3-1 Exponential Functions

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.
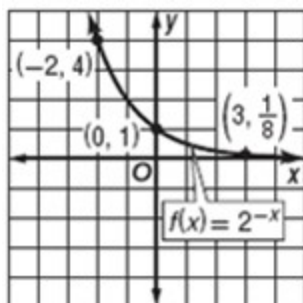
1. $f(x) = 2^{-x}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−4	−2	−1	0	1	2	4
y	16	4	2	1	0.5	0.25	0.625

Then use a smooth curve to connect each of these ordered pairs.

List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: $(0, 1)$; asymptote: x-axis; $\lim\limits_{x \to -\infty} f(x) = \infty, \lim\limits_{x \to \infty} f(x) = 0$; decreasing on $(-\infty, \infty)$.

2. $r(x) = 5^x$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
y	0.008	0.04	0.2	1	5	25	125

Then use a smooth curve to connect each of these ordered pairs.
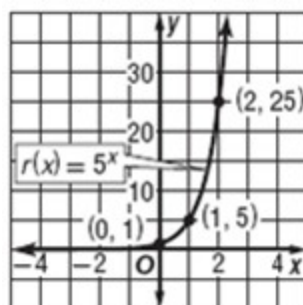
List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: $(0, 1)$; asymptote: x-axis; $\lim\limits_{x \to -\infty} r(x) = 0, \lim\limits_{x \to \infty} r(x) = \infty$; increasing on $(-\infty, \infty)$

3-1 Exponential Functions

3. $h(x) = 0.2^{x + 2}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−4	−3	−2	0	1	2
y	25	5	1	0.04	0.008	0.0016

Then use a smooth curve to connect each of these ordered pairs.

List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: $(0, 0.04)$; asymptote: x-axis; $\lim\limits_{x \to -\infty} h(x) = \infty, \lim\limits_{x \to \infty} h(x) = 0$; decreasing on $(-\infty, \infty)$

4. $k(x) = 6^x$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2
y	0.005	0.028	0.167	1	6	36

Then use a smooth curve to connect each of these ordered pairs.
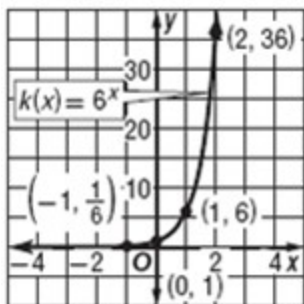
List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: $(0, 1)$; asymptote: x-axis; $\lim\limits_{x \to -\infty} k(x) = 0, \lim\limits_{x \to \infty} k(x) = \infty$; increasing on $(-\infty, \infty)$

3-1 Exponential Functions

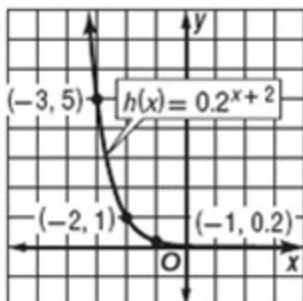5. $m(x) = -(0.25)^x$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−4	−3	−2	−1	0	1	2
y	−256	−64	−16	−4	−1	−0.25	−0.625

Then use a smooth curve to connect each of these ordered pairs.

List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.
D = $(-\infty, \infty)$; R = $(-\infty, 0)$; intercept: $(0, -1)$; asymptote: x-axis; $\lim\limits_{x \to -\infty} m(x) = -\infty, \lim\limits_{x \to \infty} m(x) = 0$; increasing on $(-\infty, \infty)$

6. $p(x) = 0.1^{-x}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
y	0.001	0.01	0.1	1	10	100	1000

Then use a smooth curve to connect each of these ordered pairs.
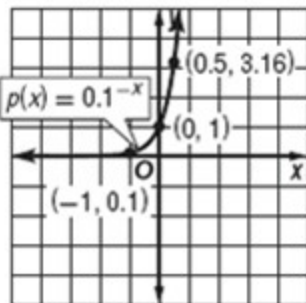
List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.
D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: $(0, 1)$; asymptote: x-axis; $\lim\limits_{x \to -\infty} p(x) = 0, \lim\limits_{x \to \infty} p(x) = \infty$; increasing for $(-\infty, \infty)$
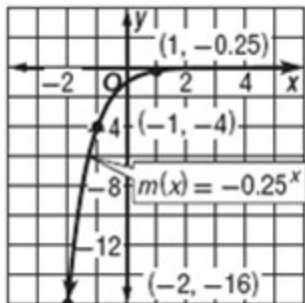
3-1 Exponential Functions

7. $q(x) = \left(\dfrac{1}{6}\right)^{x}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2
y	216	36	6	1	0.167	0.278

Then use a smooth curve to connect each of these ordered pairs.

List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

$D = (-\infty, \infty)$; $R = (0, \infty)$; intercept: (0, 1); asymptote: x-axis; $\lim\limits_{x \to -\infty} q(x) = \infty, \lim\limits_{x \to \infty} q(x) = 0$; decreasing on $(-\infty, \infty)$
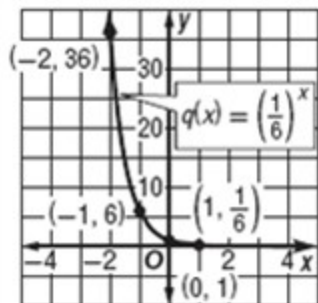
8. $g(x) = \left(\dfrac{1}{3}\right)^{x}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2
y	27	9	3	1	0.333	0.111

Then use a smooth curve to connect each of these ordered pairs.

List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

$D = (-\infty, \infty)$; $R = (0, \infty)$; intercept: (0, 1); asymptote: x-axis; $\lim\limits_{x \to -\infty} g(x) = \infty, \lim\limits_{x \to \infty} g(x) = 0$; decreasing on $(-\infty, \infty)$
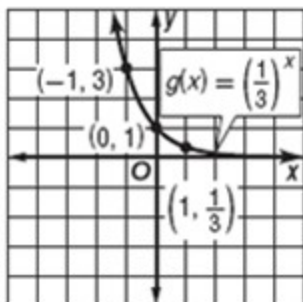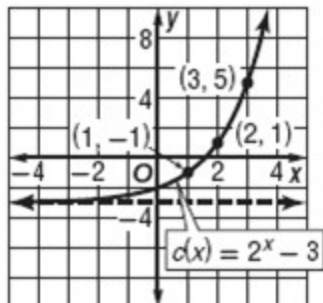
3-1 Exponential Functions

9. $c(x) = 2^x - 3$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−2	−1	0	1	2	3
y	−2.75	−2.5	−2	−1	1	5

Then use a smooth curve to connect each of these ordered pairs.

D = $(-\infty, \infty)$; R = $(-3, \infty)$; intercept: $(0, -2)$; asymptote: $y = -3$; $\lim\limits_{x \to -\infty} c(x) = -3$, $\lim\limits_{x \to \infty} c(x) = \infty$; increasing on $(-\infty, \infty)$

10. $d(x) = 5^{-x} + 2$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2
y	127	27	7	3	2.2	2.04

Then use a smooth curve to connect each of these ordered pairs.
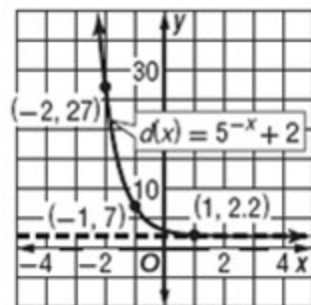
D = $(-\infty, \infty)$; R = $(2, \infty)$; intercept: $(0, 3)$; asymptote: $y = 2$; $\lim\limits_{x \to -\infty} d(x) = \infty$, $\lim\limits_{x \to \infty} d(x) = 2$; decreasing for $(-\infty, \infty)$

3-1 Exponential Functions

Use the graph of $f(x)$ to describe the transformation that results in the graph of $g(x)$. Then sketch the graphs of $f(x)$ and $g(x)$.

11. $f(x) = 4^x$; $g(x) = 4^x - 3$

SOLUTION:

This function is of the form $f(x) = 4^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ translated 3 units down. This is indicated by the subtraction of 3.

Use the graphs of the functions to confirm this transformation.
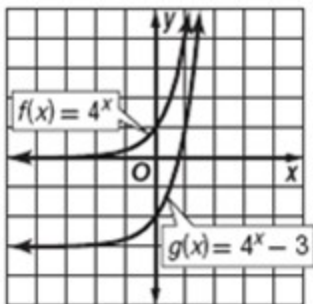
12. $f(x) = \left(\dfrac{1}{2}\right)^x$; $g(x) = \left(\dfrac{1}{2}\right)^{x+4}$

SOLUTION:

This function is of the form $f(x) = \left(\dfrac{1}{2}\right)^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ translated 4 units to the left. This is indicated by the $x + 4$ in the exponent.

Use the graphs of the functions to confirm this transformation.
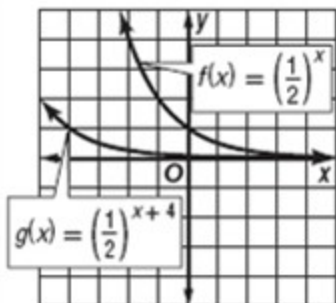
3-1 Exponential Functions

13. $f(x) = 3^x$; $g(x) = -2(3^x)$

   SOLUTION:

   This function is of the form $f(x) = 3^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ reflected across the x-axis and expanded vertically by a factor of 2. These are indicated by the negative sign and the 2 in front of the 3^x.

   Use the graphs of the functions to confirm this transformation.

14. $f(x) = 2^x$; $g(x) = 2^{x-2} + 5$

   SOLUTION:

   This function is of the form $f(x) = 2^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ translated 2 units to the right and 5 units up. These are indicated by the $x - 2$ in the exponent and the +5 at the end of the function.

   Use the graphs of the functions to confirm this transformation.

3-1 Exponential Functions

15. $f(x) = 10^x$; $g(x) = 10^{-x+3}$

SOLUTION:

This function is of the form $f(x) = 10^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ reflected in the y-axis and translated 3 units to the right. These are indicated by the $-x + 3$ in the exponent.

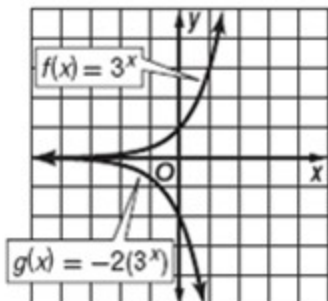Use the graphs of the functions to confirm this transformation.

16. $f(x) = e^x$; $g(x) = e^{2x}$

SOLUTION:

This function is of the form $f(x) = e^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ compressed horizontally by a factor of 2. This is indicated by the $2x$ in the exponent.

Use the graphs of the functions to confirm this transformation.

3-1 Exponential Functions

17. $f(x) = e^x$; $g(x) = e^{x+2} - 1$

    SOLUTION:

    This function is of the form $f(x) = e^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ translated 2 units to the left and 1 unit down. These are indicated by the $x + 2$ in the exponent and the subtraction of 1.

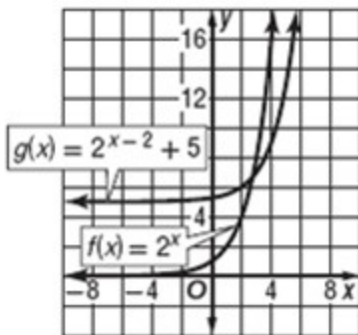   Use the graphs of the functions to confirm this transformation.

18. $f(x) = e^x$; $g(x) = e^{-x+1}$

    SOLUTION:

    This function is of the form $f(x) = e^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ reflected in the y-axis and translated 1 unit to the right. These are indicated by the $-x + 1$ in the exponent.

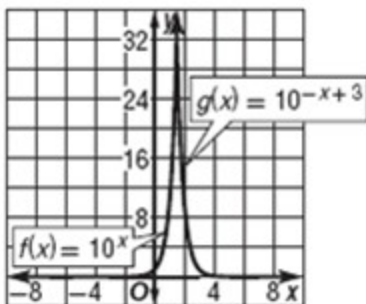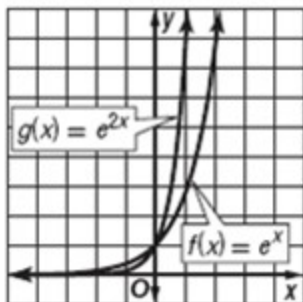   Use the graphs of the functions to confirm this transformation.

3-1 Exponential Functions

19. $f(x) = e^x$; $g(x) = 3e^x$

   SOLUTION:

   This function is of the form $f(x) = e^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ expanded vertically by a factor of 3. This is indicated by the coefficient of 3.

   Use the graphs of the functions to confirm this transformation.
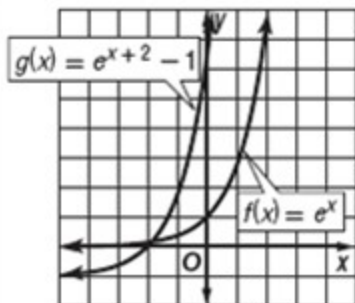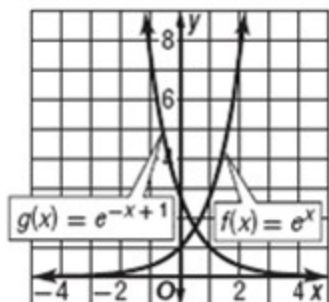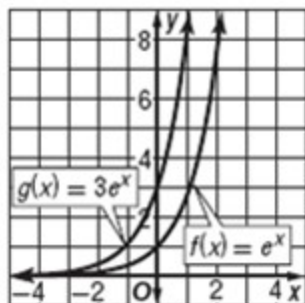

20. $f(x) = e^x$; $g(x) = -(e^x) + 4$

   SOLUTION:

   This function is of the form $f(x) = e^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ reflected across the x-axis and translated 4 units up. These are indicated by the negative coefficient and the +4 at the end of the function.

   Use the graphs of the functions to confirm this transformation.
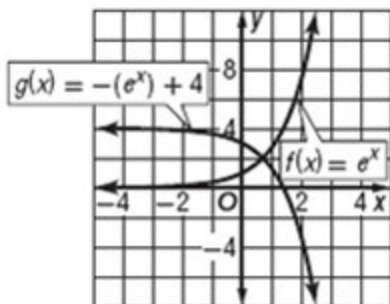
3-1 Exponential Functions

FINANCIAL LITERACY Copy and complete the table below to find the value of an investment A for the given principal P, rate r, and time t if the interest is compounded n times annually.

n	1	4	12	365	continuously
A					

21. $P = \$500$, $r = 3\%$, $t = 5$ years

SOLUTION:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 500\left(1 + \frac{0.03}{1}\right)^{5}$$

$$\approx \$579.64$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 500\left(1 + \frac{0.03}{4}\right)^{4(5)}$$

$$\approx \$580.59$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 500\left(1 + \frac{0.03}{12}\right)^{12(5)}$$

$$\approx \$580.81$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 500\left(1 + \frac{0.03}{365}\right)^{365(5)}$$

$$\approx \$580.91$$

$$A = Pe^{rt}$$

$$= 500e^{0.03(5)}$$

$$\approx \$580.92$$

n	1	4	12	365	continuously
A	\$579.64	\$580.59	\$580.81	\$580.91	\$580.92

3-1 Exponential Functions

22. $P = \$1000$, $r = 4.5\%$, $t = 10$ years

SOLUTION:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.045}{1}\right)^{1(10)}$$

$$\approx \$1552.97$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.045}{4}\right)^{4(10)}$$

$$\approx \$1564.38$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.045}{12}\right)^{12(10)}$$

$$\approx \$1566.99$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.045}{365}\right)^{365(10)}$$

$$\approx \$1568.27$$

$$A = Pe^{rt}$$

$$= 1000e^{0.045(10)}$$

$$\approx \$1568.31$$

n	1	4	12	365	continuously
A	$1552.97	$1564.38	$1566.99	$1568.27	$1568.31

3-1 Exponential Functions

23. $P = \$1000$, $r = 5\%$, $t = 20$ years

SOLUTION:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.05}{1}\right)^{1(20)}$$

$$\approx \$2653.30$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.05}{4}\right)^{4(20)}$$

$$\approx \$2701.48$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.05}{12}\right)^{12(20)}$$

$$\approx \$2712.64$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.05}{365}\right)^{365(20)}$$

$$\approx \$2718.10$$

$$A = Pe^{rt}$$

$$= 1000e^{0.05(20)}$$

$$\approx \$2718.28$$

n	1	4	12	365	continuously
A	$2653.30	$2701.48	$2712.64	$2718.10	$2718.28

3-1 Exponential Functions

24. $P = \$5000$, $r = 6\%$, $t = 30$ years

SOLUTION:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 5000\left(1 + \frac{0.06}{1}\right)^{1(30)}$$

$$\approx \$28,717.46$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 5000\left(1 + \frac{0.06}{4}\right)^{4(30)}$$

$$\approx \$29,846.61$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 5000\left(1 + \frac{0.06}{12}\right)^{12(30)}$$

$$\approx \$30,112.88$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 5000\left(1 + \frac{0.06}{365}\right)^{365(30)}$$

$$\approx \$30,243.76$$

$$A = Pe^{rt}$$

$$= 5000e^{0.06(30)}$$

$$\approx \$30,248.24$$

n	1	4	12	365	continuously
A	$28,717.46	$29,846.61	$30,112.88	$30,243.76	$30,248.24

3-1 Exponential Functions

25. **FINANCIAL LITERACY** Brady acquired an inheritance of $20,000 at age 8, but he will not have access to it until he turns 18.

a. If Brady's inheritance is placed in a savings account earning 4.6% interest compounded monthly, how much will his inheritance be worth on his 18th birthday?

b. How much will Brady's inheritance be worth if it is placed in an account earning 4.2% interest compounded continuously?

SOLUTION:

a. The inheritance is $20,000. The interest rate r is 4.6% or 0.046. The interest is compounded monthly, so $n = 12$. There are 10 years from when Brady is 8 to when he is 18. Therefore, $t = 10$. Use the compound interest formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 20,000\left(1 + \frac{0.046}{12}\right)^{12(10)}$$

$$\approx \$31,653.63$$

Brady will have $31,653.63 when he turns 18.

b. Use the continuous compound interest formula. The interest rate r is 4.6% or 0.046. The time t is 10.

$$A = Pe^{rt}$$

$$= 20,000e^{0.042(10)}$$

$$\approx \$30,439.23$$

Brady will have $30,439.23 when he turns 18.

3-1 Exponential Functions

26. **FINANCIAL LITERACY** Katrina invests $1200 in a certificate of deposit (CD). The table shows the interest rates offered by the bank on 3- and 5-year CDs.

CD Offers		
Years	3	5
Interest	3.45%	4.75%
Compounded	continuously	monthly

a. How much would her investment be worth with each option?

b. How much would her investment be worth if the 5-year CD was compounded continuously?

SOLUTION:

a. The investment P is $1200. For the 3-year CD, $t = 3$, and the interest is compounded continuously at $r = 3.45\%$.

$$A = Pe^{rt}$$
$$= 1200e^{0.0345(3)}$$
$$\approx \$1330.85$$

The 3-year CD will be worth $1330.85.

For the 5-year CD, $t = 5$, $r = 4.75\%$, and $n = 12$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 1200\left(1 + \frac{0.0475}{12}\right)^{12(5)}$$
$$\approx \$1520.98$$

The 5-year CD will be worth $1520.98.

b.

$$A = Pe^{rt}$$
$$= 1200e^{0.0475(5)}$$
$$\approx \$1521.69$$

If the 5-year CD was compounded continuously, the investment would be worth $1521.69.

3-1 Exponential Functions

POPULATION Copy and complete the table to find the population N of an endangered species after a time t given its initial population N_0 and annual rate r or continuous rate k of increase or decline.

t	5	10	15	20	50
N					

27. $N_0 = 15{,}831$, $r = -4.2\%$

SOLUTION:

The initial amount N_0 is 15,831 and the annual rate of decline is $r = -4.2\%$. Use the exponential decay formula.

$$N = N_0(1+r)^t$$
$$= 15{,}831(1-0.042)^5$$
$$\approx 12{,}774$$
$$N = N_0(1+r)^t$$
$$= 15{,}831(1-0.042)^{10}$$
$$\approx 10{,}308$$
$$N = N_0(1+r)^t$$
$$= 15{,}831(1-0.042)^{15}$$
$$\approx 8317$$
$$N = N_0(1+r)^t$$
$$= 15{,}831(1-0.042)^{20}$$
$$\approx 6711$$
$$N = N_0(1+r)^t$$
$$= 15{,}831(1-0.042)^{50}$$
$$\approx 1853$$

t	5	10	15	20	50
N	12,774	10,308	8317	6711	1853

3-1 Exponential Functions

28. $N_0 = 23{,}112$, $r = 0.8\%$

SOLUTION:

The initial amount N_0 is 23,112 and the annual rate of increase is $r = 0.8\%$. Use the exponential growth formula.

$$N = N_0(1+r)^t$$
$$\quad = 23{,}112(1+0.008)^5$$
$$\quad \approx 24{,}051$$
$$N = N_0(1+r)^t$$
$$\quad = 23{,}112(1+0.008)^{10}$$
$$\quad \approx 25{,}029$$
$$N = N_0(1+r)^t$$
$$\quad = 23{,}112(1+0.008)^{15}$$
$$\quad \approx 26{,}046$$
$$N = N_0(1+r)^t$$
$$\quad = 23{,}112(1+0.008)^{20}$$
$$\quad \approx 27{,}105$$
$$N = N_0(1+r)^t$$
$$\quad = 23{,}112(1+0.008)^{50}$$
$$\quad \approx 34{,}424$$

t	5	10	15	20	50
N	24,051	25,029	26,046	27,105	34,424

3-1 Exponential Functions

29. $N_0 = 17,692$, $k = 2.02\%$

   SOLUTION:

   The initial amount N_0 is 17,692 and the continuous rate of increase is $k = 2.02\%$. Use the continuous exponential growth formula.

   $$N = N_0 e^{kt}$$
   $$= 17,692 e^{0.0202(5)}$$
   $$\approx 19,572$$
   $$N = N_0 e^{kt}$$
   $$= 17,692 e^{0.0202(10)}$$
   $$\approx 21,652$$
   $$N = N_0 e^{kt}$$
   $$= 17,692 e^{0.0202(15)}$$
   $$\approx 23,953$$
   $$N = N_0 e^{kt}$$
   $$= 17,692 e^{0.0202(20)}$$
   $$\approx 26,499$$
   $$N = N_0 e^{kt}$$
   $$= 17,692 e^{0.0202(50)}$$
   $$\approx 48,575$$

t	5	10	15	20	50
N	19,572	21,652	23,953	26,499	48,575

3-1 Exponential Functions

30. $N_0 = 9689, k = -3.7\%$

   SOLUTION:

   The initial amount N_0 is 9689 and the continuous rate of decline is $k = -3.7\%$. Use the continuous exponential growth formula.

   $$N = N_0 e^{kt}$$
   $$= 9689 e^{-0.037(5)}$$
   $$\approx 8053$$
   $$N = N_0 e^{kt}$$
   $$= 9689 e^{-0.037(10)}$$
   $$\approx 6693$$
   $$N = N_0 e^{kt}$$
   $$= 9689 e^{-0.037(15)}$$
   $$\approx 5562$$
   $$N = N_0 e^{kt}$$
   $$= 9689 e^{-0.037(20)}$$
   $$\approx 4623$$
   $$N = N_0 e^{kt}$$
   $$= 9689 e^{-0.037(50)}$$
   $$\approx 1523$$

t	5	10	15	20	50
N	8053	6693	5562	4623	1523

3-1 Exponential Functions

31. **WATER** Worldwide water usage in 1950 was about 294.2 million gallons. If water usage has grown at the described rate, estimate the amount of water used in 2000, and predict the amount in 2050.
    a. 3% annually
    b. 3.05% continuously

SOLUTION:

a. Use the exponential growth formula. The initial amount N_0 was 294.2 million. The rate of increase r is 3%. $t = 2000 - 1950$ or 50 years.

$$N = N_0(1+r)^t$$
$$= 294.2(1+0.03)^{50}$$
$$\approx 1289.75$$

There were about 1289.75 million or 1.29 billion gallons in 2000.

$t = 2050 - 1950$ or 100 years

$$N = N_0(1+r)^t$$
$$= 294.2(1+0.03)^{100}$$
$$\approx 5654.12$$

There will be about 5654.12 million or 5.65 billion gallons in 2050.

b. Use the continuous exponential growth formula. The initial amount N_0 was 294.2 million. The rate of increase r is 3.05%. $t = 2000 - 1950$ or 50 years.

$$N = N_0 e^{kt}$$
$$= 294.2 e^{0.0305(50)}$$
$$\approx 1351.89$$

There were about 1351.89 million or 1.35 billion gallons in 2000.

$t = 2050 - 1950$ or 100 years

$$N = N_0 e^{kt}$$
$$= 294.2 e^{0.0305(100)}$$
$$\approx 6212.13$$

There will be about 6212.13 million or 6.21 billion gallons in 2050.

32. **WAGES** Jasmine receives a 3.5% raise at the end of each year from her employer to account for inflation. When she started working for the company in 1994, she was earning a salary of $31,000.
    a. What was Jasmine's salary in 2000 and 2004?
    b. If Jasmine continues to receive a raise at the end of each year, how much money will she earn during her final year if she plans on retiring in 2024?

SOLUTION:

a. Use the exponential growth formula. The growth rate r is 3.5% and the initial amount N_0 is $31,000.

$t = 2000 - 1994$ or 6

$$N = N_0(1 + r)^t$$
$$= 31,000(1 + 0.035)^6$$
$$\approx \$38,107$$

$t = 2004 - 1994$ or 10

$$N = N_0(1 + r)^t$$
$$= 31,000(1 + 0.035)^{10}$$
$$\approx \$43,729$$

b. $t = 2024 - 1994$ or 30

$$N = N_0(1 + r)^t$$
$$= 31,000(1 + 0.035)^{30}$$
$$\approx \$87,011$$

3-1 Exponential Functions

33. **PEST CONTROL** Consider the termite guarantee made by Exterm-inc in their ad below.

If the first statement in this claim is true, assess the validity of the second statement. Explain your reasoning.

SOLUTION:

Use the exponential decay formula. The decay rate is −60% and t is 3 treatments.

$$N = N_0(1+r)^t$$
$$= N_0(1-0.60)^3$$
$$= 0.064N_0$$

After 3 treatments, 6.4% of the original amount still remains, so the statement is false.

$$N = N_0(1+r)^t$$
$$= N_0(1-0.60)^4$$
$$= 0.0256N_0$$

$$N = N_0(1+r)^t$$
$$= N_0(1-0.60)^5$$
$$= 0.01024N_0$$

After 5 treatments, there will only be 1% of the colony remaining.

3-1 Exponential Functions

34. **INFLATION** The Consumer Price Index (CPI) is an index number that measures the average price of consumer goods and services. A change in the CPI indicates the growth rate of inflation. In 1958 the CPI was 28.6, and in 2008 the CPI was 211.08.

   a. Determine the growth rate of inflation between 1958 and 2008. Use this rate to write an exponential equation to model this situation.

   b. What will be the CPI in 2015? At this rate, when will the CPI exceed 350?

   SOLUTION:

   a. Use the exponential growth formula. t is $2008 - 1958$ or 50 years. The initial amount N_0 is 28.6 and the final amount N is 211.08. Solve for r.

   $$N = N_0(1+r)^t$$
   $$211.08 = 28.6(1+r)^{50}$$
   $$\frac{211.08}{28.6} = (1+r)^{50}$$
   $$\sqrt[50]{\frac{211.08}{28.6}} = 1+r$$
   $$\sqrt[50]{\frac{211.08}{28.6}} - 1 = r$$
   $$0.04079 \approx r$$

   The annual rate of growth r is about 4.079%. The equation for $N(t)$ is $N(t) = 28.6(1.04079)^t$.

   b. In 2015, $t = 2015 - 1958$ or 57 years. Use the exponential growth formula.

   $$N = N_0(1+r)^t$$
   $$N = 28.6(1.04079)^{57}$$
   $$\approx 279.3$$

   To find when the CPI will exceed 350, graph $y_1 = 28.6(1.04079)^x$ and $y_2 = 350$ and locate the intersection of the two graphs.

   Intersection
   X=62.64442   Y=350
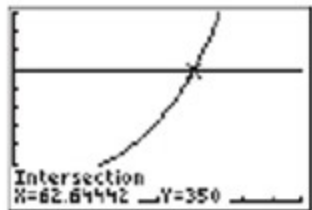
   [0, 100] scl: 10 by [0, 500] scl: 50

   $1958 + 62.6 = 2020.6$. The CPI will exceed 350 sometime during 2020-2021.

35. **GASOLINE** Jordan wrote an exponential equation to model the cost of gasoline. He found the average cost per gallon of gasoline for two years and used these data for his model.

Average Cost per Gallon of Gasoline	
Year	Cost($)
1990	1.19
2007	3.86

   a. If the average cost of gasoline increased at an exponential rate, identify the rate of increase. Write an exponential equation to model this situation.

3-1 Exponential Functions

b. Use your model to predict the average cost of a gallon of gasoline in 2011 and 2013.

c. When will the average cost per gallon of gasoline exceed $7?

d. Why might an exponential model not be an accurate representation of average gasoline prices?

SOLUTION:

a. Use the exponential growth formula. The initial amount $N_0 = 1.19$, the final amount $N = 3.86$, and $t = 2007 - 1990$ or 17. Solve for r.

$$N = N_0(1+r)^t$$
$$3.86 = 1.19(1+r)^{17}$$
$$\frac{3.86}{1.19} = (1+r)^{17}$$
$$\sqrt[17]{\frac{3.86}{1.19}} = 1+r$$
$$\sqrt[17]{\frac{3.86}{1.19}} - 1 = r$$
$$0.0717 \approx r$$

The annual rate of growth r is about 7.17%. The equation for $N(t)$ is $N(t) = 1.19(1.0717)^t$.

b. In 2011, $t = 2011 - 1990$ or 21 years. Use the exponential growth formula.

$$N = N_0(1+r)^t$$
$$= 1.19(1.0717)^{21}$$
$$\approx \$5.09$$

In 2013, $t = 2013 - 1990$ or 23 years. Use the exponential growth formula.

$$N = N_0(1+r)^t$$
$$= 1.19(1.0717)^{23}$$
$$\approx \$5.85$$

According to the model, the average cost of gasoline will be about $5.09 in 2011 and $5.85 in 2013.

c. To find when the average cost per gallon will exceed $7, graph $y_1 = 1.19(1.0717)^x$ and $y_2 = 7$ and locate the intersection of the two graphs.
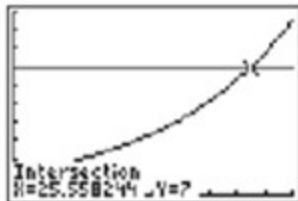
[0, 30] scl: 3 by [0, 10] scl: 1

$1990 + 25.5 = 2015.5$. The average price will exceed $7 sometime during 2015-2016.

d. Sample answer: The cost of gasoline usually does not increase or decrease at a constant rate. It fluctuates in response to many factors such as the economy, the availability of oil, etc.

3-1 Exponential Functions

36. **PHYSICS** The pressure of the atmosphere in pounds per square inch (psi) at sea level is 15.191 and decreases continuously at a rate of 0.004% as altitude increases by x feet.
    a. Write a modeling function for the continuous exponential decay representing the atmospheric pressure $a(x)$.
    b. Use the model to approximate the atmospheric pressure at the top of Mount Everest.

Highest point above sea level—peak of Mount Everest

29,035 ft

Sea Level: 15.191 psi

c. If a certain rescue helicopter can fly only in atmospheric pressures greater than 5.5 pounds per square inch, how high can it fly up Mount Everest?
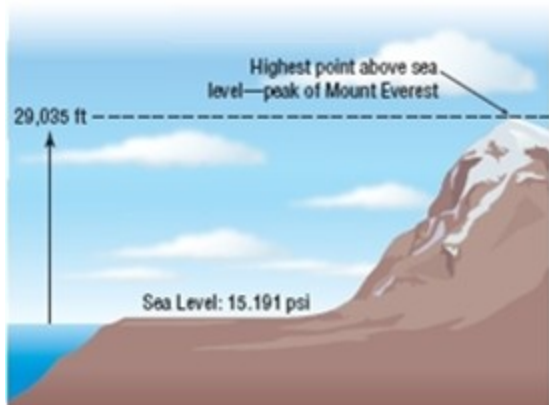
SOLUTION:

a. Use the continuous exponential growth formula.

$$N = N_0 e^{kt}$$

$$a(x) = 15.191 e^{-0.00004x}$$

b. The height x is 29,035 feet.

$$a(x) = 15.191 e^{-0.00004x}$$

$$= 15.191 e^{-0.00004(29,035)}$$

$$\approx 4.76$$

The pressure of the atmosphere is about 4.76 psi.

c. To find the height at which the atmospheric pressure will be 5.5 psi, graph $y_1 = 15.191 e^{-0.00004x}$ and $y_2 = 5.5$ and locate the intersection of the two graphs.

Intersection
X=25398.876 Y=5.5
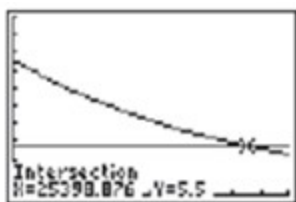
[0, 30,000] scl: 3000 by [0, 20] scl: 2

The graphs intersect at 25,398.876, so the helicopter can fly up to about 25,399 ft.

3-1 Exponential Functions

37. **RADIOACTIVITY** The half-life of a radioactive substance is the amount of time it takes for half of the atoms of the substance to disintegrate. Uranium-235 is used to fuel commercial power plants and has a half-life of 704 million years.

a. How many grams of uranium will remain after 1 million years if you start with 200 grams?

b. How many grams of uranium will remain after 4540 million years if you start with 200 grams?

SOLUTION:

a. Use the continuous exponential decay formula. If N_0 is the initial value, then the final value N should equal $0.5N_0$. The value of t is 704 million years.

$$N = N_0 e^{kt}$$
$$0.5N_0 = N_0 e^{704k}$$
$$0.5 = e^{704k}$$

Solve for the value of k by graphing $y_1 = e^{704x}$ and $y_2 = 0.5$ and locating the intersection.
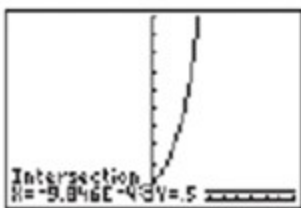
[−0.01, 0.01] scl: 0.001 by [0, 10] scl: 1

The value of k is about −0.0009846.

Use $N_0 = 200$ and $t = 1$ to determine how much is left after 1 million years.

$$N = N_0 e^{kt}$$
$$= 200e^{-0.0009846(1)}$$
$$\approx 199.8$$

b. After 4540 million years, use $t = 4540$.

$$N = N_0 e^{kt}$$
$$= 200e^{-0.0009846(4540)}$$
$$\approx 2.29$$

38. **BOTANY** Under the right growing conditions, a particular species of plant has a doubling time of 12 days. Suppose a pasture contains 46 plants of this species. How many plants will there be after 20, 65, and x days?

SOLUTION:

Use the continuous exponential growth formula. If N_0 is the initial value, then the final value N should equal $2N_0$.

The value of t is 12 days.

$$N = N_0 e^{kt}$$
$$2N_0 = N_0 e^{12k}$$
$$2 = e^{12k}$$

Solve for the value of k by graphing $y_1 = e^{12x}$ and $y_2 = 2$ and locating the intersection.
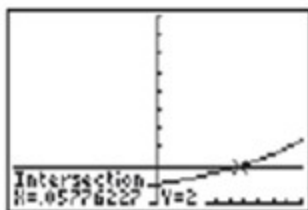
[−0.1, 0.1] scl: 0.01 by [0, 10] scl: 1

The value of k is about 0.05776227.

$t = 20$:
$$N = N_0 e^{kt}$$
$$= 46 e^{0.05776227(20)}$$
$$\approx 146$$

$t = 65$:
$$N = N_0 e^{kt}$$
$$= 46 e^{0.05776227(65)}$$
$$\approx 1965$$

$t = x$:
$$N = N_0 e^{kt}$$
$$= 46 e^{0.05776227x}$$

3-1 Exponential Functions

39. **RADIOACTIVITY** Radiocarbon dating uses carbon-14 to estimate the age of organic materials found commonly at archaeological sites. The half-life of carbon-14 is approximately 5.73 thousand years.
    a. Write a modeling equation for the exponential decay.
    b. How many grams of carbon-14 will remain after 12.82 thousand years if you start with 7 grams?
    c. Use your model to estimate when only 1 gram of the original 7 grams of carbon-14 will remain.

SOLUTION:

a. Use the continuous exponential decay formula. If N_0 is the initial value, then the final value N should equal $0.5N_0$. The value of t is 5.73 thousand years.

$$N = N_0 e^{kt}$$
$$0.5N_0 = N_0 e^{5.73k}$$
$$0.5 = e^{5.73k}$$

Solve for the value of k by graphing $y_1 = e^{5.73x}$ and $y_2 = 0.5$ and locating the intersection.
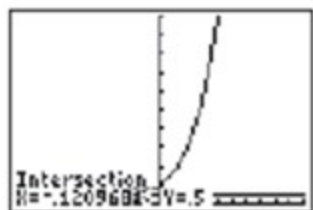
[−1, 1] scl: 0.1 by [0, 10] scl: 1

The value of k is about −0.1209681.

$$N = N_0 e^{kt}$$
$$N(t) = N_0 e^{-0.1209681t}$$

b.
Use $N_0 = 7$ and $t = 12.82$.

$$N(t) = N_0 e^{-0.1209681t}$$
$$= 7e^{-0.1209681(12.82)}$$
$$\approx 1.48$$

c. Graph $y_1 = 7e^{-0.1209681t}$ and $y_2 = 1$ and locate the intersection.
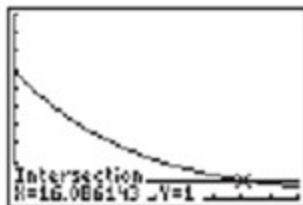
[0, 20] scl: 2 by [0, 10] scl: 1

There will be 1 gram remaining in about 16.09 thousand years

3-1 Exponential Functions

40. **MICROBIOLOGY** A certain bacterium used to treat oil spills has a doubling time of 15 minutes. Suppose a colony begins with a population of one bacterium.

a. Write a modeling equation for this exponential growth.

b. About how many bacteria will be present after 55 minutes?

c. A population of 8192 bacteria is sufficient to clean a small oil spill. Use your model to predict how long it will take for the colony to grow to this size.

SOLUTION:

a. Use the continuous exponential growth formula. If N_0 is the initial value, then the final value N should equal $2N_0$.
The value of t is 15 minutes.

$$N = N_0 e^{kt}$$
$$2N_0 = N_0 e^{15k}$$
$$2 = e^{15k}$$

Solve for the value of k by graphing $y_1 = e^{15x}$ and $y_2 = 2$ and locating the intersection.
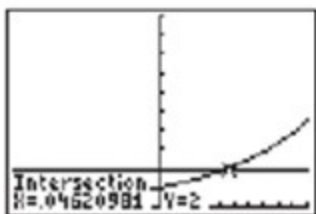
[−0.1, 0.1] scl: 0.01 by [0, 10] scl:1

The value of k is about 0.04620981.

$$N(t) = N_0 e^{0.04620981 t}$$

b.

$$N(t) = N_0 e^{0.04620981 t}$$
$$= 1 e^{0.04620981(55)}$$
$$\approx 13$$

c. Graph $y_1 = e^{0.04620981x}$ and $y_2 = 8192$ and locate the intersection.
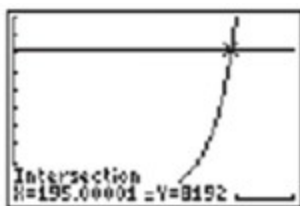
[0, 20] scl: 2 by [0, 10,000] scl: 1000

about 195 minutes

3-1 Exponential Functions

41. **ENCYCLOPEDIA** The number of articles making up an online open-content encyclopedia increased exponentially during its first few years. The number of articles, $A(t)$, t years after 2001 can be modeled by $A(t) = 16198 \cdot 2.13^t$.
   a. According to this model, how many articles made up the encyclopedia in 2001? At what percentage rate is the number of articles increasing?
   b. During which year did the encyclopedia reach 1 million articles?
   c. Predict the number of articles there will be at the beginning of 2012.

SOLUTION:

a. In 2001, $t = 0$ since t is the number of years after 2001. $2.13^0 = 1$, so there are 16,198 articles in 2001. The formula for exponential growth is $N = N_0(1 + r)^t$ so 2.13 must equal $1 + r$. The rate r is 1.13 or 113%.

b. Graph $y_1 = 16,198 \cdot 2.13^x$ and $y_2 = 1,000,000$ and locate the intersection of the two graphs.
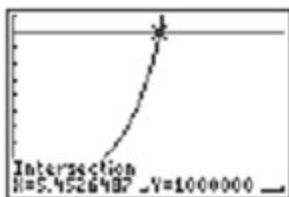
[0, 10] scl: 1 by [0, 1,100,000] scl: 100,000

$2001 + 5.45 = 2006.45$, so the encyclopedia will reach 1 million articles during 2006.

c. $t = 2012 - 2001$ or 11.
$$A(t) = 16,198 \cdot 2.13^t$$
$$A(11) = 16,198 \cdot 2.13^{11}$$
$$\approx 66,318,857$$
There will be about 66,318,857 articles at the beginning of 2012.

3-1 Exponential Functions

42. **RISK** The chance of having an automobile accident increases exponentially if the driver has consumed alcohol. The relationship can be modeled by $A(c) = 6e^{12.8c}$, where A is the percent chance of an accident and c is the driver's blood alcohol concentration (BAC).
   a. The legal BAC is 0.08. What is the percent chance of having a car accident at this concentration?
   b. What BAC would correspond to a 50% chance of having a car accident?

   SOLUTION:
   a.

   $$A(c) = 6e^{12.8c}$$
   $$A(0.08) = 6e^{12.8(0.08)}$$
   $$\approx 16.7$$
   $$= 16.7\%$$

   b. Graph $y_1 = 6e^{12.8x}$ and $y_2 = 50$ and find the intersection of the two graphs.
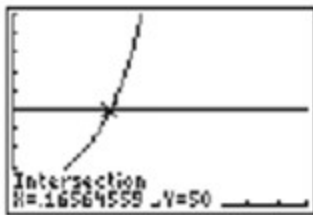
   [0, 0.5] scl: 0.05 by [0, 100] scl: 10

   According to the model, a 50% chance of having a car accident corresponds with a BAC of about 0.166.

43. **GRAPHING CALCULATOR** The table shows the number of blogs in millions semiannually from September 2003 to March 2006.

Months	1	7	13	19	25	31
Blogs	0.7	2	4	8	16	31

   a. Using the calculator's exponential regression tool, find a function that models the data.
   b. After how many months did the number of blogs reach 20 million?
   c. Predict the number of blogs after 48 months.

   SOLUTION:
   a. Enter the data in L1 and L2 of your calculator.

   Select STAT, CALC, ExpReg.

3-1 Exponential Functions

The function is $f(x) = 0.7378 \cdot 1.131^x$.

$[0, 35]$ scl: 5 by $[0, 35]$ scl: 5
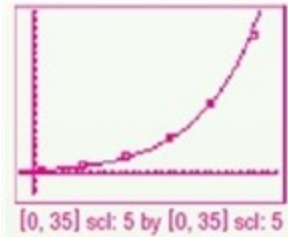
b. Graph $y_1 = 0.7378 \cdot 1.131^x$ and $y_2 = 20$ and find the intersection of the graphs.

$[0, 30]$ scl: 5 by $[0, 25]$ scl: 5
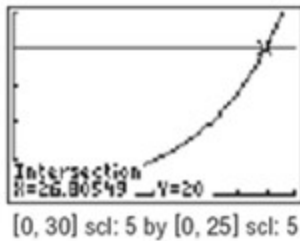
The number of blogs will reach 20 million after about 26.8 months.

c.

$$f(x) = 0.7378 \cdot 1.131^x$$
$$f(48) = 0.7378 \cdot 1.131^{48}$$
$$\approx 271.73$$

There will be about 271.7 million blogs after 48 months.

3-1 Exponential Functions

44. **LANGUAGES** *Glottochronology* is an area of linguistics that studies the divergence of languages. The equation $c = e^{-Lt}$, where c is the proportion of words that remain unchanged, t is the time since two languages diverged, and L is the rate of replacement, models this divergence.

a. If two languages diverged 5 years ago and the rate of replacement is 43.13%, what proportion of words remains unchanged?

b. After how many years will only 1% of the words remain unchanged?

SOLUTION:

a.
$$c = e^{-Lt}$$
$$= e^{-0.4313(5)}$$
$$\approx 0.1157$$

b. Graph $y_1 = e^{-0.4313x}$ and $y_2 = 0.01$ and find the intersection.
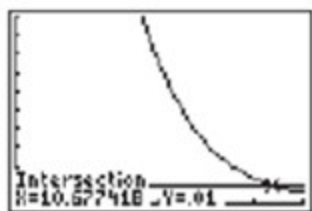
[0, 12] scl: 12 by [0, 0.01] scl: 0.001

According to the model, only 1% of the words will remain unchanged after about 10.68 years.

45. **FINANCIAL LITERACY** A couple just had a child and wants to immediately start a college fund. Use the information below to determine how much money they should invest.

$60,000

Interest Rate: 9%
Compounding: daily

0 years old   18 years old

SOLUTION:

Use the compounding interest formula and solve for P.

$$60{,}000 = P\left(1 + \frac{0.09}{365}\right)^{18(365)}$$

$$\frac{60{,}000}{\left(1 + \dfrac{0.09}{365}\right)^{18(365)}} = P$$

$$\$11{,}876{,}29 \approx P$$

3-1 Exponential Functions

GRAPHING CALCULATOR Determine the value(s) of x that makes each equation or inequality below true. Round to the nearest hundredth, if necessary.

46. $2^x < 4$

SOLUTION:

Graph $y_1 = 2^x$ and $y_2 = 4$.
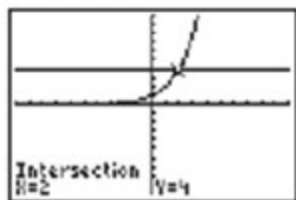
$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

As shown in the graph, $y_2 = 4$ is greater than $y_1 = 2^x$ when $x < 2$.

47. $e^{2x} = 3$

SOLUTION:

Graph $y_1 = e^{2x}$ and $y_2 = 3$.
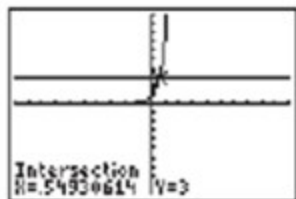
$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

As shown in the graph, the two lines intersect at about $x = 0.55$.

48. $-e^x > -2$

SOLUTION:

Graph $y_1 = -e^x$ and $y_2 = -2$.
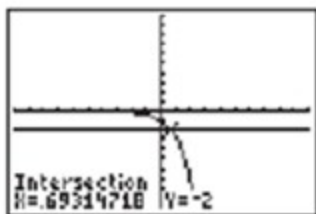
$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

As shown in the graph, $y_1 = -e^x$ is greater than $y_2 = -2$ about when $x < 0.69$.

3-1 Exponential Functions

49. $2^{-4x} \le 8$

SOLUTION:

Graph $y_1 = 2^{-4x}$ and $y_2 = 8$.

$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1
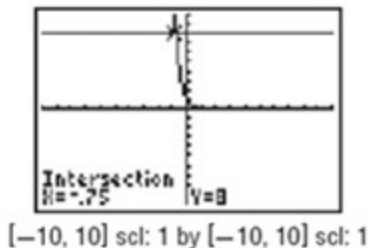
As shown in the graph, $y_2 = 8$ is greater than or equal to $y_1 = 2^{-4x}$ when $x \ge -0.75$.

Describe the domain, range, continuity, and increasing /decreasing behavior for an exponential function with the given intercept and end behavior. Then graph the function.

50. $f(0) = -1, \lim\limits_{x \to -\infty} f(x) = 0, \lim\limits_{x \to \infty} f(x) = -\infty$

SOLUTION:

The function is exponential, so D = $(-\infty, \infty)$. Since the function covers every value of the domain, it is continuous. Plot (0, −1).

Since the function approaches 0 as x approaches infinity and the function must pass through (0, −1), the function is decreasing for $(-\infty, \infty)$ and the range is R = $(-\infty, 0)$.
Sample answer:
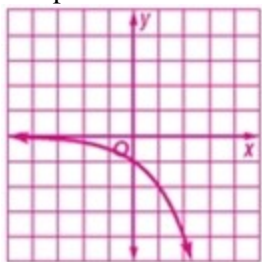
3-1 Exponential Functions

51. $f(0) = 4$, $\lim\limits_{x \to -\infty} f(x) = \infty$, $\lim\limits_{x \to \infty} f(x) = 3$

SOLUTION:

The function is exponential, so D = $(-\infty, \infty)$. Since the function covers every value of the domain, it is continuous. Plot (0, 4).

Since the function approaches infinity as x approaches negative infinity and the function must pass through (0, 4), the function is decreasing for $(-\infty, \infty)$.

As x approaches positive infinity, the function approaches 3, so the range is R = $(3, \infty)$.
Sample answer:
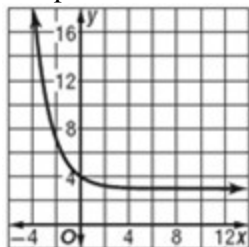
52. $f(0) = 3$, $\lim\limits_{x \to -\infty} f(x) = 2$, $\lim\limits_{x \to \infty} f(x) = \infty$

SOLUTION:

The function is exponential, so D = $(-\infty, \infty)$. Since the function covers every value of the domain, it is continuous. Plot (0, 3).

Since the function approaches infinity as x approaches infinity and the function must pass through (0, 3), the function is increasing for $(-\infty, \infty)$.

As x approaches negative infinity, the function approaches 2, so the range is R = $(2, \infty)$.

Sample answer:
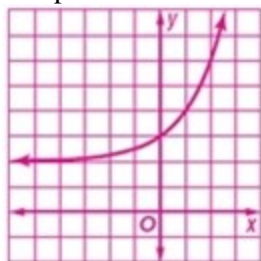
Determine the equation of each function after the given transformation of the parent function.

53. $f(x) = 5^x$ translated 3 units left and 4 units down

SOLUTION:

The original function is $f(x) = 5^x$. A horizontal translation is made by altering the exponent. Since the graph is shifting to the left, then smaller values of x are needed to obtain the result of the original function. Therefore, x becomes $x + 3$. A translation down is made by subtracting the value at the end of the function. Therefore, the equation of the new function is $f(x) = 5^{x+3} - 4$.

3-1 Exponential Functions

54. $f(x) = 0.25^x$ compressed vertically by a factor of 3 and translated 9 units left and 12 units up

SOLUTION:

The original function is $f(x) = 0.25^x$. A vertical compression makes the graph slimmer and is made by multiplying by a coefficient equal to the factor. A horizontal translation is made by altering the exponent. Since the graph is shifting to the left, then smaller values of x are needed to obtain the result of the original function. Therefore, x becomes $x + 9$. A translation up is made by adding the value at the end of the function. Therefore, the equation of the new function is $f(x) = (3)0.25^{x+9} + 12$.

55. $f(x) = 4^x$ reflected across the x-axis and translated 1 unit left and 6 units up

SOLUTION:

The original function is $f(x) = 4^x$. A reflection across the x-axis occurs when the function is multiplied by -1. Since the graph is shifting to the left, then smaller values of x are needed to obtain the result of the original function. Therefore, x becomes $x + 1$. A translation up is made by adding the value at the end of the function. Therefore, the equation of the new function is $f(x) = f(x) = -4^{x+1} + 6$

3-1 Exponential Functions

Determine the transformations of the given parent function that produce each graph.

56. $f(x) = \left(\dfrac{1}{2}\right)^x$
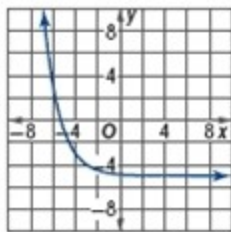
SOLUTION:

Graph $f(x) = \left(\dfrac{1}{2}\right)^x$ for [−10, 10].
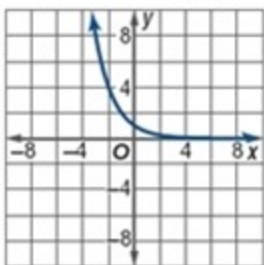
By comparing the two graphs, it appears that the new graph is translated 5 units down. Graph this translation.
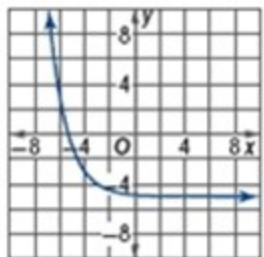
It appears that the graph is translated left as well. At $y = 0$, this graph appears to be at $x = -2$. The translation, however, appears to by at $y = 0$ when $x =$ about −5. Therefore, there is a translation left of 3 units.

3-1 Exponential Functions

57. $f(x) = 3^x$
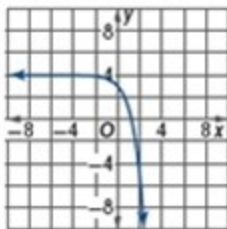
SOLUTION:

Graph $f(x) = 3^x$ for $[-10, 10]$.
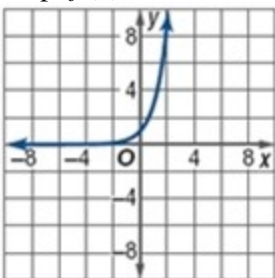
The transformation includes a reflection about the x-axis. Therefore, there is a negative coefficient. Graph this reflection.
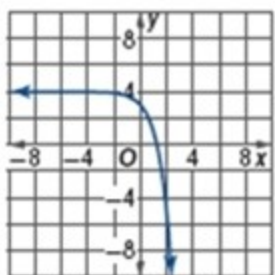
There is also a translation 4 units up. The original graph approaches 0, where the transformation approaches 4.

58. MULTIPLE REPRESENTATIONS In this problem, you will investigate the average rate of change for exponential functions.

a. GRAPHICAL Graph $f(x) = b^x$ for $b = 2, 3, 4$, or 5.

b. ANALYTICAL Find the average rate of change of each function on the interval $[0, 2]$.

c. VERBAL What can you conclude about the average rate of change of $f(x) = b^x$ as b increases? How is this shown in the graphs in part **a**?

d. GRAPHICAL Graph $f(x) = b^{-x}$ for $b = 2, 3, 4$, or 5.

e. ANALYTICAL Find the average rate of change of each function on the interval $[0, 2]$.

f. VERBAL What can you conclude about the average rate of change of $f(x) = b^{-x}$ as b increases. How is this shown in the graphs in part **d**?

SOLUTION:

a.

3-1 Exponential Functions
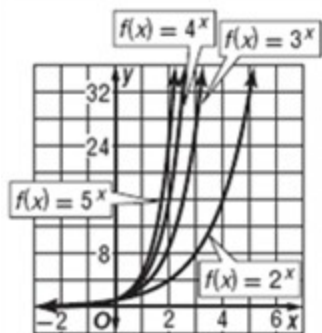
b.

$$b = 2$$

$$\frac{2^2 - 2^0}{2} = \frac{4-1}{2}$$

$$= 1.5$$

$$b = 3$$

$$\frac{3^2 - 3^0}{2} = \frac{9-1}{2}$$

$$= 4$$

$$b = 4$$

$$\frac{4^2 - 4^0}{2} = \frac{16-1}{2}$$

$$= 7.5$$

$$b = 5$$

$$\frac{5^2 - 5^0}{2} = \frac{25-1}{2}$$

$$= 12$$

c. As the values of b increase, the average rate of change increases. This results in a graph that is expanded vertically, as shown in part **a**.

d.
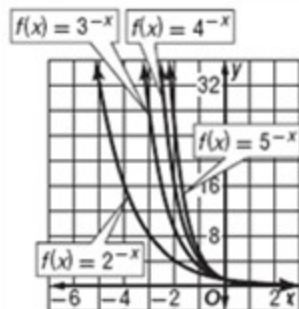
e.

3-1 Exponential Functions

$b = 2$

$$\frac{2^{-2} - 2^0}{2} = \frac{\frac{1}{4} - 1}{2}$$

$$= -0.375$$

$b = 3$

$$\frac{3^{-2} - 3^0}{2} = \frac{\frac{1}{9} - 1}{2}$$

$$\approx -0.444$$

$b = 4$

$$\frac{4^{-2} - 4^0}{2} = \frac{\frac{1}{16} - 1}{2}$$

$$\approx -0.469$$

$b = 5$

$$\frac{5^{-2} - 5^0}{2} = \frac{\frac{1}{25} - 1}{2}$$

$$= -0.48$$

f. As the values of b increase, the average rate of change decreases. Graphs that have larger values for b are greater for negative values of x, but when x is positive, the graphs decrease and approach 0 faster.

59. **ERROR ANALYSIS** Eric and Sonja are determining the worth of a $550 investment after 12 years in a savings account earning 3.5% interest compounded monthly. Eric thinks the investment is worth $837.08, while Sonja thinks it is worth $836.57. Is either of them correct? Explain.

SOLUTION:

Sample answer: Sonja; she used the equation

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 550 \cdot \left(1 + \frac{0.035}{12}\right)^{(12)(12)}$$

$$= \$836.57.$$

Eric determined the worth if the interest was compounded continuously.

$A = Pe^{rt}$
$= 550 \cdot e^{(0.035)(12)}$
$= \$837.08.$

REASONING State whether each statement is *true* or *false* . Explain your reasoning.

60. Exponential functions can never have restrictions on the domain.

SOLUTION:

False; sample answer: For a function of the form $f(x) = a^{\frac{1}{x}}$, where a is any number, the domain is restricted such that $x \neq 0$.

3-1 Exponential Functions

61. Exponential functions always have restrictions on the range.

    SOLUTION:

    True; sample answer: The range of an exponential function will always approach a value that it will never reach, so there will always be a restriction on the range.

62. Graphs of exponential functions always have an asymptote.

    SOLUTION:

    True; sample answer: Since an exponential function will always approach, but never reach, a specific range value, it will always have an asymptote.

63. **OPEN ENDED** Write an example of an increasing exponential function with a negative y-intercept.

    SOLUTION:

    Sample answer: $f(x) = 2^x - 4$. When $x = 0, f(0) = -4$ which is negative. The exponent is positive, so the function is increasing.

64. **CHALLENGE** Trina invests $1275 in an account that compounds quarterly at 8%, but at the end of each year, she takes $100 out. How much is the account worth at the end of the fifth year?

    SOLUTION:

    To solve, calculate one year at a time, subtracting the $100 at the end of each year.

    $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

    $$A(1) = 1275\left(1 + \frac{.08}{4}\right)^{4} - 100$$
    $$= 1280.10$$

    $$A(2) = 1280.10\left(1 + \frac{.08}{4}\right)^{4} - 100$$
    $$= 1285.62$$

    $$A(3) = 1285.62\left(1 + \frac{.08}{4}\right)^{4} - 100$$
    $$= 1291.60$$

    $$A(4) = 1291.60\left(1 + \frac{.08}{4}\right)^{4} - 100$$
    $$= 1298.07$$

    $$A(5) = 1298.07\left(1 + \frac{.08}{4}\right)^{4} - 100$$
    $$= 1305.07$$

3-1 Exponential Functions

65. **REASONING** Two functions of the form $f(x) = b^x$ s*ometimes*, *always*, or *never* have at least one ordered pair in common.

    SOLUTION:

    Always; sample answer: All functions of the form $f(x) = b^x$ have the point (0, 1) in common because any number b raised to the 0 power is 1.

66. *Writing in Math* Compare and contrast the domain, range, intercepts, symmetry, continuity, increasing/decreasing behavior, and end behavior of exponential and power parent functions.

    SOLUTION:

    Sample answer: We will analyze power functions of the form ax^n where a and n are positive integers and power functions of the form ab^x where a is a positive integer. The domain of both functions is $(-\infty, \infty)$ but the ranges differ. For power functions, if n is even, the range is $[0, \infty)$, but when n is odd, the range is $(-\infty, \infty)$. For exponential functions, the range is always $(0, \infty)$. Power functions have x and y-intercepts at the origin while exponential functions have a y-intercept at a and are without an x-intercept. Even power functions are symmetric in respect to the y-axis while odd power functions are symmetric in respect to the origin. Exponential functions have no symmetry. Both functions are continuous. Odd power functions increase while even power functions decrease on $(-\infty, 0)$ but increase on $(0, \infty)$. Exponential functions increase if $b > 1$ but decrease if $b < 1$. Finally, the end behavior of power functions approaches ∞ while exponential functions approach ∞ when $b > 1$ but approach 0 when $b < 1$.

    Solve each inequality.

67. $(x - 3)(x + 2) \leq 0$

    SOLUTION:

    The critical numbers of $(x - 3)(x + 2) \leq 0$ are 3 and −2. We need to test the x-values in $(-\infty, -2]$, $[-2, 3]$, and $[3, \infty)$.

    $$\text{for } (-\infty, 2], x = -4$$
    $$(-4 - 3)(-4 + 2) = (-7)(-2)$$
    $$= 14$$
    $$14 \nleq 0$$
    $$\text{for } [2, 3], x = 0$$
    $$(0 - 3)(0 + 2) = (-3)(2)$$
    $$= -6$$
    $$-6 \leq 0$$
    $$\text{for } [5, \infty), x = 5$$
    $$(5 - 3)(5 + 2) = (2)(7)$$
    $$= 14$$
    $$14 \nleq 0$$

    The solution is $[-2, 3]$

3-1 Exponential Functions

68. $x^2 + 6x \leq -x - 4$

   SOLUTION:

   Simplify the inequality.

   $$x^2 + 6x \leq -x - 4$$
   $$x^2 + 7x + 4 \leq 0$$

   Use the quadratic formula.

   $$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(4)}}{2(1)}$$
   $$= \frac{-7 \pm \sqrt{49 - 16}}{2}$$
   $$= \frac{-7 \pm \sqrt{33}}{2}$$
   $$= -0.6277 \text{ or } -6.3723$$

   Test $(-\infty, -6.3723]$. $[-6.3723, -0.6277]$, and $[-0.6277, \infty)$.

   $$\text{for } (-\infty, -6.3723], x = -10$$
   $$(-10)^2 + 7(-10) + 4 = 100 - 70 + 4$$
   $$= 34$$
   $$34 \nleq 0$$
   $$\text{for } [-6.3723, -0.6277], x = -2$$
   $$(-2)^2 + 7(-2) + 4 = 4 - 14 + 4$$
   $$= -6$$
   $$-6 \leq 0$$
   $$\text{for } [-0.6277, \infty), x = 2$$
   $$(2)^2 + 7(2) + 4 = 4 + 14 + 4$$
   $$= 22$$
   $$22 \nleq 0$$

   The solution is $[-6.3723, -0.6277]$.

3-1 Exponential Functions

69. $3x^2 + 15 \geq x^2 + 15x$

   SOLUTION:

   Simplify the inequality.

   $$3x^2 + 15 \geq x^2 + 15x$$

   $$2x^2 - 15x + 15 \geq 0$$

   Use the quadratic formula.

   $$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(15)}}{2(2)}$$

   $$= \frac{15 \pm \sqrt{225 - 120}}{4}$$

   $$= \frac{15 \pm \sqrt{105}}{4}$$

   $$= 6.3117 \text{ or } 1.1883$$

   Test $(-\infty, 1.1883]$, $[1.1883, 6.3117]$, and $[6.3117, \infty)$.

   $$for (-\infty, 1.1883], x = 0$$

   $$2(0)^2 - 15(0) + 15 = 0 - 0 + 15$$

   $$= 15$$

   $$15 \geq 0$$

   $$for [1.1883, 6.3117], x = 4$$

   $$2(4)^2 - 15(4) + 15 = 32 - 60 + 15$$

   $$= -13$$

   $$-13 \ngeq 0$$

   $$for [6.3117, \infty], x = 8$$

   $$2(8)^2 - 15(8) + 15 = 128 - 120 + 15$$

   $$= 233$$

   $$23 \geq 0$$

   The solution is $(-\infty, 1.1883] \cup [6.3117, \infty)$.

3-1 Exponential Functions

Find the domain of each function and the equations of any vertical or horizontal asymptotes, noting any holes.

70. $f(x) = \dfrac{3}{x2 - 4x + 4}$

SOLUTION:

The denominator cannot equal zero. Set the denominator equal to zero and solve for x to determine the excluded values.

$$x^2 - 4x + 4 = 0$$
$$(x - 2)^2 = 0$$
$$x = 2$$

Therefore, the domain of the function is $D = \{x \mid x \neq 2\}$.
This also identifies the location of the vertical asymptote, $x = 2$.

As x approaches positive or negative infinity, x^2 in the denominator becomes infinitely large, so the y-values approach zero. Thus, there is a horizontal asymptote at $y = 0$.

71. $f(x) = \dfrac{x - 1}{x2 + 4x - 5}$

SOLUTION:

The denominator cannot equal zero. Set the denominator equal to zero and solve for x to determine the excluded values.

$$x^2 + 4x - 5 = 0$$
$$(x + 5)(x - 1) = 0$$
$$x = -5 \; or \; x = 1$$

Therefore, the domain of the function is $D = \{x \mid x \neq -5 \text{ or } 1\}$.
This also identifies the location of the vertical asymptote, $x = -5$.

Since the $x - 1$ factor in the denominator can be canceled out by the $x - 1$ factor in the numerator, the value of $x = 1$ is a hole.

After the function is simplified to $f(x) = \dfrac{1}{x + 5}$, as x approaches positive or negative infinity, the denominator becomes infinitely large, so the y-values approach zero. Thus, there is a horizontal asymptote at $y = 0$.

3-1 Exponential Functions

72. $f(x) = \dfrac{x2 - 8x + 16}{x - 4}$

SOLUTION:

The denominator cannot equal zero. Set the denominator equal to zero and solve for x to determine the excluded values.

$$x - 4 = 0$$
$$x = 4$$

Therefore, the domain of the function is D = $\{x \mid x \neq 4\}$.

Factor the numerator and simplify the function.

$$f(x) = \dfrac{x^2 - 8x + 16}{x - 4}$$
$$= \dfrac{(x - 4)^2}{x - 4}$$
$$= x - 4$$

Since the $x - 4$ factor in the denominator can be canceled out by the $x - 4$ factor in the numerator, the value of $x = 4$ is a hole.

After the function is simplified to $f(x) = x - 4$, as x approaches positive infinity, y approaches positive infinity. Likewise, as x approaches negative infinity, y approaches negative infinity. Therefore, there are no horizontal asymptotes.

3-1 Exponential Functions

73. **TEMPERATURE** A formula for converting degrees Celsius to Fahrenheit is $F(x) = \dfrac{9}{5}x + 32$.

    a. Find the inverse $F^{-1}(x)$. Show that $F(x)$ and $F^{-1}(x)$ are inverses.

    b. Explain what purpose $F^{-1}(x)$ serves.

SOLUTION:

a. To find the inverse of the function, switch the variables and solve for y.

$$F(x) = \frac{9}{5}x + 32$$

$$y = \frac{9}{5}x + 32$$

$$x = \frac{9}{5}y + 32$$

$$x - 32 = \frac{9}{5}y$$

$$\frac{5}{9}(x - 32) = y$$

$$\frac{5}{9}(x - 32) = F^{-1}(x)$$

To show that they are inverses, show $F(F^{-1}(x)) = x$ and $F^{-1}(F(x)) = x$.

$$F(F^{-1}(x)) = \frac{9}{5}\left[\frac{5}{9}(x - 32)\right] + 32$$

$$= x - 32 + 32$$

$$= x$$

$$F^{-1}(F(x)) = \frac{5}{9}\left[\frac{9}{5}x + 32 - 32\right]$$

$$= \frac{5}{9}\left[\frac{9}{5}x\right]$$

$$= x$$

b. It can be used to convert Fahrenheit to Celsius.

$$s[p(x)] = 1.0575(0.70x)$$
$$= 0.74025x$$

$$p[s(x)] = 0.70(1.0575x)$$
$$= 0.74025x$$

3-1 Exponential Functions

74. **SHOPPING** Lily wants to buy a pair of inline skates that are on sale for 30% off the original price of $149. The sales tax is 5.75%.

   a. Express the price of the inline skates after the discount and the price of the inline skates after the sales tax using function notation. Let x represent the price of the inline skates, $p(x)$ represent the price after the 30% discount, and $s(x)$ represent the price after the sales tax.

   b. Which composition of functions represents the price of the inline skates, $p[s(x)]$ or $s[p(x)]$? Explain your reasoning.

   c. How much will Lily pay for the inline skates?

   SOLUTION:

   a. If they are on sale for 30% off, then the sale price will be 70% of the original price. The sales tax is 5.75%, so the price with tax will be $x + 0.575x$.
   $p(x) = 0.70x$; $s(x) = 1.0575x$

   b.
   $$s[p(x)] = 1.0575(0.70x)$$
   $$= 0.74025x$$
   $$p[s(x)] = 0.70(1.0575x)$$
   $$= 0.74025x$$
   Since $s[p(x)] = p[s(x)]$, either function represents the price of the skates.

   c.
   $$s[p(149)] = 0.74025(149)$$
   $$= \$110.30$$

75. **EDUCATION** The table shows the number of freshmen who applied to and the number of freshmen attending selected universities in a certain year.

   a. State the relation as a set of ordered pairs.

   b. State the domain and range of the relation.

   c. Determine whether the relation is a function. Explain.

   d. Assuming the relation is a function, is it reasonable to determine a prediction equation for this situation? Explain.

University	Applied	Attending
Auburn University	13,264	4184
University of California-Davis	27,954	4412
University of Illinois-Urbana-Champaign	21,484	6366
Florida State University	13,423	4851
State University of New York-Stony Brook	16,849	2415
The Ohio State University	19,563	5982
Texas A&M University	17,284	6949

Source: How to Get Into College

   SOLUTION:

   a. The number applied is the domain and the x-coordinate. The number attending is related to the number applying, is the range, and is the y-coordinate. {(13,264, 4184), (27,954, 4412), (21,484, 6366), (13,423, 4851), (16,849, 2415), (19,563, 5982), (17,284, 6949)}

   b. D = {13,264, 16,849, 17,284, 19,563, 21,484, 13,423, 27,954}; R = {2415, 4184, 4412, 4851, 5982, 6366, 6949}

   c. Yes; no member of the domain is paired with more than one member of the range.

   d. Sample answer: No; it is not reasonable to make a prediction equation for this situation because you cannot determine the number of attendees at a school based on the number of applications they received.

76.

   $4m$

76. **SAT/ACT** A set of n numbers has an average (arithmetic mean) of $3k$ and a sum of $12m$, where k and m are positive. What is the value of n in terms of k and m?

A $\dfrac{4m}{k}$

B $36km$

C $\dfrac{4k}{m}$

D $\dfrac{m}{4k}$

E $\dfrac{k}{4m}$

SOLUTION:

The average of a set of numbers $3k$ is the sum of the numbers $12m$ divided by the total *number* of numbers in the set n. Therefore, $3k = \dfrac{12m}{n}$. Solve for n.

$$3k = \dfrac{12m}{n}$$
$$3kn = 12m$$
$$n = \dfrac{12m}{3k}$$
$$n = \dfrac{4m}{k}$$

77.

Time	Number of Bacteria
2 P.M.	100
4 P.M.	4000

SOLUTION:

$$N = N_0(1+r)^t$$
$$4000 = 100(1+r)^2$$
$$40 = (1+r)^2$$
$$\sqrt{40} = 1+r$$
$$\sqrt{40} - 1 = r$$
$$5.3246 = r$$

$$N = N_0(1+r)^t$$
$$N = 100(1+5.3246)^5$$
$= 100(6.3246)^5$
$$\approx 1,011,929$$

$$n = \frac{12m}{3k}$$

3-1 Exponential Functions

$$\frac{}{k}$$

77. The number of bacteria in a colony were growing exponentially. Approximately how many bacteria were there at 7 P.M.?

Time	Number of Bacteria
2 P.M.	100
4 P.M.	4000

F 15,700
G 159,540
H 1,011,929
J 6,372,392

SOLUTION:

Use the formula for exponential growth. Since 2 hours elapse, $t = 2$. Solve for r.

$$N = N_0(1+r)^t$$
$$4000 = 100(1+r)^2$$
$$40 = (1+r)^2$$
$$\sqrt{40} = 1+r$$
$$\sqrt{40} - 1 = r$$
$$5.3246 = r$$

At 7 P.M., $t = 7 - 2$ or 5.

$$N = N_0(1+r)^t$$
$$N = 100(1+5.3246)^5$$
$$= 100(6.3246)^5$$
$$\approx 1,011,929$$

78. **REVIEW** If $4^{x+2} = 48$, then $4^x = ?$

A 3.0
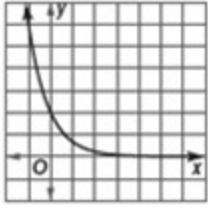**B** 6.4
C 6.9
D 12.0

SOLUTION:

$$4^{x+2} = 48$$
$$4^x \cdot 4^2 = 48$$
$$4^x \cdot 16 = 48$$
$$4^x = 3$$

79.

$$y = 2\left(\frac{1}{3}\right)^x$$

$$y = 3\left(\frac{1}{2}\right)^x$$

$$4^{\ldots} = 48$$
$$4^x \cdot 4^2 = 48$$

3-1 Exponential Functions

$$4^{\cdot} = 3$$

79. **REVIEW** What is the equation of the function?

F $y = 2(3)^x$

G $y = 2\left(\dfrac{1}{3}\right)^x$

H $y = 3\left(\dfrac{1}{2}\right)^x$

J $y = 3(2)^x$

SOLUTION:

The graph approaches 0 as x approaches infinity, so F and J are invalid choices. The y-intercept of the graph is 2. When $x = 0$, the value of the function for choice G is 2 while the value of the function for choice H is 3. The correct answer is G.