McGraw-Hill Education

Physics

United Arab Emirates Edition

GRADE **12** ADVANCED

VOLUME 1

McGraw Hill Education

Project: McGraw-Hill Education United Arab Emirates Edition Advanced Science Grade 12 Year 4 Vol 1 Physics

FM. Front Matter, from University Physics with Modern Physics, 2e by Bauer and Westfall ©2014
1. Electrostatics, Chapter 21, from University Physics with Modern Physics, 2e by Bauer and Westfall ©2014
2. Electric Fields and Gauss's Law, Chapter 22, from University Physics with Modern Physics, 2e by Bauer and Westfall ©2014
3. Electric Potential, Chapter 23, from University Physics with Modern Physics, 2e by Bauer and Westfall ©2014
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COVER: Russ Widstrand/WidStock/Alamy

mheducation.com/prek-12

McGraw Hill Education

Printed in the United Arab Emirates.

ISBN: 978-1-44-701594-9 *(Student Edition)*
MHID: 1-44-701594-0 *(Student Edition)*

ePub Edition

ISBN: 978-1-44-701598-7 *(Student Edition)*
MHID: 1-44-701598-3 *(Student Edition)*

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Chapter Sourced From: FM. Front Matter, from University Physics with Modern Physics, 2e by Bauer and Westfall ©2014

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1 Electrostatics

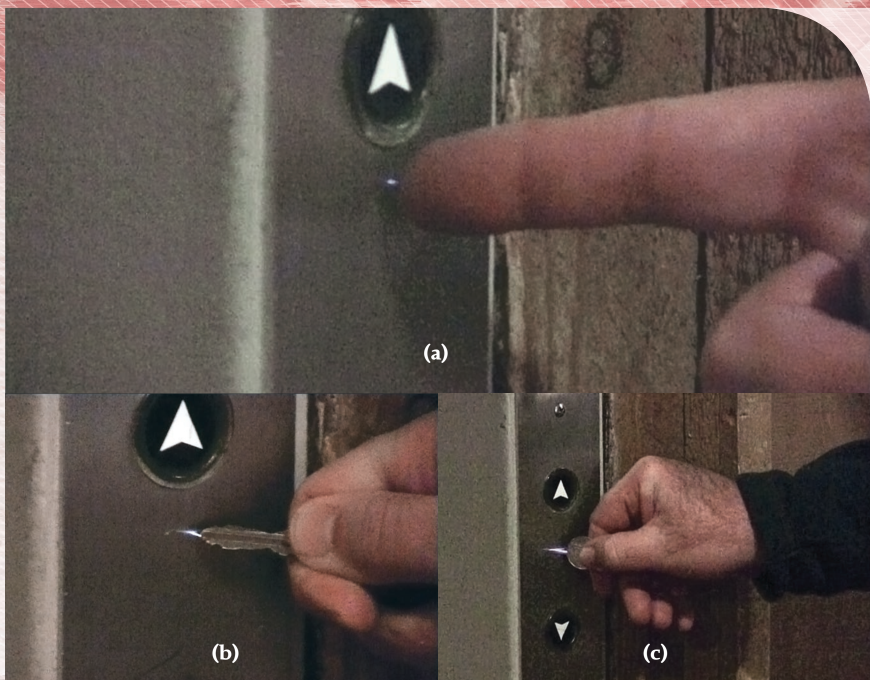


FIGURE 1.1 (a) A spark due to static electricity occurs between a person's finger and a metal surface near an elevator button. (b) and (c) Similar sparks are generated when the person holds a metal object like a car key or a coin, but are painless because the spark forms between the metal surface and the metal object.

Many people think of static electricity as the annoying spark that occurs when they reach for a metal object like a doorknob on a dry day, after they have been walking on a carpet (Figure 1.1). In fact, many electronics manufacturers place small metal plates on equipment so that users can discharge any spark on the plate and not damage the more sensitive parts of the equipment. However, static electricity is more than just an occasional annoyance; it is the starting point for any study of electricity and magnetism, forces that have changed human society as radically as anything since the discovery of fire or the wheel.

In this chapter, we examine the properties of electric charge. A moving electric charge gives rise to a separate phenomenon, called *magnetism*, which is covered in later chapters. Here we look at charged objects that are not moving–hence the term *electrostatics*. All objects have charge, since charged particles make up atoms and molecules. We often don't notice the effects of electrical charge because most objects are electrically neutral. The forces that hold atoms together and that keep objects separate even when they're in contact, are all electric in nature.

WHAT WE WILL LEARN

- Electric charge gives rise to a force between charged particles or objects.
- Electricity and magnetism together make up the electromagnetic force, one of the four fundamental forces of nature.
- There are two kinds of electric charge, positive and negative. Like charges repel, and unlike charges attract.
- Electric charge is quantized, meaning that it occurs only in integral multiples of a smallest elementary quantity. Electric charge is also conserved.
- Most materials around us are electrically neutral.
- The electron is an elementary particle, and its charge is the smallest observable quantity of electric charge.
- Insulators conduct electricity poorly or not at all. Conductors conduct electricity well but not perfectly–some energy losses occur.
- Semiconductors can be made to change between a conducting state and a nonconducting state.
- Superconductors conduct electricity perfectly.
- Objects can be charged directly by contact or indirectly by induction.
- The force that two stationary electric charges exert on each other is proportional to the product of the charges and varies as the inverse square of the distance between the two charges.
- Electrostatic forces between particles can be added as vectors by the process of superposition.

1.1 Electromagnetism

FIGURE 1.2 Lightning strikes over a city.

Perhaps no mystery puzzled ancient civilizations more than electricity, which they observed primarily in the form of lightning strikes (Figure 1.2). The destructive force inherent in lightning, which could set objects on fire and kill people and animals, puzzled people because they did not understand what caused it or where the lightening came from.

The ancient Greeks knew that if you rubbed a piece of amber with a piece of cloth, you could attract small, light objects with the amber. We now know that rubbing amber with a cloth transfers negatively charged particles called *electrons* from the cloth to the amber. (The words *electron* and *electricity* derive from the Greek word for amber.) Lightning also consists of a flow of electrons. The early Greeks and others also knew about naturally occurring magnetic objects called *lodestones*, which were found in deposits of magnetite, a mineral consisting of iron oxide. These objects were used to construct compasses as early as 300 BC.

The relationship between electricity and magnetism was not understood until the middle of the 19th century. The following chapters will reveal how electricity and magnetism can be unified into a common framework called *electromagnetism*. However, unification of forces does not stop there. During the early part of the 20th century, two more fundamental forces were discovered: the weak force, which operates in beta decay (in which an electron and a neutrino are spontaneously emitted from certain types of nuclei), and the strong force, which acts inside the atomic nucleus. Currently, the electromagnetic and weak forces are viewed as two aspects of the electroweak force (Figure 1.3). For the phenomena discussed in this and the following chapters, this electroweak unification has no influence; it becomes important in the highest-energy particle collisions. Because the energy scale for the electroweak unification is so high, most textbooks continue to speak of four fundamental forces: gravitational, electromagnetic, weak, and strong.

Today, a large number of physicists believe that the electroweak force and the strong force can also be unified, that is, described in a common framework. Several theories propose ways to accomplish this, but so far experimental evidence is missing. Interestingly, the force that has been known longer than any of the other fundamental forces, gravity, seems to be hardest to shoehorn into a unified framework with the other fundamental forces. Quantum gravity, supersymmetry, and string theory are current foci of cutting-edge physics research in which theorists are attempting to construct this grand unification and discover the (hubristically

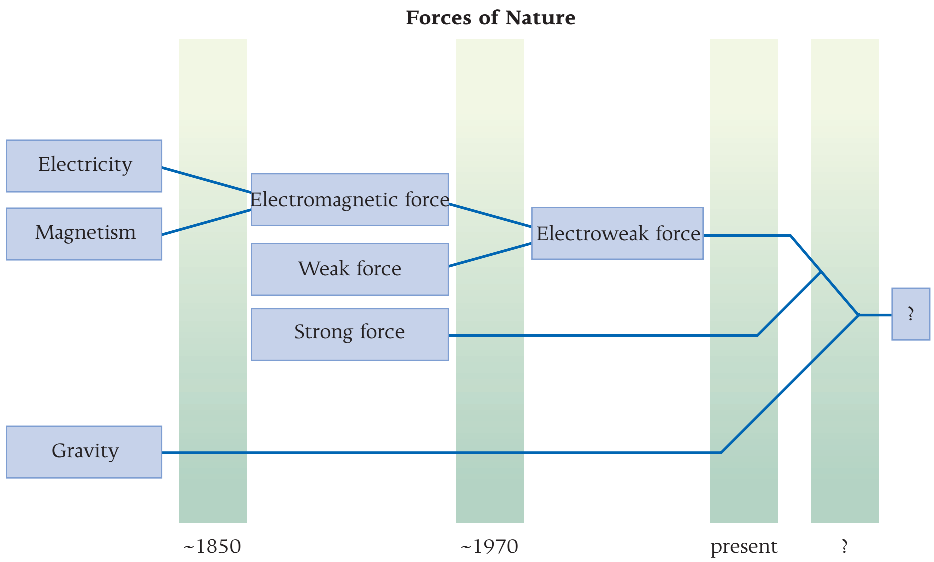


FIGURE 1.3 The history of the unification of fundamental forces.

named) Theory of Everything. They are mainly guided by symmetry principles and the conviction that nature must be elegant and simple.

In this chapter, we consider electric charge, how materials react to electric charge, static electricity, and the forces resulting from electric charges. **Electrostatics** covers situations where charges stay in place and do not move.

1.2 Electric Charge

Let's look a little deeper into the cause of the electric sparks that you occasionally receive on a dry winter day if you walk across a carpet and then touch a metal doorknob. (Electrostatic sparks have even ignited gas fumes while someone is filling the tank at a gas station. This is not an urban legend; a few of these cases have been caught on gas station surveillance cameras.) The process that causes this sparking is called **charging.** Charging consists of the transfer of negatively charged particles, called **electrons,** from the atoms and molecules of the material of the carpet to the soles of your shoes. This charge can move relatively easily through your body, including your hands. The built-up electric charge discharges through the metal of the doorknob, creating a spark.

The two types of electric charge found in nature are **positive charge** and **negative charge.** Normally, objects around us do not seem to be charged; instead, they are electrically neutral. Neutral objects contain roughly equal numbers of positive and negative charges that largely cancel each other. Only when positive and negative charges are not balanced do we observe the effects of electric charge.

If you rub a glass rod with a cloth, the glass rod becomes charged and the cloth acquires a charge of the opposite sign. If you rub a plastic rod with fur, the rod and fur also become oppositely charged. If you bring two charged glass rods together, they repel each other. Similarly, if you bring two charged plastic rods together, they also repel each other. However, a charged glass rod and a charged plastic rod will attract each other. This difference arises because the glass rod and the plastic rod have opposite charge. These observations led to the following law:

Law of Electric Charges
Like charges repel and opposite charges attract.

The unit of electric charge is the **coulomb** (C), named after the French physicist Charles-Augustine de Coulomb (1736–1806). The coulomb is defined in terms of the SI unit for current, the ampere (A), named after another French physicist, André-Marie Ampère (1775–1836). The ampere can not be derived in terms of the other SI units: meter, kilogram, and second. Instead, the ampere is another fundamental SI unit. For this reason, the SI system of units is sometimes called *MKSA (meter-kilogram-second-ampere) system*. The charge unit is defined as

$$1 \text{ C} = 1 \text{ A s} \tag{1.1}$$

The definition of the ampere must wait until we discuss current in later chapters. However, we can define the magnitude of the coulomb by simply specifying the charge of a single electron:

$$q_e = -e \tag{1.2}$$

where q_e is the charge and e has the (currently best accepted and experimentally measured) value

$$e = 1.602176565(35) \times 10^{-19} \text{ C} \tag{1.3}$$

(Usually it is enough to carry only the first two to four significant digits of this mantissa. We will use a value of 1.602 in this chapter, but you should keep in mind that equation 1.3 gives the full accuracy to which this charge has been measured.)

The charge of the electron is an intrinsic property of the electron, just like its mass. The charge of the **proton,** another basic particle of atoms, is exactly the same magnitude as that of the electron, only the proton's charge is positive:

$$q_p = +e \tag{1.4}$$

The choice of which charge is positive and which charge is negative is arbitrary. The conventional choice of $q_e < 0$ and $q_p > 0$ is due to the American statesman, scientist, and inventor Benjamin Franklin (1706–1790), who pioneered studies of electricity.

One coulomb is an extremely large unit of charge. We'll see later in this chapter just how big it is when we investigate the magnitude of the forces of charges on each other. Units of μC (microcoulombs, 10^{-6} C), nC (nanocoulombs, 10^{-9} C), and pC (picocoulombs, 10^{-12} C) are commonly used.

Benjamin Franklin also proposed that charge is conserved. Charge is not created or destroyed, simply moved from one object to another.

Concept Check 1.1

How many electrons does it take to make 1.00 C of charge?

a) 1.60×10^{19}

b) 6.60×10^{19}

c) 3.20×10^{16}

d) 6.24×10^{18}

e) 6.66×10^{17}

Law of Charge Conservation

The total electric charge of an isolated system is conserved.

This law is the fourth conservation law we have encountered so far, the first three being the conservation laws for total energy, momentum, and angular momentum. Conservation laws are a common thread that runs throughout all of physics and thus throughout this book as well.

It is important to note that there is a conservation law for charge, but *not* for mass. Mass and energy are not independent of each other. What is sometimes described in introductory chemistry as conservation of mass is not an exact conservation law, but only an approximation used to keep track of the number of atoms in chemical reactions. (It is a good approximation to a large number of significant figures but not an exact law, like charge conservation.) Conservation of charge applies to all systems, from the macroscopic system of plastic rod and fur down to systems of subatomic particles.

Elementary Charge

Electric charge occurs only in integral multiples of a minimum size. This is expressed by saying that charge is **quantized.** The smallest observable unit of electric charge is the charge of the electron, which is -1.602×10^{-19} C (as defined in equation 1.3).

The fact that electric charge is quantized was verified in an ingenious experiment carried out in 1910 by American physicist Robert A. Millikan (1868–1953) and known as the *Millikan oil drop experiment* (Figure 1.4). In this experiment, oil drops were sprayed into a chamber where electrons were knocked out of the drops by some form of radiation, usually X-rays. The resulting positively charged drops were allowed to fall between two electrically charged plates. Adjusting the charge of the plates caused the drops to stop falling and allowed their charge to be measured. What Millikan observed was that charge was quantized rather than continuous. (A quantitative analysis of this experiment will be presented in Chapter 3 on electric potential.) That is, this experiment and its subsequent refinements established that charge comes only in integer multiples of the charge of an electron. In everyday experiences with electricity, we do not notice that charge is quantized because most electrical phenomena involve huge numbers of electrons.

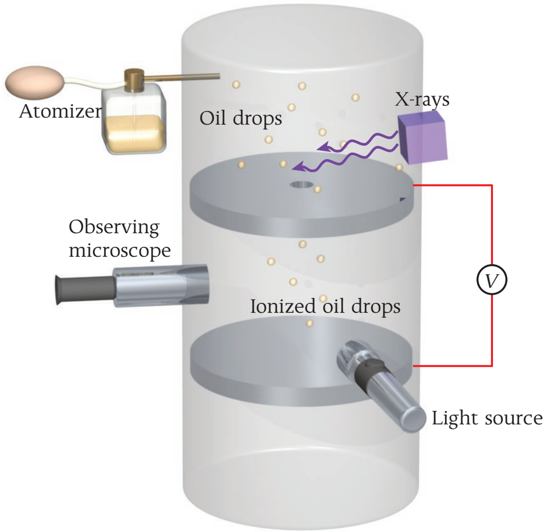


FIGURE 1.4 Schematic drawing of the Millikan oil drop experiment.

You studied earlier that matter is composed of atoms and that an atom consists of a nucleus containing charged protons and neutral neutrons. A schematic drawing of a carbon atom is shown in Figure 1.5. A carbon atom has six protons and (usually) six neutrons in its nucleus. This nucleus is surrounded by six electrons. Note that this drawing is not to scale. In the actual atom, the distance of the electrons from the nucleus is much larger (by a factor on the order of 10,000) than the size of the nucleus. In addition, the electrons are shown in circular orbits, which is also not quite correct. In Chapter 38, we'll see that the locations of electrons in the atom can be characterized only by probability distributions.

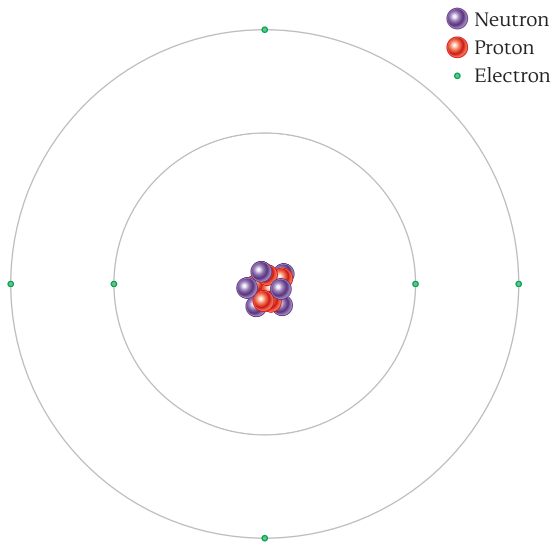


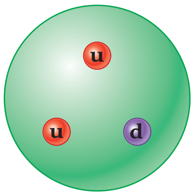
FIGURE 1.5 In a carbon atom, the nucleus contains six neutrons and six protons. The nucleus is surrounded by six electrons. Note that this drawing is schematic and not to scale.

As mentioned earlier, a proton has a positive charge with a magnitude that is *exactly* equal to the magnitude of the negative charge of an electron. In a neutral atom, the number of negatively charged electrons is equal to the number of positively charged protons. The mass of the electron is much smaller than the mass of the proton or the neutron. Therefore, most of the mass of an atom resides in the nucleus. Electrons can be removed from atoms relatively easily. For this reason, electrons are typically the carriers of electricity, rather than protons or atomic nuclei.

The electron is a fundamental particle and has no substructure: It is a point particle with zero radius (at least, according to current understanding). However, high-energy probes have been used to look inside the proton. A proton is composed of charged particles called *quarks*, held together by uncharged particles called *gluons*. Quarks have a charge of $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ times the charge of the electron. These fractionally charged particles cannot exist independently and have never been observed directly, despite numerous extensive searches. Just like the charge of an electron, the charges of quarks are intrinsic properties of these elementary particles.

A proton is composed of two *up quarks* (each with charge $+\frac{2}{3}e$) and one *down quark* (with charge $-\frac{1}{3}e$), giving the proton a charge of $q_p = (2)(+\frac{2}{3}e) + (1)(-\frac{1}{3}e) = +e$, as illustrated in Figure 1.6a. The electrically neutral neutron (hence the name!) is composed of an up quark and two down quarks, as shown in Figure 1.6b, so its charge is $q_n = (1)(+\frac{2}{3}e) + (2)(-\frac{1}{3}e) = 0$. There are also much more massive electron-like particles named *muon* and *tau*. But the basic fact remains that all of the matter in everyday experience is made up of electrons (with electrical charge $-e$), up and down quarks (with electrical charges $+\frac{2}{3}e$ and $-\frac{1}{3}e$, respectively), and gluons (zero charge).

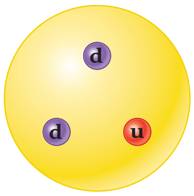
It is remarkable that the charges of the quarks inside a proton add up to *exactly* the same magnitude as the charge of the electron. This fact is still a puzzle, pointing to some deep symmetry in nature that is not yet understood.



Proton

$q_p = +\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +e$

(a)



Neutron

$q_n = +\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

(b)

FIGURE 1.6 (a) A proton contains two up quarks (u) and one down quark (d). (b) A neutron contains one up quark (u) and two down quarks (d).

Because all macroscopic objects are made of atoms, which in turn are made of electrons and atomic nuclei consisting of protons and neutrons, the charge, q, of any object can be expressed in terms of the sum of the number of protons, N_p, minus the sum of the number of electrons, N_e, that make up the object:

$$q = e\left(N_p - N_e\right) \tag{1.5}$$

EXAMPLE 1.1 Net Charge

PROBLEM

If we wanted a block of iron of mass 3.25 kg to acquire a positive charge of 0.100 C, what fraction of the electrons would we have to remove?

SOLUTION

Iron has mass number 56. Therefore, the number of iron atoms in the 3.25 kg block is

$$N_{\text{atom}} = \frac{(3.25 \text{ kg})(6.022 \times 10^{23} \text{ atoms/mole})}{0.0560 \text{ kg/mole}} = 3.495 \times 10^{25} = 3.50 \times 10^{25} \text{ atoms}$$

Note that we have used Avogadro's number, 6.022×10^{23}, and the definition of the mole, which specifies that the mass of 1 mole of a substance in grams is just the mass number of the substance–in this case, 56.

Because the atomic number of iron is 26, which equals the number of protons or electrons in an iron atom, the total number of electrons in the 3.25 kg block is

$$N_e = 26 N_{\text{atom}} = (26)(3.495 \times 10^{25}) = 9.09 \times 10^{26} \text{ electrons}$$

We use equation 1.5 to find the number of electrons, $N_{\Delta e}$, that we would have to remove. Because the number of electrons equals the number of protons in the original uncharged object, the difference in the number of protons and electrons is the number of removed electrons, $N_{\Delta e}$:

$$q = e \cdot N_{\Delta e} \Rightarrow N_{\Delta e} = \frac{q}{e} = \frac{0.100 \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 6.24 \times 10^{17}.$$

Finally, we obtain the fraction of electrons we would have to remove:

$$\frac{N_{\Delta e}}{N_e} = \frac{6.24 \times 10^{17}}{9.09 \times 10^{26}} = 6.87 \times 10^{-10}.$$

We would have to remove fewer than one in a billion electrons from the iron block in order to put the sizable positive charge of 0.100 C on it.

Self-Test Opportunity 1.1

Give the charge of the following elementary particles or atoms in terms of the elementary charge $e = 1.602 \times 10^{-19}$ C.

a) proton
b) neutron
c) helium atom (two protons, two neutron, and two electrons)
d) hydrogen atom (one proton and one electron)
e) up quark
f) down quark
g) electron
h) alpha particle (two protons and two neutrons)

1.3 Insulators, Conductors, Semiconductors, and Superconductors

Materials that conduct electricity well are called **conductors.** Materials that do not conduct electricity are called **insulators.** (Of course, there are good and poor conductors and good and poor insulators, depending on the properties of the specific materials.)

The electronic structure of a material refers to the way in which electrons are bound to nuclei, as we'll discuss in later chapters. For now, we are interested in the relative propensity of the atoms of a material to either give up or acquire electrons. For insulators, no free movement of electrons occurs because the material has no loosely bound electrons that can escape from its atoms and thereby move freely throughout the material. Even when external charge is placed on an insulator, this external charge cannot move appreciably. Typical insulators are glass, plastic, and cloth.

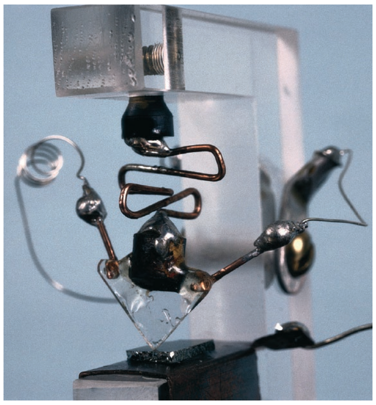
On the other hand, materials that are conductors have an electronic structure that allows the free movement of some electrons. The positive charges of the atoms of a conducting material do not move, since they reside in the heavy nuclei. Typical solid conductors are metals. Copper, for example, is a very good conductor and is therefore used in electrical wiring.

Fluids and organic tissue can also serve as conductors. Pure distilled water is not a very good conductor. However, dissolving common table salt (NaCl), for example, in water improves its conductivity tremendously, because the positively charged sodium ions (Na^+) and negatively charged chlorine ions (Cl^-) can move within the water to conduct electricity. In liquids, unlike solids, positive as well as negative charge carriers are mobile. Organic tissue is not a very good conductor, but it conducts electricity well enough to make large currents dangerous to us.

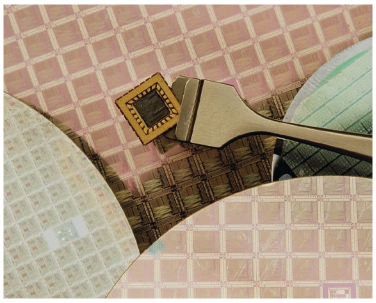
Semiconductors

A class of materials called **semiconductors** can change from being an insulator to being a conductor and back to an insulator again. Semiconductors were discovered only a little more than 50 years ago but are the backbone of the entire computer and consumer electronics industries. The first widespread use of semiconductors was in transistors (Figure 1.7a); modern computer chips (Figure 1.7b) perform the functions of millions of transistors. Computers and basically all modern consumer electronics products and devices (televisions, cameras, video game players, cell phones, etc.) would be impossible without semiconductors. Gordon Moore, cofounder of Intel, famously stated that due to advancing technology, the power of the average computer's CPU (central processing unit) doubles every 18 months, which is an empirical average over the last 5 decades. This doubling phenomenon is known as *Moore's Law*. Physicists have been and will undoubtedly continue to be the driving force behind this process of scientific discovery, invention, and improvement.

Semiconductors are of two kinds: intrinsic and extrinsic. Examples of *intrinsic semiconductors* are chemically pure crystals of gallium arsenide, germanium, or, especially, silicon. Engineers produce *extrinsic semiconductors* by *doping*, which is the addition of minute amounts (typically 1 part in 10^6) of other materials that can act as electron donors or electron receptors. Semiconductors doped with electron donors are called *n-type* (*n* stands for "negative charge"). If the doping substance acts as an electron receptor, the hole left behind by an electron that attaches to a receptor can also travel through the semiconductor and acts as an effective positive charge carrier. These semiconductors are consequently called *p-type* (*p* stand for "positive charge"). Thus, unlike normal solid conductors in which only negative charges move, semiconductors have movement of negative or positive charges (which are really electron holes, that is, missing electrons).



(a)



(b)

FIGURE 1.7 (a) Replica of the first transistor, invented in 1947 by John Bardeen, Walter H. Brattain, and William B. Shockley. (b) Modern computer chips made from silicon wafers contain many tens of millions of transistors.

Superconductors

Superconductors are materials that have zero resistance to the conduction of electricity, as opposed to normal conductors, which conduct electricity well but with some losses. Materials are superconducting only at very low temperatures. A typical superconductor is a niobium-titanium alloy that must be kept near the temperature of liquid helium (4.2 K) to retain its superconducting properties. During the last 20 years, new materials called *high-T_c superconductors* (T_c stands for "critical temperature," which is the maximum temperature that allows superconductivity) have been developed. These are superconducting at the temperature at which nitrogen can exist as a liquid (77.3 K). Materials that are superconductors at room temperature (300 K) have not yet been found, but they would be extremely useful. Research directed at developing such materials and theoretically explaining what physical phenomena cause high-T_c superconductivity is currently in progress.

1.4 Electrostatic Charging

Giving a static charge to an object is a process known as **electrostatic charging.** Electrostatic charging can be understood through a series of simple experiments. A power supply serves as a ready source of positive and negative charge. The battery in your car is a similar

FIGURE 1.8 A typical electroscope used in lecture demonstrations.

power supply; it uses chemical reactions to create a separation between positive and negative charge. Several insulating paddles can be charged with positive or negative charge from the power supply. In addition, a conducting connection is made to the Earth. The Earth is a nearly infinite reservoir of charge, capable of effectively neutralizing electrically charged objects in contact with it. This taking away of charge is called **grounding,** and an electrical connection to the Earth is called a **ground.**

An **electroscope** is a device that gives an observable response when it is charged. You can build a relatively simple electroscope by using two strips of very thin metal foil that are attached at one end and are allowed to hang straight down adjacent to each other from an isolating frame. Kitchen aluminum foil is not suitable, because it is too thick, but hobby shops sell thinner metal foils. For the isolating frame, you can use a Styrofoam coffee cup turned sideways, for example.

The lesson-demonstration-quality electroscope shown in Figure 1.8 has two conductors that in their neutral position are touching and oriented in a vertical direction. One of the conductors is hinged at its midpoint so that it will move away from the fixed conductor if a charge appears on the electroscope. These two conductors are in contact with a conducting ball on top of the electroscope, which allows charge to be applied or removed easily.

An uncharged electroscope is shown in Figure 1.9a. The power supply is used to give a negative charge to one of the insulating paddles. When the paddle is brought near the ball of the electroscope, as shown in Figure 1.9b, the electrons in the conducting ball of the electroscope are repelled, which produces a net negative charge on the conductors of the electroscope. This negative charge causes the movable conductor to rotate because the stationary conductor also has negative charge and repels it. Because the paddle did not touch the ball, the charge on the movable conductors is **induced.** If the charged paddle is then taken away, as illustrated in Figure 1.9c, the induced charge reduces to zero, and the movable conductor returns to its original position, because the total charge on the electroscope did not change in the process.

If the same process is carried out with a positively charged paddle, the electrons in the conductors are attracted to the paddle and flow into the conducting ball. This leaves a net positive charge on the conductors, causing the movable conducting arm to rotate again. Note that the net charge of the electroscope is zero in both cases and that the motion of the conductor indicates only that the paddle is charged. When the positively charged paddle is removed, the movable conductor again returns to its original position. It is important to note that we cannot determine the sign of this charge!

On the other hand, if a negatively charged insulating paddle *touches* the ball of the electroscope, as shown in Figure 1.10b, electrons will flow from the paddle to the conductor, producing a net negative charge. When the paddle is removed, the charge remains and the movable arm remains rotated, as shown in Figure 1.10c. Similarly, if a positively charged insulating paddle touches the ball of the uncharged electroscope, the electroscope transfers electrons to the positively charged paddle and becomes positively charged. Again, both a

Concept Check 1.2

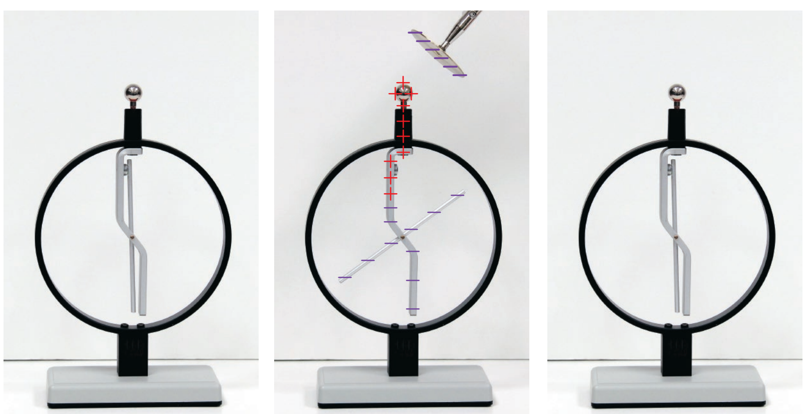
The hinged conductor moves away from the fixed conductor if a charge is applied to the electroscope, because

a) like charges repel each other.

b) like charges attract each other.

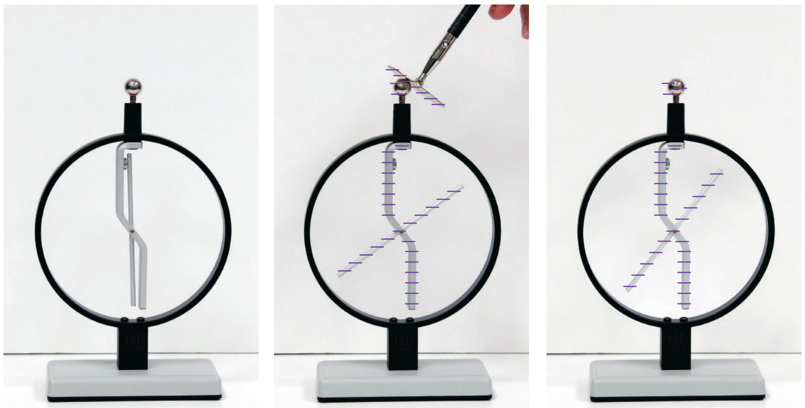
c) unlike charges attract each other.

d) unlike charges repel each other.



(a) (b) (c)

FIGURE 1.9 Inducing a charge: (a) An uncharged electroscope. (b) A negatively charged paddle is brought near the electroscope. (c) The negatively charged paddle is taken away.



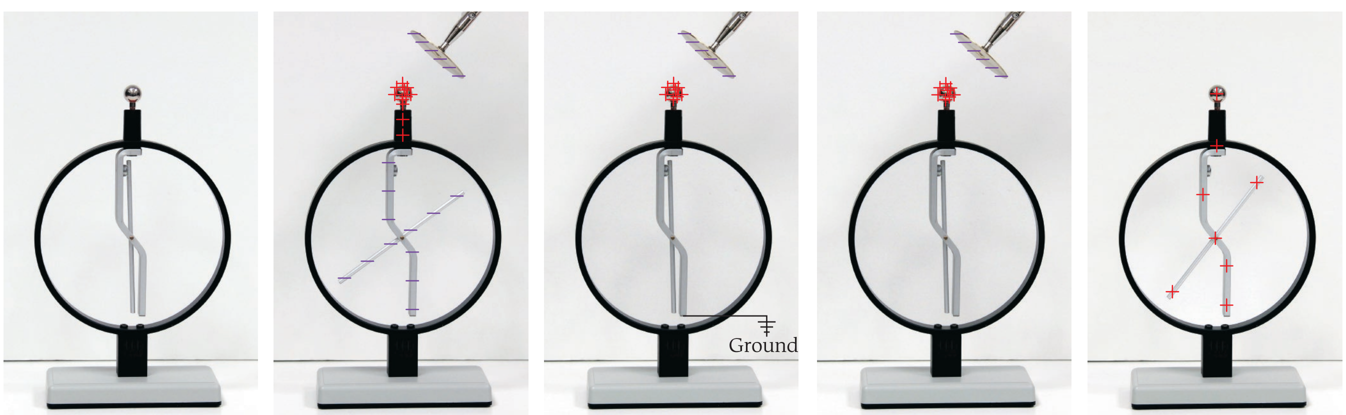
(a) (b) (c)

FIGURE 1.10 Charging by contact: (a) An uncharged electroscope. (b) A negatively charged paddle touches the electroscope. (c) The negatively charged paddle is removed.

positively charged paddle and a negatively charged paddle have the same effect on the electroscope, and we have no way of determining whether the paddles are positively charged or negatively charged. This process is called **charging by contact.**

The two different kinds of charge can be demonstrated by first touching a negatively charged paddle to the electroscope, producing a rotation of the movable arm, as shown in Figure 1.10. If a positively charged paddle is then brought into contact with the electroscope, the movable arm returns to the uncharged position. The charge is neutralized (assuming that both paddles originally had the same absolute value of charge). Thus, there are two kinds of charge. However, because charges are manifestations of mobile electrons, a negative charge is an excess of electrons and a positive charge is a deficit of electrons.

The electroscope can be given a charge without touching it with the charged paddle, as shown in Figure 1.11. The uncharged electroscope is shown in Figure 1.11a. A negatively charged paddle is brought close to the ball of the electroscope but not touching it, as shown in Figure 1.11b. In Figure 1.11c, the electroscope is connected to a ground. Then, while the charged paddle is still close to but not touching the ball of the electroscope, the ground connection is removed in Figure 1.11d. Next, when the paddle is moved away from the electroscope in Figure 1.11e, the electroscope is still positively charged (but with a smaller deflection than in Figure 1.11b). The same process also works with a positively charged paddle. This process is called **charging by induction** and yields an electroscope charge that has the opposite sign from the charge on the paddle.



(a) (b) (c) (d) (e)

FIGURE 1.11 Charging by induction: (a) An uncharged electroscope. (b) A negatively charged paddle is brought close to the electroscope. (c) A ground is connected to the electroscope. (d) The connection to the ground is removed. (e) The negatively charged paddle is taken away, leaving the electroscope positively charged.

FIGURE 1.12 Triboelectric series for some common materials.

Triboelectric Charging

As mentioned earlier, rubbing two materials together will charge them. We have not addressed two basic questions about this effect: First, what really causes it? And second, which of two given materials gets charged positively and which negatively?

Amazingly, as with many aspects of friction, the microscopic causes of triboelectric charging are still not completely understood. The prevailing theory is that when the surfaces of the two materials involved in the charging process come into contact, adhesion takes place and chemical bonds are formed between atoms at the surfaces. As the surfaces separate, some of these newly formed bonds rupture and leave more of the electrons involved in the bonding on the material with the greater work function.

Recent research results obtained by examining surfaces with atomic force microscopes suggest, however, that triboelectric charging can also occur when two pieces of the *same* material are rubbed against each other. In addition, it has been found that the charging process transfers not only electrons but sometimes also small specks (a few nanometers across) of material.

Which material gets charged positively and which negatively? This question has been answered by a long series of experiments whose results are summarized in Figure 1.12 in the form of a list of some common materials that may be rubbed together. If you rub two materials from this list against each other, the one nearer the top will receive a net positive charge and the other a net negative charge.

Finally, as a rule of thumb, more intense rubbing creates greater charge transfer. This occurs because more intense contact increases friction, which in turn creates more microscopic points of charge transfer on the surfaces of the materials.

1.5 Electrostatic Force—Coulomb's Law

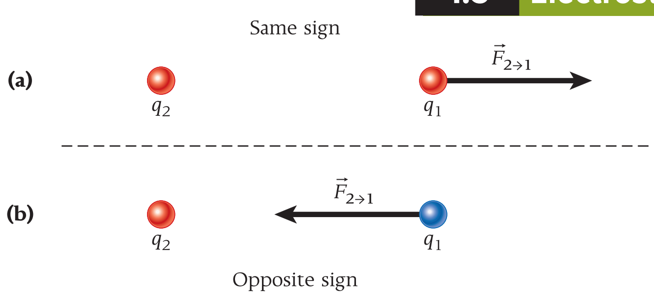


FIGURE 1.13 The force exerted by charge 2 on charge 1: (a) two charges with the same sign; (b) two charges with opposite signs.

The law of electric charges is evidence of a force between any two charges at rest. Experiments show that for the electrostatic force exerted by a charge q_2 on a charge q_1, $\vec{F}_{2\to1}$, the force on q_1 points toward q_2 if the charges have opposite signs and away from q_2 if the charges have like signs (Figure 1.13). This force on one charge due to another charge always acts along the line between the two charges. **Coulomb's Law** gives the magnitude of this force as

$$F = k\frac{|q_1q_2|}{r^2}, \tag{1.6}$$

where q_1 and q_2 are electric charges, $r = |\vec{r}_1 - \vec{r}_2|$ is the distance between them, and

$$k = 8.99\times10^9 \ \frac{\mathrm{N\,m^2}}{\mathrm{C^2}} \tag{1.7}$$

is **Coulomb's constant.** You can see that 1 coulomb is a *very* large charge. If two charges of 1 C each were at a distance of 1 m apart, the magnitude of the force they would exert on each other would be 8.99 billion N. For comparison, this force equals the weight of 450 fully loaded space shuttles!

The relationship between Coulomb's constant and another constant, ε_0, called the **electric permittivity of free space,** is

$$k = \frac{1}{4\pi\varepsilon_0}. \tag{1.8}$$

Concept Check 1.3

You place two charges a distance r apart. Then you double each charge and double the distance between the charges. How does the force between the two charges change?

a) The new force is twice as large.

b) The new force is half as large.

c) The new force is four times larger.

d) The new force is four times smaller.

e) The new force is the same.

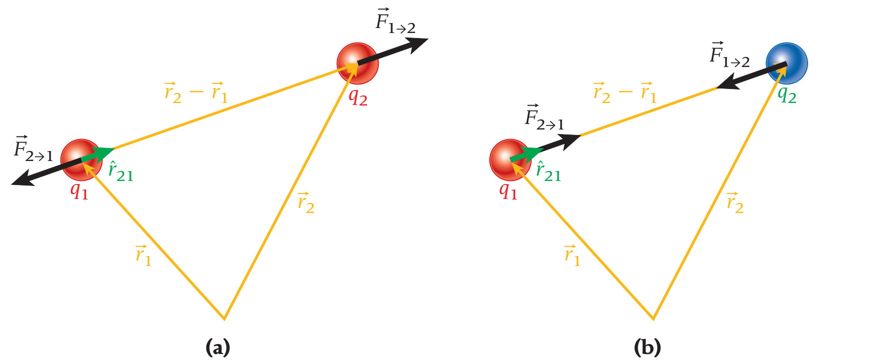


FIGURE 1.14 Vector representation of the electrostatic forces that two charges exert on each other: (a) two charges of like sign; (b) two charges of opposite sign.

Consequently, the value of ε_0 is

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N\ m}^2}. \tag{1.9}$$

An alternative way of writing equation 1.6 is then

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}. \tag{1.10}$$

As you'll see in the next few chapters, some equations in electrostatics are more convenient to write with k, while others are more easily written in terms of $1/(4\pi\varepsilon_0)$.

Note that the charges in equations 1.6 and 1.10 can be positive or negative, so the product of the charges can also be positive or negative. Since opposite charges attract and like charges repel, a negative value for the product $q_1 q_2$ signifies attraction and a positive value means repulsion.

Finally, Coulomb's Law for the force due to charge 2 on charge 1 can be written in vector form:

$$\vec{F}_{2\to1} = -k\frac{q_1 q_2}{r^3}(\vec{r}_2 - \vec{r}_1) = -k\frac{q_1 q_2}{r^2}\hat{r}_{21}. \tag{1.11}$$

In this equation, $\hat{r}_{21}$ is a unit vector pointing from q_2 to q_1 (see Figure 1.14). The negative sign indicates that the force is repulsive if both charges are positive or both charges are negative. In that case, $\vec{F}_{2\to1}$ points away from charge 2, as depicted in Figure 1.14a. On the other hand, if one of the charges is positive and the other negative, then $\vec{F}_{2\to1}$ points toward charge 2, as shown in Figure 1.14b.

If charge 2 exerts the force $\vec{F}_{2\to1}$ on charge 1, then the force $\vec{F}_{1\to2}$ that charge 1 exerts on charge 2 is simply obtained from Newton's Third Law: $\vec{F}_{1\to2} = -\vec{F}_{2\to1}$.

Concept Check 1.4

What do the forces acting on the charge q_3 in Figure 1.15 indicate about the signs of the three charges?

a) All three charges must be positive.

b) All three charges must be negative.

c) Charge q_3 must be zero.

d) Charges q_1 and q_2 must have opposite signs.

e) Charges q_1 and q_2 must have the same sign, and q_3 must have the opposite sign.

Superposition Principle

So far in this chapter, we have been dealing with two charges. Now let's consider three point charges, q_1, q_2, and q_3, at positions x_1, x_2, and x_3, respectively, as shown in Figure 1.15. The force exerted by charge 1 on charge 3, $\vec{F}_{1\to3}$, is given by

$$\vec{F}_{1\to3} = -\frac{kq_1 q_3}{(x_3 - x_1)^2}\hat{x}.$$

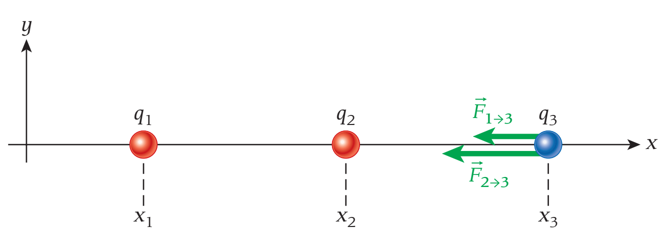


FIGURE 1.15 The forces exerted on charge 3 by charge 1 and charge 2.

Concept Check 1.5

Assuming that the lengths of the vectors in Figure 1.15 are proportional to the magnitudes of the forces they represent, what do they indicate about the magnitudes of the charges q_1 and q_2? (*Hint:* The distance between x_1 and x_2 is the same as the distance between x_2 and x_3.)

a) $|q_1| < |q_2|$

b) $|q_1| = |q_2|$

c) $|q_1| > |q_2|$

d) The answer cannot be determined from the information given in the figure.

The force exerted by charge 2 on charge 3 is

$$\vec{F}_{2\to 3} = -\frac{kq_2q_3}{(x_3 - x_2)^2}\hat{x}.$$

The force that charge 1 exerts on charge 3 is not affected by the presence of charge 2. The force that charge 2 exerts on charge 3 is not affected by the presence of charge 1. In addition, the forces exerted by charge 1 and charge 2 on charge 3 add vectorially to produce a net force on charge 3:

$$\vec{F}_{\text{net}\to 3} = \vec{F}_{1\to 3} + \vec{F}_{2\to 3}.$$

In general, we can express the electrostatic force, $\vec{F}(\vec{r})$, acting on a charge q at position $\vec{r}$ due to a collection of charges, q_i, at positions $\vec{r}_i$ as

$$\vec{F}(\vec{r}) = kq\sum_{i=1}^{n} q_i \frac{\vec{r}_i - \vec{r}}{\left|\vec{r}_i - \vec{r}\right|^3}. \tag{1.12}$$

We obtain this result using superposition of the forces and equation 1.11 for each pairwise interaction.

EXAMPLE 1.2 Electrostatic Force inside the Atom

PROBLEM 1

What is the magnitude of the electrostatic force that the two protons inside the nucleus of a helium atom exert on each other?

SOLUTION 1

The two protons and two neutrons in the nucleus of the helium atom are held together by the strong force; the electrostatic force is pushing the protons apart. The charge of each proton is $q_\text{p} = +e$. A distance of approximately $r = 2\cdot 10^{-15}$ m separates the two protons. Using Coulomb's Law, we can find the force:

$$F = k\frac{\left|q_\text{p}q_\text{p}\right|}{r^2} = \left(8.99\times 10^9\ \frac{\text{N m}^2}{\text{C}^2}\right)\frac{\left(+1.6\times 10^{-19}\ \text{C}\right)\left(+1.6\times 10^{-19}\ \text{C}\right)}{\left(2\times 10^{-15}\ \text{m}\right)^2} = 58\ \text{N}.$$

Therefore, the two protons in the atomic nucleus of a helium atom are being pushed apart with a force of 58 N (approximately the weight of a small dog). Considering the size of the nucleus, this is an astonishingly large force. Why do atomic nuclei not simply explode? The answer is that an even stronger force, the aptly named strong force, keeps them together.

PROBLEM 2

What is the magnitude of the electrostatic force between a gold nucleus and an electron of the gold atom in an orbit with radius 4.88×10^{-12} m?

SOLUTION 2

The negatively charged electron and the positively charged gold nucleus attract each other with a force whose magnitude is

$$F = k\frac{\left|q_\text{e}q_\text{N}\right|}{r^2},$$

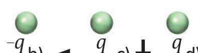
where the charge of the electron is $q_\text{e} = -e$ and the charge of the gold nucleus is $q_\text{N} = +79e$. The force between the electron and the nucleus is then

$$F = k\frac{\left|q_\text{e}q_\text{N}\right|}{r^2} = \left(8.99\times 10^9\ \frac{\text{N m}^2}{\text{C}^2}\right)\frac{\left(1.60\times 10^{-19}\ \text{C}\right)\left[(79)\left(1.60\times 10^{-19}\ \text{C}\right)\right]}{\left(4.88\times 10^{-12}\ \text{m}\right)^2} = 7.63\times 10^{-4}\ \text{N}.$$

Thus, the magnitude of the electrostatic force exerted on an electron in a gold atom by the nucleus is about 100,000 times less than that between protons inside a nucleus.

Concept Check 1.6

Three charges are arranged on a straight line as shown in the figure. What is the direction of the electrostatic force on the *middle* charge?

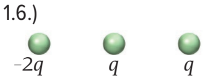


a) → b) ← c) ↓ d) ↑

e) There is no force on that charge.

Concept Check 1.7

Three charges are arranged on a straight line as shown in the figure. What is the direction of the electrostatic force on the *right* charge? (Note that the left charge is double what it was in Concept Check 1.6.)



a) → b) ← c) ↓ d) ↑

e) There is no force on that charge.

Note: The gold nucleus has a mass that is approximately 400,000 times that of the electron. But the force the gold nucleus exerts on the electron has exactly the same magnitude as the force that the electron exerts on the gold nucleus. You may say that this is obvious from Newton's Third Law, which is true. But it is worth emphasizing that this basic law holds for electrostatic forces as well.

EXAMPLE 1.3 Equilibrium Position

PROBLEM

Two charged particles are placed as shown in Figure 1.16: $q_1 = 0.15\ \mu\text{C}$ is located at the origin, and $q_2 = 0.35\ \mu\text{C}$ is located on the positive x-axis at $x_2 = 0.40$ m. Where should a third charged particle, q_3, be placed to be at an equilibrium point (such that the forces on it sum to zero)?

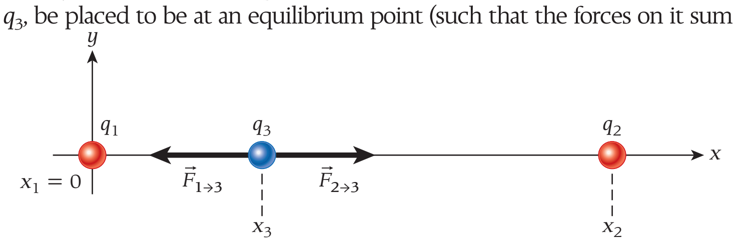


FIGURE 1.16 Placement of three charged particles. The third particle is shown as having a negative charge.

SOLUTION

Let's first determine where *not* to put the third charge. If the third charge is placed anywhere off the x-axis, there will always be a force component pointing toward or away from the x-axis. Thus, we can find an equilibrium point (a point where the forces sum to zero) only *on* the x-axis. The x-axis can be divided into three different segments: $x \le x_1 = 0$, $x_1 < x < x_2$, and $x_2 \le x$. For $x \le x_1 = 0$, the force vectors from both q_1 and q_2 acting on q_3 will point in the positive direction if the charge is negative and in the negative direction if the charge is positive. Because we are looking for a location where the two forces cancel, the segment $x \le x_1 = 0$ can be excluded. A similar argument excludes $x \ge x_2$.

In the remaining segment of the x-axis, $x_1 < x < x_2$, the forces from q_1 and q_2 on q_3 point in opposite directions. We look for the location, x_3, where the absolute magnitudes of both forces are equal and the forces thus sum to zero. We express the equality of the two forces as

$$\left|\vec{F}_{1\to3}\right| = \left|\vec{F}_{2\to3}\right|,$$

which we can rewrite as

$$k\frac{|q_1q_3|}{(x_3 - x_1)^2} = k\frac{|q_3q_2|}{(x_2 - x_3)^2}.$$

We now see that the magnitude and sign of the third charge do not matter because that charge cancels out, as does the constant k, giving us

$$\frac{q_1}{(x_3 - x_1)^2} = \frac{q_2}{(x_2 - x_3)^2}$$

or

$$q_1(x_2 - x_3)^2 = q_2(x_3 - x_1)^2. \tag{i}$$

Taking the square root of both sides and solving for x_3, we find

$$\sqrt{q_1}(x_2 - x_3) = \sqrt{q_2}(x_3 - x_1),$$

or

$$x_3 = \frac{\sqrt{q_1}x_2 + \sqrt{q_2}x_1}{\sqrt{q_1} + \sqrt{q_2}}.$$

We can take the square root of both sides of equation (i) because $x_1 < x_3 < x_2$, and so both of the roots, $x_2 - x_3$ and $x_3 - x_1$, are assured to be positive.

Inserting the numbers given in the problem statement, we obtain

$$x_3 = \frac{\sqrt{q_1}x_2 + \sqrt{q_2}x_1}{\sqrt{q_1} + \sqrt{q_2}} = \frac{\sqrt{0.15\ \mu\text{C}}(0.4\ \text{m})}{\sqrt{0.15\ \mu\text{C}} + \sqrt{0.35\ \mu\text{C}}} = 0.16\ \text{m}.$$

This result makes sense because we expect the equilibrium point to reside closer to the smaller charge.

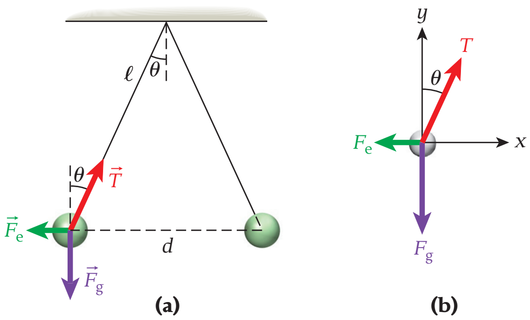


FIGURE 1.17 (a) Two charged balls hanging from the ceiling in their equilibrium position. (b) Free-body diagram for the left-hand charged ball.

SOLVED PROBLEM 1.1 Charged Balls

PROBLEM

Two identical charged balls hang from the ceiling by insulated ropes of equal length, $\ell = 1.50$ m (Figure 1.17). A charge $q = 25.0\ \mu\text{C}$ is applied to each ball. Then the two balls hang at rest, and each supporting rope has an angle of 25.0° with respect to the vertical (Figure 1.17a). What is the mass of each ball?

SOLUTION

THINK Each charged ball has three forces acting on it: the force of gravity, the repulsive electrostatic force, and the tension in the supporting rope. We can resolve the components of the three forces and set them equal to zero, allowing us to solve for the mass of the charged balls.

SKETCH A free-body diagram for the left-hand ball is shown in Figure 1.17b.

RESEARCH The condition for static equilibrium says that the sum of the x-components of the three forces acting on the ball must equal zero and the sum of y-components of these forces must equal zero. The sum of the x-components of the forces is

$$T \sin\theta - F_e = 0, \tag{i}$$

where T is the magnitude of the string tension, θ is the angle of the string relative to the vertical, and F_e is the magnitude of the electrostatic force. The sum of the y-components of the forces is

$$T \cos\theta - F_g = 0. \tag{ii}$$

The force of gravity, F_g, is just the weight of the charged ball:

$$F_g = mg, \tag{iii}$$

where m is the mass of the charged ball. The electrostatic force the two balls exert on each other is given by

$$F_e = k\frac{q^2}{d^2}, \tag{iv}$$

where d is the distance between the two balls. We can express the distance between the two balls in terms of the length of the string, ℓ, by looking at Figure 1.17a. We see that

$$\sin\theta = \frac{d/2}{\ell}.$$

We can then express the electrostatic force in terms of the angle with respect to the vertical, θ, and the length of the string, ℓ:

$$F_e = k\frac{q^2}{(2\ell\sin\theta)^2} = k\frac{q^2}{4\ell^2\sin^2\theta}. \tag{v}$$

SIMPLIFY We divide equation (i) by equation (ii):

$$\frac{T\sin\theta}{T\cos\theta} = \frac{F_e}{F_g},$$

which, after the (unknown) string tension is canceled out, becomes

$$\tan\theta = \frac{F_e}{F_g}.$$

Substituting from equations (iii) and (v) for the force of gravity and the electrostatic force, we get

$$\tan\theta = \frac{k\dfrac{q^2}{4\ell^2\sin^2\theta}}{mg} = \frac{kq^2}{4mg\ell^2\sin^2\theta}.$$

Solving for the mass of the ball, we obtain

$$m = \frac{kq^2}{4g\ell^2 \sin^2\theta \tan\theta}.$$

CALCULATE Putting in the numerical values gives

$$m = \frac{\left(8.99\times10^9 \text{ N m}^2/\text{C}^2\right)(25.0\ \mu\text{C})^2}{4\left(9.81 \text{ m/s}^2\right)(1.50 \text{ m})^2\left(\sin^2 25.0°\right)\left(\tan 25.0°\right)} = 0.764116 \text{ kg}.$$

ROUND We report our result to three significant figures:

$$m = 0.764 \text{ kg}.$$

DOUBLE-CHECK To double-check, we make the small-angle approximations that $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$. The tension in the string then approaches mg, and we can express the x-components of the forces as

$$T\sin\theta \approx mg\theta = F_e = k\frac{q^2}{d^2} \approx k\frac{q^2}{(2\ell\theta)^2}.$$

Solving for the mass of the charged ball, we get

$$m = \frac{kq^2}{4g\ell^2\theta^3} = \frac{\left(8.99\times10^9 \text{ N m}^2/\text{C}^2\right)(25.0\ \mu\text{C})^2}{4\left(9.81 \text{ m/s}^2\right)(1.50 \text{ m})^2(0.436 \text{ rad})^3} = 0.768 \text{ kg},$$

which is close to our answer.

Electrostatic Precipitator

An application of electrostatic charging and electrostatic forces is the cleaning of emissions from coal-fired power plants. A device called an **electrostatic precipitator** (ESP) is used to remove ash and other particulates produced when coal is burned to generate electricity. The operation of this device is illustrated in Figure 1.18.

The ESP consists of wires and plates, with the wires held at a high negative voltage relative to the series of plates held at a positive voltage. (Here the term *voltage* is used colloquially; in Chapter 3, the concept will be defined in terms of electric potential difference.) In Figure 1.18, the exhaust gas from the coal-burning process enters the ESP from the left. Particulates passing near the wires pick up a negative charge. These particles are then attracted to one of the positive plates and stick there. The gas continues through the ESP, leaving the ash and other particulates behind. The accumulated material is then shaken off the plates to a hamper below. This waste can be used for many purposes, including construction materials and fertilizer. Figure 1.19 shows an example of a coal-fired power plant that incorporates an ESP.

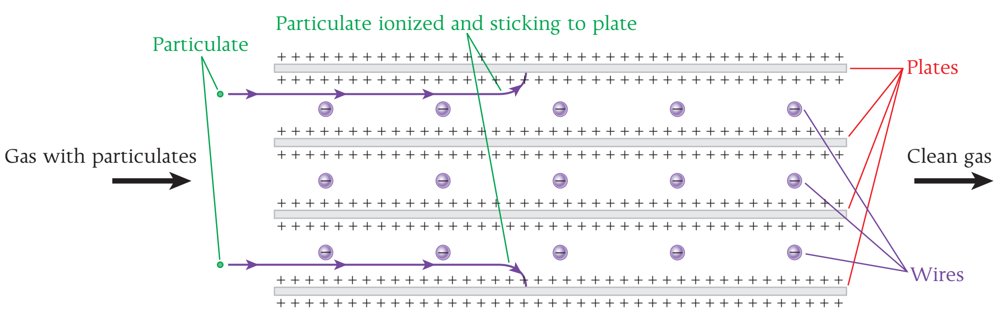
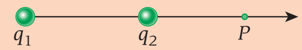


FIGURE 1.18 Operation of an electrostatic precipitator used to clean the exhaust gas of a coal-fired power plant. The view is from the top of the device.

Self-Test Opportunity 1.2

A positive point charge $+q$ is placed at point P, to the right of two charges q_1 and q_2, as shown in the figure. The net electrostatic force on the positive charge $+q$ is found to be zero. Identify each of the following statements as true or false.



a) Charge q_2 must have the opposite sign from q_1 and be smaller in magnitude.

b) The magnitude of charge q_1 must be smaller than the magnitude of charge q_2.

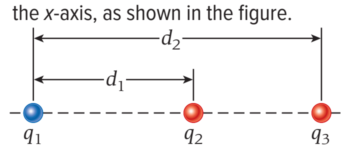
c) Charges q_1 and q_2 must have the same sign.

d) If q_1 is negative, then q_2 must be positive.

e) Either q_1 or q_2 must be positive.

Concept Check 1.8

Consider three charges placed along the x-axis, as shown in the figure.



The values of the charges are $q_1 = -8.10\ \mu\text{C}$, $q_2 = 2.16\ \mu\text{C}$, and $q_3 = 2.16$ pC. The distance between q_1 and q_2 is $d_1 = 1.71$ m. The distance between q_1 and q_3 is $d_2 = 2.62$ m. What is the magnitude of the total electrostatic force exerted on q_3 by q_1 and q_2?

a) 2.77×10^{-8} N

b) 7.92×10^{-6} N

c) 1.44×10^{-5} N

d) 2.22×10^{-4} N

e) 6.71×10^{-2} N

FIGURE 1.19 A coal-fired power plant at Michigan State University that incorporates an electrostatic precipitator to remove particulates from its emissions.

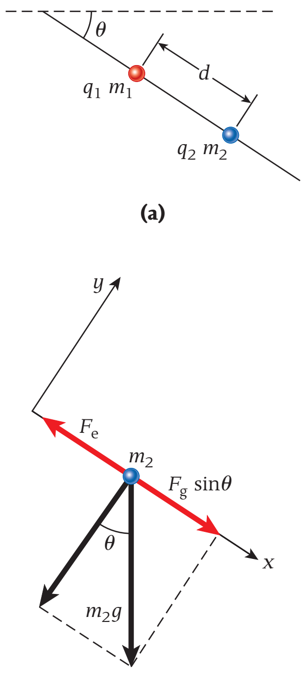


FIGURE 1.20 (a) Two charged beads on a wire. (b) Free-body diagram of the forces acting on the second bead.

SOLVED PROBLEM 1.2 Bead on a Wire

PROBLEM

A bead with charge $q_1 = +1.28\ \mu\text{C}$ is fixed in place on an insulating wire that makes an angle of $\theta = 42.3°$ with respect to the horizontal (Figure 1.20a). A second bead with charge $q_2 = -5.06\ \mu\text{C}$ slides without friction on the wire. At a distance $d = 0.380$ m between the beads, the net force on the second bead is zero. What is the mass, m_2, of the second bead?

SOLUTION

THINK The force of gravity pulling the bead of mass m_2 down the wire is offset by the attractive electrostatic force between the positive charge on the first bead and the negative charge on the second bead. The second bead can be thought of as sliding on an inclined plane.

SKETCH Figure 1.20b shows a free-body diagram of the forces acting on the second bead. We have defined a coordinate system in which the positive x-direction is down the wire. The force exerted on m_2 by the wire can be omitted because this force has only a y-component, and we can solve the problem by analyzing just the x-components of the forces.

RESEARCH The attractive electrostatic force between the two beads balances the component of the force of gravity that acts on the second bead down the wire. The electrostatic force acts in the negative x-direction and its magnitude is given by

$$F_e = k\frac{|q_1q_2|}{d^2}. \tag{i}$$

The x-component of the force of gravity acting on the second bead corresponds to the component of the weight of the second bead that is parallel to the wire. Figure 1.20b indicates that the component of the weight of the second bead down the wire is given by

$$F_g = m_2 g \sin\theta. \tag{ii}$$

SIMPLIFY For equilibrium, the electrostatic force and the gravitational force are equal: $F_e = F_g$. Substituting the expressions for these forces from equations (i) and (ii) yields

$$k\frac{|q_1q_2|}{d^2} = m_2 g \sin\theta.$$

Solving this equation for the mass of the second bead gives us

$$m_2 = \frac{k|q_1q_2|}{d^2 g \sin\theta}.$$

CALCULATE We put in the numerical values and get

$$m_2 = \frac{kq_1q_2}{d^2 g\sin\theta} = \frac{\left(8.99\times10^9\ \text{N m}^2/\text{C}^2\right)(1.28\ \mu\text{C})(5.06\ \mu\text{C})}{(0.380\ \text{m})^2\left(9.81\ \text{m/s}^2\right)(\sin 42.3°)} = 0.0610746\ \text{kg}.$$

ROUND We report our result to three significant figures:

$$m_2 = 0.0611\ \text{kg} = 61.1\ \text{g}.$$

DOUBLE-CHECK To double-check, let's calculate the mass of the second bead assuming that the wire is vertical, that is, $\theta = 90°$. We can then set the weight of the second bead equal to the electrostatic force between the two beads:

$$k\frac{|q_1q_2|}{d^2} = m_2 g.$$

Solving for the mass of the second bead, we obtain

$$m_2 = \frac{kq_1q_2}{d^2 g} = \frac{\left(8.99\times10^9\ \text{N m}^2/\text{C}^2\right)(1.28\ \mu\text{C})(5.06\ \mu\text{C})}{(0.380\ \text{m})^2\left(9.81\ \text{m/s}^2\right)} = 0.0411\ \text{kg}.$$

As the angle of the wire relative to the horizontal decreases, the calculated mass of the second bead will increase. Our result of 0.0611 kg is somewhat higher than the mass that can be supported with a vertical wire, so it seems reasonable.

Laser Printer

Another example of a device that applies electrostatic forces is the laser printer. The operation of such a printer is illustrated in Figure 1.21. The paper path follows the blue arrows. Paper is taken from the paper tray or fed manually through the alternate paper feed. The paper passes over a drum where the toner is placed on the surface of the paper and then passes through a fuser that melts the toner and permanently affixes it to the paper.

The drum consists of a metal cylinder coated with a special photosensitive material. The photosensitive surface is an insulator that retains charge in the absence of light, but discharges quickly if light is incident on the surface. The drum rotates so that its surface speed is the same as the speed of the moving paper. The basic principle of the operation of the drum is illustrated in Figure 1.22.

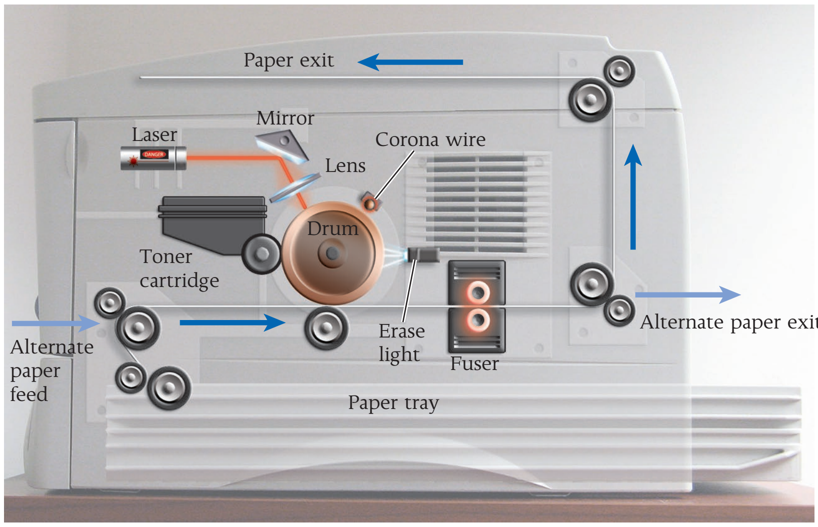


FIGURE 1.21 The operation of a typical laser printer.

The drum is negatively charged with electrons using a wire held at high voltage. Then laser light is directed at the surface of the drum. Wherever the laser light strikes the surface of the drum, the surface at that point is discharged. A laser is used because its beam is narrow and remains focused. A line of the image being printed is written one pixel (picture element or dot) at a time using a laser beam directed by a moving mirror and a lens. A typical laser printer can write 600−1200 pixels per inch. The surface of the drum then passes by a roller that picks up toner from the toner cartridge. Toner consists of small, black, insulating particles composed of a plastic-like material. The toner roller is charged to the same negative voltage as the drum. Therefore, wherever the surface of the drum has been discharged, electrostatic forces deposit toner on the surface of the drum. Any portion of the drum surface that has not been exposed to the laser will not pick up toner.

As the drum rotates, it next comes in contact with the paper. The toner is then transferred from the surface of the drum to the paper. As the drum rotates, any remaining toner is scraped off and the surface is neutralized with an erase light or a rotating erase drum in preparation for printing the next image. The paper then continues on to the fuser, which melts the toner, producing a permanent image on the paper.

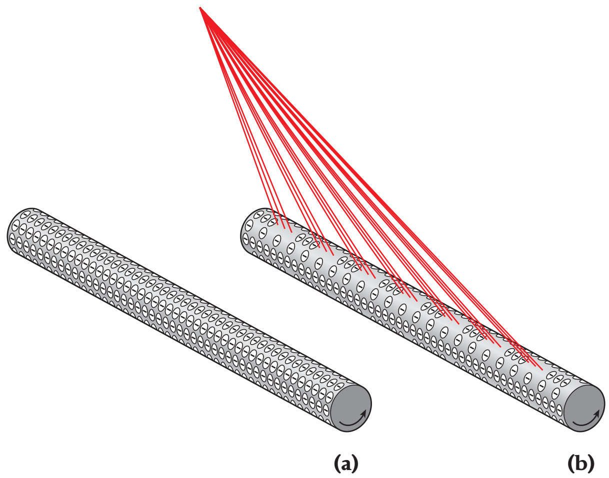


FIGURE 1.22 (a) The completely charged drum of a laser printer. This drum will produce a blank page. (b) A drum on which one line of information is being recorded by a laser. Wherever the laser strikes the charged drum, the negative charge is neutralized, and the discharged area will attract toner that will produce an image on the paper.

SOLVED PROBLEM 1.3 Four Charged Objects

Consider four charges placed at the corners of a square with side length 1.25 m, as shown in Figure 1.23a.

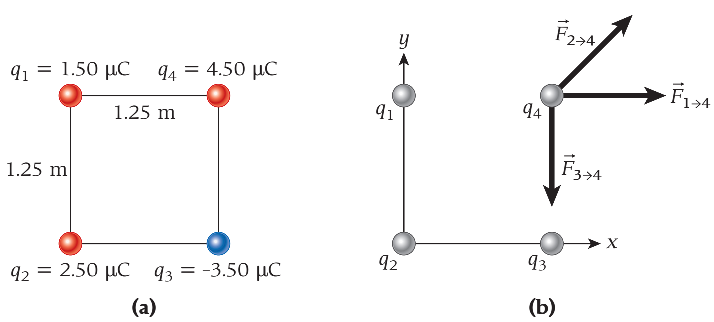


FIGURE 1.23 (a) Four charges placed at the corners of a square. (b) The forces exerted on q_4 by the other three charges.

– Continued

PROBLEM

What are the magnitude and the direction of the electrostatic force on q_4 resulting from the other three charges?

SOLUTION

THINK The electrostatic force on q_4 is the vector sum of the forces resulting from its interactions with the other three charges. Thus, it is important to avoid simply adding the individual force magnitudes algebraically. Instead we need to determine the individual force components in each spatial direction and add those to find the components of the net force vector. Then we need to calculate the length of that net force vector.

SKETCH Figure 1.23b shows the four charges in an xy-coordinate system with its origin at the location of q_2.

RESEARCH The net force on q_4 is the vector sum of the forces $\vec{F}_{1\to4}$, $\vec{F}_{2\to4}$, and $\vec{F}_{3\to4}$. The x-component of the summed forces is

$$F_x = k\frac{|q_1 q_4|}{d^2} + k\frac{|q_2 q_4|}{\left(\sqrt{2}d\right)^2}\cos 45° = \frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right), \tag{i}$$

where d is the length of a side of the square and, as Figure 1.23b indicates, the x-component of $\vec{F}_{3\to4}$ is zero. The y-component of the summed forces is

$$F_y = k\frac{|q_2 q_4|}{\left(\sqrt{2}d\right)^2}\sin 45° - k\frac{|q_3 q_4|}{d^2} = \frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right), \tag{ii}$$

where, as Figure 1.23b indicates, the y-component of $\vec{F}_{1\to4}$ is zero.

The magnitude of the net force is given by

$$F = \sqrt{F_x^2 + F_y^2}, \tag{iii}$$

and the angle of the net force is given by

$$\tan\theta = \frac{F_y}{F_x}.$$

SIMPLIFY We substitute the expressions for F_x and F_y from equations (i) and (ii) into equation (iii):

$$F = \sqrt{\left[\frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right)\right]^2 + \left[\frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right)\right]^2}.$$

We can rewrite this as

$$F = \frac{kq_4}{d^2}\sqrt{\left(q_1 + \frac{q_2}{2}\cos 45°\right)^2 + \left(\frac{q_2}{2}\sin 45° + q_3\right)^2}.$$

For the angle of the force, we get

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{\frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right)}{\frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right)}\right) = \tan^{-1}\left(\frac{\left(\frac{q_2}{2}\sin 45° + q_3\right)}{\left(q_1 + \frac{q_2}{2}\cos 45°\right)}\right).$$

CALCULATE Putting in the numerical values, we get

$$\frac{q_2}{2}\sin 45° = \frac{q_2}{2}\cos 45° = \frac{2.50\ \mu\text{C}}{2\sqrt{2}} = 0.883883\ \mu\text{C}.$$

The magnitude of the force is then

$$F = \frac{\left(8.99\times10^9\ \text{N m}^2/\text{C}^2\right)(4.50\ \mu\text{C})}{(1.25\ \text{m})^2}\sqrt{(1.50\ \mu\text{C} + 0.883883\ \mu\text{C})^2 + (0.883883\ \mu\text{C} - 3.50\ \mu\text{C})^2}$$
$$= 0.0916379\ \text{N}$$

For the direction of the force, we obtain

$$\theta = \tan^{-1}\left(\frac{\left(\frac{q_2}{2}\sin 45^\circ + q_3\right)}{\left(q_1 + \frac{q_2}{2}\cos 45^\circ\right)}\right) = \tan^{-1}\left(\frac{(0.883883\ \mu\text{C} - 3.50\ \mu\text{C})}{(1.50\ \mu\text{C} + 0.883883\ \mu\text{C})}\right) = -\,47.6593^\circ.$$

ROUND We report our results to three significant figures:

$$F = 0.0916\ \text{N}$$

and

$$\theta = -\,47.7^\circ.$$

DOUBLE-CHECK To double-check our result, we calculate the magnitude of the three forces acting on q_4. For $\vec{F}_{1\to 4}$, we get

$$F_{1\to 4} = k\frac{|q_1 q_4|}{r_{14}^2} = \frac{\left(8.99\times 10^9\ \text{N m}^2/\text{C}^2\right)(1.50\ \mu\text{C})(4.50\ \mu\text{C})}{(1.25\ \text{m})^2} = 0.0388\ \text{N}.$$

For $\vec{F}_{2\to 4}$, we get

$$F_{2\to 4} = k\frac{|q_2 q_4|}{r_{24}^2} = \frac{\left(8.99\times 10^9\ \text{N m}^2/\text{C}^2\right)(2.50\ \mu\text{C})(4.50\ \mu\text{C})}{\left[\sqrt{2}(1.25\ \text{m})\right]^2} = 0.0324\ \text{N}.$$

For $F_{3\to 4}$ we get

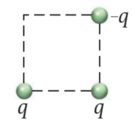
$$F_{3\to 4} = k\frac{|q_3 q_4|}{r_{34}^2} = \frac{\left(8.99\times 10^9\ \text{N m}^2/\text{C}^2\right)(3.50\ \mu\text{C})(4.50\ \mu\text{C})}{(1.25\ \text{m})^2} = 0.0906\ \text{N}$$

All three of the magnitudes of the individual forces are of the same order as our result for the net force. This gives us confidence that our answer is not off by a large factor.

The direction we obtained also seems reasonable, because it orients the resulting force downward and to the right, as could be expected from looking at Figure 1.23b.

Concept Check 1.9

Three charges are arranged at the corners of a square as shown in the figure. What is the direction of the electrostatic force on the *lower-right* charge?

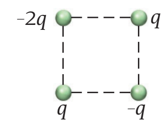


a) ↙ b) ↖ c) ↗ d) ↘

e) There is no force on that charge.

Concept Check 1.10

Four charges are arranged at the corners of a square as shown in the figure. What is the direction of the electrostatic force on the *lower-right* charge?



a) ↙ b) ↖ c) ↗ d) ↘

e) There is no force on that charge.

1.6 Coulomb's Law and Newton's Law of Gravitation

Coulomb's Law describing the electrostatic force between two electric charges, F_e, has a form similar to Newton's Law describing the gravitational force between two masses, F_g:

$$F_g = G\frac{m_1 m_2}{r^2} \quad \text{and} \quad F_e = k\frac{|q_1 q_2|}{r^2}$$

where m_1 and m_2 are the two masses, q_1 and q_2 are the two electric charges, and r is the distance of separation. Both forces vary with the inverse square of the distance. The electrostatic force can be attractive or repulsive because charges can have positive or negative signs. (See Figure 1.14a and b.) The gravitational force is always attractive because there is only one kind of mass. (For the gravitational force, only the case depicted in Figure 1.14b is possible.) The relative strengths of the forces are given by the proportionality constants k and G.

EXAMPLE 1.4 Forces between Electrons

Let's evaluate the relative strengths of the two interactions by calculating the ratio of the electrostatic force and the gravitational force that two electrons exert on each other. This ratio is given by

$$\frac{F_e}{F_g} = \frac{kq_e^2}{Gm_e^2}.$$

Because the dependence on distance is the same for both forces, there is no dependence on distance in the ratio of the two forces–it cancels out. The mass of an electron is $m_e = 9.109\text{x}10^{-31}$ kg,

– Continued

Concept Check 1.11

The proton's mass is ~2000 times larger than the electron's mass. Therefore, the ratio F_e/F_g for two protons is ___________ the value calculated in Example 1.4 for two electrons.

a) ~4 million times smaller than

b) ~2000 times smaller than

c) the same as

d) ~2000 times larger than

e) ~4 million times larger than

and its charge is $q_e = -1.602 \times 10^{-19}$ C. Using the value of Coulomb's constant given in equation 1.7, $k = 8.99 \times 10^9$ N m^2/C^2, and the value of the universal gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2/kg^2, we find numerically

$$\frac{F_e}{F_g} = \frac{(8.99\times10^9 \text{ N m}^2/\text{C}^2)(1.602\times10^{-19} \text{ C})^2}{(6.67\times10^{-11} \text{ N m}^2/\text{kg}^2)(9.109\times10^{-31} \text{ kg})^2} = 4.17\times10^{42}.$$

Therefore, the electrostatic force between electrons is stronger than the gravitational force between them by more than 42 orders of magnitude.

Despite the relative weakness of the gravitational force, it is the only force that matters on the astronomical scale. The reason for this dominance is that all stars, planets, and other objects of astronomical relevance carry no net charge. Therefore, there is no net electrostatic interaction between them, and gravity dominates.

Coulomb's Law of electrostatics applies to macroscopic systems down to the atom, though subtle effects in atomic and subatomic systems require use of a more sophisticated approach called *quantum electrodynamics*. Newton's law of gravitation fails in subatomic systems and also must be modified for some phenomena in astronomical systems, such as the precessional motion of Mercury around the Sun. These fine details of the gravitational interaction are governed by Einstein's theory of general relativity.

The similarities between the gravitational and electrostatic interactions will be covered further in the next two chapters, which address electric fields and electric potential.

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- There are two kinds of electric charge, positive and negative. Like charges repel, and unlike charges attract.
- The quantum (elementary quantity) of electric charge is $e = 1.602 \times 10^{-19}$ C.
- The electron has charge $q_e = -e$, and the proton has charge $q_p = +e$. The neutron has zero charge.
- The net charge of an object is given by e times the number of protons, N_p, minus e times the number of electrons, N_e, that make up the object: $q = e \cdot (N_p - N_e)$.
- The total charge in an isolated system is always conserved.
- Objects can be charged directly by contact or indirectly by induction.
- Coulomb's Law describes the force that two stationary charges exert on each other: $F = k\frac{|q_1q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2}$.
- The constant in Coulomb's Law is
$$k = \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9 \frac{\text{N m}^2}{\text{C}^2}.$$
- The electric permittivity of free space is
$$\varepsilon_0 = 8.85\times10^{-12} \frac{\text{C}^2}{\text{N m}^2}.$$

ANSWERS TO SELF-TEST OPPORTUNITIES

21.1 a) +1 b) 0 c) 0 d) 0 e) $+\frac{2}{3}$ f) $-\frac{1}{3}$ g) −1 h) +2

22.2 a) true b) false c) false d) true e) true

PROBLEM-SOLVING GUIDELINES

1. For problems involving Coulomb's Law, drawing a free-body diagram showing the electrostatic force vectors acting on a charged particle is often helpful. Pay careful attention to signs; a negative force between two particles indicates attraction, and a positive force indicates repulsion. Be sure that the directions of forces in the diagram match the signs of forces in the calculations.

2. Use symmetry to simplify your work. However, be careful to take account of charge magnitudes and signs as well as distances. Two charges at equal distances from a third charge do not exert equal forces on that charge if they have different magnitudes or signs.

3. Units in electrostatics often have prefixes indicating powers of 10: Distances may be given in cm or mm; charges may be given in μC, nC, or pC; masses may be given in kg or g. Other units are also common. The best way to proceed is to convert all quantities to basic SI units, to be compatible with the value of k or $1/4\pi\epsilon_0$.

MULTIPLE-CHOICE QUESTIONS

1.1 When a metal plate is given a positive charge, which of the following is taking place?

a) Protons (positive charges) are transferred to the plate from another object.

b) Electrons (negative charges) are transferred from the plate to another object.

c) Electrons (negative charges) are transferred from the plate to another object, and protons (positive charges) are also transferred to the plate from another object.

d) It depends on whether the object conveying the charge is a conductor or an insulator.

1.2 The force between a charge of 25 μC and a charge of −10 μC is 8.0 N. What is the separation between the two charges?

a) 0.28 m
b) 0.53 m
c) 0.45 m
d) 0.15 m

1.3 A charge Q_1 is positioned on the x-axis at $x = a$. Where should a charge $Q_2 = -4Q_1$ be placed to produce a net electrostatic force of zero on a third charge, $Q_3 = Q_1$, located at the origin?

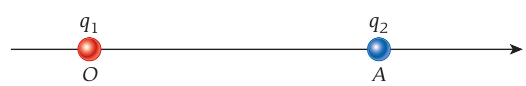
a) at the origin
b) at $x = 2a$
c) at $x = -2a$
d) at $x = -a$

1.4 Which one of these systems has the most negative charge?

a) 2 electrons
b) 3 electrons and 1 proton
c) 5 electrons and 5 protons
d) N electrons and $N - 3$ protons
e) 1 electron

1.5 Two point charges are fixed on the x-axis: $q_1 = 6.0$ μC is located at the origin, O, with $x_1 = 0.0$ cm, and $q_2 = -3.0$ μC is located at point A, with $x_2 = 8.0$ cm. Where should a third charge, q_3, be placed on the x-axis so that the total electrostatic force acting on it is zero?

a) 19 cm
b) 27 cm
c) 0.0 cm
d) 8.0 cm
e) −19 cm



1.6 Which of the following situations produces the largest net force on the charge Q?

a) Charge $Q = 1$ C is 1 m from a charge of −2 C.

b) Charge $Q = 1$ C is 0.5 m from a charge of −1 C.

c) Charge $Q = 1$ C is halfway between a charge of −1 C and a charge of 1 C that are 2 m apart.

d) Charge $Q = 1$ C is halfway between two charges of −2 C that are 2 m apart.

e) Charge $Q = 1$ C is a distance of 2 m from a charge of −4 C.

1.7 Two protons placed near one another with no other objects close by would

a) accelerate away from each other.
b) remain motionless.
c) accelerate toward each other.
d) be pulled together at constant speed.
e) move away from each other at constant speed.

1.8 Two lightweight metal spheres are suspended near each other from insulating threads. One sphere has a net charge; the other sphere has no net charge. The spheres will

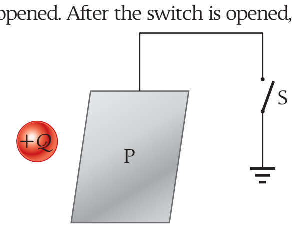
a) attract each other.

b) exert no net electrostatic force on each other.

c) repel each other.

d) do any of these things depending on the sign of the net charge on the one sphere.

1.9 A metal plate is connected by a conductor to a ground through a switch. The switch is initially closed. A charge $+Q$ is brought close to the plate without touching it, and then the switch is opened. After the switch is opened, the charge $+Q$ is removed. What is the charge on the plate then?



a) The plate is uncharged.

b) The plate is positively charged.

c) The plate is negatively charged.

d) The plate could be either positively or negatively charged, depending on the charge it had before $+Q$ was brought near.

1.10 You bring a negatively charged rubber rod close to a grounded conductor without touching it. Then you disconnect the ground. What is the sign of the charge on the conductor after you remove the charged rod?

a) negative
b) positive
c) no charge
d) cannot be determined from the information given

1.11 When a rubber rod is rubbed with rabbit fur, the rod becomes

a) negatively charged.

b) positively charged.

c) neutral.

d) either negatively charged or positively charged, depending on whether the fur is always moved in the same direction or is moved back and forth.

1.12 When a glass rod is rubbed with a polyester scarf, the rod becomes

a) negatively charged.

b) positively charged.

c) neutral.

d) either negatively charged or positively charged, depending on whether the scarf is always moved in the same direction or is moved back and forth.

1.13 Consider an electron with mass m and charge $-e$ moving in a circular orbit with radius r around a fixed proton with mass M and charge $+e$. The electron is held in orbit by the electrostatic force between itself and the proton. Which one of the following expressions for the speed of the electron is correct?

a) $v = \sqrt{\dfrac{ke^2}{mr}}$

b) $v = \sqrt{\dfrac{GM}{r}}$

c) $v = \sqrt{\dfrac{2ke^2}{mr^2}}$

d) $v = \sqrt{\dfrac{me^2}{kr}}$

e) $v = \sqrt{\dfrac{ke^2}{2Mr}}$

1.14 Consider an electron with mass m and charge $-e$ located a distance r from a fixed proton with mass M and charge $+e$. The electron is released from rest. Which one of the following expressions for the magnitude of the initial acceleration of the electron is correct?

a) $a = \dfrac{2ke^2}{mMr}$

b) $a = \sqrt{\dfrac{2e^2}{mkr}}$

c) $a = \frac{1}{2}me^2k^2$

d) $a = \dfrac{2ke^2}{mr}$

e) $a = \dfrac{ke^2}{mr^2}$

CONCEPTUAL QUESTIONS

1.15 If two charged particles (the charge on each is Q) are separated by a distance d, there is a force F between them. What is the force if the magnitude of each charge is doubled and the distance between them changes to $2d$?

1.16 Suppose the Sun and the Earth were each given an equal amount of charge of the same sign, just sufficient to cancel their gravitational attraction. How many times the charge on an electron would that charge be? Is this number a large fraction of the number of charges of either sign in the Earth?

1.17 It is apparent that the electrostatic force is *extremely* strong, compared to gravity. In fact, the electrostatic force is the basic force governing phenomena in daily life–the tension in a string, the normal forces between surfaces, friction, chemical reactions, and so forth–except weight. Why then did it take so long for scientists to understand this force? Newton came up with his gravitational law long before electricity was even crudely understood.

1.18 Occasionally, people who gain static charge by shuffling their feet on the carpet will have their hair stand on end. Why does this happen?

1.19 Two positive charges, each equal to Q, are placed a distance $2d$ apart. A third charge, $-0.2Q$, is placed exactly halfway between the two positive charges and is displaced a distance $x \ll d$ (that is, x is much smaller than d) perpendicular to the line connecting the positive charges. What is the force on this charge? For $x \ll d$, how can you approximate the motion of the negative charge?

1.20 Why does a garment taken out of a clothes dryer sometimes cling to your body when you wear it?

1.21 Two charged spheres are initially a distance d apart. The magnitude of the force on each sphere is F. They are moved closer to each other such that the magnitude of the force on each of them is $9F$. By what factor has the distance between the two spheres changed?

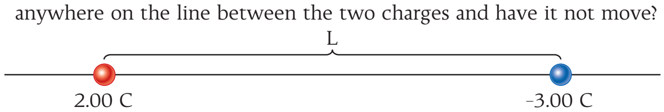
1.22 How is it possible for one electrically neutral atom to exert an electrostatic force on another electrically neutral atom?

1.23 The scientists who first contributed to the understanding of the electrostatic force in the 18th century were well aware of Newton's law of gravitation. How could they deduce that the force they were studying was *not* a variant or some manifestation of the gravitational force?

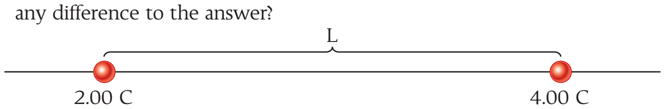
1.24 Two charged particles move solely under the influence of the electrostatic forces between them. What shapes can their trajectories have?

1.25 Rubbing a balloon causes it to become negatively charged. The balloon then tends to cling to the wall of a room. For this to happen, must the wall be positively charged?

1.26 Two electric charges are placed on a line, as shown in the figure. Is it possible to place a charged particle (that is free to move) anywhere on the line between the two charges and have it not move?



1.27 Two electric charges are placed on a line as shown in the figure. Where on the line can a third charge be placed so that the force on that charge is zero? Does the sign or the magnitude of the third charge make any difference to the answer?



1.28 When a positively charged rod is brought close to a neutral conductor without touching it, will the rod experience an attractive force, a repulsive force, or no force at all? Explain.

1.29 When you exit a car and the humidity is low, you often experience a shock from static electricity created by sliding across the seat. How can you discharge yourself without experiencing a painful shock? Why is it dangerous to get back into your car while fueling your car?

EXERCISES

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

Section 1.2

1.30 How many electrons are required to yield a total charge of 1.00 C?

1.31 The *faraday* is a unit of charge frequently encountered in electrochemical applications and named for the British physicist and chemist Michael Faraday. It consists of exactly 1 mole of elementary charges. Calculate the number of coulombs in 1.000 faraday.

1.32 Another unit of charge is the *electrostatic unit* (esu). It is defined as follows: Two point charges, each of 1 esu and separated by 1 cm, exert a force of exactly 1 dyne on each other: 1 dyne = 1 g cm/s^2 = 1×10^{-5} N.

a) Determine the relationship between the esu and the coulomb.

b) Determine the relationship between the esu and the elementary charge.

1.33 A current of 5.00 mA is enough to make your muscles twitch. Calculate how many electrons flow through your skin if you are exposed to such a current for 10.0 s.

•**1.34** How many electrons does 1.00 kg of water contain?

•**1.35** The Earth is constantly being bombarded by cosmic rays, which consist mostly of protons. These protons are incident on the Earth's atmosphere from all directions at a rate of 1245. protons per square meter per second. Assuming that the depth of Earth's atmosphere is 120.0 km, what is the total charge incident on the atmosphere in 5.000 min? Assume that the radius of the surface of the Earth is 6378. km.

•**1.36** Performing an experiment similar to Millikan's oil drop experiment, a student measures these charge magnitudes:

3.26×10^{-19} C	5.09×10^{-19} C	1.53×10^{-19} C
6.39×10^{-19} C	4.66×10^{-19} C	

Find the charge on the electron using these measurements.

Section 1.3

•**1.37** A silicon sample is doped with phosphorus at 1 part per 1.00×10^6. Phosphorus acts as an electron donor, providing one free electron per atom. The density of silicon is 2.33 g/cm^3, and its atomic mass is 28.09 g/mol.

a) Calculate the number of free (conduction) electrons per unit volume of the doped silicon.

b) Compare the result from part (a) with the number of conduction electrons per unit volume of copper wire, assuming that each copper atom produces one free (conduction) electron. The density of copper is 8.96 g/cm^3, and its atomic mass is 63.54 g/mol.

Section 1.5

1.38 Two charged spheres are 8.00 cm apart. They are moved closer to each other by enough that the force on each of them increases four times. How far apart are they now?

1.39 Two identically charged particles separated by a distance of 1.00 m repel each other with a force of 1.00 N. What is the magnitude of the charges?

1.40 How far apart must two electrons be placed on the Earth's surface for there to be an electrostatic force between them equal to the weight of one of the electrons?

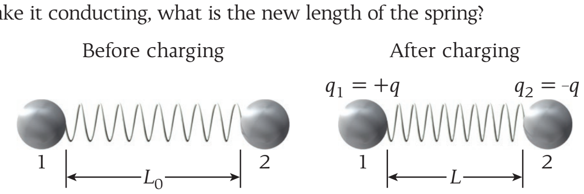
1.41 In solid sodium chloride (table salt), chloride ions have one more electron than they have protons, and sodium ions have one more proton than they have electrons. These ions are separated by about 0.28 nm. Calculate the electrostatic force between a sodium ion and a chloride ion.

1.42 In gaseous sodium chloride, chloride ions have one more electron than they have protons, and sodium ions have one more proton than they have electrons. These ions are separated by about 0.24 nm. Suppose a free electron is located 0.48 nm above the midpoint of the sodium chloride molecule. What are the magnitude and the direction of the electrostatic force the molecule exerts on it?

1.43 Calculate the magnitude of the electrostatic force the two up quarks inside a proton exert on each other if they are separated by a distance of 0.900 fm.

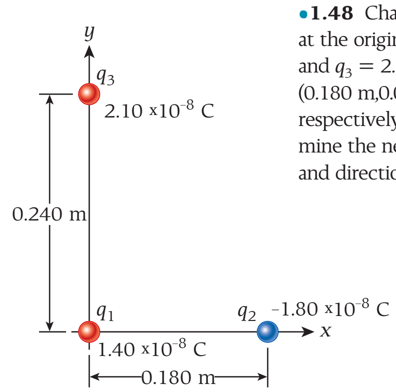
1.44 A −4.00 μC charge lies 20.0 cm to the right of a 2.00 μC charge on the x-axis. What is the force on the 2.00-μC charge?

•**1.45** Two initially uncharged identical metal spheres, 1 and 2, are connected by an insulating spring (unstretched length $L_0 = 1.00$ m, spring constant $k = 25.0$ N/m), as shown in the figure. Charges $+q$ and $-q$ are then placed on the spheres, and the spring contracts to length $L = 0.635$ m. Recall that the force exerted by a spring is $F_s = k\Delta x$, where Δx is the change in the spring's length from its equilibrium length. Determine the charge q. If the spring is coated with metal to make it conducting, what is the new length of the spring?



•**1.46** A point charge $+3q$ is located at the origin, and a point charge $-q$ is located on the x-axis at $D = 0.500$ m. At what location on the x-axis will a third charge, q_0, experience no net force from the other two charges?

•**1.47** Identical point charges Q are placed at each of the four corners of a rectangle measuring 2.00 m by 3.00 m. If $Q = 32.0$ μC, what is the magnitude of the electrostatic force on any one of the charges?

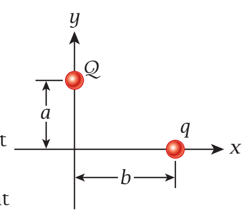


•**1.48** Charge $q_1 = 1.40 \times 10^{-8}$ C is placed at the origin. Charges $q_2 = -1.80 \times 10^{-8}$ C and $q_3 = 2.10 \times 10^{-8}$ C are placed at points (0.180 m,0.000 m) and (0.000 m,0.240 m), respectively, as shown in the figure. Determine the net electrostatic force (magnitude and direction) on charge q_3.

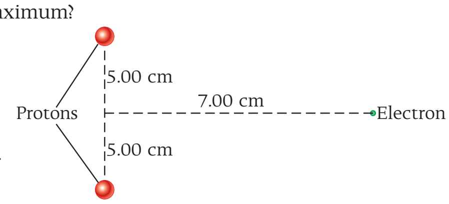
•**1.49** A positive charge Q is on the y-axis at a distance a from the origin, and another positive charge q is on the x-axis at a distance b from the origin.

a) For what value(s) of b is the x-component of the force on q a minimum?

b) For what value(s) of b is the x-component of the force on q a maximum?



•**1.50** Find the magnitude and direction of the electrostatic force acting on the electron in the figure.



•**1.51** In a region of two-dimensional space, there are three fixed charges: +1.00 mC at (0,0), −2.00 mC at (17.0 mm,−5.00 mm), and +3.00 mC at (−2.00 mm,11.0 mm). What is the net force on the −2.00 mC charge?

•**1.52** Two cylindrical glass beads each of mass $m = 10.0$ mg are set on their flat ends on a horizontal insulating surface separated by a distance $d = 2.00$ cm. The coefficient of static friction between the beads and the surface is $\mu_s = 0.200$. The beads are then given identical charges (magnitude and sign). What is the minimum charge needed to start the beads moving?

•**1.53** A small ball with a mass of 30.0 g and a charge of −0.200 μC is suspended from the ceiling by a string. The ball hangs at a distance of 5.00 cm above an insulating floor. If a second small ball with a mass of 50.0 g and a charge of 0.400 μC is rolled directly beneath the first ball, will the second ball leave the floor? What is the tension in the string when the second ball is directly beneath the first ball?

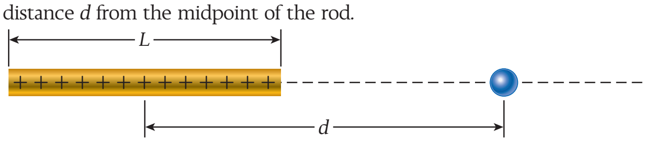
•**1.54** A +3.00 mC charge and a −4.00 mC charge are fixed in position and separated by 5.00 m.

a) Where can a +7.00 mC charge be placed so that the net force on it is zero?

b) Where can a −7.00 mC charge be placed so that the net force on it is zero?

•**1.55** Four point charges, q, are fixed to the four corners of a square that is 10.0 cm on a side. An electron is suspended above a point at which its weight is balanced by the electrostatic force due to the four electrons, at a distance of 15.0 nm above the center of the square. What is the magnitude of the fixed charges? Express the charge both in coulombs and as a multiple of the electron's charge.

••**1.56** The figure shows a uniformly charged thin rod of length L that has total charge Q. Find an expression for the magnitude of the electrostatic force acting on an electron positioned on the axis of the rod at a distance d from the midpoint of the rod.



••**1.57** A negative charge, $-q$, is fixed at the coordinate (0,0). It is exerting an attractive force on a positive charge, $+q$, that is initially at coordinate (x,0). As a result, the positive charge accelerates toward the negative charge. Use the binomial expansion $(1+x)^n \approx 1 + nx$, for $x \ll 1$, to show that when the positive charge moves a distance $\delta \ll x$ closer to the negative charge, the force that the negative charge exerts on it increases by $\Delta F = 2kq^2\delta/x^3$.

••**1.58** Two negative charges ($-q$ and $-q$) of equal magnitude are fixed at coordinates ($-d$,0) and (d,0). A positive charge of the same magnitude, q, and with mass m is placed at coordinate (0,0), midway between the two negative charges. If the positive charge is moved a distance $\delta \ll d$ in the positive y-direction and then released, the resulting motion will be that of a harmonic oscillator–the positive charge will oscillate between coordinates (0,δ) and (0,$-\delta$). Find the net force acting on the positive charge when it moves to (0,δ) and use the binomial expansion $(1+x)^n \approx 1+nx$, for $x \ll 1$, to find an expression for the frequency of the resulting oscillation. (*Hint*: Keep only terms that are linear in δ.)

Section 1.6

1.59 Suppose the Earth and the Moon carried positive charges of equal magnitude. How large would the charge need to be to produce an electrostatic repulsion equal to 1.00% of the gravitational attraction between the two bodies?

1.60 The similarity of form of Newton's law of gravitation and Coulomb's Law caused some to speculate that the force of gravity is related to the electrostatic force. Suppose that gravitation is entirely electrical in nature–that an excess charge Q on the Earth and an equal and opposite excess charge $-Q$ on the Moon are responsible for the gravitational force that causes the observed orbital motion of the Moon about the Earth. What is the required size of Q to reproduce the observed magnitude of the gravitational force?

•**1.61** In the Bohr model of the hydrogen atom, the electron moves around the one-proton nucleus on circular orbits of well-determined radii, given by $r_n = n^2 a_B$, where $n = 1, 2, 3, \ldots$ is an integer that defines the orbit and $a_B = 5.29 \times 10^{-11}$m is the radius of the first (minimum) orbit, called the *Bohr radius*. Calculate the force of electrostatic interaction between the electron and the proton in the hydrogen atom for the first four orbits. Compare the strength of this interaction to the gravitational interaction between the proton and the electron.

•**1.62** Some of the earliest atomic models held that the orbital velocity of an electron in an atom could be correlated with the radius of the atom. If the radius of the hydrogen atom is 5.29×10^{-11} m and the electrostatic force is responsible for the circular motion of the electron, what is the kinetic energy of this orbital electron?

1.63 For the atom described in Problem 1.62, what is the ratio of the gravitational force between electron and proton to the electrostatic force? How does this ratio change if the radius of the atom is doubled?

•**1.64** In general, astronomical objects are not exactly electrically neutral. Suppose the Earth and the Moon each carry a charge of -1.00×10^6 C (this is approximately correct; a more precise value is identified in Chapter 2).

a) Compare the resulting electrostatic repulsion with the gravitational attraction between the Moon and the Earth. Look up any necessary data.

b) What effects does this electrostatic force have on the size, shape, and stability of the Moon's orbit around the Earth?

Additional Exercises

1.65 Eight 1.00 μC charges are arrayed along the y-axis located every 2.00 cm starting at $y = 0$ and extending to $y = 14.0$ cm. Find the force on the charge at $y = 4.00$ cm.

1.66 In a simplified Bohr model of the hydrogen atom, an electron is assumed to be traveling in a circular orbit of radius of about 5.29×10^{-11} m around a proton. Calculate the speed of the electron in that orbit.

1.67 The nucleus of a carbon-14 atom (mass = 14 amu) has diameter of 3.01 fm. It has 6 protons and a charge of $+6e$.

a) What is the force on a proton located at 3.00 fm from the surface of this nucleus? Assume that the nucleus is a point charge.

b) What is the proton's acceleration?

1.68 Two charged objects experience a mutual repulsive force of 0.100 N.
If the charge of one of the objects is reduced by half and the distance separating the objects is doubled, what is the new force?

1.69 A particle (charge = +19.0 μC) is located on the x-axis at $x = -10.0$ cm, and a second particle (charge = −57.0 μC) is placed on the x-axis at $x = +20.0$ cm. What is the magnitude of the total electrostatic force on a third particle (charge = −3.80 μC) placed at the origin ($x = 0$)?

1.70 Three point charges are positioned on the x-axis: +64.0 μC at $x = 0.00$ cm, +80.0 μC at $x = 25.0$ cm, and −160.0 μC at $x = 50.0$ cm. What is the magnitude of the electrostatic force acting on the +64.0−μC charge?

1.71 From collisions with cosmic rays and from the solar wind, the Earth has a net electric charge of approximately -6.8×10^5 C. Find the charge that must be given to a 1.0 g object for it to be electrostatically levitated close to the Earth's surface.

•**1.72** A 10.0 g mass is suspended 5.00 cm above a nonconducting flat plate, directly above an embedded charge of q (in coulombs). If the mass has the same charge, q, how much must q be so that the mass levitates (just floats, neither rising nor falling)? If the charge q is produced by adding electrons to the mass, by how much will the mass be changed?

•**1.73** Four point charges are placed at the following xy-coordinates:

$Q_1 = -1.00$ mC, at (−3.00 cm,0.00 cm)

$Q_2 = -1.00$ mC, at (+3.00 cm,0.00 cm)

$Q_3 = +1.024$ mC, at (0.00 cm,0.00 cm)

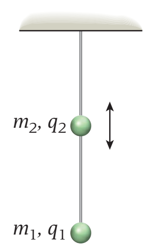
$Q_4 = +2.00$ mC, at (0.00 cm,−4.00 cm)

Calculate the net force on charge Q4 due to charges Q1, Q2, and Q3.

•**1.74** Three 5.00 g Styrofoam balls of radius 2.00 cm are coated with carbon black to make them conducting and then are tied to 1.00 m-long threads and suspended freely from a common point. Each ball is given the same charge, q. At equilibrium, the balls form an equilateral triangle with sides of length 25.0 cm in the horizontal plane. Determine q.

•**1.75** Two point charges lie on the x-axis. If one point charge is 6.00 μC and lies at the origin and the other is −2.00 μC and lies at 20.0 cm, at what position must a third charge be placed to be in equilibrium?

•**1.76** Two beads with charges $q_1 = q_2 = +2.67$ μC are on an insulating string that hangs straight down from the ceiling as shown in the figure. The lower bead is fixed in place on the end of the string and has a mass $m_1 = 0.280$ kg. The second bead slides without friction on the string. At a distance $d = 0.360$ m between the centers of



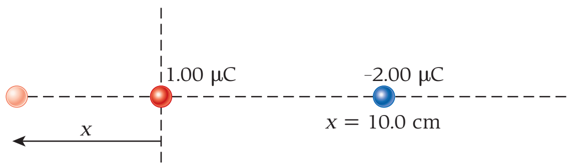
the beads, the force of the Earth's gravity on m_2 is balanced by the electrostatic force between the two beads. What is the mass, m_2, of the second bead? (*Hint:* You can neglect the gravitational interaction between the two beads.)

•**1.77** Find the net force on a +2.00-C charge at the origin of an xy-coordinate system if there is a +5.00 C charge at (3.00 m,0.00) and a −3.00 C charge at (0.00,4.00 m).

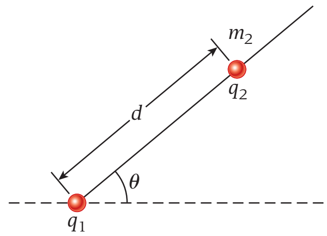
•**1.78** Two spheres, each of mass $M = 2.33$ g, are attached by pieces of string of length $L = 45.0$ cm to a common point. The strings initially hang straight down, with the spheres just touching one another. An equal amount of charge, q, is placed on each sphere. The resulting forces on the spheres cause each string to hang at an angle of $\theta = 10.0°$ from the vertical. Determine q, the amount of charge on each sphere.

•**1.79** A point charge $q_1 = 100.$ nC is at the origin of an xy-coordinate system, a point charge $q_2 = -80.0$ nC is on the x-axis at $x = 2.00$ m, and a point charge $q_3 = -60.0$ nC is on the y-axis at $y = -2.00$ m. Determine the net force (magnitude and direction) on q_1.

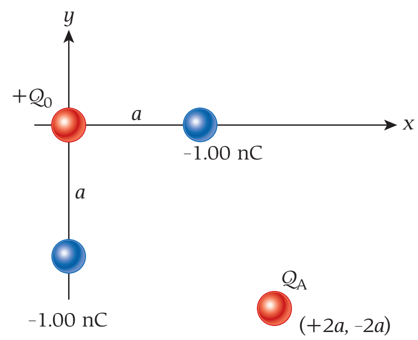
•**1.80** A positive charge $q_1 = 1.00$ μC is fixed at the origin, and a second charge $q_2 = -2.00$ μC is fixed at $x = 10.0$ cm. Where along the x-axis should a third charge be positioned so that it experiences no force?



•**1.81** A bead with charge $q_1 = 1.27$ μC is fixed in place at the end of a wire that makes an angle of $\theta = 51.3°$ with the horizontal. A second bead with mass $m_2 = 3.77$ g and a charge of 6.79 μC slides without friction on the wire. What is the distance d at which the force of the Earth's gravity on m_2 is balanced by the electrostatic force between the two beads? Neglect the gravitational interaction between the two beads.

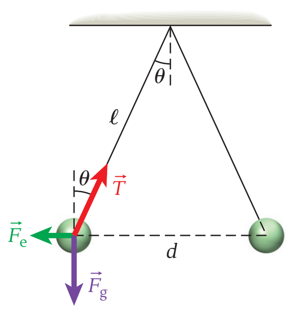


•**1.82** In the figure, the net electrostatic force on charge Q_A is zero. If $Q_A = +1.00$ nC, determine the magnitude of Q_0.



MULTI-VERSION EXERCISES

1.83 Two balls have the same mass, 0.9680 kg, and the same charge, 29.59 μC. They hang from the ceiling on strings of identical length, ℓ, as shown in the figure. If the angle of the strings with respect to the vertical is 29.79°, what is the length of the strings?



1.84 Two balls have the same mass and the same charge, 15.71 μC. They hang from the ceiling on strings of identical length, ℓ = 1.223 m, as shown in the figure. The angle of the strings with respect to the vertical is 21.07°. What is the mass of each ball?

1.85 Two balls have the same mass, 0.9935 kg, and the same charge. They hang from the ceiling on strings of identical length, ℓ = 1.235 m, as shown in the figure. The angle of the strings with respect to the vertical is 22.35°. What is the charge on each ball?

1.86 As shown in the figure, point charge q_1 is 3.979 μC and is located at $x_1 = -5.689$ m, and point charge q_2 is 8.669 μC and is located at $x_2 = 14.13$ m. What is the x-coordinate of the point at which the net force on a point charge of 5.000 μC will be zero?

1.87 As shown in the figure, point charge q_1 is 4.325 μC and is located at x_1, and point charge q_2 is 7.757 μC and is located at $x_2 = 14.33$ m. The x-coordinate of the point where the net force on a point charge of −3.000 μC is zero is 2.358 m. What is the value of x_1?

1.88 As shown in the figure, point charge q_1 is 4.671 μC and is located at $x_1 = -3.573$ m, and point charge q_2 is 6.845 μC and is located at x_2. The x-coordinate of the point where the net force on a point charge of −1.000 μC is zero is 4.625 m. What is the value of x_2?

Electric Fields and Gauss's Law

2

WHAT WE WILL LEARN 27

WHAT WE HAVE LEARNED/ EXAM STUDY GUIDE 52

FIGURE 2.1 A great white shark can detect tiny electric fields generated by its prey.

The great white shark is one of the most feared predators on Earth (Figure 2.1). It has several senses that have evolved for hunting prey; for example, it can smell tiny amounts of blood from as far away as 5 km (3 mi). Perhaps more amazing, it has developed special organs (called the *ampullae of Lorenzini*) that can detect the tiny electric fields generated by the movement of muscles in an organism, whether a fish, a seal, or a human. However, just what are electric fields? In addition, how are they related to electric charges?

The concept of vector fields is one of the most useful and productive ideas in all of physics. This chapter explains what an electric field is and how it is connected to electrostatic charges and forces and then examines how to determine the electric field due to some given distribution of charge. This study leads us to one of the most important laws of electricity–Gauss's Law–which provides a relationship between electric fields and electrostatic charge. However, Gauss's Law has practical application only when the charge distribution has enough geometric symmetry to simplify the calculation, and even then, some other concepts related to electric fields are necessary in order to apply the equations.

WHAT WE WILL LEARN

- An electric field represents the electric force at different points in space.
- Electric field lines represent the net force vectors exerted on a unit positive electric charge. They originate on positive charges and terminate on negative charges.
- The electric field of a point charge is radial, proportional to the charge, and inversely proportional to the square of the distance from the charge.
- An electric dipole consists of a positive charge and a negative charge of equal magnitude.
- The electric flux is the electric field component normal to an area times the area.
- Gauss's Law states that the electric flux through a closed surface is proportional to the net electric charge enclosed within the surface. This law provides simple ways to solve seemingly complicated electric field problems.
- The electric field inside a conductor is zero.
- The magnitude of the electric field due to a uniformly charged, infinitely long wire varies as the inverse of the perpendicular distance from the wire.
- The electric field due to an infinite sheet of charge does not depend on the distance from the sheet.
- The electric field outside a spherical distribution of charge is the same as the field of a point charge with the same total charge located at the sphere's center.

2.1 Definition of an Electric Field

You previously learned that the force between two or more point charges. When determining the net force exerted by other charges on a particular charge at some point in space, we obtain different directions for this force, depending on the sign of the charge that is the reference point. In addition, the net force is also proportional to the magnitude of the reference charge. The techniques used in Chapter 1 require us to redo the calculation for the net force each time we consider a different charge.

Dealing with this situation requires the concept of a **field,** which can be used to describe certain forces. An **electric field,** $E(r)$, is defined at any point in space, $\vec{r}$ as the net electric force on a charge, divided by that charge:

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q} \tag{2.1}$$

The units of the electric field are newtons per coulomb (N/C). This simple definition eliminates the cumbersome dependence of the electric force on the particular charge being used to measure the force. We can quickly determine the net force on any charge by using $\vec{F}(\vec{r}) = q\vec{E}(\vec{r})$ which is a trivial rearrangement of equation 2.1.

The electric force on a charge at a point is parallel or antiparallel, depending on the sign of the charge in question) to the electric field at that point and proportional to the magnitude of the charge. The magnitude of the force is given by $F = |q|E$ The direction of the force on a positive charge is along $\vec{E}(\vec{r})$ the direction of the force on a negative charge is in the direction opposite to $\vec{E}(\vec{r})$

If several sources of electric fields are present at the same time, such as several point charges, the electric field at any given point is determined by the superposition of the electric fields from all sources. This superposition follows directly from the superposition of forces introduced in our study of mechanics and discussed earlier for electrostatic forces. The **superposition principle** for the total electric field, $\vec{E}_t$ at any point in space with coordinate $\vec{r}$ due to n electric field sources can be stated as

$$\vec{E}_t(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \cdots + \vec{E}_n(\vec{r}) \tag{2.2}$$

2.2 Field Lines

An electric field can (and in most applications does) change as a function of the spatial coordinate. The changing direction and strength of the electric field can be visualized by means of **electric field lines.** These graphically represent the net vector force exerted on a

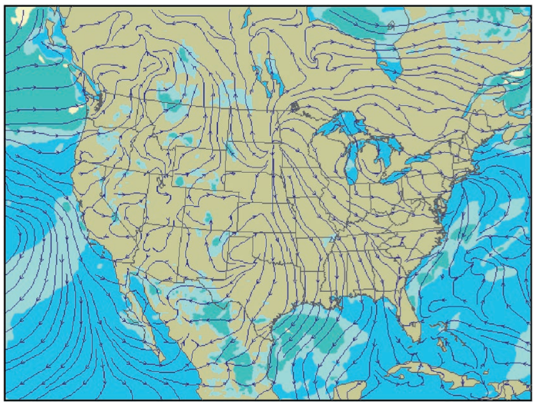


FIGURE 2.2 Streamlines of wind directions at the surface in the United States on March 23, 2008, from the National Weather Service.

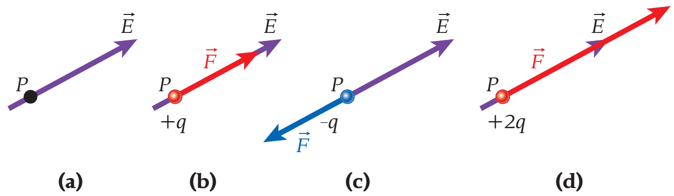


FIGURE 2.3 The force resulting from placing a charge in an electric field. (a) A point P on an electric field line. (b) A positive charge $+q$ placed at point P. (c) A negative charge $-q$ placed at point P. (d) A positive charge $+2q$ placed at point P.

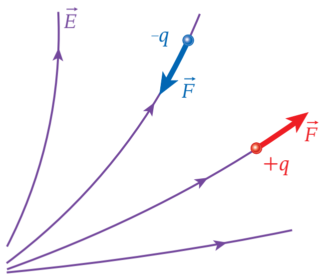


FIGURE 2.4 A nonuniform electric field. A positive charge $+q$ and a negative charge $-q$ placed in the field experience forces as shown. Each force is tangent to the electric field line.

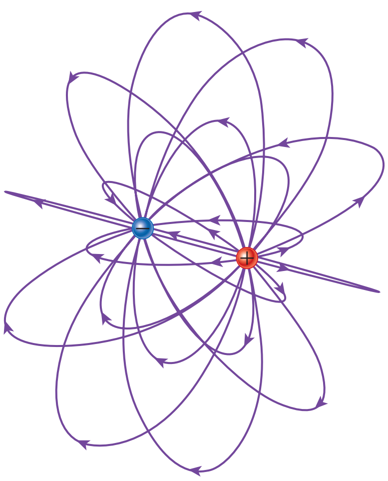


FIGURE 2.5 Three-dimensional representation of electric field lines from two point charges with opposite signs.

unit positive test charge. The representation applies separately for each point in space where the test charge might be placed. The direction of the field line at each point is the same as the direction of the force at that point, and the density of field lines is proportional to the magnitude of the force.

Electric field lines can be compared to the streamlines of wind directions, shown in Figure 2.2. These streamlines represent the force of the wind on objects at given locations, just as the electric field lines represent the electric force at specific points. A hot-air balloon can be used as a test particle for determining these wind streamlines. For example, a hot-air balloon launched in Dallas, Texas, would float from north to south in the situation depicted in Figure 2.2. Where the wind streamlines are close together, the speed of the wind is higher, so the balloon would move faster.

To draw an electric field line, we imagine placing a tiny positive charge at each point in the electric field. This charge is small enough that it does not affect the surrounding field. A small charge like this is sometimes called a **test charge.** We calculate the resultant force on the charge, and the direction of the force gives the direction of the field line. For example, Figure 2.3a shows a point in an electric field. In Figure 2.3b, a charge $+q$ is placed at point P, on an electric field line. The force on the charge is in the same direction as the electric field. In Figure 2.3c, a charge $-q$ is placed at point P, and the resulting force is in the direction opposite to the electric field. In Figure 2.3d, a charge $+2q$ is placed at point P, and the resulting force on the charge is in the direction of the electric field, with twice the magnitude of the force on the charge $+q$. We will follow the convention of depicting a positive charge as red and a negative charge as blue.

In a nonuniform electric field, the electric force at a given point is tangent to the electric field lines at that point, as illustrated in Figure 2.4. The force on a positive charge is in the direction of the electric field, and the force on a negative charge is in the direction opposite to the electric field.

Electric field lines point away from sources of positive charge and toward sources of negative charge. Each field line starts at a charge and ends at another charge. Electric field lines always originate on positive charges and terminate on negative charges.

Electric fields exist in three dimensions (Figure 2.5); however, this chapter usually presents two-dimensional depictions of electric fields for simplicity.

Point Charge

The electric field lines arising from an isolated point charge are shown in Figure 2.6. The field lines emanate in radial directions from the point charge. If the point charge is positive (Figure 2.6a), the field lines point outward, away from the charge; if the point charge is negative, the field lines point inward, toward the charge (Figure 2.6b). For an isolated positive point charge, the electric field lines originate at the charge and terminate on negative charges at infinity, and for a negative point charge, the electric field lines originate at positive charges at infinity and terminate at the charge. Note that the electric field lines are

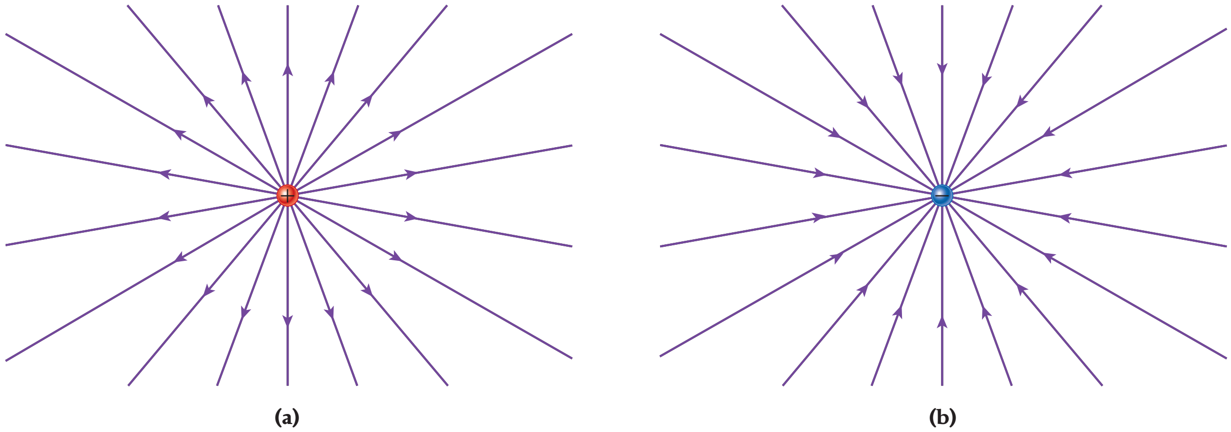


FIGURE 2.6 Electric field lines (a) from a single positive point charge and (b) to a single negative point charge.

closer together near the point charge and farther apart away from the point charge, indicating that the electric field becomes weaker with increasing distance from the charge. We'll examine the magnitude of the field quantitatively in Section 2.3.

Two Point Charges of Opposite Sign

We can use the superposition principle to determine the electric field from two point charges. Figure 2.7 shows the electric field lines for two oppositely charged point charges with the same magnitude. At each point in the plane, the electric field from the positive charge and the electric field from the negative charge add as vectors to give the magnitude and the direction of the resulting electric field. (Figure 2.5 shows the same field lines in three dimensions.)

As noted earlier, the electric field lines originate on the positive charge and terminate on the negative charge. At a point very close to either charge, the field lines are similar to those for a single point charge, since the effect of the more distant charge is small. Near the charges, the electric field lines are close together, indicating that the field is stronger in those regions. The fact that the field lines between the two charges connect indicates that an attractive force exists between the two charges.

Two Point Charges with the Same Sign

We can also apply the principle of superposition to two point charges with the same sign. Figure 2.8 shows the electric field lines for two point charges with the same sign and same magnitude. If both charges are positive (as in Figure 2.8), the electric field lines originate at the charges and terminate at infinity. If both charges are negative, the field lines originate at infinity and terminate at the charges. For two charges of the same sign, the field lines do not

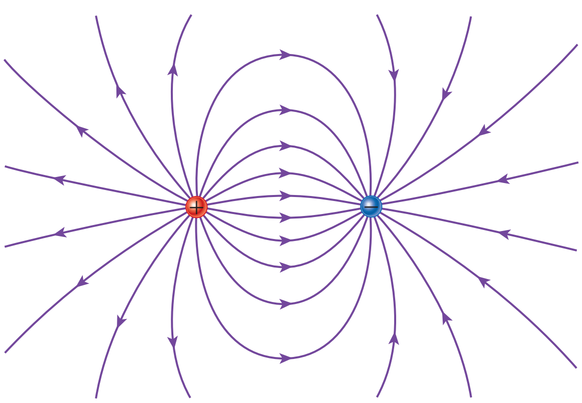


FIGURE 2.7 Electric field lines from two oppositely charged point charges. Each charge has the same magnitude.

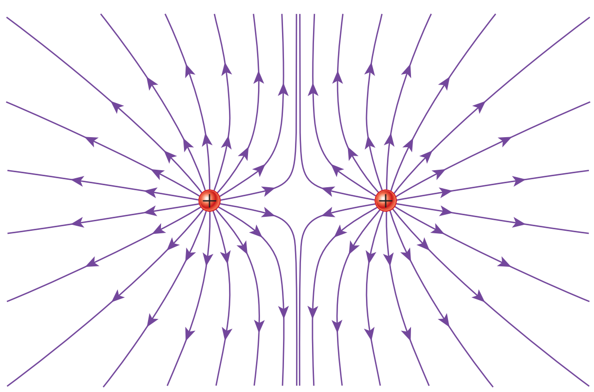
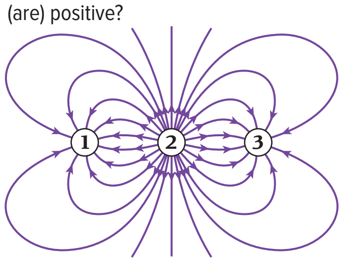


FIGURE 2.8 Electric field lines from two positive point charges with the same magnitude.

Concept Check 2.1

Which of the charges in the figure is (are) positive?



a) 1

b) 2

c) 3

d) 1 and 3

e) All three charges are positive.

connect the two charges. Rather, the field lines terminate on opposite charges at infinity or at the two charges themselves. The fact that the field lines never terminate on the other charge signifies that the charges repel each other.

General Observations

The three simplest possible cases that we just examined lead to two general rules that apply to all field lines of all charge configurations:

1. *Field lines originate at positive charges and terminate at negative charges.*
2. *Field lines never cross.* This result is a consequence of the fact that the lines represent the electric field, which in turn is proportional to the net force that acts on a charge placed at a particular point. Field lines that crossed would imply that the net force points in two different directions at the same point, which is impossible.

Concept Check 2.2

Assuming that there are no charges in the four regions shown in the figure, which of the patterns could represent an electric field?

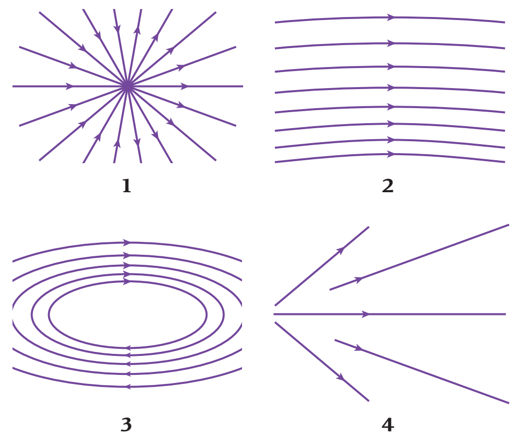
a) only 1

b) only 2

c) 2 and 3

d) 1 and 4

e) None of the patterns represent an electric field.



2.3 Electric Field due to Point Charges

The magnitude of the electric force on a point charge q_0 due to another point charge, q, is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|qq_0|}{r^2} \tag{2.3}$$

Taking q_0 to be a small test charge, we can express the magnitude of the electric field at the point where q_0 is and due to the point charge q as

$$E = \left|\frac{F}{q_0}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} \tag{2.4}$$

where r is the distance from the test charge to the point charge. The direction of this electric field is radial. The field points outward for a positive point charge and inward for a negative point charge.

An electric field is a vector quantity, and thus the components of the field must be added separately. Example 2.1 demonstrates the addition of electric fields created by three point charges.

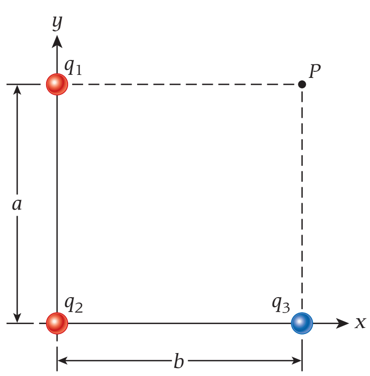


FIGURE 2.9 Locations of three point charges.

EXAMPLE 2.1 Three Charges

Figure 2.9 shows three fixed point charges: $q_1 = +1.50\ \mu\text{C}$, $q_2 = +2.50\ \mu\text{C}$, and $q_3 = -3.50\ \mu\text{C}$. Charge q_1 is located at $(0,a)$, q_2 is located at $(0,0)$, and q_3 is located at $(b,0)$, where $a = 8.00$ m and $b = 6.00$ m.

PROBLEM

What electric field, $\vec{E}$ do these three charges produce at the point $P = (b,a)$?

SOLUTION

We must sum the electric fields from the three charges using equation 2.2. We proceed by summing component by component, starting with the field due to q_1:

$$\vec{E}_1 = E_{1,x}\hat{x} + E_{1,y}\hat{y}$$

The field due to q_1 acts only in the x-direction at point (b,a), because q_1 has the same y-coordinate as P. Thus, $\vec{E}_1 = E_{1,x}\hat{x}$ We can determine $E_{1,x}$ using equation 2.4:

$$E_{1,x} = \frac{kq_1}{b^2}$$

Similarly, the field due to q_3 acts only in the y-direction at point (b,a). Thus, $\vec{E}_3 = E_{3,y}\hat{y}$ where

$$E_{3,y} = \frac{kq_3}{a^2}$$

As shown in Figure 2.10, the electric field due to q_2 at P is given by

$$\vec{E}_2 = E_{2,x}\hat{x} + E_{2,y}\hat{y}$$

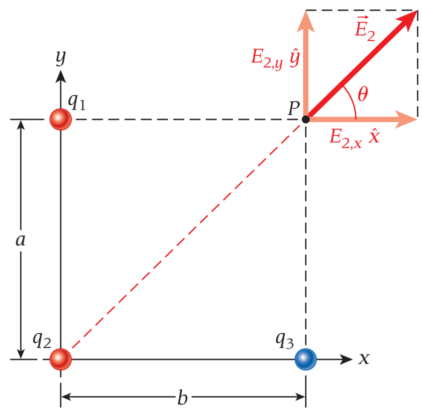


FIGURE 2.10 Electric field due to q_2 and its x- and y-components at point P.

Note that $\vec{E}_2$ the electric field due to q_2 at point P, points directly away from q_2, because $q_2 > 0$. (It would point directly toward q_2 if this charge were negative.) The magnitude of this electric field is given by

$$E_2 = \frac{k|q_2|}{a^2 + b^2}$$

The component $E_{2,x}$ is given by $E_2 \cos\theta$, where $\theta = \tan^{-1}(a/b)$, and the component $E_{2,y}$ is given by $E_2 \sin\theta$.

Adding the components, the total electric field at point P is

$$\vec{E} = \left(E_{1,x} + E_{2,x}\right)\hat{x} + \left(E_{2,y} + E_{3,y}\right)\hat{y}$$
$$= \underbrace{\left(\frac{kq_1}{b^2} + \frac{kq_2\cos\theta}{a^2+b^2}\right)}_{E_x}\hat{x} + \underbrace{\left(\frac{kq_2\sin\theta}{a^2+b^2} + \frac{kq_3}{a^2}\right)}_{E_y}\hat{y}$$

With the given values for a and b, we find $\theta = \tan^{-1}(8/6) = 53.1°$, and $a^2 + b^2 = (8.00\text{ m})^2 + (6.00\text{ m})^2 = 100\text{ m}^2$. We can then calculate the x-component of the total electric field as

$$E_x = \left(8.99\times10^9\text{ N m}^2/\text{C}^2\right)\left(\frac{1.50\times10^{-6}\text{ C}}{(6.00\text{ m})^2} + \frac{\left(2.50\times10^{-6}\text{ C}\right)\left(\cos 53.1°\right)}{100\text{ m}^2}\right) = 509\text{ N/C}$$

The y-component is

$$E_y = \left(8.99\times10^9\text{ N m}^2/\text{C}^2\right)\left(\frac{\left(2.50\times10^{-6}\text{ C}\right)\left(\sin 53.1°\right)}{100\text{ m}^2} + \frac{-3.50\times10^{-6}\text{ C}}{(8.00\text{ m})^2}\right) = -312\text{ N/C}$$

The magnitude of the field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(509\text{ N/C})^2 + (-312\text{ N/C})^2} = 597\text{ N/C}$$

The direction of the field at point P is

$$\varphi = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{-312\text{ N/C}}{509\text{ N/C}}\right) = -31.5°$$

which means that the electric field points to the right and downward.

Note that even though the charges in this example are in microcoulombs and the distances are in meters, the electric fields are still large, showing that a microcoulomb is a large amount of charge.

2.4 Electric Field due to a Dipole

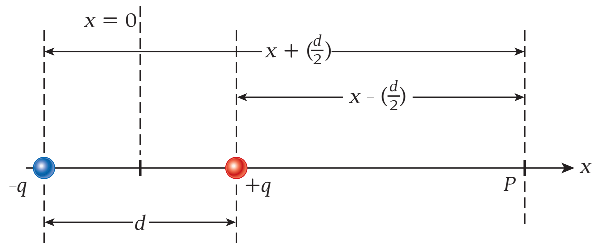


FIGURE 2.11 Calculation of the electric field from an electric dipole.

A system of two equal (in magnitude) but oppositely charged point particles is called an **electric dipole.** The electric field from an electric dipole is given by the vector sum of the electric fields from the two charges. Figure 2.7 shows the electric field lines in two dimensions for an electric dipole.

The superposition principle allows us to determine the electric field due to two point charges through vector addition of the electric fields of the two charges. Let's consider the special case of the electric field due to a dipole along the axis of the dipole, defined as the line connecting the charges. This main symmetry axis of the dipole is assumed to be oriented along the x-axis (Figure 2.11).

The electric field, $\vec{E}$ at point P on the dipole axis is the sum of the field due to $+q$, denoted as $\vec{E}_+$ and the field due to $-q$, denoted as $\vec{E}_-$

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

Using equation 2.4, we can express the magnitude of the dipole's electric field along the x-axis, for $x > d/2$, as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-^2}$$

where r_+ is the distance between P and $+q$ and r_- is the distance between P and $-q$. Absolute value bars are not needed in this equation, because the first term on the right-hand side is positive and is greater than the second (negative) term. The electric field at all points on the x-axis (except at $x = \pm d/2$, where the two charges are located) is given by

$$\vec{E} = E_x\hat{x} = \frac{1}{4\pi\varepsilon_0}\frac{q(x - d/2)}{r_+^3}\hat{x} + \frac{1}{4\pi\varepsilon_0}\frac{-q(x + d/2)}{r_-^3}\hat{x} \tag{2.5}$$

Now we examine the magnitude of $\vec{E}$ and restrict the value of x to $x > d/2$, where $E = E_x > 0$. Then we have

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x - \frac{1}{2}d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x + \frac{1}{2}d\right)^2}$$

With some rearrangement and keeping in mind that we want to obtain an expression that has the same form as the electric field from a point charge, we write the preceding equation as

$$E = \frac{q}{4\pi\varepsilon_0 x^2}\left[\left(1 - \frac{d}{2x}\right)^{-2} - \left(1 + \frac{d}{2x}\right)^{-2}\right]$$

To find an expression for the electric field at a large distance from the dipole, we can make the approximation $x \gg d$ and use the binomial expansion. (Since $x \gg d$, we can drop terms containing the square of d/x and higher powers.) We obtain

$$E \approx \frac{q}{4\pi\varepsilon_0 x^2}\left[\left(1 + \frac{d}{x} - \cdots\right) - \left(1 - \frac{d}{x} + \cdots\right)\right] = \frac{q}{4\pi\varepsilon_0 x^2}\left(\frac{2d}{x}\right)$$

which can be rewritten as

$$E \approx \frac{qd}{2\pi\varepsilon_0 x^3} \tag{2.6}$$

Equation 2.6 can be simplified by defining a vector quantity called the **electric dipole moment,** $\vec{p}$ The direction of this dipole moment is from the negative charge to the positive charge, which is opposite to the direction of the electric field lines. The magnitude, p, of the electric dipole moment is given by

$$p = qd \tag{2.7}$$

where q is the magnitude of either of the charges and d is the distance separating the two charges. With this definition, the expression for the magnitude of the electric field due to the dipole along the positive x-axis at a distance that is large compared with the separation between the two charges is

$$E = \frac{p}{2\pi\varepsilon_0 |x|^3} \tag{2.8}$$

Although not shown explicitly here, equation 2.8 is also valid for $x = \ll -d$. Also, an examination of equation 2.5 for $\vec{E}$ shows that $E_x > 0$ on either side of the dipole. In contrast to the field due to a point charge, which is inversely proportional to the square of the distance, the field due to a dipole is inversely proportional to the cube of the distance, according to equation 2.8.

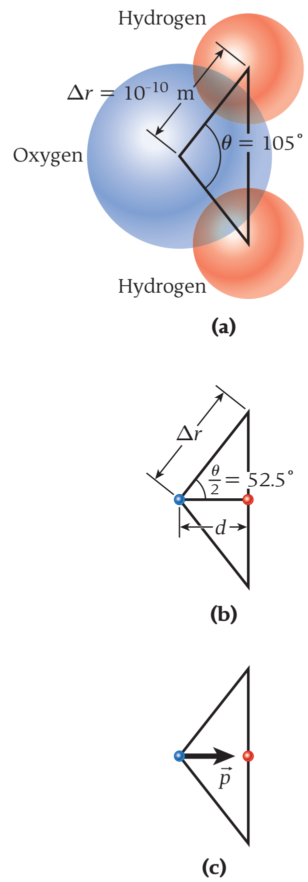


FIGURE 2.12 (a) Schematic drawing showing the geometry of a water molecule, H_2O, with atoms as spheres. (b) Diagram showing the effective positive (red dot on the right) and negative (blue dot on the left) charge centers. (c) Dipole moment assuming pointlike charges.

EXAMPLE 2.2 Water Molecule

The water molecule, H_2O, is arguably the most important one for life. It has a nonzero dipole moment, which is the basic reason why many organic molecules are able to bind to water. This dipole moment also allows water to be an excellent solvent for many inorganic and organic compounds.

Each water molecule consists of two atoms of hydrogen and one atom of oxygen, as shown in Figure 2.12a. The charge distribution of each of the individual atoms is approximately spherical. The oxygen atom tends to pull the negatively charged electrons toward itself, giving the hydrogen atoms slight positive charges. The three atoms are arranged so that the lines connecting the centers of the hydrogen atoms with the center of the oxygen atom have an angle of 105° between them (see Figure 2.12a).

PROBLEM

Suppose we approximate a water molecule by two positive charges at the locations of the two hydrogen nuclei (protons) and two negative charges at the location of the oxygen nucleus, with all charges of equal magnitude. What is the resulting electric dipole moment of water?

SOLUTION

The center of charge of the two positive charges, analogous to the center of mass of two masses, is located exactly halfway between the centers of the hydrogen atoms, as shown in Figure 2.12b. With the hydrogen-oxygen distance of $\Delta r = 10^{-10}$ m, as indicated in Figure 2.12a, the distance between the positive and negative charge centers is

$$d = \Delta r \cos\left(\frac{\theta}{2}\right) = \left(10^{-10}\ \text{m}\right)\left(\cos 52.5°\right) = 0.6\times10^{-10}\,\text{m}$$

This distance times the transferred charge, $q = 2e$, is the magnitude of the dipole moment of water:

$$p = 2ed = \left(3.2\times10^{-19}\ \text{C}\right)\left(0.6\times10^{-10}\ \text{m}\right) = 2\times10^{-29}\ \text{C m}$$

This result of an extremely oversimplified calculation actually comes close, within a factor of 3, to the measured value of 6.2×10^{-30} C m. The fact that the real dipole moment of water is smaller than this calculated result is an indication that the two electrons of the hydrogen atoms are not pulled all the way to the oxygen but, on average, only one-third of the way.

Concept Check 2.4

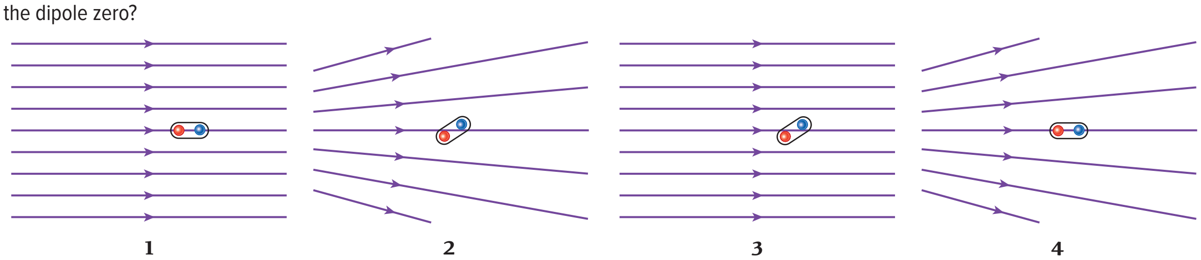
An electrically neutral dipole is placed in an external electric field as shown in the figure in Concept Check 2.3. In which situation(s) is the net *torque* on the dipole zero?

a) 1 and 3
b) 2 and 4
c) 1 and 4
d) 2 and 3
e) 1 only

Concept Check 2.3

An electrically neutral dipole is placed in an external electric field as shown in the figure. In which situation(s) is the net *force* on the dipole zero?

a) 1 and 3
b) 2 and 4
c) 1 and 4
d) 2 and 3
e) 1 only



2.5 General Charge Distributions

We have determined the electric fields of a single point charge and of two point charges (an electric dipole). What if we want to determine the electric field due to many charges? Each individual charge creates an electric field, as described by equation 2.4, and because of the superposition principle, all of these electric fields can be added to find the net field at any point in space. But we have already seen in Example 2.1 that the addition of electric field vectors can be cumbersome for a collection of only three point charges. If we had to apply this method to, say, trillions of point charges, the task would be unmanageable even if we could use a supercomputer. Since real-world applications usually involve a very large number of charges, it is clear that we need a way to simplify the calculations. This can be accomplished by using an integral, if the large number of charges are arranged in space in some regular distribution. Of particular interest are two-dimensional distributions, where charges are located on the surface of a metallic object, and one-dimensional distributions, where charges are arranged along a wire. As we will see, integration can be a surprisingly simple way to solve problems involving such charge distributions, which would be very hard to analyze by the method of direct summation.

To prepare for the integration procedure, we divide the charge into differential elements of charge, dq, and find the electric field resulting from each differential charge element as if it were a point charge. If the charge is distributed along a one-dimensional object (a line), the differential charge may be expressed in terms of a charge per unit length times a differential length, or λdx. If the charge is distributed over a surface (a two-dimensional object), dq is expressed in terms of a charge per unit area times a differential area, or σdA. And, finally, if the charge is distributed over a three-dimensional volume, then dq is written as the product of a charge per unit volume and a differential volume, or ρdV. That is,

$$\left.\begin{array}{l} dq = \lambda dx \\ dq = \sigma dA \\ dq = \rho dV \end{array}\right\} \text{ for a charge distribution } \left\{\begin{array}{l} \text{along a line;} \\ \text{over a surface;} \\ \text{throughout a volume.} \end{array}\right. \tag{2.9}$$

The magnitude of the electric field resulting from the charge distribution is then obtained from the differential charge:

$$dE = k\frac{dq}{r^2} \tag{2.10}$$

In the following example, we find the electric field due to a finite line of charge.

EXAMPLE 2.3 Finite Line of Charge

To find the electric field along a line bisecting a finite length of wire with linear charge density λ, we integrate the contributions to the electric field from all the charge in the wire. We assume that the wire lies along the x-axis (Figure 2.13).

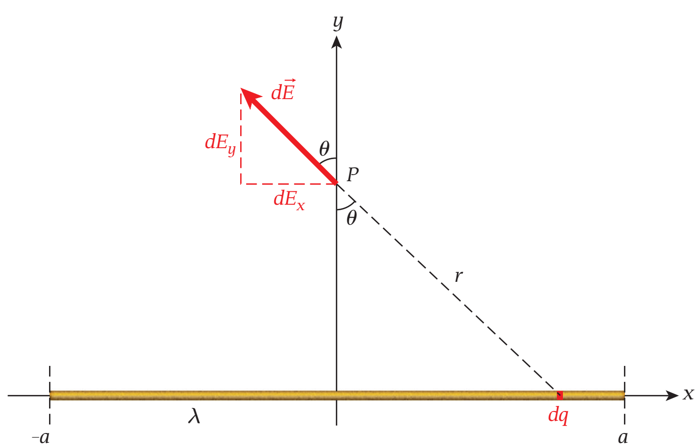


FIGURE 2.13 Calculating the electric field due to all the charge in a long wire by integrating the contributions to the electric field over the length of the wire.

We also assume that the wire is positioned with its midpoint at $x = 0$, one end at $x = a$, and the other end at $x = -a$. The symmetry of the situation then allows us to conclude that there cannot be any electric force parallel to the wire (in the x-direction) along the line bisecting the wire. Along this line, the electric field can be only in the y-direction. We can then calculate the electric field due to all the charge for $x \geq 0$ and multiply the result by 2 to get the electric field for the whole wire.

We consider a differential charge, dq, on the x-axis, as shown in Figure 2.13. The magnitude of the electric field, dE, at a point $(0,y)$ due to this charge is given by equation 2.10,

$$dE = k\frac{dq}{r^2}$$

where $r = \sqrt{x^2 + y^2}$ is the distance from dq to point P. The component of the electric field perpendicular to the wire (in the y-direction) is then given by

$$dE_y = k\frac{dq}{r^2}\cos\theta$$

where θ is the angle between the electric field produced by dq and the y-axis (see Figure 2.13). The angle θ is related to r and y because $\cos\theta = y/r$.

We can relate the differential charge to the differential distance along the x-axis through the linear charge density, λ: $dq = \lambda dx$. The electric field at a distance y from the long wire is then

$$E_y = 2\int_0^a dE_y = 2\int_0^a k\frac{dq}{r^2}\cos\theta = 2k\int_0^a \frac{\lambda dx}{r^2}\frac{y}{r} = 2k\lambda y\int_0^a \frac{dx}{\left(x^2+y^2\right)^{3/2}}$$

Evaluation of the integral on the right-hand side (with the aid of an integral table or a software package like Mathematica or Maple) gives us

$$\int_0^a \frac{dx}{\left(x^2+y^2\right)^{3/2}} = \left[\frac{1}{y^2}\frac{x}{\sqrt{x^2+y^2}}\right]_0^a = \frac{1}{y^2}\frac{a}{\sqrt{y^2+a^2}}$$

Thus, the electric field at a distance y along a line bisecting the wire is given by

$$E_y = 2k\lambda y\frac{1}{y^2}\frac{a}{\sqrt{y^2+a^2}} = \frac{2k\lambda}{y}\frac{a}{\sqrt{y^2+a^2}}$$

Finally, when $a \to \infty$, that is, the wire becomes infinitely long, $a/\sqrt{y^2+a^2} \to 1$ and we have for an infinitely long wire

$$E_y = \frac{2k\lambda}{y}$$

In other words, the electric field decreases in inverse proportion to the distance from the wire.

Now let's tackle a problem with a slightly more complicated geometry, finding the electric field due to a ring of charge along the axis of the ring.

SOLVED PROBLEM 2.1 Ring of Charge

PROBLEM

Consider a charged ring with radius $R = 0.250$ m (Figure 2.14). The ring has uniform linear charge density, and the total charge on the ring is $Q = +5.00\ \mu$C. What is the electric field at a distance $b = 0.500$ m along the axis of the ring?

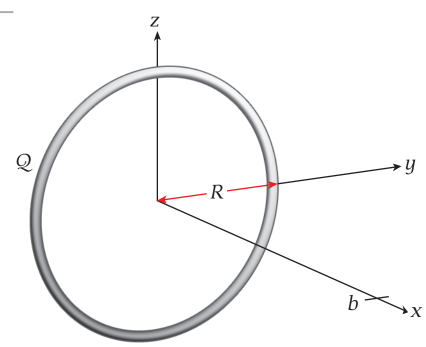


FIGURE 2.14 Charged ring with radius R and total charge Q.

SOLUTION

THINK The charge is evenly distributed around the ring. The electric field at position $x = b$ can be calculated by integrating the differential electric field due to a differential electric charge. By symmetry, the components of the electric field perpendicular to the axis of the ring integrate to zero, because the electric fields of charge elements on opposite sides of the axis cancel one another out. The resulting electric field is parallel to the axis of the circle.

– Continued

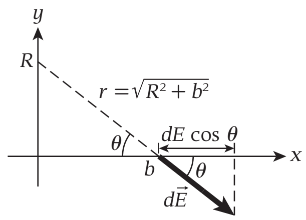


FIGURE 2.15 The geometry for the electric field along the axis of a ring of charge.

SKETCH Figure 2.15 shows the geometry for the electric field along the axis of the ring of charge.

RESEARCH The differential electric field, dE, at $x = b$ is due to a differential charge dq located at $y = R$ (see Figure 2.15). The distance from the point $(x = b, y = 0)$ to the point $(x = 0, y = R)$ is

$$r = \sqrt{R^2 + b^2}$$

Again, the magnitude of $d\vec{E}$ is given by equation 2.10:

$$dE = k\frac{dq}{r^2}$$

The magnitude of the component of $d\vec{E}$ parallel to the x-axis is given by

$$dE_x = dE\cos\theta = dE\frac{b}{r}$$

SIMPLIFY We can find the total electric field by integrating its x-components over all the charge on the ring:

$$E_x = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{b}{r} k\frac{dq}{r^2}$$

We need to integrate around the circumference of the ring of charge. We can relate the differential charge to the differential arc length, ds, as follows:

$$dq = \frac{Q}{2\pi R}ds$$

We can then express the integral over the entire ring of charge as an integral around the arc length of a circle:

$$E_x = \int_0^{2\pi R} k\left(\frac{Q}{2\pi R}ds\right)\frac{b}{r^3} = \left(\frac{kQb}{2\pi R r^3}\right)\int_0^{2\pi R} ds = kQ\frac{b}{r^3} = \frac{kQb}{\left(R^2 + b^2\right)^{3/2}}$$

CALCULATE Putting in the numerical values, we get

$$E_x = \frac{kQb}{\left(R^2 + b^2\right)^{3/2}} = \frac{\left(8.99\times10^9 \text{ N m}^2/\text{C}^2\right)\left(5.00\times10^{-6}\text{ C}\right)\left(0.500 \text{ m}\right)}{\left[\left(0.250 \text{ m}\right)^2 + \left(0.500 \text{ m}\right)^2\right]^{3/2}} = 128{,}654 \text{ N/C}$$

ROUND We report our result to three significant figures:

$$E_x = 1.29\times10^5 \text{ N/C}$$

DOUBLE-CHECK We can check the validity of the formula we derived for the electric field by using a large distance from the ring of charge, such that $b \gg R$. In this case,

$$E_x = \frac{kQb}{\left(R^2 + b^2\right)^{3/2}} \overset{b \gg R}{\Rightarrow} E_x = \frac{kQb}{b^3} = k\frac{Q}{b^2}$$

which is the expression for the electric field due to a point charge Q at a distance b. We can also check the formula with $b = 0$:

$$E_x = \frac{kQb}{\left(R^2 + b^2\right)^{3/2}} \overset{b=0}{\Rightarrow} E_x = 0$$

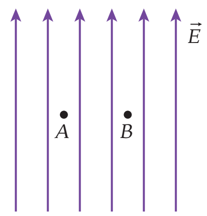
which is what we would expect at the center of a ring of charge. Thus, our result seems reasonable.

2.6 Force due to an Electric Field

The force $\vec{F}$ exerted by an electric field $\vec{E}$ on a point charge q is given by $\vec{F} = q\vec{E}$ a simple restatement of the definition of the electric field in equation 2.1. Thus, the force exerted by the electric field on a positive charge acts in the same direction as the electric field. The force vector is always tangent to the electric field lines and points in the direction of the electric field if $q > 0$.

Concept Check 2.6

A small positively charged object could be placed in a uniform electric field at position *A* or position *B* in the figure. How do the electric forces on the object at the two positions compare?



a) The magnitude of the electric force on the object is greater at position *A*.

b) The magnitude of the electric force on the object is greater at position *B*.

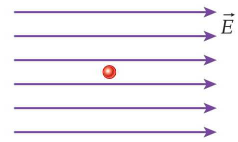
c) There is no electric force on the object at either position *A* or position *B*.

d) The electric force on the object at position *A* has the same magnitude as the force on the object at position *B* but is in the opposite direction.

e) The electric force on the object at position *A* is the same nonzero electric force as that on the object at position *B*.

Concept Check 2.5

A small positively charged object is placed at rest in a uniform electric field as shown in the figure. When the object is released, it will



a) not move.

b) begin to move with a constant speed.

c) begin to move with a constant acceleration.

d) begin to move with an increasing acceleration.

e) move back and forth in simple harmonic motion.

The force at various locations on a positive charge due to the electric field in three dimensions is shown in Figure 2.16 for the case of two oppositely charged particles. (This is the same field as in Figure 2.5, but with some representative force vectors added.) You can see that the force on the positive charge is always tangent to the field lines and points in the same direction as the electric field. The force on the negative charge would point in the opposite direction.

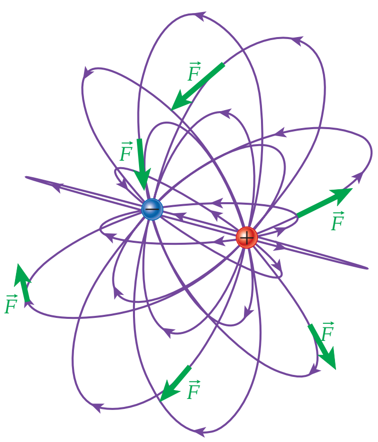
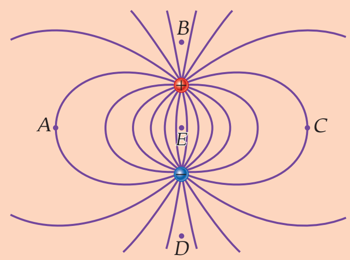


FIGURE 2.16 Direction of the force that an electric field produced by two opposite point charges exerts on a positive charge at various points in space.

Self-Test Opportunity 2.1

The figure shows a two-dimensional view of electric field lines due to two opposite charges. What is the direction of the electric field at the five points *A, B, C, D,* and *E*? At which of the five points is the magnitude of the electric field the largest?



EXAMPLE 2.4 Time Projection Chamber

Nuclear physicists study new forms of matter by colliding gold nuclei at very high energies. In particle physics, new elementary particles are created and studied by colliding protons and antiprotons at the highest energies. These collisions create many particles that stream away from the interaction point at high speeds. A simple particle detector is not sufficient to identify these particles. A device that helps physicists study these collisions is a time projection chamber (TPC), found in most large particle detectors.

– *Continued*

One example of a TPC is the STAR TPC of the Relativistic Heavy Ion Collider at Brookhaven National Laboratory on Long Island, New York. The STAR TPC consists of a large cylinder filled with a gas (90% argon, 10% methane) that allows free electrons to move within it without being captured by the gas atoms or molecules.

Figure 2.17 shows the results of a collision of two gold nuclei that occurred in the STAR TPC. In such a collision, thousands of charged particles are created that pass through the gas inside the TPC. As these charged particles pass through the gas, they ionize the atoms of the gas, releasing free electrons. A constant electric field of magnitude 13,500 N/C is applied between the center of the TPC and the caps on the ends of the cylinder, and the field exerts an electric force on the freed electrons. Because the electrons have a negative charge, the electric field exerts a force in the direction opposite to the electric field. The electrons attempt to accelerate in the direction of the electric force, but they interact with the electrons of the molecules of the gas and begin to drift toward the caps with a constant speed of 5 cm/μs $= 5 \cdot 10^4$ m/s ≈ 100,000 mph.

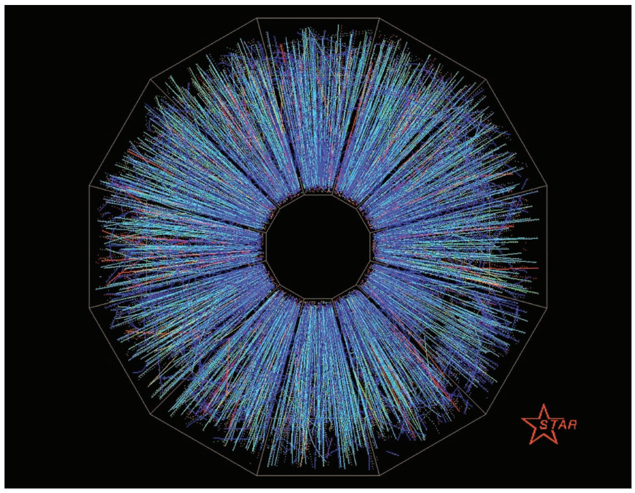


FIGURE 2.17 An event in the STAR TPC in which two gold nuclei have collided at very high energies at the point in the center of the image. Each colored line represents the track left behind by a subatomic particle produced in the collision.

Each end cap of the cylinder has 68,304 detectors that can measure the charge as a function of the drift time of the electrons from the point where they were freed. Each detector has a specific (x,y) position. From measurements of the arrival time of the charge and the known drift speed of the electrons, the z-components of their positions can be calculated. Thus, the STAR TPC can produce a complete three-dimensional representation of the ionization track of each charged particle. These tracks are shown in Figure 2.17, where the colors represent the amount of ionization produced by each track.

SOLVED PROBLEM 2.2 Electron Moving over a Charged Plate

PROBLEM

An electron with a kinetic energy of 2.00 keV (1 eV $= 1.602 \times 10^{-19}$ J) is fired horizontally across a horizontally oriented charged conducting plate with a surface charge density of $+4.00 \times 10^{-6}$ C/m^2. Taking the positive direction to be upward (away from the plate), what is the vertical deflection of the electron after it has traveled a horizontal distance of 4.00 cm?

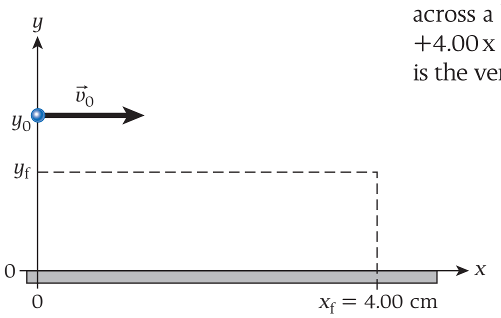


FIGURE 2.18 An electron moving to the right with initial velocity $\vec{v}_0$ over a charged conducting plate.

SOLUTION

THINK The initial velocity of the electron is horizontal. During its motion, the electron experiences a constant attractive force from the positively charged plate, which causes a constant acceleration downward. We can calculate the time it takes the electron to travel 4.00 cm in the horizontal direction and use this time to calculate the vertical deflection of the electron.

SKETCH Figure 2.18 shows the electron with initial velocity $\vec{v}_0$ in the horizontal direction. The initial position of the electron is taken to be at $x_0 = 0$ and $y = y_0$.

RESEARCH The time the electron takes to travel the given distance is

$$t = x_\mathrm{f}/v_0 \tag{i}$$

where x_f is the final horizontal position and v_0 is the initial speed of the electron. While the electron is in motion, it experiences a force from the charged conducting plate. This force is directed downward (toward the plate) and has a magnitude given by

$$F = qE = e\frac{\sigma}{\varepsilon_0} \tag{ii}$$

where σ is the charge density on the conducting plate and e is the charge of an electron. This force causes a constant acceleration in the downward direction whose magnitude is given by $a = F/m$, where m is the mass of the electron. Using the expression for the force from equation (ii), we can express the magnitude of this acceleration as

$$a = \frac{F}{m} = \frac{e\sigma}{m\varepsilon_0} \tag{iii}$$

Note that this acceleration is constant. Thus, the vertical position of the electron as a function of time is given by

$$y_\mathrm{f} = y_0 - \tfrac{1}{2}at^2 \Rightarrow y_\mathrm{f} - y_0 = -\tfrac{1}{2}at^2 \tag{iv}$$

Finally, we can relate the electron's initial kinetic energy to its initial velocity through

$$K = \tfrac{1}{2}mv_0^2 \Rightarrow v_0^2 = \frac{2K}{m} \tag{v}$$

SIMPLIFY We substitute the expressions for the time and the acceleration from equations (i) and (iii) into equation (iv) and obtain

$$y_\mathrm{f} - y_0 = -\tfrac{1}{2}at^2 = -\tfrac{1}{2}\left(\frac{e\sigma}{m\varepsilon_0}\right)\left(\frac{x_\mathrm{f}}{v_0}\right)^2 = -\frac{e\sigma x_\mathrm{f}^2}{2m\varepsilon_0 v_0^2} \tag{vi}$$

Now substituting the expression for the square of the initial speed from equation (v) into the right-hand side of equation (vi) gives us

$$y_\mathrm{f} - y_0 = -\frac{e\sigma x_\mathrm{f}^2}{2m\varepsilon_0\left(\dfrac{2K}{m}\right)} = -\frac{e\sigma x_\mathrm{f}^2}{4\varepsilon_0 K} \tag{vii}$$

CALCULATE We first convert the kinetic energy of the electron from electron-volts to joules:

$$K = (2.00 \text{ keV})\frac{1.602\times10^{-19} \text{ J}}{1 \text{ eV}} = 3.204\times10^{-16} \text{ J}$$

Putting the numerical values into equation (vii), we get

$$y_\mathrm{f} - y_0 = -\frac{e\sigma x_\mathrm{f}^2}{4\varepsilon_0 K} = -\frac{\left(1.602\times10^{-19} \text{ C}\right)\left(4.00\times10^{-6} \text{ C/m}^2\right)\left(0.0400 \text{ m}\right)^2}{4\left(8.85\times10^{-12} \text{ C}^2/(\text{N m}^2)\right)\left(3.204\times10^{-16} \text{ J}\right)} = -0.0903955 \text{ m}$$

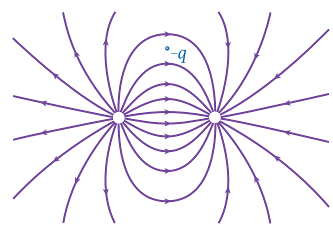
ROUND We report our result to three significant figures:

$$y_\mathrm{f} - y_0 = -0.0904 \text{ m} = -9.04 \text{ cm}$$

DOUBLE-CHECK The vertical deflection that we calculated is about twice the distance that the electron travels in the x-direction, which seems reasonable, at least in the sense of being of the same order of magnitude. Also, equation (vii) for the deflection has several features that should be present. First, the trajectory is parabolic, which we expect for a constant force and thus constant acceleration. Second, for zero surface charge density, we obtain zero deflection. Third, for very high kinetic energy, there is negligible deflection, which is also intuitively what we expect.

Concept Check 2.7

A negative charge $-q$ is placed in a nonuniform electric field as shown in the figure. What is the direction of the electric force on this negative charge?



a) $\rightarrow$

b) $\uparrow$

c) $\leftarrow$

d) $\downarrow$

e) The force is zero.

Dipole in an Electric Field

A point charge in an electric field experiences a force, given by equation 2.1. The electric force is always tangent to the electric field line passing through the point. The effect of an electric field on a dipole can be described in terms of the vector electric field, $\vec{E}$ and the

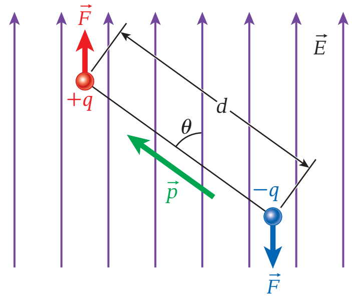


FIGURE 2.19 Electric dipole in an electric field.

Self-Test Opportunity 2.2

Use the center of mass of the dipole as the pivot point, and show that you again obtain the expression $\tau = qEd \sin\theta$ for the torque.

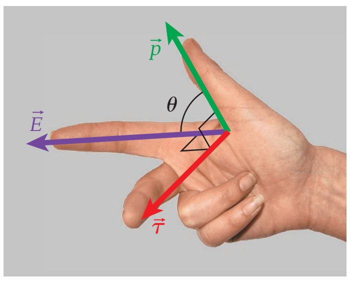


FIGURE 2.20 Right-hand rule for the vector product of the electric dipole moment and the electric field, producing the torque vector.

vector electric dipole moment, $\vec{p}$ without detailed knowledge of the charges making up the electric dipole.

To examine the behavior of an electric dipole, let's consider two charges, $+q$ and $-q$, separated by a distance d in a constant uniform electric field, $\vec{E}$ (Figure 2.19). (Note that we are now considering the forces acting on a dipole placed in an external field, as opposed to considering the field caused by the dipole, which we did in Section 2.4, and we also assume that the dipole field is small compared to $\vec{E}$ [so its effect on the uniform field can be ignored].) The electric field exerts an upward force on the positive charge and a downward force on the negative charge. Both forces have the magnitude qE. we saw that this situation gives rise to a torque, $\vec{\tau}$ given by $\vec{\tau} = \vec{r} \times \vec{F}$ where $\vec{r}$ is the moment arm and $\vec{F}$ is the force. The magnitude of the torque is $\tau = rF \sin\theta$.

As always, we can calculate the torque about any pivot point, so we can pick the location of the negative charge. Then, only the force on the positive charge contributes to the torque, and the length of the position vector is $r = d$, that is, the length of the dipole. Since, as already stated, $F = qE$, the expression for the torque on an electric dipole in an external electric field can be written as

$$\tau = qEd \sin\theta$$

Remembering that the electric dipole moment is defined as $p = qd$, we obtain the magnitude of the torque:

$$\tau = pE \sin\theta \tag{2.11}$$

Because the torque is a vector and must be perpendicular to both the electric dipole moment and the electric field, the relationship in equation 2.11 can be written as a vector product:

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{2.12}$$

As with all vector products, the direction of the torque is given by a right-hand rule. As shown in Figure 2.20, the thumb indicates the direction of the first term of the vector product, in this case $\vec{p}$ and the index finger indicates the direction of the second term, $\vec{E}$ The result of the vector product, $\vec{\tau}$ is then directed along the middle finger and is perpendicular to each of the two terms.

SOLVED PROBLEM 2.3 Electric Dipole in an Electric Field

PROBLEM

An electric dipole with dipole moment of magnitude $p = 1.40 \times 10^{-12}$ C m is placed in a uniform electric field of magnitude $E = 498$ N/C (Figure 2.21a).

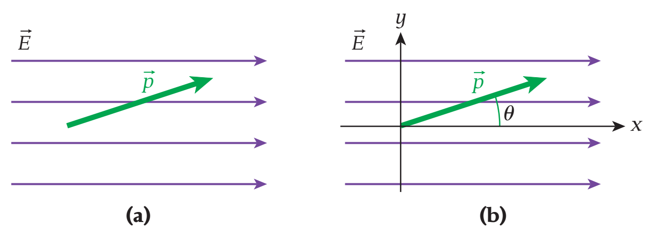


FIGURE 2.21 (a) An electric dipole in a uniform electric field. (b) The electric field oriented in the x-direction and the dipole moment in the xy-plane.

At some instant (in time) the angle between the electric dipole moment and the electric field is $\theta = 14.5°$. What are the Cartesian components of the torque on the dipole?

SOLUTION

THINK The torque on the dipole is equal to the vector product of the electric field and the electric dipole moment.

SKETCH We assume that the electric field lines point in the x-direction and the electric dipole moment is in the xy-plane (Figure 2.21b). The z-direction is perpendicular to the plane of the page.

RESEARCH The torque on the electric dipole due to the electric field is given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Since the dipole is located in the xy-plane, the Cartesian components of the electric dipole moment are

$$\vec{p} = \left(p_x, p_y, 0\right)$$

Since the electric field is acting in the x-direction, its Cartesian components are

$$\vec{E} = \left(E_x, 0, 0\right) = \left(E, 0, 0\right)$$

SIMPLIFY From the definition of the vector product, we express the Cartesian components of the torque as

$$\vec{\tau} = \left(p_y E_z - p_z E_y\right)\hat{x} + \left(p_z E_x - p_x E_z\right)\hat{y} + \left(p_x E_y - p_y E_x\right)\hat{z}$$

In this particular case, with E_y, E_z, and p_z all equal to zero, we have

$$\vec{\tau} = -p_y E_x \hat{z}$$

The y-component of the dipole moment is $p_y = p\sin\theta$, and the x-component of the electric field is simply $E_x = E$. The magnitude of the torque is then

$$\tau = (p\sin\theta)E = pE\sin\theta$$

and the direction of the torque is in the negative z-direction.

CALCULATE We insert the given numerical data and get

$$\tau = pE\sin\theta = \left(1.40\times10^{-12}\text{ C m}\right)(498\text{ N/C})(\sin 14.5°) = 1.74565\times10^{-10}\text{ N m}$$

ROUND We report our result to three significant figures:

$$\tau = 1.75\times10^{-10}\text{ N m}$$

DOUBLE-CHECK From equation 2.11, we know that the magnitude of the torque is

$$\tau = pE\sin\theta$$

which is the result we obtained using the explicit vector product. Applying the right-hand rule illustrated in Figure 2.20, we can determine the direction of the torque: With the right thumb representing the electric dipole moment and the right index finger representing the electric field, the right middle finger points into the page, which agrees with the result we found using the vector product. Thus, our result is correct.

Example 2.2 looked at the dipole moment of the water molecule. If water molecules are exposed to an external electric field, they experience a torque and thus begin to rotate. If the direction of the external electric field changes very rapidly, the water molecules perform rotational oscillations, which create heat. This is the principle of operation of a microwave oven. Microwave ovens use a frequency of 2.45 GHz for the oscillating electric field.

Electric fields also play a key role in human physiology, but these fields are time-varying and not static, like those studied in this chapter. The human brain also generates continuously changing electrical fields through the activity of the neurons. These fields can be measured by inserting electrodes through the skull and into the brain or by placing electrodes onto the surface of the exposed brain, usually during brain surgery. This technique is called electrocorticography (ECoG). An intense area of current research focuses on measuring and imaging brain electric fields noninvasively by attaching electrodes to the outside of the skull. However, since the skull itself dampens the electric fields, these techniques require great instrumental sensitivity and are still in their infancy. Perhaps the most exciting (or scary, depending on your point of view) research developments are in brain-computer interfaces. In this emerging field, electrical activity in the brain is used directly to control computers, and external stimuli are used to create electric fields inside the brain. Researchers in this area are motivated by the goal of helping people overcome physical disabilities, such as blindness or paralysis.

2.7 Electric Flux

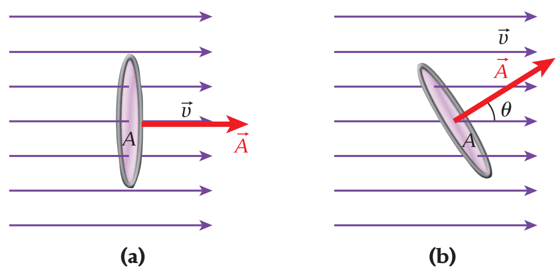


FIGURE 2.22 Water flowing with velocity of magnitude *v* through a ring of area *A*. (a) The area vector is parallel to the flow velocity. (b) The area vector is at an angle θ to the flow velocity.

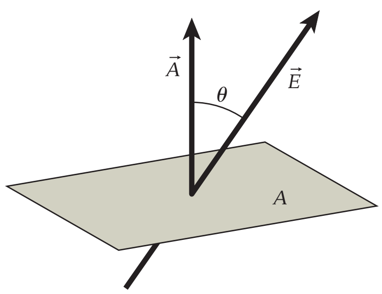


FIGURE 2.23 A uniform electric field $\vec{E}$ passing through an area $\vec{A}$

Electric field calculations, like those in Example 2.3, can require quite a bit of work. However, in many common situations, particularly those with some geometric symmetry, a powerful technique for determining electric fields without having to explicitly calculate integrals can be used. This technique is based on *Gauss's Law*, one of the fundamental relations concerning electric fields. It will allow us to solve seemingly very complicated problems involving electric fields in an amazingly straightforward and simple fashion. However, to use Gauss's Law requires understanding of a concept called *electric flux*.

Imagine holding a ring with inside area A in a stream of water flowing with velocity $\vec{v}$ as shown in Figure 22. The area vector, $\vec{A}$ of the ring is defined as a vector with magnitude A pointing in a direction perpendicular to the plane of the ring. In Figure 2.22a, the area vector of the ring is parallel to the flow velocity, and the flow velocity is perpendicular to the plane of the ring. The product Av gives the amount of water passing through the ring per unit time, where v is the magnitude of the flow velocity. If the plane of the ring is tilted with respect to the direction of the flowing water (Figure 2.22b), the amount of water flowing through the ring is given by $Av\cos\theta$, where θ is the angle between the area vector of the ring and the direction of the velocity of the flowing water. The amount of water flowing through the ring is called the *flux*, $\Phi = Av\cos\theta = \vec{A} \bullet \vec{v}$ Since flux is a measure of volume per unit time, its units are cubic meters per second (m^3/s).

An electric field is analogous to flowing water. Consider a uniform electric field of magnitude E passing through a given area A (Figure 2.23). Again, the area vector is $\vec{A}$, with a direction normal to the surface of the area and a magnitude A. The angle θ is the angle between the vector electric field and the area vector, as shown in Figure 2.23. The electric field passing through a given area A is called the **electric flux** and is given by

$$\Phi = EA\cos\theta \tag{2.13}$$

In simple terms, the electric flux is proportional to the number of electric field lines passing through the area. We'll assume that the electric field is given by $\vec{E}(\vec{r})$ and that the area is a closed surface, rather than the open surface of a simple ring in flowing water. In this closed-surface case, the total, or net, electric flux is given by an integral of the electric field over the closed surface:

$$\Phi = \oiint \vec{E} \bullet d\vec{A} \tag{2.14}$$

where $\vec{E}$ is the electric field at each differential area element $d\vec{A}$ of the closed surface. The direction of $d\vec{A}$ is outward from the closed surface. In equation 2.14, the loop on the integrals means that the integration is over a closed surface, and the two integral signs signify an integration over two variables. (*Note:* Some books use different notation for the integral over a closed surface, $\iint_S dA$ or just $\int_S dA$ but these refer to the same integration procedure as is represented in equation 2.14.) The differential area element $d\vec{A}$ must be described by two spatial variables, such as x and y in Cartesian coordinates or θ and ϕ in spherical coordinates.

Figure 2.24 shows a nonuniform electric field, $\vec{E}$ passing through a differential area element, $d\vec{A}$ A portion of the closed surface is also shown. The angle between the electric field and the differential area element is θ.

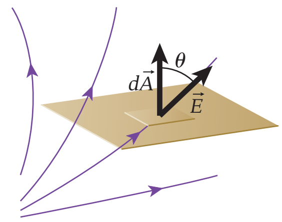


FIGURE 2.24 A nonuniform electric field, $\vec{E}$, passing through a differential area, $d\vec{A}$

EXAMPLE 2.5 Electric Flux through a Cube

Figure 2.25 shows a cube that has faces of area A in a uniform electric field, $\vec{E}$ that is perpendicular to the plane of one face of the cube.

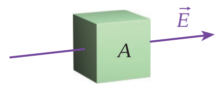


FIGURE 2.25 A cube with faces of area A in a uniform electric field, $\vec{E}$

PROBLEM

What is the net electric flux passing though the cube?

SOLUTION

The electric field in Figure 2.25 is perpendicular to the plane of one of the cube's six faces and therefore is also perpendicular to the opposite face. The area vectors of these two faces, $\vec{A}_1$ and $\vec{A}_2$ are shown in Figure 2.26a. The net electric flux passing through these two faces is

$$\Phi_{12} = \Phi_1 + \Phi_2 = \vec{E} \bullet \vec{A}_1 + \vec{E} \bullet \vec{A}_2 = -EA_1 + EA_2 = 0$$

The negative sign arises for the flux through face 1 because the electric field and the area vector, $\vec{A}_1$ are in opposite directions. The area vectors of the remaining four faces are all perpendicular to the electric field, as shown in Figure 2.26b. The net electric flux passing through these four faces is

$$\Phi_{3456} = \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = \vec{E} \bullet \vec{A}_3 + \vec{E} \bullet \vec{A}_4 + \vec{E} \bullet \vec{A}_5 + \vec{E} \bullet \vec{A}_6 = 0$$

All the scalar products are zero because the area vectors of these four faces are perpendicular to the electric field. Thus, the net electric flux passing through the cube is

$$\Phi = \Phi_{12} + \Phi_{3456} = 0$$

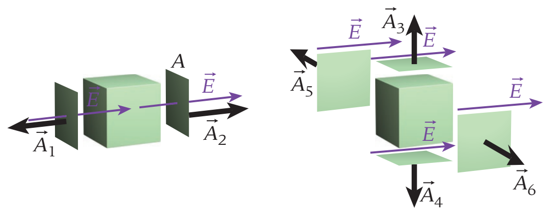
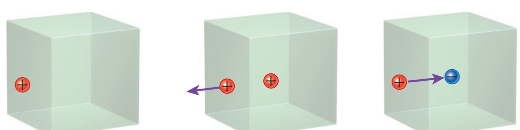


FIGURE 2.26 (a) The two faces of the cube that are perpendicular to the electric field. The area vectors are parallel and antiparallel to the electric field. (b) The four faces of the cube that are parallel to the electric field. The area vectors are perpendicular to the electric field.

2.8 Gauss's Law

To begin our discussion of Gauss's Law, let's imagine a box in the shape of a cube (Figure 2.27a), which is constructed of a material that does not affect electric fields. A positive test charge brought close to any surface of the box will experience no force. Now suppose a positive charge is inside the box and the positive test charge is brought close to the surface of the box (Figure 2.27b). The positive test charge experiences an outward force due to the positive charge inside the box. If the test charge is close to any surface of the box, it experiences the outward force. If twice as much positive charge is inside the box, the positive test charge experiences twice the outward force when brought close to any surface of the box.

Now suppose there is a negative charge inside the box (Figure 2.27c). When the positive test charge is brought close to one surface of the box, the charge experiences an inward force. If the positive test charge is close to any surface of the box, it experiences an inward force. Doubling the negative charge in the box doubles the inward force on the test charge when it is close to any surface of the box.

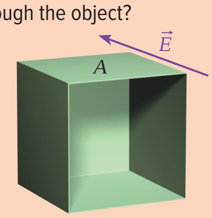


(a) (b) (c)

FIGURE 2.27 Three imaginary boxes constructed of material that does not affect electric fields. A positive test charge is brought up to the box from the left toward: (a) an empty box; (b) a box with a positive charge inside; (c) a box with a negative charge inside.

Self-Test Opportunity 2.3

The figure shows a cube with faces of area A and one face missing. This five-sided cubical object is in a uniform electric field, $\vec{E}$ perpendicular to one face. What is the net electric flux passing through the object?



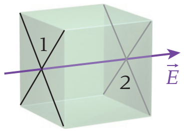
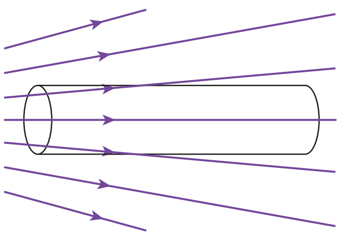


FIGURE 2.28 Imaginary empty box in a uniform electric field.

Concept Check 2.8

A cylinder made of an insulating material is placed in an electric field as shown in the figure. The net electric flux passing through the surface of the cylinder is



a) positive.

b) negative.

c) zero.

In analogy with flowing water, the electric field lines seem to be flowing out of the box containing positive charge and into the box containing negative charge.

Now let's imagine an empty box in a uniform electric field (Figure 2.28). If a positive test charge is brought close to side 1, it experiences an inward force. If the charge is close to side 2, it experiences an outward force. The electric field is parallel to the other four sides, so the positive test charge does not experience any inward or outward force when brought close to those sides. Thus, in analogy with flowing water, the net amount of electric field that seems to be flowing in and out of the box is zero.

Whenever a charge is inside the box, the electric field lines seem to be flowing in or out of the box. When there is no charge in the box, the net flow of electric field lines in or out of the box is zero. These observations and the definition of electric flux, which quantifies the concept of the flow of the electric field lines, lead to **Gauss's Law:**

$$\Phi = \frac{q}{\varepsilon_0} \tag{2.15}$$

Here q is the net charge inside a closed surface, called a **Gaussian surface.** The closed surface could be a box like that we have been discussing or any arbitrarily shaped closed surface. Usually, the shape of the Gaussian surface is chosen so as to reflect the symmetries of the problem situation.

Concept Check 2.9

The lines in the figure are electric field lines, and the circle is a Gaussian surface. For which case(s) is (are) the total electric flux nonzero?

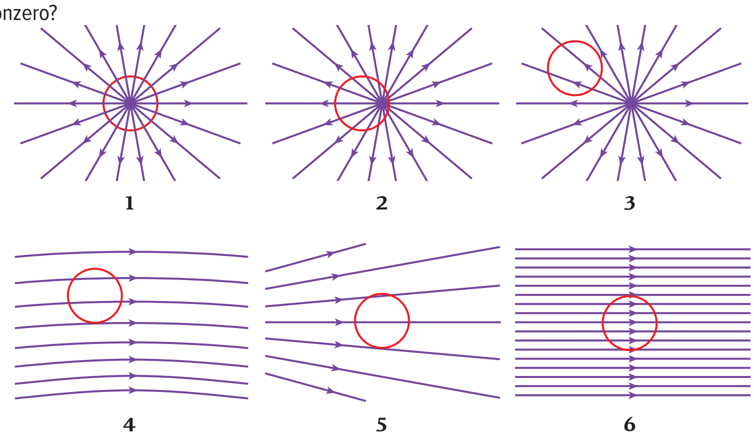
a) 1 only

b) 2 only

c) 4, 5, and 6

d) 6 only

e) 1 and 2



An alternative formulation of Gauss's Law incorporates the definition of the electric flux (equation 2.14):

$$\oiint \vec{E} \bullet d\vec{A} = \frac{q}{\varepsilon_0} \tag{2.16}$$

According to equation 2.16, Gauss's Law states that the surface integral of the electric field components perpendicular to the area times the area is proportional to the net charge within the closed surface. This expression may look daunting, but it simplifies considerably in many cases and allows us to perform very quickly calculations that would otherwise be quite complicated.

Gauss's Law and Coulomb's Law

We can derive Gauss's Law from Coulomb's Law. To do this, we start with a positive point charge, q. The electric field due to this charge is radial and pointing outward, as we saw in Section 2.3. According to Coulomb's Law (Section 1.5), the magnitude of the electric field from this charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

We now find the electric flux passing through a closed surface resulting from this point charge. For the Gaussian surface, we choose a spherical surface with radius r, with the charge at the center of the sphere, as shown in Figure 2.29. The electric field due to the positive point charge intersects each differential element of the surface of the Gaussian sphere perpendicularly. Therefore, at each point of this Gaussian surface, the electric field vector, $\vec{E}$ and the differential surface area vector, $d\vec{A}$ are parallel. The surface area vector will always point outward from the spherical Gaussian surface, but the electric field vector can point outward or inward depending on the sign of the charge. For a positive charge, the scalar product of the electric field and the surface area element is $\vec{E} \bullet d\vec{A} = E\, dA \cos 0° = E\, dA$ The electric flux in this case, according to equation 2.14, is

$$\Phi = \oiint \vec{E} \bullet d\vec{A} = \oiint E\, dA$$

Because the electric field has the same magnitude anywhere in space at a distance r from the point charge q, we can take E outside the integral:

$$\Phi = \oiint E\, dA = E \oiint dA$$

Now what we have left to evaluate is the integral of the differential area over a spherical surface, which is given by $\oiint dA = 4\pi r^2$ Therefore, we have found from Coulomb's Law for the case of a point charge

$$\Phi = (E)\left(\oiint dA\right) = \left(\frac{1}{4\pi\,\varepsilon_0}\frac{q}{r^2}\right)\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$

which is the same as the expression for Gauss's Law in equation 2.15. We have shown that Gauss's Law can be derived from Coulomb's Law for a positive point charge, but it can also be shown that Gauss's Law holds for any distribution of charge inside a closed surface.

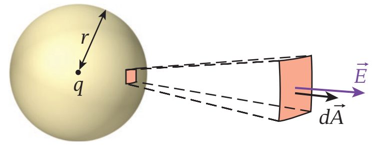


FIGURE 2.29 A spherical Gaussian surface with radius r surrounding a charge q. A closeup view of a differential surface element with area dA is shown.

Self-Test Opportunity 2.4

What changes in the preceding derivation of Gauss's Law if a negative point charge is used?

Shielding

Two important consequences of Gauss's Law are evident:

1. The electrostatic field inside any isolated conductor is always zero.
2. Cavities inside conductors are shielded from electric fields.

To examine these consequences, let's suppose a net electric field exists at some moment at some point inside an isolated conductor; see Figure 2.30a. But every conductor has free electrons inside it (blue circles in Figure 2.30b), which can move rapidly in response to any net external electric field, leaving behind positively charged ions (red circles in Figure 2.30b). The charges will move to the outer surface of the conductor, leaving no net accumulation of charge inside the volume of the conductor. These charges will in turn create an electric field inside the conductor (yellow arrows in Figure 2.30b), and they will move around until the electric field produced by them exactly cancels the external electric field. The net electric field thus becomes zero everywhere inside the conductor (Figure 2.30c).

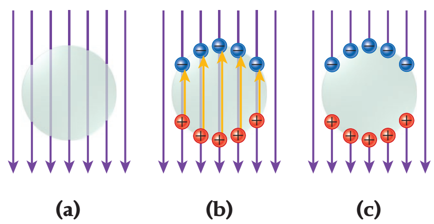


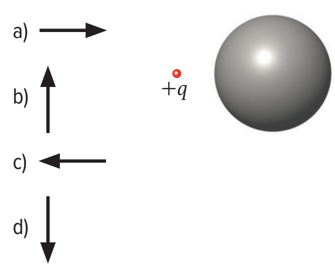
FIGURE 2.30 Shielding of an external electric field (purple vertical arrows) from the inside of a conductor.

If a cavity is scooped out of a conducting body, the net charge and thus the electric field inside this cavity is always zero, no matter how strongly the conductor is charged or how strong an external electric field acts on it. To prove this, we assume a closed Gaussian surface surrounds the cavity, completely inside the conductor. From the preceding discussion (see Figure 2.30), we know that at each point of this surface, the field is zero. Therefore, the net flux over this surface is also zero. By Gauss's Law, it then follows that this surface encloses zero net charge. If there were equal amounts of positive and negative charge on the cavity surface (and thus no net charge), this charge would not be stationary, as the positive and negative charges would be attracted to each other and would be free to move around the cavity surface to cancel each other. Therefore, any cavity inside a conductor is totally shielded from any external electric field. This effect is sometimes called **electrostatic shielding.**

A convincing demonstration of this shielding is provided by placing a plastic container filled with Styrofoam peanuts on top of a Van de Graaff generator, which serves as the source of strong electric field (Figure 2.31a). Charging the generator results in a large net charge accumulation on the dome, producing a strong electric field in the vicinity. Because

Concept Check 2.10

A hollow, conducting sphere is initially given an evenly distributed negative charge. A positive charge $+q$ is brought near the sphere and placed at rest as shown in the figure. What is the direction of the electric field inside the hollow sphere?



e) The field is zero.

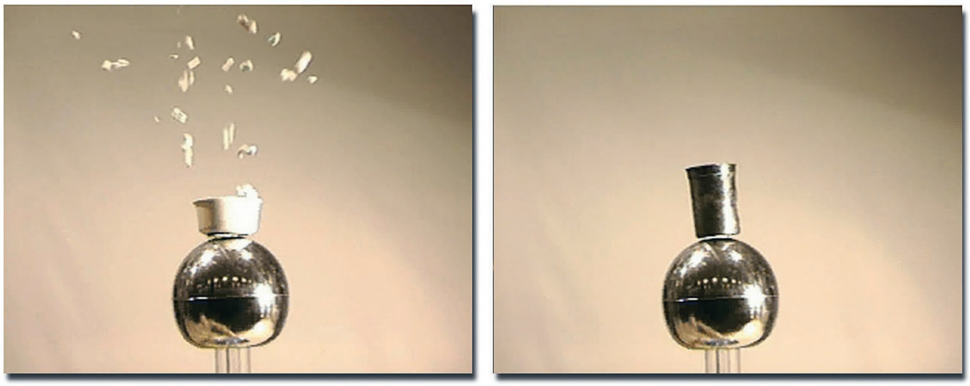


FIGURE 2.31 Styrofoam peanuts are put inside a container that is placed on top of a Van de Graaff generator, which is then charged. (a) The peanuts fly out of a nonconducting plastic container. (b) The peanuts remain within a metal can.

of this field, the charges in the Styrofoam peanuts separate slightly, and the peanuts acquire small dipole moments. If the field were uniform, there would be no force on these dipoles. However, the nonuniform electric field does exert a force, even though the peanuts are electrically neutral. The peanuts thus fly out of the container. If the same Styrofoam peanuts are placed inside an open metal can, they do not fly out when the generator is charged (Figure 2.31b). The electric field easily penetrates the walls of the plastic container and reaches the Styrofoam peanuts, whereas, in accord with Gauss's Law, the conducting metal can provide shielding inside and prevents the Styrofoam peanuts from acquiring dipole moments.

The conductor surrounding the cavity does not have to be a solid piece of metal; even a wire mesh is sufficient to provide shielding. This can be demonstrated most impressively by seating a person inside a cage and then hitting the cage with a lightning-like electrical discharge (Figure 2.32). The person inside the cage is unhurt, even if he or she touches the metal of the cage *from the inside*. (It is important to realize that severe injuries can result if any body parts stick out of the cage, for example, if hands are wrapped around the bars of the cage!) This cage is called a *Faraday cage*, after British physicist Michael Faraday (1791–1867), who invented it.

A Faraday cage has important consequences, probably the most relevant of which is the fact that your car protects you from being hit by lightning while inside it—unless you drive a convertible. The sheet metal and steel frame that surround the passenger compartment provide the necessary shielding. (But as fiberglass, plastic, and carbon fiber begin to replace sheet metal in auto bodies, this shielding is not assured any more.)

FIGURE 2.32 A person inside a Faraday cage is unharmed by a large voltage applied outside the cage, which produces a huge spark. This demonstration is performed several times daily at the Deutsches Museum in Munich, Germany.

Concept Check 2.11

A hollow, conducting sphere is initially uncharged. A positive charge, $+q_1$, is placed inside the sphere, as shown in the figure. Then, a second positive charge, $+q_2$, is placed near the sphere but outside it. Which of the following statements describes the net electric force on each charge?

a) There is a net electric force on $+q_2$ but not on $+q_1$.

b) There is a net electric force on $+q_1$ but not on $+q_2$.

c) Both charges are acted on by a net electric force with the same magnitude and in the same direction.

d) Both charges are acted on by a net electric force with the same magnitude but in opposite directions.

e) There is no net electric force on either charge.

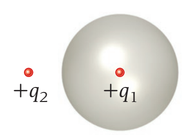


Table 2.1	Symbols for Charge Distributions	
Symbol	**Name**	**Unit**
λ	Charge per length	C/m
σ	Charge per area	C/m^2
ρ	Charge per volume	C/m^3

2.9 Special Symmetries

In this section we'll determine the electric field due to charged objects of different shapes. In Section 2.5, the charge distributions for different geometries were defined; see equation 2.9. Table 2.1 lists the symbols for these charge distributions and their units.

Cylindrical Symmetry

Using Gauss's Law, we can calculate the magnitude of the electric field due to a long straight conducting wire with uniform charge per unit length $\lambda > 0$. We first imagine a Gaussian surface in the form of a right cylinder with radius r and length L surrounding the wire so that the wire is along the axis of the cylinder (Figure 2.33). We can apply Gauss's Law to this Gaussian surface. From symmetry, we know that the electric field produced by the wire must be radial and perpendicular to the wire. What invoking symmetry means deserves further explanation because such arguments are very common.

First, we imagine rotating the wire about an axis along its length. This rotation would include all charges on the wire and their electric fields. However, the wire would still look the same after a rotation through any angle. The electric field created by the charge on the wire would therefore also be the same. From this argument, we conclude that the electric field cannot depend on the rotation angle around the wire. This conclusion is general: If an object has *rotational symmetry,* its electric field cannot depend on the rotation angle.

Second, if the wire is very long, it will look the same no matter where along its length it is viewed. If the wire is unchanged, its electric field is also unchanged. This observation means that there is no dependence on the coordinate along the wire. This symmetry is called *translational symmetry*. Since there is no preferred direction in space along the wire, there can be no electric field component parallel to the wire.

Returning to the Gaussian surface, we can see that the contribution to the integral in Gauss's Law (equation 2.16) from the ends of the cylinder is zero because the electric field is parallel to these surfaces and is thus perpendicular to the normal vectors from the surface. The electric field is perpendicular to the wall of the cylinder everywhere, so we have

$$\oiint \vec{E} \bullet d\vec{A} = EA = E(2\pi rL) = \frac{q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

where $2\pi rL$ is the area of the wall of the cylinder. Solving this equation, we find the magnitude of the electric field due to a uniformly charged long straight wire:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r} \qquad (2.17)$$

where r is the perpendicular distance to the wire. For $\lambda < 0$, equation 2.17 still applies, but the electric field points inward instead of outward. Note that this is the same result we obtained in Example 2.3 for the electric field due to a wire of infinite length–but attained here in a much simpler way!

You begin to see the great computational power contained in Gauss's Law, which can be used to calculate the electric field resulting from all kinds of charge distributions, both discrete and continuous. However, it is practical to use Gauss's Law only in situations where you can exploit some symmetry; otherwise, it is too difficult to calculate the flux.

It is instructive to compare the dependence of the electric field on the distance from a point charge and the distance from a long straight wire. For the point charge, the electric field falls off with the square of the distance, much faster than does the electric field due to the long wire, which decreases in inverse proportion to the distance.

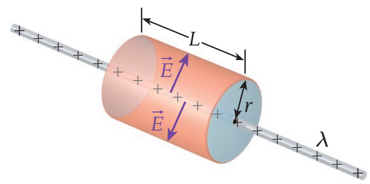


FIGURE 2.33 Long wire with charge per unit length λ surrounded by a Gaussian surface in the form of a right cylinder with radius r and length L. Representative electric field vectors are shown inside the cylinder.

Concept Check 2.12

A total of $1.45\text{x}10^6$ excess electrons are placed on an initially electrically neutral wire of length 1.13 m. What is the magnitude of the electric field at a point at a perpendicular distance of 0.401 m away from the center of wire? (*Hint:* Assume that 1.13 m is close enough to "infinitely long.")

a) $9.21\text{x}10^{-3}$ N/C

b) $2.92\text{x}10^{-1}$ N/C

c) $6.77\text{x}10^{1}$ N/C

d) $8.12\text{x}10^{2}$ N/C

e) $3.31\text{x}10^{3}$ N/C

Self-Test Opportunity 2.5

By how much does the answer to Concept Check 2.12 change if the assumption that the wire can be treated as being infinitely long is not made? (*Hint:* See Example 2.3.)

Planar Symmetry

Assume a flat thin, infinite, nonconducting sheet of positive charge (Figure 2.34), with uniform charge per unit area $\sigma > 0$. Let's find the electric field a distance r from the surface of this infinite plane of charge.

To do this, we choose a Gaussian surface in the form of a closed right cylinder with cross-sectional area A and length $2r$, which cuts through the plane perpendicularly, as shown in Figure 2.34. Because the plane is infinite and the charge is positive, the electric field must be perpendicular to the ends of the cylinder and parallel to the cylinder wall. Using Gauss's Law, we obtain

$$\oiint \vec{E} \bullet d\vec{A} = (EA + EA) = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

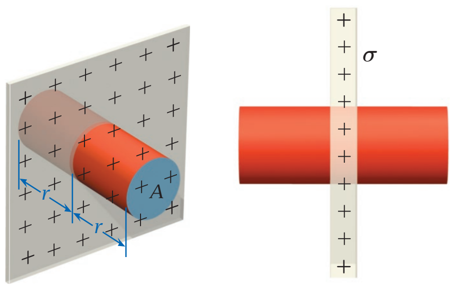


FIGURE 2.34 Infinite, flat, nonconducting sheet with charge density σ. Cutting through the plane perpendicularly is a Gaussian surface in the form of a right cylinder with cross-sectional area A parallel to the plane and height r above and below the plane.

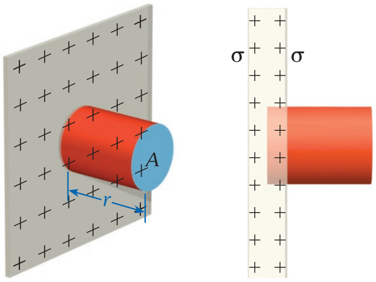


FIGURE 2.35 Infinite conducting plane with charge density σ on each surface and a Gaussian surface in the form of a right cylinder embedded in one side.

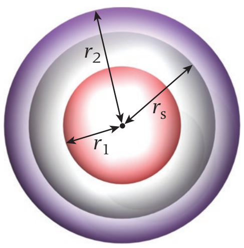


FIGURE 2.36 Spherical shell of charge with radius r_s along with a Gaussian surface with radius $r_2 > r_s$ and a second Gaussian surface with radius $r_1 < r_s$.

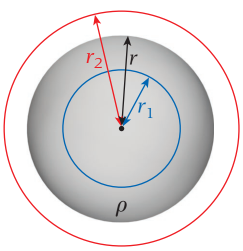


FIGURE 2.37 Spherical distribution of charge with uniform charge per unit volume ρ and radius r. Two spherical Gaussian surfaces are also shown, one with radius $r_1 < r$ and one with radius $r_2 > r$.

where σA is the charge enclosed in the cylinder. Thus, the magnitude of the electric field due to an infinite plane of charge is

$$E = \frac{\sigma}{2\varepsilon_0} \tag{2.18}$$

If $\sigma < 0$, then equation 2.18 still holds, but the electric field points toward the plane instead of away from it.

For an infinite conducting sheet with charge density $\sigma > 0$ on each surface, we can find the electric field by choosing a Gaussian surface in the form of a right cylinder. However, for this case, one end of the cylinder is embedded inside the conductor (Figure 2.35). The electric field inside the conductor is zero; therefore, there is no flux through the end of the cylinder enclosed in the conductor. The electric field outside the conductor must be perpendicular to the surface and therefore parallel to the wall of the cylinder and perpendicular to the end of the cylinder that is outside the conductor. Thus, the flux through the Gaussian surface is EA. The enclosed charge is given by σA, so Gauss's Law becomes

$$\oiint \vec{E} \bullet d\vec{A} = EA = \frac{\sigma A}{\varepsilon_0}$$

Thus, the magnitude of the electric field just outside the surface of a flat charged conductor is

$$E = \frac{\sigma}{\varepsilon_0} \tag{2.19}$$

Spherical Symmetry

To find the electric field due to a spherically symmetrical distribution of charge, we consider a thin spherical shell with charge $q > 0$ and radius r_s (Figure 2.36).

Here we use a spherical Gaussian surface with $r_2 > r_s$ that is concentric with the charged sphere. Applying Gauss's Law, we get

$$\oiint \vec{E} \bullet d\vec{A} = E\left(4\pi r_2^2\right) = \frac{q}{\varepsilon_0}$$

We can solve for the magnitude of the electric field, E, which is

$$E = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r_2^2}$$

If $q < 0$, the field points radially inward instead of radially outward from the spherical surfaces. For another spherical Gaussian surface, with $r_1 < r_s$, that is also concentric with the charged spherical shell, we obtain

$$\oiint \vec{E} \bullet d\vec{A} = E\left(4\pi r_1^2\right) = 0$$

Thus, the electric field outside a spherical shell of charge behaves as if the charge were a point charge located at the center of the sphere, whereas the electric field is zero inside the spherical shell of charge.

Now let's find the electric field due to charge that is equally distributed throughout a spherical volume, with uniform charge density $\rho > 0$ (Figure 2.37). The radius of the sphere is r. We use a Gaussian surface in the form of a sphere with radius $r_1 < r$. From the symmetry of the charge distribution, we know that the electric field resulting from the charge is perpendicular to the Gaussian surface. Thus, we can write

$$\oiint \vec{E} \bullet d\vec{A} = E\left(4\pi r_1^2\right) = \frac{q}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\left(\tfrac{4}{3}\pi r_1^3\right)$$

where $4\pi r_1^2$ is the area of the spherical Gaussian surface and $\frac{4}{3}\pi r_1^3$ is the volume enclosed by the Gaussian surface. From the preceding equation, we obtain the electric field at a radius r_1 inside a uniform distribution of charge:

$$E = \frac{\rho r_1}{3\varepsilon_0} \tag{2.20}$$

The total charge on the sphere can be called q_t, and it equals the total volume of the spherical charge distribution times the charge density:

$$q_t = \rho \tfrac{4}{3} \pi r^3$$

The charge enclosed by the Gaussian surface then is

$$q = \frac{\text{volume inside } r_1}{\text{volume of charge distribution}} q_t = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r^3} q_t = \frac{r_1^3}{r^3} q_t .$$

With this the expression for the enclosed charge, we can rewrite Gauss's Law for this case as

$$\oiint \vec{E} \bullet d\vec{A} = E\left(4\pi r_1^2\right) = \frac{q_t}{\varepsilon_0} \frac{r_1^3}{r^3}$$

which gives us

$$E = \frac{q_t r_1}{4\pi\varepsilon_0 r^3} = \frac{k q_t r_1}{r^3} \tag{2.21}$$

If we consider a Gaussian surface with a radius larger than the radius of the charge distribution, $r_2 > r$, we can apply Gauss's Law as follows:

$$\oiint \vec{E} \bullet d\vec{A} = E\left(4\pi r_2^2\right) = \frac{q_t}{\varepsilon_0}$$

or

$$E = \frac{q_t}{4\pi\varepsilon_0 r_2^2} = \frac{k q_t}{r_2^2} \tag{2.22}$$

Thus, the electric field outside a uniform spherical distribution of charge is the same as the field due to a point charge of the same magnitude located at the center of the sphere.

Self-Test Opportunity 2.6

Consider a sphere of radius *R* with charge *q* uniformly distributed throughout the volume of the sphere. What is the magnitude of the electric field at a point 2*R* away from the center of the sphere?

SOLVED PROBLEM 2.4 Nonuniform Spherical Charge Distribution

A spherically symmetrical but nonuniform charge distribution is given by

$$\rho(r) = \begin{cases} \rho_0\left(1 - \dfrac{r}{R}\right) & \text{for } r \le R \\ 0 & \text{for } r > R, \end{cases}$$

where $\rho_0 = 10.0\ \mu\text{C/m}^3$ and $R = 0.250$ m.

PROBLEM

What is the electric field produced by this charge distribution at $r_1 = 0.125$ m and at $r_2 = 0.500$ m?

SOLUTION

THINK We can use Gauss's Law to determine the electric field as a function of radius if we employ a spherical Gaussian surface. The radius $r_1 = 0.125$ m is located inside the charge distribution. The charge enclosed inside the spherical surface at $r = r_1$ is given by an integral of the charge density from $r = 0$ to $r = r_1$. Outside the spherical charge distribution, the electric field is the same as that of a point charge whose magnitude is equal to the total charge of the spherical distribution.

SKETCH The charge density, ρ, as a function of radius, r, is plotted in Figure 2.38.

RESEARCH Gauss's Law (equation 2.16) tells us that $\oiint \vec{E} \bullet d\vec{A} = q/\varepsilon_0$ Inside the nonuniform spherical charge distribution at a radius $r_1 < R$, Gauss's Law becomes

$$\varepsilon_0 E\left(4\pi r_1^2\right) = \int_0^{V_1} \rho(r)\,dV = \int_0^{r_1} \rho_0\left(1 - \frac{r}{R}\right)\left(4\pi r^2\right) dr \tag{i}$$

– Continued

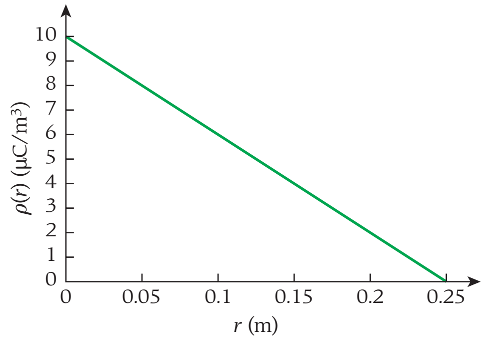


FIGURE 2.38 Charge density as a function of radius for a nonuniform spherical charge distribution.

Carrying out the integral on the right-hand side of equation (i), we obtain

$$\int_0^{r_1} \rho_0\left(1-\frac{r}{R}\right)\left(4\pi r^2\right)dr = 4\pi\rho_0\int_0^{r_1}\left(r^2-\frac{r^3}{R}\right)dr = 4\pi\rho_0\left(\frac{r_1^3}{3}-\frac{r_1^4}{4R}\right) \qquad \text{(ii)}$$

SIMPLIFY The electric field due to the charge inside $r_1 \le R$ is then given by

$$E = \frac{4\pi\rho_0\left(\dfrac{r_1^3}{3}-\dfrac{r_1^4}{4R}\right)}{\varepsilon_0\left(4\pi r_1^2\right)} = \frac{\rho_0}{\varepsilon_0}\left(\frac{r_1}{3}-\frac{r_1^2}{4R}\right) \qquad \text{(iii)}$$

In order to calculate the electric field due to the charge inside $r_2 > R$, we need the total charge contained in the spherical charge distribution. We can obtain the total charge using equation (ii) with $r_1 = R$:

$$q_t = 4\pi\rho_0\left(\frac{R^3}{3}-\frac{R^4}{4R}\right) = 4\pi\rho_0\left(\frac{R^3}{3}-\frac{R^3}{4}\right) = 4\pi\rho_0\frac{R^3}{12} = \frac{\pi\rho_0 R^3}{3}$$

The electric field outside the spherical charge distribution ($r_2 > R$) is then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_t}{r_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{1}{3}\pi\rho_0 R^3}{r_2^2} = \frac{\rho_0 R^3}{12\varepsilon_0 r_2^2} \qquad \text{(iv)}$$

CALCULATE The electric field at $r_1 = 0.125$ m is

$$E = \frac{\rho_0}{\varepsilon_0}\left(\frac{r_1}{3}-\frac{r_1^2}{4R}\right) = \frac{10.0\ \mu\text{C/m}^3}{8.85\times10^{-12}\,\text{C}^2/\text{N m}^2}\left(\frac{0.125\text{ m}}{3}-\frac{(0.125\text{ m})^2}{4(0.250\text{ m})}\right) = 29{,}425.6\text{ N/C}$$

The electric field at $r_2 = 0.500$ m is

$$E = \frac{\rho_0 R^3}{12\varepsilon_0 r_2^2} = \frac{\left(10.0\ \mu\text{C/m}^3\right)(0.250\text{ m})^3}{12\left(8.85\times10^{-12}\,\text{C}^2/\text{N m}^2\right)(0.500\text{ m})^2} = 5885.12\text{ N/C}$$

ROUND We report our results to three significant figures. The electric field at $r_1 = 0.125$ m is

$$E = 2.94\times10^4\ \text{N/C}$$

The electric field at $r_2 = 0.500$ m is

$$E = 5.89\times10^3\ \text{N/C}$$

DOUBLE-CHECK The electric field at $r_1 = R$ can be calculated using equation (iii):

$$E = \frac{\rho_0}{\varepsilon_0}\left(\frac{R}{3}-\frac{R^2}{4R}\right) = \frac{\rho_0 R}{12\varepsilon_0} = \frac{\left(10.0\ \mu\text{C/m}^3\right)(0.250\text{ m})}{12\left(8.85\times10^{-12}\,\text{C}^2/\text{N m}^2\right)} = 2.35\times10^4\ \text{N/C}$$

We can also use equation (iv) to find the electric field outside the spherical charge distribution but very close to the surface, where $r_2 \approx R$:

$$E = \frac{\rho_0 R^3}{12\varepsilon_0 R^2} = \frac{\rho_0 R}{12\varepsilon_0}$$

which is the same result we obtained using our result for $r_1 \leq R$. The calculated electric field at the surface of the charge distribution is lower than that at $r_1 = 0.125$, which may seem counterintuitive. An idea of the dependence of the magnitude of E on r is provided by the plot in Figure 2.39, which was created using equations (iii) and (iv).

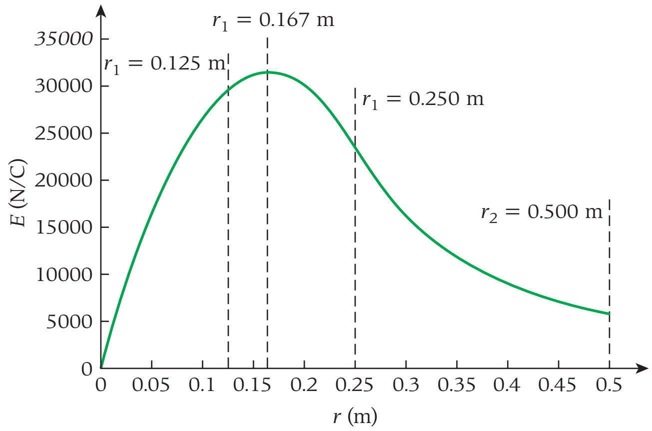


FIGURE 2.39 The electric field due to a nonuniform spherical distribution of charge as a function of the distance from the center of the sphere.

You can see that a maximum occurs in the electric field and that our result for $r_1 = 0.125$ m is less than this maximum value. We can calculate the radius at which the maximum occurs by differentiating equation (iii) with respect to r_1, setting the result equal to zero, and solving for r_1:

$$\frac{dE}{dr_1} = \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3} - \frac{r_1}{2R}\right) = 0 \Rightarrow$$

$$\frac{1}{3} = \frac{r_1}{2R} \Rightarrow r_1 = \frac{2}{3}R$$

Thus, we expect a maximum in the electric field at $r_1 = \frac{2}{3}R = 0.167$ m. The plot in Figure 2.39 does indeed show a maximum at that radius. It also shows the value of E at $r = 0.250$ to be smaller than that at $r = 0.125$ as we found in our calculation. Thus, our answers seem reasonable.

Concept Check 2.13

Suppose an uncharged solid steel ball, for example, one of the steel balls used in an old-fashioned pinball machine, is resting on a perfect insulator. Some small amount of negative charge (say, a few hundred electrons) is placed at the north pole of the ball. If you could check the distribution of the charge after a few seconds, what would you detect?

a) All of the added charge has vanished, and the ball is again electrically neutral.

b) All of the added charge has moved to the center of the ball.

c) All of the added charge is distributed uniformly over the surface of the ball.

d) The added charge is still located at or very near the north pole of the ball.

e) The added charge is performing a simple harmonic oscillation on a straight line between the south and north poles of the ball.

Concept Check 2.14

Suppose an uncharged hollow sphere made of a perfect insulator, for example a ping-pong ball, is resting on a perfect insulator. Some small amount of negative charge (say, a few hundred electrons) is placed at the north pole of the sphere. If you could check the distribution of the charge after a few seconds, what would you detect?

a) All of the added charge has vanished, and the sphere is again electrically neutral.

b) All of the added charge has moved to the center of the sphere.

c) All of the added charge is distributed uniformly over the surface of the sphere.

d) The added charge is still located at or very near the north pole of the sphere.

e) The added charge is performing a simple harmonic oscillation on a straight line between the south and north poles of the sphere.

Sharp Points and Lightning Rods

We have already seen that the electric field is perpendicular to the surface of a conductor. (To repeat, if there were a field component parallel to the surface of the conductor, then the charges inside the conductor would move until they reached equilibrium, which means no force or electric field component in the direction of motion, that is, along the surface of the conductor.) Figure 2.40a shows the distribution of charges on the surface of the end of a pointed conductor. Note that the charges are closer together at the sharp tip, where the curvature is largest. Near that sharp tip on the end of the conductor, the electric field looks much more like that due to a point charge, with the field lines spreading out radially (Figure 2.40b). Since the field lines are closer together

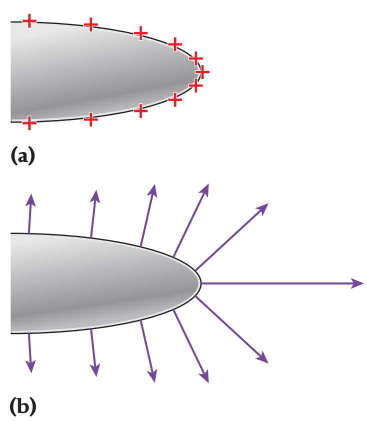


FIGURE 2.40 A sharp end of a conductor (with large curvature): (a) distribution of charges; (b) electric field at the surface of the conductor.

near a sharp point on a conductor, the field is stronger near the sharp tip than on the flat part of the conductor.

Benjamin Franklin proposed metal rods with sharp points as lightning rods. He reasoned that the sharp points would dissipate the electric charge built up in a storm, preventing the discharge of lightning. When Franklin installed such lightning rods, they were struck by lightning instead of the buildings to which they were attached. However, recent findings indicate that lightning rods used to protect structures from lightning should have blunt, rounded ends. When charged during thunderstorm conditions, a lightning rod with a sharp point creates a strong electric field that locally ionizes the air, producing a condition that actually causes lightning. Conversely, round-ended lightning rods are just as effective in protecting structures from lightning and do not increase lightning strikes. Any lightning rod should be carefully grounded to carry charge from a lightning strike away from the structure on which the lightning rod is mounted.

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- The electric force, $\vec{F}(\vec{r})$ on a charge, q, due to an electric field, $\vec{E}(\vec{r})$ is given by $\vec{F}(\vec{r}) = q\vec{E}(\vec{r})$
- The electric field at any point is equal to the sum of the electric fields from all sources: $\vec{E}_t(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \cdots + \vec{E}_n(\vec{r})$
- The magnitude of the electric field due to a point charge q at a distance r is given by $E(r) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2} = \dfrac{k|q|}{r^2}$
  The electric field points radially away from a positive point charge and radially toward a negative charge.
- A system of two equal (in magnitude) but oppositely charged point particles is an electric dipole. The magnitude, p, of the electric dipole moment is given by $p = qd$, where q is the magnitude of either of the charges and d is the distance separating them. The electric dipole moment is a vector pointing from the negative toward the positive charge. On the dipole axis, the dipole produces an electric field of magnitude $E = \dfrac{p}{2\pi\varepsilon_0|x|^3}$ where $|x| \gg d$.
- Gauss's Law states that the electric flux over an entire closed surface is equal to the enclosed charge divided by ϵ_0: $\oiint \vec{E} \bullet d\vec{A} = \dfrac{q}{\varepsilon_0}$
- The differential electrical field is given by $dE = k\dfrac{dq}{r^2}$ and the differential charge is

$$\left.\begin{aligned} dq &= \lambda\,dx \\ dq &= \sigma\,dA \\ dq &= \rho\,dV \end{aligned}\right\} \text{ for a charge distribution } \begin{cases} \text{along a line;} \\ \text{over a surface;} \\ \text{throughout a volume.} \end{cases}$$

- The magnitude of the electric field at a distance r from a long straight wire with uniform linear charge density $\lambda > 0$ is given by $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{2k\lambda}{r}$
- The magnitude of the electric field produced by an infinite nonconducting plane that has uniform charge density $\sigma > 0$ is $E = \tfrac{1}{2}\sigma/\epsilon_0$.
- The magnitude of the electric field produced by an infinite conducting plane that has uniform charge density $\sigma > 0$ on each side is $E = \sigma/\epsilon_0$.
- The electric field inside a closed conductor is zero.
- The electric field outside a charged spherical conductor is the same as that due to a point charge of the same magnitude located at the center of the sphere.

ANSWERS TO SELF-TEST OPPORTUNITIES

2.1 The direction of the electric field is downward at points A, C, and E and upward at points B and D. (There is an electric field at point E, even though there is no line drawn there; the field lines are only sample representations of the electric field, which also exists between the field lines.) The field is largest in magnitude at point E, which can be inferred from the fact that it is located where the field lines have the highest density.

2.2 The two forces acting on the two charges in the electric field create a torque on the electric dipole around its center of mass, given by

$$\tau = (\text{force}_+)(\text{moment arm}_+)(\sin\theta) + (\text{force}_-)(\text{moment arm}_-)(\sin\theta).$$

The length of the moment arm in both cases is $\tfrac{1}{2}d$, and the magnitude of the force is $F = qE$ for both charges. Thus, the torque on the electric dipole is

$$\tau = qE\left(\frac{d}{2}\sin\theta\right) + qE\left(\frac{d}{2}\sin\theta\right) = qEd\sin\theta$$

2.3 The net electric flux passing though the object is EA. Remember, the object is not a closed surface; otherwise, the result would be zero.

2.4 The sign of the scalar product changes, because the electric field points radially inward: $\vec{E} \bullet d\vec{A} = EdA\cos 180^\circ = -EdA$. But the magnitude of the electric field due to the negative charge is $E = \frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}$ The two minus signs cancel, giving the same results for Coulomb's and Gauss's Laws for a point charge, independent of the sign of the charge.

2.5 For a wire of infinite length, $E_y = \frac{2k\lambda}{y}$

for a wire of finite length, $E_y = \frac{2k\lambda}{y}\frac{a}{\sqrt{y^2+a^2}}$

With the values given in Concept Check 2.12, $\frac{a}{\sqrt{y^2+a^2}} = \frac{0.565}{\sqrt{0.401^2+0.565^2}} = 0.815$ Thus, the "infinitely long" approximation is off by ~18%.

2.6 The charged sphere acts like a point charge, so the electric field at $2R$ is

$$E = k\frac{q}{(2R)^2} = k\frac{q}{4R^2}$$

PROBLEM-SOLVING GUIDELINES

1. Be sure to distinguish between the point where an electric field is being generated and the point where the electric field is being determined.

2. Some of the same guidelines for dealing with electrostatic charges and forces also apply to electric fields: Use symmetry to simplify your calculations; remember that the field is composed of vectors and thus you have to use vector operations instead of simple addition, multiplication, and so on; convert units to meters and coulombs for consistency with the given values of constants.

3. Remember to use the correct form of the charge density for field calculations: λ for linear charge density, σ for surface charge density, and ρ for volume charge density.

4. The key to using Gauss's Law is to choose the right shape for the Gaussian surface to exploit the symmetry of the problem situation. Cubical, cylindrical, and spherical Gaussian surfaces are typically useful.

5. Often, you can break a Gaussian surface into surface elements that are either perpendicular to or parallel to the electric field lines. If the field lines are perpendicular to the surface, the electric flux is simply the field strength times the area, EA, or $-EA$ if the field points inward instead of outward. If the field lines are parallel to the surface, the flux through that surface is zero. The total flux is the sum of the flux through each surface element of the Gaussian surface. Remember that zero flux through a Gaussian surface does not necessarily mean that the electric field is zero.

MULTIPLE-CHOICE QUESTIONS

2.1 In order to use Gauss's Law to calculate the electric field created by a known distribution of charge, which of the following *must* be true?

a) The charge distribution must be in a nonconducting medium.

b) The charge distribution must be in a conducting medium.

c) The charge distribution must have spherical or cylindrical symmetry.

d) The charge distribution must be uniform.

e) The charge distribution must have a high degree of symmetry that allows assumptions about the symmetry of its electric field to be made.

2.2 An electric dipole consists of two equal and opposite charges situated a small distance from each other. When the dipole is placed in a uniform electric field, which of the following statements is (are) true?

a) The dipole will not experience any net force from the electric field; since the charges are equal and have opposite signs, the individual effects will cancel out.

b) There will be no net force and no net torque acting on the dipole.

c) There will be a net force but no net torque acting on the dipole.

d) There will be no net force, but there will (in general) be a net torque acting on dipole.

2.3 A point charge, $+Q$, is located on the x-axis at $x = a$, and a second point charge, $-Q$, is located on the x-axis at $x = -a$. A Gaussian surface with radius $r = 2a$ is centered at the origin. The flux through this Gaussian surface is

a) zero.

b) greater than zero.

c) less than zero.

d) none of the above.

2.4 A charge of $+2q$ is placed at the center of an uncharged conducting shell. What will be the charges on the inner and outer surfaces of the shell, respectively?

a) $-2q$, $+2q$

b) $-q$, $+q$

c) $-2q$, $-2q$

d) $-2q$, $+4q$

2.5 Two infinite nonconducting plates are parallel to each other, with a distance $d = 10.0$ cm between them, as shown in the figure. Each plate carries a uniform charge distribution of $\sigma = 4.5\ \mu\text{C/m}^2$. What is the electric field, $\vec{E}$ at point P (with $x_P = 20.0$ cm)?

a) 0 N/C

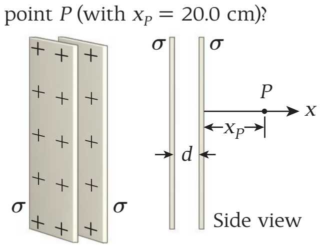
b) $2.54\hat{x}$ N/C

c) $(-5.08 \times 10^5)\hat{x}$ N/C

d) $(5.08 \times 10^5)\hat{x}$ N/C

e) $(-1.02 \times 10^6)\hat{x}$ N/C

f) $(1.02 \times 10^6)\hat{x}$ N/C



2.6 At which of the following locations is the electric field the strongest?

a) a point 1 m from a 1-C point charge

b) a point 1 m (perpendicular distance) from the center of a 1-m-long wire with 1 C of charge distributed on it

c) a point 1 m (perpendicular distance) from the center of a 1-m^2 sheet of charge with 1 C of charge distributed on it

d) a point 1 m from the surface of a charged spherical shell with a radius of 1 m

e) a point 1 m from the surface of a charged spherical shell with a radius of 0.5 m and a charge of 1 C

2.7 The electric flux through a spherical Gaussian surface of radius R centered on a charge Q is 1200 N/(C m^2). What is the electric flux through a cubic Gaussian surface of side R centered on the same charge Q?

a) less than 1200 N/(C m^2)

b) more than 1200 N/(C m^2)

c) equal to 1200 N/(C m^2)

d) cannot be determined from the information given

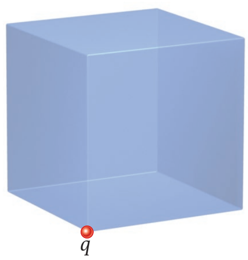
2.8 A single positive point charge, q, is at one corner of a cube with sides of length L, as shown in the figure. The net electric flux through the three adjacent sides is zero. The net electric flux through *each* of the other three sides is

a) $q/3\epsilon_0$.

b) $q/6\epsilon_0$.

c) $q/24\epsilon_0$.

d) $q/8\epsilon_0$.



2.9 Three −9mC point charges are located at (0,0), (3 m,3 m), and (3 m,−3 m). What is the magnitude of the electric field at (3 m,0)?

a) 0.9 x 10^7 N/C

b) 1.2 x 10^7 N/C

c) 1.8x 10^7 N/C

d) 2.4 x 10^7 N/C

e) 3.6 x 10^7 N/C

f) 5.4 x 10^7 N/C

g) 10.8 x 10^7 N/C

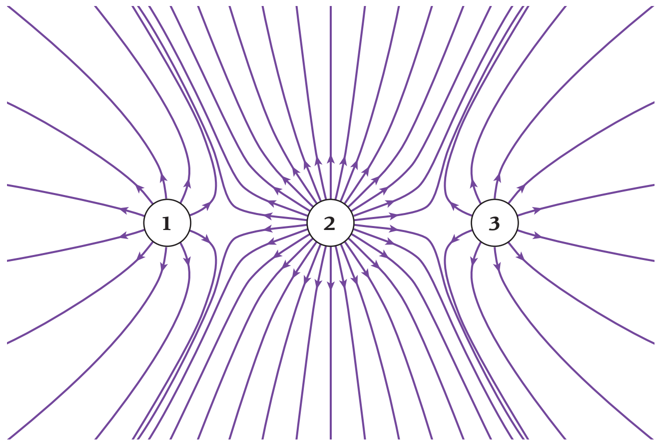
2.10 Which of the following statements is (are) true?

a) There will be *no* change in the charge on the inner surface of a hollow conducting sphere if additional charge is placed on the outer surface.

b) There will be some change in the charge on the inner surface of a hollow conducting sphere if additional charge is placed on the outer surface.

c) There will be *no* change in the charge on the inner surface of a hollow conducting sphere if additional charge is placed at the center of the sphere.

d) There will be some change in the charge on the inner surface of a hollow conducting sphere if additional charge is placed at the center of the sphere.



2.11 What are the signs of the charges in the configuration shown in the figure?

a) Charges 1, 2, and 3 are negative.

b) Charges 1, 2, and 3 are positive.

c) Charges 1 and 3 are positive, and 2 is negative.

d) Charges 1 and 3 are negative, and 2 is positive.

e) All that can be said is that the charges have the same sign.

2.12 Which of the following statements is (are) true?

a) Electric field lines point inward toward negative charges.

b) Electric field lines form circles around positive charges.

c) Electric field lines may cross.

d) Electric field lines point outward from positive charges.

e) A positive point charge released from rest will initially accelerate along a tangent to the electric field line at that point.

CONCEPTUAL QUESTIONS

2.13 Many people have been sitting in a car when it was struck by lightning. Why were they able to survive such an experience?

2.14 Why is it a bad idea to stand under a tree in a thunderstorm? What should one do instead to avoid getting struck by lightning?

2.15 Why do electric field lines never cross?

2.16 How is it possible that the flux through a closed surface does not depend on where inside the surface the charge is located (that is, the charge can be moved around inside the surface with no effect whatsoever on the flux)? If the charge is moved from just inside to just outside the surface, the flux changes discontinuously to zero, according to Gauss's Law. Does this really happen? Explain.

2.17 A solid conducting sphere of radius r_1 has a total charge of $+3Q$. It is placed inside (and concentric with) a conducting spherical shell of inner radius r_2 and outer radius r_3. Find the electric field in these regions: $r < r_1$, $r_1 < r < r_2$, $r_2 < r < r_3$, and $r > r_3$.

2.18 A thin rod has end points at $x = \pm 100$ cm. There is a total charge Q uniformly distributed along the rod.

a) What is the electric field very close to the midpoint of the rod?

b) What is the electric field a few centimeters (perpendicularly) from the midpoint of the rod?

c) What is the electric field very far (perpendicularly) from the midpoint of the rod?

2.19 A dipole is completely enclosed by a spherical surface. Describe how the total electric flux through this surface varies with the strength of the dipole.

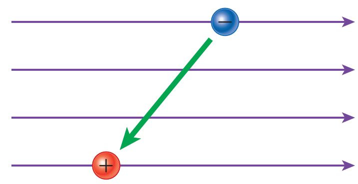
2.20 Repeat Example 2.3, assuming that the charge distribution is $-\lambda$ for $-a < x < 0$ and $+\lambda$ for $0 < x < a$.

2.21 A negative charge is placed on a solid prolate spheroidal conductor (shown in cross section in the figure). Sketch the distribution of the charge on the conductor and the electric field lines due to the charge.

2.2 *Saint Elmo's fire* is an eerie glow that appears at the tips of masts and yardarms of sailing ships in stormy weather and at the tips and edges of the wings of aircraft in flight. St. Elmo's fire is an electrical phenomenon. Explain it, concisely.

2.23 A charge placed on a conductor of any shape forms a layer on the outer surface of the conductor. Mutual repulsion of the individual charge elements creates an outward pressure on this layer, called *electrostatic stress*. Treating the infinitesimal charge elements like tiles of a mosaic, calculate the magnitude of this electrostatic stress in terms of the surface charge density, σ. Note that σ need not be uniform over the surface.

2.24 An electric dipole is placed in a uniform electric field as shown in the figure. What motion will the dipole have in the electric field? Which way will it move? Which way will it rotate?



EXERCISES

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

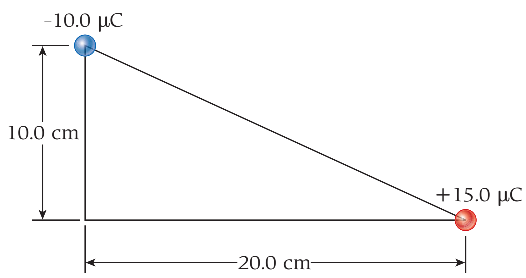
Section 2.3

2.25 A point charge, $q = 4.00 \times 10^{-9}$ C, is placed on the x-axis at the origin. What is the electric field produced at $x = 25.0$ cm?

2.26 A +1.60 nC point charge is placed at one corner of a square (1.00 m on a side), and a −2.40 nC charge is placed on the corner diagonally opposite. What is the magnitude of the electric field at either of the other two corners?

2.27 A +48.00 nC point charge is placed on the x-axis at $x = 4.000$ m, and a −24.00-nC point charge is placed on the y-axis at $y = -6.000$ m. What is the direction of the electric field at the origin?

•2.28 Two point charges are placed at two of the corners of a triangle as shown in the figure. Find the magnitude and the direction of the electric field at the third corner of the triangle.

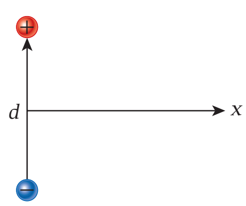


•2.29 A +5.00 C charge is located at the origin. A −3.00 C charge is placed at $x = 1.00$ m. At what finite distance(s) along the x-axis will the electric field be equal to zero?

•2.30 Three charges are on the y-axis. Two of the charges, each $-q$, are located $y = \pm d$, and the third charge, $+2q$, is located at $y = 0$. Derive an expression for the electric field at a point P on the x-axis.

Section 2.4

2.31 For the electric dipole shown in the figure, express the magnitude of the resulting electric field as a function of the perpendicular distance x from the center of the dipole axis. Comment on what the magnitude is when $x \gg d$.



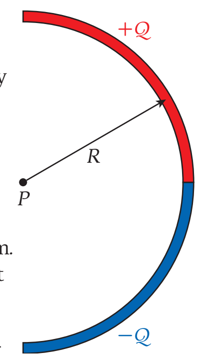
•2.32 Consider an electric dipole on the x-axis and centered at the origin. At a distance h along the positive x-axis, the magnitude of electric field due to the electric dipole is given by $k(2qd)/h^3$. Find a distance perpendicular to the x-axis and measured from the origin at which the magnitude of the electric field is the same.

Section 2.5

•2.33 A small metal ball with a mass of 4.00 g and a charge of 5.00 mC is located at a distance of 0.700 m above the ground in an electric field of 12.0 N/C directed to the east. The ball is then released from rest. What is the velocity of the ball after it has moved downward a vertical distance of 0.300 m?

•2.34 A charge per unit length $+\lambda$ is uniformly distributed along the positive y-axis from $y = 0$ to $y = +a$. A charge per unit length $-\lambda$ is uniformly distributed along the negative y-axis from $y = 0$ to $y = -a$. Write an expression for the electric field (magnitude and direction) at a point on the x-axis a distance x from the origin.

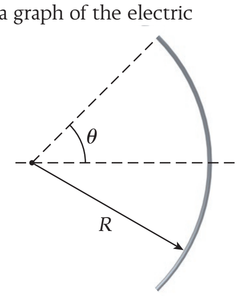
•2.35 A thin glass rod is bent into a semicircle of radius R. A charge $+Q$ is uniformly distributed along the upper half, and a charge $-Q$ is uniformly distributed along the lower half as shown in the figure. Find the magnitude and direction of the electric field $\vec{E}$ (in component form) at point P, the center of the semicircle.



•2.36 Two uniformly charged insulating rods are bent in a semicircular shape with radius $r = 10.0$ cm. If they are positioned so that they form a circle but do not touch and if they have opposite charges of +1.00 μC and −1.00 μC, find the magnitude and the direction of the electric field at the center of the composite circular charge configuration.

•2.37 A uniformly charged rod of length L with total charge Q lies along the y-axis, from $y = 0$ to $y = L$. Find an expression for the electric field at the point $(d,0)$ (that is, the point at $x = d$ on the x-axis).

••2.38 A charge Q is distributed evenly on a wire bent into an arc of radius R, as shown in the figure. What is the electric field at the center of the arc as a function of the angle θ? Sketch a graph of the electric field as a function of θ for $0 < \theta < 180°$.



••2.39 A thin, flat washer is a disk with an outer diameter of 10.0 cm and a hole of diameter 4.00 cm in the center. The washer has a uniform charge distribution and a total charge of 7.00 nC. What is the electric field on the axis of the washer at a distance of 30.0 cm from the center of the washer?

Section 2.6

2.40 Research suggests that the electric fields in some thunderstorm clouds can be on the order of 10.3 kN/C. Calculate the magnitude of the electric force acting on a particle with two excess electrons in the presence of a 10.0 kN/C field.

2.41 An electric dipole has opposite charges of 5.00×10^{-15} C separated by a distance of 0.400 mm. It is oriented at 60.0° with respect to a uniform electric field of magnitude 2.00×10^3 N/C. Determine the magnitude of the torque exerted on the dipole by the electric field.

2.42 Electric dipole moments of molecules are often measured in debyes (D), where 1 D $= 3.34 \times 10^{-30}$ C m. For instance, the dipole moment of hydrogen chloride gas molecules is 1.05 D. Calculate the maximum torque such a molecule can experience in the presence of an electric field of magnitude 160.0 N/C.

2.43 An electron is observed traveling at a speed of 27.5×10^6 m/s parallel to an electric field of magnitude 11,400 N/C. How far will the electron travel before coming to a stop?

2.44 Two charges, $+e$ and $-e$, are a distance of 0.680 nm apart in an electric field, E, that has a magnitude of 4.40 kN/C and is directed at an angle of 45.0° with respect to the dipole axis. Calculate the dipole moment and thus the torque on the dipole in the electric field.

•2.45 A body of mass M, carrying charge Q, falls from rest from a height h (above the ground) near the surface of the Earth, where the gravitational acceleration is g and there is an electric field with a constant component E in the vertical direction.

a) Find an expression for the speed, v, of the body when it reaches the ground, in terms of M, Q, h, g, and E.

b) The expression from part (a) is not meaningful for certain values of M, g, Q, and E. Explain what happens in such cases.

•2.46 A water molecule, which is electrically neutral but has a dipole moment of magnitude $p = 6.20 \times 10^{-30}$ C m, is 1.00 cm away from a point charge $q = +1.00$ μC. The dipole will align with the electric field due to the charge. It will also experience a net force, since the field is not uniform.

a) Calculate the magnitude of the net force. (*Hint:* You do not need to know the precise size of the molecule, only that it is much smaller than 1 cm.)

b) Is the molecule attracted to or repelled by the point charge? Explain.

•2.47 A total of 3.05×10^6 electrons are placed on an initially uncharged wire of length 1.33 m.

a) What is the magnitude of the electric field a perpendicular distance of 0.401 m away from the midpoint of the wire?

b) What is the magnitude of the acceleration of a proton placed at that point in space?

c) In which direction does the electric field force point in this case?

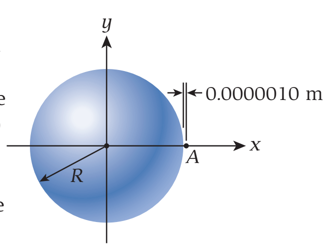
Sections 2.7 and 2.8

2.48 Four charges are placed in three-dimensional space. The charges have magnitudes $+3q$, $-q$, $+2q$, and $-7q$. If a Gaussian surface encloses all the charges, what will be the electric flux through that surface?

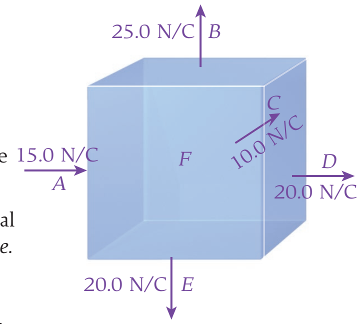
2.49 The six faces of a cubical box each measure 20.0 cm by 20.0 cm, and the faces are numbered such that faces 1 and 6 are opposite to each other, as are faces 2 and 5, and faces 3 and 4. The flux through each face is given in the table. Find the net charge inside the cube.

Face	Flux (N m^2/C)
1	−70.0
2	−300.0
3	−300.0
4	+300.0
5	−400.0
6	−500.0

2.50 A conducting solid sphere ($R = 0.15$ m, $q = 6.1 \times 10^{-6}$ C) is shown in the figure. Using Gauss's Law and two different Gaussian surfaces, determine the electric field (magnitude and direction) at point A, which is 0.0000010 m outside the conducting sphere. (*Hint:* One Gaussian surface is a sphere, and the other is a small right cylinder.)



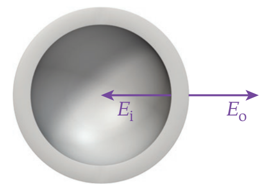
2.51 Electric fields of varying magnitudes are directed either inward or outward at right angles on the faces of a cube, as shown in the figure. What is the strength and direction of the field on the face F?



2.52 Consider a hollow spherical conductor with total charge $+5e$. The outer and inner radii are a and b, respectively. (a) Calculate the charge on the sphere's inner and outer surfaces if a charge of $-3e$ is placed at the center of the sphere. (b) What is the total net charge of the sphere?

•2.53 A spherical aluminized Mylar balloon carries a charge Q on its surface. You are measuring the electric field at a distance R from the balloon's center. The balloon is slowly inflated, and its radius approaches but never reaches R. What happens to the electric field you measure as the balloon increases in radius? Explain.

•2.54 A hollow conducting spherical shell has an inner radius of 8.00 cm and an outer radius of 10.0 cm. The electric field at the inner surface of the shell, E_i, has a magnitude of 80.0 N/C and points toward the center of the sphere, and the electric field at the outer surface, E_o, has a magnitude of 80.0 N/C and points away from the center of the sphere (see the figure). Determine the magnitude of the charge on the inner surface and on the outer surface of the spherical shell.



•2.55 A −6.00 nC point charge is located at the center of a conducting spherical shell. The shell has an inner radius of 2.00 m, an outer radius of 4.00 m, and a charge of +7.00 nC.

a) What is the electric field at $r = 1.00$ m?

b) What is the electric field at $r = 3.00$ m?

c) What is the electric field at $r = 5.00$ m?

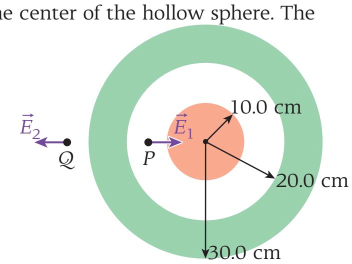
d) What is the surface charge distribution, σ, on the outside surface of the shell?

Section 2.9

2.56 A solid, nonconducting sphere of radius a has total charge Q and a uniform charge distribution. Using Gauss's Law, determine the electric field (as a vector) in the regions $r < a$ and $r > a$ in terms of Q.

2.57 There is an electric field of magnitude 150.0 N/C, directed downward, near the surface of the Earth. What is the net electric charge on the Earth? You can treat the Earth as a spherical conductor of radius 6371 km.

2.58 A hollow metal sphere has inner and outer radii of 20.0 cm and 30.0 cm, respectively. As shown in the figure, a solid metal sphere of radius 10.0 cm is located at the center of the hollow sphere. The electric field at a point P, a distance of 15.0 cm from the center, is found to be $E_1 = 1.00 \times 10^4$ N/C, directed radially inward. At point Q, a distance of 35.0 cm from the center, the electric field is found to be $E_2 = 1.00 \times 10^4$ N/C, directed radially outward. Determine the total charge on (a) the surface of the inner sphere, (b) the inner surface of the hollow sphere, and (c) the outer surface of the hollow sphere.



2.59 Two parallel, infinite, nonconducting plates are 10.0 cm apart and have charge distributions of $+1.00$ μC/m^2 and -1.00 μC/m^2. What is the force on an electron in the space between the plates? What is the force on an electron located outside the two plates near the surface of one of the two plates?

2.60 An infinitely long charged wire produces an electric field of magnitude 1.23×10^3 N/C at a distance of 50.0 cm perpendicular to the wire. The direction of the electric field is toward the wire.

a) What is the charge distribution?

b) How many electrons per unit length are on the wire?

•**2.61** A solid sphere of radius R has a nonuniform charge distribution $\rho = Ar^2$, where A is a constant. Determine the total charge, Q, within the volume of the sphere.

•**2.62** Two parallel, uniformly charged, infinitely long wires are 6.00 cm apart and carry opposite charges with a linear charge density of $\lambda = 1.00$ μC/m. What are the magnitude and the direction of the electric field at a point midway between the two wires and 40.0 cm above the plane containing them?

•**2.63** A sphere centered at the origin has a volume charge distribution of 120. nC/cm^3 and a radius of 12.0 cm. The sphere is centered inside a conducting spherical shell with an inner radius of 30.0 cm and an outer radius of 50.0 cm. The charge on the spherical shell is −2.00 mC. What are the magnitude and the direction of the electric field at each of the following distances from the origin?

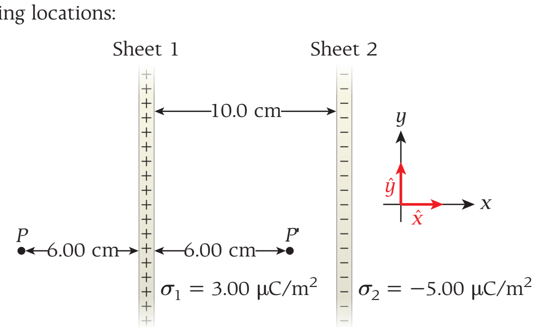
a) at $r = 10.0$ cm c) at $r = 40.0$ cm

b) at $r = 20.0$ cm d) at $r = 80.0$ cm

•**2.64** A thin, hollow, metal cylinder of radius R has a surface charge distribution σ. A long, thin wire with a linear charge density $\lambda/2$ runs through the center of the cylinder. Find an expression for the electric field and determine the direction of the field at each of the following locations:

a) $r \leq R$ b) $r \geq R$

•**2.65** Two infinite sheets of charge are separated by 10.0 cm as shown in the figure. Sheet 1 has a surface charge distribution of $\sigma_1 = 3.00$ μC/m^2 and sheet 2 has a surface charge distribution of $\sigma_2 = -5.00$ μC/m^2. Find the total electric field (magnitude and direction) at each of the following locations:



a) at point P, 6.00 cm to the left of sheet 1

b) at point P', 6.00 cm to the right of sheet 1

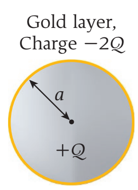
•**2.66** A conducting solid sphere of radius 20.0 cm is located with its center at the origin of a three-dimensional coordinate system. A charge of 0.271 nC is placed on the sphere.

a) What is the magnitude of the electric field at point $(x,y,z) = (23.1$ cm$,1.10$ cm$,0.00$ cm$)$?

b) What is the angle of this electric field with the x-axis at this point?

c) What is the magnitude of the electric field at point $(x,y,z) = (4.10$ cm$,1.10$ cm$,0.00$ cm$)$?

••**2.67** A solid nonconducting sphere of radius a has a total charge $+Q$ uniformly distributed throughout its volume. The surface of the sphere is coated with a very thin (negligible thickness) conducting layer of gold. A total charge of $-2Q$ is placed on this conducting layer. Use Gauss's Law to do the following.



a) Find the electric field $E(r)$ for $r < a$ (inside the sphere, up to and excluding the gold layer).

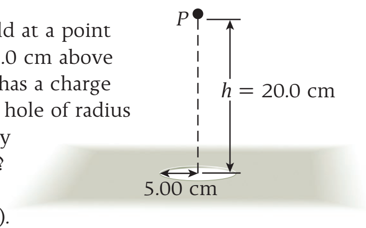
b) Find the electric field $E(r)$ for $r > a$ (outside the coated sphere, beyond the sphere and the gold layer).

c) Sketch the graph of $E(r)$ versus r. Comment on the continuity or discontinuity of the electric field, and relate this to the surface charge distribution on the gold layer.

••**2.68** A solid nonconducting sphere has a volume charge distribution given by $\rho(r) = (\beta/r)\sin(\pi r/2R)$. Find the total charge contained in the spherical volume and the electric field in the regions $r < R$ and $r > R$. Show that the two expressions for the electric field equal each other at $r = R$.

••**2.69** A very long cylindrical rod of nonconducting material with a 3.00-cm radius is given a uniformly distributed positive charge of 6.00 nC per centimeter of its length. Then a cylindrical cavity is drilled all the way through the rod, of radius 1 cm, with its axis located 1.50 cm from the axis of the rod. That is, if, at some cross section of the rod, x- and y-axes are placed so that the center of the rod is at $(x,y) = (0,0)$; then the center of the cylindrical cavity is at $(x,y) = (0,1.50)$. The creation of the cavity does not disturb the charge on the remainder of the rod that has not been drilled away; it just removes the charge from the region in the cavity. Find the electric field at the point $(x,y) = (2.00,1.00)$.

••**2.70** What is the electric field at a point P, which is at a distance $h = 20.0$ cm above an infinite sheet of charge that has a charge distribution of 1.30 C/m^2 and a hole of radius 5.00 cm whose center is directly below P, as shown in the figure? Plot the electric field as a function of h in terms of $\sigma/(2e_0)$.



Additional Exercises

2.71 A cube has an edge length of 1.00 m. An electric field acting on the cube from outside has a constant magnitude of 150 N/C and its direction is also constant but unspecified (not necessarily along any edges of the cube). What is the total charge within the cube?

2.72 A carbon monoxide (CO) molecule has a dipole moment of approximately 8.0×10^{-30} C m. If the carbon and oxygen atoms are separated by 1.2×10^{-10} m, find the net charge on each atom and the maximum amount of torque the molecule would experience in an electric field of 500.0 N/C.

2.73 An infinitely long, solid cylinder of radius $R = 9.00$ cm, with a uniform charge per unit of volume of $\rho = 6.40 \times 10^{-8}$ C/m^3, is centered about the y-axis. Find the magnitude of the electric field at a radius $r = 4.00$ cm from the center of this cylinder.

2.74 Find the magnitudes and the directions of the electric fields needed to counteract the weight of (a) an electron and (b) a proton at the Earth's surface.

2.75 A solid metal sphere of radius 8.00 cm, with a total charge of 10.0 μC, is surrounded by a metallic shell with a radius of 15.0 cm carrying a −5.00 μC charge. The sphere and the shell are both inside a larger metallic shell of inner radius 20.0 cm and outer radius 24.0 cm. The sphere and the two shells are concentric.

a) What is the charge on the inner wall of the larger shell?

b) If the electric field outside the larger shell is zero, what is the charge on the outer wall of the shell?

2.76 Two infinite, uniformly charged, flat, nonconducting surfaces are mutually perpendicular. One of the surfaces has a charge distribution of +30.0 pC/m^2, and the other has a charge distribution of −40.0 pC/m^2. What is the magnitude of the electric field at any point not on either surface?

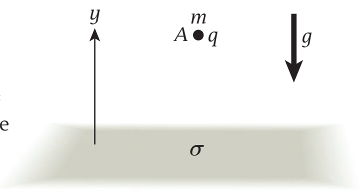
2.77 There is an electric field of magnitude 150. N/C, directed vertically downward, near the surface of the Earth. Find the acceleration (magnitude and direction) of an electron released near the Earth's surface.

2.78 Suppose you have a large spherical balloon and you are able to measure the component E_n of the electric field normal to its surface. If you sum $E_n\,dA$ over the whole surface area of the balloon and obtain a magnitude of 10.0 N m^2/C, what is the electric charge enclosed by the balloon?

2.79 A 30.0 cm-long uniformly charged rod is sealed in a container. The total electric flux leaving the container is 1.46×10^6 N m^2/C. Determine the linear charge distribution on the rod.

•**2.80** A long conducting wire with charge distribution λ and radius r produces an electric field of 2.73 N/C just outside its surface. What is the magnitude of the electric field just outside the surface of another wire with charge distribution 0.810λ and radius $6.50r$?

•**2.81** An object with mass $m = 1.00$ g and charge q is placed at point A, which is 0.0500 m above an infinitely large, uniformly charged, nonconducting sheet ($\sigma = -3.50 \times 10^{-5}$ C/m^2), as shown in the figure. Gravity is acting downward ($g = 9.81$ m/s^2). Determine the number, N, of electrons that must be added to or removed from the object for the object to remain motionless above the charged plane.



•**2.82** A proton enters the gap between a pair of metal plates (an electrostatic separator) that produce a uniform, vertical electric field between them. Ignore the effect of gravity on the proton.

a) Assuming that the length of the plates is 15.0 cm and that the proton approaches the plates with a speed of 15.0 km/s, what electric field strength should the plates be designed to provide so that the proton will be deflected vertically by 1.50×10^{-3} rad?

b) What speed will the proton have after exiting the electric field?

c) Suppose the proton is one in a beam of protons that has been contaminated with positively charged kaons, particles whose mass is 494 MeV/c^2 (8.81×10^{-28} kg), while the mass of the proton is 938 MeV/c^2 (1.67×10^{-27} kg). The kaons have a charge of $+1e$, just like the protons. If the electrostatic separator is designed to give the protons a deflection of 1.20×10^{-3} rad, what deflection will kaons with the same momentum as the protons experience?

•**2.83** Consider a uniform nonconducting sphere with a surface charge density $\rho = 3.57 \times 10^{-6}$ C/m^3 and a radius $R = 1.72$ m. What is the magnitude of the electric field 0.530 m from the center of the sphere?

••**2.84** A uniform sphere has a radius R and a total charge $+Q$, uniformly distributed throughout its volume. It is surrounded by a thick spherical shell carrying a total charge $-Q$, also uniformly distributed, and having an outer radius of $2R$. What is the electric field as a function of R?

••**2.85** If a charge is held in place above a large, flat, grounded, conducting slab, such as a floor, it will experience a downward force toward the floor. In fact, the electric field in the room above the floor will be exactly the same as that produced by the original charge plus a "mirror image" charge, equal in magnitude and opposite in sign, as far below the floor as the original charge is above it. Of course, there is no charge below the floor; the effect is produced by the surface charge distribution induced on the floor by the original charge.

a) Describe or sketch the electric field lines in the room above the floor.

b) If the original charge is 1.00 μC at a distance of 50.0 cm above the floor, calculate the downward force on this charge.

c) Find the electric field at (just above) the floor, as a function of the horizontal distance from the point on the floor directly under the original charge. Assume that the original charge is a point charge, $+q$, at a distance a above the floor. Ignore any effects of walls or ceiling.

d) Find the surface charge distribution $\sigma(\rho)$ induced on the floor.

e) Calculate the total surface charge induced on the floor.

MULTI-VERSION EXERCISES

2.86 A long, horizontal, conducting wire has the charge density $\lambda = 2.849 \times 10^{-12}$ C/m. A proton (mass $= 1.673 \times 10^{-27}$ kg) is placed 0.6815 m above the wire and released. What is the magnitude of the initial acceleration of the proton?

2.87 A long, horizontal, conducting wire has the charge density λ. A proton (mass $= 1.673 \times 10^{-27}$ kg) is placed 0.6897 m above the wire and released. The magnitude of the initial acceleration of the proton is 1.111×10^7 m/s^2. What is the charge density on the wire?

2.88 A long, horizontal, conducting wire has the charge density $\lambda = 6.055 \times 10^{-12}$ C/m. A proton (mass $= 1.673 \times 10^{-27}$ kg) is placed a distance d above the wire and released. The magnitude of the initial acceleration of the proton is 1.494×10^7 m/s^2. What is the distance d?

2.89 There is a uniform charge distribution of $\lambda = 5.635 \times 10^{-8}$ C/m along a thin wire of length $L = 2.13$ cm. The wire is then curved into a semicircle that is centered at the origin and has a radius of $R = L/\pi$. Find the magnitude of the electric field at the center of the semicircle.

2.90 There is a uniform charge distribution λ along a thin wire of length $L = 10.55$ cm. The wire is then curved into a semicircle that is centered at the origin and has a radius of $R = L/\pi$. The magnitude of the electric field at the center of the semicircle is 3.117×10^4 N/C. What is the value of λ?

2.91 There is a uniform charge distribution of $\lambda = 6.005 \times 10^{-8}$ C/m along a thin wire of length L. The wire is then curved into a semicircle that is centered at the origin and has a radius of $R = L/\pi$. The magnitude of the electric field at the center of the semicircle is 2.425×10^4 N/C. What is the value of L?

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