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Methods of Legal Reasoning

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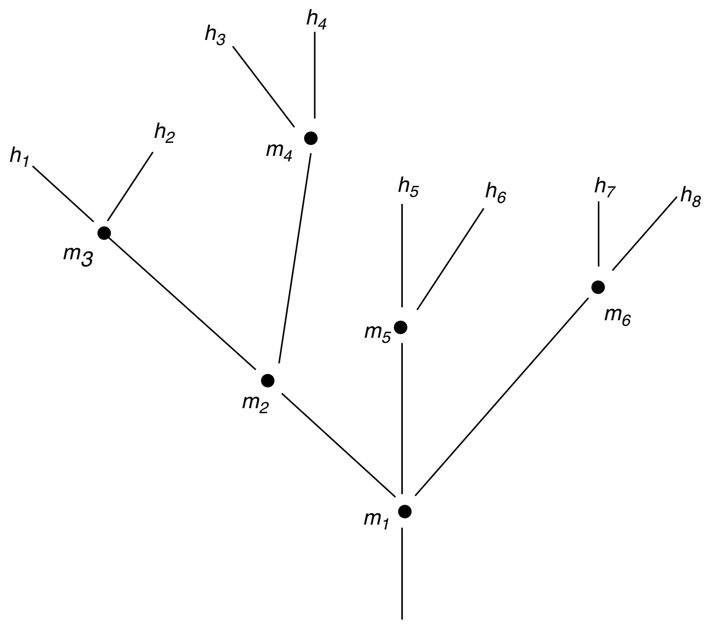
capture the *ought-to-do*. G.H. von Wright in 1951 proposed a system equipped with symbols A, B, C,. . . to which deontic operators, like *O* ("it is obligatory that. . .") were added. A, B, C,. . . stood for "general actions", e.g., theft, sale, etc. Contemporarily, various strategies of constructing deontic logics of action are employed.[26] We will look more closely at two of them.

The following intuition is behind the first of the strategies. Human actions bring about changes in the world. For instance, if the action is building a bridge, the change in the world consists in the appearance of a bridge. In describing the state of the world prior to the action, the sentence "there is a bridge here" is false, whereas after the action it is true. Human actions lead us, therefore, from one state of the world to another. Or, in other words, they constitute a move from one possible world to another. This simple analysis prompts the following idea: on the semantic level actions are represented by pairs of possible worlds – the first world of the pair is the situation in which the action is undertaken, and the second is the situation in which the action ends. Observe that an expression denoting an action is not defined by a single pair – "world before action – world after action" – but by all such possible pairs. Therefore, the action "John builds a bridge" is, on the semantic level, a set of all pairs of possible worlds, of which the first is a world in which there is no bridge and the second is a world in which there is a bridge. Each pair can be labeled an *execution* of the formalized action.[27]

Now, it suffices to apply a procedure similar to that of SDL in order to define which actions are obligatory, which are forbidden and which are allowed. Recall that in the logic described in the previous section the act of creating a norm consisted in identifying a set of possible worlds which we called deontically perfect. In the deontic logic of action "moves" between the worlds (ordered pairs of worlds) are divided into legal (the Leg set) and illegal (the Illeg set). We shall say that an action A is forbidden in world *w* if the set of all executions of A in *w* is included in Illeg. Similarly, an action A is allowed in world *w* if at least one execution of A in *w* belongs to Leg.[28] In order to define obligation let us assume that *Om*A means nonexecution of A, i.e., it is an execution of any action which is not A. We shall say that the action A is obligatory in *w* if all executions of *Om*A in *w* belong to Illeg.

One can query whether a logic thus constructed is better – and in what respects – than "normal" deontic logics. The first reason to claim this is the philosophical motivation that stands behind the proposed system – the distinction between *ought-to-be* and *ought-to-do*. Another advantage of this system over "normal" logics is connected to the fact that, in the logic of action, certain kinds of obligation can be expressed that cannot be reconstructed in SDL. The latter concerns only "ideal" situations but cannot deal with "sub-ideal" ones, i.e., obligations which must be fulfilled in situations in which other obligations have already been violated. In order to make such a reconstruction in our logic of action executions of certain actions that lead from one sub-ideal world to another sub-ideal must be included in the set Leg. Another desirable feature of the present system is that it can easily be "personalized", i.e., obligations can easily be ascribed here to specific persons.

Our second example of a formal system that tries to capture the "ought-to-do" is a deontic logic developed with the use of STIT logic, created in the 1980s by N. Belnap.[29] STIT is a logic that includes the operator "*See To It That*". This operator is defined in a very rich semantic structure, constructed with the use of a technique called *branching*. The basic semantic ideas of STIT are extremely simple and intuitive. Two fundamental concepts of STIT are that of a *moment* and a *history*. Moments are ordered (they form transitive and nonreflexive relation). Two moments can belong to the same, or two different, histories. This is depicted in the figure below as a tree, which, from the bottom-up represents the direction of the flow of time.

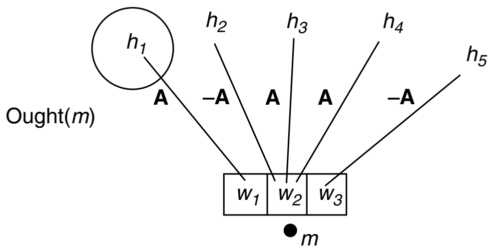


Let moment m_1 be the actual moment. As we can see, "below" m_1 the tree does not branch. Intuitively, this corresponds to the thesis that the past is fixed (fully determined). However, "above" m_1 our tree has several branches that, taken together, represent the undetermined future. Every "maximal" branch of the tree represents a certain history. For instance, the branch that goes through m_1, m_2, m_3 and onward, constitutes the history h_1 (or h_2), and the branch that goes through m_1, m_2 and m_4 constitutes the history h_3 (or h_4). It is useful to denote by H_m the set of all histories "going through" the moment m (therefore, for instance, $H_{m1} = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}$, and $H_{m4} = \{h_3, h_4\}$). From the perspective of the logical systems we have already presented, every moment is a certain possible world (possible state of affairs); hence, in a propositional logic a moment is represented by the valuation function that ascribes to all the atomic expressions of a language the values of truth or falsehood.

In such semantics the concept of action is encoded by defining the functor "see to it that", which we write formally:

$$[\alpha \; stit \; A]$$

this is read: person $\propto$ sees to it that A. In order to give a semantic characteristic to $[\alpha \; stit \; A]$ we must introduce the concept of choice. From the intuitive point of view this is simple. In every moment m person α can choose from various actions and her choice determines the future, i.e., determines which history will be realized. It is not the case, however, that the choices of a given person determine the future univocally. The below figure illustrates this. In moment m person α has three possible choices: w_1, w_2 and w_3. Choices w_1 and w_3 determine one, concrete history (in the case of the former – h_1, and of the latter – h_5). The choice of w_2 does not determine the future univocally, reducing only the set of possible histories to h_2, h_3 and h_4.



An important feature of STIT is that, in it, we ascribe to different people the choices they can make at moment m. If person α has at m three possible choices, it does not mean that person β also has three choices; moreover, the choices of β can "divide" the set of possible histories in a different way than the choices of α can.[30] Let us use $Choice_{\alpha}^{m}$ to denote the set of all choices of α at m. Let $Choice_{\alpha}^{m}(h_i)$ stand for the choice that includes history h_i. Therefore, e.g., $Choice_{\alpha}^{m}(h_1) = w_1$.

We can now define the functor [. . . *stit* . . .]. The expression [α *stit A*] ("Person α sees to it that *A*") is true at moment *m* and history *h*, if and only if A is true for every history *h'*, which belongs to $Choice_{\alpha}^{m}(h)$. Look at the example depicted in the figure above. [α *stit A*] is true at moment *m* for h_1 (since h_1 is the only history belonging to w_1 and *A* is true in h_1); in the case of the remaining histories [α *stit A*] is false. For h_5 this is obvious. The falsity of [α *stit A*] at *m* for h_3 and h_4 results from the fact that in one of the histories (h_2) belonging to the same choice as h_3 and h_4, *A* is false. It seems that this definition of the truth of [α *stit A*] is sound. Person α sees to it that *A*, if her choice results in A being true.

In order to take the last step – introduce to our semantic structure the functor of obligation *O* – we need, as in the case of the deontic logics discussed above, a norming function. This time "norming" means picking out for every moment *m* a set Ought(*m*) of those histories that are desirable from the point of view of a legislator. We will say that at a moment *m* and for a history *h* it ought to be that *X* (we write: *OX*), if and only if *X* is true at moment *m* and a history h_i for every history h_i belonging to Ought(*m*). For instance, in the situation depicted in the figure, the sentence *OA* is true at *m* and h_1, for *A* is true at *m* for the only history belonging to Ought(*m*), i.e., for h_1.

From our point of view the most interesting question is when the expression *O*[α *stit A*] ("it ought to be the case that person α sees to it that *A*") is true. According to the above definition *O*[α *stit A*] is true at moment *m* and history *h*, if [α *stit A*] is true at *m* for every history belonging to Ought(*m*). In our example, only h_1 belongs to Ought(*m*), and we have already determined that [α *stit A*] is true at *m* for h_1. Therefore, *O*[α *stit A*] is true at *m* for all the histories h_1–h_5. If, however, the set Ought(*m*) included, apart from h_1, also h_3, the expression *O*[α *stit A*] would be false at *m* for all the histories, since at m/h_3 [α *stit* A] is false. What is interesting, is that in the same situation *OA* would be true (for *A* is true both in h_1 and in h_3). This shows that, in the semantics under consideration, the expression "it ought to be the case that A" is not equivalent to "it ought to be the case that person α sees to it that A".

The system described above will not be developed any further.[31] We would like, however, to point out several facts. First, the expression *O*[α *stit A*] is not yet the *ought-to-do* obligation. J.F. Horty, the author of the presented conception, indicates that even the operator O, as now defined, suffices to solve the "Fred and Ginger puzzle". Observe that the expression "it ought to be the case that person α sees to it that A" cannot be reduced to the expression "it ought to be the case that A". The difference between OA and $O[\alpha\ stit\ A]$ is based on the fact that, in the latter case the obligation is in a way personalized. This supports our diagnosis that the paradoxical character of the Fred and Ginger case stems from the fact that obligations in traditional deontic logics are not "personalized"; *a contrario*, it is not a result of reducing *ought-to-do* to *ought-to-be*. Horty shows, however,[32] that there exist situations which cannot be adequately accounted for with the use of the functor $O[\ldots stit \ldots]$. In order to describe those situations he proposes – in the framework provided by STIT – another deontic operator which encodes *ought-to-do*.[33]

Secondly, the semantics of STIT enables Horty to build a system that takes into account the obligations of many persons. Such logic has important consequences for the problem of the "group ought". One can identify at least a few such obligations. Let us look at two examples. A group of pupils has two obligations: to clean the blackboard before the lesson begins, and to keep quiet during the lesson. The first of the obligations will be fulfilled if any of the pupils cleans the blackboard. Fulfillment of the second requires that all the pupils stay quiet. The distinction between those two kinds of obligation is possible only when the language of our logic can not only express the fact that there are different obligations on different persons, but can also account for the complicated relations between those obligations.

The logics of action constitute, as we have seen, an interesting alternative for traditional deontic logics. They may be characterized, first and foremost, as using rich and intuitively sound semantic structures. The search for such structures, which are able to model more and more complex situations, is one of the most important directions of contemporary logical research.[34]

2.4.3 Jørgensen Dilemma

Up to now the focus has been on when sentences which take the form "it ought to be the case that p", can be labeled true. But can such sentences be true or false at all? It seems that one can ascribe truth or falsehood to descriptive sentences that inform us about facts. Questions, orders and norms, on the other hand, do not seem to fall into categories possessing truth values. One may maintain that this is not an important problem; but the fact is that contemporary logic – or at least the commonly accepted part of it – concerns expressions that are true or false. If we reflect on these two observations, i.e., that:

(1) Only true or false sentences can serve as premises or conclusions in logically valid arguments.

and

(2) Norms cannot be ascribed truth values.

then we should conclude that:

(3) Norms cannot serve as premises or conclusions in logically valid arguments.

Our conclusion (3) puts into doubt the possibility of developing any logic of legal reasoning. However, we do put forward many legal arguments every day, and they seem intuitively correct. Therefore, we can note:

(4) Intuitively correct normative arguments do exist.

Theses (1) – (4) constitute a dilemma that was first described by Jørgen Jørgensen in a paper published in 1938.[35] It should be added that, in the original formulation, the Jørgensen Dilemma concerned imperatives, not norms.

It is not difficult to argue that the Jørgensen Dilemma poses a fundamental challenge for any formal reconstruction of legal, or, more generally, normative, discourse. As we have already observed, the acceptance of thesis (3) of the Dilemma leads directly to questioning the very possibility of a logic of norms. For that reason, it is necessary to devote some space to analyzing possible solutions to the Dilemma.

Thesis (3) of the Jørgensen Dilemma is a logical consequence of theses (1) and (2). Therefore, in order to resolve the Dilemma one can question thesis (1), (2) or (4). We would like to start with the latter possibility, observing that thesis (4) can be understood in two ways. First, the expression "normative arguments" can mean "arguments using norms as premises and conclusions". On the other hand, however, "normative arguments" can consist of other expressions than norms. We have therefore two versions of thesis (4): (a) when "normative arguments" means "arguments using norms"; or (b) when we maintain that intuitively correct normative arguments are built of some other expressions. The distinction between (4a) and (4b) is useless if we cannot identify those "other expressions". Philosophers and logicians have named and defined such a category of expressions called deontic sentences. A deontic sentence is an expression stating the existence of an obligation relative to a certain deontic system. The following is an example of a deontic sentence: "According to valid law, John ought not to steal". It is usually held that such sentences, which describe only what is obligatory, prohibited or permitted relative to a certain normative system can be ascribed truth values. If, however, deontic sentences are true or false, then there can be logical relationships between them.[36]

Therefore, if we choose to understand thesis (4) of the Dilemma in the (4b) sense, then the Dilemma can be resolved, assuming that the "normative arguments" in question are arguments made up of deontic sentences. If we accept (4a), however, the problem remains. Of course, the distinction between norms and deontic sentences is problematic. We will not go into the details of the debate on this issue. However, we must address a terminological problem. Sometimes deontic logic is distinguished from the logic of norms. The former is thought to concern deontic sentences, i.e., expressions that are true or false. The latter concerns norms, i.e., expressions which are neither true nor false. This conception – that norms cannot be ascribed the value of truth – is called noncognitivism. From the perspective of noncognitivism it is essential to distinguish carefully between deontic logic and the logic of norms, whilst in the rival theory – cognitivism – this distinction is not required.

As already observed, the Jørgensen Dilemma can also be resolved by questioning thesis (1) or thesis (2). According to thesis (1) logical relations obtain only between sentences that are true or false. It must be conceded that not all logicians and philosophers agree with this stance. This is proven by some logical systems that are not based on truth values. The proponents of such logics have to deal with very serious problems, mainly related to the fact that basic and commonly accepted metalogical concepts, such as satisfaction or soundness, are based on the notion of truth. The adaptation of those concepts to a logic that makes no use of notions of truth and falsehood is not a trivial task. Therefore, the solution to the Jørgensen Dilemma that consists in abandoning thesis (1) remains highly problematic.

The last of the proposed solutions to the Dilemma is to abandon thesis (2). This thesis is questioned by cognitivists, i.e., those who ascribe truth values to norms. There is some agreement that the cognitivist theory of Aquinas is one of the most coherent. According to him, the norms that we should follow in our lives are only an inferior copy of eternal divine law. Because of their pedigree, those norms can be ascribed truth (and norms incompatible with them can be designated as false). A closer analysis of Thomistic philosophy reveals, however, that one can, with a sufficient degree of certainty, express only one – the most general – norm: *bonum est faciendum*, good should be done. To deduce more precise rules of behavior from this general norm is a question of individual decision, rather than of well established logical operations.[37] It is therefore difficult to "transfer" truth from the general norm (which is true because of its pedigree) to the specific rules we use every day. Other versions of cognitivism are also troublesome. Usually, the notions of truth they apply are problematic. Therefore, this solution to the Jørgensen Dilemma is not commonly accepted either.

In recent years the discussion surrounding the Jørgensen Dilemma has become less and less intense, although there emerge, from time to time – new attempts to deal with it.[38] Despite the fact that there is still no commonly accepted solution to the Dilemma, the research on logical reconstruction of legal reasoning goes on, and each year new deontic logics or logics of norms are developed. This may well stem from the fact that the Jørgensen Dilemma continues to be a challenge for logicians and thus forces them to search for new formalisms and ideas. Most of the constructed logics of normative discourse are based on notions of truth and falsehood. This is also true of the logical systems presented above. This does not mean that we claim the impossibility of a logic of norms in which expressions cannot be ascribed the value of truth or falsehood. Our choice was motivated only by the importance the presented systems enjoy in contemporary legal theory. Whether we can treat them as proper logics of norms, or "only" as deontic logics depends on the accepted solution to the Jørgensen Dilemma, which, in turn, is based on some ontological choices.

2.5 DEFEASIBLE LOGIC

2.5.1 The Concept of Defeasibility

We would like to turn now to a discussion of defeasible logic. Research on this type of logical system began in the 1970s. The concept of defeasibility, however, was introduced much earlier. It appeared in H.L.A. Hart's paper "The Ascription of Responsibility and Rights", published in 1948. Hart writes:

> When the student has learnt that in English law there are positive conditions required for the existence of a valid contract, (. . .) his understanding of the legal concept of a contract is still incomplete (. . .). For these conditions, although necessary, are not always sufficient and he has still to learn what can *defeat* a claim that there is a valid contract, even though all these conditions are satisfied. The student has still to learn what can follow on the word "unless", which should accompany the statement of these conditions. This characteristic of legal concepts is one for which no word exists in ordinary English. The words "conditional" and "negative" have the wrong implications, but the law has a word which with some hesitation I borrow and extend: this is the word "*defeasible*", used of a legal interest in property which is subject to termination or defeat in a number of different contingencies but remains intact if no such contingencies mature. In this sense, then, contract is a defeasible concept.[39]

Hart's idea amounts to a declaration that certain legal concepts, like that of a binding contract, lack definite conditions of application. Unforeseen circumstances can always occur, causing us to withdraw the claim that we are dealing with a valid contract, although the usual conditions for its validity are fulfilled.[40] An important point must be stressed here. We call certain contracts "valid" because they were concluded in accordance with certain legal rules. Therefore, the ascription of a legal concept hangs together with the fulfillment of a legal norm. If we say – after Hart – that legal concepts are defeasible, then so are legal norms. A legal norm is defeasible if there are situations in which the conditions of that norm's application obtain, but the norm is not applied.

Defeasibility thus defined leads to some logical problems. If we reconstruct a legal norm, as we did above, with the use of material implication:

$$h \rightarrow d$$

(h stands for the norm's antecedent, and d for the consequent), we will not be able to say that the norm is defeasible. This is because in the case of defeasible norms, it is possible that $h \rightarrow d$ is valid, h obtains, but we cannot deduce d. In classical logic (including deontic logic based on classical calculi) this cannot be the case, since if we have $h \rightarrow d$ together with h, d follows on the basis of *modus ponens*.

It is clear from the above that acceptance of the thesis that legal rules are defeasible forces us to look for an alternative logic of legal discourse. Such logic has been developed, not in the field of legal theory, but within research on artificial intelligence. It turns out that the problem of defeasibility is important not only for legal or normative reasoning, but also in theoretical discourse. Logicians developing artificial languages for computer systems encountered the following problem.[41] It happens that a man (or a computer system) has to reason with uncertainty as to whether all relevant information has been collected. For instance, when we know that Tweety is a bird, it is reasonable to say that Tweety flies. If, however, we had additional information that Tweety is a penguin or a baby bird, then we would have to withdraw from saying that Tweety flies (for we know from elsewhere that if a bird is a penguin or a baby bird it does not fly). It follows from this example that such conditionals as "if x is a bird, then x flies" are simply false, but nevertheless, we sometimes use them in our reasoning. Naturally, such conditionals cannot be formalized as a material implication. A new, nonclassical implication needs to be sought.

Defeasible logic constitutes such a nonclassical system. It is an example of nonmonotonic logic.[42] It is instructive to expand here on the meaning of "nonmonotonic". Classical logic is monotonic. This means that if a sentence *p* follows from a set of premises A, then *p* follows also from a set B, which is a superset of A. Every logic which lacks this feature is nonmonotonic. It is easy to show that our Tweety example requires a nonmonotonic logic. In the example we first infer from two premises – "if x is a bird, then x flies" and "Tweety is a bird" – that "Tweety flies". Later, we add the information that "Tweety is a penguin" (and we know that penguins do not fly). From this extended set of premises the conclusion that Tweety flies no longer follows.

2.5.2 Defeasible Logic

There are many defeasible logics.[43] In this section we would like to present one of them,[44] concentrating on its main ideas and omitting technical details.

Our defeasible logic (in short: DL) operates on two levels. On the first level *arguments* are built from a given set of premises; on the second level the arguments are compared in order to decide which of them prevails. The conclusion of which argument is "best" becomes the conclusion of the given set of premises.

The language of DL is the language of first order predicate logic, extended by the addition of a new functor, the defeasible implication, for which we will use the symbol⇒. For defeasible implication there exists a defeasible *modus ponens*, analogous to that of the material implication:

$$A \Rightarrow B$$

$$\frac{A}{B}$$

The difference between material and defeasible implications is visible only on the second level of DL.

The language of DL serves the building of arguments. In our Tweety example we have two situations. In the first, three sentences belong to our set of premises: "if x is a bird then x flies", "Tweety is a bird" and "if x is a penguin then x does not fly". The first of the premises can be formalized in the following way:

`bird`(x) ⇒ `flies`(x)

The second premise is, of course:

`bird`(*tweety*)

And the third:

`penguin`(x) ⇒ ¬(`flies`(x))[45]

This set of premises enables us to construct only one argument. With the help of defeasible *modus ponens* we obtain:

> `bird`(x) ⇒ `flies`(x)
>
> `bird`(*tweety*)
>
> `flies`(*tweety*)

The addition of a fourth premise:

> `penguin`(*tweety*)

enables us to build the following argument:

> `penguin`(x) ⇒ ¬(`flies`(x)
>
> `penguin`(*tweety*)
>
> ---
>
> ¬`flies`(*tweety*)

Having those two arguments we can move to the second level of DL, in which the arguments are compared in order to decide which is better, and in consequence which of the sentences – `flies`(*tweety*) or ¬`flies`(*tweety*) – should be regarded as the conclusion of our set of four premises.

In the second level of DL two concepts play a crucial role: *attack* and *defeat*. We shall say that an argument A attacks an argument B if the conclusions of both arguments are logically inconsistent.[46] In our example that is the case since `flies`(*tweety*) and ¬`flies`(*tweety*) are contradictory. If two arguments compete with one another, one must know how to decide which argument prevails, i.e., which *defeats* the other. Various ways of comparing attacking arguments have been developed.[47] The easiest and most flexible is the following. One checks what the defeasible implications that served to build the attacking arguments are. It is assumed that those implications are ordered. In a comparison an argument wins when it is built with the use of a defeasible implication that is higher in the order. In our example the first argument is based on the implication `bird`(x) ⇒ `flies`(x), whilst the second is based on `penguin`(x) ⇒¬`flies`(x). It is reasonable to assume that the second implication is higher in the ordering, since it represents a stronger tie – there are exceptions to the rule that if something is a bird then it flies, but the second rule that penguins do not fly, is exceptionless. If `penguin`(x) ⇒ ¬`flies`(x) is higher in the ordering than `bird`(x) ⇒ `flies`(x), then the second argument defeats the first.

The conclusion of which argument prevails in a comparison of all competing arguments built from the given set of premises, is the logical conclusion of this set. In the first situation our set of premises contained only three sentences (`bird`(x) ⇒ `flies`(x), `penguin`(x) ⇒ ¬`flies`(x), `bird`(*tweety*)), which enabled us to build only one argument. The conclusion of this argument, `flies`(*tweety*), is the logical conclusion in the first situation. In the second situation another sentence is added to our premises: `penguin`(*tweety*). This made it possible to construct the second argument. Both arguments attack one another and the second argument wins. Therefore, its conclusion, ¬`flies`(*tweety*), and not the conclusion of the first argument, follows logically in the second situation. It is clear from this that DL is nonmonotonic. In the first situation `flies`(*tweety*) was the logical conclusion, but in the second, in which the set of premises is extended, `flies`(*tweety*) no longer follows.

2.5.3 Objections Against Nonmonotonic Logic

At the beginning of this chapter we attempted to define what logic is. The definition we proposed poses a serious challenge for defeasible (or, more generally, nonmonotonic) logics. It prompts doubts as to whether these systems are logics at all.

As already observed, the key insight regarding the nature of logic was formulated by Tarski in his definition of logical consequence. The definition may be somewhat boldly presented as follows:

A sentence A follows logically from the set of premises Γ if and only if in every case in which the premises of Γ are true, A is also true.

A short reflection enables us to say that Tarski's analysis shows our intuitive notion of logical consequence to be monotonic (even if we extend the set of premises Γ, it still will be a case in which all the sentences of Γ are true; therefore if after the extension of Γ, A ceases to follow from it, as is the case with nonmonotonic logics, such a notion of logical consequence is incompatible with Tarski's analysis). Moreover, the concept of truth also seems to be "monotonic" (it is difficult to assume that the addition of a new premise can make false a conclusion thus far considered true). In such a situation, the idea of logic as a set of rules for the "transmission of truth" must be abandoned. Instead some theoreticians are inclined to speak of the "transmission of justification". The role of nonmonotonic logic would be to determine which forms of reasoning lead from justified premises to a justified conclusion.

Abandoning Tarski's analysis also results in abandoning the intuitively appealing soundness theorems. This is problematic as regards the question of whether one can "trust" nonmonotonic logic. On the other hand, the concepts of the second level of DL seem sound and the analyses carried out using DL demonstrate the flexibility and usefulness of this formal system.

Let us look more closely now at some examples of formalizations in DL. This will enable us to formulate several arguments in favor of nonmonotonic logic. We will identify two important features of DL formalizations: modularity and structural resemblance between legal texts and their formal counterparts. Further, we will show how DL deals with some hard cases. A comparison of defeasible and classical techniques will highlight some additional problems of the formal reconstruction of legal reasoning.

2.5.4 Examples

Some peculiar logical problems are connected with the structure of legal texts. Let us look at the following example. Let Article 1 say that the full capacity to perform legal acts is granted once a person is 18 years old; Article 2, in turn, constitutes an exception to Article 1, stating that persons declared mentally ill by a court do not have the capacity to perform legal acts. An attempt to formalize those two provisions in classical monotonic logic leads to the following results:

A1: $\forall x((\texttt{18\_years}(x) \wedge \neg\texttt{mentally\_ill}(x)) \rightarrow \texttt{capacity}(x))$

A2: $\forall x(\texttt{mentally\_ill}(x) \rightarrow \neg\texttt{capacity}(x))$

A distinctive feature of this formalization is that the formula representing Article 1 includes the predicate "mentally_ill", and therefore it takes into account the exception stated in Article 2. Our formalization mixes up, then, information from two different provisions. Such circumstances do not occur when a nonmonotonic system is used. In DL Articles 1 and 2 take the following form:

A1: $\texttt{18\_years}(x) \Rightarrow \texttt{capacity}(x)$

A2: $\texttt{mentally\_ill}(x) \Rightarrow \neg\texttt{capacity}(x)$[48]

In DL the information contained in Articles 1 and 2 is not "mixed up". Therefore, the defeasible formalization *resembles structurally* legal texts.

The presented formalization, apart from being structurally similar to legal texts, displays *modularity*. Imagine introducing Article 3, stating another exception to Article 1, for instance that married men do not have the capacity to perform legal acts. In the classical formalization this causes a revision of the formula representing Article 1:

$$\text{A1: } \forall x((\texttt{18\_years}(x) \wedge \neg\texttt{mentally\_ill}(x) \wedge \neg\texttt{married}(x)) \rightarrow \texttt{capacity}(x))$$

A formula representing the new provision is also needed:

$$\text{A3: } \forall x(\texttt{married}(x) \rightarrow \neg\texttt{capacity}(x))$$

In DL the introduction of Article 3 is much easier. It suffices to add:

$$\text{A3: } \texttt{married}(x) \Rightarrow \neg\texttt{capacity}(x)$$

The formalization in DL displays modularity because adding a new provision does not lead to the revision of the formulas formulated earlier.

Modularity and structural resemblance in nonmonotonic systems, such as DL, can be fully appreciated when we imagine that, together with Article 3, the legislator enacts also Article 4, which states an exception to Article 3 saying that those married men whose last names begin with C have the capacity to perform legal acts.

Let us recall that the formalization of Articles 1–3 in classical first order predicate logic looks like this:

$$\text{A1: } \forall x((\texttt{18\_years}(x) \wedge \neg\texttt{mentally\_ill}(x) \wedge \neg\texttt{married}(x)) \rightarrow \texttt{capacity}(x))$$

$$\text{A2: } \forall x(\texttt{mentally\_ill}(x) \rightarrow \neg\texttt{capacity}(x))$$

$$\text{A3: } \forall x(\texttt{married}(x) \rightarrow \neg\texttt{capacity}(x))$$

Assume that `name_C` stands for the predicate from Article 4. Then, this article can be formulated as follows:

$$\text{A4: } \forall x((\texttt{married}(x) \wedge \texttt{name\_C}(x)) \rightarrow \texttt{capacity}(x))$$

However, we have to change also the formalization of Article 3:

$$\text{A3: } \forall x((\texttt{married}(x) \wedge \neg\texttt{name\_C}(x)) \rightarrow \neg\texttt{capacity}(x))$$

In DL, in which we had:

$$\text{A1: } \texttt{18\_years}(x) \Rightarrow \texttt{capacity}(x)$$

$$\text{A2: } \texttt{mentally\_ill}(x) \Rightarrow \neg\texttt{capacity}(x)$$

$$\text{A3: } \texttt{married}(x) \Rightarrow \neg\texttt{capacity}(x)$$

we only need to add:

A4: (`married`(x) ∧ `name_C` (x)) ⇒ `capacity`(x)[49]

Modularity and structural resemblance may seem weak arguments in favor of nonmonotonic systems. There are some facts, however, which testify to the contrary. The nonmonotonic formalizations lead not only to simpler results as regards "quality", but also as regards "quantity". For instance, an attempt to formalize the provisions of the Polish penal code concerning killing in classical logic results in more than 100 formulas. A similar formalization in DL requires only 33 formulas.[50]

In order to illustrate this, and to formulate one more argument in favor of nonmonotonic logics, let us try to formalize Article 148§1 of the Polish penal code (kk). The provision says that whoever kills a man shall be imprisoned for at least 8 years. This can be formalized in classical logic the following way:

148§1 kk: ∀x(`kills`(x) → `punishment`(x))

This is not a complete formalization, however. It does not take into account, for instance, the exception stated in Article 148§2 kk, which qualifies some types of killing. If we cover them with the predicate qualified, our formalization must be changed in the following way:

148§1 kk: ∀x((`kills`(x) ∧ ¬`qualified`(x)) →
`punishment`(x))

One must add to this a formula representing Article 148§2. Exceptions to Article 148§1 kk can be found also in the remaining part of Article 148 and in Articles 149–151 kk. It should also not be forgotten that in the general part of the penal code there are provisions concerning guilt and self-defense that also constitute exceptions to Article 148 kk. As a result, a formalization of Article 148 kk in classical logic – due to its lack of modularity and structural resemblance – requires that at least ten, if not more, exceptions be taken into account. This causes the following problem: if this formalization is accepted as the basis of a judge's decision, the judge would be required to check whether any of the exceptions to Article 148 kk have occurred in making a decision; so, the judge would have to question whether the killing in question is an act of euthanasia, killing with particular cruelty, etc. In actual cases such justifications do not exist. The judge tackles directly only those questions, which are obviously relevant. It seems that such a process of the application of law can be successfully modeled using nonmonotonic systems, for they offer formalizations that are modular and display structural resemblance.

Even more important theoretical and logical problems are connected to hard cases. The most widely popularized such case seems to be Riggs vs. Palmer, described in *Taking Rights Seriously* by R. Dworkin.[51] These are the facts: Elmer Palmer murdered his grandfather, Francis Palmer. According to the applicable law of succession, Elmer was to inherit part of Francis' property. The law in question did not contain any provision that would deprive Elmer of his right to inherit because of what he had done. The New York Court of Appeals decided, however, that Elmer had no right to the inheritance, because "no man should profit from his own wrong".

Dworkin interprets the court's decision in the following way: in a legal system there are two types of legal norms – rules and principles. Legal rules, such as the norm that gave Elmer the right to inherit are applied in an "all-or-nothing" fashion: they are either fulfilled or not, *tertium non datur*. Legal principles, on the other hand, have the "dimension of weight", i.e., they may be taken into account to greater or smaller degrees. Moreover, principles can, in particular cases, "produce" exceptions to legal rules. In Riggs vs. Palmer we are dealing with such a case. The legal principle "No man shall profit from his own wrong" 'produces' an exception to the rule that gives Elmer his right to inherit.

Let us try to look at the situation from a logical point of view. We have the following predicates: `dies`, `grandfather` and `inherits`. Observe that both `grandfather` and `inherits` are two-place predicates for we will not say "Francis is a grandfather", but "Francis is Elmer's grandfather"; similarly, we will say "Elmer inherits from Francis", and not "Elmer inherits". In classical first order predicate logic the rule of the law of succession, which determines that if someone dies and has a grandson, the grandson has a right to the inheritance, can be formalized in the following way:

$$R: (\forall x)\ (\forall y)\ ((\mathtt{dies}(x) \wedge \mathtt{grandfather}(x,y)) \rightarrow \mathtt{inherits}(y,x))$$

The principle, in turn, which says that "No man shall profit from his own wrong" can be written:

$$P: (\forall x)\ (\mathtt{wrong}(x) \rightarrow \neg\mathtt{profit}(x))$$

Since Francis died (`dies`(*francis*)), and he was Elmer's grandfather (`grandfather`(*francis*,*elmer*)), then on the basis of *modus ponens* we can conclude that Elmer benefits from Francis' inheritance (`inherits`(*elmer*, *francis*)):

$$(\forall x)\ (\forall y)\ ((\mathtt{dies}(x) \wedge \mathtt{grandfather}(x, y)) \rightarrow \mathtt{inherits}\ (y, x))$$

$$\begin{array}{l}\texttt{dies}(\textit{francis})\\ \texttt{grandfather}(\textit{francis}, \textit{elmer})\\ \hline \texttt{inherits}(\textit{elmer}, \textit{francis})\end{array}$$

On the other hand, Elmer did wrong (killing Francis) and therefore, according to the principle we formulated, he shall not profit from his act ($\neg\texttt{profit}(\textit{elmer})$):

$$\begin{array}{l}(\forall x)\ (\texttt{wrong}(x) \rightarrow \neg\texttt{profit}(x))\\ \texttt{wrong}(\textit{elmer})\\ \hline \neg\texttt{profit}(\textit{elmer})\end{array}$$

If we assume what seems obvious – that the fact of inheriting is an instance of profit (it can be formalized as: $(\forall x)\ (\forall y)\ (\texttt{inherits}(yx) \rightarrow \texttt{profit}(y)))$, then our formalization of the rule R and the principle P produces a contradiction. Using the rule we obtain $\texttt{inherits}(\textit{elmer}, \textit{francis})$, and hence, on the basis of the just formulated relationship, $\texttt{profit}(\textit{elmer})$; applying the principle, on the other hand, leads us to the conclusion $\neg\texttt{profit}(\textit{elmer})$.

Our analyses suggest a way out of this problem: in the formalization of the rule R we must include the exception "produced" by the principle P. R thus becomes:

$$\begin{array}{l}(\forall x)\ (\forall y)\ ((\texttt{dies}(x) \wedge \texttt{grandfather}(x,y) \wedge \neg\texttt{wrong}(x)) \rightarrow\\ \texttt{inherits}(y,x))\end{array}$$

Now, the argument leading to the conclusion that Elmer benefits from Francis' inheritance ($\texttt{inherits}(\textit{elmer}, \textit{francis})$) is blocked:

$$\begin{array}{l}(\forall x)\ (\forall y)\ ((\texttt{dies}(x) \wedge \texttt{grandfather}(x,y) \wedge \neg\texttt{wrong}(x)) \rightarrow\\ \texttt{inherits}(y,x))\\ \texttt{dies}(\textit{francis})\\ \texttt{grandfather}(\textit{francis}, \textit{elmer})\\ \texttt{wrong}(\textit{elmer})\end{array}$$

Now we cannot apply *modus ponens* to R for $\texttt{wrong}(\textit{elmer})$ obtains, and not $\neg\texttt{wrong}(\textit{elmer})$.

Is the presented solution satisfactory? It is easy to observe that R does not resemble structurally the rule it stands for. This formalization is not modular either. One can easily imagine that the norm saying that a grandson benefits from the inheritance of his late grandfather, could "lose" against some other principle. This other exception would also have to be included in the formulation of R. If there is interaction between rules and principles, the lack of modularity has, however, catastrophic consequences. Principles can "produce" exceptions to rules in *particular* cases and the number of those exceptions is theoretically unforeseeable and potentially infinite. Therefore, one can never construct "the final" formalization of any legal rule, for there is always a possibility that in a certain case a principle will "produce" an additional exception.

Those problems are omitted when one shifts to nonmonotonic logic. In DL, R becomes:

R: (`dies`(x) ∧ `grandfather`(x, y)) ⇒ `inherits`(y, x)

and the principle:

P: `wrong`(x) ⇒ ¬`profit`(x)

We have to add, as in the case of the classical formalization, that:

(∀ x) (∀ y) (`inherits`(y, x) → `profit`(x))

Modularity in the nonmonotonic formalization makes it possible to deal with the potentially endless list of exceptions to R "produced" by different principles very easily. Those exceptions do not have to be included in the formulation of R.

Up to now, we have not looked at how legal norms are applied in DL. This process is highly characteristic and may even be deemed problematic.

Let us recall, first, the classical, monotonic formalization of Articles 1, 2 and 3 introduced above (for the sake of simplicity we omit Article 4): A1:

A1: ∀x((`18_years`(x) ∧ ¬`mentally_ill`(x) ∧
¬`married`(x)) → `capacity`(x))

A2: ∀x(`mentally_ill`(x) → ¬`capacity`(x))

A3: ∀x(`married`(x) → ¬`capacity`(x))

Imagine two situations. In the first John is more than 18 years old, is not mentally ill and is not married. On the basis of Article 1 we conclude that John has the capacity to perform legal acts (`capacity`(*john*)):

∀x((`18_years`(x) ∧ ¬`mentally_ill`(x) ∧ ¬`married`(x))
→ `capacity`(x))

$$\frac{\begin{array}{l}\texttt{18\_years}(\textit{john})\\ \neg\texttt{mentally\_ill}(\textit{john})\\ \neg\texttt{married}(\textit{john})\end{array}}{\texttt{capacity}(\textit{john})}$$

We reached this conclusion by applying to the formalization of Article 1 and to the known facts the simple scheme of *modus ponens*. The same scheme can be applied in the second situation, in which John, in addition to being over 18 years old, is married. This time we conclude on the basis of Article 3 that John does not have the capacity to perform legal acts:

$$\frac{\begin{array}{l}\forall x(\texttt{married}(x) \rightarrow \neg\texttt{capacity}(x))\\ \texttt{married}(\textit{john})\end{array}}{\neg\texttt{capacity}(\textit{john})}$$

Determination of the logical consequences of legal norms in both situations is more complicated in the case of the nonmonotonic formalization. In DL the first situation looks as follows. We have three legal norms:

A1: $\texttt{18\_years}(x) \Rightarrow \texttt{capacity}(x)$

A 2: $\texttt{mentally\_ill}(x) \Rightarrow \neg\texttt{capacity}(x)$

A3: $\texttt{married}(x) \Rightarrow \neg\texttt{capacity}(x)$

and the following facts obtain:

$\texttt{18\_years}(\textit{john})$

$\neg\texttt{mentally\_ill}(\textit{john})$

$\neg\texttt{married}(\textit{john})$

From those premises only one argument can be built

$$\frac{\begin{array}{l}\texttt{18\_years}(x) \Rightarrow \texttt{capacity}(x)\\ \texttt{18\_years}(\textit{john})\end{array}}{\texttt{capacity}(\textit{john})}$$

Since we have only one argument, its conclusion – $\texttt{capacity}(\textit{john})$ – is the logical consequence in the first situation.

In the second situation, besides the formulas representing our three norms, we have also:

18_years(*john*)

married(*john*)

We can now construct two arguments leading to contradictory conclusions:

(A)

$$\frac{\texttt{18\_years}(x) \Rightarrow \texttt{capacity}(x) \qquad \texttt{18\_years}(\textit{john})}{\texttt{capacity}(\textit{john})}$$

and

(B)

$$\frac{\texttt{married}(x) \Rightarrow \neg\texttt{capacity}(x) \qquad \texttt{married}(\textit{john})}{\neg\texttt{capacity}(\textit{john})}$$

In order to determine the logical consequence in the second situation we must compare arguments (A) and (B), or, more precisely, "weigh" two legal norms occurring in the arguments: Article 1 and Article 3. As the second provision constitutes an exception to the first, it can be placed "higher" in the ordering, and hence argument (B) prevails over argument (A). Therefore, it is the conclusion of argument (B) – ¬capacity(*john*) – that is the required logical conclusion in the second situation.

It turns out, then, that DL, which displays structural resemblance and modularity, leads to relatively complicated application of legal norms (determining the logical consequences in the given case). Classical formalizations are simpler in this regard. This advantage of classical calculi diminishes, however, as soon as cases more difficult than the application of Articles 1–3 are at stake. For instance, let us look at Riggs vs. Palmer. In the classical formalization, after the exception resulting from the principle "No man shall profit from his own wrong" has been introduced, we have the following, complex formula:

R: $(\forall x)\,(\forall y)\,((\texttt{dies}(x) \wedge \texttt{grandfather}(x, y) \wedge \neg\texttt{wrong}(x)) \rightarrow \texttt{inherits}(y, x))$

We will not apply this norm in Riggs vs. Palmer, since one of the conjuncts is not fulfilled, i.e., ¬wrong(x) does not obtain (for Elmer did wrong).

In DL we have:

R: $(\mathtt{dies}(x) \wedge \mathtt{grandfather}(x, y)) \Rightarrow \mathtt{inherits}(yx)$

P: $\mathtt{wrong}(x) \Rightarrow \neg\mathtt{profit}(x)$

In the analyzed case the following facts obtain:

$\mathtt{dies}(\mathit{francis})$

$\mathtt{grandfather}(\mathit{francis}, \mathit{elmer})$

$\mathtt{wrong}(\mathit{elmer})$

this enables us to construct two arguments:

(A)

$$\frac{\begin{array}{l}(\mathtt{dies}(x) \wedge \mathtt{grandfather}(x, y)) \Rightarrow \mathtt{inherits}(yx)\\ \mathtt{dies}(\mathit{francis})\\ \mathtt{grandfather}(\mathit{francis}, \mathit{elmer})\end{array}}{\mathtt{inherits}(\mathit{elmer}, \mathit{francis})}$$

and

(B)

$$\frac{\begin{array}{l}\mathtt{wrong}(x) \Rightarrow \neg\mathtt{profit}(x)\\ \mathtt{wrong}(\mathit{elmer})\end{array}}{\neg\mathtt{profit}(\mathit{elmer})}$$

and since benefiting from inheritance is profitable $((\forall x)\ (\forall y)\ (\mathtt{inherits}(y, x) \rightarrow \mathtt{profit}(x)))$, the conclusions of both arguments contradict one another. Comparing arguments (A) and (B) we "weigh" the norms $(\mathtt{dies}(x) \wedge \mathtt{grandfather}(x, y)) \Rightarrow \mathtt{inherits}(yx)$ and $\mathtt{wrong}(x) \Rightarrow \neg\mathtt{profit}(x)$. The New York Court of Appeals gave priority to the latter norm and concluded that in Riggs vs. Palmer the logical conclusion of argument (B) prevails.

Let us modify the case slightly and imagine that Elmer killed his grandfather but did it unintentionally. He most certainly did wrong and, according to the principle employed by the court, he should not benefit from his act. The application of rule R as formalized in classical logic leads to the conclusion that, in the modified circumstances, Elmer does not benefit from Francis' inheritance. It could be argued, however, that the modified case is different from the original and that it is unjust to deprive Elmer of his rights. Such reasoning can easily be represented in

DL. Here, in the process of "weighing" the norms of arguments (A) and (B), priority would be given to the first of the norms. It is clear, then, that the flexibility of the "complicated" application of norms in DL may have profound practical consequences.

2.5.5 Two Remarks

At the end of our presentation of DL we would like to add two remarks. First, the substitution of the idea of "transmission of truth" with the idea of "transmission of justification" enables one to regard DL as a logic that captures some pragmatic aspects of legal reasoning, and to look for a pragmatic notion of logical consequence. Second, nonmonotonic systems may serve as a basis for questioning the thesis that the role of logic is confined to the context of justification. The complicated procedure of applying norms in DL can be seen as an attempt to capture the formal aspects of the context of discovery.

2.6 SUMMARY

Our analyses of the logical reconstruction of legal reasoning, although not all-embracing, may serve as a basis for some conclusions regarding the nature and limits of applying logical methods. First and foremost, they show that there is no common agreement over what the logic of legal discourse looks like. It should be added that the formalisms that we presented are not complementary. These are, in most cases, formal systems that are incompatible. For instance, the proponents of defeasible logics put forward arguments against classical logic, whilst the constructors of the deontic logic of action oppose the way obligation is defined in SDL.

Secondly, every attempt to develop a logic of legal discourse faces two kinds of problem. On the one hand, there are issues of general, philosophical nature; the Jørgensen Dilemma, considerations of various kinds of obligation, and objections against labeling nonmonotonic systems "logics" are cases in point. On the other hand, there are more specific problems, such as the various paradoxes of deontic logic. What is important, however, is that these problems do not result in the abandonment of attempts to construct a logic of legal discourse; on the contrary, they only encourage new research in the field.

We would like to stress one more thing: the important role intuition plays in constructing normative logics. It is intuition that stands behind the feeling of "a paradox" in certain situations. This is not to say that intuition decides everything, but its role should not be underestimated.

There is one more characteristic feature of contemporary research on normative logic: the way in which new systems develop to overcome recurring paradoxes leads through more and more complex semantics. This "semantic strategy" has recently been extended by the addition of a pragmatic ingredient. We must stress that this feature of contemporary normative logic – intuition plus semantics, with a bit of pragmatics – can be found in almost any logical research carried out nowadays.

It is necessary yet to ask what the conclusions of our analyses of the limits of applying logical methods should be. It should be observed that the contemporary logic of legal discourse aims to "conquer" more than classical logic did. It is appropriate to recall the question of whether the role of logic should be confined to the context of justification, or attempts to analyze logically such hard cases as Riggs vs. Palmer. This shows that there is no such thing as issues that cannot be analyzed from a logical point of view. Even hard cases have a logical dimension. Naturally, it is not the case that logic establishes algorithms for solving every legal case imaginable. However, with the expansion of logical methods, it is impossible to identify any strict limits on the application of formal tools. The only indication of such limits may be the fact that the role of logic remains to point out when, on the basis of given premises, we can accept some conclusion. However, this is also the aim – at least *prima facie* – of analysis and argumentation and, one could even argue, also of hermeneutics.

In concluding, we would like to mention those logics, which have not been discussed above: the logic of induction and probability logic. Our omission of those logics does not mean that they are unimportant for modeling logically legal reasoning. They can serve well the reconstruction of some arguments regarding evidence. We have decided, however, not to present them because there is nothing "peculiar" about their application in legal discourse. In other words, these formalisms are not connected with practical discourse in any special way.

Apart from those mentioned above, it is possible to find other kinds of "logic" in literature: informal, discursive, dialectical, etc. We have deliberately inserted quotation marks around "logic", for the theories in question have nothing to do with how logic is understood in this chapter. We do not want to say that we regard those conceptions as useless. Their introduction would ruin, however, the coherence of our presentation. Furthermore, they are based on ideas that resemble those on which theories of legal argumentation, discussed in Chapter 4, are based.

NOTES

1. Cf. A. Tarski, "O pojęciu wynikania logicznego" [On the Concept of Logical Consequence], *Przegląd Filozoficzny*, vol. 39, 1936, pp. 58–68.
2. The logical form of ordinary language expressions is not usually "visible at first sight". Thus, in order to judge the logical validity of arguments carried out in ordinary language, they are usually "translated" (paraphrased) into the chosen logical equation. As we will see, such paraphrasing is rarely universal or unproblematic.
3. Cf. J. Etchemedy, *The Concept of Logical Consequence*, Harvard University Press, 1990, p. 5 ff.
4. It seems obvious that one can reconstruct logically legal arguments (e.g., judicial reasoning) only from the point of view of justification; what "really happens" in the judge's head must be disregarded, whilst what is intersubjectively controllable is taken into account.
5. These are not, of course, all the possible functors. In a two-valued logic there are 16 possible functors.
6. One can demonstrate this with the following example: if we assumed that norms have the form of material implication, then all norms that had a false (or contradictory) hypothesis would be true (valid).
7. W.V.O. Quine, *Methods of Logic*, 4th edition, Cambridge, Massachussets., 1982, p. 45 ff.
8. Cf. G. Priest, *An Introduction to Non-classical Logic*, Cambridge, 2001, p. 13.
9. Cf. K. Ajdukiewicz, "Okres warunkowy a implikacja materialna" [Conditionals and Material Implications], *Studia Logica*, IV, 1956.
10. Cf. G. Restall, *Introduction to Substructural Logics*, London–New York, 2000.
11. In this argument we apply, of course, *modus ponens,* but for the sake of simplicity we omit the step of universal instantiation, as we do also in the examples below.
12. The terminology used is due to J. Wróblewski. Cf. his *Sądowe stosowanie prawa* [Judicial Application of Law], 2nd edition, Warszawa, 1988.
13. A survey of modal logics can be found in: G.E. Hughes, M.J. Cresswell, *A New Introduction to Modal Logic*, London–New York, 1996.
14. And Stig Kanger and Jaakko Hintikka.
15. That is how it looks in the propositional calculus. In modal predicate logic semantics is, of course, more complex, but the main ideas are the same.
16. Aristotle, *Etyka Nikomachejska* [Nicomachean Ethics], 1147a, in Aristotle, *Dziela Wszystkie* [Collected Works], vol. V, Wydawnictwo Naukowe PWN, Warszawa, 2000, p. 216.
17. Cf. J. Kalinowski, *Logika norm* [Logic of Norms], Daimonion, Lublin 1993, pp. 48–63.
18. This shows that Pp is the so-called weak permission, which means that $\neg p$ is not obligatory, but does not guarantee that p is not obligatory, as is the case with strong permissions.
19. Cf. R. Hilpinen, "Deontic Logic", in L. Goble (ed.), *The Blackwell Guide to Philosophical Logic*, Malden–Oxford, 2001, pp. 159–182.
20. Cf. J. Wolenski, *Logiczne problemy wykladni prawa* [Logical Problems of Legal Interpretation], Zeszyty Naukowe UJ, Warszawa–Kraków, 1972.
21. See J. Carmo, A.J.I. Jones, "Deontic Logic and Contrary-to-Duties", in D. Gabbay (ed.), *Handbook of Philosophical Logic*, 2nd edition, vol. IV, Dordrecht, 2001, pp. 287–366.

22. Cf. G.H. von Wright, "Ought to be – Ought-to-do", in E.G. Valdes, W. Krawietz, G.H. von Wright and R. Zimmerling (ed.), *Normative Systems in Legal and Moral Theory – Festschrift for Carlos E. Alchourrón and Eugenio Bulygin*, Berlin, 1997, pp. 427–438 and J.W. Forrester, *Being Good and Being Logical – Philosophical Groundwork for a New Deontic Logic*, New York, 1996.
23. For instance: Meinong, Hartmann and Chisholm, cf. J.F. Horty, *Agency and Deontic Logic*, Oxford, 2001.
24. Cf. *ibidem*.
25. It is not Geach's original example but its revised version proposed by J.F. Horty and N. Belnap in "The deliberative *stit*: a study of action, omission, ability, and obligation", *Journal of Philosophical Logic*, 24, 1995, pp. 583–644.
26. Cf. K. Segerberg, "Getting started: beginnings in the logic of action", *Studia Logica*, 51, 1992.
27. From the mathematical point of view an action in the universe of possible worlds W is therefore a two-argument relationship, i.e., a set of ordered pairs $<u, w>$, such that $u, w \in W$.
28. Here, we are dealing, once again, with the weak permission.
29. Cf. J.F. Horty, *op. cit*.
30. Therefore, we have the following structure: $<Tree, <, Agent, Choice>$, where *Tree* is a set of moments, $<$ is the relation that orders the moments, *Agent* is the set of agents and *Choice* is a function ascribing to every agent α at the moment m a subset of the set H_m of all the histories "going through" m.
31. For the details, see J.F. Horty, *op. cit*.
32. *Ibid*., p. 55 ff.
33. *Ibid*., p. 59 ff.
34. We would not like to suggest that logics of action are developed only in order to solve the "Fred and Ginger problem". There are also other problems in which those logics are developed. One can point out, for instance, the definition of other-than-standard deontic operators, the analysis of mutual relations between those operators or the problem of expressing the conflict between obligations, etc.
35. Cf. J. Jørgensen, "Imperatives and Logic", *Erkenntnis* 7, 1938, pp. 288–296.
36. See for instance J. Woleński, *Z zagadnień analitycznej filozofii prawa* [Issues in the Analytical Philosophy of Law], Zeszyty Naukowe UJ, Prace Prawnicze, Warszawa–Kraków, 1980.
37. J. Kalinowski in *Le problème de la vérité en morale et en droit*, Lyon 1967 argues to contrary.
38. Cf. the discussion in *Ratio Juris*, caused by R. Walter's paper "Jørgensen's Dilemma and How to Face It", *Ratio Juris* 9, pp. 168–171.
39. H.L.A. Hart, "Ascription of responsibility and rights", in A. Flew (ed.), *Logic and Language*, Blackwell, 1951, p. 152.
40. Such a thesis seems more justified in relation to common law systems than continental systems. However, as shown below, the idea of defeasibility can be useful for analyzing certain aspects of legal reasoning as carried out within the continental tradition.
41. On other problems that caused the development of nonmonotonic systems see J.F. Horty, "Nonmonotonic Logic", in L. Goble (ed.), *Blackwell Guide to Philosophical Logic*, Malden–Oxford, 2001, pp. 336–361.