كل ما يحتاجه الطالب في جميع الصفوف من أوراق عمل واختبارات ومذكرات، يجده هنا في الروابط التالية لأفضل مواقع تعليمي إماراتي 100 %

تطبيق المناهج الإماراتية	الاجتماعيات	الرياضيات
الصفحة الرسمية على التلغرام	الاسلامية	العلوم
الصفحة الرسمية على الفيسبوك	الانجليزية	
التربية الاخلاقية لجميع الصفوف	اللغة العربية	
التربية الرياضية		

مجموعات التلغرام.	مجموعات الفيسبوك	قنوات تلغرام	قنوات الفيسبوك
الصف الأول	الصف الأول	الصف الأول	الصف الأول
الصف الثاني	الصف الثاني	الصف الثاني	الصف الثاني
الصف الثالث	الصف الثالث	الصف الثالث	الصف الثالث
الصف الرابع	الصف الرابع	الصف الرابع	الصف الرابع
الصف الخامس	الصف الخامس	الصف الخامس	الصف الخامس
الصف السادس	الصف السادس	الصف السادس	الصف السادس
الصف السابع	الصف السابع	الصف السابع	الصف السابع
الصف الثامن	الصف الثامن	الصف الثامن	الصف الثامن
الصف التاسع عام	الصف التاسع عام	الصف التاسع عام	الصف التاسع عام
الصف التاسع متقدم	الصف التاسع متقدم	الصف التاسع متقدم	تاسع متقدم
الصف العاشر عام	الصف العاشر عام	الصف العاشر عام	عاشر عام
الصف العاشر متقدم	الصف العاشر متقدم	الصف العاشر متقدم	عاشر متقدم
الحادي عشر عام	الحادي عشر عام	الحادي عشر عام	حادي عشر عام
الحادي عشر متقدم	الحادي عشر متقدم	الحادي عشر متقدم	حادي عشر متقدم
ثاني عشر عام	الثاني عشر عام	الثاني عشر عام	الثاني عشر عام
ثاني عشر متقدم	الثاني عشر متقدم	ثاني عشر متقدم	ثاني عشر متقدم

Chapter 9: Circular Motion

Concept Checks

9.1. e **9.2.** b **9.3.** a **9.4.** d **9.5.** b **9.6.** d

Multiple-Choice Questions

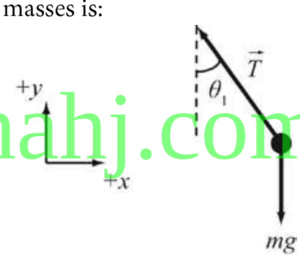
9.1. d **9.2.** c **9.3.** b **9.4.** d **9.5.** c **9.6.** a **9.7.** c **9.8.** a **9.9.** a **9.10.** c **9.11.** a **9.12.** a **9.13.** d **9.14.** d **9.15.** d **9.16.** a

Conceptual Questions

9.17. A ceiling fan is rotating in the clockwise direction, as viewed from below. This also means that the direction of angular velocity of the fan is in the clockwise direction. The angular velocity is decreasing or slowing down. This indicates that the angular acceleration is negative or in the opposite direction of the angular velocity. Therefore, the angular acceleration is in the counter-clockwise direction.

9.18. No, it will not. This is because when the actor swings across the stage there will be an additional tension on the rope needed to hold the action in circular motion. Note that the total mass of the rope and the actor is 3 lb + 147 lb = 150 lb. This is the maximum mass that can be supported by the hook. Therefore, the additional tension on the rope will break the hook.

9.19. The force body diagram for one of the masses is:



The force of tension in the x-axis is equal to the centripetal force, $T\sin\theta_1 = m\omega^2 r$. The force of the tension along the y-axis must be equal to the force of gravity, $T\cos\theta_1 = mg$. This means $\frac{T\sin\theta_1}{T\cos\theta_1} = \tan\theta_1 = \frac{\omega^2 r}{g}$; therefore, both θ_1 and θ_2 are the same, since they don't depend on the mass.

9.20. For the two points of interest, there are two forces acting on the person; the force of gravity and the normal force. These two forces combine to create the centripetal force. In case A: $F_{\text{c,A}} = F_{\text{N,A}} - F_{\text{g}}$ and case B: $-F_{\text{c,A}} = F_{\text{N,B}} - F_{\text{g}}$. This means that the normal force is $F_{\text{N,A}} = F_{\text{c,A}} + F_{\text{g}} = m\omega^2 r + mg$ and $F_{\text{N,B}} = F_{\text{g}} - F_{\text{c,A}} = mg - m\omega^2 r$. Therefore, $F_{\text{N,A}}$ is greater than $F_{\text{N,B}}$.

9.21. The linear speed of the bicycle is given by $v = r\omega$. The smaller the diameter, D, the lower the linear speed for the same angular speed because $r = D/2$ so tires with a lower diameter than 25 cm will have a velocity too slow to be practical transportation.

9.22. Both the angular velocity and acceleration are independent of the radius. This means they are the same at the edge and halfway between the edge and center. The linear velocity and acceleration, however, do change with radius, r. At the edge $v_{\text{e}} = r\omega$ and $a_{\text{c}} = R\omega^2$. The halfway point gives $v_{1/2} = \frac{r\omega}{2}$ and $a_{1/2} = \frac{1}{2}R\omega^2$. Comparing the two points, it can be seen that $v_{\text{e}} = 2v_{1/2}$ and $a_{\text{e}} = 2a_{1/2}$.

9.23. For the car to stay on the road there must be a force in the direction of the centripetal force. In this case, the road is not banked, leaving the force of friction as the only possible choice. Since the car travels with constant speed, the force of friction holding the car on a circular path points in radial inward direction towards the center of the circle.

9.24. As the car makes the turn, both strings have a new angular position, θ. From the discussion of the conical pendulum on page 290 and 291, you can see that this angle is given by $\tan\theta = r\omega^2/g$, where r is the distance to the center of the circle. This means that the pendulum that is further away from the center has a larger angle. A larger angle means a larger sideways deflection of the pendulum, and thus the distance between the two pendula increase during the turn, both for a right turn and for a left turn. If the distance d between the two pendula is small compared to the turning radius r, however, this effect is hard to measure or see.

9.25. The kinetic energy when the point mass gets to the top of the loop is equal to the difference in potential energy between the height h and $2R$. Rearranging $\frac{mv^2}{2} = mgh - 2mgR$ for v^2 gives $v^2 = 2g(h-2R)$. For the particle to stay connected to the loop, the centripetal force has to be greater than or equal to the force of gravity. This requirement means $v^2 = Rg$. Using these two equations, the height, h, can be determined:

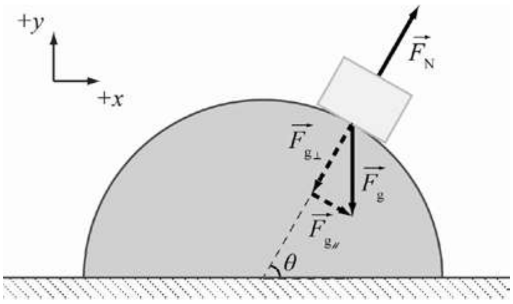
$$2g(h-2R) = Rg \Rightarrow h = 2R + \frac{1}{2}R = \frac{5}{2}R.$$

The height should be $5R/2$ or greater for the point to complete the loop.

9.26. The bob is moving in a horizontal circle at constant speed. This means that the bob experiences a net force equal to the centripetal force inwards. This force is equal to the horizontal component of the tension. The vertical component of the tension must be balanced by the force of gravity. The two forces acting on the bob are the tension and the force of gravity.

9.27. From our discussion of the conical pendulum on page 271, you can see that this angle is given by $\tan\theta = r\omega^2/g$. As the angular speed assumes larger and larger values, the angle *approaches* a value of $90°$, which is the condition that the string is parallel to the ground. However, the *exact* value of $90°$ cannot be reached, because it would correspond to an infinitely high value of the angular speed, which cannot be achieved.

9.28. A picture of the situation is as follows:



This picture tells us that the normal force can be related to the force of gravity by $F_N = F_{g\perp} = F_g \sin\theta = mg\sin\theta$. In this situation, the normal force provides the centripetal force, so $F_c = mg\sin\theta$ and $a_c = g\sin\theta$. As θ decreases, $\sin\theta$ decreases, and therefore a_c decreases. The acceleration vector for circular motion has two components; the centripetal acceleration, a_c, and the tangential acceleration, $a_t = \cos\theta$, which increases as θ decreases to zero. This satisfies the requirement that $\vec{a} = a_t\hat{t} - a_c\hat{r}$.

9.29. The forces that you feel at the top and bottom of the loop are equal to the normal force. At the top and bottom of the loop, the normal force are along the vertical direction and are $N_{\text{T}} = mg - \dfrac{mv_{\text{T}}^2}{R}$ and $N_{\text{B}} = mg + \dfrac{mv_B^2}{R}$. If you experience weightlessness at the top, then $N_{\text{T}} = 0 \Rightarrow mg = mv_{\text{T}}^2 / R$. Energy conservation tells us that $\frac{1}{2}mv_{\text{B}}^2 = 2mgR + \frac{1}{2}mv_{\text{T}}^2$. Insert both of these results into the expression for the normal force at the bottom and find:

$$N_{\text{B}} = mg + mv_{\text{B}}^2 / R = mg + (4mgR + mv_{\text{T}}^2) / R = mg + 4mg + mg = 6mg$$

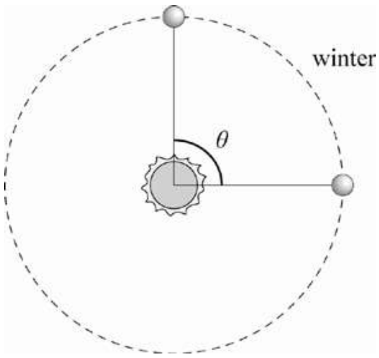
This means that the normal force exerted by the seat on you, your apparent weight, is indeed 6 times your weight at the top of the loop.

9.30. The combined weight of the five daredevils is W. To determine the strength of the rope needed, the tension at the bottom of the arc must be determined. At this point the centripetal force is equal to the difference between the tension and the force of gravity. The tension is equal to $T = \dfrac{mv^2}{R} + mg$ $= \dfrac{Wv^2}{Rg} + W$. The kinetic energy at the bottom of the arc is equal to the potential energy at the level of the bridge, $\dfrac{1}{2}mv^2 = mgR$ or $v^2 = 2gR$. Using this, $T = \dfrac{W2gR}{Rg} + W = 3W$. The rope must be able to withstand a tension equal to three times the combined weight of the daredevils.

Exercises

9.31. **THINK:** Determine the change in the angular position in radians. Winter lasts roughly a fourth of a year. There are 2π radians in a circle. Consider the orbit of Earth to be circular.

SKETCH:



RESEARCH: The angular velocity of the earth is $\omega = 2\pi / \text{yr}$. The angular position is given by $\theta = \theta_0 + \omega_0 t$.

SIMPLIFY: $\Delta\theta = \theta - \theta_0 = \omega_0 t$

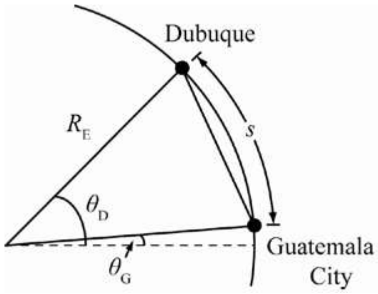
CALCULATE: $\Delta\theta = \dfrac{2\pi \text{ rad}}{\text{yr}}\left(\dfrac{1}{4}\text{ yr}\right) = \dfrac{\pi \text{ rad}}{2} = \dfrac{3.14 \text{ rad}}{2} = 1.57 \text{ rad}$

ROUND: Since π is used to three significant figures, the angle the Earth sweeps over winter is 1.57 rad. It would also be entirely reasonable to leave the answer as $\pi / 2$ radians.

DOUBLE-CHECK: This value makes sense, since there are four seasons of about equal length, so the angle should be a quarter of a circle.

9.32. **THINK:** Determine the arc length between Dubuque and Guatemala City. The angular positions of Dubuque and Guatemala City are $\theta_{\text{D}} = 42.50°$ and $\theta_{\text{G}} = 14.62°$, respectively. The radius of the Earth is $R_{\text{E}} = 6.37 \cdot 10^6$ m.

SKETCH:



RESEARCH: The length of an arc is given by $s = r\theta$, where r is the radius of the circle, and θ is the arc angle given in radians.

SIMPLIFY: $s = R_{\text{E}}(\theta_{\text{D}} - \theta_{\text{G}})\dfrac{2\pi}{360°}$, where the units of the angles are degrees.

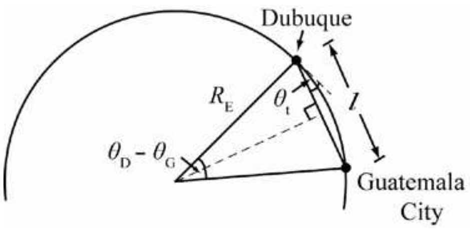
CALCULATE: $s = \left(6.37 \cdot 10^6 \text{ m}\right)\left(42.50° - 14.62°\right)\dfrac{2\pi}{360°} = 3.0996 \cdot 10^6 \text{ m}$

ROUND: The arc length's accuracy is given by the least accurate value used to determine it. In this case, the least accurate value is the radius of Earth, given to three significant figures, so the arc length is $3.10 \cdot 10^6$ m.

DOUBLE-CHECK: This is equal to 3100 km, a reasonable distance between the northern United States and Central America.

9.33. **THINK:** Determine the linear distance between Dubuque and Guatemala city. Also, determine the angle below the horizontal for a tunnel that connects the two. The angular positions of Dubuque and Guatemala City are $\theta_{\text{D}} = 42.50°$ and $\theta_{\text{G}} = 14.62°$, respectively. The radius of the Earth is $R_{\text{E}} = 6.37 \cdot 10^6$ m.

SKETCH:



RESEARCH: Use the triangle of the drawing to relate $\theta_{\text{D}} - \theta_{\text{G}}$, R_{E} and $l/2$. The right triangle gives rise to the equation $\sin\dfrac{\theta_{\text{D}} - \theta_{\text{G}}}{2} = \dfrac{l/2}{R_{\text{E}}}$. The angle of the tunnel is $\theta_{\text{t}} = \left(\dfrac{\theta_{\text{D}} - \theta_{\text{G}}}{2}\right)$.

SIMPLIFY: $l = 2R_{\text{E}}\sin\left(\dfrac{\theta_{\text{D}} - \theta_{\text{G}}}{2}\right)$

CALCULATE: $l = 2\left(6.37 \cdot 10^6 \text{ m}\right)\sin\left(\dfrac{42.50° - 14.62°}{2}\right) = 3.06914 \cdot 10^6 \text{ m}$ $\theta_{\text{t}} = \left(\dfrac{42.50° - 14.62°}{2}\right) = 13.94°$

ROUND: The length will have the same accuracy as the radius of Earth. The angle of the tunnel will be as accurate as the latitude of the cities. Therefore, the length of the tunnel is $3.07 \cdot 10^6$ m, with an angle of 13.94° below the surface of the Earth.

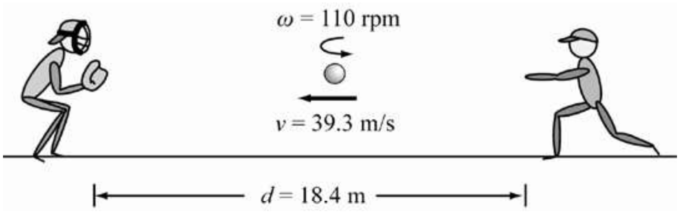
DOUBLE-CHECK: The length of the tunnel is a bit shorter than the arc length, which is expected. See the solution to Problem 9.32.

9.34. **THINK:** Determine the number of rotations the ball will make as it travels to the catcher's glove. The linear and angular speeds of the ball are $v = 88$ mph and $\omega = 110$ rpm. In SI units, these are

$$v = 88 \text{ mph}\left(\frac{0.447 \text{m/s}}{\text{mph}}\right) = 39.3 \text{ m/s} \quad \text{and} \quad \omega = 110 \text{ rpm}\left(\frac{2\pi \text{ rad}}{1 \text{ rot}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 11.52 \text{ rad/s}.$$

The ball travels a distance, $d = 60.5$ ft or 18.4 m.

SKETCH:



RESEARCH: The time it takes for the ball to reach the catcher is given by $t = v/d$. This time will then be used to calculate the number of rotations, given by $n = \omega t$. This number n will be in radians which will then have to be converted to rotations, where $1 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi}\right) = 0.16$ revolution.

SIMPLIFY: $n = \omega\left(\frac{d}{v}\right)$

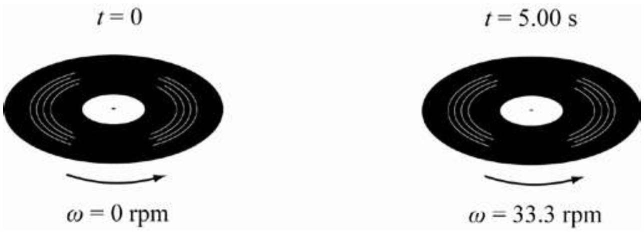
CALCULATE: $n = \dfrac{18.4 \text{ m}(11.52 \text{ rad/s})}{39.3 \text{ m/s}} = 5.394 \text{ rad}\left(\dfrac{1 \text{ rev}}{2\pi}\right) = 0.8586$ rotations

ROUND: The linear speed of the ball, the distance traveled, and the angular speed of the ball are all given to three significant figures, so the number of rotations should be 0.859.

DOUBLE-CHECK: Dimensional analysis: $[n] = \text{rad/s} \cdot \dfrac{\text{m}}{\text{m/s}} \cdot \dfrac{\text{revolution}}{2\pi \text{ rad}}$. All units cancel giving a dimensionless quantity, as expected.

9.35. **THINK:** Determine the average angular acceleration of the record and its angular position after reaching full speed. The initial and final angular speeds are 0 rpm to 33.3 rpm. The time of acceleration is 5.00 s.

SKETCH:



RESEARCH: The equation for angular acceleration is $\alpha = (\omega_f - \omega_i)/\Delta t$. The angular position of an object under constant angular acceleration is given by $\theta = \frac{1}{2}\alpha t^2$.

SIMPLIFY: There is no need to simplify the equation.

CALCULATE: $\alpha = \dfrac{33.3 \text{ rpm} - 0 \text{ rpm}}{5.00 \text{ s}(60 \text{ s/min})} = 0.111 \text{ rev/s}^2 \cdot \dfrac{2\pi \text{ rad}}{1 \text{ rev}} = 0.6974 \text{ rad/s}^2$

$$\theta = \frac{1}{2}(0.111 \text{ rev/s})(5.00 \text{ s})^2 = 1.3875 \text{ rev} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 8.718 \text{ rad}$$

ROUND: To three significant figures, the angular acceleration and position are:

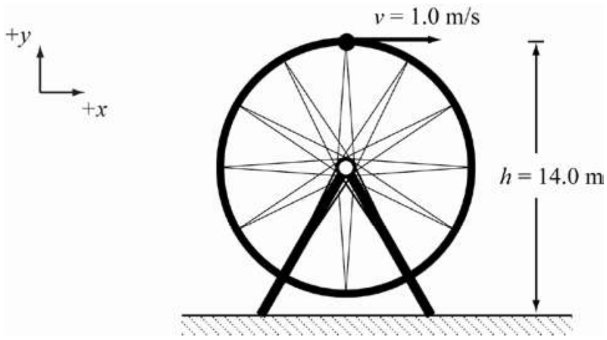
(a) $\alpha = 0.697 \text{ rad/s}^2$

(b) $\theta = 8.72 \text{ rad}$

DOUBLE-CHECK: The calculations yield the correct units of radians and rad/s^2.

9.36. **THINK:** Determine the horizontal distance the teddy bear travels during its fall. In order to do this, the height and the horizontal speed of the bear must be determined. The diameter of the wheel is 12.0 m, the bottom of which is 2.0 m above the ground. The rim of the wheel travels at a speed of $v = 1.0 \text{ m/s}$. The height of the bear is 14.0 m from the ground and is traveling at a speed of 1.0 m/s in the horizontal direction when it falls.

SKETCH:



RESEARCH: The horizontal distance is given by $x = vt$. The time is not yet known but can be determined from $h = \frac{1}{2}gt^2$.

SIMPLIFY: The time it takes the bear to fall is $t^2 = \frac{2h}{g}$ or $t = \sqrt{\frac{2h}{g}}$. The horizontal distance traveled is $x = vt = v\sqrt{\frac{2h}{g}}$.

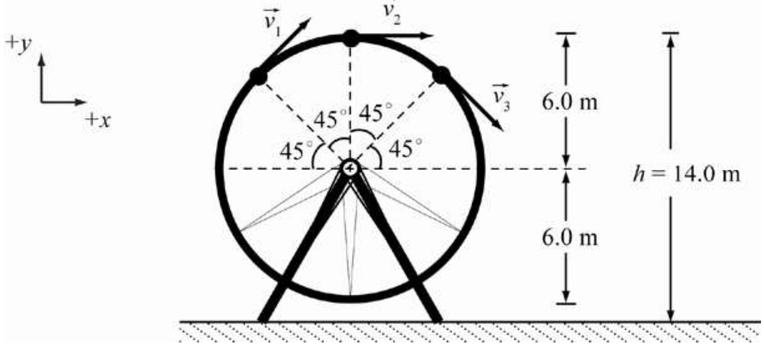
CALCULATE: $x = (1.00 \text{ m/s})\sqrt{\frac{2(14.0 \text{ m})}{9.81 \text{ m/s}^2}} = 1.6894 \text{ m}$

ROUND: The velocity is given to three significant figures, so round the distance to 1.69 m.

DOUBLE-CHECK: The bear lands a short distance from the base of the wheel, as one would expect given its small initial velocity.

9.37. **THINK:** Determine the distance between the three teddy bears. The bears will be traveling at 1.00 m/s but will have different directions and distances from the ground. The angle between adjacent bears is 45.0°. The diameter of the wheel is 12.0 m and the bottom of the wheel is 2.00 m above the ground.

SKETCH:



RESEARCH: The height of bear 1 and 3 is the same and is $h_1 = (8.00 + 6.00\sin(45.0°))$ m. The second bear is $h_2 = 14.0$ m above the ground. The velocities of each bear in the horizontal and vertical directions are $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where $\theta_1 = 45.0°$, $\theta_2 = 0°$ and $\theta_3 = -45.0°$. The distance between each bear before they are dropped is $\Delta d = 6.00\sin(45.0°)$ m. Use the regular equations for projectile motion: $\Delta x = v_x t$ and $\Delta y = v_y t - \frac{1}{2}gt^2$.

SIMPLIFY: For different initial heights, H, the time of the fall can be determined from $h(t) = v\sin(\theta)t - \frac{1}{2}gt^2 + H$. This is a quadratic equation with solution $t = \frac{v\sin\theta}{g} \pm \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}$. Choose the positive root, that is, $t = \frac{v\sin\theta}{g} + \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}$. The change in distance is $\Delta x = v\cos(\theta)t = v\cos\theta\left(\frac{v\sin\theta}{g} + \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}\right)$, which means that the value of x is given by the equation $x = x_0 + v\cos\theta\left(\frac{v\sin\theta}{g} + \sqrt{\frac{v^2\sin^2\theta}{g^2} + \frac{2H}{g}}\right)$.

CALCULATE: For the first bear $x_0 = 0$, $H = h_1$, and $\theta_1 = 45.0°$. Recall $\cos 45.0° = \sin 45.0° = 1/\sqrt{2}$.

$$x_1 = 0 + \left((1.00 \text{ m/s})\left(\frac{1}{\sqrt{2}}\right)\right)\left[\frac{1.00 \text{ m/s}}{9.81 \text{ m/s}^2}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{\frac{(1.00 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)^2} + \frac{2(8.00 + 6.00/\sqrt{2}) \text{ m}}{9.81 \text{ m/s}^2}}\right] = 1.1693 \text{ m}$$

The initial velocity of the second bear is horizontal, so the bear travels a horizontal distance of $\Delta x = 1.6894$ m (see solution to question 9.36). The second bear's position is $x_2 = x_0 + \Delta x = \Delta d + \Delta x = \frac{6.00 \text{ m}}{\sqrt{2}} + 1.6894 \text{ m} = 5.9320 \text{ m}$ from the origin. For the third bear, $x_0 = 2\Delta d$, $H = h_1$, and $\theta_2 = -45.0°$.

$$x_3 = 2\left(\frac{6.00 \text{ m}}{\sqrt{2}}\right) + \frac{(1.00 \text{ m/s})}{\sqrt{2}}\left[\frac{-1.00 \text{ m/s}}{\sqrt{2}(9.81 \text{ m/s}^2)} + \sqrt{\frac{(1.00 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)^2} + \frac{2(8.00 + 6.00/\sqrt{2}) \text{ m}}{9.81 \text{ m/s}^2}}\right] = 9.5526 \text{ m}$$

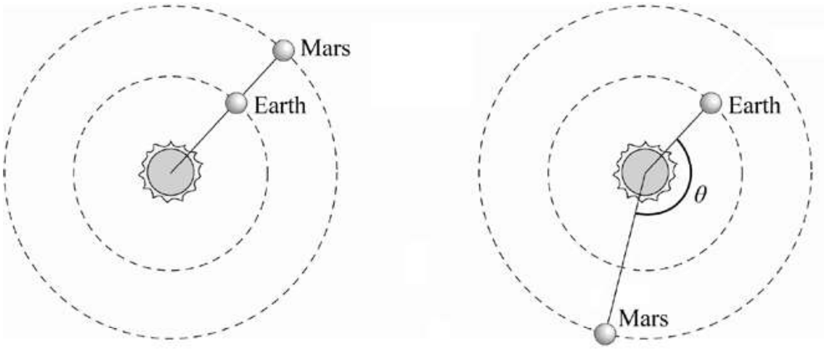
The distance between the first 2 bears is $\Delta d_{12} = 4.7627$ m. The distance between the last two bears is $\Delta d_{23} = 3.6206$ m.

ROUND: The velocity has three significant figures, so the results should also have three significant figures. The distances between the bears once they hit the ground are $\Delta d_{12} = 4.76$ m and $\Delta d_{23} = 3.62$ m.

DOUBLE-CHECK: The result is reasonable since $\Delta d_{12} > \Delta d_{23}$. This must be so since the third bear is in the air for a shorter time because the original horizontal velocity points towards the ground.

9.38. **THINK:** Determine (a) the angular distance between the two planets a year later, (b) the time it takes the two planets to align again and (c) the angular position the alignment occurs at. The radius and period of each planet's orbit are $r_M = 228 \cdot 10^6$ km, $T_M = 687$ days, $r_E = 149.6 \cdot 10^6$ km and $T_E = 365.26$ days.

SKETCH:



RESEARCH: The questions can be answered using $\theta = \omega t$ and $\omega = 2\pi / T$.

SIMPLIFY: The angular distance is

$$\Delta\theta = \theta_{\text{E}} - \theta_{\text{M}} = \omega_{\text{E}} T_{\text{E}} - \omega_{\text{M}} T_{\text{E}} = 2\pi\left(\frac{T_{\text{E}}}{T_{\text{E}}} - \frac{T_{\text{E}}}{T_{\text{M}}}\right) = 2\pi\left(1 - \frac{T_{\text{E}}}{T_{\text{M}}}\right).$$

The time it takes the planets to realign occurs when $\theta_{\text{E}} = \theta_{\text{M}} + 2\pi$ or $\omega_{\text{E}}\Delta t = \omega_{\text{M}}\Delta t + 2\pi$, so

$$\Delta t = \frac{2\pi}{\omega_{\text{E}} - \omega_{\text{M}}} = \frac{2\pi}{\dfrac{2\pi}{T_{\text{E}}} - \dfrac{2\pi}{T_{\text{M}}}} = \frac{T_{\text{E}} T_{\text{M}}}{T_{\text{M}} - T_{\text{E}}}.$$

The angular position is found by solving for the angle instead of the time. $\theta_{\text{M}} = \omega_{\text{M}}\Delta t \;\Rightarrow\; \Delta t = \theta_{\text{M}} / \omega_{\text{M}}$, so: $\theta_{\text{E}} = \omega_{\text{E}}\Delta t = \dfrac{\omega_{\text{E}}\theta_{\text{M}}}{\omega_{\text{M}}} = \theta_{\text{M}} + 2\pi \;\Rightarrow\; \theta_{\text{M}} = \dfrac{2\pi}{\dfrac{\omega_{\text{E}}}{\omega_{\text{M}}} - 1} - 2\pi = \dfrac{2\pi}{\dfrac{T_{\text{M}}}{T_{\text{E}}} - 1} - 2\pi = \dfrac{2\pi T_{\text{E}}}{T_{\text{M}} - T_{\text{E}}} - 2\pi$. Subtract 2π from

the answer, so that $\theta \le 2\pi$.

CALCULATE: $\Delta\theta = 2\pi\left(1 - \dfrac{365.26}{687}\right) = 2.9426$ rad, $\Delta t = \dfrac{687(365.26)}{687 - 365.26} = 779.93$ days,

$$\theta = \frac{2\pi(365.26)}{687 - 365.26} - 2\pi = 0.84989 \text{ rad}$$

ROUND: The periods of Mars and Earth have three significant figures, so the results should be rounded accordingly.

(a) $\Delta\theta = 2.94$ rad

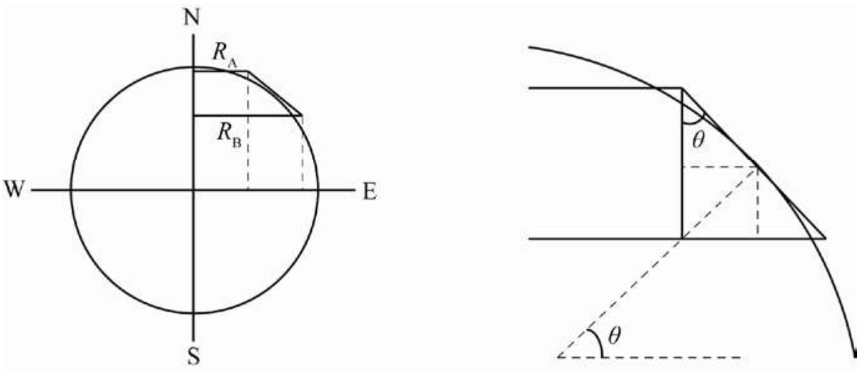
(b) $\Delta t = 780.$ days

(c) $\theta = 0.850$ rad

DOUBLE-CHECK: The numbers are of the correct order for this solar system.

9.39. **THINK:** Determine (a) the magnitude and direction of the velocities of the pendulum at position *A* and *B*, (b) the angular speed of the pendulum motion, (c) the period of the rotation and (d) the effects of moving the pendulum to the equator. The latitude of the pendulum is 55.0° above the equator. The pendulum swings over a distance of d = 20.0 m. The period of the Earth's rotation is $T_{\text{E}} = 23 \text{ hr} + 56 \text{ min} = 86160 \text{ s}$ and the Earth's radius is $R_{\text{E}} = 6.37 \cdot 10^6$ m.

SKETCH:



RESEARCH: The following equations can be used: $\omega = \dfrac{2\pi}{T}$, $v = r\omega$, $R_A = R_E \cos\theta - \left(\dfrac{d}{2}\sin\theta\right)$ and $R_B = R_E \cos\theta + \left(\dfrac{d}{2}\sin\theta\right)$.

SIMPLIFY: The magnitudes of the velocities are:

$$v_A = R_A \omega_A = \frac{2\pi R_A}{T_E} = \frac{2\pi\left(R_E \cos\theta - \left(\frac{d}{2}\sin\theta\right)\right)}{T_E} \text{ and } v_B = \frac{2\pi\left(R_E \cos\theta + \left(\frac{d}{2}\sin\theta\right)\right)}{T_E}.$$

The angular speed of the rotation is related to the linear speed by $\Delta v = \omega_R d$. Rearranging gives:

$$\omega_R = \frac{\Delta v}{d} = \left(\frac{1}{d}\right)\frac{2\pi}{T_E}\left(\left(R_E \cos\theta + \left(\frac{d}{2}\sin\theta\right)\right) - \left(R_E \cos\theta - \left(\frac{d}{2}\sin\theta\right)\right)\right) = \frac{2\pi}{dT_E} d\sin\theta = \frac{2\pi}{T_E}\sin\theta.$$

The period is then $T_R = \dfrac{2\pi}{w_R} = \dfrac{2\pi}{\dfrac{2\pi}{T_E}\sin\theta} = \dfrac{T_E}{\sin\theta}$. At the equator, $\theta = 0°$.

CALCULATE:

(a) $v_A = 2\pi\left(\dfrac{(6.37 \cdot 10^6 \text{ m})\cos(55.0°) - (10.0 \text{ m})\sin(55.0°)}{86{,}160 \text{ s}}\right) = 266.44277 \text{ m/s}$

$v_B = 2\pi\left(\dfrac{(6.37 \cdot 10^6 \text{ m})\cos(55.0°) + (10.0 \text{ m})\sin(55.0°)}{86{,}160 \text{ s}}\right) = 266.44396 \text{ m/s}$

$\Delta v = v_B - v_A = 266.44396 \text{ m/s} - 266.44277 \text{ m/s} = 0.00119 \text{ m/s or } 1.19 \text{ mm/s}$

(b) $\omega_R = \dfrac{2\pi\sin(55.0°)}{86{,}160 \text{ s}} = 5.97 \cdot 10^{-5} \text{ rad/s}$

(c) $T_R = \dfrac{86{,}160 \text{ s}}{\sin(55.0°)} = 105{,}182 \text{ s or about 29.2 hours}$

(d) At the equator, $T_R = \lim\limits_{\theta \to 0} \dfrac{T_E}{\sin\theta} = \infty$.

ROUND: The values given in the question have three significant figures, so the answers should also be rounded to three significant figures:

(a) The velocities are $v_A = 266.44277$ m/s and $v_B = 266.44396$ m/s, are in the direction of the Earth's rotation eastward. This means the difference between the velocities is $\Delta v = 1.19$ mm/s.

(b) The angular speed of rotation is $w_R = 1.19 \cdot 10^{-4}$ rad/s.

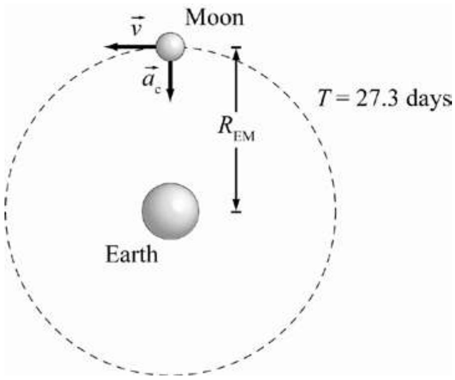
(c) The period of rotation is about 29.2 hours.

(d) At the equator there is no difference between the velocities at *A* and *B*, so the period is $T_R = \infty$. This means the pendulum does not rotate.

DOUBLE-CHECK: These are reasonable answers. If the difference in velocities was larger, these effects would be seen in everyday life but they are not. These are things pilots deal with when planning a flight path.

9.40. **THINK:** Determine the centripetal acceleration of the Moon around the Earth. The period of the orbit is $T = 27.3$ days and the orbit radius is $R = 3.85 \cdot 10^8$ m.

SKETCH:



RESEARCH: The centripetal acceleration is given by $a_c = \frac{v^2}{R}$. The radius, *R*, is known, so the speed, *v*, can be determined by making use of the period and noting that in this period the moon travels a distance equal to the circumference of a circle of radius *R*. Therefore,

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R_{EM}}{T}.$$

SIMPLIFY: $a_c = \frac{v^2}{R} = \left(\frac{2\pi R_{EM}}{T}\right)^2 \left(\frac{1}{R_{EM}}\right) = \frac{4\pi^2 R_{EM}}{T^2}$

CALCULATE: Convert the period to seconds: 27.3 days = $2.3587 \cdot 10^6$ seconds. Therefore,

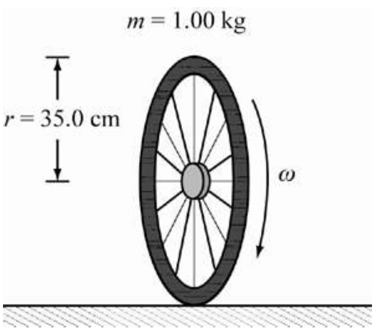
$$a_c = \frac{4\pi^2 \left(3.85 \cdot 10^8 \text{ m}\right)}{\left(23.587 \cdot 10^5 \text{ s}\right)^2} = 2.732 \cdot 10^{-3} \text{ m/s}^2.$$

ROUND: Since the values are given to three significant figures, $a_c = 2.73 \cdot 10^{-3}$ m/s^2.

DOUBLE-CHECK: This is reasonable for a body in uniform circular motion with the given values.

9.41. **THINK:** Determine the angular acceleration of a wheel given that it takes 1.20 seconds to stop when put in contact with the ground after rotating at 75.0 rpm. The wheel has a radius 35.0 cm and a mass of 1.00 kg.

SKETCH:



RESEARCH: Consider the angular speed of the wheel, and the necessary acceleration to bring that speed to zero in the given time. The angular acceleration is given by $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(\omega - \omega_0)}{t}$ and the rotational speed is given by:

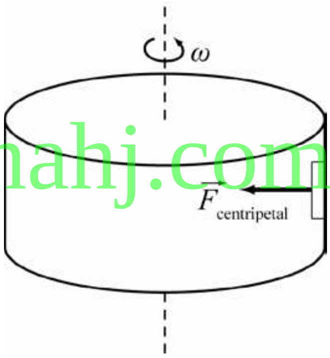
$$\omega = (\text{rpm})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right).$$

SIMPLIFY: Since the final rotational speed is zero, $\alpha = \frac{-\omega_0}{t} = -\frac{(\text{rpm})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right)}{t}$.

CALCULATE: $\alpha = -\frac{(75.0 \text{ rpm})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{(1.20 \text{ s})} = -6.54 \text{ rad/s}^2$

ROUND: Since the values are given to three significant figures, the result is $\alpha = -6.54 \text{ rad/s}^2$.
DOUBLE-CHECK: It is important that the acceleration is negative since it is slowing down the wheel. The magnitude seems reasonable based on the given values.

9.42. **THINK:** Determine the frequency of rotation required to produce an acceleration of $1.00 \cdot 10^5 g$. The radius is R = 10.0 cm.
SKETCH:



RESEARCH: Recall that the centripetal acceleration is given by $a_c = \omega^2 R$. Also, $\omega = 2\pi f$. Therefore, $a_c = (2\pi f)^2 R = 4\pi^2 f^2 R$.

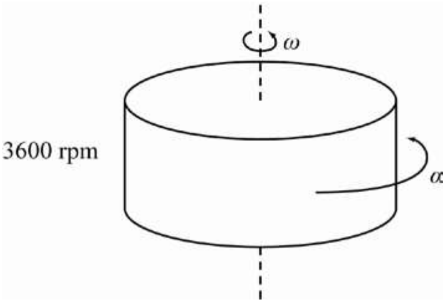
SIMPLIFY: Solving for *f*, $f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$.

CALCULATE: $f = \frac{1}{2\pi}\sqrt{\frac{(1.00 \cdot 10^5)(9.81 \text{ m/s}^2)}{0.100 \text{ m}}} = 498.49 \text{ Hz}$

ROUND: Since all values are given to three significant figures, the result is $f = 498$ Hz.
DOUBLE-CHECK: A frequency of about 500 Hz seems reasonable to try to obtain an acceleration five orders of magnitude greater than *g*.

9.43. **THINK:** The initial angular speed is $\omega_0 = 3600. \text{ rpm} = 3600. \text{ rpm} \cdot \frac{2\pi \text{ rad}}{1 \text{ rotation}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 120\pi \text{ rad/s}$. Calculate the time, t_1, it takes for the centrifuge to come to a stop $(\omega_1 = \omega(t_1) = 0)$ by using the average angular speed, $\bar{\omega}$, and the fact that that it completes $n = 60.0$ rotations. Use the time taken to stop to find the angular acceleration.

SKETCH:



RESEARCH: The average angular speed is given by $\bar{\omega} = \frac{1}{2}(\omega_f + \omega_0)$. Since the centrifuge completes 60.0 turns while decelerating, it turns through an angle of $\Delta\theta = 60.0 \text{ turns} \cdot \frac{2\pi \text{ rad}}{\text{turn}} = 120\pi \text{ rad}$. Use the two previous calculated values in the formula $\Delta\theta = \bar{\omega}t_1$ to obtain the time taken to come to a stop. Then, use the equation $\omega(t) = \omega_0 + \alpha t$ to compute the angular acceleration, α.

SIMPLIFY: The time to decelerate is given by, $t_1 = \frac{\Delta\theta}{\bar{\omega}} = \frac{\Delta\theta}{(\omega_1 + \omega_0)/2}$. Substituting this into the last equation given in the research step gives the equation, $\omega(t_1) = \omega_1 = \omega_0 + \alpha \frac{2\Delta\theta}{(\omega_1 + \omega_0)}$. Solving for α yields the equation: $\alpha = \frac{(\omega_1 - \omega_0)(\omega_1 + \omega_0)}{2\Delta\theta}$.

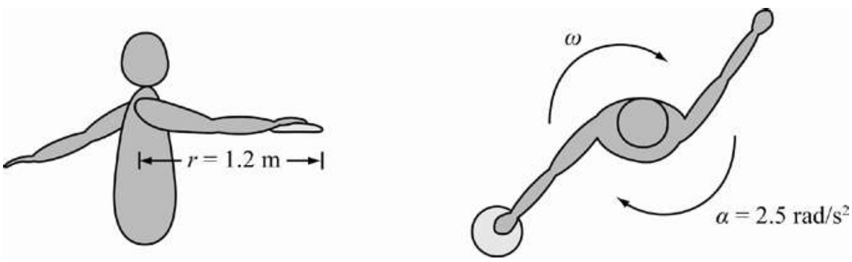
CALCULATE: $\alpha = \frac{(0 - 120\pi \text{ rad/s})(0 + 120\pi \text{ rad/s})}{2(120\pi \text{ rad})} = -60\pi \text{ rad/s}^2 = -188.496 \text{ rad/s}^2$

ROUND: Since the number of rotations is given to three significant figures, the final result should be also rounded to three significant figures: $\alpha = -188 \text{ rad/s}^2$.

DOUBLE-CHECK: The negative sign of α indicates deceleration, which is appropriate since the centrifuge is coming to a stop. The centrifuge decelerates from 120π rad/s to rest in $t_1 = \frac{2\Delta\theta}{(\omega_1 + \omega_0)} = \frac{2(120\pi \text{ rad})}{(0 + 120\pi \text{ rad/s})} = 2$ s, and since the angular deceleration is constant, it must be the case that the deceleration is $60\pi \text{ rad/s}^2$. The answer is therefore reasonable.

9.44. **THINK:** A circular motion has a constant angular acceleration of $\alpha = 2.5 \text{ rad/s}^2$ and a radius of $r = 1.2$ m. Determine (a) the time required for the angular speed to reach 4.7 rad/s, (b) the number of revolutions to reach this angular speed of 4.7 rad/s, (c) the linear speed when the angular speed is 4.7 rad/s, (d) the linear acceleration when the angular speed is 4.7 rad/s, (e) the magnitude of the centripetal acceleration when the angular speed is 4.7 rad/s and (f) the magnitude of the discus' total acceleration.

SKETCH:



RESEARCH:

(a) Since the angular acceleration is constant, the time required to reach the final angular speed can be determined by means of the kinematic equation, $\omega = \omega_0 + \alpha t$, where $\omega_0 = 0.0$ rad/s.

(b) Once the time required to reach the angular speed, ω, is determined, the number of revolutions can be determined by setting $1 \text{ rev} = 2\pi$ rad, where the number of radians is obtained from $d[\text{rad}] = \frac{1}{2}(\omega + \omega_0)t$.

(c) The linear speed, v, can be determined from the angular speed, ω, by the relation $v = \omega r$.

(d) The linear acceleration, a_t, can be obtained from the angular acceleration, α, by the relation $a_t = \alpha r$.

(e) The magnitude of the centripetal acceleration can be determined from the linear speed by the relation $a_c = \frac{v^2}{r}$.

(f) The total acceleration, a_T, can be found as the hypotenuse of a right angle triangle where the sides are the linear (tangential) acceleration, a_t, and the angular acceleration, α. The relationship is $a_T = \sqrt{a_t^2 + \alpha^2}$.

SIMPLIFY:

(a) $\omega = \omega_0 + \alpha t = \alpha t \Rightarrow t = \frac{\omega}{\alpha}$

(b) $d[\text{rad}] = \frac{1}{2}(\omega + \omega_0)t = \frac{1}{2}(\omega t)$, and to convert to the number of revolutions, $\text{rev} = \frac{d}{(2\pi)}$.

(c) $v = \omega r$

(d) $a_t = \alpha r$

(e) $a_c = \frac{v^2}{r}$

(f) $a_T = \sqrt{a_t^2 + \alpha^2}$

CALCULATE:

(a) $t = \frac{4.70 \text{ rad/s}}{2.50 \text{ rad/s}^2} = 1.88 \text{ s}$

(b) $d[\text{rad}] = \frac{(4.70 \text{ rad/s})(1.88 \text{ s})}{2} = 4.42 \text{ rad} \text{ or } 4.42 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 0.70314 \text{ rev}$

(c) $v = (4.70 \text{ rad/s})(1.20 \text{ m}) = 5.64 \text{ m/s}$

(d) $a_t = (2.50 \text{ rad/s}^2)(1.20 \text{ m}) = 3.00 \text{ m/s}^2$

(e) $a_c = \frac{(5.64 \text{ m/s})^2}{1.20 \text{ m}} = 26.5 \text{ m/s}^2$

(f) $a_T = \sqrt{(2.88 \text{ m/s}^2)^2 + (26.5 \text{ m/s}^2)^2} = 26.656 \text{ m/s}^2$

ROUND: Rounding to three significant figures:

(a) $t = 1.88$ s

(b) 0.703 revolutions

(c) $v = 5.64$ m/s

(d) $a_t = 3.00 \text{ m/s}^2$

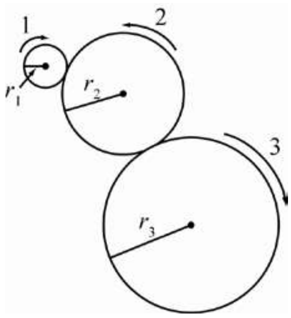
(e) $a_c = 26.5 \text{ m/s}^2$

(f) $a_T = 26.7 \text{ m/s}^2$

DOUBLE-CHECK: Based on the given values, these results are reasonable.

9.45. **THINK:** Three coupled disks have radii $r_1 = 0.100$ m, $r_2 = 0.500$ m, $r_3 = 1.00$ m. The rotation rate of disk 3 is one revolution every 30.0 seconds. Determine (a) the angular speed of disk 3, (b) the tangential velocities of the three disks, (c) the angular speeds of disks 1 and 2 and (d) if the angular acceleration of disk 1 is $\alpha_1 = 0.100 \text{ rad/s}^2$, what are the angular accelerations of disks 2 and 3?

SKETCH:



RESEARCH:

(a) To obtain the angular speed of disk 3, use its rotation rate, $T = 30.0$ s, and the relationship between revolutions and radians, 2π rad/rev. Therefore, $\omega_3 = 2\pi / T$.

(b) Since the three disks are touching each other and there is no slipping, they all have the same tangential speed. Therefore, only one tangential speed must be determined. Since the angular speed of disk 3 is known, the tangential speed can be determined from $v = \omega_3 r_3$.

(c) Calculate the angular speed for disks 1 and 2 from the tangential speeds and the radii. That is, $\omega_1 = v / r_1$, and $\omega_2 = v / r_2$.

(d) Since the angular acceleration of disk 1 is known, its tangential acceleration can be determined. Since the disks are touching each other, and no slipping occurs, this tangential acceleration is common to all disks. The angular acceleration for disks 2 and 3 can be determined from this tangential acceleration and the radii. Therefore, $\alpha_1 = a_1 / r_1$, implies $a_1 = \alpha_1 r_1$. Since $a_1 = a_2 = a_3 = a$, $\alpha_2 = a / r_2$ and $\alpha_3 = a / r_3$.

SIMPLIFY:

(a) $\omega_3 = 2\pi / T$

(b) $v = \omega_3 r_3$

(c) $\omega_1 = v / r_1$, and $\omega_2 = v / r_2$.

(d) $\alpha_2 = a / r_2$ and $\alpha_3 = a / r_3$, where $a = \alpha_1 r_1$.

CALCULATE:

(a) $\omega_3 = \dfrac{(2\pi \text{ rad/rev})}{30.0 \text{ s}} = 0.209 \text{ rad/s}$

(b) $v = (0.209 \text{ rad/s})(1.00 \text{ m}) = 0.209 \text{ m/s}$ for all three disks.

(c) $\omega_1 = \dfrac{0.209 \text{ m/s}}{0.100 \text{ m}} = 2.09 \text{ rad/s}$ and $\omega_2 = \dfrac{0.209 \text{ m/s}}{0.500 \text{ m}} = 0.419 \text{ rad/s}$.

(d) $a = (0.100 \text{ rad/s}^2)(0.100 \text{ m}) = 1.00 \cdot 10^{-2} \text{ m/s}^2$.

Therefore, $\alpha_2 = \dfrac{1.00 \cdot 10^{-2} \text{ m/s}^2}{0.500 \text{ m}} = 2.00 \cdot 10^{-2} \text{ rad/s}^2$ and $\alpha_3 = \dfrac{1.00 \cdot 10^{-2} \text{ m/s}^2}{1.00 \text{ m}} = 1.00 \cdot 10^{-2} \text{ rad/s}^2$.

ROUND: Keeping three significant figures, the results are:

(a) $\omega_3 = 0.209 \text{ rad/s}$

(b) $v = 0.209 \text{ m/s}$ for all three disks

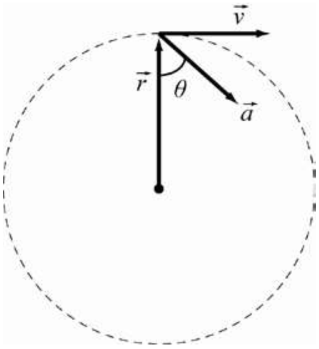
(c) $\omega_1 = 2.09$ rad/s and $\omega_2 = 0.419$ rad/s

(d) $\alpha_2 = 2.00 \cdot 10^{-2}$ rad/s^2 and $\alpha_3 = 1.00 \cdot 10^{-2}$ rad/s^2

DOUBLE-CHECK: Based on the given values, all the results are reasonable.

9.46. **THINK:** Determine the speed of a particle whose acceleration has a magnitude of $a = 25.0 \text{ m/s}^2$ and makes an angle of $\theta = 50.0°$ with the radial vector.

SKETCH:



RESEARCH: To determine the tangential speed, v, recall that the centripetal acceleration is given by $a_c = \frac{v^2}{r}$. The centripetal acceleration is the projection of the total acceleration on the radial axis, i.e. $a_c = a_T \cos\theta$.

SIMPLIFY: Therefore, the tangential speed is given by $v = \sqrt{a_c r} = \sqrt{a_T r \cos\theta}$.

CALCULATE: $v = \sqrt{(25.0 \text{ m/s}^2)\cos(50.0°)(1.00 \text{ m})} = 4.01 \text{ m/s}$

ROUND: The values are given to three significant figures, therefore the result is $v = 4.01$ m/s.

DOUBLE-CHECK: This result is reasonable based on the magnitudes of the given values.

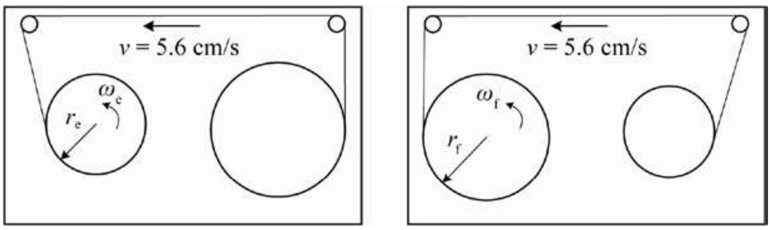
9.47. **THINK:** Determine the angular speed of the take-up spool in a tape recorder in the following cases:

(a) When the take-up spool is empty with radius, $r_e = 0.800$ cm.

(b) When the take-up spool is full with radius, $r_f = 2.20$ cm.

(c) Determine the average angular acceleration of the take-up spool if the length of the tape is $l = 100.80$ m. The magnetic tape has a constant linear speed of $v = 5.60$ cm/s.

SKETCH:



RESEARCH:

(a) & (b) To determine the angular speed, make use of the relationship $v = \omega r \Rightarrow \omega = \frac{v}{r}$.

(c) To determine an average angular acceleration, use the definition, $\alpha = \Delta w / \Delta t$, where the time is determined from $\Delta t = \frac{(\text{distance})}{(\text{speed})} = \frac{l}{v}$.

SIMPLIFY:

(a) $\omega_e = \dfrac{v}{r_e}$

(b) $\omega_f = \dfrac{v}{r_f}$

(c) $\alpha = \dfrac{\Delta\omega}{\Delta T} = \dfrac{\omega_f - \omega_e}{l/v} = \dfrac{v(\omega_f - \omega_e)}{l}$

CALCULATE:

(a) $\omega_e = \dfrac{5.60 \cdot 10^{-2}\text{ m/s}}{8.00 \cdot 10^{-3}\text{ m}} = 7.00\text{ rad/s}$

(b) $\omega_f = \dfrac{5.60 \cdot 10^{-2}\text{ m/s}}{2.20 \cdot 10^{-2}\text{ m}} = 2.54\text{ rad/s}$

(c) $\alpha = \dfrac{(5.60 \cdot 10^{-2}\text{ m/s})(2.54 - 7.00)}{100.80\text{ m}} = -2.48 \cdot 10^{-3}\text{ rad/s}^2$

ROUND: Keep three significant figures:

(a) $\omega_e = 7.00$ rad/s

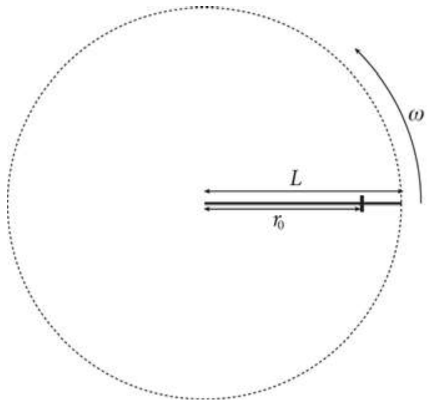
(b) $\omega_f = 2.54$ rad/s

(c) $\alpha = -2.48 \cdot 10^{-3}\text{ rad/s}^2$

DOUBLE-CHECK: It is reasonable that the angular speed of the spool when it's empty is greater than when it's full. Also, it is expected that the angular acceleration is negative since the angular speed is decreasing as the spool gets full.

9.48. **THINK:** Determine the radial velocity of a ring fitted around a rod as it reaches the end of the rod. The rod is spun in a horizontal circle at a constant angular velocity. The given values are the length of the rod, l = 0.50 m, the initial distance of the ring from the fixed end of the rod, $r_0 = 0.30$ m, and the constant angular velocity, $\omega = 4.0$ rad/s.

SKETCH:



RESEARCH: For the ring to move in a circular path at a fixed distance along the rod, it would require a centripetal acceleration of $a_c = \omega^2 r$ directed toward the center of the path. However, there is no force on the ring that will supply this acceleration, thus the inertia of the ring will tend to pull it outward along the rod. The resulting radial acceleration is equal to the missing centripetal acceleration, $a_c = \omega^2 r$. Since this radial acceleration depends on the radial position, the differential kinematic relations must be used:

$$\frac{dv_r}{dt} = \omega^2 r \Rightarrow \left(\frac{dv_r}{dr}\right)\left(\frac{dr}{dt}\right) = \omega^2 r,$$

where the second equation follows from using the chain rule of calculus.

SIMPLIFY: Since $\frac{dr}{dt} = v_\text{r}$, use separation of variables to set up the integral:

$$v_\text{r} dv_\text{r} = \omega^2 r dr \;\Rightarrow\; \int_0^v v_\text{r}' dv_\text{r}' = \omega^2 \int_{r_0}^{l} r' dr'$$

$$\left.\frac{\left(v_\text{r}'\right)^2}{2}\right|_0^{v_\text{r}} = \omega^2 \left.\frac{\left(r'\right)^2}{2}\right|_{r_0}^{l}$$

$$\frac{v_\text{r}^2}{2} = \frac{\omega^2\left(l^2 - r_0^2\right)}{2} \;\rightarrow\; v_\text{r} = \omega\sqrt{l^2 - r_0^2}.$$

CALCULATE: The speed is therefore, $v_\text{r} = (4.00 \text{ rad/s})\sqrt{(0.500 \text{ m})^2 - (0.300 \text{ m})^2} = 1.60 \text{ m/s}.$

ROUND: Since the angular velocity is given to three significant figures, the result remains $v_\text{r} = 1.60$ m/s.

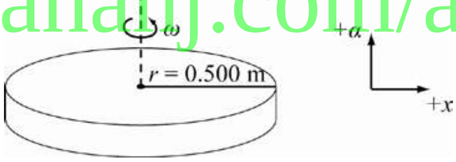
DOUBLE-CHECK: Based on the given values, the resulting radial velocity is reasonable.

9.49. **THINK:** A flywheel with a diameter of 1.00 m is initially at rest, and has an angular acceleration in terms of time as $\alpha(t) = 0.1t^2$, and has units of rad/s^2. Determine:

(a) The angular separation between the initial and final positions of a point on the rim 8.00 seconds after the rotations begin.

(b) Find the linear position, velocity, and acceleration of a point 8.00 seconds after the wheel starts rotating, where the starting position of the point is at $\theta = 0$. Use the known equations relating the position and velocity to the acceleration.

SKETCH:



RESEARCH:

(a) The angular separation can be determined by first considering the change in angular speed through the time period:

$$\Delta\omega = \int_\text{i}^\text{f} \alpha(t)\, dt.$$

Since the initial angular speed is zero, $\Delta\omega = \omega$. Then consider the change in the angle through the time period:

$$\Delta\theta = \int_\text{i}^\text{f} \omega\, dt.$$

(b) The angular acceleration and angular velocity are known and can be related to the tangential component of the linear acceleration and to the velocity through the equations $a_t = \alpha(t) r$ and $v = \omega r$. The radial component of the acceleration vector is the centripetal acceleration, $a_r = \text{v}^2/r$. The position will be on the circumference, given by $\vec{r} = r(\cos\theta)\hat{x} + r(\sin\theta)\hat{y}$ where the angle is known from (a). Note that in this case, the question indicates that $v_0 = 0$ and $\theta_0 = 0$. By convention, θ is measured counterclockwise from the positive x-axis.

SIMPLIFY:

(a) $\Delta\omega = \int_\text{i}^\text{f} \alpha\, dt$ and $\Delta\theta = \int_\text{i}^\text{f} \omega\, dt$.

(b) $a = \alpha(t) r$, $v = \omega(t) r$, $\vec{r} = r(\cos\theta)\hat{x} + r(\sin\theta)\hat{y}$ do not need simplifying.

CALCULATE:

(a) $\Delta\omega = \int_0^t (0.1)t^2\ dt = \frac{0.1t^3}{3}$, $\Delta\theta = \int_i^f \omega\ dt = \int_0^8 \frac{0.1t^3}{3}\ dt = \frac{0.1t^4}{12}\Big|_0^8 = 34.13333 \text{ rad} = 5.43249 \text{ rev}$

This is 5 complete revolutions plus an additional 0.43249 of a revolution. Therefore the angular separation is given by $(0.43249)(2\pi) = 2.717$ rad.

(b) For the linear velocity, $v = \frac{0.1t^3 r}{3} = \frac{(0.1)(8.00 \text{ s})^3(0.500 \text{ m})}{3} = 8.53333$ m/s. The linear position,

$\vec{r} = (0.500 \text{ m})[\cos(2.717 \text{ rad})]\hat{x} + (0.500 \text{ m})[\sin(2.717 \text{ rad})]\hat{y} = -(0.4556 \text{ m})\hat{x} + (0.20597 \text{ m})\hat{y}$

For the tangential acceleration, $a_t = 0.1t^2 r = (0.1)(8.00 \text{ s})^2(0.500 \text{ m}) = 3.200 \text{ m/s}^2$.

For the radial acceleration: $a_r = (8.53333 \text{ m/s})^2 / (0.500 \text{ m}) = 145.635 \text{ m/s}^2$.

Therefore the magnitude of the total acceleration is dominated by the centripetal acceleration:

$a = \sqrt{a_r^2 + a_t^2} = \sqrt{145.635^2 + 3.2^2} \text{ m/s}^2 = 145.671 \text{ m/s}^2$.

ROUND: The constant 0.1 in the function for α is treated as precise.

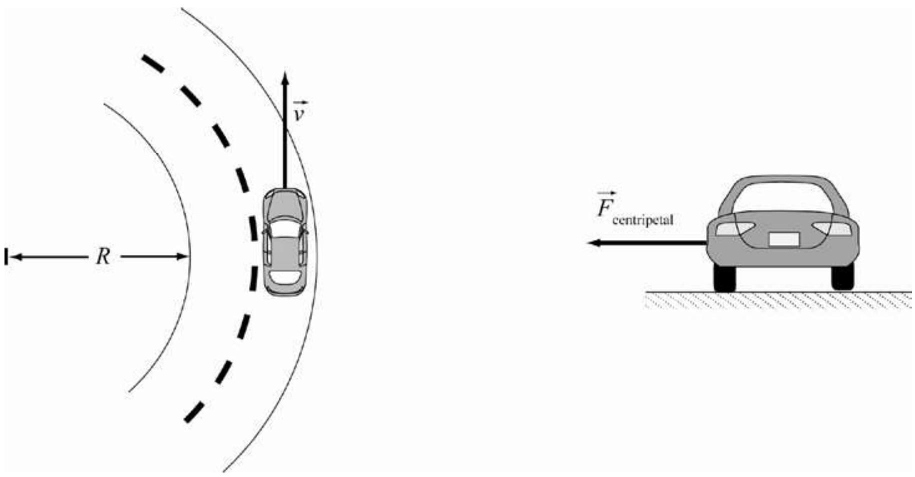
(a) Since all values are given to three significant figures, the result is $\Delta\theta = 2.72$ rad.

(b) To three significant figures, the results are $a_t = 3.20 \text{ m/s}^2$, $a_r = 146 \text{ m/s}^2$, and $v = 8.53$ m/s. The position of the point is $-0.456\hat{x} + 0.206\hat{y}$ (or 0.500 m from the center at an angle of $+2.72$ rad from its initial position).

DOUBLE-CHECK: Based on the given values, these results are reasonable. The magnitude of the linear position vector is $|\vec{r}| = \sqrt{(-0.4556 \text{ m})^2 + (0.20597 \text{ m})^2} = 0.500$ m, which is consistent with the requirement that the point is at the edge of the wheel.

9.50. **THINK:** Determine the force that plays the role of and has the value of the centripetal force on a vehicle of mass $m = 1500.$ kg, with speed $v = 15.0$ m/s around a curve of radius $R = 400.$ m.

SKETCH:



RESEARCH: The force that keeps the vehicle from slipping out of the curve is the force of static friction. The force can be calculated by recalling the form of the centripetal force,

$$F_c = m\frac{v^2}{R}.$$

SIMPLIFY: The equation is in its simplest form.

CALCULATE: $F_c = (1500.\text{ kg})\dfrac{(15.0\text{ m})^2}{400.\text{ m}} = 843.75\text{ N}$

ROUND: To three significant figures: $F_c = 844\text{ N}$

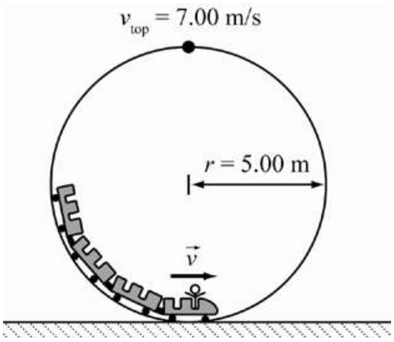
DOUBLE-CHECK: The coefficient of static friction can be determined from the equation $F_c = \mu_s mg$:

$$\Rightarrow \mu_s = \frac{F_c}{mg} = \frac{800\text{ N}}{1500\text{ kg}\left(9.81\text{ m/s}^2\right)} = 0.05.$$

This is within the expected values before slipping occurs. Therefore this is a reasonable force to obtain for the centripetal force.

9.51. **THINK:** The apparent weight of a rider on a roller coaster at the bottom of the loop is to be determined. From Solved Problem 9.1, the radius is $r = 5.00$ m, and the speed at the top of the loop is 7.00 m/s.

SKETCH:



RESEARCH: The apparent weight is the normal force from the seat acting on the rider. At the bottom of the loop the normal force is the force of gravity plus the centripetal force:

$$N = F_g + \frac{mv^2}{r} = mg + \frac{mv^2}{r}.$$

The velocity at the bottom of the loop can be determined by considering energy conservation between the configuration at the top and that at the bottom:

$$\frac{1}{2}mv^2 = mgh + \frac{1}{2}mv_t^2$$

where $h = 2r$. In Solved Problem 9.1 it as determined that the feeling of weightlessness at the top is achieved if $\dfrac{mv_t^2}{r} = mg$.

SIMPLIFY: Multiply the equation for energy conservation by a factor of $2/r$ and find:

$$\frac{mv^2}{r} = \frac{2mgh}{r} + \frac{mv_t^2}{r}.$$

Since $h = 2r$, this results in:

$$\frac{mv^2}{r} = 4mg + \frac{mv_t^2}{r}.$$

Insert this for the normal force and see

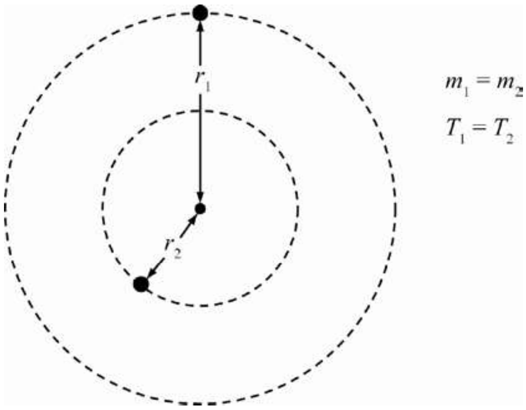
$$N = mg + \frac{mv^2}{r} = mg + 4mg + \frac{mv_t^2}{r} = mg + 4mg + mg = 6mg.$$

CALCULATE: Not needed.

ROUND: Not needed.

DOUBLE-CHECK: Our result means that you experience 6*g* of acceleration at the bottom of the loop, which seems like a large number, if you consider that the maximum acceleration during the launch of a Space Shuttle is kept to 3*g*. However, if you have ever had the opportunity to ride on such a roller coaster, then our result does not seem unreasonable.

9.52. **THINK:** Two skaters have equal masses and periods of rotation but the radius of one is half of the other. Determine:
(a) The ratio of their speeds.
(b) The ratio of the magnitudes of the forces on each skater.
SKETCH:



RESEARCH:
(a) The ratio of the speeds, v_1 / v_2, can be determined by considering the period of rotation, given by $T = 2\pi r / v$. Since the two skaters have the same period, $T = 2\pi r_1 / v_1 = 2\pi r_2 / v_2$.
(b) The force acting on each skater has only a centripetal component whose magnitude is mv^2 / r. Therefore the ratio of the magnitudes is simply $\frac{F_2}{F_1}$, and $\frac{F_2}{F_1} = \frac{m_2\left(v_2^2 / r_2\right)}{m_2\left(v_1^2 / r_1\right)}$.

SIMPLIFY:
(a) $\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} \Rightarrow \frac{r_1}{v_1} = \frac{r_2}{v_2} \Rightarrow \frac{r_1}{r_2} = \frac{v_1}{v_2}$

(b) $\frac{F_2}{F_1} = \frac{m_2\left(v_2^2 / r_2\right)}{m_2\left(v_1^2 / r_1\right)} = \frac{\left(v_2^2 / r_2\right)}{\left(v_1^2 / r_1\right)} = \frac{\left(r_2 / T_2^2\right)}{\left(r_1 / T_1^2\right)} = \frac{r_2}{r_1}$

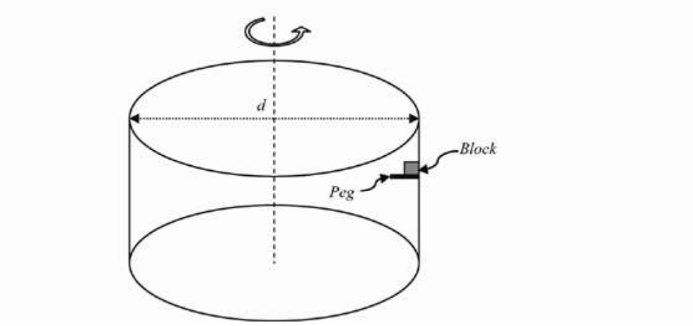
CALCULATE:
(a) Since $r_2 = \frac{r_1}{2}$, $\frac{r_2}{r_1} = \frac{v_2}{v_1} = \frac{1}{2}$.

(b) $\frac{F_2}{F_1} = \frac{r_2}{r_1} = \frac{v_2}{v_1} = \frac{1}{2}$

ROUND: It is not necessary to round. The result for both parts (a) and (b) is a ratio of 1/2.
DOUBLE-CHECK: It is reasonable that by doubling the radius, both the speed of rotation and centripetal force also double.

9.53. **THINK:** Determine the minimum time required for a block held by a peg inside a cylinder to stay in place once the cylinder starts rotating with angular acceleration, α. The coefficient of static friction is given as μ. To avoid slipping in the vertical direction, balance the force due to gravity with the force due to friction between the block and the cylinder. For large values of the angular acceleration, we also obtain a significant force in tangential direction. However, we restrict our considerations to the case of small angular acceleration and neglect the tangential force.

SKETCH:



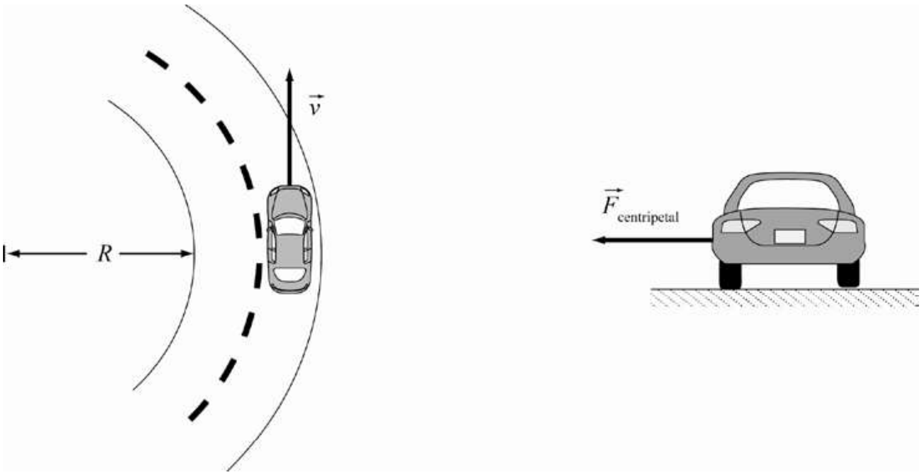
RESEARCH: The force due to friction is given by $f = \mu N$, and in this case N is simply the centripetal force, $F_c = mv^2/(d/2)$. The time required to reach a suitable centripetal force can be determined by means of the angular speed, $\omega = v/r$, and the angular acceleration, $\alpha = \omega/t$.

SIMPLIFY: The centripetal force can be rewritten as $F_c = mv^2/r = m(\omega r)^2/(d/2) = m\omega^2 d/2$. Thus, the force of static friction is given by $f_f = \mu m(d/2)\omega^2$. Therefore, from the balancing of the vertical forces: $f = F_g$, or $\mu m\omega^2 d/2 = mg \Rightarrow \omega^2 = 2g/(\mu d)$. Since $t = \omega/\alpha$, the time interval is:

$$t = \sqrt{2g/\mu d}/\alpha = \sqrt{\frac{2g}{\mu d\alpha^2}}.$$

CALCULATE: There are no numbers to insert in this problem.
ROUND: There is nothing to round since there are no numerical values.
DOUBLE-CHECK: An easy check we can perform right away is to make sure that the units on the right-hand side of our formula indeed work out to be seconds.

9.54. **THINK:** The maximum velocity such that the car performs uniform circular motion without slipping must be determined. The coefficient of static friction is $\mu_s = 1.20$ and the radius of the circular path is r = 10.0 m.
SKETCH:



RESEARCH: Consider which force is providing the centripetal force. Since the car is not sliding, it is the force of static friction. Those two forces must be related to determine the maximum velocity. That is,

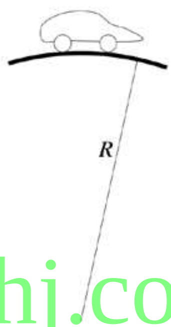
$$F_{\text{friction}} = F_{\text{centripetal}} \Rightarrow \mu_s mg = \frac{mv^2}{r}.$$

SIMPLIFY: $\mu_s mg = \frac{mv^2}{r} \Rightarrow v = \sqrt{\mu_s gr}$

CALCULATE: $v = \sqrt{\mu_s gr} = \sqrt{(1.20)(9.81\text{ m/s}^2)(10.0\text{ m})} = 10.84988\text{ m/s}$

ROUND: Since the values given have three significant figures, the result is then $v = 10.8$ m/s. **DOUBLE-CHECK:** This result may seem quite small for a racecar. But, consider that 10.8 m/s are ~24 mph, and that this is a very tight curve with a diameter of less than the length of a basketball court. It then seems reasonable that a car cannot go very fast through such a tight curve. Also, note that as expected, the maximum velocity is independent of the mass of the car.

9.55. **THINK:** Determine the maximum speed of a car as it goes over the top of a hill such that the car always touches the ground. The radius of curvature of the hill is 9.00 m. As the car travels over the top of the hill it undergoes circular motion in the vertical plane. The only force that can provide the centripetal force for this motion is gravity. Clearly, for small speeds the car remains in contact with the road due to gravity. But the car will lose contact if the centripetal acceleration exceeds gravity.
SKETCH:



RESEARCH: In the limiting case of the maximum speed we can set the centripetal acceleration equal to g: $g = v_{\text{max}}^2 / r$.

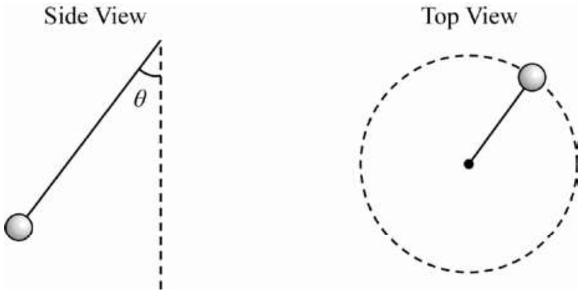
SIMPLIFY: Solve for the maximum speed and find $v_{\text{max}} = \sqrt{gr}$.

CALCULATE: $v_{\text{max}} = \sqrt{gr} = \sqrt{(9.81\text{ m/s}^2)(9.00\text{ m})} = 9.40\text{ m/s}$

ROUND: Since the radius is given to three significant figures, the result is $v_{\text{max}} = 9.40$ m/s.
DOUBLE-CHECK: This speed of 9.40 m/s, which is approximately 21.0 mph, seems very small. But on the other hand, this is a very significant curvature at the top of the hill, equivalent to a good-sized speed bump. Going over this type of bump at more than 21 mph makes it likely that your car will lose contact with the road surface.

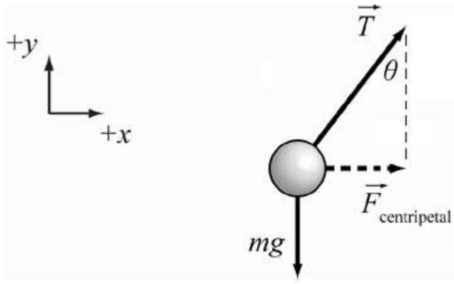
9.56. **THINK:** A ball attached to a string is in circular motion as described by the sketch. Determine:
(a) The free-body diagram for the ball.
(b) The force acting as the centripetal force.
(c) The required speed of the ball such that $\theta = 45.0°$.
(d) The tension on the string.

SKETCH:



RESEARCH:

(a)



(b) As shown in the sketch, the projection of the tension onto the horizontal plane provides the centripetal force. Therefore, $mv^2/r = T\sin\theta$.

(c) From the sketch, the force due to gravity is balanced by the projection of the tension on the vertical axis, i.e. $mg = T\cos\theta$. From part (b), the centripetal force is given by $mv^2/r = T\sin\theta$. By solving both equations for T and then equating them, the speed for the given angle can be determined.

(d) The tension on the string can most easily be found from $mg = T\cos\theta$, for the given angle, θ.

SIMPLIFY:

(a) Not applicable.

(b) Not applicable.

(c) $mg = T\cos\theta \Rightarrow T = \dfrac{mg}{\cos\theta}$, and $\dfrac{mv^2}{r} = T\sin\theta \Rightarrow T = \dfrac{mv^2}{r\sin\theta}$.

Equating the above equations gives $\dfrac{mg}{\cos\theta} = \dfrac{mv^2}{r\sin\theta} \Rightarrow v = \sqrt{gr\tan\theta}$, where $r = L\sin\theta$.

(d) $mg = T\cos\theta \Rightarrow T = \dfrac{mg}{\cos\theta}$

CALCULATE:

(a) Not applicable.

(b) Not applicable.

(c) $v = \sqrt{(9.81 \text{ m/s}^2)((1.00 \text{ m})\sin(45.0°))\tan 45.0°} = 2.63376 \text{ m/s}$

(d) $T = \dfrac{(0.200 \text{ kg})(9.81 \text{ m/s}^2)}{\cos(45.0°)} = 2.7747 \text{ N}$

ROUND:

(a) Not applicable.

(b) Not applicable.

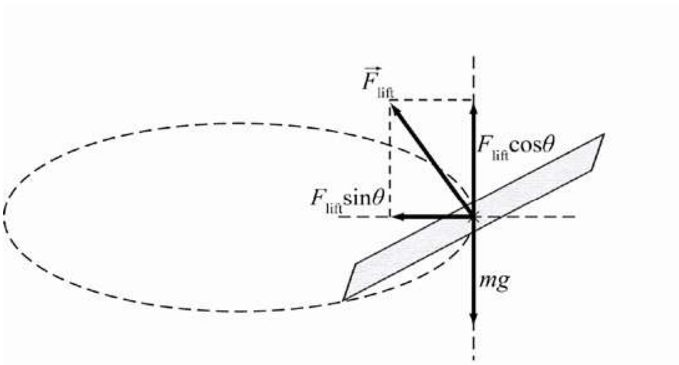
(c) Since values are given to three significant figures, the result is $v = 2.63$ m/s.

(d) Keeping three significant figures, $T = 2.77$ N.

DOUBLE-CHECK: All results are reasonable based on the given values. It is expected that the tension on the string will be greater than the weight of the ball.

9.57. **THINK:** Determine the banking angle for a plane performing uniform circular motion. The radius is 7.00 miles ($1.12654\cdot 10^4$ meters), the speed is 360. mph (160.93 m/s), the height is $2.00\cdot 10^4$ ft (6096 meters) and the plane length is 275 ft (83.82 meters).

SKETCH:



RESEARCH: Suppose F is the lift force, which makes an angle, θ, with the vertical as shown in the sketch. Also, suppose the weight of the plane is mg. Now, $F\cos\theta$ balances the weight of the plane when the plane is banked with the horizontal and $F\sin\theta$ provides the necessary centripetal force for the circular motion. Therefore,

$$F\sin\theta = \frac{mv^2}{r} \text{ and } F\cos\theta = mg.$$

SIMPLIFY: Dividing the two equations gives $\tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$.

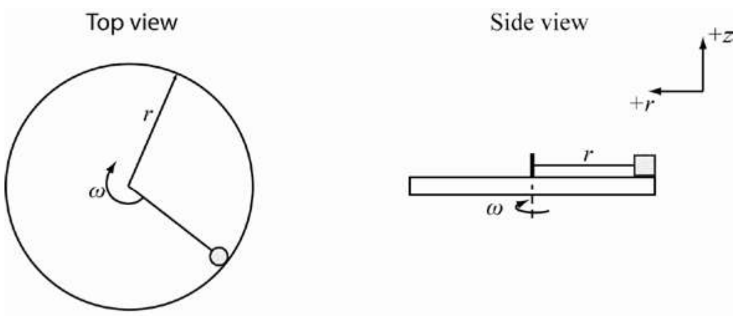
CALCULATE: $\theta = \tan^{-1}\left(\frac{(160.93 \text{ m/s})^2}{(1.12654\cdot 10^4 \text{ m})(9.81 \text{ m/s}^2)}\right) = 13.189°$

ROUND: Rounding to three significant figures, the result is an angle of approximately 13.2°.

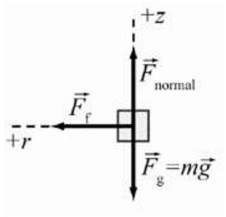
DOUBLE-CHECK: Based on the given values, the result is reasonable.

9.58. **THINK:** Determine the tension on the string attaching a cylinder (m = 20.0 g) to the center of a turntable as the angular velocity increases up to 60.0 rpm. The coefficient of static friction is $\mu_s = 0.800$ and the distance between the center of the turntable and the cylinder of l = 80.0 cm.

SKETCH:



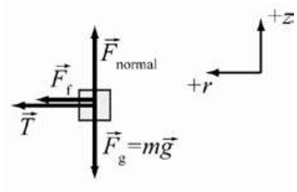
RESEARCH: As the turntable speeds up from the rest, the static friction force provides the centripetal force and no tension is built into the string for a while. The corresponding free body diagram for the cylinder under these conditions is presented. (Since the turntable speeds up very slowly, the tangential static friction force that acts on the cylinder from the turntable and keeps it moving with the turntable is important physically, but negligible in magnitude).



At a critical value, ω_1, of the angular velocity, the static friction force reaches its maximum value, so $F_f = \mu_s mg$ becomes

$$\frac{mv^2}{r} = m\omega_1^2 r = \mu_s mg \Rightarrow \omega_1 = \sqrt{\frac{\mu_s g}{r}}.$$

Once the angular velocity exceeds ω_1, static friction alone is not enough to provide the required centripetal force, and a tension is built into the string. The corresponding free body diagram is presented.



SIMPLIFY: The tension in the string when the angular velocity of the turntable is $\omega_2 = 60.0 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 2.00\pi \text{ rad/s}$ is calculated from the centripetal force at this velocity, $F_{c2} = m\omega_2^2 r$, and the tension is given by $T = F_{c2} - F_f = m\omega_2^2 r - \mu_s mg = m\left(\omega_2^2 r - \mu_s g\right)$.

CALCULATE: $T = (0.0200 \text{ kg})\left[(2.00\pi \text{ rad/s})^2 (0.800 \text{ m}) - (0.800)(9.81 \text{ m/s}^2)\right] = 0.475 \text{ N}$

ROUND: To three significant figures, $T = 0.475$ N.

DOUBLE-CHECK: This is a reasonable tension for the small system described.

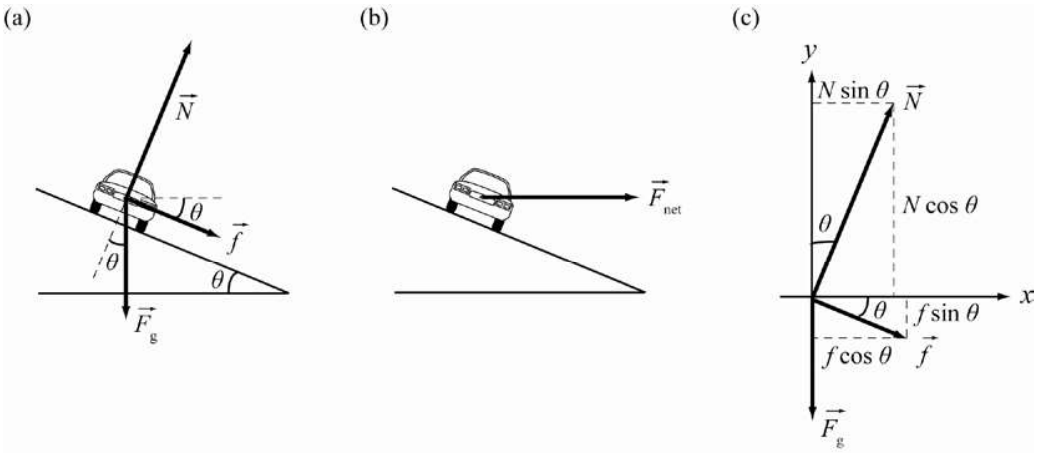
9.59. **THINK:** A speedway turn has a radius, R, and is banked at an angle of θ above the horizontal. This problem is a special case of Solved Problem 9.4, and the results of that solved problem will be used to obtain a solution to this problem. Determine:

(a) The optimal speed to take the turn when there is little friction present.

(b) The maximum and minimum speeds at which to take the turn if there is now a coefficient of static friction, μ_s.

(c) The value for parts (a) and (b) if $R = 400.$ m, $\theta = 45.0°$, and $\mu_s = 0.700$.

SKETCH:



RESEARCH: It was found in Solved Problem 9.4 that the maximum speed a car can go through the banked curve is given by

$$v_{\text{max}} = \sqrt{\frac{Rg(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta}}.$$

SIMPLIFY:

(a) For the case of zero friction the case above approaches the limit of $v_{\text{zero friction}} = \sqrt{\dfrac{Rg\sin\theta}{\cos\theta}} = \sqrt{Rg\tan\theta}$.

(b) For the maximum speed we can use the formula already quoted above. The minimum speed that the car can travel through the curve is given by reversing the direction of the friction force. In this case the friction force points up the bank, because it needs to prevent the car from sliding down. Reversing the sign of the friction force leads to $v_{\text{min}} = \sqrt{\dfrac{Rg(\sin\theta - \mu_s \cos\theta)}{\cos\theta + \mu_s \sin\theta}}$.

CALCULATE:

(c) For the results from part (a):

$$v_{\text{zero friction}} = \sqrt{(400.\text{ m})(9.81\text{ m/s}^2)\tan 45.0^\circ} = 62.64184\text{ m/s}.$$

For the results from part (b), the minimum speed is:

$$v_{\text{min}} = \sqrt{\frac{(400.\text{ m})(9.81\text{ m/s}^2)(\sin 45.0^\circ - 0.700\cos 45.0^\circ)}{\cos 45.0^\circ + 0.700\sin 45.0^\circ}} = 26.31484\text{ m/s}.$$

and the maximum speed is:

$$v_{\text{max}} = \sqrt{\frac{(400.\text{ m})(9.81\text{ m/s}^2)(\sin 45.0^\circ + 0.700\cos 45.0^\circ)}{\cos 45.0^\circ - 0.700\sin 45.0^\circ}} = 149.1174\text{ m/s}.$$

ROUND:

(a) Not applicable.

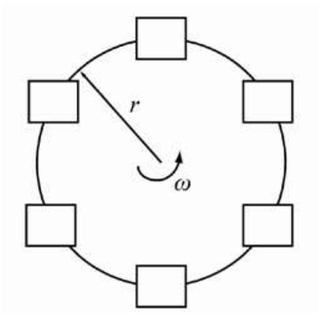
(b) Not applicable.

(c) $v_{\text{zero friction}} = 62.6\text{ m/s}$, $v_{\text{min}} = 26.3\text{ m/s}$ and $v_{\text{max}} = 149\text{ m/s}$.

DOUBLE-CHECK: The results are reasonable considering that the friction-free speed should be within the minimum and maximum speed. The values for the given parameters are consistent with experiment.

9.60. **THINK:** A Ferris wheel has a radius of 9.00 m, and a period of revolution of T = 12.0 s. Let's start with part (a) and solve it all the way.

SKETCH:



RESEARCH: The constant speed of the riders can be determined by the equation for the speed, v = distance/time, where the distance is calculated from the circumference of the path.

SIMPLIFY: $v = \frac{d}{T} = \frac{2\pi r}{T}$

CALCULATE: $v = \frac{2\pi(9.00 \text{ m})}{12.0 \text{ s}} = 4.712 \text{ m/s}$

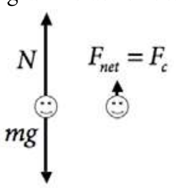
ROUND: Since the input values are given to two significant figures, the result for the linear speed is: $v = 4.7$ m/s.

DOUBLE-CHECK: For part (b) and part (c), realize that there is an essential difference between a Ferris wheel and a loop in a roller coaster: the speed of the Ferris wheel is gentle enough so that the riders do not get lifted out of their seats at the top. However, we need to check that the speed is actually sufficiently small so that this does not happen. In Solved Problem 9.1 we found that the minimum speed to experience weightlessness (i.e. zero normal force from the seat) at the top of the loop is $v_{N=0} = \sqrt{Rg}$. For the given value of R this speed works out to 9.4 m/s. Since our result is below this value, it is at least possible that a Ferris wheel could exist, which uses the values given here. Note that the centripetal acceleration from the speed use here is:

$$a_c = \frac{v^2}{R} = \frac{(4.71 \text{ m/s})^2}{9 \text{ m}} = 2.47 \text{ m/s}^2 = 0.25g.$$

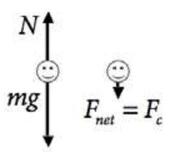
With the above information from our double-check we can solve parts (b) and (c):

(b) At the bottom of the ride the normal force has to balance gravity and in addition provide the centripetal force of 0.25 mg. The free-body diagram is as follows:



The normal force at the bottom of the path is thus: $N = mg + 0.25mg = 1.25mg$.

(c) At the top of the Ferris wheel gravity points in the direction of the centripetal force. The free-body diagram at the top is therefore:

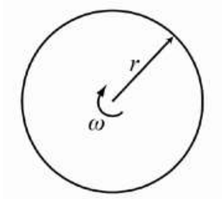


The normal force is in this case: $N = mg - 0.25mg = 0.75mg$.

Note the essential difference in parts (b) and (c): in part (b) the magnitude of the vector for the normal force is greater than that of the gravitational force, and in part (c) it is smaller.

9.61. **THINK:** The radius of the Ferris wheel is $r = 9.00$ m and its period is $T = 12.0$ s. Use these values to calculate ω. $\Delta\omega$ and $\Delta\theta$ are known when stopping at a uniform rate, which is sufficient to determine α. Also, the time it takes to stop, Δt, can be determined and with this, the tangential acceleration, a_t, can be determined.

SKETCH:



RESEARCH:

(a) $\omega = \dfrac{2\pi \text{ rad}}{T}$

(b) $\Delta\theta = \omega_i \Delta t + \dfrac{1}{2}\alpha \Delta t^2$, $\Delta\omega = \alpha\Delta t$

(c) $a_t = r\alpha$

SIMPLIFY:

(a) $\omega = \dfrac{2\pi}{T}$

(b) $\Delta\theta = \omega_i \Delta t + \dfrac{1}{2}\alpha \Delta t^2$ and $\Delta t = \dfrac{\Delta\omega}{\alpha} = \dfrac{\omega_f - \omega_i}{\alpha} = \dfrac{-\omega_i}{\alpha}$, since $\omega_f = 0$.

$$\Rightarrow \Delta\theta = \omega_i\left(\frac{-\omega_i}{\alpha}\right) + \frac{1}{2}\alpha\left(\frac{\omega_i^2}{\alpha^2}\right) = -\frac{\omega_i^2}{\alpha} + \frac{\omega_i^2}{2\alpha} = -\frac{\omega_i^2}{2\alpha} \Rightarrow \alpha = \frac{-\omega_i^2}{2\Delta\theta}$$

(c) $a_t = r\alpha$

CALCULATE:

(a) $\omega = \dfrac{2\pi \text{ rad}}{12.0 \text{ s}} = 0.5236$ rad/s

(b) $\alpha = \dfrac{-(0.5236 \text{ rad/s})^2}{2(\pi/2) \text{ rad}} = -0.08727 \text{ rad/s}^2$

(c) $a_t = (-0.08727 \text{ rad/s}^2)(9.00 \text{ m}) = -0.785 \text{ m/s}^2$

ROUND: The given values have three significant figures, so the results should be rounded accordingly.

(a) $\omega = 0.524$ rad/s

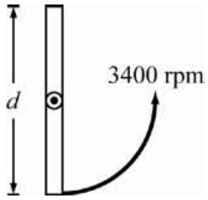
(b) $\alpha = -0.0873 \text{ rad/s}^2$

(c) $a_t = -0.785 \text{ m/s}^2$

DOUBLE-CHECK: These numbers are reasonable for a Ferris wheel. Note that the radius is only required for part (c). As expected, the value for the tangential acceleration is small compared to the gravitational acceleration *g*.

9.62. **THINK:** Determine the linear speed, given the blade's rotation speed and its diameter. To help determine the constant (negative) acceleration, it is given that it takes a time interval of 3.00 s for the blade to stop. The known values are ω = 3400. rpm, $d = 53.0$ cm.

SKETCH:



RESEARCH: $1 \text{ rpm} = \dfrac{2\pi \text{ rad}}{60 \text{ s}}$

(a) $v = \omega r = \dfrac{1}{2}\omega d$

(b) $\alpha = \dfrac{\Delta w}{\Delta t} = \dfrac{w_f - w_i}{\Delta t}$

SIMPLIFY: It is not necessary to simplify.

CALCULATE:

(a) $v = \left(3400.\left(\dfrac{2\pi}{60}\right)\right) \text{rad/s}\left(\dfrac{0.530 \text{ m}}{2}\right) = 94.35 \text{ m/s}$

(b) $\omega_f = 0$, $\omega_i = 3400.\left(\dfrac{2\pi}{60} \text{ rad/s}\right) = 356 \text{ rad/s}$ and $\Delta t = 3$ s, so $\alpha = \dfrac{-356 \text{ rad/s}}{3.00 \text{ s}} = -118.7 \text{ rad/s}^2$.

ROUND: The results should be rounded to three significant figures.

(a) $v = 94.3$ m/s

(b) $\alpha = -119 \text{ rad/s}^2$

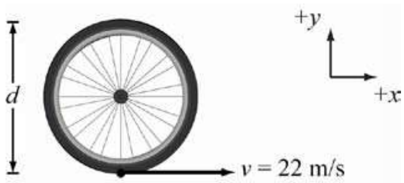
DOUBLE-CHECK: For lawn mower blades, these are reasonable values.

9.63. **THINK:**

(a) If the distance traveled can be determined, then the number of revolutions the tires made can be determined, since the diameter of the tires is known.

(b) The linear speed of the tires and the diameter of the tires are known, so the angular speed can be determined. The known variables are $v_i = 0$, $v_f = 22.0$ m/s, $\Delta t = 9.00$ s, $d = 58.0$ cm. Use $1\dfrac{\text{rev}}{\text{s}} = 2\pi\dfrac{\text{rad}}{\text{s}} \Rightarrow 1\dfrac{\text{rad}}{\text{s}} = \dfrac{1}{2\pi}\dfrac{\text{rev}}{\text{s}}$.

SKETCH:



RESEARCH: The circumference of a circle is given by $C = 2\pi r = \pi d$. The displacement at constant acceleration is $\Delta x = v_i\Delta t + \dfrac{1}{2}a\Delta t^2$, where $v = \omega r$.

SIMPLIFY:

(a) $v_i = 0 \Rightarrow \Delta x = \dfrac{1}{2}a\Delta t^2$, $a = \dfrac{\Delta v}{\Delta t} \Rightarrow \Delta x = \dfrac{1}{2}\dfrac{\Delta v}{\Delta t}\Delta t^2 = \dfrac{1}{2}\Delta v\Delta t$

Let N = number of revolutions and the displacement is given by $\Delta x = \left(\dfrac{\text{displacement}}{\text{revolution}}\right)N$. The displacement per revolution is simply the circumference, C, so

$$\Delta x = CN \;\Rightarrow\; N = \frac{\Delta x}{C} = \frac{1}{\pi d}\left(\frac{1}{2}\Delta v \Delta t\right) = \frac{\Delta v \Delta t}{2\pi d}.$$

(b) $\omega = \frac{v}{r} = \frac{v}{d/2} = \frac{2v}{d}$

CALCULATE:

(a) $N = \frac{(22.0 \text{ m/s})(9.00 \text{ s})}{2\pi(0.58 \text{ m})} = 54.33$ revolutions

(b) $\omega = \frac{2(22.0 \text{ m/s})}{0.58 \text{ m}} = 75.86 \text{ rad/s} = \frac{75.86}{2\pi} \text{ rev/s} = 12.07 \text{ rev/s}$

ROUND: The results should be rounded to three significant figures.

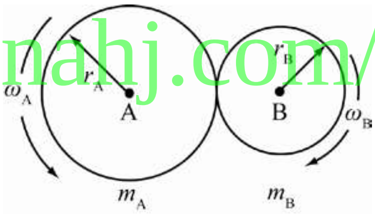
(a) $N = 54.3$ revolutions

(b) $\omega = 12.1$ rev/s

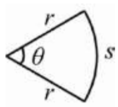
DOUBLE-CHECK: For the given values, these results are reasonable.

9.64. **THINK:** First, determine the number of revolutions gear A undergoes while slowing down. From this, determine the total arc-length of gear A. Gear B must have the same arc-length, from which the number of rotations undergone by gear B can be determined. The following values are given: $\omega_{\text{i,A}} = 120.$ rpm, $\omega_{\text{f,A}} = 60.0$ rpm, $\Delta t = 3.00$ s, $r_{\text{A}} = 55.0$ cm, $r_{\text{B}} = 30.0$ cm, $m_{\text{A}} = 1.00$ kg, $m_{\text{B}} = 0.500$ kg and $\Delta\omega_{\text{A}} = \omega_{\text{f,A}} - \omega_{\text{i,A}} = -60.0$ rpm. Use the conversion factor $1 \text{ rpm} = \frac{2\pi \text{ rad}}{60 \text{ s}}$.

SKETCH:



RESEARCH: The arc-length is given by $s = r\theta$.



The angular displacement is $\Delta\theta = \omega_{\text{i}}\Delta t + \frac{1}{2}\alpha t^2$ The angular acceleration is $\alpha = \frac{\Delta\omega}{\Delta t}$.

SIMPLIFY: The arc-length of gear A is given by $s_{\text{A}} = r_{\text{A}}\Delta\theta_{\text{A}} = r_{\text{A}}\left(\omega_{\text{i,A}}\Delta t + \frac{1}{2}\alpha_{\text{A}} t^2\right)$.

$$\alpha_{\text{A}} = \frac{\Delta\omega_{\text{A}}}{\Delta t} \;\Rightarrow\; s_{\text{A}} = r_{\text{A}}\left(\omega_{\text{i,A}}\Delta t + \frac{\Delta\omega_{\text{A}}\Delta t}{2}\right)$$

Gear B has the same arc-length, $s_{\text{A}} = s_{\text{B}}$. The angular displacement of gear B is $s_{\text{B}} = r_{\text{B}}\Delta\theta_{\text{B}}$, so $\Delta\theta_{\text{B}} = \frac{s_{\text{B}}}{r_{\text{B}}} = \frac{s_{\text{A}}}{r_{\text{B}}}$ (since $s_{\text{A}} = s_{\text{B}}$).

The number of rotations, n, of gear B is $n = \frac{\Delta\theta_{\text{B}}}{2\pi}$, so $n = \frac{s_{\text{A}}}{2\pi r_{\text{B}}} = \frac{1}{2\pi}\frac{r_{\text{A}}}{r_{\text{B}}}\left(\omega_{\text{i,A}}\Delta t + \frac{1}{2}\Delta\omega_{\text{A}}\Delta t\right)$

$$n = \frac{1}{2\pi}\frac{r_{\text{A}}}{r_{\text{B}}}\Delta t\left(\omega_{\text{i,A}} + \frac{1}{2}\Delta\omega_{\text{A}}\right)$$

CALCULATE: $n=\frac{1}{2\pi}\left(\frac{0.550\text{ m}}{0.300\text{ m}}\right)\left[\left(120.\left(\frac{2\pi}{60}\right)\right)\text{ s}^{-1}+\frac{1}{2}\left(-60.0\left(\frac{2\pi}{60}\right)\right)\text{ s}^{-1}\right](3.00\text{ s})$

$$=\frac{1}{2\pi}\left(\frac{0.550\text{ m}}{0.300\text{ m}}\right)\left[4\pi-\pi\right](3.00)=\frac{9.00}{2}\left(\frac{0.550}{0.300}\right)=8.250$$

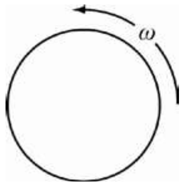
ROUND: Rounding the result to three significant figures gives $n = 8.25$ rotations.

DOUBLE-CHECK: There is an alternate solution. The average angular speed of A during the slowing down is $(120+60)/2\text{ rpm}=90\text{ rpm}$. In 3 s, A undergoes $90(3/60)=4.5$ rotations. Since B has a smaller radius, it undergoes a proportionally greater number of rotations. The proportionality is the ratio of the radii:

$$n=4.5\left(\frac{0.55\text{ m}}{0.30\text{ m}}\right)=8.25,\text{ as before.}$$

9.65. **THINK:** The angular acceleration is constant, so the uniform angular acceleration equations can be used directly. The known quantities are $\omega_{\text{i}}=10.0\text{ rev/s}$, $\omega_{\text{f}}=0$ and $\Delta t=10.0\text{ min}$.

SKETCH:



RESEARCH: $\alpha=\frac{\Delta\omega}{\Delta t},\quad \Delta\theta=\omega_{\text{i}}\Delta t+\frac{1}{2}\alpha t^2$

SIMPLIFY: Simplification is not necessary.

CALCULATE: $\alpha=\frac{-10.0\text{ rev/s}(2\pi\text{ rad/rev})}{10.0\text{ min}(60\text{ s/min})}=-\frac{20\pi}{600}\text{ rad/s}^2=-\frac{\pi}{30}\text{ rad/s}^2=-0.1047\text{ rad/s}^2$

$$\Delta\theta=\left(10.0\text{ rev/s}(2\pi\text{ rad/rev})\right)\left(10.0\text{ min}(60\text{ s/min})\right)-\frac{1}{2}\left(\frac{\pi}{30}\text{rad/s}^2\right)\left(10.0\text{ min}(60\text{ s/min})\right)^2$$

$$=(20\pi\text{ rad/s})(600\text{ s})-\frac{\pi}{60}\text{ rad/s}^2(600\text{ s})^2=3.77\cdot10^4\text{ rad}-1.885\cdot10^4\text{ rad}=1.885\cdot10^4\text{ rad}$$

ROUND: Rounding each result to three significant figures gives $\alpha=-0.105\text{ rad/s}^2$ and $\Delta\theta=1.88\cdot10^4\text{ rad}$.

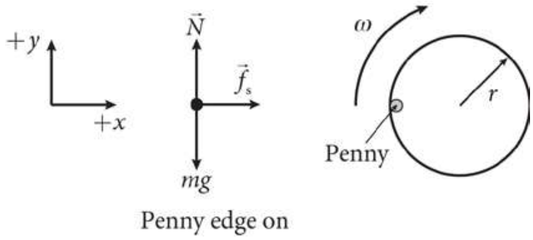
DOUBLE-CHECK: The average angular speed is

$$\frac{0+10.0\text{ rev/s}}{2}=5\text{ rev/s}=5(2\pi\text{ rad/s}).$$

The displacement during this time interval for the average speed is $\Delta\theta=\omega_{\text{avg}}\Delta t$ $=(10\pi\text{ rad/s})(600\text{ s})=1.885\cdot10^4\text{ rad}$, as above. The results are consistent and reasonable.

9.66. **THINK:** The force of static friction between the penny and the phonograph disk provides the centripetal force to keep the penny moving in a circle.

SKETCH:



RESEARCH: The maximum force of static friction between the penny and the photograph disk is $f_s = \mu_s mg$. The centripetal force required to keep the penny moving in a circle is $F_c = mr\omega^2$. Frequency is related to angular frequency by $\omega = 2\pi f$.

SIMPLIFY: $mr\omega^2 = \mu_s mg \Rightarrow \mu_s = \dfrac{\omega^2 r}{g} \Rightarrow \mu_s = \dfrac{(2\pi f)^2 r}{g}$.

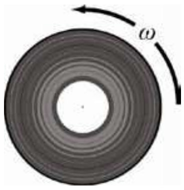
CALCULATE: $f = 33\dfrac{\text{rev}}{\text{min}}\dfrac{\text{min}}{60\text{ s}} = 0.5500\text{ s}^{-1}$, $r = \dfrac{12\text{ in}}{2}\dfrac{2.54\text{ cm}}{\text{in}}\dfrac{1\text{ m}}{100\text{ cm}} = 0.1524\text{ m}$,

$$\mu_s = \frac{\left[2\pi\left(0.5500\text{ s}^{-1}\right)\right]^2(0.1524\text{ m})}{9.81\text{ m/s}^2} = 0.1855.$$

ROUND: Rounding the result to two significant figures gives $\mu_s = 0.19$.

DOUBLE-CHECK: The results are reasonable for the given values.

9.67. **THINK:** The acceleration is uniform during the given time interval. The average angular speed during this time interval can be determined and from this, the angular displacement can be determined.
SKETCH:



RESEARCH: $\omega_{avg} = \dfrac{\omega_f + \omega_i}{2}$, $\Delta\theta = \omega_{avg}\Delta t$

SIMPLIFY: Simplification is not necessary.

CALCULATE: $\omega_i = 33.33\text{ rpm} = 33.33\text{ rpm}\left(\dfrac{2\pi}{60\text{ s}}\right) = 3.491\text{ rad/s}$, $\omega_f = 0$

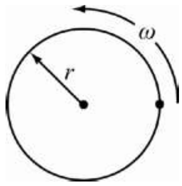
$$\Delta\theta = \left(\frac{3.491}{2}\text{ rad s}^{-1}\right)(15.0\text{ s}) = 26.18\text{ rad}$$

$$\text{number of rotations} = \frac{\Delta\theta}{2\pi} = 4.167$$

ROUND: Rounding the result to three significant figures gives the number of rotations to be 4.17 rotations.

DOUBLE-CHECK: These are reasonable results for a turntable.

9.68. **THINK:** Given the radius (2.0 cm) and rotation speed (250 rpm), the linear and angular speeds and acceleration can be determined.
SKETCH:



RESEARCH: $\omega = \text{rpm}\left(\dfrac{2\pi}{60}\text{ rad/s}\right)$, $v = \omega r$, $a = \omega^2 r$, $\alpha = 0$

SIMPLIFY: Simplification is not necessary.

CALCULATE: $\omega = 250.\left(\dfrac{2\pi}{60}\text{ rad/s}\right) = 26.18\text{ rad/s}$, $v = \omega r = (26.18)(0.0200)\text{ m/s} = 0.5236\text{ m/s}$

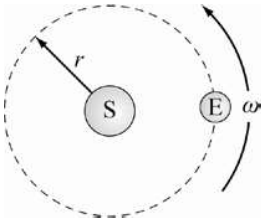
$a = \omega^2 r = (26.18)^2 (0.020)\text{ m/s}^2 = 13.71\text{ m/s}^2$, $\alpha = 0$

ROUND: Rounding the results to three significant figures gives $\omega = 26.2$ rad/s, $v = 0.524$ m/s, $a = 14\text{ m/s}^2$, and $\alpha = 0$.

DOUBLE-CHECK: The rotation speed is constant, so $\alpha = 0$. The other values are likewise reasonable.

9.69. **THINK:** The angular acceleration of the Earth is zero. The linear acceleration is simply the centripetal acceleration. $r = 1$ AU or $r = 1.50 \cdot 10^{11}$ m and $\omega = 2\pi$ rad / year.

SKETCH:



RESEARCH: $a = \omega^2 r$, $1\text{ yr} = 3.16 \cdot 10^7$ s

SIMPLIFY: Simplification is not necessary.

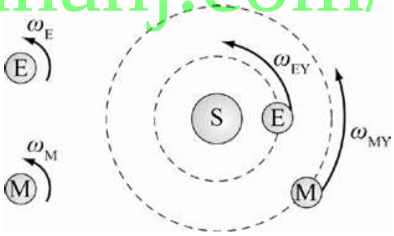
CALCULATE: $a = \left(\dfrac{2\pi}{3.16 \cdot 10^7\text{ s}}\right)^2 \left(1.50 \cdot 10^{11}\text{ m}\right) = 5.93 \cdot 10^{-3}\text{ m/s}^2$

ROUND: Keeping three significant figures, $a = 5.93 \cdot 10^{-3}\text{ m/s}^2$.

DOUBLE-CHECK: The linear acceleration is rather small because the distance to the Sun is so great.

9.70. **THINK:** From the given data, the ratio of the angular accelerations of Mars and Earth can be determined.

SKETCH:



RESEARCH: $\omega_{\text{day}} = \dfrac{2\pi\text{ rad}}{1\text{ day}}$, $\omega_{\text{yr}} = \dfrac{2\pi\text{ rad}}{1\text{ yr}}$

SIMPLIFY: $\omega_{\text{M}} = \dfrac{2\pi\text{ rad}}{24.6\text{ hr}}$, $\omega_{\text{E}} = \dfrac{2\pi\text{ rad}}{24\text{ hr}}$, $\omega_{\text{My}} = \dfrac{2\pi\text{ rad}}{687\text{ Earth-days}}$, $\omega_{\text{Ey}} = \dfrac{2\pi\text{ rad}}{365\text{ Earth-days}}$

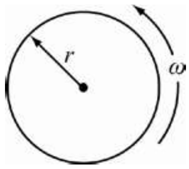
CALCULATE: $\dfrac{\omega_{\text{M}}}{\omega_{\text{E}}} = \dfrac{24.0\text{ hr}}{24.6} = 0.9756$, $\dfrac{\omega_{\text{My}}}{\omega_{\text{Ey}}} = \dfrac{365}{687} = 0.5319$

ROUND: Rounding the results to three significant figures gives $\dfrac{\omega_{\text{M}}}{\omega_{\text{E}}} = 0.976$ and $\dfrac{\omega_{\text{My}}}{\omega_{\text{Ey}}} = 0.532$.

DOUBLE-CHECK: The angular speed of Mars' orbit is 0.532 that of Earth. The latter is reasonable given that Mars is further from the Sun than Earth, as we will learn in Chapter 12.

9.71. **THINK:** Parts (a) and (b) can be solved using the constant angular acceleration equations. For part (c), calculate the angular displacement and, from this, compute the total arc-length, which is equal to the distance traveled.

SKETCH:



RESEARCH:

(a) $v = \omega r$

(b) $\alpha = \frac{\Delta\omega}{\Delta t}$, $\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$

(c) $s = r\Delta\theta$, $\Delta\theta = 2\pi(\text{total revs.})$

SIMPLIFY:

(a) $\omega_{\text{i}} = \frac{v_{\text{i}}}{r}$

(b) $\Delta\omega = \omega_{\text{f}} - \omega_{\text{i}} = 0 - \frac{v_{\text{i}}}{r} = -\omega_{\text{i}}$, $\Delta t = \frac{\Delta\omega}{\alpha} = -\frac{\omega_{\text{i}}}{\alpha}$

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2 = \omega_i\left(\frac{-\omega_i}{\alpha}\right) + \frac{\alpha}{2}\left(\frac{-\omega_i}{\alpha}\right)^2 = \frac{-\omega_i^2}{\alpha} + \frac{\omega_i^2}{2\alpha} = -\frac{\omega_i^2}{2\alpha} \Rightarrow \alpha = \frac{-\omega_i^2}{2\Delta\theta}$$

(c) $s = r\Delta\theta$

CALCULATE:

(a) $\omega_i = \frac{35.8 \text{ m/s}}{0.550 \text{ m}} = 65.09 \text{ s}^{-1}$

(b) $\alpha = \frac{-\left(65.09 \text{ s}^{-1}\right)^2}{2\left(2\pi(40.2)\right)} = -8.387 \text{ s}^{-2}$

(c) $s = (0.550 \text{ m})\left(2\pi(40.2)\right) = 138.92 \text{ m}$

ROUND: Rounding the results to three significant figures:

(a) $\omega_i = 65.1 \text{ s}^{-1}$

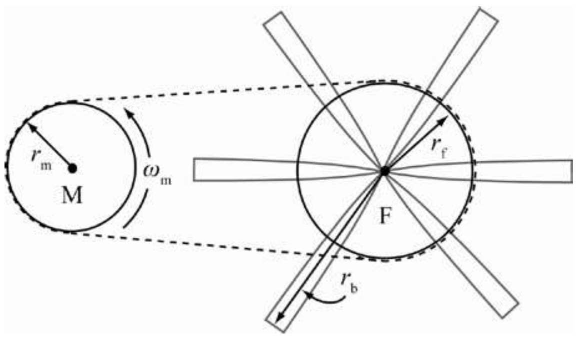
(b) $\alpha = -8.39 \text{ s}^{-2}$

(c) $s = 139 \text{ m}$

DOUBLE-CHECK: For the parameters given, these are reasonable results.

9.72. **THINK:** Everything in the problem rotates at constant angular speed. The two wheels have radii of $r_{\text{m}} = 2.00$ cm and $r_{\text{f}} = 3.00$ cm and rotate at the same linear speed.

SKETCH:



RESEARCH: $v = \omega r$

SIMPLIFY: $v_{\text{m}} = \omega_{\text{m}} r_{\text{m}}$, $v_{\text{f}} = \omega_{\text{f}} r_{\text{f}}$, $v_{\text{b}} = \omega_{\text{b}} r_{\text{b}}$

The wheels are attached by a belt, so $v_{\text{m}} = v_{\text{f}} \Rightarrow \omega_{\text{m}} r_{\text{m}} = \omega_{\text{f}} r_{\text{f}} \Rightarrow \omega_{\text{f}} = \dfrac{\omega_{\text{m}} r_{\text{m}}}{r_{\text{f}}}$. The blades are attached to wheel *F*, so $\omega_{\text{b}} = \omega_{\text{f}} \Rightarrow v_{\text{b}} = \omega_{\text{f}} r_{\text{b}} = \dfrac{\omega_{\text{m}} r_{\text{m}}}{r_{\text{f}}} r_{\text{b}}$.

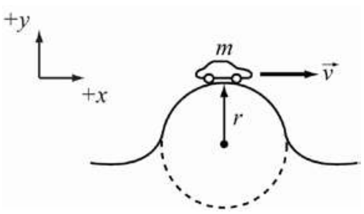
CALCULATE: $v_{\text{b}} = \dfrac{1}{0.03 \text{ m}}\left(1200\left(\dfrac{2\pi}{60 \text{ s}}\right)\right)(0.02 \text{ m})(0.15 \text{ m}) = 12.57 \text{ m/s}$

ROUND: Rounding the result to three significant figures gives $v_{\text{b}} = 12.6$ m/s.

DOUBLE-CHECK: From $v_{\text{b}} = \dfrac{\omega_{\text{m}} r_{\text{m}} r_{\text{b}}}{r_{\text{f}}}$, it can be seen that v_{b} grows with ω_{m}, r_{m}, r_{b}, and v_{b} decreases as r_{f} grows. All these relations are reasonable.

9.73. **THINK:** The net force due to gravity (down) and normal force from the hill (upward) equals the centripetal force determined by the car's speed and the path's radius of curvature. The force the car exerts on the hill is equal and opposite to the force of the hill on the car.

SKETCH:



RESEARCH: $F_{\text{g}} = mg$ acts downward, and let *N* be the upward force of the hill on the car. The net force, F_{net}, which is the centripetal force $F_{\text{c}} = m\dfrac{v^2}{r}$, acts downward.

SIMPLIFY: Taking upward force as positive and downward force as negative,

$$-F_{\text{net}} = N - F_{\text{g}} = N - mg = -F_{\text{c}} = -m\frac{v^2}{r} \Rightarrow N = m\left(g - \frac{v^2}{r}\right)$$

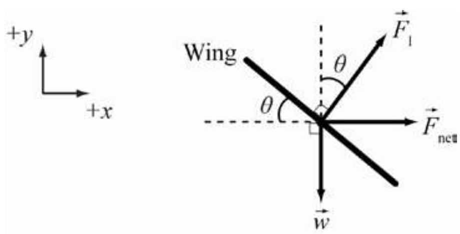
CALCULATE: $N = (1000.\text{ kg})\left(9.81 \text{ m/s}^2 - \dfrac{(60.0 \text{ m/s})^2}{370. \text{ m}} - \right) = 80.270 \text{ N}$

ROUND: To three significant figures, $N = 80.3$ N.

DOUBLE-CHECK: The equation confirms what we know from observation, namely that if *v* is large enough, then the normal force will go to zero and the car will lose contact with the ground.

9.74. **THINK:** A free body diagram will show all the forces acting on the plane. The net force is horizontal, directed towards the center of the radius of curvature. The speed is *v* = 4800 km/h and the turning radius is *r* = 290 km. The banking angle, θ, must be determined.

SKETCH:



RESEARCH: $\vec{F}_{\text{net}} = m\vec{a},\ \vec{F}_{\text{net}} = \vec{w} + \vec{F}_1$

SIMPLIFY: $F_{\text{net}} = ma = m\dfrac{v^2}{r},\ w = mg$

$$\sum F_y = mg - F_1\cos\theta = 0 \Rightarrow mg = F_1\cos\theta \Rightarrow \frac{F_1}{m} = \frac{g}{\cos\theta}, \qquad \sum F_x = F_1\sin\theta = m\frac{v^2}{r} \Rightarrow \frac{F_1}{m} = \frac{v^2}{r\sin\theta}$$

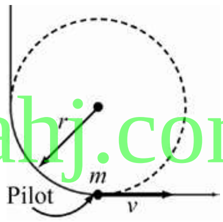
Equating the two above equations gives $\dfrac{g}{\cos\theta} = \dfrac{v^2}{r\sin\theta} \Rightarrow \tan\theta = \dfrac{v^2}{gr} \Rightarrow \theta = \tan^{-1}\left(\dfrac{v^2}{gr}\right)$.

CALCULATE: $v = 4800 \text{ km/h} = 4800\left(\dfrac{10^3 \text{ m}}{3600 \text{ s}}\right) = 1.333\cdot 10^3 \text{ m/s},\quad r = 290 \text{ km} = 2.9\cdot 10^5 \text{ m}$

$$\theta = \tan^{-1}\frac{\left(1.333\cdot 10^3 \text{ m/s}\right)^2}{\left(9.81 \text{ m/s}^2\right)\left(2.9\cdot 10^5 \text{ m}\right)} = 32.00^\circ$$

ROUND: The speed is given to three significant figures, so the result should be $\theta = 32.0^\circ$.
DOUBLE-CHECK: A banking angle of 32° is reasonable for the SR-71.

9.75. **THINK:** From the linear speed and the radius, the centripetal acceleration can be determined. With the pilots' mass, the centripetal force can also be determined. The pilot's apparent weight is the combined effect of gravitational and centripetal accelerations.
SKETCH:



RESEARCH:

(a) $a_c = \dfrac{v^2}{r},\ F_c = \dfrac{mv^2}{r}$

(b) $F_c = \dfrac{mv^2}{r},\ F_g = mg,\ w = \dfrac{mv^2}{r} + mg$

SIMPLIFY: Simplification is not necessary.
CALCULATE:

(a) $a_c = \dfrac{\left(500.\text{ m/s}\right)^2}{4000.\text{ m}} = 62.50 \text{ m/s}^2,\quad F_c = ma_c = \left(80.0 \text{ kg}\right)\left(62.50 \text{ m/s}^2\right) = 5.00\cdot 10^3 \text{ N}$

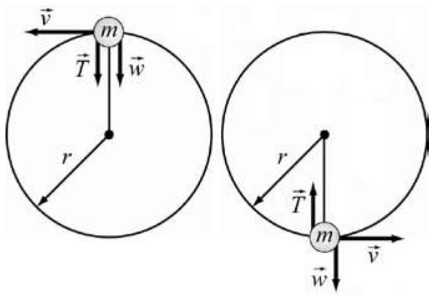
(b) $w = 5.00\cdot 10^3 \text{ N} + \left(80.0 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right) = 5784.8 \text{ N}$

ROUND: Round the results to three significant figures:

(a) $a_c = 62.5 \text{ m/s}^2$ and $F_c = 5.00\cdot 10^3 \text{ N}$
(b) $w = 5780 \text{ N}$
DOUBLE-CHECK: These are all reasonable values.

9.76. **THINK:** The net force on the ball is the centripetal force. Gravity and tension sum to produce this force. m = 1.00 kg, r = 1.00 m and v = 10.0 m/s. At the top of the circle, gravity and tension both point down. At the bottom of the circle, gravity still points down, but the tension points up.

SKETCH:



RESEARCH: $F_{\text{net}} = \dfrac{mv^2}{r}$

(a) $F_{\text{net}} = T + w$

(b) $F_{\text{net}} = T - w$

SIMPLIFY:

(a) $T = F_{\text{net}} - w = \dfrac{mv^2}{r} - mg$

(b) $T = F_{\text{net}} + w = \dfrac{mv^2}{r} + mg$

CALCULATE: $\dfrac{mv^2}{r} = \dfrac{(1.00 \text{ kg})(10.0 \text{ m/s})^2}{1.00 \text{ m}} = 100. \text{ N}, \;\; mg = (1.00 \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \text{ N}$

(a) $T = 100. \text{ N} - 9.81 \text{ N} = 90.19 \text{ N}$

(b) $T = 100. \text{ N} + 9.81 \text{ N} = 109.8 \text{ N}$

(c) The tension in the string is greatest at the bottom of the circle. As the ball moves away from the bottom, the tension decreases to its minimum value at the top of the circle. It then increases until the ball again reaches the bottom.

ROUND: Round the results to three significant figures.

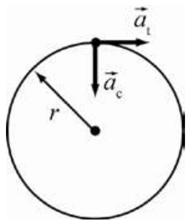
(a) $T = 90.2$ N

(b) $T = 110.$ N

DOUBLE-CHECK: If you are swinging the ball with a high speed like in this problem, the weight becomes almost negligible, and thus we should expect that the tensions at the bottom and top become almost identical. The tension is still highest at the bottom, as would be reasonably expected.

9.77. **THINK:** The car start slipping at the point where the magnitude of the total acceleration exceeds the maximum acceleration that can be provided by the friction force. The total acceleration of the car is composed of contributions from the centripetal and the tangential acceleration, which have to be added as vectors. Given here are $R = 36.0$ m, $a_{\text{t}} = 3.30 \text{ m/s}^2$, $v_{\text{i}} = 0$ and $\mu = 0.950$.

SKETCH:



RESEARCH: The magnitude of the total acceleration is given by the tangential and radial acceleration, $a = \sqrt{a_{\text{t}}^2 + a_{\text{c}}^2}$. The centripetal acceleration is $a_{\text{c}} = v^2 / R$. Since the car accelerates at constant linear acceleration starting from rest, the speed as a function of time is $v = a_{\text{t}} t$. The maximum force of friction is given by $f=\mu mg$. So the maximum acceleration due to friction is $a_f=\mu g$. The distance traveled by then is $d=\frac{1}{2}a_t t^2$.

SIMPLIFY: Slippage occurs when $a_f=a$; so $a_f=\mu g=a=\sqrt{a_t^2+a_c^2}$

$\Rightarrow \mu^2 g^2 = a_t^2+a_c^2 = a_t^2+(v^2/R)^2 = a_t^2+(a_t^2t^2/R)^2 \quad \Rightarrow t^2 = R\sqrt{\mu^2g^2-a_t^2}/a_t^2$

$$\Rightarrow d=\frac{1}{2}a_t t^2 = \frac{\frac{1}{2}R\sqrt{\mu^2g^2-a_t^2}}{a_t}$$

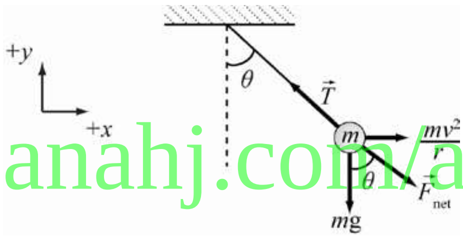
CALCULATE: $d=\dfrac{\frac{1}{2}(36.0\text{ m})\sqrt{0.950^2(9.81\text{ m/s}^2)^2-(3.30\text{ m/s}^2)^2}}{(3.30\text{ m/s}^2)}=47.5401\text{ m}$

ROUND: Rounding to three significant figures gives $d=47.5$ m.

DOUBLE-CHECK: d is proportional to R. This makes sense because a larger R implies less curvature and thus less centripetal force. d is also inversely proportional to a_t, which also makes sense since a smaller tangential acceleration implies a greater distance traveled before the maximum speed is attained.

9.78. **THINK:** The pendulum experiences a vertical force due to gravity and a horizontal centripetal force. These forces are balanced by the tension in the pendulum string. $r=6.0$ m and $\omega=0.020$ rev/s.

SKETCH:



RESEARCH: $F_{net}\sin\theta=\dfrac{mv^2}{r}$, $F_{net}\cos\theta=mg$, $v=\omega r$, $1\text{ rev/s}=2\pi\text{ rad/s}$

SIMPLIFY: $\dfrac{F_{net}}{m}=\dfrac{v^2}{r\sin\theta}$, $\dfrac{F_{net}}{m}=\dfrac{g}{\cos\theta}$

Equating the equations, $\dfrac{v^2}{r\sin\theta}=\dfrac{g}{\cos\theta}\Rightarrow\tan\theta=\dfrac{v^2}{rg}=\dfrac{\omega^2r^2}{rg}=g\dfrac{\omega^2 r}{g}\Rightarrow\theta=\tan^{-1}\left(\dfrac{\omega^2 r}{g}\right)$.

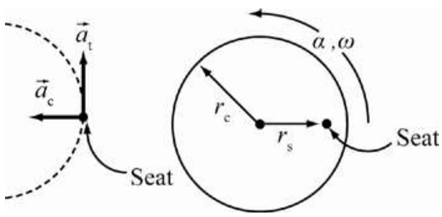
CALCULATE: $\theta=\tan^{-1}\left(\dfrac{\left(0.0200(2\pi)\text{s}^{-1}\right)^2(6.00\text{ m})}{9.81\text{ m/s}^2}\right)=0.5533°$

ROUND: Rounding to three significant figures, $\theta=0.553°$.

DOUBLE-CHECK: Such a small deviation is reasonable, given that the rotation is so slow.

9.79. **THINK:** Use the relationship between angular and centripetal acceleration. The given values are $r_s=2.75$ m, $r_c=6.00$ m, $\omega_i=0$, $\omega_f=0.600$ rev/s and $\Delta t=8.00$ s.

SKETCH:



RESEARCH:

(a) $\alpha = \dfrac{\Delta\omega}{\Delta t}$

(b) $a_c = \dfrac{v_s^{\,2}}{r_s}, \quad v_s = \omega_s r_s$

(c) $\vec{a} = \vec{a}_c + \vec{a}_t, \quad a_t = \alpha r_s$

SIMPLIFY:

(a) Simplification is not necessary.

(b) $a_c = \omega_s^{\,2} r_s$

(c) $a = \sqrt{a_c^{\,2} + a_t^{\,2}}, \quad \tan\theta = \left(\dfrac{a_t}{a_c}\right)$

CALCULATE:

(a) $\alpha = \dfrac{0.600(2\pi) \text{ rad/s}}{8.00 \text{ s}} = 0.4712 \text{ rad/s}^2$

(b) At 8.00 s, $\omega_s = 0.600$ rev/s, so $a_c = \left(0.600(2\pi) \text{ s}^{-1}\right)^2 (2.75 \text{ m}) = 39.08 \text{ m/s}^2$ and $\alpha = 0.4712 \text{ rad/s}^2$.

(c) $a = \sqrt{\left(39.08 \text{ m/s}^2\right)^2 + \left(0.4712 \text{ s}^{-2}\right)^2 (2.75 \text{ m})^2} = 39.10 \text{ m/s}^2 \quad \theta = \tan^{-1}\dfrac{\left(0.4712 \text{ s}^{-2}\right)(2.75 \text{ m})}{39.08 \text{ m/s}^2} = 1.899°$

If the centripetal acceleration is along the positive x axis, then the direction of the total acceleration is $1.90°$ along the horizontal (rounded to three significant figures).

ROUND: Values are given to three significant figures, so the results should be rounded accordingly.

(a) $\alpha = 0.471 \text{ rad/s}^2$

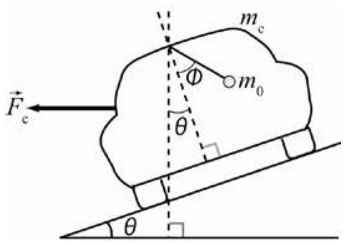
(b) $a_c = 39.1 \text{ m/s}^2$ and $\alpha = 0.471 \text{ rad/s}^2$.

(c) $a = 39.1 \text{ m/s}^2$ at $\theta = 1.90°$.

DOUBLE-CHECK: The total acceleration is quite close to the centripetal acceleration, since the tangential acceleration and the angular acceleration are both quite small.

9.80. **THINK:** The forces acting on the ornament are the tension on the string and the force of gravity. The net force is the centripetal force acting towards the center of the track. The centripetal force is close to the car's friction with the ground. $m_c g = 10.0$ kN, $\theta = 20.0°$ and $\phi = 30.0°$. F_f is the frictional force acting on the car.

SKETCH:



RESEARCH: $T\cos(\theta+\phi) = m_0 g, \quad T\sin(\theta+\phi) = m_0\dfrac{v^2}{r}, \quad F_c = F_f = m_c\dfrac{v^2}{r}$

SIMPLIFY: $\dfrac{T}{m_0} = \dfrac{g}{\cos(\theta+\phi)}$

$\dfrac{T}{m_0} = \dfrac{v^2}{r\sin(\theta+\phi)}$

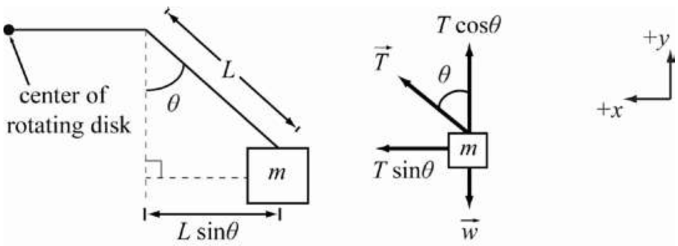
Equating the equations above gives $\dfrac{g}{\cos(\theta+\phi)} = \dfrac{v^2}{r\sin(\theta+\phi)} \Rightarrow \tan(\theta+\phi) = \dfrac{v^2}{rg} \Rightarrow \dfrac{v^2}{r} = g\tan(\theta+\phi)$.

The force of friction then becomes $F_\text{f} = m_\text{c}\dfrac{v^2}{r} = m_\text{c} g\tan(\theta+\phi)$.

CALCULATE: $F_\text{f} = (10.0\cdot 10^3\text{ N})\tan(20.0° + 30.0°) = 1.192\cdot 10^4\text{ N}$

ROUND: Rounding to three significant figures gives $F_\text{f} = 1.19\cdot 10^4\text{ N}$.
DOUBLE-CHECK: This is a reasonable value for a car of this weight.

9.81. **THINK:** Both gravity and tension act on the passenger. The net force is the centripetal force acting towards the center. The given values are as follows: $\theta = 30.0°$, $m = 65.0$ kg, $L = 3.20$ m and $R_0 = 3.00$ m.
SKETCH:



RESEARCH: $T\cos\theta = w,\ w = mg,\ T\sin\theta = \dfrac{mv^2}{r},\ \ r = R_0 + L\sin\theta$

SIMPLIFY: $v^2 = \dfrac{rT\sin\theta}{m},\ \ T = \dfrac{w}{\cos\theta} = \dfrac{mg}{\cos\theta}$

(a) $v^2 = \dfrac{r\sin\theta}{m}\left(\dfrac{mg}{\cos\theta}\right) = rg\tan\theta \Rightarrow v = \sqrt{rg\tan\theta}$

(b) $T = \dfrac{mg}{\cos\theta}$ or $T = \dfrac{mv^2}{r\sin\theta}$.

CALCULATE:

(a) $v = \sqrt{(3.00\text{ m} + 3.20\sin 30.0°\text{ m})(9.81\text{ m/s}^2)(\tan 30.0°)} = 5.104\text{ m/s}$

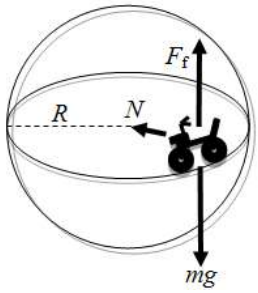
(b) $T = \dfrac{(65.0\text{ kg})(9.81\text{ m/s}^2)}{\cos 30.0°} = 736.3\text{ N}$

ROUND: All values are given to three significant figures, so the results should be rounded accordingly.
(a) $v = 5.10$ m/s
(b) $T = 736$ N
DOUBLE-CHECK: Note that the speed increases if the main disk, R_0, increases, or the length of the cable, L, increases, as it should.

Multi-Version Exercises

9.82. **THINK:** The only values given in this problem are the radius of the sphere and the coefficient of static friction between the motorcycle and the sphere. The motorcycle will stay on the surface as long as the vertical force exerted by the force of friction is at least as much as the weight of the motorcycle. The friction force is proportional to the normal force exerted by the wall of the dome, which is given by the centripetal force. Combine these to solve for the minimum velocity.

SKETCH:



RESEARCH: The centripetal force required to keep the motorcycle moving in a circle is $F_c = \frac{mv^2}{R}$. The friction force is $F_f = \mu_s N$, and it must support the weight of the motorcycle, s o $F_f \geq mg$.

SIMPLIFY: Since the normal force equals the centripetal force in this case, substitute F_c for N in the equation $F_f = \mu_s N$ to get $F_f = \mu_s F_c = \mu_s \frac{mv^2}{R}$. Combine this with the fact that the frictional force must be enough to support the weight of the motorcycle, so $mg \leq F_f = \mu_s \frac{mv^2}{R}$. Finally, solve the inequality for the velocity (keep in mind that the letters represent positive values):

$$\mu_s \frac{mv^2}{R} \geq mg \Rightarrow$$

$$\frac{R}{\mu_s m} \cdot \mu_s \frac{mv^2}{R} \geq \frac{R}{\mu_s m} \cdot mg \Rightarrow$$

$$v^2 \geq \frac{Rg}{\mu_s}$$

$$v \geq \sqrt{\frac{Rg}{\mu_s}}$$

CALCULATE: The radius of the sphere is 12.61 m, and the coefficient of static friction is 0.4601. The gravitational acceleration near the surface of the earth is about 9.81 m/s^2, so the speed must be:

$$v \geq \sqrt{\frac{Rg}{\mu_s}}$$

$$v \geq \sqrt{\frac{12.61 \text{ m} \cdot 9.81 \text{ m/s}^2}{0.4601}}$$

$$v \geq 16.3970579 \text{ m/s}$$

ROUND: Since the measured values are all given to four significant figures, the final answer will also have four figures. The minimum velocity is 16.40 m/s.

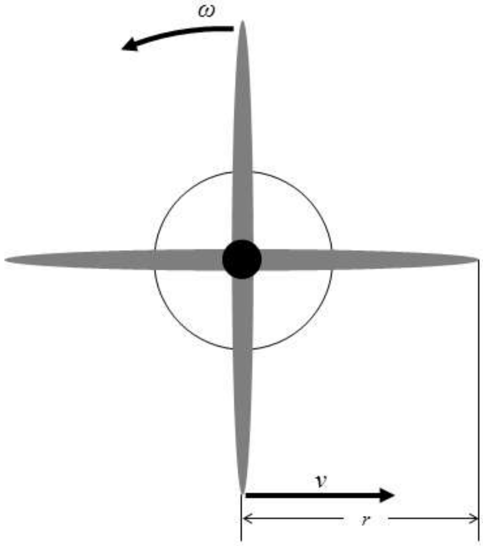
DOUBLE-CHECK: In this case, the motorcycle is traveling at 16.40 m/s, or about 59 kilometers per hour, which is a reasonable speed based on how fast motorcycles *can* go. It needs to travel 12.61(2π) = 79.23 meters to go all the way around the sphere, so it makes one revolution every 4.83 seconds, or between 12 and 13 revolutions per minute. These values all seem reasonable based on past experience with motorcycles.

9.83. $$\mu_s = \frac{Rg}{v^2} = \frac{(13.75 \text{ m})(9.81 \text{ m/s}^2)}{(17.01 \text{ m/s})^2} = 0.4662$$

9.84. $$R = \frac{\mu_s v^2}{g} = \frac{(0.4741)(15.11)^2}{9.81 \text{ m/s}^2} = 11.03 \text{ m}$$

9.85. **THINK:** The speed of a point on the tip of the propeller can be calculated from the angular speed and the length of the propeller blade. The angular speed of the propeller can be calculated from the frequency. Find the maximum length of the propeller blade such that the angular speed at the tip of the propeller blade is less than the indicated speed of sound.

SKETCH: A view, looking towards the airplane from the front, is shown.



RESEARCH: The linear velocity should be less than the speed of sound $v \leq v_{\text{sound}}$. The magnitude of the linear velocity v is equal to the product of the radius of rotation r and the angular speed ω: $v = r\omega$. The angular speed is related to the rotation frequency by $\omega = 2\pi f$. The length of the propeller blade is twice the radius of the propeller ($d = 2r$). Finally, note that the rotation frequency is given in revolutions per minute and the speed of sound is given in meters per second, so a conversion factor of 60 seconds / minute will be needed.

SIMPLIFY: Use the equation for the linear speed ($v = r\omega$) and the equation for the rotation frequency to get $v = 2\pi f \cdot r$. Use this in the inequality $v \leq v_{\text{sound}}$ to find that $2\pi f \cdot r \leq v_{\text{sound}}$. Solve this for the length of the propeller blade r (note that ω is a positiv e number of revolutions per minute) to get $r \leq \dfrac{v_{\text{sound}}}{2\pi f}$. The maximum length of the propeller blade is two times the largest possible value of $d = 2r = \dfrac{v_{\text{sound}}}{\pi f}$.

CALCULATE: The angular frequency f is given in the problem as 2403 rpm and the speed of sound is 343.0 m/s. The maximum length of the propeller blade is thus $d = \dfrac{343.0 \text{ m/s} \cdot 60 \text{ s/min}}{\pi \cdot 2403 \text{ rev/min}} = 2.726099649 \text{ m}$.

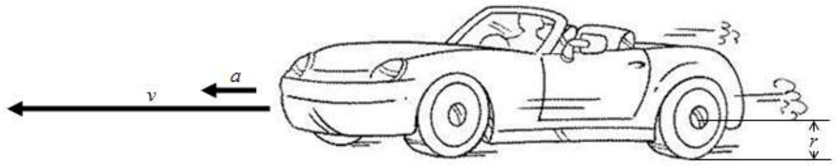
ROUND: The measured values from the problem (the angular frequency and speed of sound) are given to four significant figures, so the final answer should also have four significant figures. The maximum length of a propeller blade is 2.726 m.

DOUBLE-CHECK: For those familiar with propeller-driven aircraft, a total propeller length of about 2.7 m seems reasonable. Working backwards, if the propeller blade is 2.726 m and the linear speed at the tip of the propeller is 343.0 m/s, then the angular speed is $\omega = \dfrac{v}{r} = \dfrac{343.0 \text{ m/s}}{1.363 \text{ m}}$. The angular frequency is then $f = \dfrac{\omega}{2\pi} = \dfrac{343.0 \text{ m/s}}{2\pi \cdot 1.363 \text{ m}} = 40.05 \text{ rev/sec}$. Since there are 60 seconds in a minute, this agrees with the value of 2403 rev/min given in the problem, and the calculations were correct.

9.86. $f = \dfrac{v}{\pi d} = \dfrac{343.0\text{ m/s}}{\pi(2.601\text{ m})} = \left(41.98\dfrac{1}{\text{s}}\right)\left(\dfrac{60\text{ s}}{\text{min}}\right) = 2519\text{ rpm}$

9.87. **THINK:** The linear acceleration can be computed from the change in the speed of the car and the time required to accelerate, both of which are given in the problem. The angular acceleration can be calculated from the linear acceleration and the radius of the tires. Since the car's acceleration is constant and it starts at rest, the motion of the car occurs in only one direction, which can be taken to be the +x direction, and the time that the car starts moving can be taken as time zero.

SKETCH: The car starts at rest, so the constant acceleration and velocity are in the same direction.



RESEARCH: The constant linear acceleration is the change in speed per unit time $a = \dfrac{\Delta v}{\Delta t}$. The relationship between linear acceleration a and angular acceleration α is given by $a = r\alpha$, where r is the radius of the rotating object.

SIMPLIFY: Since there are two expressions for the linear acceleration, $a = \dfrac{\Delta v}{\Delta t}$ and $a = r\alpha$, they must be equal to one another: $r\alpha = \dfrac{\Delta v}{\Delta t}$. Solve for the angular acceleration α to get $\alpha = \dfrac{\Delta v}{r\Delta t}$. The car starts at rest at time zero, the final velocity is equal to Δv and the total time is equal to Δt, giving $\alpha = \dfrac{v}{rt}$.

CALCULATE: After 3.945 seconds, the car's final speed is 29.13 m/s. The rear wheels have a radius of 46.65 cm, or $46.65\cdot10^{-2}$ m. The angular acceleration is then

$$\alpha = \frac{29.13\text{ m/s}}{46.65\cdot10^{-2}\text{ m}\cdot3.945\text{ s}} = 15.82857539\text{ s}^{-2}$$

ROUND: The time in seconds, radius of the tires, and speed of the car are all given to four significant figures, so the final answer should also have four figures. The angular acceleration of the car is 15.83 s^{-2}.

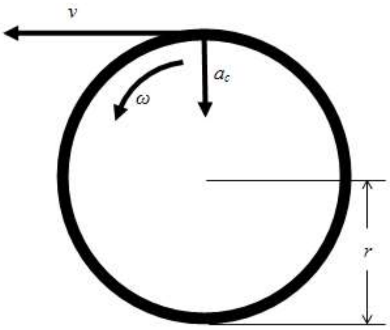
DOUBLE-CHECK: First note that the units (per second per second) are correct for angular acceleration. Working backwards, if the sports car accelerates with an angular acceleration of 15.83 s^{-2} for 3.945 seconds, it will have a final angular speed of (15.83·3.945) s^{-1}. With a tire radius of 46.65 cm, this means that the car's final speed will be (46.65·15.83·3.945) cm/s, or 29.13 m/s (when rounded to four significant figures), which agrees with the problem statement. This confirms that the first set of calculations was correct.

9.88. $v = r\alpha t = (0.4895\text{ m})(14.99\text{ s}^{-2})(3.997\text{ s}) = 29.33\text{ m/s}$

9.89. $r = \dfrac{v}{\alpha t} = \dfrac{29.53\text{ m/s}}{(17.71\text{ s}^{-2})(4.047\text{ s})} = 0.4120\text{ m} = 41.20\text{ cm}$

9.90. **THINK:** The frequency and radius of the flywheel can be used to calculate the speed at the edge of the flywheel. The centripetal acceleration can be calculated from the linear speed and the radius of the flywheel.

SKETCH:



RESEARCH: The centripetal acceleration at the edge of the flywheel is $a_c = \frac{v^2}{r}$, where v is the linear speed at the edge of the flywheel and r is the flywheel's radius. The linear speed v is equal to the angular speed times the radius of the flywheel ($v = r\omega$), and the angular speed ω is related to the frequency f by the equation $\omega = 2\pi f$. The numbers are given in centimeters and revolutions per minute, so conversion factors of $\frac{1 \text{ m}}{100 \text{ cm}}$ and $\frac{1 \text{ min}}{60 \text{ sec}}$ may be needed.

SIMPLIFY: First, find the equation for the velocity in terms of the angular frequency to get $v = r\omega = r(2\pi f)$. Use this in the equation for centripetal acceleration to find

$$a_c = \frac{v^2}{r} = \frac{(2\pi rf)^2}{r} = 4r(\pi f)^2.$$

CALCULATE: The radius is 27.01 cm, or 0.2701 m and the frequency of the flywheel is 4949 rpm. So the angular acceleration is $4 \cdot 27.01 \text{ cm}\left(\pi \cdot 4949 \frac{\text{rev}}{\text{min}}\right)^2 = 2.611675581 \cdot 10^{10} \frac{\text{cm}}{\text{min}^2}$. Converting to more familiar units, this becomes

$$2.611675581 \cdot 10^{10} \frac{\text{cm}}{\text{min}^2} \cdot \frac{1 \text{ m}}{100 \text{ cm}} \cdot \left(\frac{1 \text{ min}}{60 \text{ sec}}\right)^2 = 7.254654393 \cdot 10^4 \text{ m/s}^2.$$

ROUND: The radius and frequency of the flywheel both have four significant figures, so the final answer should also have four figures. The centripetal acceleration at a point on the edge of the flywheel is $7.255 \cdot 10^4$ m/s^2.

DOUBLE-CHECK: Work backwards to find the frequency from the centripetal acceleration and the radius of the flywheel. The linear velocity is $v = \sqrt{a_c r}$, the angular speed is $\omega = v / r = \frac{\sqrt{a_c r}}{r}$, and the frequency $f = \frac{\omega}{2\pi} = \frac{\sqrt{a_c r}}{2\pi r}$. The radius of the flywheel is 0.2701 m and the centripetal acceleration is $7.255 \cdot 10^4$ m/s^2, so the frequency is

$$\begin{aligned} f &= \frac{\sqrt{a_c r}}{2\pi r} \\ &= \frac{\sqrt{7.255 \cdot 10^4 \text{ m/s}^2 \cdot 0.2701 \text{ m}}}{2\pi \cdot 0.2701 \text{ m}} \\ &= 82.4853 \text{ s}^{-1} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \\ &= 4949.117882 \text{ min}^{-1} \end{aligned}$$

After rounding to four significant figures, this agrees with the frequency given in the problem of 4949 rpm (revolutions per minute).

9.91. $a_c = r(2\pi f)^2$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{r}} = \frac{1}{2\pi}\sqrt{\frac{8.629 \cdot 10^4 \text{ m/s}^2}{0.3159 \text{ m}}} = \left(83.18 \frac{1}{\text{s}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 4991 \text{ rpm}$$