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# REAL-TIME MEASUREMENT OF COKE AGGREGATE SIZE AND VIBRATED BULK DENSITY USING IMAGE TEXTURE ANALYSIS

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## Abstract

The objective of this work is to develop a rapid and non-destructive machine vision sensor to predict Vibrated Bulk Density (VBD) of coke aggregate samples based on their surface textural characteristics obtained by imaging. This information could be useful for ultimately making real-time process adjustments to reduce green anode variability. Coke samples from different sources and sieved in a number of size classes were investigated individually and in blends. Wavelet Texture Analysis (WTA) was used to extract textural features of coke samples, and these were related to their VBD using Partial Least Squares (PLS) regression. It is shown that WTA captures variations in coke size and source and leads to good VBD predictions.

## Introduction

Petroleum coke is a by-product of crude oil processing and is considered by the refineries as a waste product [1]. For the aluminum industry, however, coke is a raw material for the production of baked carbon anodes and its increasing variability is an important issue for carbon plants [2]. Variations in coke physical and chemical properties through time and from one supplier to another have a direct impact on anode performance in the aluminium reduction cells in terms of energy efficiency, carbon consumption, emissions of greenhouse gases and metal purity.

Particle size distribution, particle shape, internal porosity of the material and structure of petroleum coke (e.g., sponge or shot coke) affect the pitch penetration in the dry aggregate and in turn, the properties of green and bake anodes (e.g. density). The frequent coke supplier changes also contribute to increasing incoming coke variability at the carbon plant. Therefore, smelters need to track changes in coke properties in order to make appropriate and timely process adjustments to reduce anode density variation.

Methods for measuring the Vibrated Bulk Density (VBD) of coke materials have received a lot of attention in recent years because it is an easy and fast way to indirectly measure coke porosity. The porosity of the calcined coke is one of the most important physical properties affecting pitch demand and anode density. Hence, finding the optimum amount of pitch is necessary for optimizing anode performance when facing variations in coke porosity [3].

The VBD is typically measured off-line by collecting samples from the dry aggregate mix from conveyor belts just before adding pitch. The samples are then sent to a laboratory for testing using a standard procedure involving a vibration table [4]. Measurements are typically collected a few times per day because even if the procedure is simple and quick, it requires operator intervention. Unfortunately, this sampling frequency may not be sufficient for tracking changes in VBD and making timely corrective actions when needed. Coke properties vary because of the intrinsic heterogeneity of incoming coke shipments and frequent supplier changes (i.e. low frequency fluctuations). In addition, the silo management strategy when using multiple cokes, segregation within silos, and operational problems with size reduction or sizing equipment contribute to higher frequency variations. A method for real-time measurement of VBD is required to capture the range of variations, from low to high frequencies.

In this work, we investigate a rapid and non-destructive machine vision sensor for tracking VBD variations. It is proposed to extract textural features from coke sample images and to use them to predict the sample VBD. The rationale behind this is as follows. VBD is a function of the real coke density and its internal porosity, as well as the inter-particle porosity due to packing. The latter is mainly influenced by particle size and shape which can be measured by imaging. Inter-particle porosity variations due to packing most likely happen at a higher frequency (changes in coke particle size and shape) compared with real coke density and internal porosity which mainly occur when the coke supply changes (i.e. lower frequency). Thus, extracting coke image features in real-time (ultimately on conveyor belts) and combining them with infrequently measured coke properties could provide a very efficient way of tracking VBD variations automatically. This new measurement could be used passively to monitor VBD, but also actively in a feedback control loop. For example, aggregate size distribution could be adjusted to maintain VBD at a desired set-point. The measurement could also enable feedforward control corrections to anode paste formulation (i.e. pitch ratio) when deemed necessary.

This paper reports on a preliminary study aimed at investigating the use of coke image texture for predicting VBD. Coke textural features are extracted using Wavelet Texture Analysis (WTA), the efficiency and performance of which was demonstrated in several industrial product quality control applications [5]. Partial Least Squares (PLS) regression is used to model the relationship between coke textural features obtained by imaging and sample VBD. Several sources of cokes sieved in different size classes are used. Changes in coke particle size and source can be detected by WTA and used for predicting the VBD of coke samples for each source or in blends.

## Experimental

Commercially available sponge cokes from five different sources (i.e. suppliers) were used in this analysis. These are identified by using letters A-E. These cokes were selected to span a range of properties.

Two cases were investigated to represent situations that may occur in practice when formulating the coke aggregates for anode manufacturing. First, each coke was imaged and analyzed separately (unmixed samples) as if anodes were made from a single coke source at a time. Then blends of two different cokes (mixed samples) were formulated to represent the situation where multiple cokes are used in anode making.

Unmixed samples

The coke from each source was sieved into seven size fractions as shown in Table I. These fractions can also be grouped into three categories: coarse, intermediate and fines. The sizing ranges for the coke fractions are approximately as follows: coarse (-4 / +30); intermediate (-30 / +100) and fine (-100 US Mesh) [6]. ASTM D5709-09 standard test method [7] was followed to analyze the particle size distribution.

Table I. Particle size distribution of coke samples

| Fraction | Size class | Opening (mm) | US Mesh (USM) |
|---|---|---|---|
| Coarse | 1 | - 4.75 / + 2.38 | - 4 / + 8 |
| | 2 | - 2.38 / + 1.41 | - 8 / + 14 |
| | 3 | - 1.41 / + 0.6 | - 16 / + 30 |
| Intermediate | 4 | - 0.6 / + 0.3 | - 30 / + 50 |
| | 5 | - 0.3 / + 0.15 | - 50 / + 100 |
| Fine | 6 | - 0.15 / + 0.074 | - 100 / + 200 |
| | 7 | - 0.074 | - 200 |

(-) The particles pass through the sieve.
(+) The particles are retained by the sieve.

The Vibrated Bulk Density (VBD) was measured separately on the first five size fractions of each coke (i.e., from 4 to 100 USM) to evaluate the effect of coke source and particle size on VBD, and to assess the ability of the machine vision system to capture these changes. ASTM D4292-10 standard test method [4] with vibration time of 2 minutes was used for the VBD test.

Mixed samples

A set of 25 synthetic blends of two cokes (A and E) were produced to span a range of particle sizes and compositions. These blends consist of 5 different size distributions, each of them made with 5 distinct proportions (%w/w) of the two cokes (i.e. each size distribution was replicated 5 times but with different ratios between the two cokes). Coke A and E were selected because they have the most different VBD values amongst those tested in this study.

The size distributions were selected according to a multivariate design of experiments in order to represent the range of aggregate size distributions observed at the Alcoa Deschambault smelter in Quebec (ADQ) from 2009 to 2014. Basically, historical data on 7 size fractions collected on 2000 samples was retrieved from the data historian at ADQ. Principal Component Analysis (PCA) was applied to reduce the dimensionality of the data and visualize the clustering patterns in the size distributions. More details on PCA can be found in [8]. Figure 1 shows the multivariate distribution of aggregate sizes by plotting the first two Principal Components ($t_1$ vs $t_2$) against each other. A color map was applied to show the density of points in the various regions of the plot, from blue (low density) to red (high density). The 5 distributions identified as P1-P5 were selected to represent the range of size distributions commonly observed at the plant, including different proportions (% w/w) of coarse, intermediate and fine particles.

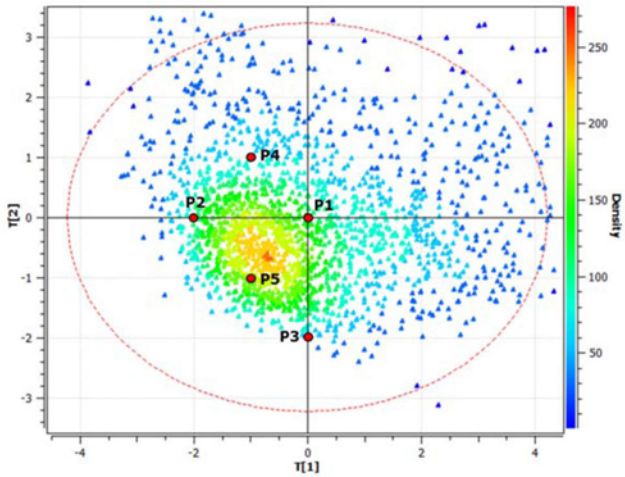


Figure 1. Multivariate distribution of coke aggregate size distributions historically observed at ADQ (2009-2014).

The desired size distributions P1-P5 were formulated in the laboratory by mixing the appropriate proportions (% w/w) of the size classes presented in Table I. The targeted compositions were reached by combining coke A and E in the required (% w/w) ratio in each size classes.

The VBD of each blend was measured on the -30+50 USM size fraction according to the standard procedure described previously. This size fraction was used because it is the dominant component of the blend (by weight).

Imaging set-up

All coke samples (both unmixed and mixed) were imaged using the set-up shown in Figure 2. The apparatus consists of a high-resolution (6 megapixels) CCD camera (Prosilica GX2750C, Allied Vision Technologies) taking digital color (RGB) images. A 50 mm Kowa lens (LM50HC) was used in front of the camera. Uniform lighting of the material was obtained by using two 4.5 W LED light bulbs and Fresnel Lenses. The lighting incident angle and the camera height can be adjusted between some limits in order to optimize the imaging conditions. Lighting the coke samples at an angle is important to enhance its surface texture, and hence it size and shape.

Each coke sample was first poured into an aluminium container until it overflowed and a pile of coke was formed on top of the container. The excess material was removed carefully using the edge of a spatula before the image was collected. The coke sample was then put back in the mixing bowl, given a small hand mixing and poured again in the container for imaging a second time. This procedure was repeated a third time in such a way that three replicate images were collected for each coke sample to account for sampling variability in coke visual appearance. Note that the RGB images were converted to grey levels prior to applying texture analysis.

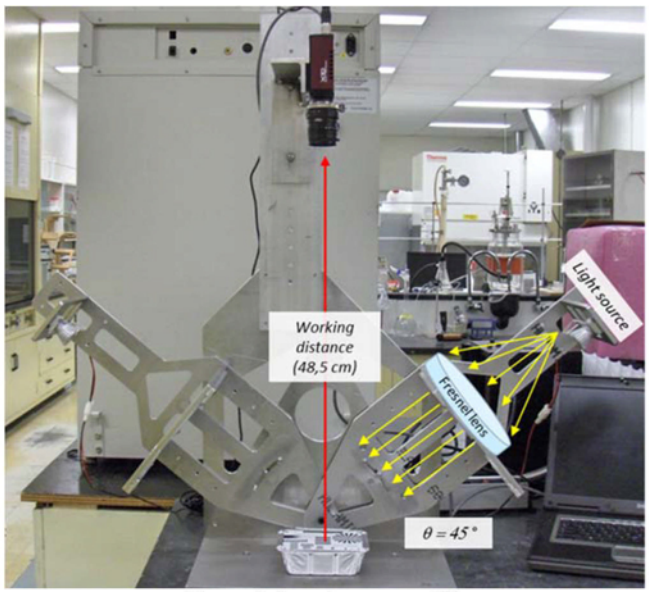

Figure 2. Imaging set-up [9].

**Machine vision approach for texture analysis**

Wavelet Texture Analysis (WTA)

A grey level image is defined as a two-dimensional function *f(x, y)*, where *x* and *y* correspond to the spatial coordinates of a pixel in the image. The amplitude of that function is the light intensity at that point [10]. The light intensity values range from 0 to 255 (i.e., image quantized with 8-bits of resolution in this case). Hence, grey levels vary from black (0) to white (255).

Image texture can be defined as a function of the spatial variation in pixel intensities (i.e., grey levels) [11]. Texture is a property that represents the surface and structure of an image. It can be defined as a regular repetition of an element or pattern on a surface [12]. Textural descriptors (i.e., statistics) are used to measure properties such as smoothness, coarseness and surface roughness in an image, which can be used to characterize the aggregate size and shape.

Image texture is highly scale dependent, therefore a texture can be described in multiple resolutions and an appropriate scale within the image can be chosen to achieve the maximum texture discrimination. For example, high frequency variations from pixel-to-pixel produce a fine texture while lower frequency variations lead to coarser textures. Petroleum coke images are good candidates for texture analysis because aggregates are composed of a mixture of solid particles of different sizes and shapes creating a variety of textures (from fine to coarse) depending on the coke source.

Transform-based texture analysis techniques are used to represent the image information in a new space whose coordinates are related to the texture (such as frequency, scale or size). Whereas the two-dimensional Fast Fourier Transform (2D-FFT) performs a frequency decomposition only, the 2D Discrete Wavelet Transform (2D-DWT) enables a space-frequency decomposition of an image, which is more suitable for texture analysis because the wavelet transform maintain good space and frequency localization when discretized [13]. For this reason the Wavelet Texture Analysis (WTA) is considered to be the current state-of-the-art multi-resolution textural method in process industries.

The WTA is used to decompose greyscale images into the space-frequency domain, and converts the textural information of an image into a series of so-called wavelet coefficients at each resolution. Different textural descriptors (i.e., statistic) are extracted from these coefficients to characterize the texture contained within an image (e.g., mean, variance, energy and entropy) [14-15]. The WTA method allows analysis of fine texture at high frequency and coarse textures at low frequency.

The first step of this approach consists of choosing a wavelet function, also called mother wavelet, in which its shape coincides with the image signals (i.e. pixel-to-pixel intensity variations in a given direction). Various wavelet families can be found in literature in different texture analysis applications. The Daubechies wavelet [16] was selected due to its orthonormal characteristic and similarity with the image signals. Also, it has continuous derivatives that respond well to signal discontinuities.

The mother wavelet can be dilated in order to capture textural information at lower spatial frequency. This is modulated by a scaling function which is an integral part of the wavelet equation. The wavelet at a given scale is then convoluted with the image signal in the horizontal, vertical and diagonal directions. Having a finite duration, the wavelet is translated across the image in all three directions to capture the spatial information through the correlation between the wavelet and the image signal. Then, the wavelet coefficients or so-called detail coefficients are computed for each pixel. Therefore, the 2D-DWT extracts the horizontal (H), vertical (V) and diagonal (D) textural features respectively, which are represented by a set of three detail sub-images $d_j^h$, $d_j^v$ and $d_j^d$ at the $j^{th}$ decomposition level of a greyscale image as shown in Figure 3.

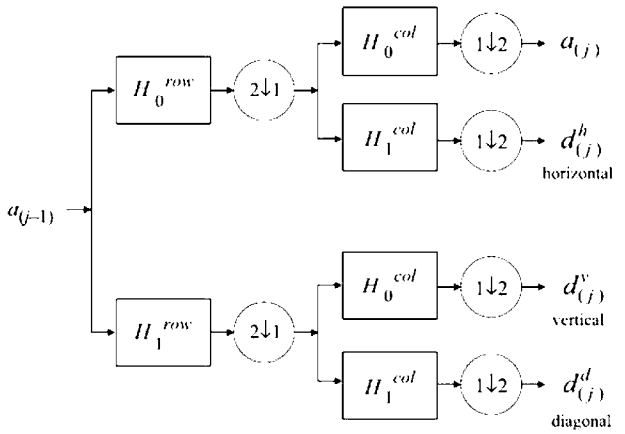

Figure 3. Schematic of DWT decomposition. Adapted from [13].

The 2D-DWT can be viewed as a filtering approach where the detail sub-images contain most texture information at different frequency scales or resolutions. Successive low-pass ($H_0$) and high-pass ($H_1$) filters allows the extraction of textural features simultaneously at each scale (see Figure 3). For instance, the detail sub-images at the first scale will capture high frequency information (fine texture), whereas higher scales will contain lower frequency information (coarser texture). After filtering the image at a selected number of scales ($j=1,2,..,J$), the residual image is called the approximation sub-image ($a_j$), which contains

the information of all the lower frequency textures (e.g. due to uneven lighting).

The physical appearance of the material is affected by the imaging conditions (e.g., working distance and lighting) since the images may contain noise during acquisition. Therefore, texture information to the higher frequency (i.e., first decomposition level *j=1*) can be removed in subsequent analyses.

To analyze and compare the texture of a set of images, it is a common practice to compute a row vector of textural descriptors from the detail sub-images ($d_j^h$, $d_j^v$ and $d_j^d$) of each original image. These vectors are then collected row-wise in a feature matrix **X** for further analyses, such as classification or regression modelling [5,15].

The energy of each detail sub-image was used as the textural descriptor (i.e. feature) in this analysis. The energy is a measure of the variance of the detail coefficients at each decomposition level. On the other hand, it can be interpreted as the amount of signal contained in an image in a given frequency band which is useful for tracking changes in size distributions of aggregates. For example, a coarser size distribution should have more energy at higher scales (lower frequencies) compared to a finer distribution. The energy of detail sub-image is defined by:

$$E_j^k = \frac{1}{XY}\sum_{x=1}^{X}\sum_{y=1}^{Y}\left|d_j^k(x,y)\right|^2 \quad (1)$$

Where $d_j^k$ is the detail sub-image of size $(x,y)$ at decomposition level $j$, and $k = h,v,d$ direction (see Figure 3). The energy of each detail coefficients is normalized by the number of pixels in the original image of size (X,Y). When these energies are used as elements of the textural feature vector, it is called the *wavelet energy signature*, which is one of the most commonly textural features used in WTA [15].

Multivariate analysis of image textural features

Data classification using the latent variable space of multivariate statistical methods like Principal Component Analysis (PCA) and Partial Least Squares (PLS) has been widely used in chemometrics literature [8]. In this work, PLS regression was used to correlate the coke image textural features collected in the **X** matrix with the corresponding VBD data stored in response matrix **Y**. PLS was selected because it is designed to handle the high level of collinearity between the columns of **X** (i.e., textural features are highly correlated). The PLS model relates the two groups of variables (i.e., **X** and **Y**) through a set of new orthogonal latent variables **T** (i.e., score vectors) while also explaining variations in both data blocks. The number of score vectors or the so-called components of the PLS model is typically selected in such a way to maximize the predictive ability of the model via some cross-validation procedure [8]. Since the number of components is typically much smaller than the number of X and Y variables, a dimensionality reduction is also achieved. The structure of the PLS model is as follows:

$$\mathbf{X} = \mathbf{T}\mathbf{P}^T + \mathbf{E} \quad (2)$$
$$\mathbf{Y} = \mathbf{T}\mathbf{C}^T + \mathbf{F} \quad (3)$$
$$\mathbf{T} = \mathbf{X}\mathbf{W}^* \quad (4)$$

Where the **P** and **C** matrices contain the loading vectors that best represent the **X** and **Y** spaces, respectively. **W*** contains the weight vectors that define the relationship between the **X** and **Y** spaces. The **E** and **F** matrices contain the residuals of each space. PLS is often seen as a regression extension of PCA for the analysis of two groups of variables having a lower dimensional structure (i.e., collinearity between the columns of **X** and **Y**).

The PLS weight vectors **W*** (linear combinations of the X-variables) are chosen to maximize the covariance between **X** and **Y** instead of maximizing the explained variance of each space separately as PCA does. The loading and score vectors of each latent dimension (or PCs) are orthogonal and independent from each other. They are usually calculated sequentially using the non-linear iterative partial least squares (NIPALS) algorithm.

The components of the PLS model are ordered in such a way that the first component is the one that explains the greatest amount of covariance between **X** and **Y**, the second explains the greatest amount of covariance orthogonal to the first component, and so on. More details on PLS can be found in reference [8].

## Results and discussion

Unmixed samples

This section presents the results of the impact of coke source and size on VBD. In particular, the ability of the proposed machine vision approach to detect changes in coke source and size and to predict their impact on VBD is investigated. The different size classes of each coke source were imaged separately in this analysis (i.e. size classes and coke sources were not mixed). Texture analysis of grey level images collected for each coke source and size class was performed by applying WTA using the orthogonal Daubechies wavelet (*db1*) at 8 decomposition levels (*J=8*) and three directions (*K=3* or H,V,D). This yielded 3×8=24 detail sub-images from which the energy signatures were calculated. Hence, the feature matrix **X** had 24 columns (energies at each scale j and direction k) and 25 rows corresponding to each coke image (5 sources × 5 size classes).

Note that the textural features calculated for the 3 replicate images of each sample were averaged because the mean standard error (MSE) of the predictions made from the image replicates was smaller than the MSE of the measured VBD. The VBD measured on each of these 25 coke samples was collected in **Y**. PLS regression was applied on **X** and **Y**. Two PLS components (PC's) were found to be statistically significant by a cross-validation procedure. The model explains 74% of the total variation in VBD ($PC_1$=69% and $PC_2$=5%). The PLS model results are presented in Figure 4 and 5.

The $t_1$-$t_2$ score plot shown in Figure 4 is used to visualize the clustering patterns of the coke images based on their textural features. Each point on this plot represents a single coke image from a given source and size class (from 4 to 100 USM). They are identified by a letter followed by a number. The former correspond to the coke source and the latter to the size class listed in Table I (e.g. C2 means coke source C and size class 2 or - 8 / + 14 USM). Coke samples having similar textural features cluster together (i.e. have similar values of $t_1$ and $t_2$) in the score plot but in a different location to those coke samples having distinct textural characteristics. The 1st PC mainly explains the variations

related with coke particle size (i.e. each size class is indicated using black ellipses) whereas the 2nd PC mainly explains variations associated with coke sources. Interpretation of the model using the PLS loadings and weights (not shown) reveals that the latter PC captures differences in the light reflection intensity from the surface of the materials and will be explored further as a mean to detect changes in coke supply in real-time.

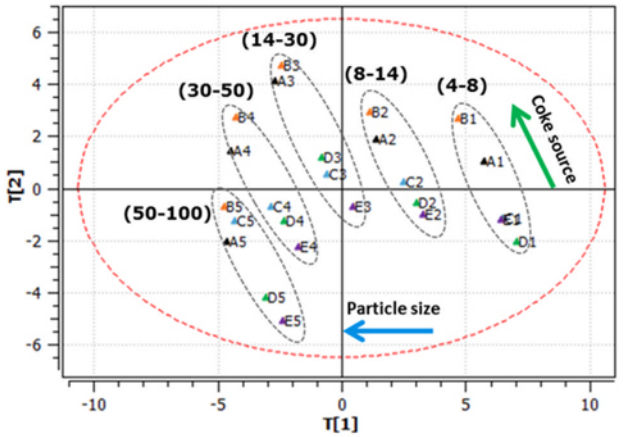


Figure 4. PLS $t_1$-$t_2$ score plot of unmixed coke images.

The PLS regression model was also used to predict the Vibrated Bulk Density (VBD) from textural characteristics of each coke fraction. The VBD is strongly related with particle size distribution and coke source as shown in Figure 5. The VBD increases with decreasing particle size, in agreement with previous studies on the effect of particle size on VBD [17]. Cokes from different sources also have a different VBD. This difference is also captured by the model. Overall, the predicted VBD shows a good agreement with the observed values

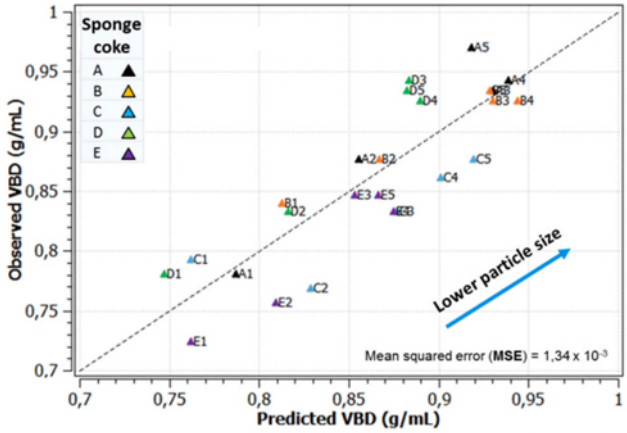


Figure 5. Predicted VBD of unmixed cokes images ($R^2$ =0.74).

Mixed samples

Based on the above results, it was decided to investigate whether the imaging sensor would provide as good results on blends of cokes from different sources as typically occur in anode manufacturing plants. Blends of coke A and E were formulated as described in the experimental section. Each blend was imaged (in triplicate then averaged) and textural features were calculated from each image using the same procedure as described previously. A new PLS model was built to relate the textural features of the blends (i.e. introduced by changes in size distribution and composition) and their VBD.

Figure 6 shows again the $t_1$-$t_2$ latent variable score space for the first two components PLS model built on coke blends. Note that each blend is identified by its size distribution (e.g. P1) and the proportions of both cokes (e.g. M4) as listed in the legend of Figure 6. In this case, the two arrows indicate the two main effects studied, that is the effect of aggregate size distribution and blend composition on VBD. A total of 4 PLS components were found significant and together explain 93% of the total variation in VBD, but only the first two are shown and interpreted. The 1st PC mainly represents the variation related with the proportion of coke sponge (A and E) in the mix, whereas the 2nd PC explains the variation according to the particle size distribution.

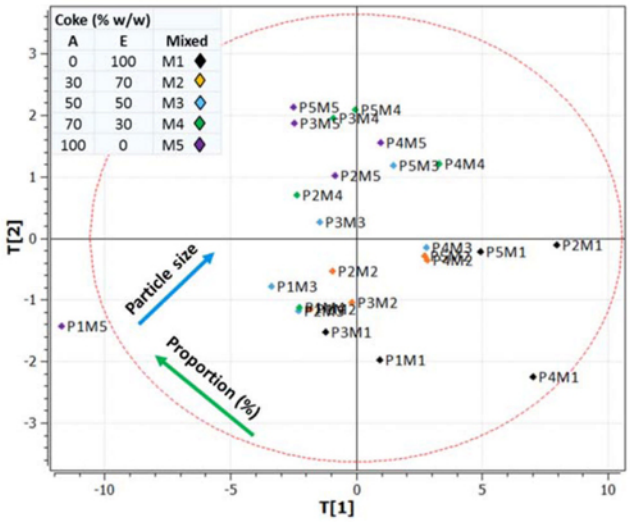


Figure 6. PLS $t_1$-$t_2$ score plot of mixed coke images.

Figure 7 shows the PLS model predictions against measured VBD. In this case, only the energies from decomposition levels (J = 2, 3 and 4) were kept in the feature matrix **X**. Each scale or decomposition level in WTA captures a certain frequency band from the image signals which in turn can be related to size of objects in the spatial domain. It was calculated that decomposition levels 2-4 extracted information about objects of sizes closely matching the size fraction used to measure VBD (i.e. the 30-50 USM fraction or around 0.3-0.6 mm). Thus, by using more specific image features, the PLS model showed excellent prediction of VBD and very small mean squared prediction errors (MSE = 6 x $10^{-5}$). The effect of aggregate size and composition on VBD was very well captured by the image texture analysis.

## Conclusions

This work investigated a new machine vision sensor for estimating the Vibrated Bulk Density (VBD) of coke aggregate samples based on their surface texture. Wavelet Texture Analysis (WTA) was shown to detect changes in coke particle size and source (i.e. different suppliers) from images of aggregates. This is due to its ability to analyze textures at different frequencies or resolutions. Partial Least Squares (PLS) regression was used to correlate the VBD measured on various coke samples (from a single source or in blends) to the sample image textural features. Very good prediction results were obtained ($R^2$ values of 0.74 and

0.93 for unmixed and mixed coke samples). Future work will concentrate on validating the robustness of this approach for estimating the dry aggregate VBD from industrial carbon plant samples. Ultimately, the proposed analysis would be performed directly on the dry aggregate mix conveyor belt. The real-time VBD measurement could be used passively to monitor the mix or actively in feedback/feedforward control schemes.

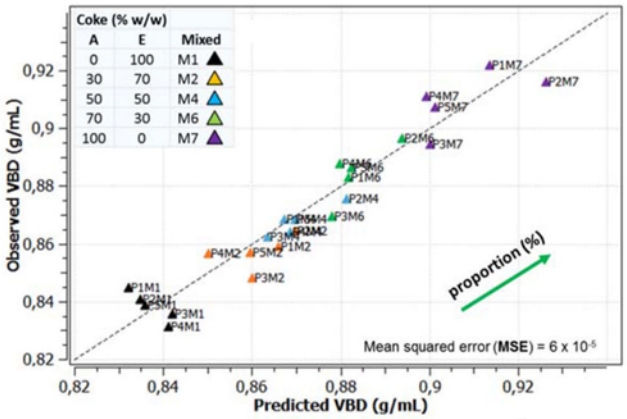


Figure 7. Predicted VBD of mixed cokes images ($R^2$ = 0.93).


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## References


1. U. Mannweiler, W.K. Fischer, and R.C. Perruchoud, "Carbon Products: A Major Concern to Aluminum Smelters", *Light Metals*, (2009), 909-911.

2. L. Edwards, N. Backhouse, H. Darmstadt, and M.J. Dion, "Evolution of Anode Grade Coke Quality", *Light Metals*, (2012), 1207-1212.

3. P. Jignesh, W. Mark, and R. Jeffrey, "Historical and Future Challenges with the Vibrated Bulk Density Test Methods for Determining Porosity of Calcined Petroleum Coke", *Light Metals*, (2011), 925-930.

4. ASTM D4292-10, "Standard Test Method for Determination of Vibrated Bulk Density of Calcined Petroleum Coke, Annual Book of ASTM Standards, D02.05 (2010), 1-5.

5. C. Duchesne, J.J. Liu, and J.F. MacGregor, "Multivariate Image Analysis in the Process Industries: A Review", *Chemometrics and Intelligent Laboratory Systems*, 117 (2012) 116-128.

6. S. Jones Samuel, Anode-Carbon Usage in the Aluminum Industry, Petroleum-Derived Carbons, American Chemical Society1986, pp. 234-250.

7. ASTM D5709-09, "Standard Test Method for Sieve Analysis of Petroleum Coke", Annual Book of ASTM Standards, D02.05 (2009), 1-4.

8. L. Eriksson et al., *Multi and Megavariate Data Analysis Part I Basic Principles and Applications, 2nd ed.* (Umetrics Academy, Umea, Sweden, 2006), 39-101.

9. J. Lauzon-Gauthier, C. Duchesne, and J. Tessier, "Texture Analysis of Anode Paste Images" (Paper presented at the 143th TMS Annual Meeting, San Diego, California, 18 February 2014).

10. R.C. Gonzalez, and R.E. Woods, *Digital Image Processing* (Harlow: Prentice Hall 2007).

11. M. Tuceryan and A. K. Jain, "Texture Analysis", in: C.H. Chen, L.F. Pau, P.S.P Wang (Eds.), *The Handbook of Pattern Recognition and Computer Vision, 2nd ed.* (World Scientific Publishing Co, 1998), 207-248.

12. G.N. Srinivasan, and G. Shobha, "Statistical Texture Analysis", *Proceedings of World Academy of Science, Engineering and Technology*, 36 (2008), 1264-1269.

13. S.G. Mallat, "A Theory for Multiresolution Signal Decomposition: the Wavelet Representation", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11 (1989), 674-693.

14. S. Borah, E.L. Hines, and M. Bhuyan, "Wavelet Transform Based Image Texture Analysis for Size Estimation Applied to the Sorting of Tea Granules", *Journal of Food Engineering*, 79 (2007), 629-639.

15. M.H. Bharati, J.J. Liu, and J.F. MacGregor, "Image Texture Analysis: Methods and Comparisons", *Chemometrics and Intelligent Laboratory Systems*, 72 (2004), 57-71.

16. I. Daubechies, "The wavelet transform, time-frequency localization and signal analysis", *IEEE Transactions on Information Theory*, 36 (1990), 961-1005.

17. F. Cannova, M. Canada, B. Vitchus, "Calcined coke particle size and crushing steps affect its VBD result", Light Metals, (2011), 937-939.