mm}$, $t = 63.0 \text{ s}$, $m = 0.100 \text{ g}$ and $2r = 1.00 \text{ mm}$.

SKETCH:

RESEARCH: $I = P/A$, $A = \pi r^2$, $P_r = \frac{2I}{c} = \frac{ma}{A}$, $d = \frac{1}{2}at^2$
SIMPLIFY: $P = IA = I\pi r^2$, $I = \frac{mca}{2A} = \frac{mca}{2\pi r^2}$, $a = \frac{2d}{t^2} \Rightarrow P = \frac{mca}{2\pi r^2} \pi r^2 = \frac{mca}{2} = \frac{mc}{2} \left(\frac{2d}{t^2} \right) = \frac{mcd}{t^2}$
CALCULATE: $P = \frac{(0.100 \cdot 10^{-3} \text{ kg})(3.00 \cdot 10^8 \text{ m/s})(2.00 \cdot 10^{-3} \text{ m})}{(63.0 \text{ s})^2} = 1.512 \cdot 10^{-2} \text{ W}$
ROUND: $P = 1.51 \cdot 10^{-2} \text{ W} = 15.1 \text{ mW}$
DOUBLE-CHECK: Note that the result does not depend on the spot size of the laser. The power is all that matters. This result is reasonable.

- 31.50. THINK:** The radius of the particle can be determined if the required mass of the particle is known because the density is given. The mass of the particle can be determined by comparing the radiation force and the gravitational force. $\rho = 2000. \text{ kg/m}^3$, $d = 1.50 \cdot 10^{11} \text{ m}$, $M = 2.00 \cdot 10^{30} \text{ kg}$, $F_{\text{rad}} / F_{\text{grav}} = 1.00\% = 0.0100$, $I = 1400. \text{ W/m}^2$.

SKETCH:



RESEARCH: $F_{\text{grav}} = \frac{GMm}{d^2}$, $m = \frac{4}{3}\pi r^3 \rho$, $F_{\text{rad}} = P_r A$, $A = \pi r^2$, $P_r = 2I/c$.

SIMPLIFY: $F_{\text{grav}} = \frac{GMm}{d^2} = \frac{GM}{d^2} \left(\frac{4}{3}\pi r^3 \rho \right)$, $F_{\text{rad}} = P_r A = \frac{2I}{c} (\pi r^2)$

$$F_{\text{rad}} = 0.01 F_{\text{grav}} \Rightarrow \frac{2\pi r^2 I}{c} = 0.01 \frac{\left(\frac{4}{3}\pi r^3 \rho GM \right)}{d^2} \Rightarrow r = \left(\frac{2\pi I}{c} \right) \frac{d^2}{0.01 \left(\frac{4}{3}\pi \rho GM \right)} = \frac{3Id^2}{(0.01)2c\rho GM}$$

CALCULATE:

$$r = \frac{3(1400. \text{ W/m}^2)(1.500 \cdot 10^{11} \text{ m})^2}{(0.0100)2(3.00 \cdot 10^8 \text{ m/s})(2000. \text{ kg/m}^3)(6.673 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2})(2.00 \cdot 10^{30} \text{ kg})} = 5.901 \cdot 10^{-5} \text{ m}$$

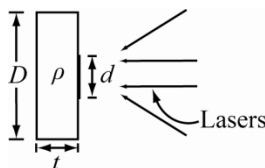
ROUND: $r = 59.0 \mu\text{m}$

DOUBLE-CHECK: If the radiation pressure on the particle is to be equal to the gravitational force on it, its radius would have to be smaller by a factor of 100. This indicates that very small particles would be pushed away from the sun, while more massive objects would be pulled toward the sun. This is consistent with observation. The result makes sense.

- 31.51. THINK:** Given the density and volume, the mass can be determined. Given the power and spot size of the laser, the intensity and the radiation pressure of the laser can be determined. To determine how many lasers are needed, calculate the total force required and divide this by the force per laser applied.

$$\rho = 1.00 \text{ mg/cm}^3, D = 2.00 \text{ mm}, t = 0.100 \text{ mm}, P = 5.00 \text{ mW}, d = 2.00 \text{ mm}.$$

SKETCH:



RESEARCH:

(a) The weight is given by $w = mg$, where $m = \pi(D/2)^2 t \rho$. Note that $1 \text{ mg/cm}^3 = 1 \text{ kg/m}^3$.

(b) $P_r = \frac{I}{c}$ (absorbing material), $I = \frac{P}{A}$, $A = \pi \left(\frac{d}{2} \right)^2$.

(c) $F_{\text{las}} = P_r A$. The number of lasers needed is given by $N = w / F_{\text{las}}$.

SIMPLIFY:

(a) $w = \frac{\pi D^2 t \rho g}{4}$

(b) $I = P \frac{4}{\pi d^2} = \frac{4P}{\pi d^2}$, $P_r = \frac{I}{c} = \frac{4P}{\pi d^2 c}$

(c) $F_{\text{las}} = P_r A = \frac{IA}{c} = \frac{P}{cA} A = \frac{P}{c}$, $N = \frac{w}{F_{\text{las}}} = \frac{wc}{P}$

CALCULATE:

$$(a) \quad w = \frac{\pi(2.00 \cdot 10^{-3} \text{ m})^2 (0.100 \cdot 10^{-3} \text{ m})(1.00 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}{4} = 3.082 \cdot 10^{-9} \text{ N}$$

$$(b) \quad I = \frac{4(5.00 \cdot 10^{-3} \text{ W})}{\pi(2.00 \cdot 10^{-3} \text{ m})^2} = 1.592 \cdot 10^3 \text{ W/m}^2, \quad P_r = \frac{I}{c} = \frac{1.592 \cdot 10^3 \text{ W/m}^2}{3.00 \cdot 10^8 \text{ m/s}} = 5.30510^{-6} \text{ N/m}^2$$

$$(c) \quad N = \frac{(3.082 \cdot 10^{-9} \text{ N})(3.00 \cdot 10^8 \text{ m/s})}{(5.00 \cdot 10^{-3} \text{ W})} = 184.9$$

ROUND:

$$(a) \quad w = 3.08 \cdot 10^{-9} \text{ N} = 3.08 \text{ nN}$$

$$(b) \quad I = 1.59 \text{ kW/m}^2, \quad P_r = 5.31 \mu\text{N/m}^2$$

$$(c) \quad N = 185 \text{ lasers}$$

DOUBLE-CHECK: Even though the object is very light, it would still require a large power output to produce enough radiation pressure to overcome the force of gravity.

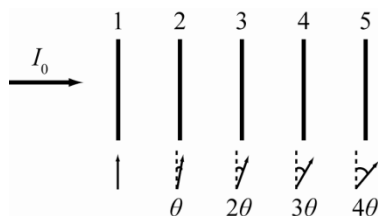
- 31.52.** The first filter is out of alignment by 15.0° with the incident light. The second filter is out of alignment by 30.0° with the incident light. The intensity of the transmitted light is $I = I_0 \cos^2 \theta_1 \cos^2 \theta_2 = (1.00) \cos^2(15.0^\circ) \cos^2(30.0^\circ) = 0.69976 \text{ W/m}^2 \approx 0.700 \text{ W/m}^2$.

31.53.



$$I = I_0 \cos^2 \theta = (10.0 \cdot 10^{-3} \text{ W}) \cos^2(90.0^\circ - 30.0^\circ) = (10^{-2} \text{ W}) \cos^2 60.0^\circ = 2.50 \text{ mW}$$

- 31.54. THINK:** Only half the intensity gets through the first polarizer since the incident light is un-polarized. After this, multiply the transmission for each polarizer to obtain the final intensity. $\theta = 10^\circ$.

SKETCH:

$$\text{RESEARCH: } I_1 = I_0 / 2, \quad I_n = I_{n-1} \cos^2 \theta \quad (n = 2, 3, 4, 5)$$

$$\text{SIMPLIFY: } I_2 = I_1 \cos^2 \theta = (I_0 \cos^2 \theta) / 2, \quad I_3 = I_2 \cos^2 \theta = (I_0 \cos^4 \theta) / 2, \quad I_4 = I_3 \cos^2 \theta = (I_0 \cos^6 \theta) / 2,$$

$$I_5 = I_4 \cos^2 \theta = (I_0 \cos^8 \theta) / 2$$

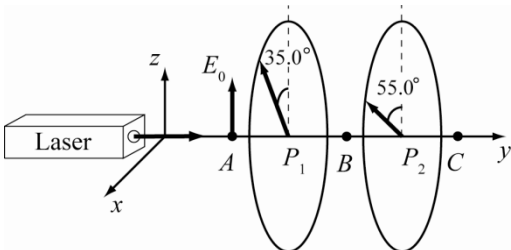
$$\text{CALCULATE: } I_5 = (I_0 \cos^8 10^\circ) / 2 = 0.4424 I_0$$

$$\text{ROUND: } I_5 = 0.442 I_0$$

DOUBLE-CHECK: Only 44.2% of the original intensity passes through the polarizers. The first polarizer decreases the intensity by 50%, but the subsequent polarizers allow the majority of the light to pass through due to the smaller angles. This is a reasonable result.

- 31.55. THINK:** First calculate the intensity of the light after it first passes through the two polarizers. Once the intensity is calculated, the magnitude of the electric and magnetic fields can be determined. The angles of the first and second polarizers are $\theta_1 = 35^\circ$ and $\theta_2 = 55^\circ$, respectively. The laser spot size diameter is $d = 1.00$ mm and the laser power is $P = 15.0$ mW.

SKETCH:



RESEARCH: $I_1 = I_0 \cos^2 \theta_1$, $I_2 = I_1 \cos^2 (\theta_2 - \theta_1)$, $I_0 = \frac{P}{A}$, $A = \pi \left(\frac{d}{2} \right)^2$, $I = \frac{1}{2} \left(\frac{E^2}{c\mu_0} \right)$, $\frac{E}{B} = c$

SIMPLIFY: $I_2 = I_1 \cos^2 (\theta_2 - \theta_1) = I_0 \cos^2 \theta_1 \cos^2 (\theta_2 - \theta_1)$, $I_0 = \frac{P}{A} = \frac{4P}{\pi d^2} \Rightarrow I_2 = \frac{4P}{\pi d^2} \cos^2 \theta_1 \cos^2 (\theta_2 - \theta_1)$

$E = \sqrt{2Ic\mu_0}$, $B = E/c$

CALCULATE: $I_2 = \frac{4(15.0 \cdot 10^{-3} \text{ W})}{\pi(1.00 \cdot 10^{-3} \text{ m})^2} \cos^2 (35^\circ) \cos^2 (55^\circ - 35^\circ) = 1.132 \cdot 10^4 \text{ W/m}^2$

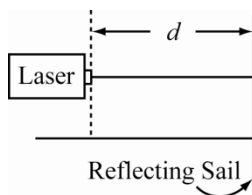
$E = \sqrt{2(1.132 \cdot 10^4 \text{ W/m}^2)(3.00 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ T m A}^{-1})} = 2.921 \cdot 10^3 \text{ V/m}$

$B = \frac{2.921 \cdot 10^3 \text{ V/m}}{3.00 \cdot 10^8 \text{ m/s}} = 9.737 \cdot 10^{-6} \text{ T}$

ROUND: $I_2 = 1.13 \cdot 10^4 \text{ W/m}^2$, $E = 2.92 \cdot 10^3 \text{ V/m}$, $B = 9.74 \cdot 10^{-6} \text{ T}$

DOUBLE-CHECK: The initial intensity of the laser light is about $1.9 \cdot 10^4 \text{ W/m}^2$. The initial electric and magnetic fields are also significantly larger. It is expected that some of the intensity of the laser beam would be blocked by the polarizers.

- 31.56.**



The total distance traveled by the beam is $2d$.

$$v = c = \frac{2d}{t} \Rightarrow d = \frac{ct}{2} = \frac{(3.00 \cdot 10^8 \text{ m/s})(50.0 \cdot 10^{-3} \text{ s})}{2} = 7.50 \cdot 10^6 \text{ m}$$

- 31.57.** The total average power on all of the photovoltaic panels is equal to the average power times the area:

$$(300. \text{ W/m}^2)(3.00 \text{ m})(8.00 \text{ m}) = 7.20 \text{ kW.}$$

The total electrical energy for 30 days is the product of the total power times 30 days times the efficiency for converting the solar power into electricity: $E_{\text{total}} = (7.20 \text{ kW})(720 \text{ h})(0.100) = 518 \text{ kW h}$. This is enough for a small energy-efficient home.

- 31.58.** Use the fact that radiation pressure scales with intensity, and intensity goes linearly with the intrinsic power of the star, and inversely as the square of the distance away. Thus, if it is known that at the Earth's orbit, the intensity is 1.35 kW/m^2 , this intensity can be related to the radiation pressure at Earth's orbit, and then to the hypothetical distance of Uranus' orbit away from Betelgeuse. Note that the radius of the earth's orbit is $r_E \approx 1.50 \cdot 10^{11} \text{ m} = 1 \text{ AU}$ and Uranus's orbit is $r_U \approx 2.88 \cdot 10^{12} \text{ m} = 19.2 \text{ AU}$.

$$P_{rE} \propto \frac{P_S}{r_E^2}, \quad P_{rB} \propto \frac{P_B}{r_U^2}$$

$$R = \frac{P_{rU}}{P_{rE}} = \frac{P_B r_E^2}{r_U^2 P_S} = \frac{P_B}{P_S} \left(\frac{r_E}{r_U} \right)^2 = 10^4 \left(\frac{1}{19.2} \right)^2 = 27.1 \Rightarrow I_{rU} = 27.1 \cdot I_{rE} = (27.1)(1350 \text{ W/m}^2) \\ = 3.66 \cdot 10^4 \text{ W/m}^2$$

For a perfect absorber we have: $P_{rU} = \frac{I_{rU}}{c} = \frac{36600 \text{ W/m}^2}{3.00 \cdot 10^8 \text{ m/s}} = 1.22 \cdot 10^{-4} \text{ N/m}^2$, and for a perfect reflector we

have: $P_{rU} = \frac{2I_{rU}}{c} = \frac{2 \cdot 36600 \text{ W/m}^2}{3.00 \cdot 10^8 \text{ m/s}} = 2.44 \cdot 10^{-4} \text{ N/m}^2$. Since no information is provided on the reflectivity

of the surface being acted on by the radiation, the final solution is: $1 \cdot 10^{-4} \text{ N/m}^2 < P_{rU} < 2 \cdot 10^{-4} \text{ N/m}^2$,

31.59. $S = \left(\frac{\text{power}}{\text{area}} \right) = \left(\frac{200. \text{ W}}{1.00 \cdot 10^6 \text{ m}^2} \right) = 2.00 \cdot 10^8 \text{ W/m}^2 = \frac{E^2}{2c\mu_0} \Rightarrow E = \sqrt{2c\mu_0 S} = 3.88 \cdot 10^5 \text{ V/m}$

Note the 2 is in the denominator from: $E_{rms} = \frac{E}{\sqrt{2}}$. Therefore, $E_{rms}^2 = \frac{E^2}{2}$ and $S = \frac{E_{rms}^2}{c\mu_0} = \frac{E^2}{2c\mu_0}$. The wavelength has nothing to do with the solution.

31.60. $f\lambda = c \Rightarrow \lambda = \frac{c}{f} = \frac{2.9979 \cdot 10^8 \text{ m/s}}{848.97 \cdot 10^6 \text{ Hz}} = 0.34901 \text{ m} = 34.901 \text{ cm}$.

31.61. $P = IA\varepsilon$, $I = \frac{1}{2} \left(\frac{E^2}{c\mu_0} \right)$, $A = LW$, and $\varepsilon = 0.18$.

$$\Rightarrow P = \frac{E^2 LW}{2c\mu_0} \varepsilon = \frac{(673 \text{ V/m})^2 (1.40 \text{ m})(0.900 \text{ m})(0.180)}{2(3.00 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ T m/A})} = 136 \text{ W}$$

- 31.62.** The displacement current between the capacitor plates is the same as the conventional current in the rest of the circuit.

$$i = \frac{V_{emf}}{R} e^{-t/RC} = \left(\frac{25.0 \text{ V}}{24,300 \Omega} \right) \exp \left[\frac{-0.3621 \text{ s}}{24,300 \Omega (14.9 \cdot 10^{-6} \text{ F})} \right] = (1.0288 \cdot 10^{-3})(0.36785) = 0.37845 \text{ mA}$$

$$i_d = \varepsilon_0 A \frac{dE}{dt} \Rightarrow \frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{0.37845 \text{ mA}}{(8.85 \cdot 10^{-12} \text{ C}^2 / (\text{N m}^2))(1.00 \cdot 10^{-4} \text{ m}^2)} = 4.28 \cdot 10^{11} \text{ V}/(\text{m s})$$

31.63. $E_{rms} = \sqrt{c\mu_0 I} = \sqrt{c\mu_0 \left(\frac{P}{A} \right)} = \sqrt{\frac{c\mu_0 P}{\pi (d/2)^2}} = \sqrt{\frac{4c\mu_0 P}{\pi d^2}}$

$$= \sqrt{\frac{4(3.00 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ T m/A})(0.40 \cdot 300. \text{ W})}{\pi (2 \text{ m})^2}} = 100 \text{ V/m}.$$

31.64. $\frac{E}{B} = c \Rightarrow E = cB = (3.00 \cdot 10^8 \text{ m/s})(5.00 \cdot 10^{-3} \text{ T}) = 1.50 \text{ MV/m}$

- 31.65. The antinodes are spaced half a wavelength apart, $d = \lambda/2$, where $\lambda = c/f$.

$$\Rightarrow d = \frac{c}{2f} = \frac{3.00 \cdot 10^8 \text{ m/s}}{2(2.4 \cdot 10^9 \text{ Hz})} = 6.25 \cdot 10^{-2} \text{ m} = 6.3 \text{ cm}$$

- 31.66. $I = 1400 \text{ W/m}^2$

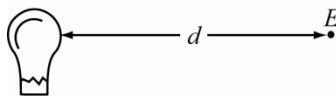
$$(a) I = \frac{E_{\text{rms}}^2}{c\mu_0} \Rightarrow E_{\text{rms}} = \sqrt{Ic\mu_0} = \sqrt{(1400 \text{ W/m}^2)(3.00 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ T m/A})} = 726.5 \text{ V/m}$$

$$E_{\text{max}} = \sqrt{2}E_{\text{rms}} = \sqrt{2}(726.5 \text{ V/m}) = 1027 \text{ V/m} = 1.0 \cdot 10^3 \text{ V/m}$$

$$(b) B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{1027 \text{ V/m}}{3.00 \cdot 10^8 \text{ m/s}} = 3.425 \cdot 10^{-6} \text{ T} = 3.4 \cdot 10^{-6} \text{ T}$$

- 31.67. **THINK:** The peak magnetic field can be determined from the speed of light and the peak electric field. The power of the bulb can be determined from its intensity, which can be determined from the electric and magnetic fields. Use the values $r = 2.25 \text{ m}$ and $E = 21.2 \text{ V/m}$.

SKETCH:



$$\text{RESEARCH: } \frac{E}{B} = c, \quad I = \frac{P}{A}, \quad I = \frac{E^2}{2c\mu_0}$$

SIMPLIFY:

$$(a) B = E/c$$

$$(b) P = IA = \frac{E^2}{2c\mu_0} A. \text{ Light from a light-bulb is emitted isotropically, that is equally in all directions. To}$$

determine the power a distance, d , away from the light-bulb, the intensity at all points a distance, d , from the light-bulb must be summed. Hence, A should be the surface area of a sphere of radius, r :

$$A = 4\pi r^2 \Rightarrow P = \frac{4\pi r^2 E^2}{2c\mu_0}.$$

CALCULATE:

$$(a) B = \frac{21.2 \text{ V/m}}{3 \cdot 10^8 \text{ m/s}} = 7.067 \cdot 10^{-8} \text{ T}$$

$$(b) P = \frac{4\pi(2.25 \text{ m})^2(21.2 \text{ V/m})^2}{2(3.00 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ T m/A})} = 37.92 \text{ W}$$

ROUND:

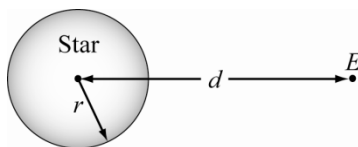
$$(a) B = 7.07 \cdot 10^{-8} \text{ T}$$

$$(b) P = 37.9 \text{ W}$$

DOUBLE-CHECK: These values are consistent with the power output for a regular household light bulb.

- 31.68. **THINK:** To determine the temperature of the star, the power radiated by the star must be known. To determine the power, the intensity is needed. The intensity can be determined from the electric field and the distance. $D = 15 \text{ AU}$, $E = 0.015 \text{ V/m}$, $r = 2r_s$, $r_s = 6.955 \cdot 10^5 \text{ km}$.

SKETCH:



RESEARCH: $P = A\sigma T^4 = 4\pi r^2 \sigma T^4$, $I = \frac{P}{4\pi d^2} = \frac{E^2}{2\mu_0 c}$

SIMPLIFY: $P = \frac{4\pi d^2 E^2}{2\mu_0 c} = \frac{2\pi d^2 E^2}{\mu_0 c} = 4\pi(2r_s)^2 \sigma T^4$, $T = \left[\frac{2\pi d^2 E^2}{8\pi r_s^2 \sigma \mu_0 c} \right]^{1/4} = \left[\frac{d^2 E^2}{4r_s^2 \sigma \mu_0 c} \right]^{1/4}$

CALCULATE:

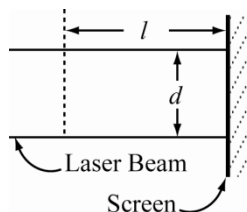
$$T = \left[\frac{((15)1.49598 \cdot 10^{11} \text{ m})^2 (44.0 \text{ V/m})^2}{4((6.955 \cdot 10^5 \cdot 10^3 \text{ m})^2 (5.670 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4})(4\pi \cdot 10^{-7} \text{ N/A}^{-2})(3.00 \cdot 10^8 \text{ m/s}))} \right]^{1/4} = 3918 \text{ K}$$

ROUND: $T = 3920 \text{ K}$

DOUBLE-CHECK: This temperature is realistic for a K-class star.

- 31.69. THINK:** From the power and the spot size, the intensity of the beam can be determined. From the intensity, the electric field can be determined. For the total energy, multiply the energy density by the volume of the beam. $P = 5.00 \text{ mW}$, $d = 2.00 \text{ mm}$, $l = 1.00 \text{ m}$.

SKETCH:



RESEARCH: $I = \frac{E_{\text{rms}}^2}{\mu_0 c} = \frac{P}{A}$, $A = \pi d^2$, $u_E = \frac{1}{2} \epsilon_0 E_{\text{max}}^2$

SIMPLIFY:

(a) $E_{\text{rms}} = \sqrt{\frac{\mu_0 c P}{\pi (d/2)^2}}$

(b) $U = u_E V = u_E A l = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 A l = \epsilon_0 E_{\text{rms}}^2 A l = \epsilon_0 (P \mu_0 c) l = (\epsilon_0 \mu_0 c) P l$.

$$c^2 = \frac{1}{\epsilon_0 \mu_0} \rightarrow \epsilon_0 \mu_0 = \frac{1}{c^2}$$

$$U = P l / c$$

CALCULATE:

(a) $E_{\text{rms}} = \sqrt{\frac{(4\pi \cdot 10^{-7} \text{ N/A}^2)(3.00 \cdot 10^8 \text{ m/s})(5.00 \cdot 10^{-3} \text{ W})}{\pi(1.00 \cdot 10^{-3} \text{ m})^2}} = 774.6 \text{ V/m}$

(b) $U = \frac{(5.00 \cdot 10^{-3} \text{ W})(1.00 \text{ m})}{3.00 \cdot 10^8 \text{ m/s}} = 1.6667 \cdot 10^{-11} \text{ J}$.

ROUND:

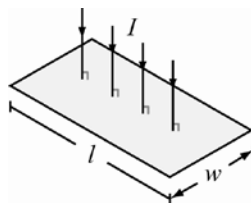
(a) $E_{\text{rms}} = 775 \text{ V/m}$.

(b) $U = 1.67 \cdot 10^{-11} \text{ J}$.

DOUBLE-CHECK: It is expected that a laser pointer would have small electric field and generate a small amount of energy, considering it's intended use. A laser pointer with more power would be dangerous.

- 31.70. THINK:** The total power incident on the roof is the intensity of the light times the area of the roof. From the intensity, the radiation pressure can be determined, and from this the force can be determined. $I = 1.00 \text{ kW/m}^2$, $l = 30.0 \text{ m}$, $w = 10.0 \text{ m}$.

SKETCH:



RESEARCH: $P = IA$, $A = lw$, $p_r = I/c$.

SIMPLIFY:

(a) $P = Ilw$

(b) $p_r = I/c$

CALCULATE:

(a) $P = (1.00 \cdot 10^3 \text{ W/m}^2)(30.0 \text{ m})(10.0 \text{ m}) = 3.00 \cdot 10^5 \text{ W}$

(b) $p_r = \left(\frac{1.00 \cdot 10^3 \text{ W/m}^2}{3.00 \cdot 10^8 \text{ m/s}} \right) = 3.33 \cdot 10^{-6} \text{ N/m}^2$

ROUND:

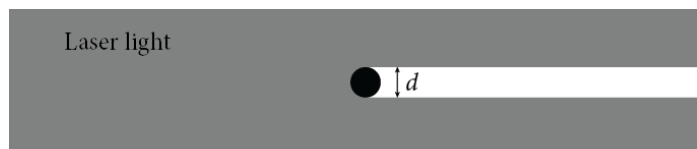
(a) $P = 3.00 \cdot 10^5 \text{ W}$

(b) $p_r = 3.33 \cdot 10^{-6} \text{ N/m}^2$

DOUBLE-CHECK: The radiation force is small, as expected, while the amount of power incident on the roof is fairly large. This large source of energy can be harnessed by the use of solar panels.

- 31.71. THINK:** The laser will apply a force to the particle. Assume the particle starts from rest and 2.00% of the laser light is absorbed. The laser applies a force to a known mass for a time interval Δt from which we can calculate the impulse applied by the laser. Use the values: $P = 500./192 \text{ TW}$, $d = 2.00 \text{ mm}$, $\rho = 2.00 \text{ g/cm}^3 = 2.00 \cdot 10^3 \text{ kg/m}^3$, $\Delta t = 1.00 \cdot 10^{-9} \text{ s}$, and $\varepsilon = 0.0200$.

SKETCH:



RESEARCH: $F\Delta t = m\Delta v = p_r A \Delta t \Rightarrow \frac{\Delta v}{\Delta t} = \frac{p_r A}{m} = a$, $p_r = \frac{\varepsilon I}{c}$, $I = \frac{P}{A}$, $m = \frac{4}{3}\pi \left(\frac{d}{2}\right)^3 \rho = \frac{\pi d^3 \rho}{6}$

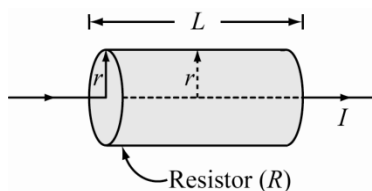
SIMPLIFY: $a = \frac{\Delta v}{\Delta t} = \frac{p_r A}{m} = \frac{A \left(\frac{\varepsilon I}{c}\right)}{m} = \frac{A \varepsilon \left(\frac{P}{A}\right)}{mc} = \frac{\varepsilon P}{mc} = \frac{\varepsilon P}{c \left(\frac{\pi d^3 \rho}{6}\right)} = \frac{6\varepsilon P}{\pi d^3 \rho c}$.

CALCULATE: $a = \frac{6(0.0200)(500. \cdot 10^{12} \text{ W}/192)}{\pi(2.00 \cdot 10^{-3} \text{ m})^3 (2.00 \cdot 10^3 \text{ kg/m}^3)(3.00 \cdot 10^8 \text{ m/s})} = 2.0723 \cdot 10^7 \text{ m/s}^2$.

ROUND: To three significant figures, $a = 2.07 \cdot 10^7 \text{ m/s}^2$.

DOUBLE-CHECK: This is a reasonable result for 2% of the power of one very powerful laser.

- 31.72. THINK:** Power will be dissipated out of the curved surface of the resistor. The result should be $P = i^2 R$. This can be derived by determining the expressions for the electric and magnetic fields at the surface of the resistor and using the Poynting vector definition of power/area.

SKETCH:


RESEARCH: $S = \frac{EB}{\mu_0}$, $P = SA$, $A = 2\pi rL$, $V = iR$, $E = \frac{V}{L}$, $B = \frac{\mu_0 i}{2\pi r}$

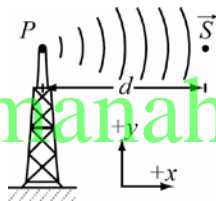
SIMPLIFY: $P = SA = \frac{EB}{\mu_0}(2\pi rL) = \frac{2\pi rL}{\mu_0} \left(\frac{V}{L} \right) \left(\frac{\mu_0 i}{2\pi r} \right) = Vi = (iR)i = i^2 R$

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: This is consistent with previous results for the power dissipated by a resistor.

- 31.73. **THINK:** The direction of the Poynting vector is the direction in which energy is transported, in this case radially away from the antenna. From the definition of the Poynting vector as power/area, the magnitude can be calculated given the power and the radial distance. Once the Poynting vector is known, the electric field can be determined. The power emitted toward the ground is reflected so we assume that the power is emitted into a hemisphere rather than a sphere. $P = 3.00 \cdot 10^4 \text{ W}$, $d = 12.0 \text{ km}$.

SKETCH:


RESEARCH: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$, $|\vec{S}| = S = \frac{P}{A} = \frac{E^2}{\mu_0 c}$, $E_{\text{rms}} = \frac{E}{\sqrt{2}}$, $A = 2\pi d^2$, $F_{\text{rms}} = qE_{\text{rms}}$

SIMPLIFY: (a) $S = \frac{P}{2\pi d^2}$

(b) $E = \sqrt{\mu_0 c S} \Rightarrow E_{\text{rms}} = \sqrt{\mu_0 c S / 2}$, $F_{\text{rms}} = q_e \sqrt{\mu_0 c S / 2}$

CALCULATE

(a) $S = \frac{3.00 \cdot 10^4 \text{ W}}{2\pi (12.0 \cdot 10^3 \text{ m})^2} = 3.3157 \cdot 10^{-5} \text{ W/m}^2$.

(b) $F_{\text{rms}} = (1.602 \cdot 10^{-19} \text{ C}) \sqrt{(4\pi \cdot 10^{-7} \text{ T mA}) (3.00 \cdot 10^8 \text{ m/s}) (3.3157 \cdot 10^{-5} \text{ W/m}^2) / 2} = 1.26649 \cdot 10^{-20} \text{ N}$.

ROUND:

(a) $S = 3.32 \cdot 10^{-5} \text{ W/m}^2$.

(b) $F_{\text{rms}} = 1.27 \cdot 10^{-20} \text{ N}$.

DOUBLE-CHECK: Dimensional analysis shows the results all have the correct units.

- 31.74. THINK:** To answer these questions, use the classical equation for the momentum and angular momentum, and use the quantum equation for the energy.

SKETCH:



RESEARCH: $p = \frac{U}{c} = \frac{E}{c}$, $L = \frac{U}{\omega} = \frac{E}{\omega}$

SIMPLIFY:

(a) $p = \frac{E}{c} = \frac{\hbar\omega}{c}$

From the dispersion relation of light (see Exercise 29.61): $|\vec{k}| = \omega\sqrt{\mu_0\epsilon_0} = \omega/c$. Then the vector momentum can be written:

$$|\vec{p}| = \hbar|\vec{k}| \Rightarrow \vec{p} = \hbar\vec{k}.$$

(b) $L = \frac{E}{\omega} = \frac{\hbar\omega}{\omega} = \hbar = |\vec{L}|$

The angular momentum of a photon is constant!

(c) The spin quantum number is given by: $s = \pm L/\hbar \Rightarrow s = \pm 1$.

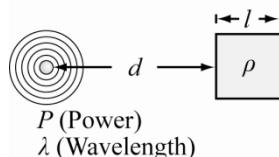
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: These results are classically unreasonable. However, they are correct nonetheless. They all stem from the purely quantum fact that light is quantized in units of \hbar . For light of frequency, ω , $\hbar\omega$ is the smallest amount of energy the electromagnetic wave can be measured to have.

- 31.75. THINK:** To determine how long it takes the ice to melt, first determine how much total energy is required to melt the ice cube, and then determine the intensity of the microwaves at the location of the ice cube. To determine the number of photons hitting the ice per second, the energy of one photon must be calculated and compared to the total radiation power incident on the ice. $P_0 = 250$ W, $l = 2.00$ cm, $d = 10.0$ cm, $\rho = 0.960$ g/cm³, $\lambda = 10.0$ cm. The fraction of incident light absorbed by ice is $\epsilon = 0.100$.

SKETCH:



RESEARCH: The energy required to melt the ice is $c_f = 334$ J/g. The intensity of light at the cube is $I = P_0/4\pi d^2$. The radiation power incident on the cube is $I l^2$. The power absorbed by the cube is $P = \epsilon I l^2 = E/t$. The mass of the ice is given by $m = \rho l^3$. The energy of one photon is given by $E_{\text{ph}} = hf = hc/\lambda$.

SIMPLIFY: The energy required to melt the ice is given by $E_m = mc_f = \rho l^3 c_f$. The power absorbed by the cube is given by:

$$P = \epsilon I l^2 = \epsilon l^2 \left(\frac{P_0}{4\pi d^2} \right) = \frac{E}{t}.$$

The time required to melt the cube can be determined as follows:

$$\frac{E}{t} = \frac{\epsilon l^2 P_0}{4\pi d^2} \Rightarrow t = \frac{4\pi E d^2}{\epsilon l^2 P_0}, \quad E = E_m = \rho l^3 c_f \Rightarrow t = \frac{4\pi \rho l^3 c_f d^2}{\epsilon l^2 P_0} = \frac{4\pi \rho l d^2 c_f}{\epsilon P_0}.$$

The total power incident on the cube is given by: $I^2 = P_0 l^2 / 4\pi d^2 = x \text{ J/s}$. $x \text{ J/s}$ is supplied by N photons of energy E_{ph} every second:

$$N E_{\text{ph}} = \frac{Nhc}{\lambda} \Rightarrow \frac{Nhc}{s} = x \text{ J/s} \Rightarrow \frac{Nhc}{\lambda} = x \text{ J} \Rightarrow N = (x \text{ J}) \frac{\lambda}{hc},$$

$$x \text{ J} = \frac{P_0 l^2}{4\pi d^2} \text{ s} \Rightarrow N = \frac{P_0 l^2 \lambda}{4\pi h d^2 c} \text{ s}.$$

CALCULATE: $t = \frac{4\pi(0.960 \text{ g/cm}^3)(2.00 \text{ cm})(10.0 \text{ cm})^2(334 \text{ J/g})}{(0.100)250. \text{ J/s}} = 3.223 \cdot 10^4 \text{ s} = 8.954 \text{ h}$

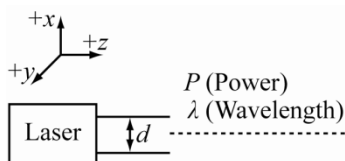
$$N = \frac{(250. \text{ J/s})(2.00 \text{ cm})^2(10.0 \text{ cm}) \text{ s}}{4\pi(6.626 \cdot 10^{-34} \text{ J s})(10.0 \text{ cm})^2(3.00 \cdot 10^{10} \text{ cm/s})} = 4.003 \cdot 10^{23}$$

ROUND: $t = 8.95 \text{ h}$ (or 8 hours 57 minutes), $N = 4.00 \cdot 10^{23}$

DOUBLE-CHECK: The number of photons per second, N , is reasonable. The time is correct, although a real microwave will work much faster. This is because a real microwave is not a single point source. Also, a microwave has shielding which serves to reflect all waves hitting the walls, which keeps the intensity of the radiation high.

- 31.76. THINK:** The Poynting vector is directed along the propagation axis, \hat{z} . Its magnitude is given by the laser power and the beam's cross-sectional area. The electric field is along the polarization direction, and the magnetic field is directed in the y -direction, perpendicular to both \vec{E} and \vec{S} . $P = 6.00 \text{ kW}$, $\lambda = 10.6 \mu\text{m}$ and $d = 100 \mu\text{m}$. Note that due to the properties of electromagnetic waves the relative phase between the magnetic field and electric field is 0 degrees.

SKETCH:



RESEARCH: $\vec{E} = E_0 \sin(kz - \omega t) \hat{x}$, $k = 2\pi / \lambda$, $\vec{B} = B_0 \sin(kz - \omega t) \hat{y}$, $\omega / k = c$, $S = \frac{E_0 B_0}{\mu_0} = \frac{2P}{A}$, where P

is the average power, $A = \pi \left(\frac{d}{2}\right)^2$, and $\frac{E_0}{B_0} = c$.

SIMPLIFY: $S = \frac{2P}{A} = \frac{8P}{\pi d^2}$ and $S = \frac{E_0(E_0/c)}{\mu_0} = \frac{E_0^2}{\mu_0 c}$. Set these expressions equal to each other to get:

$$\frac{8P}{\pi d^2} = \frac{E_0^2}{\mu_0 c} \Rightarrow E_0 = \sqrt{\frac{8P\mu_0 c}{\pi d^2}}, \quad B_0 = \frac{E_0}{c} = \sqrt{\frac{8P\mu_0}{\pi d^2 c}}.$$

Then, with $k = \frac{2\pi}{\lambda}$, $\omega = ck = \frac{2\pi c}{\lambda}$: $\vec{E} = \sqrt{\frac{8P\mu_0 c}{\pi d^2}} \sin\left(\frac{2\pi z}{\lambda} - \frac{2\pi ct}{\lambda}\right) \hat{x}$ and $\vec{B} = \sqrt{\frac{8P\mu_0}{\pi d^2 c}} \sin\left(\frac{2\pi z}{\lambda} - \frac{2\pi ct}{\lambda}\right) \hat{y}$.

CALCULATE: $E_0 = \sqrt{\frac{8(6.00 \cdot 10^3 \text{ W})(4\pi \cdot 10^{-7} \text{ T m/A})(3.00 \cdot 10^8 \text{ m/s})}{\pi(1.000 \cdot 10^{-4} \text{ m})^2}} = 2.400 \cdot 10^7 \text{ V/m}$

$$B_0 = \frac{2.400 \cdot 10^7 \text{ V/m}}{3 \cdot 10^8 \text{ m/s}} = 8 \cdot 10^{-2} \text{ T}, \quad \frac{2\pi}{\lambda} = \frac{2\pi}{10.6 \cdot 10^{-6} \text{ m}} = 5.928 \cdot 10^5 \text{ m}^{-1}$$

$$\frac{2\pi c}{\lambda} = 5.928 \cdot 10^5 \text{ m}^{-1} (3 \cdot 10^8 \text{ m/s}) = 1.778 \cdot 10^{14} \text{ s}^{-1}$$

$$\text{ROUND: } \vec{E} = (2.40 \cdot 10^7 \text{ V/m}) \sin(5.93 \cdot 10^5 z \text{ m}^{-1} - 1.78 \cdot 10^{14} t \text{ s}^{-1}) \hat{x}$$

$$\vec{B} = (8.00 \cdot 10^{-2} \text{ T}) \sin(5.93 \cdot 10^5 z \text{ m}^{-1} - 1.78 \cdot 10^{14} t \text{ s}^{-1}) \hat{y}$$

$$\text{DOUBLE-CHECK: Check that } \frac{E_0}{B_0} = c: \frac{2.40 \cdot 10^7 \text{ V/m}}{8.00 \cdot 10^{-2} \text{ T}} = 3.00 \cdot 10^8 \text{ m/s. Also, it is necessary that } \frac{\omega}{k} = c:$$

$$\frac{1.78 \cdot 10^{14} \text{ s}^{-1}}{5.93 \cdot 10^5 \text{ m}^{-1}} = 3.00 \cdot 10^8 \text{ m/s. These results are reasonable.}$$

Multi-Version Exercises

$$31.77. \quad I = \frac{E_{\text{rms}}^2}{c\mu_0} \Rightarrow E_{\text{rms}} = \sqrt{Ic\mu_0} = \sqrt{(182.9 \text{ W/m}^2)(2.998 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ Tm/A})} = 262.5 \text{ V/m}$$

$$31.78. \quad I = B_{\text{rms}}^2 c / \mu_0 \Rightarrow B_{\text{rms}} = \sqrt{I\mu_0 / c} = \sqrt{(191.4 \text{ W/m}^2)(4\pi \cdot 10^{-7} \text{ Tm/A}) / (2.998 \cdot 10^8 \text{ m/s})} = 8.957 \cdot 10^{-7} \text{ T}$$

$$31.79. \quad I = B_{\text{rms}}^2 c / \mu_0 = (9.142 \cdot 10^{-7} \text{ T})^2 (2.998 \cdot 10^8 \text{ m/s}) / (4\pi \cdot 10^{-7} \text{ Tm/A}) = 199.3 \text{ W/m}^2$$

$$31.80. \quad I = E_{\text{rms}}^2 / (c\mu_0) = (279.9 \text{ V/m})^2 / [(2.998 \cdot 10^8 \text{ m/s})(4\pi \cdot 10^{-7} \text{ Tm/A})] = 208.0 \text{ W/m}^2$$

31.81. If I_0 is the intensity of the incoming sunlight, then the light passing through the first polarizer has intensity $I_1 = \frac{1}{2}I_0$. The intensity of the light passing through the second polarizer is given by $I_2 = I_1 \cos^2(\theta_2 - \theta_1)$, so that $I_2 = \frac{1}{2}I_0 \cos^2(\theta_2 - \theta_1)$. The reduction in intensity, then, is

$$R = \frac{I_0 - I_2}{I_0} = 1 - \frac{1}{2} \cos^2(\theta_2 - \theta_1) = 1 - \frac{1}{2} \cos^2(88.6^\circ - 28.1^\circ) = 87.9\%.$$

31.82. As in the preceding problem, reduction of initial intensity $R = 1 - \frac{1}{2} \cos^2(\theta_2 - \theta_1)$.

$$\theta_2 = \theta_1 + \cos^{-1}(\sqrt{2 - 2R}) = 38.3^\circ + \cos^{-1}(\sqrt{2 - 2 \cdot 0.7584}) = 84.3^\circ$$

31.83. As above, reduction of initial intensity $R = 1 - \frac{1}{2} \cos^2(\theta_2 - \theta_1)$.

$$\theta_1 = \theta_2 - \cos^{-1}(\sqrt{2 - 2R}) = 110.6^\circ - \cos^{-1}(\sqrt{2 - 2 \cdot 0.7645}) = 63.9^\circ$$