1-

(b) $ny \tan x$ (1)

2-

(d) -3 (1)

3-

$\because \frac{dv}{dt} = K(4\pi r^2)$ (1) $\frac{1}{2}$, K is constant

$\because V = \frac{4}{3}\pi r^3$ $\frac{1}{2}$

$\therefore \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ (2) 1

From (1), (2) $\therefore \frac{dr}{dt} = K$ 1

i.e the radius decreases at constant rate

3

4-

$$\therefore y = \frac{10 - \cos x}{x}$$

$$\therefore xy = 10 - \cos x \quad \left(\frac{1}{2}\right)$$

by differentiation

$$\therefore x\frac{dy}{dx} + y = \sin x \quad \left(\frac{1}{2}\right)$$

by differentiation

$$\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = \cos x \quad \left(\frac{1}{2}\right)$$

$$\therefore x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = \cos x \quad \left(\frac{1}{2}\right)$$

(2)

(تراعى الحلول الأخرى)

5-

(c) $2 \ln a$ (1)

6-

(a) $e^{-x}\left(\frac{1}{x} - \ln x\right)$ (1)

7-

$\therefore y = 3x^2 - \ln x$

$\therefore \frac{dy}{dx} = 6x - \frac{1}{x}$ (1)

at $x = 1 \Rightarrow \frac{dy}{dx} = 6(1) - 1 = 5$ ($\frac{1}{2}$)

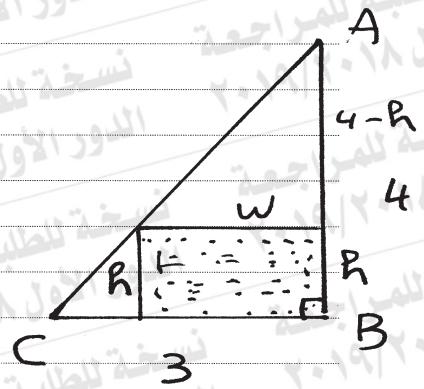
$\therefore$ Equation of Tangent is

$y - 3 = 5(x - 1)$ ($\frac{1}{2}$)

$\therefore 5x - y - 2 = 0$

(2)

8-



$\therefore \frac{w}{3} = \frac{4-h}{4}$

$\therefore w = \frac{12-3h}{4}$ (1) ½

Area $A = hw$

$= \frac{h(12-3h)}{4}$

$\therefore A = \frac{1}{4}(12h - 3h^2)$ ½

$\therefore \frac{dA}{dh} = \frac{1}{4}(12 - 6h)$ ½

$\frac{dA}{dh} = 0 \Rightarrow h = 2$ A' + + max − − ½

$\therefore h = 2$ makes the area max ½

From (1) $\therefore w = \frac{3}{2}$ cm ½

3

(تراعى الحلول الأخرى)

9-

(a) $3x + C$ (1)

10-

(d) $]-\infty, -1[$ and $]0, 1[$ (1)

11-

$\therefore \frac{dy}{dx} = a \csc^2 x$

$\therefore y = \int a \csc^2 x \, dx$ ½

$\therefore y = -a \cot x + C$ ½

$\because (\frac{\pi}{4}, 5) \in$ curve $\therefore -a + C = 5$ (1) } ½

, $\because (\frac{3\pi}{4}, 1) \in$ curve $\therefore a + C = 1$ (2) }

From (1), (2) $\therefore 2C = 6 \Rightarrow C = 3$ ½

, $a = -2$ ½

$\therefore y = 2 \cot x + 3$ ½

(3)

12-

$\because |x-4| = \begin{cases} x-4, & x \geq 4 \\ 4-x, & x < 4 \end{cases}$ cont. at $x=4$

$\therefore \int_0^6 |x-4|\,dx = \int_0^4 (4-x)\,dx + \int_4^6 (x-4)\,dx$ ½

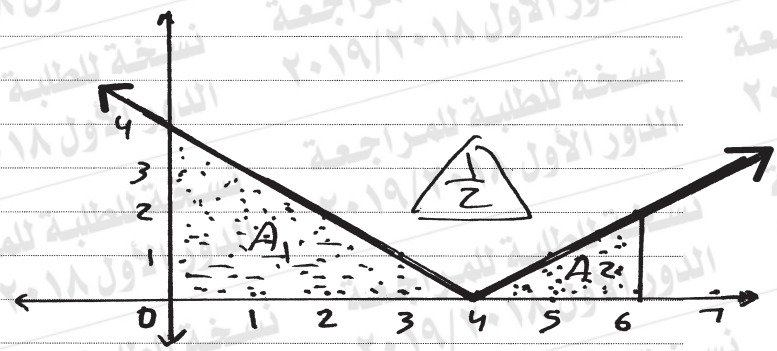
$= \left[4x - \frac{x^2}{2}\right]_0^4 + \left[\frac{x^2}{2} - 4x\right]_4^6$ 1

$= 16 - 8 + (18 - 24) - (8 - 16)$

$= 10$ ½

2

Another solution:



$\int_0^6 |x-4|\,dx = A_1 + A_2$

$= \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 2 \times 2$ 1

$= 10$ ½

2

13-

(d) 2 local min values and 1 local max. value (١)

14-

(a) e (١)

15-

(A) $\because y = x^3 + ax^2 + bx$ (1)

$\therefore y' = 3x^2 + 2ax + b$, $y'' = 6x + 2a$ (½)

$\therefore y''(3) = 0 \Rightarrow 18 + 2a = 0$

$\therefore a = -9$ (½)

$\because (3, -9) \in$ curve

$\therefore -9 = 27 - 9 \times 9 + 3b$

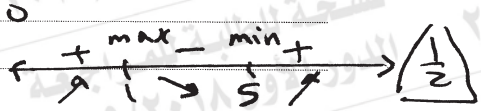
$\therefore b = 15$ (½)

$\therefore y' = 3x^2 - 18x + 15$

$y' = 0 \Rightarrow x^2 - 6x + 5 = 0$

$\therefore (x - 1)(x - 5) = 0$

$\therefore x = 1$ or $x = 5$



(½)

$\therefore y = x^3 - 9x^2 + 15x$

$\therefore y(1) = 7$ is the local max. value (½)

$\therefore y(5) = -25$ is the local min. value (½)

(3)

(B) $\because F(x) = 2x^2 e^x$ (1)

$\therefore F'(x) = 2x^2 e^x + 4x e^x$ ½

$\therefore F'(x) = 2x e^x (x+2)$

$F' = 0 \Rightarrow x+2 = 0$ or $x = 0$

$\therefore x = -2 \in [-3,1]$ or $x = 0 \in [-3,1]$ 2 ½

$\therefore f(-3) = \frac{18}{e^3} \simeq 0.9$

$f(-2) = \frac{8}{e^2} \simeq 1.08$ ½

$f(1) = 2e \simeq 5.44$

$f(0) = 0$ ½

$\therefore$ The absolute max value is $2e$ ½

, The absolute min value is 0 ½

3

(تراعى الحلول الأخرى)

16-

(c) $\pi \int_0^4 (64x^2 - 4x^4)\, dx$ — 1

17-

(b) 4 — 1

18-

(A) $\int x^3 \sqrt{4-x^2}\, dx$

let $u = x^2 \Rightarrow du = 2x\, dx$ ½

$dv = x\sqrt{4-x^2} \Rightarrow v = \int x\sqrt{4-x^2}\, dx$

$= -\frac{1}{2} \int (-2x)(4-x^2)^{\frac{1}{2}} dx$

$= -\frac{1}{2} \cdot \frac{2}{3}(4-x^2)^{\frac{3}{2}} = -\frac{1}{3}(4-x^2)^{\frac{3}{2}}$ ½

$\therefore \int x^3 \sqrt{4-x^2}\, dx$

$= -\frac{1}{3} x^2 (4-x^2)^{\frac{3}{2}} - \int -\frac{1}{3}(4-x^2)^{\frac{3}{2}} \cdot 2x\, dx$ ½

$= -\frac{1}{3} x^2 (4-x^2)^{\frac{3}{2}} - \frac{1}{3} \times \frac{2}{5}(4-x^2)^{\frac{5}{2}} + c$

$= -\frac{1}{3} x^2 (4-x^2)^{\frac{3}{2}} - \frac{2}{15}(4-x^2)^{\frac{5}{2}} + C$ ½

2

(B) $\int \sin^3 x \, dx$

$I = \int \sin x (1 - \cos^2 x) \, dx$ ½

let $u = \cos x \Rightarrow du = -\sin x \, dx$

$\therefore dx = -\frac{du}{\sin x}$

$\therefore I = \int (1 - u^2) \cdot \sin x \left(\frac{-du}{\sin x}\right)$ ½

$= -\int (1 - u^2) \, du$

$= \int (u^2 - 1) \, du$

$= \frac{1}{3} u^3 - u + C$ ½

$= \frac{1}{3} \cos^3 x - \cos x + C$ ½

Another solution

$I = \int (\sin x - \cos^2 x \cdot \sin x) \, dx$ 1

$= -\cos x + \frac{1}{3} \cos^3 x + C$ 1

2

(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)