نموذج إجابة مادة التفاضل والتكامل (باللغة الإنجليزية) لشهادة إتمام الدراسة الثانوية العامة - الدور الثانى - العام الدراسي ٢٠١٧/ ٢٠١٨

النموذج (ب)

1-

(b) $3t$ 1

2-

(d) $]0,\infty[$ 1

3-

$\sin x = xy$ (derivative with respect to x)

$\cos x = x y' + y$ $\frac{1}{2}$ $\Rightarrow y' = \dfrac{\cos x - y}{x}$ $\frac{1}{2}$

$-\sin x = x y'' + y' + y'$ $\frac{1}{2}$

$-\sin x = x y'' + 2y'$

$-xy = x y'' + 2 \times \dfrac{\cos x - y}{x}$ $\frac{1}{2}$

$-x^2 y = x^2 y'' + 2\cos x - 2y$ $\frac{1}{2}$

$x^2 y + x^2 y'' + 2\cos x = 2y$

$x^2 (y + y'') + 2\cos x = 2y$ $\frac{1}{2}$

4-

$x e^{y} = 2 - \ln 2 + \ln x$

$x e^{y} \frac{dy}{dt} + e^{y} \frac{dx}{dt} = \frac{1}{x} \frac{dx}{dt}$ 1

$\because \frac{dx}{dt} = 6 \quad , \quad x = 2 \quad , \quad y = 0$

$\therefore 2 e^{0} \frac{dy}{dt} + e^{0} \times 6 = \frac{1}{2} \times 6$ $\frac{1}{2}$

$\therefore 2 \frac{dy}{dt} = 3 - 6$

$2 \frac{dy}{dt} = -3$

$\frac{dy}{dt} = \frac{-3}{2}$ $\frac{1}{2}$

(تراعى الحلول الأخرى)

5-

(a) $\sqrt{2}$ [1]

6-

(d) 10 [1]

7-

$$Z(n) = 20\left(\frac{n}{12} - \ln\left(\frac{n}{12}\right)\right) + 30$$

$$Z'(n) = 20\left(\frac{1}{12} - \frac{12}{n} \times \frac{1}{12}\right)$$ [1]

$$Z'(n) = 20\left(\frac{n-12}{12n}\right)$$

at $Z'(n) = 0 \Rightarrow n - 12 = 0 \Rightarrow n = 12$ [1/2]

(i) from the discussion of the sign of the function $Z'(n)$ [1/2]

Z'	$- - -$	12	$+ + +$
Z	$\searrow$		$\nearrow$

the number of bacteria be minimum when $n = 12$ days. [1/2]

(ii) The least number of bacteria $= Z(12)$

$$= 20\left(\frac{12}{12} - \ln\left(\frac{12}{12}\right)\right) + 30 = 50 \text{ cm}^3$$ [1/2]

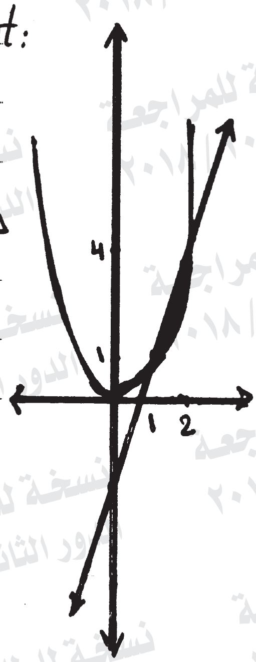
8-

To determine the intersection point:

let $y_2 = y_1 \Rightarrow x^2 = 3x - 2$

$\therefore x^2 - 3x + 2 = 0 \Rightarrow (x-2)(x-1) = 0$

$\therefore x = 1$ or $x = 2$ $\frac{1}{2}$



$\text{Volume} = \pi \int_1^2 \left[(3x-2)^2 - (x^2)^2\right] dx$ $\frac{1}{2}$

$= \pi \int_1^2 \left((3x-2)^2 - x^4\right) dx$ $\frac{1}{2}$

$= \pi \left[\frac{(3x-2)^3}{3 \times 3} - \frac{1}{5} x^5\right]_1^2$ $\frac{1}{2}$

$= \pi \left[\left(\frac{32}{45}\right) - \left(\frac{-4}{45}\right)\right]$

$= \frac{4}{5} \pi$ cubic unit $\frac{1}{2}$

(تراعى الحلول الأخرى)

9-

(C) e 1

10-

(b) Zero 1

11-

ⓐ $\int x(x+2)^6 dx$

let $y = x+2 \Rightarrow x = y-2$ & $dx = dy$ $\frac{1}{2}$

$\therefore \int x(x+2)^6 dx = \int (y-2) y^6 dy$ $\frac{1}{2}$

$= \int (y^7 - 2y^6) dy$

$= \frac{1}{8} y^8 - \frac{2}{7} y^7 + C$ $\frac{1}{2}$

$= \frac{1}{8}(x+2)^8 - \frac{2}{7}(x+2)^7 + C$ $\frac{1}{2}$

ⓑ $\int (x+5) e^x dx$

$= (x+5)e^x - \int e^x dx$ $\frac{1}{2}$

$= (x+5) e^x - e^x + C$ $\frac{1}{2}$

$= e^x (x+4) + C$

where

$\frac{1}{2}$ $u = x+5$ $\xrightarrow{(\times)}$ $dv = e^x dx$

$\therefore du = dx$ $\xleftarrow{(-)}$ $v = e^x$ $\frac{1}{2}$

(تراعى الحلول الأخرى)

12-

(d) $x + \ln|x+1| + c$ $\triangle 1$

13-

(b) $\frac{1}{2}$ $\triangle 1$

14-

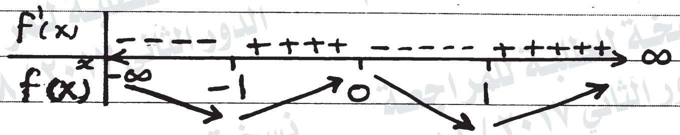
ⓐ $f(x) = x^4 - 2x^2$

$\therefore f'(x) = 4x^3 - 4x = 4x(x^2 - 1)$ $\frac{1}{2}$

at $f'(x) = 0$ $\quad 4x(x-1)(x+1) = 0$

$\therefore x = 0$ or $x = 1$ or $x = -1$ $\frac{1}{2}$

by discussing the sign of the function:



$\frac{1}{2}$

at $x = -1$, the function has a local min. value $f(-1) = -1$ $\frac{1}{2}$

at $x = 0$, the function has a local max. value $f(0) = 0$ $\frac{1}{2}$

at $x = 1$, the function has a local min. value $f(1) = -1$ $\frac{1}{2}$

$\boxed{b}$ $f(x) = \frac{4x}{x^2+1}$

$\therefore f'(x) = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2}$ $\frac{1}{2}$

$f'(x) = \frac{-4x^2+4}{(x^2+1)^2}$ $\frac{1}{2}$

at $f'(x) = 0 \Rightarrow -4x^2 + 4 = 0$ $\frac{1}{2}$

$-4(x-1)(x+1) = 0$ $\quad \therefore x = 1$ or $x = -1$ $\frac{1}{2}$

$f(-1) = -2$, $f(1) = 2$, $f(3) = \frac{6}{5}$

$\therefore$ The absolute max. value $= 2$ $\frac{1}{2}$

The absolute min. value $= -2$ $\frac{1}{2}$

(تراعى الحلول الأخرى)

15-

(d) e^3 1

16-

(b) -3 1

17-

$y = 3 + \sec x$

at $x = \frac{2\pi}{3} \Rightarrow y = 3 + \sec\frac{2\pi}{3} = 1$

$\therefore \left(\frac{2\pi}{3}, 1\right) \in$ the Curve $\frac{1}{2}$

$\because \frac{dy}{dx} = \sec x \tan x$ $\frac{1}{2}$

$\therefore$ the slope at $x = \frac{2\pi}{3}$ $= \sec\frac{2\pi}{3}\tan\frac{2\pi}{3}$ $\frac{1}{2}$

$= (-2)(-\sqrt{3}) = 2\sqrt{3}$ $\frac{1}{2}$

The equation of the tangent:

$y - 1 = 2\sqrt{3}\left(x - \frac{2\pi}{3}\right)$ $\frac{1}{2}$

The equation of the normal:

$y - 1 = \frac{-1}{2\sqrt{3}}\left(x - \frac{2\pi}{3}\right)$ $\frac{1}{2}$

18-

To determine the intersection point,

let $y_1 = y_2 \Rightarrow x = \sqrt{2x}$

$\therefore x^2 = 2x \quad \therefore x^2 - 2x = 0$

$x(x-2) = 0 \quad \therefore x = 0$ or $x = 2$ $\quad \frac{1}{2}$

$\therefore$ The area $= \int_0^2 (\sqrt{2x} - x)\, dx$ $\quad \frac{1}{2}$

$= \left[\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} - \frac{1}{2} x^2\right]_0^2$ $\quad \frac{1}{2}$

$= \left[\left(\frac{2\sqrt{2}}{3}(2)^{\frac{3}{2}} - \frac{1}{2}(2)^2\right) - 0\right] = \frac{2}{3}$ area unit $\quad \frac{1}{2}$

(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)