Chapter 6: Potential Energy and Energy Conservation

Concept Checks

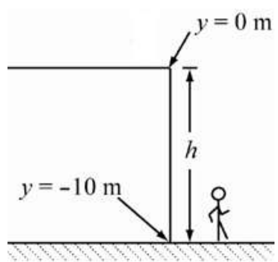
6.1. b **6.2.** a **6.3.** c **6.4.** e **6.5.** e **6.6.** d **6.7.** b

Multiple-Choice Questions

6.1. a **6.2.** c **6.3.** e **6.4.** e **6.5.** d **6.6.** e **6.7.** d **6.8.** e **6.9.** a **6.10.** d **6.11.** c **6.12.** c **6.13.** b

Conceptual Questions

6.14. The kinetic energy, *K*, of an object is proportional to the mass, *m*, of the object and the square of its speed, *v*. The formula is $K = mv^2/2.$ The mass is always positive, and the square of the velocity is non-negative. Since the product of non-negative numbers is non-negative, the kinetic energy of an object cannot be negative. However, the potential energy of an object can be negative because it is a relative value. An example of negative potential energy is gravitational potential energy, given by the formula $U = mgh$, where *m* is the mass of the object, *g* is the acceleration due to gravity, and *h* is the vertical distance above the ground. Consider a person standing at the base of a bridge as in the figure below.



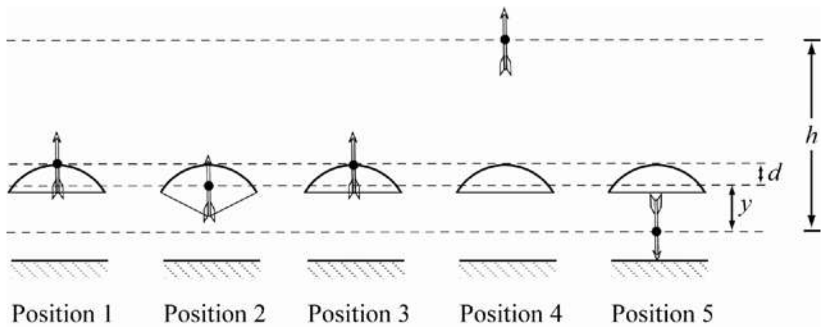
In this coordinate system, the person has potential energy due to gravity of $U = m\left(9.81 \text{ m/s}^2\right)\left(-10 \text{ m}\right)$ relative to the reference point of the bridge. Since a mass is always positive, the potential energy of the person standing on the ground relative to the bridge above has a negative value.

6.15. (a) If a person jumps off a table onto the floor, mechanical energy is not conserved. Mechanical energy is conserved while you are falling towards the floor (assuming energy lost to air resistance is ignored) because gravitational potential energy is being converted to kinetic energy. However, once you land on the floor all of the kinetic energy is absorbed by your body on impact. The energy is lost to non-conservative forces such as friction within your body and heat expelled by your muscles.

(b) The car's mechanical energy is not conserved. Assume a car is on a level plane so it has no gravitational potential energy. The car is in motion so its energy is in the form of kinetic energy. The energy is lost to non-conservative forces such as friction on the tires, thermal energy on the car's brakes and energy dissipated as the car's body is bent by the tree.

6.16. Work is defined as the dot product of force and displacement. This is indicated in the formula $W = \vec{F}\cdot\Delta\vec{r} = \left|\vec{F}\right|\left|\Delta\vec{r}\right|\cos\theta$, where $\vec{F}$ is the applied force, $\vec{r}$ is the displacement of the object, and θ is the angle between the vectors $\vec{F}$ and $\vec{r}$. When you are standing still, the bag of groceries does not travel any distance, i.e. $\left|\vec{r}\right| = 0$, so there is no work done. Assuming that you do not lift or lower the bag of groceries when you carry the bag a displacement $\vec{r}$ across the parking lot, then you do not do any work. This is because the applied force $\vec{F}$ is perpendicular to the displacement $\vec{r}$. Using $\theta = 90°$ in the formula gives $W = \left|\vec{F}\right|\left|\Delta\vec{r}\right|\cos 90° = \left|\vec{F}\right|\left|\Delta\vec{r}\right|\cdot 0 = 0 \text{ J}.$

6.17. The energy in the system, E, is the sum of the energy stored in the bow by flexing it, E_b, the kinetic energy of the arrow, K, and the gravitational potential energy of the arrow, U. Let the arrow have mass m and the bow have spring constant k. Five separate positions of the arrow and bow system will be considered. Position 1 is where the arrow is put in the bow. Position 2 is where the arrow is pulled back in the bow. Position 3 is where the bow has returned to its relaxed position and the arrow is leaving the bowstring. Position 4 is where the arrow has reached its maximum height h. Position 5 is where the arrow has stuck in the ground.



At position 1 the arrow has gravitational potential energy $U = mg(y+d)$ (refer to diagram) relative to the ground. The total energy in the system at this position is $E_1 = mg(y+d)$. At position 2, the arrow now has gravitational potential energy $U = mgy$ and the elastic energy stored in the bow is $E_b = kd^2/2$ due to the downward displacement d. The total energy in the system at this position is $E_2 = mgy + \left(kd^2/2\right)$. The work done by the bowstring during this displacement is $E_{tot} = 2.0$ J At position 3, the bow's tension is released and the arrow is launched with a velocity, v. The total energy is given by $E_3 = \left(mv^2/2\right) + mg\left(y+d\right)$. The work done on the arrow by the bow is $W_3 = kd^2/2$. At position 4, the arrow has reached its maximum height, h. At this position, the velocity of the arrow is zero, so the kinetic energy is zero. The total energy is given by $E_4 = mgh$. The work done on the arrow by gravity is equal to the change in kinetic energy, $W_4 = \Delta K = 0 - mv^2/2$. At position 5, the arrow has hit the ground and stuck in. The total energy is $E_5 = 0$. When the arrow hits the ground the energy of the system is dissipated by friction between the arrow and the ground. The work done on the arrow by gravity during its fall is given by $W_5 = \Delta K = \left(mv^2/2\right) - 0$. This is equal to the kinetic energy of the arrow just before it strikes the ground.

6.18. (a) Assuming both billiard balls have the same mass, m, the initial energies, E_{Ai} and E_{Bi} are given by $E_{Ai} = mgh$ and $E_{Bi} = mgh$. The final energy is all due to kinetic energy, so the final energies are $E_{Af} = \left(mv_A{}^2\right)/2$ and $E_{Bf} = \left(mv_B{}^2\right)/2$. By conservation of energy (assuming no loss due to friction), $E_i = E_f$. For each ball the initial and final energies are equal. This means $mgh = \left(mv_A^2\right)/2 \Rightarrow v_A = \sqrt{2gh}$ and $mgh = \left(mv_B{}^2\right)/2 \Rightarrow v_B = \sqrt{2gh}$. Therefore, $v_A = v_B$. The billiard balls have the same speed at the end.

(b) Ball B undergoes an acceleration of a and a deceleration of $-a$ due to the dip in the track. The effects of the acceleration and deceleration ultimately cancel. However, the ball rolling on track B will have a greater speed over of the lowest section of track. Therefore, ball B will win the race.

6.19. Because the girl/swing system swings out, then returns to the same point, the girl/swing system has moved over a closed path and the work done is zero. Therefore the forces acting on the girl/swing system are conservative. Assuming no friction, the only forces acting on the girl/swing system are the tension in the ropes holding up the girl/swing system and the force of gravity. Assume that the ropes cannot be stretched so that the tension in the ropes is conservative. Gravity is a conservative force, so it is expected that all forces are conservative for the girl/swing system.

6.20. No. Friction is a dissipative force (non-conservative). The work done by friction cannot be stored in a potential form.

6.21. No. The mathematical expression for the potential energy of a spring is $U = \left(kx^2\right)/2.$ The spring constant, k is a positive constant. The square of the displacement of the spring, x, will always be non-negative. Hence, the potential energy of a spring will always be non-negative.

6.22. The elastic force is given by $\vec{F} = -k\vec{r}$, where $\vec{r}$ is the displacement of the spring. The force is therefore a function of displacement, so denote that the force by $\vec{F}\left(\vec{r}\right).$ The sum of the inner product between $\vec{F}\left(\vec{r}\right)$ and the local displacements Δr can be expressed as $\sum_{i=1}^{n}\vec{F}\left(\vec{r}_i\right)\cdot\Delta r.$ If the local displacements are chosen so they are infinitesimally small, the sum can be expressed as an integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\vec{F}\left(\vec{r}_i\right)\cdot\Delta r = \int_a^b\vec{F}\left(\vec{r}\right)\cdot d\vec{r}.$$

If the trajectory is a closed loop, then $a = b$ and the integral becomes $\int_a^b\vec{F}\left(\vec{r}\right)\cdot d\vec{r} = 0$ because

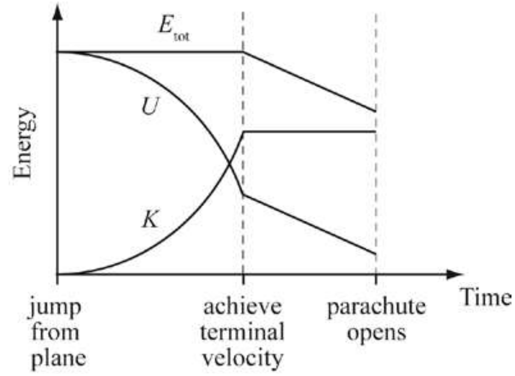
$$\int_a^b\vec{F}\left(\vec{r}\right)\cdot d\vec{r} = f\left(b\right) - f\left(a\right) = f(a) - f(a) = 0.$$

It should be noted that work, $W = \int\vec{F}\cdot d\vec{r}$, is independent of a path because the force is conservative. If there was a dissipative force, such as friction present, the force would be non-conservative and therefore be path-dependent.

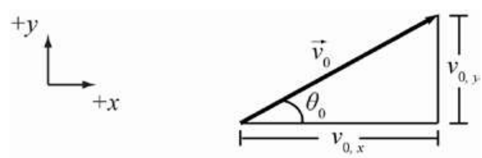
6.23. No. There is not a 1-1 correspondence between potential energy functions and conservative forces. The conservative force is the negative gradient of the potential energy. Therefore, two conservative forces will have the same potential energy function U if they differ by a constant. For example, consider the force $F = 0.$ The corresponding potential function is a constant, but it could be any constant depending on the situation. Therefore there is not necessarily a unique potential function corresponding to a conservative force.

6.24. When the person first steps out of the plane, all of the energy is potential energy and as they fall, the potential energy is converted to kinetic energy. In the first stage, before they reach the terminal velocity, they accelerate at a constant rate, so their velocity increases at a linear rate, and so $K = \frac{1}{2}mv^2$ increases at a quadratic rate. On the other hand, their height decreases at a quadratic rate, so $U = mgh$ decreases at a quadratic rate. Because there is no air resistance in the first stage of the model, the total energy, $E_{\text{tot}} = K + U$, remains constant. In the second stage, their acceleration becomes zero, and their velocity becomes constant. This means that $K = \frac{1}{2}mv^2$ is constant, and $U = mgh$ decreases at a linear rate. The sum of the energies is no longer constant. The lost energy is due to the air resistance that counter-balances the acceleration due to gravity.

The rate of decrease of energy in the system is equal to the rate of decrease of potential energy.



6.25.



The lengths of the component vectors of v_0 are $v_{0,x} = v_0 \cos\theta_0$ and $v_{0,y} = v_0 \sin\theta_0$. Velocity is a vector quantity, so $\vec{v} = v_x \hat{x} + v_y \hat{y}$. Let $v = |\vec{v}|$. Then, $v^2 = v_x^{\ 2} + v_y^{\ 2}$. The velocity vector $\vec{v}$ has component vectors $v_x = v_0 \cos\theta_0$ (horizontal component is constant) and $v_y = v_0 \sin\theta_0 - gt$ (which changes relative to time). To compute the kinetic energy, use the formula $K = mv^2/2$. First, compute

$$\begin{aligned} v^2 &= v_0^{\ 2} \cos\theta_0^{\ 2} + v_0^{\ 2} \sin^2\theta_0 - 2v_0 \sin\theta_0 gt + g^2t^2 \\ &= v_0^{\ 2}\left(\cos^2\theta_0 + \sin^2\theta_0\right) - 2v_0 \sin\theta_0 gt + g^2t^2 \\ &= v_0^2 - 2v_0 \sin\theta_0 gt + g^2t^2. \end{aligned}$$

So, $K(t) = \left[m\left(v_0^{\ 2} - 2v_0 \sin\theta_0 gt + g^2t^2\right)\right]/2.$ The potential energy only changes with displacement in the vertical direction. The gravitational potential energy is given by $U = mgy$. From kinematics equations, $y = y_0 + v_{0,y}t - \left(gt^2\right)/2$. Because the projectile was launched from the ground, $y_0 = 0$. Substitute $v_{0,y} = v_0 \sin\theta_0$ into the equation to get $y = v_0 \sin\theta_0 t - \left(gt^2\right)/2$. Substituting this into the expression for U yields $U(t) = mg\left(v_0 \sin\theta_0 t - \left(gt^2\right)/2\right)$. The total energy of the projectile is $E(t) = K(t) + U(t)$. This equation can be written as

$$E(t) = \frac{m\left(v_0^{\ 2} - 2v_0 \sin\theta_0 gt + g^2t^2\right)}{2} + mg\left(v_0 \sin\theta_0 t - \frac{1}{2}gt^2\right).$$

Grouping like terms, the equation can be simplified:

$$E(t) = \frac{m}{2}\left(g^2t^2 - g^2t^2\right) - mgv_0 \sin\theta_0 t + mgv_0 \sin\theta_0 t + \frac{1}{2}mv_0^{\ 2} \Rightarrow E(t) = \frac{1}{2}mv_0^{\ 2}.$$

Notice that E is actually not time dependent. This is due to the conservation of energy.

6.26. (a) The total energy is given by the sum of the kinetic energy, $K = mv^2/2$, and potential energy, $U = mgh$. This gives the formula $E = \frac{1}{2}mv^2 + mgh$ for total energy. Therefore,

$$H(m,h,v) = \frac{\frac{1}{2}mv^2 + mgh}{mg} = \frac{\frac{1}{2}v^2 + gh}{g} = \frac{v^2}{2g} + h.$$

(b) The aircraft has a mass of $m = 3.5\cdot 10^5$ kg , a velocity of v = 250.0 m/s and a height of $h = 1.00\cdot 10^4$ m Substituting these values gives

$$H = \frac{(250.0\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 1.00\cdot 10^4\text{ m} = 13{,}185.5\text{ m} \approx 13{,}200\text{ m}.$$

6.27. (a) The energy in the system is the sum of the kinetic energy and the gravitational potential energy, $E = K + U$. For motion in the x-direction, $U = 0$ and $K = mv^2/2$. So, $E = mv^2/2$. Newton's second law is $\vec{F} = m\vec{a}$, which can also be written as $\vec{F} = m\,d\vec{v}/dt$. By the work-kinetic energy theorem, $W = \Delta K$ and $W = \vec{F}\bullet\vec{x}$. If the work on the body as a function of position is to be determined,

$$W = \sum_{i=1}^{n}\vec{F}(x_i)\Delta\vec{x}.$$

If the motion is continuous, let the intervals become infinitesimal so that the sum becomes an integral, $W = \int_a^b \vec{F}\bullet d\vec{x}$. Since $\vec{F} = m\,d\vec{v}/dt$ and $\vec{v} = d\vec{x}/dt$, it must be that $d\vec{x} = \vec{v}\,dt$. Substituting these values into the equation:

$$W = \int m\frac{d\vec{v}}{dt}\bullet\vec{v}\,dt = \int m\vec{v}\bullet d\vec{v}.$$

Work is also equal to the change in kinetic energy, therefore,

$$\Delta K = \int_{v_\text{i}}^{v_\text{f}} m\vec{v}\bullet d\vec{v} = \frac{1}{2}mv^2\Big|_{v_\text{i}}^{v_\text{f}} = \frac{1}{2}m\left(v_\text{f}^2 - v_\text{i}^2\right).$$

(b) Newton's second law, expressed as $\vec{F} = m\vec{a}$, does not hold for objects on the subatomic scale or for objects approaching the speed of light. The law of conservation of energy holds under all known circumstances.

6.28. (a) The force function is $F(x) = -\dfrac{dU(x)}{dx} = 4U_0\left[\dfrac{12x_0^{12}}{x^{13}} - \dfrac{6x_0^6}{x^7}\right]$.

(b) The two atoms experience zero force from each other when F = 0, which is when $\left[12\dfrac{x_0^{12}}{x^{13}} - \dfrac{6x_0^6}{x^7}\right] = 0.$

Solving for x yields $\dfrac{6x_0^6}{x^7} = \dfrac{12x_0^{12}}{x^{13}} \Rightarrow x^6 = 2x_0^6$ or $x = \pm\sqrt[6]{2}x_0$. Since x is the separation, $x = \sqrt[6]{2}x_0$.

(c) For separations larger than $x = \sqrt[6]{2}x_0$, let $x = 3x_0$:

$$U(3x_0) = 4U_0\left[\left(\frac{x_0}{3x_0}\right)^{12} - \left(\frac{x_0}{3x_0}\right)^6\right] = 4U_0\left[\left(\frac{1}{3}\right)^{12} - \left(\frac{1}{3}\right)^6\right].$$

The factor $\left[(1/3^{12}) - (1/3^6)\right]$ is negative and the potential is negative. Therefore, for $x > \sqrt[6]{2}x_0$, the nuclei attract. For separations smaller than $x = \sqrt[6]{2}x_0$, let $x = x_0/2$:

$$U(x_0/2) = 4U_0\left[\left(\frac{2x_0}{x_0}\right)^{12} - \left(\frac{2x_0}{x_0}\right)^6\right] = 4U_0\left[2^{12} - 2^6\right].$$

The term $\left[2^{12} - 2^6\right]$ is positive and the potential is positive. So, when $x < \sqrt[6]{2}x_0$, the potential is positive and the nuclei repel.

6.29. (a) In two-dimensional situations, the force components can be obtained from the potential energy using the equations $F_x = -\dfrac{\partial U(x,y)}{\partial x}$ and $W_a = (10.0 \text{ N/cm})\left((5.00 \text{ cm})^2 - (-5.00 \text{ cm})^2\right)/2 = 0 \text{J}$ The net force is given by:

$$\vec{F} = F_x\hat{x} + F_y\hat{y} = -\frac{\partial U(x,y)}{\partial x}\hat{x} - \frac{\partial U(x,y)}{\partial y}\hat{y} = -\frac{1}{2}k\left(\frac{\partial}{\partial x}(x^2+y^2)\hat{x} + \frac{\partial}{\partial y}(x^2+y^2)\hat{y}\right)$$

$$= -\frac{1}{2}k(2x\hat{x} + 2y\hat{y}) = -k(x\hat{x} + y\hat{y}).$$

(b) The equilibrium point will be where $\vec{F} = 0$. This occurs if and only if x and y are both zero.

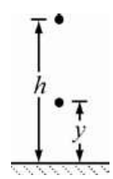
(c) These forces will accelerate the mass in the $-\hat{x}$ and $-\hat{y}$ directions for positive values of x and y and vice versa for negative values of x and y.

(d) $|\vec{F}| = \left[(F_x)^2 + (F_y)^2\right]^{\frac{1}{2}}$. For x = 3.00 cm, y = 4.00 cm and k = 10.0 N/cm:

$$|\vec{F}| = \left[\left(-(10.0 \text{ N/cm})(3.00 \text{ cm})\right)^2 + \left(-(10.0 \text{ N/cm})(4.00 \text{ cm})\right)^2\right]^{\frac{1}{2}} = 50.0 \text{ N}.$$

(e) A turning point is a place where the kinetic energy, K is zero. Since $K = E - U$, the turning point will occur when $U = E$, so the turning points occurs when U = 10 J. Solve $U(x,y) = 10 \text{ J} = \frac{1}{2}k(x^2+y^2)$. This gives $20.0 \text{ J} = \frac{10.0 \text{ N}}{\text{cm}} \cdot \frac{100 \text{ cm}}{\text{m}}(x^2+y^2)$, or $x^2 + y^2 = 0.0200 \text{ m}^2$. The turning points are the points on the circle centered at the origin of radius 0.141 m.

6.30. Setting the kinetic energy equal to the potential energy will normally not yield useful information. To use the example in the problem, if the rock is dropped from a height, h above the ground, then solving for the speed at two different locations:

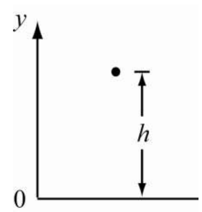


Point 1: $U_1 = mgh$ and $K_1 = (mv_1^2)/2$. If $mgh = mv^2/2$, then solving for v_1: $v_1 = \sqrt{2gh}$. But the rock has not been dropped yet so in fact v_1 is really zero. Point 2: just before the rock hits the ground. In this case, the rock's height above the ground, y, is almost zero. If $U_2 = K_2$, then $mgy = mv_2^2/2$ or $v_2 = \sqrt{2gy}$. But if y is about 0 m, then $v_2 \approx 0$ m/s. At point 2, the rock's velocity is reaching its maximum value, so by setting the potential and kinetic energy equal to one another at this point, the wrong value is calculated for the rock's speed.

Exercises

6.31. **THINK:** The mass of the book is m = 2.00 kg and its height above the floor is h = 1.50 m. Determine the gravitational potential energy, U_g.

SKETCH:



RESEARCH: Taking the floor's height as $U_g = 0$, U_g for the book can be determined from the formula $U_g = mgh.$

SIMPLIFY: It is not necessary to simplify.

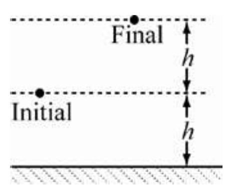
CALCULATE: $U_g = (2.00 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m}) = 29.43 \text{ J}$

ROUND: The given initial values have three significant figures, so the result should be rounded to $U_g = 29.4$ J.

DOUBLE-CHECK: This is a reasonable value for a small mass held a small distance above the floor.

6.32. **THINK:** The rock's mass is m = 40.0 kg and the gravitational potential energy is $U_g = 500.$ J. Determine:
(a) the height of the rock, h, and
(b) the change, ΔU_g if the rock is raised to twice its original height, $2h$.

SKETCH:



RESEARCH: Use the equation $U_g = mgh.$ Note: $\Delta U_g = U_g - U_{g,0}$.

SIMPLIFY:

(a) $U_g = mgh \Rightarrow h = \dfrac{U_g}{mg}$

(b)
$$\begin{aligned}\Delta U_g &= U_g - U_{g,0}\\ &= mg(2h) - mgh\\ &= mgh\\ &= U_g\end{aligned}$$

CALCULATE:

(a) $h = \dfrac{500. \text{ J}}{40.0 \text{ kg}(9.81 \text{ m/s}^2)} = 1.274 \text{ m}$

(b) $\Delta U_g = 500.$ J

ROUND:

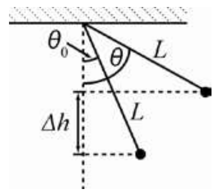
(a) $h = 1.27$ m

(b) $\Delta U_g = 500.$ J does not need to be rounded.

DOUBLE-CHECK: The initial height is reasonable for such a large mass, despite the large U_g. Since the potential energy is proportional to height, it should double when the height is doubled.

6.33. **THINK:** The rock's mass is m = 0.773 kg. The length of the string is L = 2.45 m. The gravitational acceleration on the Moon is $g_{\text{M}} = g/6$. The initial and final angles are $\theta_0 = 3.31°$ and $\theta = 14.01°$, respectively. Determine the rock's change in gravitational potential energy, ΔU.

SKETCH:



RESEARCH: To determine ΔU, the change in height of the rock, Δh, is needed. This can be determined using trigonometry. Then $\Delta U = mg_{\text{M}}\Delta h.$

SIMPLIFY: To determine Δh: $\Delta h = L\cos\theta_0 - L\cos\theta = L(\cos\theta_0 - \cos\theta)$. Then

$$\Delta U = mg_{\text{M}}\Delta h = \frac{1}{6}mgL(\cos\theta_0 - \cos\theta).$$

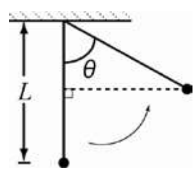
CALCULATE: $\Delta U = \frac{1}{6}(0.773 \text{ kg})(9.81 \text{ m/s}^2)(2.45 \text{ m})(\cos(3.31°) - \cos(14.01°)) = 0.08694 \text{ J}$

ROUND: With three significant figures in the values, the result should be rounded to $\Delta U = 0.0869$ J.

DOUBLE-CHECK: ΔU is small, as it should be considering the smaller gravitational acceleration and the small change in height.

6.34. **THINK:** The child's mass is m = 20.0 kg. Each rope has a length of L = 1.50 m. Determine (a) U_{g} at the lowest point of the swing's trajectory, (b) U_{g} when the ropes are $\theta = 45.0°$ from the vertical and (c) the position with the higher potential energy.

SKETCH:



RESEARCH: Use $U_{\text{g}} = mgh.$

SIMPLIFY:

(a) Relative to the point where $U_{\text{g}} = 0$, the height of the swing is $-L$. Then $U_{\text{g}} = -mgL.$

(b) Now, the height of the swing is $-L\cos\theta$. Then $U_{\text{g}} = -mgL\cos\theta.$

CALCULATE:

(a) $U_{\text{g}} = -(20.0 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m}) = -294.3 \text{ J}$

(b) $U_{\text{g}} = -(20.0 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m})\cos 45.0° = -208.1 \text{ J}$

(c) Relative to the point $U_{\text{g}} = 0$, the position in part (b) has greater potential energy.

ROUND: With three significant figures in m and L:

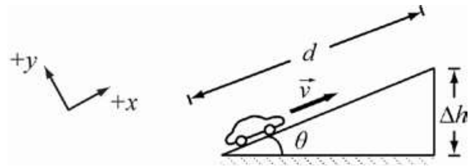
(a) $U_{\text{g}} = -294$ J

(b) $U_{\text{g}} = -208$ J

DOUBLE-CHECK: Had $U_{\text{g}} = 0$ been set at the lowest point of the swing's trajectory, the potential energy in part (b) would still be greater than the potential energy in part (a), as it should be.

6.35. **THINK:** The mass of the car is $m = 1.50 \cdot 10^3$ kg. The distance traveled is $d = 2.50$ km $= 2.50 \cdot 10^3$ m. The angle of inclination is $\theta = 3.00°$. The car travels at a constant velocity. Determine the change in the car's potential energy, ΔU and the net work done on the car, W_{net}.

SKETCH:



RESEARCH: To determine ΔU the change of height of the car Δh must be known. From trigonometry, the change in height is $\Delta h = d\sin\theta$. Then, $\Delta U = mg\Delta h$. To determine W_{net} use the work-kinetic energy theorem. Despite the fact that non-conservative forces are at work (friction force on the vehicle), it is true that $W_{\text{net}} = \Delta K$.

SIMPLIFY: $\Delta U = mg\Delta h = mgd\sin\theta$

$$W_{\text{net}} = \Delta K = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(v_{\text{f}}^2 - v_0^2\right)$$

CALCULATE: $\Delta U = \left(1.50\cdot 10^3 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\left(2.50\cdot 10^3 \text{ m}\right)\sin\left(3.00°\right) = 1925309 \text{ J}$

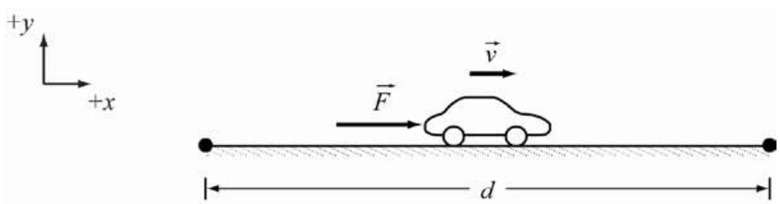
$$W_{\text{net}} = \frac{1}{2}m\left(v_{\text{f}}^2 - v_0^2\right) = \frac{1}{2}m(0) = 0$$

ROUND: Since θ has two significant figures, $\Delta U = 1.93\cdot 10^6$ J, and there is no net work done on the car.

DOUBLE-CHECK: The change in potential energy is large, as the car has a large mass and a large change in height, $\Delta h = \left(2.50\cdot 10^3 \text{ m}\right)\sin\left(3.00°\right) = 131$ m. The fact that the net work done is zero while there is a change in potential energy means that non-conservative forces did work on the car (friction, in this case).

6.36. **THINK:** The constant force is F = 40.0 N. The distance traveled is $d = 5.0\cdot 10^3$ m. Assume the force is parallel to the distance traveled. Determine how much work is done, and if it is done on or by the car. The car's speed is constant.

SKETCH:



RESEARCH: In general,

$$W = \int_{x_0}^{x} F(r)\,dr \text{ (in one dimension).}$$

Here the force is constant, so $F(r) = F$. Bearing in mind that $W_{\text{net}} = \Delta K = 0$, due to the constant speed, the work done by the constant force, F can still be calculated.

SIMPLIFY: $W = \int_{x_0}^{x_1} F\,dr = F\int_{x_0}^{x_1} dr = F\Delta x = Fd$

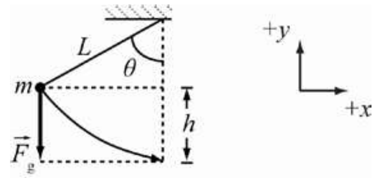
CALCULATE: $W = \left(40.0 \text{ N}\right)\left(5.0\cdot 10^3 \text{ m}\right) = 200{,}000 \text{ J}$. This is the work done on the car by the constant force, as it is a positive value.

ROUND: With two significant figures in d, $W = 2.0\cdot 10^5$ J.

DOUBLE-CHECK: This is a reasonable amount of work done by F, given the large distance the force acts over.

6.37. **THINK:** The piñata's mass is m = 3.27 kg. The string length is L = 0.810 m. Let h be the height of the piñata at its initial position, at an initial angle of $\theta = 56.5°$ to the vertical. Determine the work done by gravity, W_{g}, by the time the string reaches a vertical position for the first time.

SKETCH:



RESEARCH: Since the force of gravity is constant, the work is given by $W_g = \vec{F}_g h = mgh$.

SIMPLIFY: $W_g = mgh = mg(L - L\cos\theta) = mgL(1-\cos\theta)$

CALCULATE: $W_g = (3.27 \text{ kg})(9.81 \text{ m/s}^2)(0.810 \text{ m})(1-\cos(56.5°)) = 11.642 \text{ J}$

ROUND: With three significant figures in *L*, $W_g = 11.6$ J.

DOUBLE-CHECK: The work done by gravity should be positive because F_g pulls the piñata downward.

6.38. **THINK:** $U(x) = \frac{1}{x} + x^2 + x - 1$. Determine (a) a function which describes the force on the particle, and (b) a plot of the force and the potential functions and (c) the force on the particle when $x =$ 2.00 m.

SKETCH: A sketch will be provided when part (b) is completed.

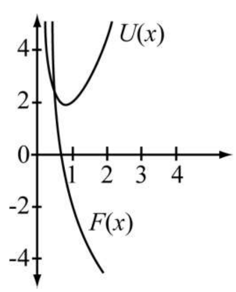
RESEARCH: The relationship between *F* and *U*, in one dimension, is $F(x) = -\frac{d}{dx}U(x)$.

SIMPLIFY: (a) $F(x) = -\frac{d}{dx}\left(\frac{1}{x} + x^2 + x - 1\right) = -\left(-x^{-2} + 2x + 1\right) = \frac{1}{x^2} - 2x - 1$

CALCULATE:

(a) Not necessary.

(b) Plotting yields:



(c) At *x* = 2.00 m, $F(2.00) = \frac{1}{(2.00)^2} - 2(2.00) - 1 = -4.75$ N (SI units are assumed).

ROUND: $F(2.00 \text{ m}) = -4.75$ N

DOUBLE-CHECK: $F(x)$ is the negative of the slope of $U(x)$. *F*(*x*) crosses the *x*-axis where *U*(*x*) has a local minimum, as would be expected.

6.39. **THINK:** The potential energy functions are (a) $U(y) = ay^3 - by^2$ and (b) $U(y) = U_0 \sin(cy)$. Determine $F(y)$ from $U(y)$.

SKETCH: A sketch is not necessary.

RESEARCH: $F(y) = -\frac{\partial U(y)}{\partial y}$

SIMPLIFY:

(a) $F(y) = -\frac{\partial\left(ay^3 - by^2\right)}{\partial y} = 2by - 3ay^2$

(b) $F(y) = -\dfrac{\partial\left(U_0 \sin(cy)\right)}{\partial y} = -cU_0\cos(cy)$

CALCULATE: There are no numerical calculations to perform.
ROUND: It is not necessary to round.
DOUBLE-CHECK: The derivative of a cubic polynomial should be a quadratic, so the answer obtained for (a) makes sense. The derivative of a sine function is a cosine function, so it makes sense that the answer obtained for (b) involves a cosine function.

6.40. **THINK:** The potential energy function is of the form $U(x,z) = ax^2 + bz^3$. Determine the force vector, $\vec{F}$, associated with U.
SKETCH: Not applicable.
RESEARCH: $\vec{F}(x,y,z) = -\vec{\nabla}U(x,y,z) = -\left(\dfrac{\partial}{\partial x}U\hat{x} + \dfrac{\partial}{\partial y}U\hat{y} + \dfrac{\partial}{\partial z}U\hat{z}\right)$
SIMPLIFY: The expression cannot be further simplified.
CALCULATE: $\vec{F} = -\dfrac{\partial\left(ax^2+bz^3\right)\hat{x}}{\partial x} - \dfrac{\partial\left(ax^2+bz^3\right)\hat{y}}{\partial y} - \dfrac{\partial\left(ax^2+bz^3\right)\hat{z}}{\partial z}$

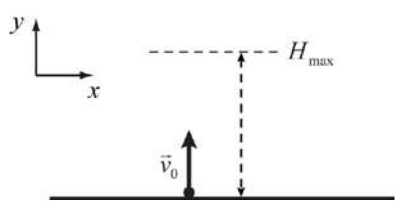
$$= -(2ax)\hat{x} - 0\hat{y} - \left(3bz^2\right)\hat{z}$$
$$= -(2ax)\hat{x} - \left(3bz^2\right)\hat{z}$$

ROUND: Not applicable.
DOUBLE-CHECK: Notice that U is the sum of a function of x and a function of z, namely,

if $G(x) = ax^2$ and $H(z) = bz^3$ then $U(x,z) = G(x) + H(z)$.

Since $G(x)$ has a critical point at $x = 0$ and $H(z)$ has a critical point at $z = 0$, we may expect that $\vec{F} = 0$. And in fact, $\vec{F} = -(2a(0))\hat{x} - \left(3b(0)^2\right)\hat{z} = 0$. Therefore, the answer is reasonable.

6.41. **THINK:** The maximum height achieved is $H_{\text{max}} = 5.00$ m, while the initial height h_0 is zero. The speed of the ball when it reaches its maximum height is $v = 0$. Determine the initial speed.
SKETCH:



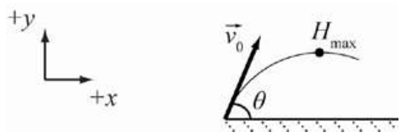
RESEARCH: In an isolated system with only conservative forces, $\Delta E_{\text{mec}} = 0$. Then, $\Delta K = -\Delta U$. Use $U = mgH_{\text{max}}$ and $K = mv^2/2$.
SIMPLIFY: $K_f - K_i = -\left(U_f - U_i\right) = U_i - U_f$, so $\dfrac{1}{2}mv^2 - \dfrac{1}{2}mv_0^{\,2} = mgh_0 - mgH_{\text{max}}$.

Substituting $v = 0$ and $h_0 = 0$ gives the equation $-\dfrac{1}{2}mv_0^{\,2} = -mgH_{\text{max}}$. Therefore, $v_0 = \sqrt{2gH_{\text{max}}}$.

CALCULATE: $v_0 = \sqrt{2\left(9.81 \text{ m/s}^2\right)\left(5.00 \text{ m}\right)} = 9.9045 \text{ m/s}$
ROUND: With three significant figures in H_{max}, $v_0 = 9.90$ m/s.
DOUBLE-CHECK: This is a reasonable speed to throw a ball that reaches a maximum height of 5 m.

6.42. **THINK:** The cannonball's mass is m = 5.99 kg. The launch angle is $\theta = 50.21°$ above the horizontal. The initial speed is $v_0 = 52.61$ m/s and the final vertical speed is $v_y = 0$. The initial height is zero. Determine the gain in potential energy, ΔU.

SKETCH:



RESEARCH: Neglecting air resistance, there are only conservative forces at work. Then, $\Delta K = -\Delta U$ or $\Delta U = -\Delta K$. Determine ΔK from $K = mv^2/2$. From trigonometry, $v_x = v_0 \cos\theta$.

SIMPLIFY: $\Delta U = -\Delta K = -(K_f - K_i) = K_i - K_f$. Note that initially the ball has a horizontal speed v_x (which is constant throughout the cannonball's motion) and a vertical speed v_{y0}. At its maximum height, $v_y = 0$. Then, $\Delta U = \frac{1}{2}mv_0{}^2 - \frac{1}{2}m(v_0\cos\theta)^2 = \frac{1}{2}mv_0{}^2(1-\cos^2\theta)$.

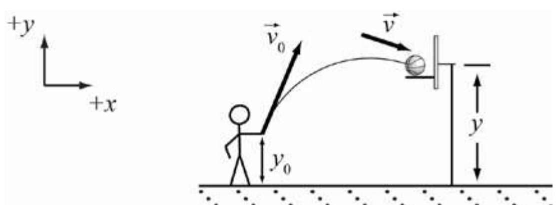
CALCULATE: $\Delta U = \frac{1}{2}(5.99 \text{ kg})(52.61 \text{ m/s})^2(1-\cos^2(50.21°)) = 4894.4 \text{ J}$

ROUND: With three significant figures in m, $\Delta U = 4890$ J.

DOUBLE-CHECK: The change in potential energy is positive, implying that the ball gained potential energy, which it would if raised any height above its initial point. Since the horizontal velocity of the cannonball is constant, it makes sense that the initial vertical velocity is converted entirely into potential energy when the cannonball reaches the highest point.

6.43. **THINK:** The initial height of the basketball is $y_0 = 1.20$ m. The initial speed of the basketball is $v_0 = 20.0$ m/s. The final height is $y = 3.05$ m. Determine the speed of the ball at this point.

SKETCH:



RESEARCH: Neglecting air resistance, there are only conservative forces, so $\Delta K = -\Delta U$. The kinetic energy K can be determined from $K = mv^2/2$ and U from $U = mgh$.

SIMPLIFY: $K_f - K_i = U_i - U_f$, so $(1/2)mv^2 - (1/2)mv_0^2 = mgy_0 - mgy$. Dividing through by the mass m yields the equation $(1/2)v^2 - (1/2)v_0{}^2 = gy_0 - gy$. Then solving for v gives

$$v = \sqrt{2\left(g(y_0 - y) + \frac{1}{2}v_0{}^2\right)}.$$

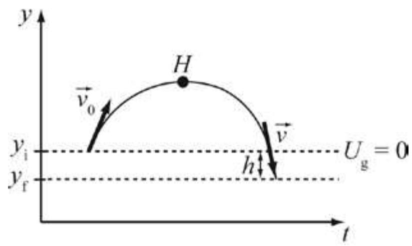
CALCULATE:
$$v = \sqrt{2\left((9.81 \text{ m/s}^2)(1.20 \text{ m} - 3.05 \text{ m}) + \frac{1}{2}(20.0 \text{ m/s})^2\right)}$$
$$= \sqrt{2(-18.1485 \text{ m}^2/\text{s}^2 + 200.0 \text{ m}^2/\text{s}^2)}$$
$$= 19.071 \text{ m/s}$$

ROUND: The initial height is given with the fewest number of significant figures. Since it has three significant figures the value of v needs to be rounded to three significant figures: v = 19.1 m/s.

DOUBLE-CHECK: The final speed should be less than the initial speed since the final height is greater than the initial one.

6.44. **THINK:** The book's mass is m = 1.0 kg. The initial height is $y_0 = 1.0$ m, where $U_g = 0$, the maximum height is H = 3.0 m, and the final height is $y_f = 0$ m. Determine (a) the potential energy of the book when it hits the ground, U_g, and (b) the velocity of the book as it hits the ground, v_f. The book is thrown straight up into the air, so the launch angle is vertical. The sketch is not a plot of the trajectory of the book, but a plot of height versus time.

SKETCH:



RESEARCH:

(a) Gravitational potential energy is given by $U_g = mgh$. To compute the final energy, consider the height relative to the height of zero potential, $y_i = 1.0$ m.

(b) To determine v_f, consider the initial point to be at $y = H$ (where $v = 0$), and the final point to be at the point of impact $y = y_f = 0$. Assume there are only conservative forces, so that $\Delta K = -\Delta U$. ΔU between H and y_f is unaffected by the choice of reference point.

SIMPLIFY:

(a) Relative to $U_g = 0$ at y_i, the potential energy of the book when it hits the ground is given by $U_g = mgh = mg(y_f - y_i)$.

(b) $\Delta K = -\Delta U \Rightarrow K_f - K_i = -(U_f - U_i)$. With $v = 0$ at the initial point, $K_f = U_i - U_f$ and $(1/2)mv^2 = mgH - mgy_f = mgH$. Solving for v_f gives the equation: $v_f = -\sqrt{2gH}$. The negative root is chosen because the book is falling.

CALCULATE:

(a) $U_g = (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0 - 1.0 \text{ m}) = -9.81 \text{ J}$

(b) $v_f = -\sqrt{2(9.81 \text{ m/s}^2)(3.0 \text{ m})} = -7.6720 \text{ m/s}$

ROUND: With two significant figures in m, y_i and H:

(a) $U_g = -9.8$ J

(b) $v_f = -7.7$ m/s, or 7.7 m/s downward.

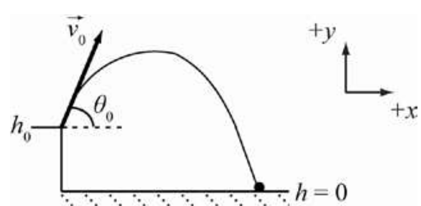
DOUBLE-CHECK: U_g should be negative at y_f, relative to $U_g = 0$ at y_0, because there should be a loss of potential energy. Also, it is sensible for the final velocity of the book to be directed downward.

6.45. **THINK:** The ball's mass is m = 0.0520 kg. The initial speed is $v_0 = 10.0$ m/s. The launch angle is $\theta_0 = 30.0°$. The initial height is $h_0 = 12.0$ m. Determine:

(a) kinetic energy of the ball when it hits the ground, K_f and

(b) the ball's speed when it hits the ground, v.

SKETCH:



RESEARCH: Assuming only conservative forces act on the ball (and neglecting air resistance), $\Delta K = -\Delta U$. K_f can be determined using the equations $\Delta K = -\Delta U$, $K = mv^2/2$ and $U = mgh$. Note that $U_f = 0$, as $h = 0$. With K_f known, v can be determined.

SIMPLIFY:

(a) $\Delta K = -\Delta U \Rightarrow K_f - K_i = U_i - U_f = U_i \Rightarrow K_f = U_i + K_i = mgh_0 + \frac{1}{2}mv_0^2$

(b) $K_f = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2K_f / m}$

CALCULATE:

(a) $K_f = (0.0520 \text{ kg})(9.81 \text{ m/s}^2)(12.0 \text{ m}) + \frac{1}{2}(0.0520 \text{ kg})(10.0 \text{ m/s})^2 = 6.121 \text{ J} + 2.60 \text{ J} = 8.721 \text{ J}$

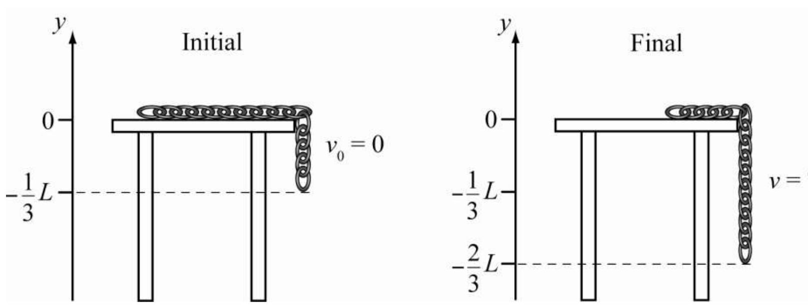
(b) $v = \sqrt{2(8.721 \text{ J})/(0.0520 \text{ kg})} = 18.32 \text{ m/s}$

ROUND: With m having three significant figures, $K_f = 8.72$ J and v = 18.3 m/s.

DOUBLE-CHECK: The amount of kinetic energy computed is a reasonable amount for a ball. The final speed should be greater than the initial speed because the mechanical energy has been completely transformed to kinetic energy. It is, so the calculated value is reasonable.

6.46. **THINK:** The chain's mass is m and has a length of L = 1.00 m. A third of the chain hangs over the edge of the table and held stationary. After the chain is released, determine its speed, v, when two thirds of the chain hangs over the edge.

SKETCH:



RESEARCH: Consider the center of mass (com) location for the part of the chain that hangs over the edge. Since the chain is a rigid body, and it is laid out straight (no slack in the chain), $v_{com} = v$. $\Delta K = -\Delta U$, $K = (mv^2)/2$ and $U = mgh$.

SIMPLIFY: Initially, 1/3 of the chain is hanging over the edge and then $m_{com,0} = m/3$, and $h_{com,i} = -L/6$. When 2/3 of the chain is hanging over the edge, the hanging mass is $m_{com} = 2m/3$. Then, $\Delta K = -\Delta U \Rightarrow K_f - K_i = U_i - U_f$ and $K_i = 0$, so $K_f = U_i - U_f$. Substituting gives $(1/2)mv_{com}^2 = m_{com,i}gh_{com,i} - m_{com}gh_{com}$, so $\frac{1}{2}mv^2 = \left(\frac{m}{3}\right)g\left(-\frac{L}{6}\right) - \left(\frac{2m}{3}\right)g\left(-\frac{L}{3}\right)$, and dividing through by m gives the equation $\frac{1}{2}v^2 = -\frac{1}{18}gL + \frac{2}{9}gL$. Solving for v yields $v = \sqrt{gL/3}$.

CALCULATE: $v = \sqrt{(9.81 \text{ m/s}^2)(1.00 \text{ m})/3} = 1.808 \text{ m/s}$

ROUND: With three significant figures in L, v = 1.81 m/s.

DOUBLE-CHECK: This is a reasonable speed for the chain to achieve while sliding off the table.

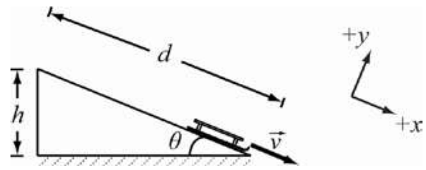
6.47. **THINK:** The initial height is h = 40.0 m. Determine:

(a) the speed v_f at the bottom, neglecting friction,

(b) if the steepness affects the final speed; and

(c) if the steepness affects the final speed when friction is considered.

SKETCH:



RESEARCH:

(a) With conservative forces, $\Delta K = -\Delta U$. v can be determined from $K = \left(m v_{\text{f}}^{2}\right)/2$ and $U = mgh$.

(b and c) Note that the change in the angle θ affects the distance, *d*, traveled by the toboggan: as θ gets larger (the incline steeper), d gets smaller.

(c) The change in thermal energy due to friction is proportional to the distance traveled: $\Delta E_{\text{th}} = \mu_{\text{k}} N d$. The total change in energy of an isolated system is $\Delta E_{\text{tot}} = 0$, where $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{th}}$, and ΔE_{th} denotes the non-conservative energy of the toboggan-hill system (in this case, friction).

SIMPLIFY:

(a) With $K_i = 0$ (assuming $v_0 = 0$) and $U_{\text{f}} = 0$ (taking the bottom to be $h = 0$):

$$K_{\text{f}} = U_{\text{i}} \Rightarrow \frac{1}{2} m v_{\text{f}}^{2} = mgh \Rightarrow v_{\text{f}} = \sqrt{2gh}$$

(b) The steepness does not affect the final speed, in a system with only conservative forces, the distance traveled is not used when conservation of mechanical energy is considered.

(c) With friction considered, then for the toboggan-hill system,

$$\Delta E = \Delta K + \Delta U + \Delta E_{\text{th}} = 0 \Rightarrow \Delta K = -\Delta U - \Delta E_{\text{th}} \Rightarrow K_{\text{f}} = U_{\text{i}} - \Delta E_{\text{th}} = mgh - \mu_{\text{k}} N d$$

The normal force *N* is given by $N = mg\cos\theta$, while on the hill. With $d = h/\sin\theta$,

$$K_{\text{f}} = mgh - \mu_{\text{k}}\left(mg\cos\theta\right)\left(\frac{h}{\sin\theta}\right) = mgh\left(1 - \mu_{\text{k}}\cot\theta\right).$$

The steepness of the hill does affect K_{f} and therefore v at the bottom of the hill.

CALCULATE:

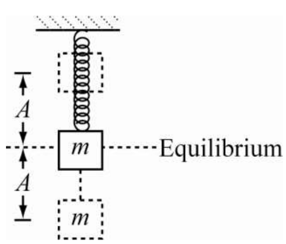
(a) $v_{\text{f}} = \sqrt{2\left(9.81 \text{ m/s}^2\right)\left(40.0 \text{ m}\right)} = 28.01 \text{ m/s}$

ROUND: Since *h* has three significant figures, *v* = 28.0 m/s.

DOUBLE-CHECK: This is a very fast, but not unrealistic speed for the toboggan to achieve.

6.48. **THINK:** The block's mass is m = 0.773 kg, the spring constant is k = 239.5 N/m and the amplitude is A = 0.551 m. The block oscillates vertically. Determine the speed v of the block when it is at x = 0.331 m from equilibrium.

SKETCH:



RESEARCH: The force of gravity in this system displaces the equilibrium position of the hanging block by *mg*/*k*. Since the distance from equilibrium is given, the following equation can be used to determine *v*:

$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2.$

SIMPLIFY: $v=\sqrt{\dfrac{k}{m}\left(A^2-x^2\right)}$

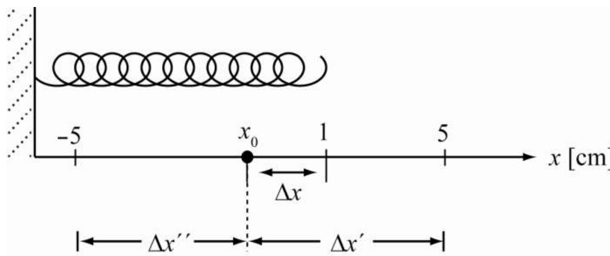
CALCULATE: $v=\sqrt{\dfrac{239.5\text{ N/m}}{0.773\text{ kg}}\left((0.551\text{ m})^2-(0.331\text{ m})^2\right)}=7.7537\text{ m/s}$

ROUND: The least precise value has three significant figures, so round the answer to three significant figures: v = 7.75 m/s.

DOUBLE-CHECK: This is a reasonable speed for the block on the spring.

6.49. **THINK:** It is known that $k=$ 10.0 N/cm and $\Delta x=1.00$ cm. Determine (a) the energy needed to further stretch the spring to $\Delta x'=5.00$ cm and (b) the energy needed to compress the spring from $\Delta x'=5.00$ cm to $\Delta x''=-5.00$ cm.

SKETCH:



RESEARCH: Assume the spring is stationary at all positions given above. The energy required to stretch the spring is the work applied to the spring, W_{a}, and $W_{\text{a}}=-W_{\text{s}}$ for $\Delta k=0$. It is known that $W_{\text{s}}=\left[\left(kx_{\text{i}}^2\right)/2\right]-\left[\left(kx_{\text{f}}^2\right)/2\right]$.

SIMPLIFY: $W_{\text{a}}=-W_{\text{s}}=-\left[\left(kx_{\text{i}}^2\right)/2\right]+\left[\left(kx_{\text{f}}^2\right)/2\right]=k\left(x_{\text{f}}^2-x_{\text{i}}^2\right)/2$

CALCULATE:

(a) $W_{\text{a}}=(10.0\text{ N/cm})\left((5.00\text{ cm})^2-(1.00\text{ cm})^2\right)/2=120.\text{ N cm}=1.20\text{ J}$

(b) $W_{\text{a}}=(10.0\text{ N/cm})\left((5.00\text{ cm})^2-(-5.00\text{ cm})^2\right)/2=0\text{J}$

ROUND: With three significant figures in each given value, (a) $W_{\text{a}}=1.20$ J and (b) $W_{\text{a}}=0$. Take this zero to be precise.

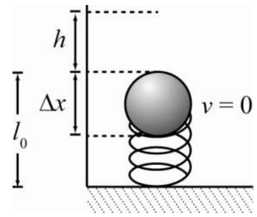
DOUBLE-CHECK:

(a) W_{a} should be positive because energy must be transferred to the spring to stretch it further from equilibrium.

(b) The spring is the same distance from the equilibrium point, so the net energy transferred to the spring must be zero.

6.50. **THINK:** The mass of the ball is m = 5.00 kg. The initial height is h = 3.00 m. The initial speed is $v_0=5.00$ m/s. The spring constant is k = 1600. N/m. The final speed of the ball is zero. Determine (a) the maximum compression Δx of the spring and (b) the total work done on the ball while the spring compresses. The spring is initially at equilibrium, so the height given is the height above the equilibrium point of the spring.

SKETCH:



RESEARCH:

(a) There are no non-conservative forces, so $\Delta K = -\Delta U$, $U_{\text{s}} = \left(kx^2\right)/2$, $U_{\text{g}} = mgh$ and $K = \left(mv^2\right)/2.$

(b) Use the work-kinetic energy theorem to find the net work done on the ball while the spring compresses Δx by $W_{\text{net}} = \Delta K.$

SIMPLIFY:

(a) $\Delta K = -\Delta U$ so $K_{\text{f}} - K_{\text{i}} = U_{\text{si}} - U_{\text{sf}} + U_{\text{gi}} - U_{\text{gf}}$. Note that the equilibrium position of the spring is l_0. Since K_{f} and U_{si} are zero, $0 - K_{\text{i}} = 0 - U_{\text{sf}} + U_{\text{gi}} - U_{\text{gf}}$, and $-\frac{1}{2}mv_0^2 = -\frac{1}{2}k\left(\Delta x\right)^2 + mg\left(l_0 + h\right) - mg\left(l_0 - \Delta x\right)$, which simplifies to $-\frac{1}{2}mv_0^2 = -\frac{1}{2}k\left(\Delta x\right)^2 + mgh + mg\Delta x$ and subsequently $\frac{1}{2}k\left(\Delta x\right)^2 - mg\Delta x - \frac{1}{2}mv_0^2 - mgh = 0$. Solving the quadratic equation gives

$$\Delta x = \frac{mg \pm \sqrt{\left(-mg\right)^2 + 2k\left(\frac{1}{2}mv_0^2 + mgh\right)}}{k} = \frac{mg \pm \sqrt{\left(mg\right)^2 + mk\left(v_0^2 + 2gh\right)}}{k}.$$

(b) $W_{\text{net}} = \Delta K = -\Delta U = -\Delta U_{\text{s}} - \Delta U_{\text{g}}$

$$\begin{aligned} W_{\text{net}} &= U_{\text{si}} - U_{\text{sf}} + U_{\text{gi}} - U_{\text{gf}} \\ &= 0 - \frac{1}{2}k\left(\Delta x\right)^2 + mgl_0 - mg\left(l_0 - \Delta x\right) \\ &= -\frac{1}{2}k\left(\Delta x\right)^2 + mg\Delta x \end{aligned}$$

CALCULATE:

(a) $$\Delta x = \frac{\left(5.00\text{ kg}\right)\left(9.81\text{ m/s}^2\right)}{1600.\text{ N/m}} \pm \frac{\sqrt{\left(5.00\text{ kg}\right)^2\left(9.81\text{ m/s}^2\right)^2 + \left(1600.\text{ N/m}\right)\left(5.00\text{ kg}\right)\left(\left(5.00\text{ m/s}\right)^2 + 2\left(9.81\text{ m/s}^2\right)\left(3.00\text{ m}\right)\right)}}{1600.\text{ N/m}}$$

$$= 0.54349\text{ m},\ -0.48218\text{ m}$$

Since Δx is defined as a positive distance (not a displacement), the solution must be positive. Take $\Delta x = 0.54349$ m.

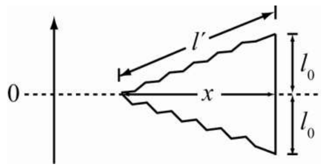
(b) $W_{\text{net}} = -\frac{1}{2}\left(1600.\text{ N/m}\right)\left(0.54349\text{ m}\right)^2 + \left(5.00\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(0.54349\text{ m}\right) = -209.6\text{ J}$

ROUND: Since the least precise value given in the question has three significant figures, both answers will have three significant figures: $\Delta x = 0.543$ m and $W_{\text{net}} = -210.$ J.

DOUBLE-CHECK: Δx should be positive. Relative to the height, *h*, the value of Δx is reasonable. Because the net work is negative, and since $\left|\Delta U_{\text{s}}\right| > \left|\Delta U_{\text{g}}\right|$ for the distance Δx, the clay ball does positive work on the spring and the spring does negative work on the clay ball. This makes sense for spring compression.

6.51. **THINK:** The spring constant for each spring is k = 30.0 N/m. The stone's mass is m = 1.00 kg. The equilibrium length of the springs is $l_0 = 0.500$ m. The displacement to the left is x = 0.700 m. Determine the system's total mechanical energy, E_{mec} and (b) the stone's speed, v, at x = 0.

SKETCH:



Note: The sketch is a side view. The word "vertical" means that the springs are oriented vertically above the ground. The path the stone takes while in the slingshot is completely horizontal so that gravity is neglected.

RESEARCH:

(a) In order to determine E_{mec}, consider all kinetic and potential energies in the system. Since the system is at rest, the only form of mechanical energy is spring potential energy, $U_s = \left(kx^2\right)/2$.

(b) By energy conservation, ΔE_{mec} (no non-conservative forces). v can be determined by considering $\Delta E_{\text{mec}} = 0$.

SIMPLIFY:

(a) $E_{\text{mec}} = K + U = U_s = U_{s1} + U_{s2} = \frac{1}{2}k_1\left(l_0 - l'\right)^2 + \frac{1}{2}k_2\left(l_0 - l'\right)^2 = k\left(l_0 - l'\right)^2$. To determine l', use the Pythagorean theorem, $l' = \sqrt{l_0^2 + x^2}$. Then, $E_{\text{mec}} = k\left(l_0 - \sqrt{l_0^2 + x^2}\right)^2$.

(b) As the mechanical energy is conserved, $E_{\text{mec f}} = E_{\text{mec i}}$ so $K_{\text{f}} + U_{s\text{f}} = E_{\text{mec}}$ (with $U_{\text{f}} = 0$), and therefore $K_{\text{f}} = E_{\text{mec}}$. Solving the equation for kinetic energy, $\frac{1}{2}mv^2 = E_{\text{mec}} \Rightarrow v = \sqrt{2E_{\text{mec}} / m}.$

CALCULATE:

(a) $E_{\text{mec}} = 30.0 \text{ N/m}\left(0.500 \text{ m} - \sqrt{\left(0.500 \text{ m}\right)^2 + \left(0.700 \text{ m}\right)^2}\right)^2 = 3.893 \text{ J}$

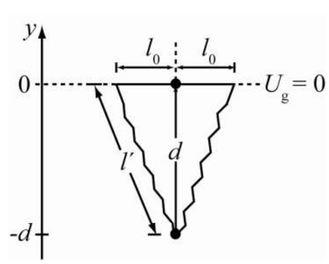
(b) $v = \sqrt{2\left(3.893 \text{ J}\right)/1.00 \text{ kg}} = 2.790 \text{ m/s}$

ROUND: Since all of the given values have three significant figures, the results should be rounded to $E_{\text{mec}} = 3.89$ J and v = 2.79 m/s.

DOUBLE-CHECK: The values are reasonable considering the small spring constant.

6.52. **THINK:** The spring constant for each spring is k = 30.0 N/m. The stone's mass is m = 0.100 kg. The equilibrium length of each spring is $l_0 = 0.500$ m. The initial vertical displacement is d = 0.700 m. Determine (a) the total mechanical energy, E_{mec} and (b) the stone's speed, v, when it passes the equilibrium point.

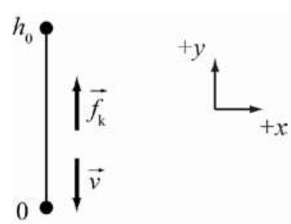
SKETCH:



RESEARCH:
(a) To determine E_{mec}, all forms of kinetic and potential energy must be calculated for the system. Note that initially $K = 0$. Use the equations $U_s = \left(kx^2\right)/2$ and $U_g = mgh$.
(b) As there are no non-conservative forces, E_{mec} is conserved. The speed, v, can be determined from $E_{\text{mec f}} = E_{\text{mec i}}$, using $K = \left(mv^2\right)/2$.
SIMPLIFY:
(a) $E_{\text{mec}} = K + U = U_g + U_{s1} + U_{s2} = mg(-d) + 2\left(\frac{1}{2}k\left(l' - l_0\right)^2\right)$. Note $l' = \sqrt{l_0^2 + d^2}$ from Pythagorean's theorem. Then, $E_{\text{mec}} = k\left(\sqrt{l_0^2 + d^2} - l_0^2\right)^2 - mgd$.
(b) $E_{\text{mec i}} = E_{\text{mec f}} = E_{\text{mec}}$ so $K_f = E_{\text{mec}}$ (as $U_{\text{gf}} = U_{\text{sf}} = 0$) and therefore $\frac{1}{2}mv^2 = E_{\text{mec}} \Rightarrow v = \sqrt{2E_{\text{mec}} / m}$.

CALCULATE:
(a)
$$E_{\text{mec}} = (30.0 \text{ N/m})\left(\sqrt{(0.500 \text{ m})^2 + (0.700 \text{ m})^2} - 0.500 \text{ m}\right)^2 - (0.100 \text{ kg})\left(9.81 \text{ m/s}^2\right)(0.700 \text{ m})$$
$$= 3.893 \text{ J} - 0.6867 \text{ J}$$
$$= 3.206 \text{ J}$$
(b) $v = \sqrt{2(3.206 \text{ J})/(0.100 \text{ kg})} = 8.0075 \text{ m/s}$
ROUND: As each given value has three significant figures, the results should be rounded to $E_{\text{mec}} = 3.21$ J and $v = 8.01$ m/s.
DOUBLE-CHECK: E_{mec} is decreased by the gravitational potential energy. The stone's speed is reasonable considering its small mass.

6.53. **THINK:** The mass of the man is $m = 80.0$ kg. His initial height is $h_0 = 3.00$ m. The applied frictional force is $f_k = 400.$ N. His initial speed is $v_0 = 0$. What is his final speed, v?
SKETCH:

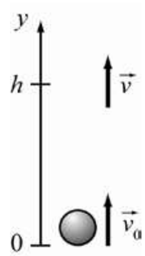


RESEARCH: In an isolated system, the total energy is conserved. $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{th}} = 0$. Using $K = \left(mv^2\right)/2$, $U_g = mgh_0$ and $\Delta E_{\text{th}} = f_k d$, v can be determined.
SIMPLIFY: Note $K_i = U_{\text{gf}} = 0$. Then, $\Delta K + \Delta U + \Delta E_{\text{th}} = 0 \Rightarrow K_f - U_{\text{gi}} + \Delta E_{\text{th}} = 0$. Note that the force of friction acts over the length of the pole, h_0. Then, $\frac{1}{2}mv^2 - mgh_0 + f_k h_0 = 0 \Rightarrow v = \sqrt{2\left(gh_0 - f_k h_0 / m\right)} = \sqrt{2h_0\left(g - f_k / m\right)}$.
CALCULATE:
$$v = \sqrt{2\left(\left(9.81 \text{ m/s}^2\right)(3.00 \text{ m}) - (400. \text{ N})(3.00 \text{ m})/(80.0 \text{ kg})\right)}$$
$$= \sqrt{2\left(29.43 \text{ m}^2/\text{s}^2 - 15.0 \text{ m}^2/\text{s}^2\right)}$$
$$= 5.372 \text{ m/s}$$
ROUND: With three significant figures in each given value, the result should be rounded to $v = 5.37$ m/s.

DOUBLE-CHECK: This velocity is less than it would be if the man had slid without friction, in which case v would be $\sqrt{2gh_0} \approx 8$ m/s.

6.54. **THINK:** The ball's mass is $m = 0.100$ kg. The initial speed is $v_0 = 10.0$ m/s. The final height is $h = 3.00$ m and the final speed is $v = 3.00$ m/s. Determine the fraction of the original energy lost to air friction. Note that the initial height is taken to be zero.

SKETCH:



RESEARCH: For an isolated system, $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{other}} = 0$. The fraction that must be determined is as follows: $\dfrac{\Delta E_{\text{friction}}}{E_{\text{initial}}} = \dfrac{\Delta E_{\text{f}}}{E_{\text{i}}}$.

SIMPLIFY: $\Delta K + \Delta U + \Delta E_{\text{f}} = 0$. Note $U_i = 0$. This means that

$$\Delta E_{\text{f}} = -\Delta K - \Delta U = -\left(\frac{1}{2}m\left(v^2 - v_0^{\,2}\right) + mgh\right) \text{ and } E_i = K_i + U_i = K_i = \frac{1}{2}mv_0^{\,2}.$$

Then, $\dfrac{\Delta E_{\text{f}}}{E_{\text{i}}} = \dfrac{\frac{1}{2}m\left(v_0^{\,2} - v^2 - 2gh\right)}{\frac{1}{2}mv_0^{\,2}} = \dfrac{\left(v_0^{\,2} - v^2 - 2gh\right)}{v_0^{\,2}}.$

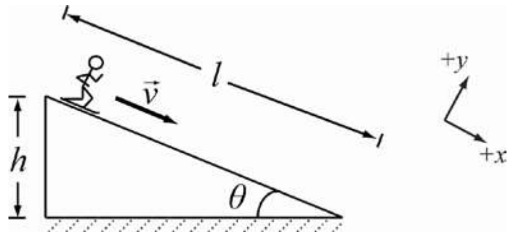
CALCULATE: $\dfrac{\Delta E_{\text{f}}}{E_{\text{i}}} = \dfrac{(10.0 \text{ m/s})^2 - (3.00 \text{ m/s})^2 - 2\left(9.81 \text{ m/s}^2\right)(3.00 \text{ m})}{(10.0 \text{ m/s})^2} = 0.3214$

ROUND: Each given value has three significant figures, so the result should be rounded as $\Delta E_{\text{f}} = 0.321 E_{\text{i}}$. The final answer is 32.1% of E_i is lost to air friction.

DOUBLE-CHECK: If there were no friction and the ball started upward with an initial speed of $v_0 = 10$ m/s, its speed at a height of 3 m would be using kinematics $v = \sqrt{v_0^2 - 2gh} = \sqrt{(10 \text{ m/s})^2 - 2\left(9.81 \text{ m/s}^2\right)(3 \text{ m})} = 6.41$ m/s. This corresponds to a mechanical energy of $E = \frac{1}{2}(0.1 \text{ kg})(6.41 \text{ m/s})^2 + (0.1 \text{ kg})\left(9.81 \text{ m/s}^2\right)(3 \text{ m}) = 5.00$ J. The ball actually had a mechanical energy of $E = \frac{1}{2}(0.1 \text{ kg})(3 \text{ m/s})^2 + (0.1 \text{ kg})\left(9.81 \text{ m/s}^2\right)(3 \text{ m}) = 3.93$ J, which corresponds to a 32.1% loss, which agrees with the result using energy concepts.

6.55. **THINK:** The skier's mass is m = 55.0 kg. The constant speed is v = 14.4 m/s. The slope length is $l = 123.5$ m and the angle of the incline is $\theta = 14.7°$. Determine the mechanical energy lost to friction, ΔE_{th}.

SKETCH:



RESEARCH: The skier and the ski slope form an isolated system. This implies that $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{th}} = 0$. Note that $\Delta K = 0$ since v is constant. Use the equation $U = mgh$, where the height of the ski slope can be found using trigonometry: $h = l\sin\theta$.

SIMPLIFY: At the bottom of the slope, $U_{\text{f}} = 0$. Then,

$$\Delta E_{\text{th}} = -\Delta U = -\left(U_{\text{f}} - U_{\text{i}}\right) = U_{\text{i}} = mgh = mgl\sin\theta.$$

CALCULATE: $\Delta E_{\text{th}} = \left(55.0 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\left(123.5 \text{ m}\right)\sin 14.7° = 16909 \text{ J}$

ROUND: With three significant figures in m, g and θ, the result should be rounded to $\Delta E_{\text{th}} = 16.9$ kJ.

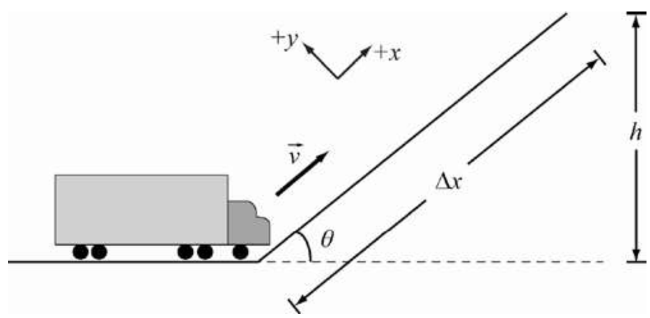
DOUBLE-CHECK: If this energy had been transformed completely to kinetic energy (no friction), and if the skier had started from rest, their final velocity would have been 24.8 m/s at the bottom of the slope. This is a reasonable amount of energy transferred to thermal energy generated by friction.

6.56. **THINK:** The truck's mass is m = 10,212 kg. The initial speed is

$$v_0 = 61.2 \text{ mph}\left(\frac{1609.3 \text{ m}}{1 \text{ mile}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 27.3581 \text{ m/s}.$$

The incline angle is $\theta = 40.15°$ and the coefficient of friction is $\mu_{\text{k}} = 0.634$. Determine the distance traveled along the incline, Δx, before the truck stops (when $v = 0$).

SKETCH:



RESEARCH: The truck and the gravel incline form an isolated system. Use energy conservation to determine Δx. The initial energy is purely kinetic, $K = \left(mv^2\right)/2$. The final energies are thermal, $\Delta E_{\text{th}} = f_{\text{k}}d$ and gravitational potential, $U = mgh$.

SIMPLIFY:

$$\Delta E_{\text{tot}} = 0$$

$$\Delta K + \Delta U + \Delta E_{\text{th}} = 0$$

$$-K_{\text{i}} + U_{\text{f}} + \Delta E_{\text{th}} = 0$$

$$-\frac{1}{2}mv_0^2 + mgh + f_{\text{k}}\Delta x = 0$$

$$-\frac{1}{2}mv_0^2 + mg\Delta x\sin\theta + \mu_{\text{k}}N\Delta x = 0$$

Note that $N = mg\cos\theta$ on the incline. This gives:

$$-\frac{1}{2}mv_0^2 + mg\Delta x\sin\theta + \mu_k mg\cos\theta\Delta x = 0$$

$$\Delta x\left(g\sin\theta + \mu_k g\cos\theta\right) = \frac{1}{2}v_0^2$$

$$\Delta x = \frac{v_0^2}{2g\left(\sin\theta + \mu_k\cos\theta\right)}$$

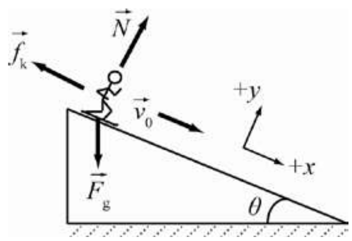
CALCULATE: $\Delta x = \dfrac{\left(27.3581 \text{ m/s}\right)^2}{2\left(9.81 \text{ m/s}^2\right)\left(\sin\left(40.15°\right) + 0.634\cos\left(40.15°\right)\right)} = 33.777 \text{ m}$

ROUND: With three significant figures in v_0, the result should be rounded to $\Delta x = 33.8$ m.

DOUBLE-CHECK: This is a reasonable stopping distance given the incline angle and high coefficient of friction.

6.57. **THINK:** The snowboarder's mass is m = 70.1 kg. The initial speed is $v_0 = 5.10$ m/s. The slope angle is $\theta = 37.1°$. The coefficient of kinetic friction is $\mu_k = 0.116$. Determine the net work, W_{net} done on the snowboarder in the first t = 5.72 s.

SKETCH:



RESEARCH: It is known that $W_{net} = \Delta K$. By considering the forces acting on the skier, and assuming constant acceleration, v_f can be determined at t = 5.72 s. Use $f_k = \mu_k N$, $F_{x\text{ net}} = \sum F_x$ and $v = v_0 + at$.

SIMPLIFY: In the x-direction (along the slope), $F_{x\text{ net}} = F_{gx} - f_k$. Since $N = mg\cos\theta$, the force equation is expanded to

$$ma_{net} = mg\sin\theta - \mu_k mg\cos\theta \Rightarrow a_{net} = g\left(\sin\theta - \mu_k\cos\theta\right).$$

Then, the velocity is given by the formula $v = v_0 + at = v_0 + g\left(\sin\theta - \mu_k\cos\theta\right)t$, and $W_{net} = K_f - K_i = \frac{1}{2}m\left(\left(v_0 + g\left(\sin\theta - \mu_k\cos\theta\right)t\right)^2 - v_0^2\right)$.

CALCULATE:

$$\begin{aligned}
W_{net} &= \frac{1}{2}\left(70.1 \text{ kg}\right)\left(\left(5.10 \text{ m/s} + \left(9.81 \text{ m/s}^2\right)\left(\sin\left(37.1°\right) - 0.116\cos\left(37.1°\right)\right)\left(5.72 \text{ s}\right)\right)^2 - \left(5.10 \text{ m/s}\right)^2\right) \\
&= \frac{1}{2}\left(70.1 \text{ kg}\right)\left(\left(5.10 \text{ m/s} + 28.66 \text{ m/s}\right)^2 - \left(5.10 \text{ m/s}\right)^2\right) \\
&= \frac{1}{2}\left(70.1 \text{ kg}\right)\left(1139.5 \text{ m}^2/\text{s}^2 - 26.01 \text{ m}^2/\text{s}^2\right) \\
&= 39027.5 \text{ J}
\end{aligned}$$

ROUND: Because the m and v_0 have three significant figures, the result should be rounded to $W_{net} = 39.0$ kJ.

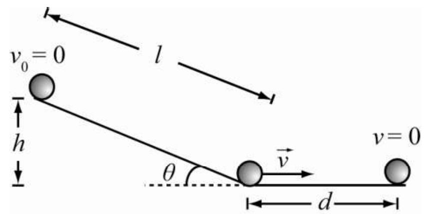
DOUBLE-CHECK: This is a reasonable energy required to change the snowboarder's speed.

6.58. **THINK:** The ball's mass is m = 0.0459 kg. The length of the bar is $l = 30.0 \text{ in}\left(0.0254 \text{ m/in}\right) = 0.762 \text{ m}$. The incline angle is $\theta = 20.0°$. The distance traveled on the green is

$$d = 11.1 \text{ ft}\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 3.38328 \text{ m}.$$

Determine the coefficient of friction between the green and the ball. Assume the bar is frictionless.

SKETCH:



RESEARCH: The ball-bar-green system is isolated, so $\Delta E_{\text{tot}} = 0$. Take the initial point to be when the ball starts to roll down the bar, and the final point where the ball has stopped rolling on the green after traveling a distance, d, on the green. $K_{\text{i}} = K_{\text{f}} = U_{\text{f}} = 0$. Then, $\Delta K + \Delta U + \Delta E_{\text{th}} = 0$, with $U = mgh$ and $\Delta E_{\text{th}} = f_{\text{k}}d$ can be used to determine μ_{k}.

SIMPLIFY: $\Delta K + \Delta U + \Delta E_{\text{th}} = 0 \Rightarrow -U_{\text{i}} + \Delta E_{\text{th}} = 0 \Rightarrow \Delta E_{\text{th}} = U_{\text{i}} \Rightarrow f_{\text{k}}d = mgh \Rightarrow \mu_{\text{k}}mgd = mgl\sin\theta$

$$\Rightarrow \mu_{\text{k}} = \frac{l\sin\theta}{d}$$

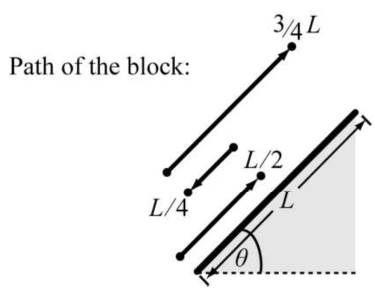
CALCULATE: $\mu_{\text{k}} = \dfrac{\left(0.762 \text{ m}\right)\sin\left(20.0°\right)}{3.38328 \text{ m}} = 0.0770316$

ROUND: With three significant figures in each given value, the result should be rounded to $\mu_{\text{k}} = 0.0770$.

DOUBLE-CHECK: μ_{k} has no units and has a small value, which is reasonable for golf greens.

6.59. **THINK:** The block's mass is m = 1.00 kg. The length of the plank is L = 2.00 m. The incline angle is $\theta = 30.0°$. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.300$. The path taken by the block is $L/2$ upward, $L/4$ downward, then up to the top of the plank. Determine the work, W_b, done by the block against friction.

SKETCH:



RESEARCH: Friction is a non-conservative force. The work done by friction, W_{f}, is therefore dependent on the path. It is known that $W_{\text{f}} = -f_{\text{k}}d$, and with $W_{\text{b}} = -W_{\text{f}}$, the equation is $W_{\text{b}} = f_{\text{k}}d$. The total path of the block is $d = L/2 + L/4 + 3L/4 = 1.5L$.

SIMPLIFY: $W_{\text{b}} = f_{\text{k}}d = \mu_{\text{k}}Nd = \mu_{\text{k}}mg\left(\cos\theta\right)d$ ($N = mg\cos\theta$ on the incline)

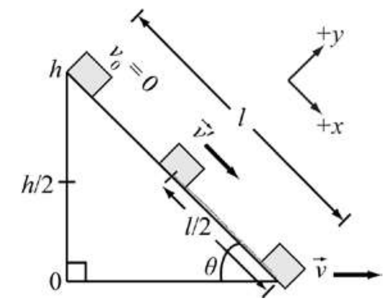
CALCULATE: $W_b = \left(0.300\right)\left(1.00 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\cos\left(30.0°\right)\left(1.50\left(2.00 \text{ m}\right)\right) = 7.646 \text{ J}$

ROUND: Each given value has three significant figures, so the result should be rounded to $W_{\text{b}} = 7.65$ J.

DOUBLE-CHECK: This is a reasonable amount of work done against friction considering the short distance traveled.

6.60. **THINK:** The block's mass is m = 1.00 kg. The initial velocity is $v_0 = 0$ m/s. The incline's length is $l = 4.00$ m. The angle of the incline is $\theta = 45.0°$. The coefficient of friction is $\mu_k = 0.300$ for the lower half of the incline. Determine (a) the block's speed just before the rough section, v', and (b) the block's speed at the bottom, v.

SKETCH:



RESEARCH: Energy is conserved in the block/incline system. Recall $K = (mv^2)/2$, U = mgh and $\Delta E_{th} = f_k d = \mu_k N d$.

(a) With no friction, $\Delta K + \Delta U = 0$.

(b) With friction, $\Delta K + \Delta U + \Delta E_{th} = 0$.

SIMPLIFY:

(a) With $v_0 = 0$ m/s and $K_i = 0$, $K_f - K_i + U_f - U_i = 0$ becomes $K_f = U_i - U_f$.

$$\frac{1}{2}mv'^2 = mgh - mg\left(\frac{h}{2}\right) = \frac{mgh}{2} \Rightarrow \frac{1}{2}v'^2 = \frac{1}{2}gl\sin\theta \Rightarrow v' = \sqrt{gl\sin\theta}$$

(b) Consider the initial point to be halfway down l (when the velocity is v'), and the final point where $U_f = 0$: $\Delta K + \Delta U + \Delta E_{th} = 0 \quad \Rightarrow K_f - K_i + U_f - U_i + \Delta E_{th} = 0 \quad \Rightarrow K_f = K_i + U_i - \Delta E_{th}$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mg\left(\frac{h}{2}\right) - f_k d \quad \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mgl\sin\theta - \mu_k mg\cos\theta\left(\frac{l}{2}\right)$ since $N = mg\cos\theta$ and $f_k = \mu_k N$. So $v^2 = v'^2 + gl\sin\theta - \mu_k lg\cos\theta$. Since $v'^2 = gl\sin\theta$, $v = \sqrt{gl(2\sin\theta - \mu_k\cos\theta)}$.

CALCULATE:

(a) $v' = \sqrt{(9.81 \text{ m/s}^2)(4.00 \text{ m})\sin(45.0°)} = 5.2675$ m/s

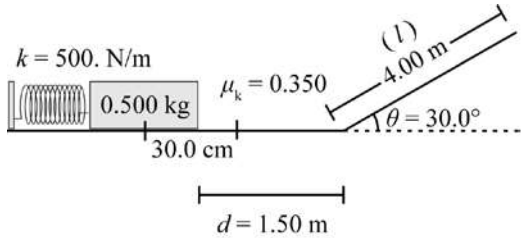
(b) $v = \sqrt{(9.81 \text{ m/s}^2)(4.00 \text{ m})(2\sin(45.0°) - 0.300\cos(45.0°))} = 6.868$ m/s

ROUND: With l having three significant figures, the results should be rounded to $v' = 5.27$ m/s and $v = 6.87$ m/s.

DOUBLE-CHECK: In the complete absence of friction, the speed at the bottom would be $v = \sqrt{2gh} = 7.45$ m/s. The velocity calculated in part (b) is less than this due to the thermal energy dissipated by friction.

6.61. **THINK:** The spring constant is k = 500. N/m. The mass is m = 0.500 kg. The spring compression is $x = 30.0$ cm. The length of the plane is $l = 4.00$ m. The incline angle is $\theta = 30.0°$. The coefficient of kinetic friction is $\mu_k = 0.350$. With the spring compressed, the mass is 1.50 m from the bottom of the inclined plane. Determine (a) the speed of the mass at the bottom of the inclined plane, (b) the speed of the mass at the top of the inclined plane, and (c) the total work done by friction from beginning to end.

SKETCH:



RESEARCH:

(a) The elastic potential energy is $U_{\text{spring}} = \left(kx^2\right)/2.$ The mass loses energy $W_{\text{f}} = -F_{\text{f}}d = -\mu_k mgd$ due to friction. Therefore, the kinetic energy at the bottom is given by $K_{\text{b}} = \frac{1}{2}mv_{\text{b}}^2 = \frac{1}{2}kx^2 - \mu_{\text{k}}mgd.$

(b) To reach the top of the incline, the gravitational potential energy must also be considered: $\Delta U_{\text{gravity}} = U_{\text{top}} - U_{\text{bottom}}.$ Since the plane has length, *l,* and incline angle, θ, $\Delta U_{\text{gravity}} = mgl\sin\theta.$ The kinetic energy at the top (and thus the speed) can then be calculated by subtracting the gravitational potential energy and work due to friction from the kinetic energy at the bottom: $K_{\text{top}} = K_{\text{b}} - \mu_{\text{k}}mgl\cos\theta - mgl\sin\theta.$

(c) The total work due to friction is given by $W_{\text{f}} = -F_{\text{f}}\left(d + l\right).$

SIMPLIFY:

(a) $K_{\text{b}} = \frac{1}{2}mv_{\text{b}}^2 = \frac{1}{2}kx^2 - \mu_{\text{k}}mgd \Rightarrow v_b = \sqrt{\frac{kx^2}{m} - 2\mu_{\text{k}}gd}$

(b) $K_{\text{top}} = \frac{1}{2}mv_{\text{top}}^2 = K_{\text{b}} - \mu_{\text{k}}mgl\cos\theta - mgl\sin\theta \Rightarrow v_{\text{top}} = \sqrt{\frac{2}{m}\left[K_{\text{b}} - mgl\left(\mu_{\text{k}}\cos\theta + \sin\theta\right)\right]}$

(c) $W_{\text{f}} = -\mu_{\text{k}}mgd - \mu_{\text{k}}mg\left(\cos\theta\right)l$

CALCULATE:

(a) $v_{\text{b}} = \sqrt{\frac{\left(500.\text{ N/m}\right)\left(0.300\text{ m}\right)^2}{0.500\text{ kg}} - 2\left(0.350\right)\left(9.81\text{ m/s}^2\right)\left(1.50\text{ m}\right)} = 8.927\text{ m/s}$

$K_{\text{b}} = \frac{1}{2}\left(0.500\text{ kg}\right)\left(8.927\text{ m/s}\right)^2 = 19.92\text{ J}$

(b) $v_{\text{top}} = \sqrt{\frac{2}{0.500\text{ kg}}\left[\left(19.92\text{ J}\right) - \left(0.500\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(4.00\text{ m}\right)\left(0.350\cos 30.0° + \sin 30.0°\right)\right]} = 4.08\text{ m/s}$

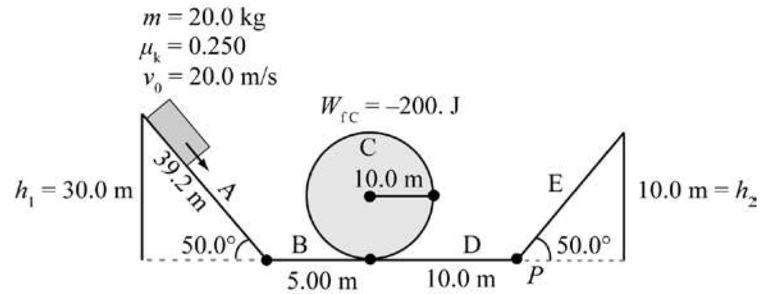
(c) $W_{\text{f}} = -\left(0.350\right)\left(0.500\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(1.50\text{ m} + 4.00\cos\left(30.0°\right)\text{ m}\right) = -8.52\text{ J}$

ROUND: Rounding to three significant figures, $v_{\text{b}} = 8.93\text{ m/s}$, $v_{\text{top}} = 4.08\text{ m/s}$ and $W_{\text{f}} = -8.52\text{ J}.$

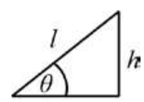
DOUBLE-CHECK: The results are reasonable for the given values.

6.62. **THINK:** Determine the speed of the sled at the end of the track or the maximum height it reaches if it stops before reaching the end. The initial velocity is $v_0 = 20.0$ m/s.

SKETCH:



RESEARCH: The total initial energy is given by $E_0 = K_0 - U_0$. When the sled reaches point *p* at the bottom of the second incline, it has lost energy due to friction given by $W_p = W_A + W_B + W_C + W_D$, where $W_A = A\mu_k mg\cos\theta$, $W_B = B\mu_k mg$, $W_C = 200.$ J, and $W_D = D\mu_k mg$. As the sled reaches point *p*, it has kinetic energy $K_p = E_0 - W_p$. In order for the sled to reach the end of the incline, it needs to have enough energy to cover the work due to friction as well as the gravitational potential energy at the top. Therefore, if $K_p > U_E + W_E$, then it does reach the top and the speed can be determined from the kinetic energy at the top: $K_{top} = K_p - P_E - W_E$. If $K_p < U_E + W_{fE}$, then it stops before reaching the top and the height the sled reaches can be determined by considering the gravitational potential energy equation: $U = mgh = K_p - W_{fE}$, where W_{fE} is the work due to friction for the section of the incline up to *h*. The height can be related to the distance covered on the incline by recalling that $h = l\sin\theta \Rightarrow l = h/\sin\theta$.



Therefore,

$$W_{fE} = \mu_k mg(\cos\theta)l = \mu_k mg\frac{(\cos\theta)h}{\sin\theta} = \mu_k mg(\cot\theta)h.$$

SIMPLIFY: It is convenient to evaluate the following terms separately:
E_0, W_A, W_B, W_C, W_D, U_E, W_E and W_{fE}.

$$E_0 = \frac{1}{2}mv_0^2 + mgh_1,\ U_E = mgh_2,\ W_E = E\mu_k mg\cos\theta = h_2\mu_k mg\cot\theta.$$

CALCULATE: $E_0 = \frac{1}{2}(20.0 \text{ kg})(20.0 \text{ m/s})^2 + (20.0 \text{ kg})(9.81 \text{ m/s}^2)(30.0 \text{ m}) = 9886 \text{ J}$

$W_A = (39.2 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2)\cos(50.0°) = 1236 \text{ J}$

$W_B = (5.00 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2) = 245 \text{ J}$, $\quad W_C = 200.$ J

$W_D = (10.0 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2) = 491 \text{ J}$, $\quad U_E = (20.0 \text{ kg})(9.81 \text{ m/s}^2)(10.0 \text{ m}) = 1962 \text{ J}$

$W_E = (10.0 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2)\cot(50.0°) = 412 \text{ J}$

Therefore, $K_p = (9886 \text{ J}) - (1236 \text{ J}) - (245 \text{ J}) - 200. \text{ J} - (491 \text{ J}) = 7714 \text{ J}$, and
$U_E + W_E = (1962 \text{ J}) + (412 \text{ J}) = 2374 \text{ J}$. Therefore, since $K_p > U_E + W_E$, the sled will reach the top and have speed: $K_{top} = K_p - U_E - W_E \Rightarrow \frac{1}{2}mv_{top}^2 = K_p - U_E - W_E$

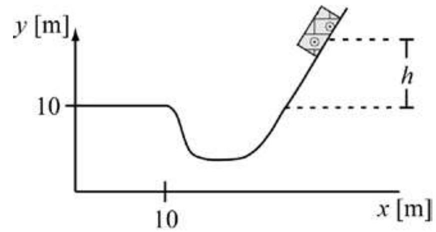
$$\Rightarrow v_{top} = \sqrt{\frac{2}{m}(K_p - U_E - W_E)} = \sqrt{\left(\frac{2}{20.0 \text{ kg}}\right)((7714 \text{ J}) - (2374 \text{ J}))} = 23.11 \text{ m/s}.$$

ROUND: Rounding to three significant figures, $v_{\text{top}} = 23.1\text{ m/s}$.

DOUBLE-CHECK: The fact that the sled reaches the top of the second ramp is reasonable given how much higher the second ramp is than the first. The value of the velocity is of the same order of magnitude as the initial velocity so it is reasonable.

6.63. **THINK:** The mass of the cart is 237.5 kg. The initial velocity is $v_0 = 16.5\text{ m/s}$. The surface is frictionless. Determine the turning point shown on the graph in the question, sketched below.

SKETCH:



RESEARCH: Since the system is conservative, $E_{\text{tot}} = \text{constant} = U_{\text{max}} = K_{\text{max}}$. Therefore, the kinetic energy at x = 0, y = 10. m is the same as the kinetic energy whenever the track is at y = 10. m again. Set y = 10. m as the origin for gravitational potential energy. Therefore,

$$E_{\text{tot}} = K_{\text{max}} = \frac{mv_0^2}{2}.$$

This is the available energy to climb the track from y = 10. m. The turning point is when v = 0 and

$$U_{\text{max}} = K_{\text{max}} \Rightarrow mgh = \frac{mv_0^2}{2}.$$

SIMPLIFY: $h = \dfrac{v_0^2}{2g}$, $y = 10.\text{ m} + h$

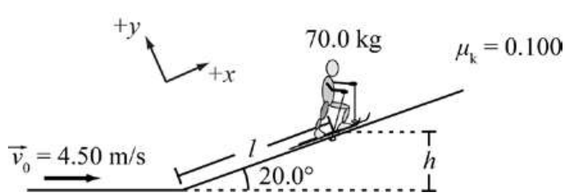
CALCULATE: $h = \dfrac{(16.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 13.9\text{ m}$, $y = 10.\text{ m} + 13.9\text{ m} = 23.9\text{ m}$

ROUND: Reading off the graph is accurate to the nearest integer, so round the value of y to 24 m.
Reading off the graph, the value of x at $y = 24$ m is x = 42 m.

DOUBLE-CHECK: It is reasonable that the cart will climb about 18 m with an initial velocity of $v_0 = 16.5\text{ m/s}$.

6.64. **THINK:** A 70.0 kg skier's initial velocity is $v_0 = 4.50\text{ m/s}$ towards a $20.0°$ incline. Determine (a) the range up the incline if there is no friction and (b) the range up the incline if $\mu_{\text{k}} = 0.100$.

SKETCH:



RESEARCH:

(a) Since the system is conservative, $E_{\text{tot}} = K_{\text{max}} = U_{\text{max}} \Rightarrow \left(mv_0^2\right)/2 = mgh_1 = mgl_1\sin\theta$.

(b) The work due to friction is determined by $W_{\text{f}} = F_{\text{f}}l_2 = \mu_{\text{k}}mgl_2\cos\theta$. Therefore,

$$K_{\text{bottom}} = U_{\text{top}} - W_{\text{f}}.$$

SIMPLIFY:

(a) $\frac{1}{2}mv_0^2 = mgl_1\sin\theta \Rightarrow l_1 = \frac{v_0^2}{2g\sin\theta}$

(b) $\frac{1}{2}mv_0^2 = mgl_2\sin\theta + \mu_k mgl_2\cos\theta \Rightarrow \frac{v_0^2}{2} = l_2 g(\sin\theta + \mu_k\cos\theta) \Rightarrow l_2 = \frac{v_0^2}{2g(\sin\theta + \mu_k\cos\theta)}$

CALCULATE:

(a) $l = \frac{(4.50 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)\sin(20.0°)} = 3.0177 \text{ m}$

(b) $l_2 = \frac{(4.50 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)(\sin(20.0°) + 0.100\cos(20.0°))} = 2.3672 \text{ m}$

ROUND: The final rounded answer should contain 3 significant figures:

(a) $l = 3.02$ m

(b) $l = 2.37$ m

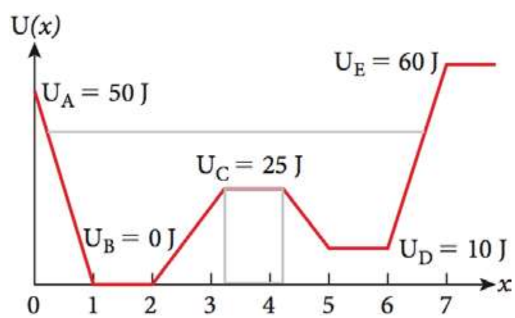
DOUBLE-CHECK: As expected, introducing friction into the system will decrease the available mechanical energy.

6.65. **THINK:** The particle has a total energy of $E_{\text{tot}} = 40.0$ J at its initial position and retains it everywhere. Thus we can draw a horizontal line (gray) for its total energy, approximately at 4/5 of the value of the potential energy at point A (= 50.0 J) for this value of the total energy. The locations of the turning points are here this horizontal line intersects the potential energy curve (red).

Further, we can determine the shape of the potential curve in a more analytical form. From the drawing we can clearly see that it is piecewise linear, falling from 50.0 J at x = 0 to 0 J at x = 1, rising from 0 J at x = 2 to 25.0 J at x = 3.25, falling again from 25.0 J at x = 4.25 to 10.0 J at x = 5, and finally rising from 10.0 J at x = 6 to 60.0 J at x = 7. (We have drawn in a gray rectangle; this way it is easier to see at what x-values the slopes change.)

The turning points are where v = 0, which is where the total energy is equal to the potential energy.

SKETCH:



RESEARCH: Assume a conservative system and $E_{\text{tot}} = K + U$.

(a) Consider the potential energy at the point x = 3 m and call it U_3:

$$E_{\text{tot}} = K + U \Rightarrow K_3 = E_{\text{tot}} - U_3 \text{, and } K_3 = \frac{mv_3^2}{2}.$$

(b) Similarly, $K_{4.5} = E_{\text{tot}} - U_{4.5}$, and $K_{4.5} = (mv_{4.5}^2)/2$.

(c) Since $E_{\text{tot}} = 40.0$ J at x = 4.00 and $U_C = 25.0$ J, then $E_{\text{tot}} - U_C = K_C$. This kinetic energy will become potential energy to reach the turning point.

SIMPLIFY:

(a) $\frac{1}{2}mv_3^2 = E_{\text{tot}} - U_3 \Rightarrow v_3 = \sqrt{\frac{2}{m}(E_{\text{tot}} - U_3)}$. U_3 is obtained from the graph.

(b) $v_{4.5} = \sqrt{\frac{2}{m}\left(E_{\text{tot}} - U_{4.5}\right)}$. $U_{4.5}$ is obtained from the graph.

(c) $E_{\text{tot}} - U_{\text{C}} = K_{\text{C}} = U_4$. Therefore, $U_{\text{turning}} = U_{\text{C}} + U_{\text{t}} = E_{\text{tot}}$.

CALCULATE:

(a) Interpolation between $x = 2$ and $x = 3.25$ yields

$$U(x) = U_{\text{C}}(x-2)/(3.25-2) \Rightarrow U_3 \equiv U(x{=}3) = (25.0 \text{ J})(3-2)/(3.25-2) = 20.0 \text{ J}$$

$$v_3 = \sqrt{\left(\frac{2}{0.200 \text{ kg}}\right)\left(40.0 \text{ J} - 20.0 \text{ J}\right)} = 14.14 \text{ m/s}$$

(b) Interpolation between $x = 4.25$ and $x = 5$ yields

$$U(x) = U_{\text{C}} - (U_{\text{C}} - U_{\text{D}})(x-4.25)/(5-4.25) \Rightarrow U_{4.5} \equiv U(x{=}4.5) = (25.0 \text{ J}) - (15.0 \text{ J})(4.5-4.25)/(0.75) = 20.0 \text{ J}$$

$$v_{4.5} = \sqrt{\left(\frac{2}{0.200 \text{ kg}}\right)\left(40.0 \text{ J} - 20.0 \text{ J}\right)} = 14.14 \text{ m/s}$$

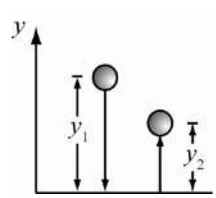
(c) Graphical interpolation between 0 and 1 and between 6 and 7 then results in turning points results in $x_L = 1 - E/U_A = 1 - (40.0 \text{ J})/(50.0 \text{ J}) = 0.2$ for the left turning point, and $x_R = 6 + (E - U_D)/(U_E - U_D) = 6 + (30.0 \text{ J})/(50.0 \text{ J}) = 6.6$ for the right one.

ROUND: Since we are reading data of a graph, we should probably round to two figures and state our results as $v_3 = v_{4.5} = 14$ m/s and $x_L = 0.2$ m and $x_R = 6.6$ m.

DOUBLE-CHECK: Our numerical findings for the turning points agree with our graphical estimation, within the uncertainties stated here.

6.66. **THINK:** The mass of the ball is m = 1.84 kg. The initial height is $y_1 = 1.49$ m and the second height is $y_2 = 0.87$ m. Determine the energy lost in the bounce.

SKETCH:



RESEARCH: Consider the changes in the potential energy from y_1 to y_2. The energy lost in the bounce is given by $U_1 - U_2$.

SIMPLIFY: $E_{\text{lost}} = mgy_1 - mgy_2 = mg\left(y_1 - y_2\right)$

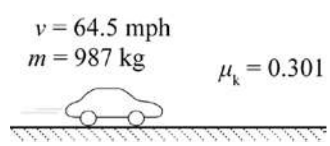
CALCULATE: $E_{\text{lost}} = \left(1.84 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\left(1.49 \text{ m} - 0.87 \text{ m}\right) = 11.2 \text{ J}$

ROUND: Since the least precise value is given to two significant figures, the result is $E_{\text{lost}} = 11$ J.

DOUBLE-CHECK: The ball lost roughly half of its height, so it makes sense that it lost roughly half of its potential energy (which was about 27 J).

6.67. **THINK:** The mass of the car is m = 987 kg. The speed is v = 64.5 mph. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.301$. Determine the mechanical energy lost.

SKETCH:



RESEARCH: Since all of the mechanical energy is considered in the form of kinetic energy, the energy lost is equal to the kinetic energy before applying the brakes. Using the conversion 1 mph is equal to 0.447 m/s, the speed can be converted to SI units. Convert the speed: $v=(64.5\text{ mph})\left(\frac{0.447\text{ m/s}}{1\text{ mph}}\right)=28.8\text{ m/s}$.

SIMPLIFY: $E_{\text{lost}}=\frac{1}{2}mv^2$

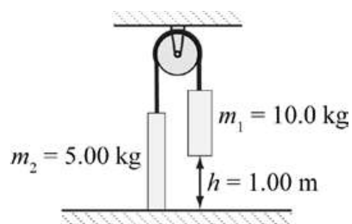
CALCULATE: $E_{\text{lost}}=\frac{1}{2}(987\text{ kg})(28.8\text{ m/s})^2=4.10\cdot 10^5\text{ J}$

ROUND: Rounding to three significant figures, $E_{\text{lost}}=4.10\cdot 10^5$ J.

DOUBLE-CHECK: For an object this massive, it is reasonable that it requires such a large amount of energy to stop it.

6.68. **THINK:** Two masses, $m_1=10.0\text{ kg}$ and $m_2=5.00\text{ kg}$ are attached to a frictionless pulley. The first mass drops h = 1.00 m. Determine (a) the speed of the 5.00 kg mass before the 10.0 kg mass hits the ground and (b) the maximum height of the 5.00 kg mass.

SKETCH:



RESEARCH:

(a) Since energy is conserved, $\Delta K=-\Delta U$. Since the masses are attached to each other, their speeds are the same before one touches the ground.

(b) When m_1 hits the ground, m_2 is at h = 1.00 m with a speed v. The kinetic energy for m_2 is then $(m_2v^2)/2$ and this is given to potential energy for a height above h = 1.00 m. Let h_t be the height where the potential and kinetic energies are equal. When the kinetic energies are equal, $U=K \Rightarrow m_2gh_t=(m_2v^2)/2 \Rightarrow h_t=v^2/2g$. Therefore, the maximum height is $h_{\text{max}}=h+h_t$.

SIMPLIFY:

(a)
$$K_{\text{f}}-K_{\text{i}}=U_{\text{i}}-U_{\text{f}}$$
$$\left(\frac{1}{2}m_1v^2+\frac{1}{2}m_2v^2\right)-0=(m_1gh+m_2gh)-0$$
$$(m_1+m_2)v^2=2gh(m_1-m_2)$$
$$v^2=2gh\left(\frac{m_1-m_2}{m_1+m_2}\right)$$
$$v=\pm\sqrt{2gh\left(\frac{m_1-m_2}{m_1+m_2}\right)}$$

(b) $h_{\text{max}}=h+\frac{v^2}{2g}$

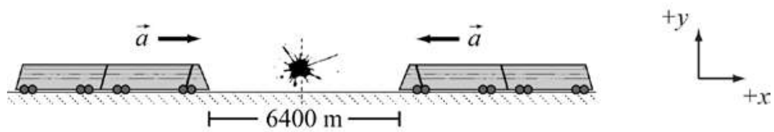
CALCULATE:

(a) $v=\sqrt{2(9.81\text{ m/s}^2)(1.00\text{ m})\left(\frac{10.0\text{ kg}-5.00\text{ kg}}{10.0\text{ kg}+5.00\text{ kg}}\right)}=2.557\text{ m/s}$

(b) $h_{\text{max}} = 1.00 \text{ m} + \dfrac{(2.557 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 1.333 \text{ m}$

ROUND: Rounding to three significant figures, v = 2.56 m/s and $h_{\text{max}} = 1.33$ m.
DOUBLE-CHECK: The calculated values have appropriate units and are of reasonable orders of magnitude for a system of this size.

6.69. **THINK:** The distance that each train covered is $\Delta x = 3200$ m. The weight of each train is $w = 1.2 \cdot 10^6$ N. Their accelerations have a magnitude of $a = 0.26 \text{ m/s}^2$, but are in opposite directions. Determine the total kinetic energy of the two trains just before the collision. The trains start from rest.
SKETCH:



RESEARCH: The total kinetic energy will be twice the kinetic energy for one train. With $K = (mv^2)/2$, m can be determined from $w = mg$ and v from $v^2 = v_0^2 + 2a\Delta x$.
SIMPLIFY: $m = w/g$. Then, $K_{\text{tot}} = 2K = 2((mv^2)/2) = w(2a\Delta x)/g$.

CALCULATE: $K_{\text{tot}} = \dfrac{2(1.2 \cdot 10^6 \text{ N})(0.26 \text{ m/s}^2)(3200 \text{ m})}{(9.81 \text{ m/s}^2)} = 2.035 \cdot 10^8 \text{ J}$

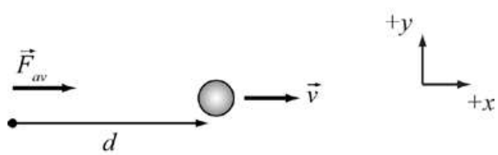
ROUND: With two significant figures in each given value, $K_{\text{tot}} = 2.0 \cdot 10^8$ J.
DOUBLE-CHECK: For such a horrific explosion, a very large kinetic energy is expected before impact.

6.70. **THINK:** The ball's mass is $m = 5.00 \text{ oz}(0.02835 \text{ kg/oz}) = 0.14175$ kg. The final speed is

$$v = 90.0 \, \frac{\text{miles}}{\text{h}} \left(\frac{1609.3 \text{ m}}{1 \text{ mile}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 40.2325 \text{ m/s}.$$

The distance traveled is $d = 2(28.0 \text{ in}) \left(\dfrac{0.0254 \text{ m}}{1 \text{ in}} \right) = 1.4224$ m. Determine the average force, F_{av}.

SKETCH:



RESEARCH: There are no non-conservative forces in the system. So, $\Delta K = -\Delta U$. With F_{av} as a conservative force, the work it does is given by $W_c = -\Delta U$ and $W_c = \vec{F} \bullet \vec{d}$. From this, F_{av} can be determined.
SIMPLIFY: Note $\vec{F}_{\text{av}}$ and $\vec{d}$ are in the same direction, so $W_c = F_{\text{av}} d$ and
$\Delta K = -\Delta U = W_c \Rightarrow K_f - K_i = W_c$. Since $K_i = 0$, $K_f = W_c$.

$$\frac{1}{2} mv^2 = F_{\text{av}} d \Rightarrow F_{\text{av}} = \frac{mv^2}{2d}$$

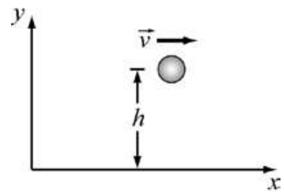
CALCULATE: $F_{\text{av}} = \dfrac{(0.14175 \text{ kg})(40.2325 \text{ m/s})^2}{2(1.4224 \text{ m})} = 80.654 \text{ N}$

ROUND: Since the values are given to three significant figures, $F_{\text{av}} = 80.7$ N.

DOUBLE-CHECK: This average force is equal to holding an object that has a mass of 14.8 kg ($m = F/g = (145\text{ N})/\left(9.81\text{ m/s}^2\right)$), so it is reasonable.

6.71. **THINK:** The mass of the ball is m = 1.50 kg. Its speed is v = 20.0 m/s and its height is h = 15.0 m. Determine the ball's total energy, E_{tot}.

SKETCH:



RESEARCH: Total energy is the sum of the mechanical energy and other forms of energy. As there are no non-conservative forces (neglecting air resistance), the total energy is the total mechanical energy. $E_{\text{tot}} = K + U$. Use $K = \left(mv^2\right)/2$ and $U = mgh$.

SIMPLIFY: $E_{\text{tot}} = m\left(\frac{1}{2}v^2 + gh\right)$

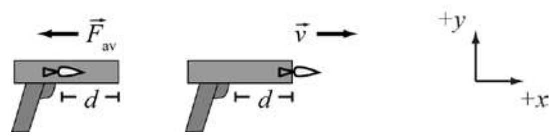
CALCULATE:
$$E_{\text{tot}} = 1.50\text{ kg}\left(\frac{1}{2}(20.0\text{ m/s})^2 + \left(9.81\text{ m/s}^2\right)(15.0\text{ m})\right)$$
$$= 1.50\text{ kg}\left(200\text{ m}^2/\text{s}^2 + 147.15\text{ m}^2/\text{s}^2\right)$$
$$= 520.725\text{ J}$$

ROUND: As the speed has three significant figures, the result should be rounded to $E_{\text{tot}} = 521$ J.

DOUBLE-CHECK: The energy is positive and has the correct unit of measurement. It is also on the right order of magnitude for the given values. This is a reasonable energy for a ball.

6.72. **THINK:** The average force used to load the dart gun is $F_{\text{av}} = 5.5$ N. The dart's mass is $m = 4.5 \cdot 10^{-3}$ kg and the distance the dart is inserted into the gun is d = 0.060 m. Determine the speed of the dart, v, as it exits the gun.

SKETCH:



RESEARCH: Assuming the barrel is frictionless, and neglecting air resistance, the conservation of mechanical energy can be used to determine v. Use $\Delta K = -\Delta U$, $K = \left(mv^2\right)/2$ and $W_{\text{c}} = -\Delta U = \vec{F} \bullet \vec{d}$ (W_c is work done by a conservative force).

SIMPLIFY: Note $\vec{F}$ and $\vec{d}$ are in the same direction so the equation can be reduced to $-\Delta U = W_{\text{c}} = F_{\text{av}}d$.

$$\Delta K = -\Delta U \Rightarrow K_f - K_i = F_{\text{av}}d \;\Rightarrow\; \frac{1}{2}mv^2 = F_{\text{av}}d \;\left(\text{as } v_0 = 0\right) \;\Rightarrow\; v = \sqrt{2F_{\text{av}}d/m}$$

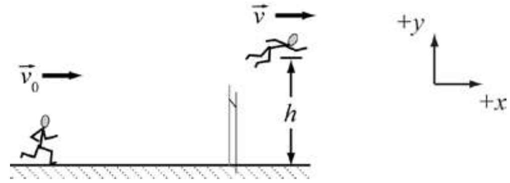
CALCULATE: $v = \sqrt{2(5.5\text{ N})(0.060\text{ m})/\left(4.5 \cdot 10^{-3}\text{ kg}\right)} = 12.111\text{ m/s}$

ROUND: All given values have two significant figures, so the result should be rounded to v = 12 m/s.

DOUBLE-CHECK: This is a reasonable velocity for a dart to exit a dart gun.

6.73. **THINK:** The jumper's initial speed is $v_0 = 9.00$ m/s and his final speed as he goes over the bar is $v = 7.00$ m/s. Determine his highest altitude, h.

SKETCH:



RESEARCH: As there are no non-conservative forces in the system, the conservation of mechanical energy can be used to solve for h as follows, $\Delta K = -\Delta U$.

SIMPLIFY: $K_\text{f} - K_\text{i} = U_\text{i} - U_\text{f} \;\Rightarrow\; \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = -mgh$

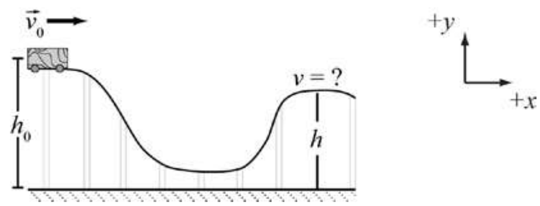
CALCULATE: $h = \dfrac{(9.00 \text{ m/s})^2 - (7.00 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 1.63099 \text{ m}$

ROUND: There are three significant figures in v_0 and v, so the result should be rounded to $h = 1.63$ m.

DOUBLE-CHECK: Since $v < v_0$, it is necessary that $h > h_0$ to conserve mechanical energy.

6.74. **THINK:** The initial speed of the roller coaster is $v_0 = 2.00$ m/s and its initial height is $h_0 = 40.0$ m. Determine the speed, v at the top of the second peak at a height of $h = 15.0$ m.

SKETCH:



RESEARCH: As there are no non-conservative forces in this system, to solve for v, the conservation of mechanical energy can be used: $\Delta K = -\Delta U$, where $K = (mv^2)/2$ and $U = mgh$.

SIMPLIFY:

$$\Delta K = -\Delta U$$

$$K_\text{f} - K_\text{i} = U_\text{i} - U_\text{f}$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgh_0 - mgh$$

$$v = \sqrt{2g(h_0 - h) + v_0^2}$$

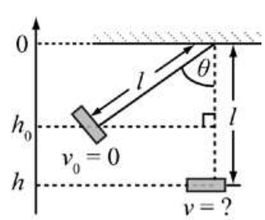
CALCULATE: $v = \sqrt{2(9.81 \text{ m/s}^2)(40.0 \text{ m} - 15.0 \text{ m}) + (2.00 \text{ m/s})^2} = 22.24 \text{ m/s}$

ROUND: As h_0 has three significant figures, the result should be rounded to $v = 22.2$ m/s.

DOUBLE-CHECK: The speed on the lower hill must be greater than the speed on the higher hill.

6.75. **THINK:** The length of the chain is l = 4.00 m and the maximum displacement angle is $\theta = 35°$. Determine the speed of the swing, v, at the bottom of the arc.

SKETCH:



RESEARCH: At the maximum displacement angle, the speed of the swing is zero. Assuming there are no non-conservative forces, to determine the speed, v, the conservation of mechanical energy can be used: $\Delta K = -\Delta U$. Use $K = \left(mv^2\right)/2$ and $U = mgh$. The initial height can be determined using trigonometry. Take the top of the swing to be $h = 0$.

SIMPLIFY: $v_0 = 0$ and $K_\text{i} = 0$. From the sketch, $h_0 = -l\cos\theta$ and $h = -l$. Then,

$$K_\text{f} = U_\text{i} - U_\text{f} \Rightarrow \frac{1}{2}mv^2 = mg\left(-l\cos\theta\right) - mg\left(-l\right) \Rightarrow \frac{1}{2}v^2 = \text{g}\left(l - l\cos\theta\right) \Rightarrow v = \sqrt{2\text{g}\left(l - l\cos\theta\right)}$$

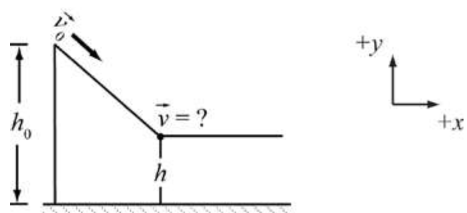
CALCULATE: $v = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(4.00\text{ m} - \left(4.00\text{ m}\right)\cos 35.0°\right)} = 3.767\text{ m/s}$

ROUND: l and θ have two significant figures, so the result should be rounded to $v = 3.77$ m/s.

DOUBLE-CHECK: This is a reasonable speed for a swing to achieve when initially displaced from the vertical by $35°$.

6.76. **THINK:** The initial height of the truck is $h_0 = 680$ m. The initial speed is $v_0 = 15$ m/s and the final height is $h = 550$ m. Determine the maximum final speed, v.

SKETCH:



RESEARCH: The maximum final speed, v, can be determined by neglecting non-conservative forces and using the conservation of mechanical energy, $\Delta K = -\Delta U$. Use $K = \left(mv^2\right)/2$ and $U = mgh$.

SIMPLIFY: $K_\text{f} - K_\text{i} = U_\text{i} - U_\text{f} \;\Rightarrow\; \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgh_0 - mgh \;\Rightarrow\; v = \sqrt{2g\left(h_0 - h\right) + v_0^2}$

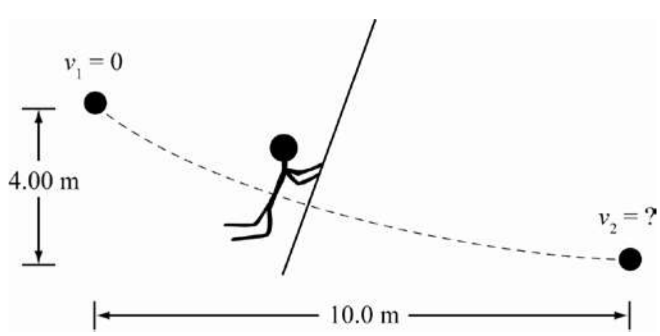
CALCULATE: $v = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(680\text{ m} - 550\text{ m}\right) + \left(15.0\text{ m/s}\right)^2} = 52.68\text{ m/s}$

ROUND: Each initial value has two significant figures, so the result should be rounded to $v = 53$ m/s.

DOUBLE-CHECK: Since the truck is going downhill, its final speed must be greater than its initial speed in the absence of non-conservative forces.

6.77. **THINK:** Determine Tarzan's speed when he reaches a limb on a tree. He starts with a speed of $v_0 = 0$ and reaches a limb on a tree which is 10.0 m away and 4.00 m below his starting point. Consider the change in potential energy as he moves to the final point and relate this to the change in kinetic energy. The velocity can be determined from the kinetic energy.

SKETCH:



RESEARCH: Gravitational potential energy is given by $U = mgh$. The change in potential energy is given by $\Delta U = mgh_2 - mgh_1$. Kinetic energy is given by $K = (mv^2)/2$. The change in kinetic energy is given by $\Delta K = (mv_2^2)/2 - (mv_1^2)/2$.

SIMPLIFY: Assume the system is conservative. The change in potential energy must be equal to the negative of the change in kinetic energy:

$$\Delta U = -\Delta K$$

$$mgh_2 - mgh_1 = -\left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)$$

$$g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2)$$

$$(v_2^2 - v_1^2) = 2g(h_1 - h_2)$$

$$v_2 = \sqrt{2g(h_1 - h_2) + v_1^2}.$$

CALCULATE: $v_2 = \sqrt{2(9.81\text{ m/s}^2)(4.00\text{ m})} = 8.86\text{ m/s}$

ROUND: Since the values are given to three significant figures, the result remains $v_2 = 8.86\text{ m/s}$.

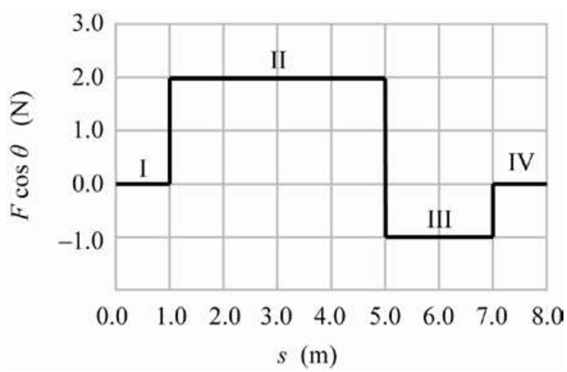
DOUBLE-CHECK: This speed is reasonable for swinging on a vine.

6.78. **THINK:**

(a) Determine the net work done on the block given a varying applied force, $F\cos\theta$. The mass of the block is 2.0 kg.

(b) Given an initial speed of zero at *s* = 0, determine the final speed at the end of the trajectory.

SKETCH:



RESEARCH:

(a) The net work is given by $W_{\text{net}} = \sum_i W_i$ and $W_{\text{i}} = F_{\text{i}} d_{\text{i}}$.

(b) By the work-energy theorem, $W_{\text{net}} = \Delta K$, where $\Delta K = (mv_2^2)/2 - (mv_1^2)/2$.

SIMPLIFY:

(a) $W_{\text{net}} = F_{\text{I}} d_{\text{I}} + F_{\text{II}} d_{\text{II}} + F_{\text{III}} d_{\text{III}} + F_{\text{IV}} d_{\text{IV}}$.

(b) $v_2 = \sqrt{\frac{2}{m}\left(W_{\text{net}} + \frac{1}{2}mv_1^2\right)}$

CALCULATE:

(a) $W_{\text{net}} = (0.0\text{ N})(1.0\text{ m}) + (2.0\text{ N})(4.0\text{ m}) + (-1.0\text{ N})(2.0\text{ m}) + (0.0\text{ N})(1.0\text{ m})$

$= 8.0\text{ N m} - 2.0\text{ N m}$

$= 6.0\text{ N m}$

(b) $v_2 = \sqrt{\left(\dfrac{2}{2.0\text{ kg}}\right)\left(6.0\text{ N m} + \dfrac{1}{2}(2.0\text{ kg})(0.0\text{ m/s})^2\right)}$

$= \sqrt{6.0\ \dfrac{\text{N m}}{\text{kg}}}$

$= \sqrt{6.0\ \dfrac{\text{m}^2}{\text{s}^2}}$

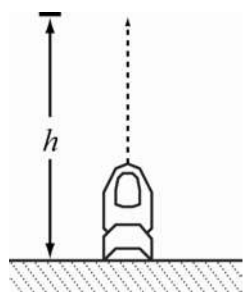
$= 2.4\text{ m/s}$

ROUND: Since all values are given to two significant figures, the results remain $W_{\text{net}} = 6.0\text{ N m}$ and $v_2 = 2.4\text{ m/s}$.

DOUBLE-CHECK: An increase of speed of 2.4 m/s after doing $6.0\text{ N}\cdot\text{m}$ of work is reasonable.

6.79. **THINK:** A rocket that has a mass of $m = 3.00$ kg reaches a height of $1.00\cdot 10^2$ m in the presence of air resistance which takes $8.00\cdot 10^2$ J of energy away from the rocket, so $W_{\text{air}} = -8.00\cdot 10^2$ J. Determine the height the rocket would reach if air resistance could be neglected.

SKETCH:



RESEARCH: Air resistance performs $-8.00\cdot 10^2$ J of work on the rocket. The absence of air resistance would then provide an extra $8.00\cdot 10^2$ J of energy to the system. If this energy is converted into potential energy, the increase in height of the rocket can be determined.

SIMPLIFY: $U_{\text{t}} = -W_{\text{air}} \Rightarrow mgh_{\text{t}} = -W_{\text{air}} \Rightarrow h_{\text{t}} = \dfrac{-W_{\text{air}}}{mg}$, where h_{t} is the added height.

CALCULATE: $h_{\text{t}} = \dfrac{-(-8.00\cdot 10^2\text{ J})}{(3.00\text{ kg})(9.81\text{ m/s}^2)} = 27.183\text{ J/kg}\cdot\text{m/s}^2 = 27.183\text{ m}$

Therefore, the total height reached by the rocket in the absence of air resistance is

$$h_{\text{tot}} = h_0 + h_{\text{t}} = 1.00\cdot 10^2\text{ m} + 0.27183\cdot 10^2\text{ m} = 1.27183\cdot 10^2\text{ m}.$$

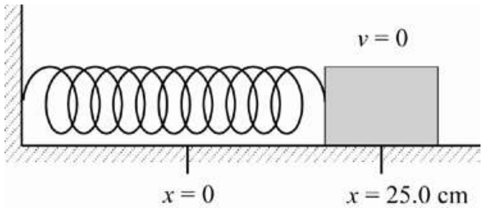
ROUND: Since the values are given to three significant figures, the result should be rounded to $h_{\text{tot}} = 1.27\cdot 10^2$ m.

DOUBLE-CHECK: It is reasonable that air resistance will decrease the total height by approximately a fifth.

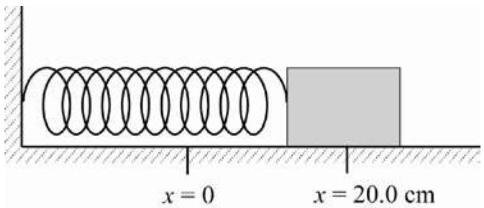
6.80. **THINK:** The mass-spring system is frictionless. The spring constant is k = 100. N/m and the mass is 0.500 kg. For a stretch of 25.0 cm, determine (a) the total mechanical energy of the system, (b) the speed of the mass after it has moved 5.0 cm (at x = 20.0 cm) and (c) the maximum speed of the mass.

SKETCH:

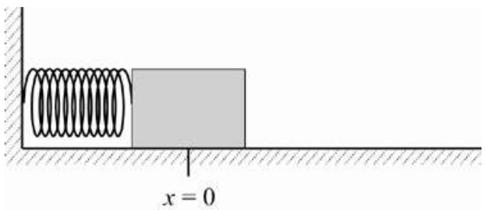
(a)



(b)



(c)



RESEARCH:

(a) The total mechanical energy of the system is given by $E_{\text{tot}} = K + U$. For a conservative system, it is known that $E_{\text{tot}} = \text{constant} = K_{\max} = U_{\max}$. The maximum potential energy can be calculated so the total mechanical energy can be determined:

$$E_{\text{tot}} = U_{\max} = \frac{1}{2}kx_{\max}^2.$$

(b) The speed at any point can be determined by considering the difference in potential energy and relating this to the kinetic energy. Kinetic energy at *x* is given by

$$K(x) = -\Delta U = \frac{kx_{\max}^2}{2} - \frac{kx^2}{2}, \text{ and } K = \frac{mv^2}{2}.$$

(c) Speed, and therefore kinetic energy, is at its maximum when potential energy is zero, i.e., at the equilibrium position $x = 0$. Since $K_{\max} = U_{\max}$, $\left(mv_{\max}^2\right)/2 = \left(kx_{\max}^2\right)/2.$

SIMPLIFY:

(a) $E_{\text{tot}} = \frac{1}{2}kx_{\max}^2$

(b) $K(x) = \frac{1}{2}mv_x^2 = \frac{1}{2}kx_{\max}^2 - \frac{1}{2}kx^2 \;\Rightarrow\; v_x = \sqrt{\frac{k}{m}\left(x_{\max}^2 - x^2\right)}$

(c) $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2 \;\Rightarrow\; v_{\max} = \sqrt{\frac{k}{m}x_{\max}^2} = x_{\max}\sqrt{\frac{k}{m}}$

CALCULATE:

(a) $E_{\text{tot}} = \frac{1}{2}(100.\text{ N/m})\left(2.50 \cdot 10^{-1}\text{m}\right)^2 = 3.125\text{ J}$

(b) $v_x = \sqrt{\left(\frac{100.\text{ N/m}}{0.500\text{ kg}}\right)\left[\left(2.50\cdot 10^{-1}\text{m}\right)^2 - \left(2.00\cdot 10^{-1}\text{m}\right)^2\right]} = \sqrt{4.5\text{ m}^2/\text{s}^2} = 2.1213\text{ m/s}$

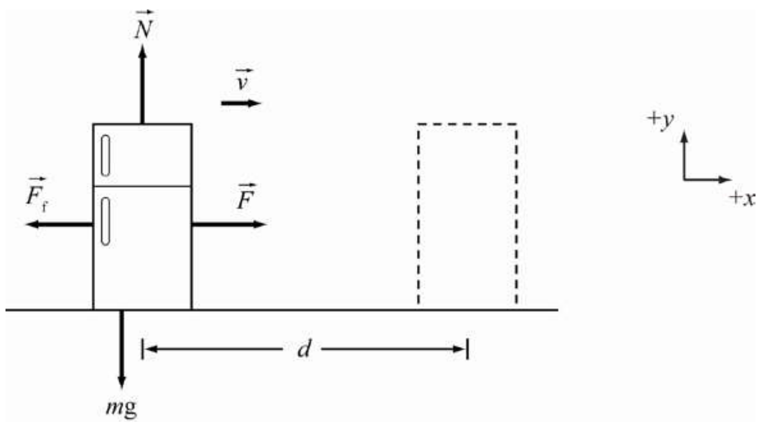
(c) $v_{\text{max}} = \left(2.50\cdot 10^{-1}\text{m}\right)\sqrt{\frac{(100.\text{ N/m})}{0.500\text{ kg}}} = 3.5355\text{ m/s}$

ROUND: The results should be rounded to three significant figures: $E_{\text{tot}} = 3.13\text{ J}$, $v_x = 2.12\text{ m/s}$ and $v_{\text{max}} = 3.54\text{ m/s}$.

DOUBLE-CHECK: A total mechanical energy of 3 J is reasonable for this system, based on the given values. A speed anywhere other than at $x = 0$ must be less than at $x = 0$. In this case, v_x must be less than v_{max}. At $x = 0$ the potential energy is zero. Therefore, all of the energy is kinetic energy, so the velocity is maximized. This value is greater than the value found in part (b), as expected.

6.81. **THINK:** The mass of a refrigerator is m = 81.3 kg. The displacement is d = 6.35 m. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.437$.

SKETCH:



RESEARCH: The force of friction is given by $F_{\text{f}} = \mu_{\text{k}} N$. Use Newton's second law and $W = Fd$. This net mechanical work is the work done by you. The net mechanical work done by the roommate is zero, since he/she lifts the refrigerator up and then puts it back down. Therefore, $\Delta E = 0$.

SIMPLIFY: $\sum F_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg$, $\sum F_x = 0 \Rightarrow F - F_{\text{f}} = 0 \Rightarrow F = \mu_{\text{k}} N \Rightarrow F = \mu_{\text{k}} mg$

The work done is given by $W = Fd = \mu_{\text{k}} mgd$.

CALCULATE: $W = (0.437)(81.3\text{ kg})(9.81\text{ m/s}^2)(6.35\text{ m}) = 2213.17\text{ J}$

ROUND: Rounding to three significant figures, $W = 2.21\text{ kJ}$.

DOUBLE-CHECK: Joules are a usual unit for work. One kilogram is equivalent to about 10 Newtons on Earth, and the fridge weighs about 100 kilograms. The fridge is being moved about 5 meters with a coefficient of friction around a half, so the work should be roughly $0.5\cdot 100\cdot 10\cdot 5 = 2500$ J. The calculated value is reasonable close to this approximation, so the calculated value is reasonable.

6.82. **THINK:** A 1.00 kg block is moving between two springs with constants $k_1 = 100.$ N/m and $k_2 = 50.0$ N/m. If the block is compressed against spring 1 by 20.0 cm, determine

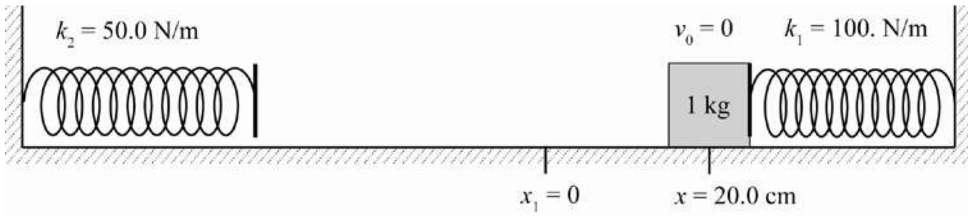
(a) the total energy in the system,

(b) the speed of the block as it moves from one spring to the other and

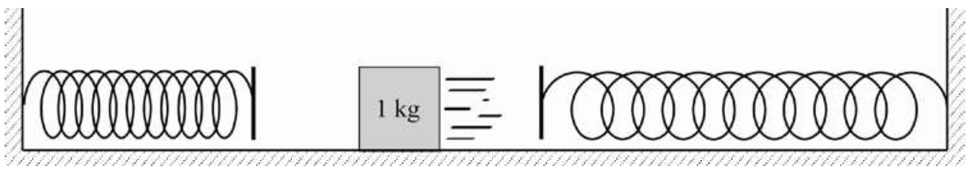
(c) the maximum compression on spring 2.

SKETCH:

(a)



(b)



(c)

RESEARCH:

(a) The total mechanical energy can be determined by recalling that in a conservative system $E_{\text{tot}} = \text{constant} = U_{\max} = K_{\max}$. $U_{\max}$ can be determined from spring 1: $U_{\max} = \frac{1}{2}k_1 x_{\max}^2 = E_{\text{tot}}$.

(b) $K_{\max} = U_{\max} \Rightarrow \left(mv_{\max}^2\right)/2 = \left(k_1 v_{\max,1}^2\right)/2$. Since the system is conservative, the speed of the block is $v_{\max}$ anytime it is not touching a spring.

(c) The compression on spring 2 can be determined by the following relation:

$$U_{\max,2} = K_{\max} \Rightarrow \frac{1}{2}k_2 v_{\max,\,2}^2 = K_{\max}.$$

SIMPLIFY:

(a) $E_{\text{tot}} = \frac{1}{2}k_1 x_{\max,1}^2$

(b) $v_{\max} = \sqrt{\frac{k_1}{m}x_{\max,1}^2} = x_{\max,1}\sqrt{\frac{k_1}{m}}$

(c) $x_{\max,2} = \sqrt{\frac{2K_{\max}}{k_2}}$

CALCULATE:

(a) $E_{\text{tot}} = \frac{1}{2}(100.\text{ N/m})\left(20.0\cdot 10^{-2}\text{ m}\right)^2 = 2.00\text{ J}$

(b) $v_{\max} = \left(20.0\cdot 10^{-2}\text{ m}\right)\sqrt{\frac{(100.\text{ N/m})}{1.00\text{ kg}}} = 2.00\text{ m/s}$

(c) $x_{\max,2} = \sqrt{\frac{2(2.00\text{ J})}{50.0\text{ N/m}}} = 2.83\cdot 10^{-1}\text{ m} = 28.3\text{ cm}$

ROUND: Since the least number of significant figures in the given values is three, so the results should be rounded to $E_{\text{tot}} = 2.00$ J, $v_{\text{max}} = 2.00$ m/s and $x_{\text{max},2} = 28.3$ cm.

DOUBLE-CHECK: It can be seen that $U_{\text{max},1} = U_{\text{max},2} = K_{\text{max}}$

$$U_{\text{max},1} = \frac{1}{2}k_1 x_{\text{max},1}^2 = \frac{1}{2}(100.\text{ N/m})\left(20.0\cdot 10^{-2}\text{ m}\right)^2 = 2.00\text{ J}$$

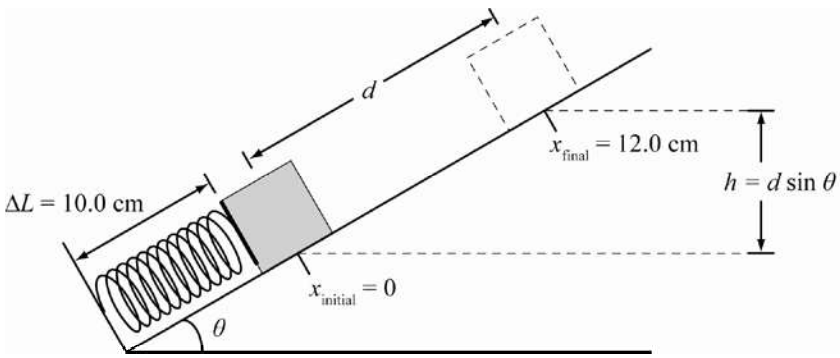
$$U_{\text{max},2} = \frac{1}{2}k_1 x_{\text{max},1}^2 = \frac{1}{2}(50.\text{ N/m})\left(28.3\cdot 10^{-2}\text{ m}\right)^2 = 2.00\text{ J}$$

$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}(1.00\text{ kg})(2.00\text{ m/s})^2 = 2.00\text{ J}$$

and all results are reasonable for the given values.

6.83. **THINK:** A block of mass, m = 1.00 kg is against a spring on an inclined plane of angle, $\theta = 30.0°$. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.100$. The spring is initially compressed 10.0 cm and the block moves to 2.00 cm beyond the springs normal length after release (therefore the block moves d = 12.0 cm after it is released). Determine (a) the change in the total mechanical energy and (b) the spring constant.

SKETCH:



RESEARCH:

(a) Since this is not a conservative system, the change in the total mechanical energy can be related to the energy lost due to friction. This energy can be determined by calculating the work done by the force of friction: $W_{\text{friction}} = F_{\text{friction}}d = \mu_{\text{k}}mg(\cos\theta)d$, and $\Delta E_{\text{tot}} = -W_{\text{friction}} = -\mu_{\text{k}}mg(\cos\theta)d$.

(b) From conservation of energy, the change in total energy, ΔE_{tot} determined in (a), is equal to $\Delta K + \Delta U$. Since $K = 0$ at both the initial and final points it follows that

$$\Delta E_{\text{tot}} = U_{\text{final}} - U_{\text{initial}} = mgd\sin\theta - \frac{1}{2}k\Delta L^2.$$

SIMPLIFY:

(a) $\Delta E_{\text{tot}} = -\mu_k mg(\cos\theta)d$

(b) $k = 2\dfrac{(mgd\sin\theta - \Delta E_{\text{tot}})}{\Delta L^2}$

CALCULATE:

(a) $\Delta E_{\text{tot}} = -(0.100)(1.00\text{ kg})(9.81\text{ m/s}^2)\cos(30.0°)(12.0\cdot 10^{-2}\text{ m}) = -0.1019$ J

(b) $k = -2\dfrac{(1.00\text{ kg})(9.81\text{ m/s}^2)(0.120\text{ m})\sin(30.0°) - (-0.1019\text{ J})}{(0.100\text{ m})^2} = 138.1$ N/m

ROUND:

(a) Since the lowest number of significant figures is three, the result should be rounded to $\Delta E_{\text{tot}} = -1.02\cdot 10^{-1}$ J (lost to friction).

(b) Since the mass is given to three significant figures, the result should be rounded to $k = 138$ N/m.

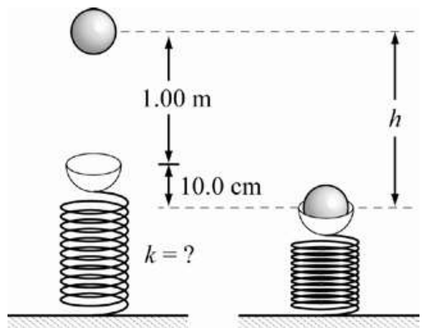
DOUBLE-CHECK:

(a) A change of about 0.1 J given away to friction for a distance of 12 cm and with this particular coefficient of friction is reasonable.

(b) The spring constant is in agreement with the expected values.

6.84. **THINK:** A 0.100 kg ball is dropped from a height of 1.00 m. If the spring compresses 10.0 cm, determine (a) the spring constant and (b) the percent difference between a spring constant calculated by neglecting a change in U_{gravity} while compressing the spring, and the result in part (a).

SKETCH:



RESEARCH:

(a) Determine the spring constant by relating the gravitational potential energy, given to the system, to the elastic potential energy stored by the spring: $U_{\text{gravity}} = U_{\text{spring}} \Rightarrow mgh = (1/2)kx^2$.

(b) If the change in gravitational potential energy is ignored during the compression:

$$mg(h-x) = \frac{1}{2}kx^2.$$

To calculate the percent difference, use $\%\text{ difference} = \frac{|k_1 - k_2|}{(k_1 + k_2)/2}(100\%)$.

SIMPLIFY:

(a) $mgh = \frac{1}{2}k_1x^2 \Rightarrow k_1 = \frac{2mgh}{x^2}$

(b) $mg(h-x) = \frac{1}{2}k_2x^2 \Rightarrow k_2 = \frac{2mg(h-x)}{x^2}$

Therefore,

$$\%\text{ difference} = \frac{\left|\frac{2mgh}{x^2} - \frac{2mg(h-x)}{x^2}\right|}{\left(\frac{2mgh}{x^2} + \frac{2mg(h-x)}{x^2}\right)/2} = \frac{|h-(h-x)|}{(h+h-x)/2} = \frac{2x}{2h-x}.$$

CALCULATE:

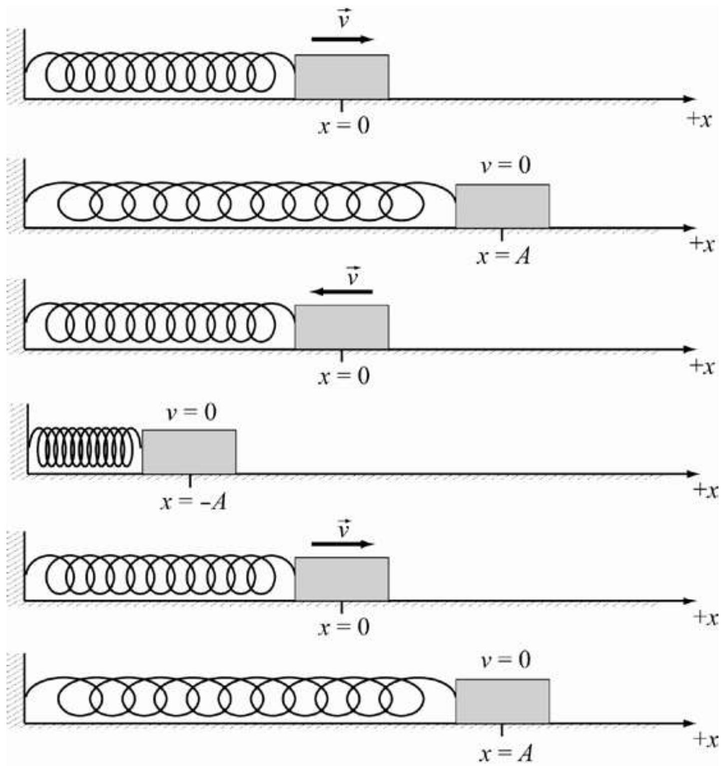
(a) $k_1 = \frac{2(0.100 \text{ kg})(9.81 \text{ m/s}^2)(1.10 \text{ m})}{(0.100 \text{ m})^2} = 215.82$ N/m

(b) $\%\text{ difference} = \frac{2(0.100 \text{ m})}{2(1.10 \text{ m}) - 0.100 \text{ m}} = 9.52\%$

ROUND: Rounding to three significant figures, $k_1 = 216$ N/m and the % difference is 9.52 %.

DOUBLE-CHECK: The percent difference is reasonable.

6.85. **THINK:** The mass is m = 1.00 kg, k = 100. N/m, the amplitude is A = 0.500 m and $x_1 = 0.250$ m. Determine:
(a) the total mechanical energy,
(b) the potential energy for the system and the kinetic energy of the mass at x_1,
(c) the kinetic energy of the mass at x = 0, that is $K_{\max}$,
(d) the change in kinetic energy of the mass if the amplitude is cut in half due to friction, and
(e) the change in potential energy if the amplitude is cut in half due to friction.
SKETCH:



RESEARCH:
(a) Assume a frictionless table and write $E_{tot} = U_{\max} = K_{\max}$ and calculate $U_{\max} = \left(kA^2\right)/2$.
(b) At x_1, the potential energy is $U_{x_1} = \left(\mathrm{k}x_1^{\ 2}\right)/2$ and the kinetic energy will be given by:

$$K_{x_1} = U_{\max} - U_{x_1}.$$

(c) At x = 0, all the energy is in the form of kinetic energy, therefore $K_{x=0} = K_{\max} = U_{\max}$.
(d) Let $K_{\max}$ * denote that the maximum kinetic energy of the mass if there was friction between the mass and the table. At the moment when the amplitude is cut in half, the maximum kinetic energy is obtained by the maximum potential energy:

$$K_{\max} = U_{\max} = \frac{1}{2}kA^2 \Rightarrow K_{\max}{}^* = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right).$$

(e) As described in part (d), $U_{\max}{}^* = \frac{1}{4}U_{\max}$.

SIMPLIFY:
(a) $E_{\text{tot}} = \frac{1}{2}kA^2$
(b) $K_{x_1} = U_{\max} - U_{x_1} = \frac{1}{2}kA^2 - \frac{1}{2}kx_1^{\ 2} = \frac{1}{2}k\left(A^2 - x_1^{\ 2}\right)$

(c) $K_{max} = U_{max} = \frac{1}{2}kA^2$

(d) $K_{max}{}^* = \frac{1}{4}K_{max}$

(e) $U_{max}{}^* = \frac{1}{4}U_{max}$

CALCULATE:

(a) $E_{tot} = \frac{1}{2}(100.\text{ N/m})(0.500\text{ m})^2 = 12.5\text{ J}$

(b) $U_{x_1} = \frac{1}{2}(100.\text{N/m})(0.250\text{m})^2 = 3.125\text{J},\ K_{x_1} = \frac{1}{2}(100.\text{ N/m})\left[(0.500\text{ m})^2 - (0.250\text{m})^2\right] = 9.375\text{ J}$

(c) $K_{max} = E_{tot} = 12.5\text{ J}$

(d) A factor of ¼.

(e) A factor of ¼.

ROUND: Rounding to three significant figures:

(a) $E_{tot} = 12.5\text{ J}$

(b) $U_{x_1} = 3.13\text{J}\ K_{x_1} = 9.38\text{ J}$

(c) $K_{max} = 12.5\text{ J}$

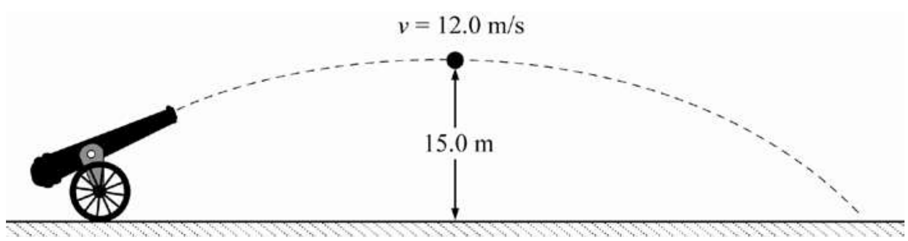
(d) K_{max} changes by a factor of ¼.

(e) U_{max} changes by a factor of ¼.

DOUBLE-CHECK: As expected, the kinetic energy at any point other than $x = 0$ is less than the maximum kinetic energy.

6.86. **THINK:** Bolo has a mass of 80.0 kg and is projected from a 3.50 m long barrel. Determine the average force exerted on him in the barrel in order to reach a speed of 12.0 m/s at the top of the trajectory at 15.0 m above the ground.

SKETCH:



RESEARCH: When Bolo is at the top of the trajectory, his total energy (neglecting air friction) is $E_{tot} = U + K$. This energy can be related to the force exerted by the cannon by means of the work done on Bolo by the cannon: $W = Fd \Rightarrow F = W/d$. Since all the energy was provided by the cannon, $W = E_{tot} \Rightarrow F = E_{tot}/d$.

SIMPLIFY: $F = \frac{E_{tot}}{d} = \frac{U+K}{d} = \frac{mgh + \frac{1}{2}mv^2}{d} = \frac{m}{d}\left(gh + \frac{v^2}{2}\right)$

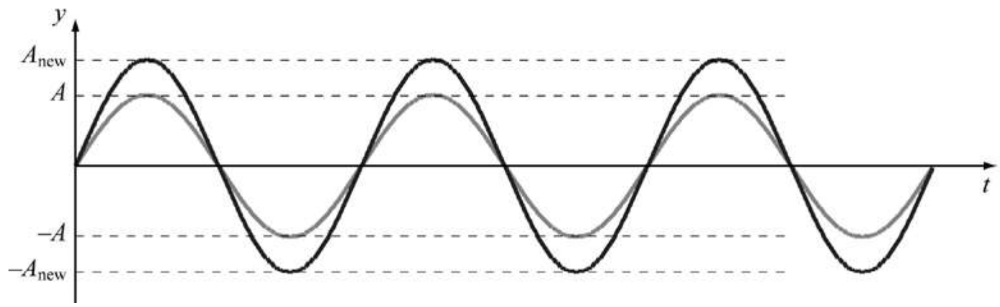
CALCULATE: $F = \left(\frac{80.0\text{ kg}}{3.50\text{ m}}\right)\left((9.81\text{ m/s}^2)(15.0\text{ m}) + \frac{(12.0\text{ m/s})^2}{2}\right) = 5009.14\text{ N}$

ROUND: Since the number of significant figures in the calculation is three, the result rounds to $F = 5010\text{ N}$.

DOUBLE-CHECK: That a force of about 5000 N is required to propel an 80 kg object through such a distance is reasonable.

6.87. **THINK:** The mass hanging vertically from a spring can be treated using a method that is independent of gravitational effects on the mass (see page 185 in the text). The mechanical energy of the mass on a spring is defined in terms of the amplitude of the oscillation and the spring constant. When the mass is pushed, the system gains mechanical energy. This new mechanical energy can be used to calculate the new velocity of the mass at the equilibrium position (b) and the new amplitude (c).

SKETCH: Before the mass is hit, the amplitude of the oscillation is A. After the mass is hit, the amplitude of the oscillation is A_{new}.



RESEARCH: The total mechanical energy before the hit is $E = \frac{1}{2}kA^2$. After the hit, the total mechanical energy is given by $E_{\text{new}} = \frac{1}{2}kA^2 + \frac{1}{2}mv_{\text{push}}^2$ where v_{push} is the speed with which the mass is pushed. The new speed at equilibrium is given by $\frac{1}{2}mv_{\text{new}}^2 = E_{\text{new}}$ and the new amplitude of oscillation is given by $\frac{1}{2}kA_{\text{new}}^2 = E_{\text{new}}$.

SIMPLIFY:

(a) $E_{\text{new}} = \frac{1}{2}kA^2 + \frac{1}{2}mv_{\text{push}}^2$

(b) $v_{\text{new}} = \sqrt{\frac{2E_{\text{new}}}{m}}$

(c) $A_{\text{new}} = \sqrt{\frac{2E_{\text{new}}}{k}}$

CALCULATE:

(a) $E_{\text{new}} = \frac{1}{2}kA^2 + \frac{1}{2}mv_{\text{push}}^2 = \frac{1}{2}(100.\text{ N/m})(0.200\text{ m})^2 + \frac{1}{2}(1.00\text{ kg})(1.00\text{ m/s})^2 = 2.50\text{ J}$

(b) $v_{\text{new}} = \sqrt{\frac{2E_{\text{new}}}{m}} = \sqrt{\frac{2(2.50\text{ J})}{1.00\text{ kg}}} = 2.236\text{ m/s}$

(c) $A_{\text{new}} = \sqrt{\frac{2E_{\text{new}}}{k}} = \sqrt{\frac{2(2.50\text{ J})}{100.\text{ N/m}}} = 0.2236\text{ m}$

ROUND: Rounding to three significant figures: $E_{\text{new}} = 2.50\text{ J}$, $v_{\text{max},2} = 2.24\text{ m/s}$ and $A_2 = 22.4\text{ cm}$.

DOUBLE-CHECK: The mechanical energy before the hit was

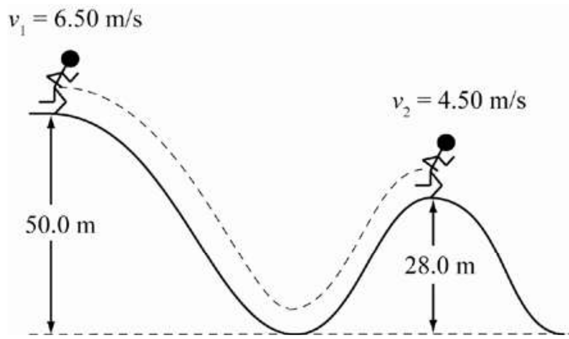
$$E = (1/2)kA^2 = (1/2)(100.\text{ N/m})(0.200\text{ m})^2 = 2.00\text{ J}.$$

The speed of the mass passing the equilibrium point before the hit was $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(2.00\text{ J})}{1.00\text{ kg}}} = 2.00\text{ m/s}$.

It is reasonable that adding 0.5 J to the total energy by means of a hit results in an increase of the speed of the mass at the equilibrium point of 0.24 m/s and an increase of about 2.4 cm to the amplitude.

6.88. **THINK:** Determine the total work done by a runner on a track where the initial speed is $v_1 = 6.50$ m/s at a height of 50.0 m and the final speed is $v_2 = 4.50$ m/s at a different hill with a height of 28.0 m. The runner has a mass of 83.0 kg, there is a constant resistance of 9.00 N and the total distance covered is 400. m.

SKETCH:



RESEARCH: Let the force of resistance be denoted F_r The total work done by the runner can be determined by considering the change in kinetic and potential energy and by considering the work done by the resistance force: $W_1 = \Delta K$, $W_2 = \Delta U$ and $W_3 = F_r d$.

SIMPLIFY: $W_1 = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}m\left(v_1^2 - v_2^2\right)$, $W_2 = mg\left(h_1 - h_2\right)$, and $W_3 = F_r d$. The total energy at point 1: $E_{\text{tot},1} = \frac{1}{2}mv_1^2 + mgh_1$. The total energy at point 2: $E_{\text{tot},2} = \frac{1}{2}mv_2^2 + mgh_2$.

CALCULATE: $E_{\text{tot},1} = \frac{1}{2}(83.0\text{ kg})(6.50\text{ m/s})^2 + (83.0\text{ kg})(9.81\text{ m/s}^2)(50.0\text{ m}) = 4.25\cdot 10^4\text{ J}$

$E_{\text{tot},2} = \frac{1}{2}(83.0\text{ kg})(4.50\text{ m/s})^2 + (83.0\text{ kg})(9.81\text{ m/s}^2)(28.0\text{ m}) = 2.36\cdot 10^4\text{ J}$

Therefore, $\Delta E_{\text{tot}} = 4.25\cdot 10^4\text{ J} - 2.36\cdot 10^4\text{ J} = 1.89\cdot 10^4\text{ J}$.

$W_{\text{friction}} = (9.00\text{ N})(400.\text{ m}) = 3600\text{ J}$.

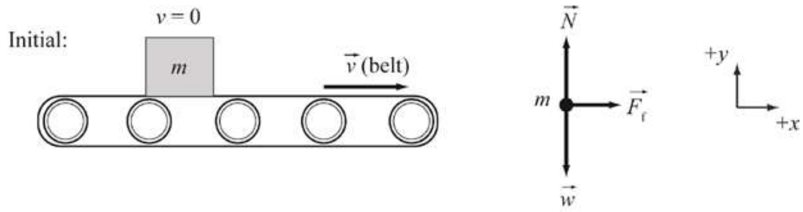
Therefore, $E_{\text{lost}} = \Delta E_{\text{tot}} + W_{\text{friction}} = 1.89\cdot 10^4\text{ J} + 3.60\cdot 10^3\text{ J} = 2.25\cdot 10^4\text{ J}$.

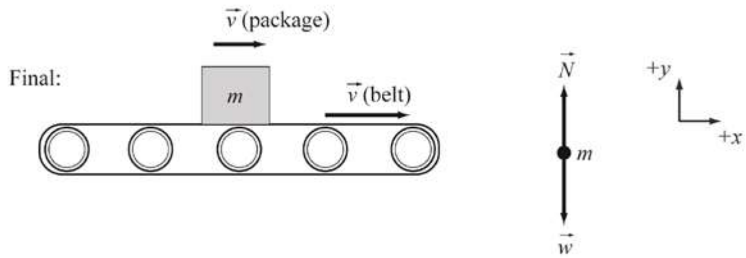
ROUND: Rounding to three significant figures, $E_{\text{lost}} = 2.25\cdot 10^4$ J.

DOUBLE-CHECK: This is a reasonable value for the energy exerted by a runner with the given values.

6.89. **THINK:** Once the package is dropped on the left, the only horizontal force acting on the package is friction. The speed the package is moving relative to the belt is known, so the constant acceleration expressions can be used to determine the time taken for the package to stop sliding on the belt, i.e. the time it takes for the package to stop moving relative to the belt (part (a)). For the remaining problems, the principles of work and conservation of energy can be used to determine the required values. The known quantities are: v (the speed of the belt relative to the package), m (the mass of the package), μ_k (the coefficient of kinetic friction).

SKETCH:





RESEARCH: Work is given by $W = Fd$ ($\vec{F}$ is parallel to $\vec{d}$). Kinetic energy is given by $K = \left(mv^2\right)/2$. The constant acceleration equations are: $v_{\text{f}} = v_{\text{i}} + at$ and $v_{\text{f}}^2 = v_{\text{i}}^2 + 2ax$.

SIMPLIFY:

(a) $v_{\text{f}} = v_{\text{i}} + at$, $v_{\text{i}} = 0 \Rightarrow t = \dfrac{v_{\text{f}}}{a}$, $v_{\text{f}} = v$, $ma = F_{\text{f}} = \mu_{\text{k}} mg \Rightarrow a = \mu_{\text{k}} g$, and $t = \dfrac{v_{\text{f}}}{a} = \dfrac{v}{\mu_{\text{k}} g}$.

(b) $v_{\text{f}}^2 = v_{\text{i}}^2 + 2ax$, $v_{\text{i}} = 0$, $v_{\text{f}} = v$, $a = \mu_{\text{k}} g$, and $x = \dfrac{v_{\text{f}}^2}{2a} = \dfrac{v^2}{2\mu_{\text{k}} g}$.

(c) The energy dissipated is equal to the work done by the belt minus the change in kinetic energy:

$$W - \Delta E = Fd - \left(mv^2\right)/2 = \left(\mu_{\text{k}} mg\right)\left(vt\right) - \left(mv^2\right)/2 = \left(\mu_{\text{k}} mg\right)\left(v^2 / \mu_{\text{k}} g\right) - \left(mv^2\right)/2 = \left(mv^2\right)/2$$

(d) The total work done by the belt is $W = Fd = \left(\mu_{\text{k}} mg\right)\left(vt\right) = \left(\mu_{\text{k}} mg\right)\left(v^2 / \mu_{\text{k}} g\right) = mv^2$

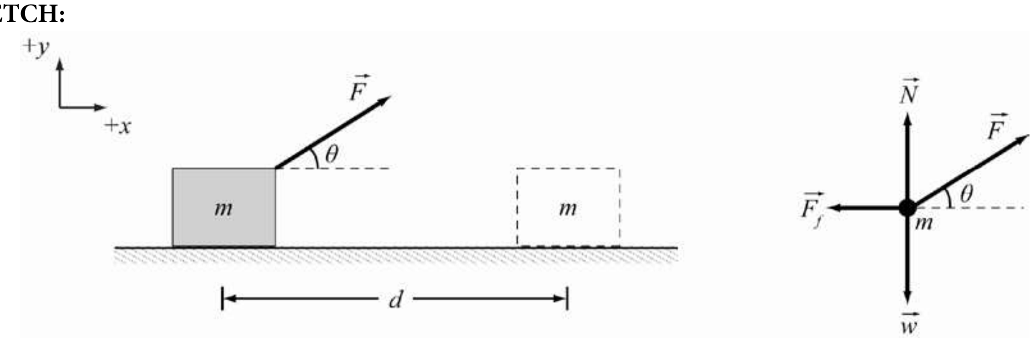
CALCULATE: It is not necessary to calculate any values.

ROUND: This step is not necessary.

DOUBLE-CHECK: Of the work done by the conveyor belt, half has ended up as kinetic energy of the package and the other half has been dissipated as friction heat. This seems reasonable, since the package transitioned steadily from a state ($v_{\text{i}} = 0$) where all the belt work was being dissipated as friction to a state ($v_{\text{f}} = v$) where none of it was.

6.90. **THINK:** There is enough information to determine all the forces. From the forces, the work can be determined. The given values are as follows: m = 85.0 kg, d = 8.00 m, $\theta = 20.0°$, $\left|\vec{F}\right| = 2.40 \cdot 10^2$ N and $\mu_{\text{k}} = 0.200$.

SKETCH:



RESEARCH: $W = \vec{F} \cdot \vec{d} = Fd\cos\theta \Rightarrow W_{\text{tot}} = F_{\text{net}} d\cos\theta$

SIMPLIFY:

(a) $W_{\text{father}} = F_{\text{father}} d\cos\theta$

(b) $W_{\text{friction}} = F_{\text{friction}} d$ (the force is parallel to the displacement), $F_{\text{friction}} = \mu_{\text{k}}(mg - F\sin\theta)$

(c) $W_{\text{total}} = W_{\text{father}} + W_{\text{friction}}$

CALCULATE:

(a) $W_{\text{father}} = \left(2.40 \cdot 10^2 \text{ N}\right)\left(8.00 \text{ m}\right)\cos\left(20.0°\right) = 1.8042 \cdot 10^3 \text{ J}$

(b) $F_{\text{friction}} = \left(0.200\right)\left(\left(85.0 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right) - \left(2.40 \cdot 10^2 \text{ N}\right)\sin\left(20.0°\right)\right) = 150.35 \text{ N}$

$W_{\text{friction}} = \left(1.5035 \cdot 10^2 \text{ N}\right)\left(8.00 \text{ m}\right)\cos\left(180°\right) = -1.2028 \cdot 10^3 \text{ J}$

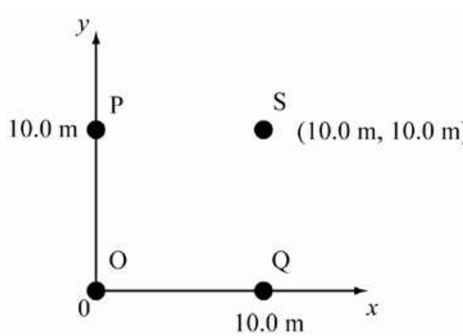
(c) $W_{\text{total}} = \left(1.8042 \cdot 10^3 \text{ J}\right) - \left(1.2028 \cdot 10^3 \text{ J}\right) = 6.014 \cdot 10^2 \text{ J}$

ROUND: The given quantities have three significant figures, so the results should be rounded to $W_{\text{father}} = 1.80 \cdot 10^3 \text{ J}$, $W_{\text{friction}} = -1.20 \cdot 10^3 \text{ J}$ and $W_{\text{total}} = 601 \text{ J}$.

DOUBLE-CHECK: Note also that the total work can be calculated using the net force, $W_{\text{tot}} = F_{\text{net}} d\cos\theta$, which gives the same result.

6.91. **THINK:** The total work can be determined if the path taken and the force applied are known. These are both given as follows: $\vec{F}(x,y) = \left(x^2\hat{x} + y^2\hat{y}\right)$ N and the points are S(10.0 m,10.0 m), P(0 m,10.0 m), Q(10.0 m,0 m) and O(0 m,0 m).

SKETCH:



RESEARCH: Work is given by:

$$W = \int_a^b d\vec{l} \bullet \vec{F} = \int_a^b \left(x^2 dx + y^2 dy\right).$$

The equations of the paths are: along OP, $x = 0$, $dx = 0$; along OQ, $y = 0$, $dy = 0$; along OS, $y = x$, $dy = dx$; along PS, $y = 10$, $dy = 0$; along QS, $x = 10$, $dx = 0$.

SIMPLIFY:

(a) OPS: $W = \int_O^P \left(x^2 dx + y^2 dy\right) + \int_P^S \left(x^2 dx + y^2 dy\right)$

$= \int_0^{10} y^2 dy + \int_0^{10} x^2 dx = \frac{1}{3} y^3\Big|_0^{10} + \frac{1}{3} x^3\Big|_0^{10}$

$= \frac{1}{3}(10)^3 + \frac{1}{3}(10)^3 = \frac{2}{3}(10)^3$

$= W_{\text{OP}} + W_{\text{PS}}$