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CHAPTER 2

LOGIC

2.1 INTRODUCTION

Logical studies have a very long and rich tradition that dates back to antiquity. Despite this it is not easy to define logic. A consensus exists, however, on the fact that logic is about reasoning: it helps us to evaluate the validity of arguments. The question "which arguments are valid?" is usually, however, answered in the following way: "the ones in which the conclusion follows logically from the premises". In this way we come back to the question of the nature of logic, or – more precisely – of logical consequence.

A famous analysis of the notion of logical consequence was presented by A. Tarski.[1] Disregarding the details, one may summarize Tarski's findings in the following sentence:

A sentence A follows logically from the set of premises Γ if and only if in every case in which the premises of Γ are true, A is also true.

The idea behind this analysis is that logic is a theory that describes the "transmission of truth". The "transmission" begins with the premises of an argument, and ends with the conclusion. The aim of logic, therefore, is to identify forms of argument that guarantee the transmission of truth: if the premises of those arguments are true, their conclusions will also be true.

In the previous sentence we used another notion that needs explanation. We said that the aim of logic is to indicate valid "forms of argument". But what are these "forms of argument"? Let us look at the following two arguments:

(1) If John is intelligent and hard working, he will succeed as a lawyer.
(2) John is intelligent and hard working.
Therefore: (3) John will succeed as a lawyer.

(1) If the weather is good, John will go swimming.
(2) The weather is good.
Therefore: (3) John will go swimming.

Let us substitute the sentence "John is intelligent and hard working" with *p*, and "John will succeed as a lawyer" with *q*. Similarly, let *p* denote "The weather is good" and *q* – "John will go swimming". The first of the arguments can now be presented as follows:

(1) If *p*, then *q*.
(2) *p*
(3) *q*

Naturally, using this substitution the second argument looks the same. We shall say that both arguments have something in common: they have the same *form*. It is relatively easy to speak of logical form in particular cases; it is however much more difficult to define it *in abstracto*. One may say that the logical form of an argument is determined by some key terms in natural language. In our example these are "If . . ., then . . ." and "Therefore". The set of key terms also includes "and", "or", "either . . ., or . . .". It is clear, therefore, that the key terms shall have special counterparts in the logical language. The counterparts are known as logical connectives.[2]

The role of logic is to designate certain forms of argument as valid. How can one know, however, that the designated forms are indeed valid? The first of the possible answers is that acceptance of a given logic is ultimately based on intuition: if the forms of argument indicated as valid by a given logic are in accordance with our intuitive understanding of what is valid, the logic in question is adequate. The problem with this solution is that, if intuition determines whether a given argument is valid or not, we do not need logic at all. One may try to overcome this difficulty by saying that the intuition in question is not just any "subjective intuition", but an intuition that is shared by many reasonable people. One may further observe that we are able to judge intuitively only relatively simple arguments; more complicated cases have to be analyzed using logical tools and we can trust logical theory in the complex matters when it does not lead to counter-intuitive solutions in the simple.

An intuitive judgment is fortunately not the only way to demonstrate the adequacy of a logical theory. Tarski's analysis of logical consequence, mentioned above, is of help here. It forms a basis for the so-called soundness and completeness theorems.[3] Almost every logical system consists of two parts: syntax and semantics. Syntax is a language-resembling structure, consisting of an alphabet, rules for constructing well formed formulas, rules of inference and axioms. Semantics, on the other hand, may be regarded as a mathematical model of the world. If every sentence that can be proved on the basis of the axioms and inference rules of a given logic is true in all possible “world models” (and *vice versa*), the logic is said to be sound (complete). Speaking metaphorically, and somewhat loosely, the soundness and completeness theorems show that the language of a given logic “fits together” with the world, and therefore we have grounds for believing that the logic adequately indicates valid forms of argument.

This is not the end of the trouble, however. There exist various logics with different syntaxes and semantics, identifying different forms of argument as valid; moreover, the soundness and completeness theorems hold for each of them. Thus the problem arises of choosing between different logics that are sound and complete. The choice must be made according to certain criteria. One can, for example, try to evaluate which logic uses the semantics that best reflects the world. Intuition can also help in picking out the correct logic.

At the end of this short introduction, two additional problems should be addressed. Firstly, we have said that logical validity guarantees the “transmission of truth” from the premises to the conclusion of an argument. This does not mean, however, that each argument, which is logically correct, has a true conclusion. For this to be true it is necessary for the premises of the argument to be true. If the premises are not true, there can be no “transmission of truth”. In this context internal and external justifications are differentiated. We shall say that an argument is internally justified if its conclusion follows logically from its premises. An argument is externally justified, on the other hand, when it is: (a) internally justified, and (b) its premises are true.

Secondly, one must mention the role that is usually ascribed to logic. In the philosophy of science one distinguishes between the context of discovery and the context of justification; the distinction is made in at least two different ways. First, a scientific discovery could be divided into two stages: the first ends with the formulation of a hypothesis of the analyzed phenomena (context of discovery), and the second serves to justify (test, falsify) the hypothesis (context of justification). Second, in every scientific discovery one may differentiate between two aspects: socio-psychological (context of discovery) and logical (context of justification). The socio-psychological aspect consists of all factors that influence the discovery – everything described by psychologists and sociologists may be of interest here. The second aspect – the logical – enables one to look at the scientific discovery as a purely rational undertaking, which meets certain criteria for accepting and refuting scientific theories. Irrespective of which version of the distinction is chosen, logic has a certain function only in the context of justification. Additionally, it is easy to relate the distinction between both contexts to arguments other than scientific ones. In this way the theory of the logical character of the context of justification is valid universally, i.e., it concerns all kinds of argument.[4]

Below we present a kind of "history" of the search for the logic of normative discourse (including legal discourse). We shall not, however, pay much attention to chronology. We will rather try to show the basic ideas behind different logics of normative discourse; we will also concentrate on the criteria for comparing different logics, and the reasons for constructing new normative logics. We will discuss classical logic (propositional logic and first order predicate logic), deontic logic, the logic of agency and, finally, defeasible logic.

2.2 CLASSICAL LOGIC: PROPOSITIONAL LOGIC AND FIRST ORDER PREDICATE LOGIC

2.2.1 Presentation of Calculi

The history of contemporary logic began just over a hundred years ago with the publication of the works of G. Boole, C.S. Peirce, and – first and foremost – B. Russell and A.N. Whitehead. *Begriffschrift* (1902) by Frege, and *Principia Mathematica* (1910–13) by Russell and Whitehead constituted the turning point in the history of logic; both those works set the stage for the incredible development of logic in the twentieth century.

The two basic logics elaborated by Frege, Russell and Whitehead are classical propositional logic and first order predicate logic. The propositional calculus takes into account only those forms of argument, in which elementary sentences are basic elements. The elementary sentences can be, with just a few exceptions, identified with (grammatically) simple (not compound) sentences. The compound sentences have also a complex logical structure. Because of that, the sentence connectives must have logical counterparts. Those counterparts are called truth-functional functors (or sentential connectives).

The alphabet of propositional logic consists of propositional variables that are usually denoted by small letters p, q, r, etc. A propositional variable denotes an arbitrary elementary sentence. In the alphabet of propositional calculus one can also find symbols denoting the truth-functional functors (sentential connectives): negation ($\neg$), implication ($\rightarrow$), conjunction ($\wedge$) and disjunction ($\vee$).[5] In order to provide a full syntactic characterization of propositional calculus it is necessary to recall the rules of forming formulas, the rules of inference and axioms. According to the rules of forming formulas, all propositional variables are well formed formulas of propositional calculus. Additionally, if A and B are (arbitrary) well formed formulas of propositional logic, $\neg A$, $A \wedge B$, $A \rightarrow B$, and $A \vee B$ are also well formed formulas of propositional calculus.

We will not present here the axioms of propositional logic, for the metalogical features of this logical system will not be analyzed. We will limit ourselves to mentioning only one rule of inference that plays an important role in the considerations below. The rule is *modus ponens*, according to which an implication ($A \rightarrow B$) and its antecedent (A) logically imply its consequent (B). Schematically, this rule may be depicted in the following way:

$$\frac{\begin{array}{c} A \rightarrow B \\ A \end{array}}{B}$$

Besides the syntactical, a semantic characterization of propositional calculus is also needed. Interpretation in this calculus amounts to ascribing the Boolean values of truth (1) or falsehood (0) to propositional variables. The sentential connectives are defined as follows:

$\neg A$ is true, if and only if A is false, otherwise it is false.
$A \wedge B$ is true if and only if A is true and B is true, otherwise it is false.
$A \rightarrow B$ is false if and only if A is true and B is false, otherwise it is true.
$A \vee B$ is false if and only if A is false and B is false, otherwise it is true.

The above semantic characterization of sentential connectives is usually presented in the form of truth-tables. The following is an example of a table for implication:

→	**1**	**0**
1	1	0
0	1	1

Elementary sentences are the most basic elements taken into account when analyzing the validity of arguments using propositional logic. The other type of logic developed by Frege and Russell – first order predicate logic – enables one to take into account the inner structure of elementary sentences. In the structure predicates (corresponding to the verb-part of a sentence), the predicate's arguments (corresponding to the subject) and quantifiers (statements that the predicate refers to some or all objects in the language's domain) are distinguished.

The alphabet of first order predicate logic looks as follows. The predicates are usually denoted by capital letters: P, Q, R, S. In order to increase the readability of the formalizations, however, we will denote predicates with words, such as `read, father_of, charged_with`. Variables referring to the predicate's arguments will be denoted by small letters x, y, z. Arguments will also sometimes be referred to with names written in italics, e.g., *john, car_of_peter, house_of_hanna*, etc. The difference between names and variables is clear: a given name (e.g., *john*) denotes a particular object (e.g., a particular person), whilst a variable x is not ascribed to a particular object, and only refers to an object determined in a general, abstract way.

In the alphabet of first order predicate logic there are also logical constants: truth-functional functors (the same as in propositional logic, i.e., $\neg$, $\wedge$, $\rightarrow$ and $\vee$) and two quantifiers: general $\forall$, and existential $\exists$.

In the above presented symbolism the sentence:

John is convicted of murder

could be written as follows:

`convicted_of_murder`(*john*)

whilst the sentence:

Some people are convicted of murder

is:

$\exists$ x(`convicted_of_murder`(x))

The rules for forming compound expressions of first order predicate logic are as follows. First, atomic formulas, i.e., an *n*-ary predicate with *n* individual variables or names (e.g., `convicted_of_murder`(*john*), `convicted_of_murder`(x), `father_of`(*john, bill*), etc.) are well formed formulas of first order predicate logic. Second, if A and B are well formed formulas of first order predicate logic, $\neg$A, A $\wedge$ B, A $\rightarrow$ B, A $\vee$ B as well as $\forall$ x A and $\exists$ x A are also well formed formulas.

As in the case of propositional logic, we shall not present the axioms of first order predicate logic. Furthermore, there exists a version of *modus ponens* for first order predicate logic.

The semantic definitions (truth-tables) of functors in first order predicate logic are not different from definitions of the same functors in propositional logic. Two additional logical constants – $\forall$ and $\exists$ – are defined in the following way: the expression $\forall$ x(`predicate`(x)) is true if and only if all the objects belonging to the domain of the discourse can truly be said to be `predicate`. The expression $\exists$ x(`predicate`(x)) in turn, is true if and only if in the discursive domain there exists at least one object that can truly be said to be `predicate`.

Interpretation in first order predicate logic is as follows. First, a set constituting the discourse's domain is chosen (intuitively this set is comprised of the objects existing in the world). To the 1-ary predicates there are ascribed subsets of the discourse domain (containing only those objects of which the given predicate can truly be predicated); the 2-ary predicates are ascribed sets of ordered pairs of objects from the domain; the 3-ary predicates – sets of ordered triples, etc. In addition, every individual constant (name) is ascribed a determined object from the domain of the discourse.

It is worth adding that both propositional logic and first order predicate logic are sound and complete.

2.2.2 Paradoxes of Material Implication

Before we attempt to show how legal reasoning is reconstructed with the use of propositional logic and first order predicate logic, it is necessary to mention the controversies surrounding material implication. As is well known, different functors (sentential connectives) are designed to "correspond" to different connectives of natural language: disjunction to "or", conjunction to "and", etc. It is usually held that the semantic characterization of $\wedge$ and $\vee$ is in accordance with the use of "and" and "or" in natural language. For example, the sentences in which "and" occurs are held to be true only when both sentences connected by use of "and" are true.

A number of controversies are connected, however, to implication. This functor is said to correspond to conditionals of the form "if . . ., then . . .". The semantic characterization of implication, however, seems not to meet – in certain circumstances at least – the criteria for using conditionals. This is an extremely important problem as it is clear that conditionals are the natural way of expressing legal norms.

Let us look once more at the truth-table for implication:

→	**1**	**0**
1	1	0
0	1	1

The last row is troublesome. According to it, an implication is true in each case in which its antecedent is false. This leads to the following sentences being established as true:

(1) If New York is the capital of the USA, then water boils at 30°C.
(2) If 2 + 2 = 5, then Washington, D.C. is the capital of the USA.

The ascription of truth to the two above sentences may seem counter-intuitive.[6] However, as W.V.O. Quine observes rightly,[7] the following sentence may seem similarly counter-intuitive:

(3) If 2 + 2 = 4, then water boils at 100°C.

With (3) we still feel that "something is wrong", although both the antecedent and consequent of the implication are true. From this we may conclude that sentences (1)–(3) are counter-intuitive because of what they say (there is no connection between the meaning of the antecedent and the consequent), and not because of the rules put forward in the truth-table for implication.

Quine's solution to the problems of material implication is not, however, fully satisfactory. The semantic characterization of implication leads to acceptance of the following expressions as tautologies of the classical propositional calculus (these are the paradoxes of the material implication)[8]:

$$((p \to q) \land (r \to s)) \to ((p \to r) \lor (q \to s))$$
$$\neg (p \to q) \to p$$

According to the former, the following argument is logically valid:

If John is in Paris, then he is in France; and if John is in London, then he is in England. Therefore, if John is in Paris, he is in London or if he is in France, he is in England.

The second of the tautologies leads us to accept the following chain of reasoning:

It is not true, that if there exists God, the prayers of evil people will be heard. Therefore there exists God.

It is difficult to apply Quine's solution to the two presented arguments. Should we say, then, that material implication does not correspond to conditionals?

Both positive and negative answers to this question have been advocated.[9] The positive answer leads usually to the development of new functors that fit better the criteria for using conditionals in natural language; a case in point is the development of relevant logics.[10] We will not, however, discuss these formalisms in any detail. It is sufficient to note that significant doubts exist over the relevance of material implication for representing natural language conditionals. This is of great importance for us, because – as noted above – it is usually held that every legal norm can be expressed in an "If . . ., then . . ." sentence.

2.2.3 Examples

We will start our reconstruction of legal reasoning by use of classical logic with a simple example. According to Article 278§1 of the Polish penal code (kk), "whoever takes somebody else's property shall be imprisoned for term between 3 months and 5 years". Let us imagine that the accused, John, stole Adam's bicycle. What is the argument that leads to John's conviction? It may look something like this:

(1) Whoever takes somebody else's property shall be imprisoned for a term between 3 months and 5 years.

(2) John has taken somebody else's property (for he has stolen Adam's bicycle).

Therefore: (3) John shall be imprisoned for a term between 3 months and 5 years.

This argument may be reconstructed in classical propositional logic, assuming that (1) is an implication:

(1) $p \rightarrow q$

(2) p

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(3) q

A valid form of argument has been applied here – *modus ponens*. What are the variables substituted for? If (2) is substituted with p, then p must stand for "John has taken somebody else's property"; q is, of course, "John shall be imprisoned for a term between 3 months and 5 years". If so, "$p \rightarrow q$" should be read: "If John has taken somebody else's property, then John shall be imprisoned for a term between 3 months and 5 years". This sentence is not, however, equivalent to Article 278§1 kk! That is clearly visible when one analyzes some other case, let us say a situation in which Adam has stolen Ted's car. Adam bears responsibility according to the same norm as John, namely Article 278§1 kk. The judge applies then the same form of argument as in the previous case:

(1) Whoever takes somebody else's property shall be imprisoned for a term between 3 months and 5 years.

(2) Adam has taken somebody else's property (for he has stolen Ted's car).

Therefore: (3) Adam shall be imprisoned for a term between 3 months and 5 years.

This time (2) cannot be substituted with p (for p stands already for "John has taken somebody else's property"); similarly (3) cannot be written as q. Let us therefore choose other variables, r and s. Now (1) in our formalization is $r \rightarrow s$. Our reconstructions show that, in classical propositional logic, it is difficult to formalize the first premise, which is a formal counterpart of Article 278§1 kk, of either argument. In the case of John's crime we obtain the sentence: "If John has taken somebody else's property, then John shall be imprisoned for a term between 3 months and 5 years", whilst in Adam's: "If Adam has taken somebody else's property, then Adam shall be imprisoned for a term between 3 months and 5 years".

One can apply here the following trick to maintain that both arguments have the same form:

(1) If the accused takes somebody else's property, he shall be imprisoned for a term between 3 months and 5 years.
(2) The accused has taken somebody else's property.
Therefore: (3) The accused shall be imprisoned for a term between 3 months and 5 years.

Substituting *p* with "The accused has taken somebody else's thing", and *q* for "The accused shall be imprisoned for a term between 3 months and 5 years", we obtain the same logical schema for the cases of both Adam and John. This solution is, however, not acceptable. First, it is based on the fact that the term "accused" refers to different persons in different contexts. Second, whilst premise (1) of both arguments is – as required – identical, premise (2) is likewise– and counter-intuitively – identical.

The indicated problems occur because of the fact that, in propositional logic, one cannot reconstruct the inner structure of the sentences composing the analyzed arguments. Much more can be done in first order predicate logic. Applying this calculus enables the following reconstruction of our examples:

(1) $\forall x(\texttt{takes}(x) \rightarrow \texttt{imprisoned}(x))$
(2) $\texttt{takes}(\textit{john})$

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(3) $\texttt{imprisoned}(\textit{john})$[11]

`takes` stands here for "takes somebody else's property", `imprisoned` – "shall be imprisoned for a term between 3 months and 5 years", and *john* is a name for John. If we use *adam* as a name for Adam the judge's reasoning in Adam's case may be presented as follows:

(1) $\forall x(\texttt{takes}(x) \rightarrow \texttt{imprisoned}(x))$
(2) $\texttt{takes}(\textit{adam})$

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(3) $\texttt{imprisoned}(\textit{adam})$

The presented formalization has the required features. In both arguments premise (1) is identical, but premise (2) is different; in other words, the structure of the analyzed examples, as reconstructed with the use of first order predicate logic, seems to resonate with our intuitions.

Arguments of the type presented above are traditionally called legal syllogisms. The concept of a legal syllogism played a crucial role in legal positivism. The continental positivists held that legal reasoning has (or should have) the form of legal syllogism. Every such syllogism consists of two premises and a conclusion. The first premise is a general and abstract legal norm, as e.g., Article 278§1 kk from our example: "Whoever takes somebody else's property shall be imprisoned for a term between 3 months and 5 years". The second premise describes a state of affairs, e.g., "John has taken somebody else's property". Finally, the conclusion is an individual and concrete legal norm; in the case of our example: "John shall be imprisoned for a term between 3 months and 5 years". Therefore, the logical reconstruction presented below:

(1) $\forall x(\mathtt{takes}(x) \rightarrow \mathtt{imprisoned}(x))$
(2) $\mathtt{takes}(\mathit{john})$
———
(3) $\mathtt{imprisoned}(\mathit{john})$

is an instance of a legal syllogism.

Various arguments have been put forward against legal syllogism as a correct reconstruction of legal reasoning. It has been maintained, for example, that the syllogism is impossible, for the logic of norms is impossible. It has also been held that the essence of legal reasoning is the process of valuation and not logical consequence, as positivists and other adherents of formal logic seem to suggest. These problems will be dealt with below (see Section 2.4). Here, another issue must be addressed, namely the thesis that legal syllogism is trivial because it can be applied only after all the significant problems of legal reasoning have already been solved.

The core of this objection may easily be displayed using our earlier example. In describing the state of affairs we said that John has stolen Adam's bicycle. Meanwhile, premise (2) says that John has taken somebody else's property. These are certainly two different sentences. In one of our informal reconstructions we dealt with this problem saying: "John has taken somebody else's property (for he has stolen Adam's bicycle)". This formulation indicates that we are concerned here with an additional stage in reasoning (please note our use of the word "for"), which has not been accounted for in our logical reconstruction of legal syllogism. It is relatively easy, however, to fix this problem, e.g., in the following way:

(1) $\forall x(\mathtt{steals\_bicycle}(x) \rightarrow \mathtt{takes}(x))$
(2) $\mathtt{steals\_bicycle}(\mathit{john})$
(3) $\mathtt{takes}(\mathit{john})$
———

This argument follows from two premises, i.e., "Whoever steals Adam's bicycle takes somebody else's property" and "John has stolen Adam's bicycle" to the conclusion that John has taken somebody else's property.

In legal theoretic literature, the conclusion of the kind of argument that we have just presented is called the interpretational decision.[12] Here, Article 278§1 kk is interpreted; we say that the expression it contains ("to take somebody else's property") refers, *inter alia*, to the act of stealing Adam's bicycle. The argument leading to the interpretational decision can of course be adduced to our main syllogism:

(1) $\forall x(\mathtt{takes}(x) \rightarrow \mathtt{imprisoned}(x))$ [general and abstract legal norm]

(2) ∀ x(steals_bicycle(x) → takes(x)) [premise of the interpretational decision]
(3) steals_bicycle(*john*) [a description of the state of affairs]
(4) takes(*john*) [from (2) and (3), *modus ponens*]

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(5) imprisoned(*john*) [from (1) and (4), *modus ponens*]

When we look more closely at the above reconstruction, some other troublesome elements can be found. A case in point is, for example, the conclusion of our argument: "John shall be imprisoned for a period between 3 months and 5 years". Judges never formulate their sentences in such a manner, but stipulate exactly the period of imprisonment, saying, for example, that John shall be imprisoned for 2 years. The argument of the judge which leads to the determination of the duration of imprisonment may also be reconstructed logically. Let us denote all the circumstances that make a 2 year sentence a just one by circumstances; two_years means "shall be imprisoned for 2 years".The judge's argument may be reconstructed as follows:

(1) ∀ x((imprisoned(x) ∧ circumstances(x)) → two_years(x))
(2) imprisoned(*john*)
(3) circumstances(*john*)

---

(4) two_years(*john*)

The key premise (1) says that when somebody should be imprisoned for a period between 3 months and 5 years (imprisoned(x)), and there are circumstances justifying a two year sentence (circumstances(x)), the person shall be imprisoned for two years (two_years (x)). Premise (2) is the conclusion of our syllogism in its earlier version. Finally, premise (3) says that the circumstances justifying the two year sentence do obtain in John's case. The conclusion of the analyzed argument, which Jerzy Wróblewski calls *consequences choice decision*, says: John shall be imprisoned for two years. We may now present our syllogism in a more complete form:

(1) ∀ x(takes(x) → imprisoned(x)) [general and abstract legal norm]
(2) ∀ x(steals_bicycle(x) → takes(x)) [premise of the interpretational decision]
(3) steals_bicycle(*john*) [description of the state of affairs]
(4) takes(*john*) [from (2) and (3), *modus ponens*]
(5) imprisoned(*john*) [from (1) and (4), *modus ponens*]
(6) ∀ x((imprisoned(x) ∧ circumstances(x)) → two_years(x)) [premise of the consequences choice decision]
(7) circumstances(*john*) [further description of the state of affairs]

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(8) two_years(*john*) [from (6), (5) and (7), *modus ponens*]

In this way we obtain a complex and logically valid schema. The argumentative structure presented can be further elaborated upon. For example, one can take into account the *validity decision*, i.e., the decision leading to the establishment of the validity of Article 278§1 kk (that has been reconstructed as (1)). Similarly, the arguments leading to the establishment of (3) and (7) (the *evidential decisions*) can be reconstructed logically.

Legal syllogism, as it is usually presented, could be called trivial. It is not, however, a complete logical reconstruction of legal reasoning. Every decision made during a judge's reasoning can be analyzed with the use of logical tools, resulting in complicated argumentative structures.

2.3 DEONTIC LOGIC

2.3.1 Possible World Semantics

From our perspective, i.e., in the context of developing an adequate "legal logic", one cannot overlook modal logics. These are logics that, besides traditional logical connectives, like negation or implication, offer also modal functors – aletic ("it is possible that", "it is necessary that"), epistemic ("it is known that", "it is believed that") or deontic ("it is forbidden that", "it is obligatory that", "it is permitted that"). Aletic modal logics were first described by the American logician C.I. Lewis. Their development is, however, connected with the creation of possible world semantics in the 1950s.[13]

The notion of a possible world is intuitively clear. In 2002 Brazil won the World Cup. But one can easily imagine a situation in which they failed to reach the final. They did not, and for that reason the world in which Brazil lost is not an actual world; but it is a possible one. If you are sitting in a chair right now, then in one minute you may still be sitting, but you may also be standing. These are two possible worlds: in one of them you are sitting and in the other – standing. But if you are in Kraków now, in one minute you may be sitting or standing in Kraków, but not in New York. There exists, of course, such a possible world, in which you may be standing in New York in one minute's time; but this world is not possible relative to the actual world, in which you are in Kraków. Therefore, one can differentiate between an "absolutely" possible world – i.e., every consistent state of affairs (the fact that you are in New York in a minute is not *logically* inconsistent) – and a "relatively" possible world, i.e., a world that is possible relative to the actual world (we will say that the possible world is *accessible* from the actual world). Brazil winning the 1998 World Cup is possible in the absolute sense. Today it is not, however, possible "relatively", for the 1998 World Cup was won by another team. At the beginning of 1998, however, it was possible not only "absolutely", but also "relatively".

It is due to Saul Kripke[14] that the above presented intuitions were encapsulated in a very elegant mathematical form. Kripke showed how one can build an adequate mathematical object that takes into account the mentioned differentiations, and serves as a semantic model for certain logical calculi. The object is an ordered quadruple:

$$\langle w_a, W, R, v \rangle$$

w_a stands here for the actual (our) world. W is a set of the possible worlds. R is the accessibility relation. If the relation R holds between two worlds (say w_i and w_j) – we will refer to this fact by (w_iRw_j) – it means that the world w_j is accessible from the world w_i. Finally, v is the interpretation function that ascribes truth or falsehood to every sentence in the given possible world ($v_w(p) = 1$ or $v_w(p) = 0$).[15] Therefore, if we substitute p for "Brazil won the World Cup in 2002", then the interpretation function ascribes to p in the actual world w_a, the value of truth ($v_{w_a}(p) = 1$), whilst in some other possible world, w_i, in which Brazil failed to win, $v_{w_i}(p) = 0$.

The semantic structure thus constructed enables necessity and possibility to be defined. We will say that p is necessary in the actual world $w_a(\Box p)$, if p is true in all the worlds accessible from w_a; p is possible in $w_a(\Diamond p)$, if p is true at least in one world accessible from w_a. It is clear that $\Box p$ is true if and only if $\sim\Diamond\sim p$ is also true, i.e., if there does not exist such a possible world accessible form w_a in which $\sim p$ is true. The last remark shows that necessity and possibility are mutually definable ($\Box p \Leftrightarrow \sim\Diamond\sim p$).

2.3.2 Deontic Logic

S. Kripke developed possible world semantics with the aim of analyzing the concepts of necessity and possibility. Rather quickly, however, it turned out that Kripke's mathematical tool could serve perfectly well the analysis of other notions, such as "to know" and "to believe" or – important for us – "obligatory", "forbidden" and "permitted".

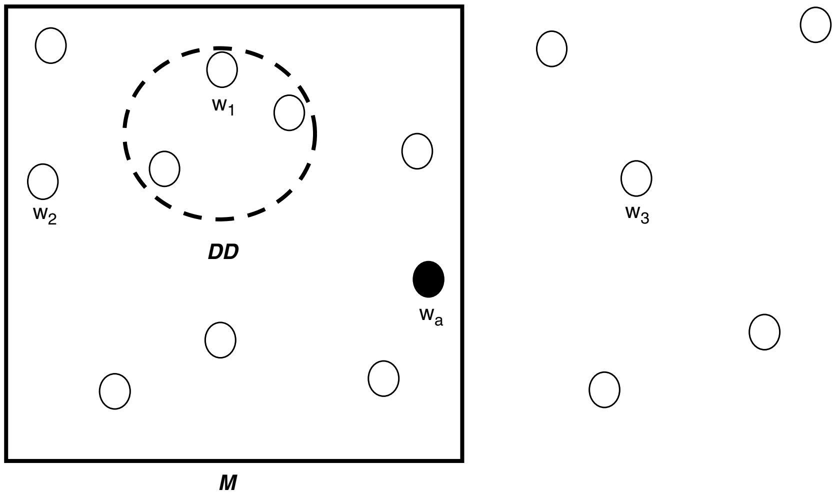
This does not mean, however, that the logic of obligations, i.e., *deontic logic*, originated with the development of possible world semantics in the mid-twentieth century. In "Nicomachean Ethics", Aristotle had already analyzed arguments like the following: "if 'everything sweet should be tasted', and 'a given thing is sweet', i.e., it is one of the sweet things, then a man who is capable should taste this given thing".[16] This is an example of applying the practical syllogism that formalizes normative reasoning. Another, much later work that includes some considerations of deontic logic is Leibniz's "Elementa iuris naturalis".

In twentieth century philosophy analyses of normative sentences can be found in the work of B. Bolzano, A. Höfler and E. Husserl. More advanced conceptions of practical discourse were developed by E. Lapie and E. Mally (logic of will), and by E. Menger (logic of habits). They were created in 1902, 1926 and 1934 respectively. Only small parts of those works were devoted to deontic logic. A different story can be told about the works of W. Dubislav, J. Jørgensen, A. Hofstadter, J.C.C. McKinsey, R.M. Hare and R. Rand, who each tried to formalize arguments that are normative *par excellence*.[17]

It is usually held, however, that the birth of deontic logic took place in 1951, the year of publication of G.H. von Wright's paper "Deontic Logic". At the same time similar problems were investigated by J. Kalinowski and O. Becker. The former published the results of his research in 1953, the latter in 1952.

The first systems of deontic logic can be characterized as syntactic. The situation changed with the above mentioned development of possible world semantics. Semantics enables one to define in a very intuitive way the notion of obligation. Let *O* stand, as usually, for "it is obligatory that . . .". What does it mean if the sentence *Op*, "it is obligatory that *p*", is true in the actual world (w_a)? One can imagine that enacting a norm of behavior simply involves a legislator picking out a subset of the worlds that are possible relative to w_a. In the chosen worlds things stand as the legislator wishes them to. Let us notice (see the figure) that the set DD of deontically perfect worlds has to be a subset of the set M that contains worlds possible relative to w_a. This condition reflects the basic principle of law, i.e., *impossibilium nulla obligatio est*. The legislator cannot make obligatory (include into DD) what is not possible (i.e., what does not belong to M).

With DD established by the legislator we can give the conditions of truth for *Op*. *Op* is true in w_a if and only if *p* is true in every world of DD. As an illustration consider the sentence "It is obligatory that John does not steal". This sentence is true in w_a, if the sentence "John does not steal" is true in every world of DD. Were the latter sentence true only in one of the worlds from DD, e.g., in w_1, "It is obligatory that John does not steal" would not be true; therefore, John would not be obliged to refrain from stealing. The world w_2 is a world that is accessible from the actual world, but the legislator does not regard it as deontically perfect. Finally, w_3 is not accessible from w_a, which amounts to saying that it cannot become an actual world. Therefore, w_3 cannot be included in DD.



In deontic logic, in addition to the functor of obligation *O*, there are also two other functors: the functor of prohibition *F* (forbidden) and the functor of permission *P* (permitted). Those functors can be defined in a natural way with the use of functor *O*. If *p* is forbidden, i.e., *Fp*, then it is obligatory that $\neg p$:

$$Fp \equiv O\neg p$$

Furthermore, if *p* is permitted, *Pp*, then it is not true that it is obligatory that $\neg p$:

$$Pp \equiv \neg\, O\neg p$$

Let us address how these definitions work in practice. Suppose that a legislator forbids John to steal. Let *p* stand for "John steals". According to the definition *Fp* is equivalent to $O\neg p$, i.e., John is under an obligation not to steal. Therefore, in order to make the sentence "it is forbidden for John to steal" true in w_a, the sentence "John does not steal" has to be true in every single world of DD. Let us now turn our attention to the sentence "It is permitted for John to steal" – *Pp*. According to the definition this sentence is equivalent to $\neg O \neg p$. And the latter sentence is true only if the sentence $O\neg p$ is false. As we know, the falsity of $O\neg p$ requires that, in at least one possible world of DD, the sentence $\neg p$ is false – which equates to *p* being true. In the described situation, the sentence "it is permitted for John to steal" is true, if there exists a world in DD in which the sentence "John steals" is true. Let us observe that it is possible that the sentence "John steals" is true in every world belonging to DD. Then, both *Pp* ("It is permitted for John to steal") and *Op* ("It is obligatory for John to steal") are true.[18]

The semantic ideas presented above are realized in various systems of deontic logic, as, for example, in standard deontic logic (SDL).[19] It should be noted that the soundness and completeness theorems have been proved for SDL.

2.3.3 Paradoxes of Deontic Logic

There are numerous deontic logics that differ to greater or lesser degrees. There are various reasons for the search for new logics of obligation, one of the most important being the paradoxes of deontic logic.

Amongst those paradoxes one can list the Ross paradox. This has to do with the fact that, in SDL, the argument from

$$Op$$

to

$$O(p \vee q)$$

is valid.

Let us substitute *p* with "send the letter" and *q* with "burn it". Thus, a paradoxical reasoning is constructed: "if it is obligatory to send the letter, it is obligatory to send it or burn it".

There is no consensus as to whether the Ross Paradox can be called a real problem. Some adhere to the thesis that there is a problem in the reasoning, since the acceptance of the second norm seems counter-intuitive. Others deny this, saying that the mere fact that one norm ($O(p \vee q)$) follows logically from another norm (Op) does not mean that the latter ceases to be binding. Therefore, if the norm "it is obligatory to send the letter or to burn it" is fulfilled by burning the letter, the norm "it is obligatory to send the letter" is broken. In other words, the fulfillment of a norm B that follows logically from a norm A, does not necessarily lead to the fulfillment of norm A.[20]

It is not the Ross Paradox, however, nor problems of its kind that constitute the major puzzle of deontic logic. It is usually held that the most difficult problems facing deontic logicians are contrary-to-duty (CTD) paradoxes. Those paradoxes arise in connection with CTD-norms, i.e., norms that require the breaking of another norm as a condition of their application. Let us look more closely at a famous example known as the Chisholm Paradox.[21] This paradox is connected with the following four sentences:

(1) It is obligatory for a certain man to help his neighbors.
(2) It is obligatory that if he helps them, he tells them about it.
(3) If he does not help them, he should not tell them he helps them.
(4) The man does not help his neighbors.

It is easy to observe that the norm expressed in (3) is a CTD-norm, for its antecedent ("if he does not help them") describes the fact of breaking another norm (norm (1)). It is usually held that, intuitively, sentences (1)–(4) are mutually consistent and logically independent of each other (none of the sentences follows logically from others). Let us try to formalize Chisholm's example. If the sentence "A certain man helps his neighbors" is substituted with p, and "He tells them about it" with q, sentences (1)–(4) can be formalized in SDL in the following manner:

(1) Op
(2) $O(p \rightarrow q)$
(3) $\neg p \rightarrow O \neg q$
(4) $\neg p$

Unfortunately, this formalization is inconsistent. In SDL it follows from sentences (1) and (2) that Oq, and from (3) and (4) – $O \neg q$. The intuitively consistent set of sentences turns out in our formalization to be inconsistent.

It is easy to observe that the above presented formalization is not the only way of reconstructing sentences (1)–(4) in SDL. The issue here is how the conditional duties should be formalized. Let us notice that legal norms usually take the form of a conditional; traditionally it is maintained that every norm consists of an antecedent and a consequent. It is held, moreover (as already noticed during the discussion of first order predicate logic) that this very structure is captured by the material implication. In the language of deontic logic, however, there occurs a problem. If p is the antecedent and q the consequent, the given norm can be formalized in two ways – either as:

$$p \rightarrow Oq$$

or

$$O(p \rightarrow q)$$

In the analyzed example norm (3) was formalized in the former manner, and norm (2) in the latter. One may consider it incorrect that the two conditional norms were treated differently (even though their natural language formulation encouraged this). In light of the above observation, two solutions are possible: either represent norm (2) as $p \rightarrow Oq$, or norm (3) as $O(\neg p \rightarrow \neg q)$. It turns out, however, that neither of those solutions is acceptable. In the new formalizations some of the analyzed sentences are logically dependent on others. In the case of the former, norm (2) follows logically from sentence (4), and in the case of the latter, norm (3) follows from (1). It can therefore be concluded that attempts to formalize Chisholm's example lead to results that are intuitively unacceptable.

The CTD paradoxes, together with some other problems, serve as a reason for creating deontic logics that differ to a greater or lesser degree from SDL. We will not attempt to describe them here. We would only like to indicate the mechanism that leads to developing new logical systems: usually, when a new logical system is built, it is provided with an intuitively sound semantics and is tested on different sets of examples. Sometimes such problems as CTD paradoxes occur. They highlight the weaknesses of the developed systems and suggest that a search for other solutions may be needed. It is vitally important, then, to consider when and why something may seem paradoxical. Clear intuitions decide this matter. Chisholm's example is paradoxical, because we intuitively held the sentences that constitute it to be consistent and logically independent. However, it seems impossible to obtain a consistent and logically independent formalization of those sentences in SDL.

2.3.4 Examples

The examples presented in Section 2.2.3 may seem atypical as regards legal reasoning. The way in which Article 278§1 kk is formulated does not make it immediately obvious that we are dealing with a legal norm. In that provision such phrases as "ought to", "it is forbidden" or "it is allowed" do not occur. This is, however, the common way of formulating legal text: legal provisions are usually expressed in an indicative mood. This does not mean, however, that at the logical level they should be reconstructed without the use of deontic operators. If the legislator says that "whoever takes somebody else's property shall be imprisoned for a period between 3 months and 5 years", it seems intuitive to reconstruct that statement in the following way: "it ought to be the case that whoever takes somebody else's property shall be imprisoned for a period between 3 months and 5 years". The following represents another way of introducing the deontic operator into Article 278§1 kk: whoever takes somebody else's property ought to be imprisoned for a period between 3 months and 5 years. In the first case Article 278§1 kk obtains the following symbolic form:

$$O\ (\forall\ x(\texttt{takes}(x) \rightarrow \texttt{imprisoned}(x)))$$

and in the second:

$$\forall\ x(\texttt{takes}(x) \rightarrow O(\texttt{imprisoned}(x)))$$

If we choose the second option, our basic legal syllogism in John's case will look as follows:

(1) $\forall\ x(\texttt{takes}(x) \rightarrow O(\texttt{imprisoned}(x)))$
(2) $\texttt{takes}(\textit{john})$

---

(3) $O(\texttt{imprisoned}(\textit{john}))$

In this formalization it is clear that both (1) and (3) are normative in character.

In the Article 278§1 kk example, one more thing may seem counter-intuitive. Whilst reading a penal code one would expect to find norms of behavior: statements of what we should and should not do. Article 278§1 kk, however, says nothing of this kind. It is often maintained that penal codes contain only sanctioning and no sanctioned norms. And it is the former that determine the obligations and rights of a citizen, whilst the addressee of a sanctioning norm is a state authority. Such a norm obliges the authority to act in a specific way where a sanctioned norm has been broken. For instance, in the case of Article 278§1 kk, the action of the authority consists in imprisoning the person who broke the sanctioned norm for a term of 3 months to 5 years.

In principle, in penal codes only sanctioning norms are expressed. It is sometimes held that sanctioned norms are "outside the code". This is, of course, a metaphor. Sanctioned norms are not directly stated in penal codes, but we can reconstruct them on the basis of directly stated sanctioning norms. If Article 278§1 kk says that "whoever takes somebody else's property shall be imprisoned for a term between 3 months and 5 years", it is a basis for formulating the following sanctioned norm: one should not in any circumstances take somebody else's property.

It may be observed that a judge does not need the norm "one should not in any circumstances take somebody else's property" in order to give her judgment. This is true, but one can easily imagine intuitively correct arguments in which sanctioned norms serve as premises or conclusions. For instance: one should not in any circumstances take somebody else's property, and because stealing a bicycle constitutes an instance of taking somebody else's property, therefore one should not in any circumstances steal a bicycle. In order to put forward such an argument one has to know that there exists a norm stating that one should not in any circumstances take somebody else's property. This norm is not expressed directly in the penal code. It has to be reconstructed and such reconstruction is not always a trivial task. We shall not analyze this problem here in any detail. It is worth observing, however, that there are no *a priori* reasons excluding the search for logical schemata of such reconstruction.

One does not find similar problems in civil codes. Let us consider, as an example, Article 415 kc of the Polish civil code: "whoever intentionally causes damage to someone has to redress it". This provision is not only addressed to "normal citizens", but also includes a deontic operator "has to". One can formalize Article 415 in the following way:

$$\forall x(\texttt{causes\_damage}(x) \rightarrow O(\texttt{redresses}(x)))$$

Let us consider now whether there exist reasons to formalize legal reasoning with the use of deontic logic as opposed to first order predicate logic. It can be maintained that, from a practical point of view, there is no difference between the two. Let us look once again at Article 415 kc. In our deontic-logic formalization it has the form:

∀ x(causes_damage(x) → O(redresses(x)))

The following formalization that uses the predicate “has to redress the damage” and sticks to first order predicate logic seems equally good:

∀ x(causes_damage(x) → has_to_redress(x))

If a judge finds that John has intentionally caused damage to someone (causes_damage(*john*)), she would conclude – in the first formalization- that *O*(redresses(*john*)), and in the second: has_to_redress(*john*). Both conclusions are the same: John is obliged to redress the damage. Therefore, from a practical perspective, there is no difference here.

There are, however, at least two reasons for using deontic logic. Let us try, first, to formalize both Article 415 kc and, e.g., Article 728§1 kc: “the bank is obliged to inform the account holder about every change in the account status”. We can formalize this in a deontic calculus as follows:

∀ x((bank(x) ∧ change(x)) → *O*(inform(x)))

In first order predicate logic it becomes:

∀ x((bank(x) ∧ change(x)) → obliged_to_inform(x))

Let us compare now both formalizations of Articles 415 kc and 728§1 kc. In the former case we have:

∀ x(causes_damage(x) → has_to_redress(x))

and

∀ x((bank(x) ∧ change(x)) → obliged_to_inform(x))

whilst in the latter we have:

∀ x(causes_damage(x) → *O*(redresses(x)))

and

∀ x((bank(x) ∧ change(x)) → *O*(inform(x)))

Only in the latter case is it clearly visible that we are dealing with the same notion of obligation (the deontic operator *O*). The formalization in classical logic forces us to include the notion of obligation in the predicate letters. This may seem counter-intuitive.

The second reason why deontic logic is better for formalizing legal reasoning than classical logic is even more profound. There are situations in which we infer one norm from another. Here is a simple example: if one should not kill then one should not kill on Sundays. Or: if it is obligatory that a judge behaves responsibly and it is obligatory that a judge is honest, then it is obligatory that a judge behaves responsibly and is honest. Finally: if it is obligatory that Adam does not steal Sven's skies, then it is forbidden for Adam to steal Sven's skies.

Let us try to formalize those three arguments using different logical systems. For the sake of simplicity we will confine ourselves to propositional logic: classical and deontic. Let us begin with classical logic. Assume that p stands for "one should not kill" and q for "one should not kill on Sundays". The problem is that from just p, q does not follow. We need some other paraphrase. The norm "one should not kill" can be formulated – as above – as a conditional norm, in which the conditions of applications are tautological (T):

$$T \rightarrow p$$

Because of the fact that T is true in any circumstances, $T \rightarrow p$ expresses an unconditional obligation described by p. From this formula, it follows by the rule of the strengthening of antecedent that:

$$(T \wedge q) \rightarrow p$$

Naturally, q does not stand here for "one should not kill on Sundays"; it can be expressed by something like "acts on Sundays". The norm "one should not kill on Sundays" is represented by the whole expression $(T \wedge q) \rightarrow p$.

Let us look, in turn, at the second argument. Let p stand for "it is obligatory that a judge behaves responsibly", q for "it is obligatory that a judge is honest" and r for "it is obligatory that a judge behaves responsibly and is honest". From p and q it does not follow that r. This time we can also try another, perhaps slightly counter-intuitive, paraphrase. Let us assume that we are dealing with conditional norms: "if someone is a judge, then she should behave in a responsible way" and "if someone is a judge, then she should be honest". Let us formalize them in the following way:

$$p \rightarrow q$$
$$p \rightarrow r$$

From those two premises a sentence follows:

$$p \rightarrow (q \wedge r)$$

which can be read: if someone is a judge then she should behave responsibly and should be honest. It is not exactly what we have been looking for (in the consequent of the norm there are two sentences connected by a conjunction, and not a single sentence in which there is the compound predicate "behaves in a responsible way and is honest"). This problem, however, results from using propositional logic instead of first order predicate logic. It has nothing to do with the fact that we abstained from using deontic logic here.

The last of the three arguments poses the biggest challenge for classical logic. If we substitute *p* for "it is obligatory that Adam does not steal Sven's skies" and *q* for "it is prohibited for Adam to steal Sven's skies", then *q*, of course, does not follow from p. In the analyzed case, however, no paraphrase can be found that would enable us to deal with the problem.

In SDL there is no problem whatsoever! Let *p* stand for "Adam steals Sven's skies". We can write now:

$$\frac{O \neg p}{Fp}$$

This is a valid reasoning which leads – as we desire – from the sentence $O \neg p$ (it is obligatory that Adam does not steal Sven's skies) to the sentence Fp (it is forbidden for Adam to steal Sven's skies). We apply here simply the definition of the functor *F*.

SDL deals similarly elegantly with the two previous examples. The second of them especially takes a simpler form than in the case of classical logic. Let us substitute *p* for "a judge behaves in a responsible way", and *q* for "a judge is honest". Let us formalize now both norms that serve as premises of our argument, i.e., "it is obligatory that a judge behaves responsibly" and "it is obligatory that a judge is honest":

$$Op$$
$$Oq$$

From those two sentences it follows in SDL that:

$$O(p \wedge q)$$

which reads: it is obligatory that a judge behaves responsibly and that a judge is honest.

One can reasonably question whether such arguments, having norms as premises and conclusions, are important. It seems that the answer should be positive. First, such arguments are actually carried out, therefore a complete theory of legal reasoning should account for them. Second, the notion of the set of logical consequences of a given set of norms is used to define the set of valid legal norms. According to the systemic conception of the validity of law, valid norms are the norms explicitly enacted by a legislator, plus what follows logically from them.

The analyzed examples allow us to say that formalizations in deontic logic are better suited to our intuitions than classical formalizations. Similarly, possible world semantics, as used in deontic logic, seems intuitive. Moreover, SDL is an extension of classical logic. This means that, as far as the arguments that do not include the operators of obligation, permission or prohibition are concerned, deontic logic is equivalent to classical logic. It does not necessarily follow from all this that deontic logic is not problematic. The most important problems are connected with paradoxes. But the role of paradoxes is positive: they indicate what is wrong and encourage the search for new, better deontic systems.

2.4 LOGIC OF ACTION AND LOGIC OF NORMS

2.4.1 Two Types of Obligation

Amongst objections against deontic logics, in addition to the problem of paradoxes, there are several problematic questions of a more general, philosophical nature. We will try to look more closely at two such objections: first, the thesis that deontic logic formalizes the notion of *ought-to-be*, and does not take into account the notion of *ought-to-do*; second, the thesis that deontic logic is not a logic of norms because we cannot say that norms are either true or false. The former problem will serve as a pretext for discussion of the logic of action. The latter, in turn, will allow us to comment on the Jørgensen Dilemma.

Philosophers sometimes distinguish between two concepts of obligation: the first stating what ought to be the case and the second stating what ought to be done.[22] The importance of this distinction is questioned by those who claim that the latter can be reduced to the former. They insist[23] that the sentence "person α ought to do p" is equivalent to the sentence "it ought to be the case that person α does p". If one approves of this reduction then deontic logic, as described in the previous sections, which is a logic of the *ought-to-be* operator, is adequate. There are, however, strong objections against reducing *ought-to-do* to *ought-to-be*.[24] Let us present one of them. P. Geach suggested analyzing the following sentence[25]: "Fred ought to dance with Ginger". According to the reductionist conception, that sentence is equivalent to this: "it ought to be the case that Fred dances with Ginger". The sentence "Fred dances with Ginger" is, however, equivalent to "Ginger dances with Fred" (for the relations of dancing are symmetrical). Instead of saying "it ought to be the case that Fred dances with Ginger" we could also say: "it ought to be the case that Ginger dances with Fred". But now, reversing the direction of the first transformation, we can write that "Ginger ought to dance with Fred". This seems counter-intuitive. From the sentence that Fred ought to dance with Ginger it does not necessarily follow that Ginger ought to dance with Fred.

This problem may also be illustrated from a legal perspective. Let us imagine two persons, John and Adam, concluding an agreement according to which, when a certain condition is fulfilled, John will be under an obligation to sell Adam his car, but Adam will have the right to choose whether he wants to conclude the final agreement and buy the car or not. If a similar analysis to that carried out in the case of Fred and Ginger is carried out here, from the sentence "John ought to conclude a sale agreement with Adam", and from the fact that concluding an agreement is a symmetrical relations, it would follow that Adam ought to conclude a sale agreement with John, which in the described circumstances is paradoxical.

Both presented examples show that the idea of reducing "α ought to do p" to "it ought to be the case that α does p" can lead to counter-intuitive results. One may wonder, however, whether the problems are really connected with the failure to distinguish between *ought-to-be* and *ought-to-do*. The Fred and Ginger example indicates one more feature of SDL: that the concept of obligation involved is impersonal. In our example we started with the sentence "Fred ought to dance with Ginger" and substituted it with "it ought to be the case that Fred dances with Ginger". It is suggested, sometimes, that distinguishing the "obligation from the point of view of Fred" from the "obligation from the point of view of Ginger" suffices to solve the puzzle in question. The sentence "it ought to be the case that Fred dances with Ginger" and the equivalent sentence "it ought to be the case that Ginger dances with Fred" express obligations from the point of view of Fred. In consequence, from the sentence "(From the point of view of Fred) it ought to be the case that Ginger dances with Fred" one cannot derive that Ginger ought to dance with Fred. In this way we concede that obligations differ (for they always relate to a specific person), but we are not forced to say that *ought-to-do* cannot be reduced to *ought-to-be*. This observation will be confirmed when we look in greater detail at the deontic logic of action.

2.4.2 Logic of Action

We would like to analyze now some formalizations of *ought-to-do*. It is interesting that the first attempts at constructing deontic logic aimed to