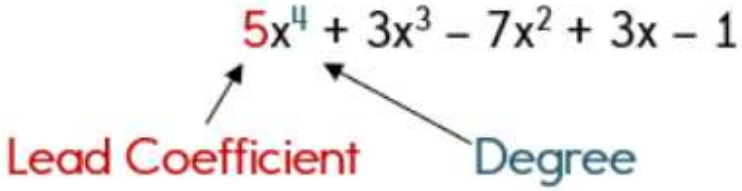
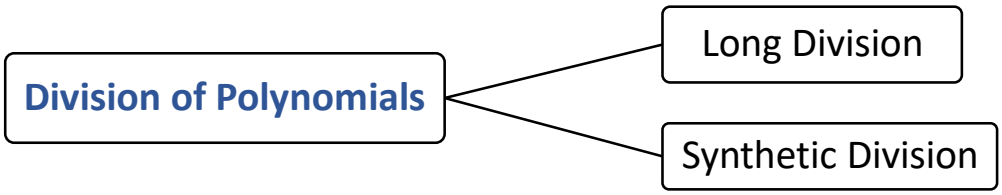
3. Polynomials & Polynomial Functions

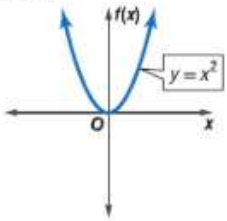
Degree – Highest power / exponent of the variable
Leading Coefficient – Number with the variable of the highest power





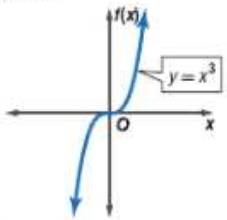
KeyConcept End Behavior of a Polynomial Function

Degree: even
Leading Coefficient: positive
End Behavior:
$f(x) \to +\infty$ as $x \to -\infty$
$f(x) \to +\infty$ as $x \to +\infty$



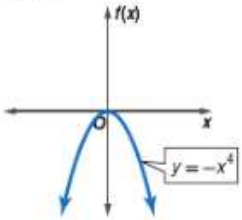
Domain: all real numbers
Range: all real numbers ≥ minimum

Degree: odd
Leading Coefficient: positive
End Behavior:
$f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to +\infty$ as $x \to +\infty$



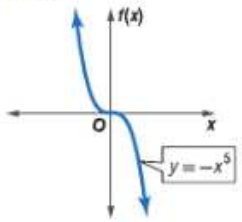
Domain: all real numbers
Range: all real numbers

Degree: even
Leading Coefficient: negative
End Behavior:
$f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to +\infty$



Domain: all real numbers
Range: all real numbers ≤ maximum

Degree: odd
Leading Coefficient: negative
End Behavior:
$f(x) \to +\infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to +\infty$



Domain: all real numbers
Range: all real numbers

Analyzing Graphs

$f(x) = x^3 - 4x^2 - 2x + 3$
Step 1 : Make a table of values
Step 2: Graph the function
Step 3 : sign change in the y coordinates indicate zero (value of zero will be x coordinates)
Step 4: Locate the **Relative maximum and minimum**

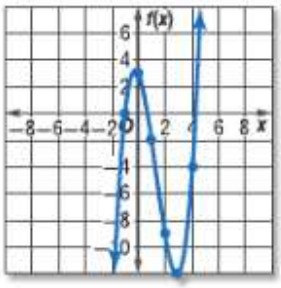
x	$f(x)$
−2	−17
−1	0
0	3
1	−2
2	−9
3	−12
4	−4
5	18

zero

indicates a relative maximum

indicates a relative minimum

zero between 4 and 5



Solving polynomial Equations

ConceptSummary Factoring Techniques

Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$4a^3b^2 - 8ab = 4ab(a^2b - 2)$
two	Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
	Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
	Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$
		$a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials	$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)$

The Remainder Theorem

If the polynomial $f(x)$ is divided by $x - k$, then the remainder is $f(k)$.

The Factor Theorem

Let $f(x)$ be a polynomial.
If $f(k) = 0$, then $x - k$ is a factor of $f(x)$.
If $x - k$ is a factor of $f(x)$, then $f(k) = 0$.

A polynomial equation of degree n has exactly n roots in the set of complex numbers, including repeated roots.

$x^3 + 2x^2 + 6$	$4x^4 - 3x^3 + 5x - 6$	$-2x^5 - 3x^2 + 8$
3 roots	4 roots	5 roots

Total Number of Zeros = Positive Real Zeros + Negative Real Zeros + Imaginary Zeros

Possible Rational Roots

Ex: $6x^3 + 8x^2 - 7x - 3 = 0$

q $\qquad$ p

Factors of p: $\pm 1, \pm 3$

Factors of q: $\pm 1, \pm 2, \pm 3, \pm 6$

Possible zeros: $\pm \frac{p}{q}$

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 3, \pm \frac{3}{2}$ (12 possible zeros)

3.1 Operations with Polynomials

Simplify

i. $3t\,(tn - 5)$

ii. $3b(2b - 1) + 2b(b + 3)$

iii. $(x^4)^3$

iv. $\frac{14x^4y}{2x^3y^2}$

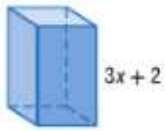
3.2 Dividing Polynomials

Simplify

i. $(6y^3 + 13y^2 - 10y - 24) \div (y + 2)$

ii. $(a^4 + 5a^3 + 2a^2 - 6a + 4)\,(a + 2)^{-1}$

iii. The volume of the rectangular prism is $3x^3 + 11x^2 - 114x - 80$ cubic units. What is the area of the base?



3.3 Polynomial Functions

State the degree and the leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

$5x^6 - 3x^4 + x^3 - 9x^2 + 1$

$12x^3 - 5x^4 = 6x^8 - 3x - 3$

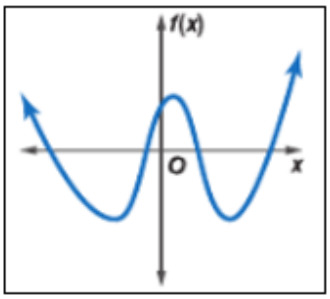
$6xy^2 - xy + y^2$

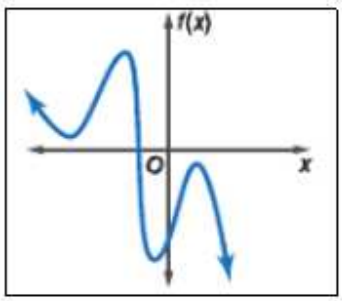
$-6x^6 - 4x^5 + 13xy$

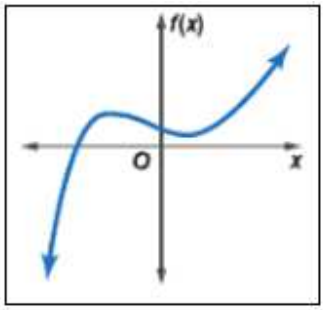
Find p(-2) and p(x+h) for each function.

$p(x) = x^2 + 2x - 3$	$p(x) = 3x^2 - x$

For each graph, a. Describe the end behavior
b. Determine whether it represents an odd degree or an even degree function
c. State the number of real zeros.





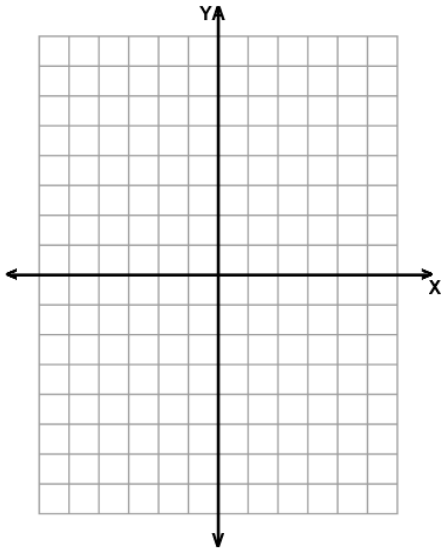


3.4 Analyzing Graphs

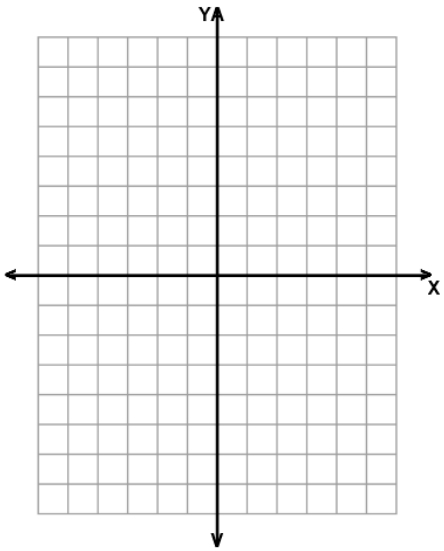
a. Graph each function by making a table of values.
b. Determine the consecutive integer values of x between which each real zero is located.
c. Estimate the x-coordinates at which the relative maxima and minima occur

$f(x) = x^3 - 4x^2 - 7x + 10$

$f(x) = 4x^4 - 21x^2 + 5$



$f(x) = 4x^4 - 21x^2 + 5$



3.5 Solving Polynomial Functions

Factor completely. If the polynomial is not factorable, write prime

$a8 – a^2b6$	$18x6 + 5y6$
$12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$	$a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x – 64b^3$

Solve each equation

$x^3 + 2x^2 – 35x = 0$ Hint: Take x common $x (x^2 + 2x – 35)$ Factorize the equation in bracket	$8x^4 – 10x^2 + 3 = 0$ $8x^4 – 6\_\_ – 4\_\_ + 3 = 0$ $2x^2(4\_\_ – 3) – 1(4\_\_ – 3) = 0$ $(\_\_\_ – 1)\,)(4x^2 – 3) = 0$ $\_\_\_ – 1 = 0$ or $4\_\_ – 3 = 0$ $x^2 =$ ____ or $x^2 =$ ____ $x = \pm$_____ or $x = \pm$_____

3.6 Remainder and Factor Theorem

Use synthetic substitution to find f (-2) and f (4) for each function.

$f(x) = x^2 - 3$	$f(x) = x^2 - 5x + 4$

Given a polynomial and one of its factors, find the remaining factors of the polynomial	
$3x^3 + 20x^2 + 23x - 10$; $(x + 5)$	$x^3 + 2x^2 - 23x - 60$; $(x - 5)$
3.7 Roots and Zeros	
State the possible number of positive real zeros, negative real zeros and imaginary real zeros of each function	
$f(x) = -2x^3 + 11x^2 - 3x + 2$	$f(x) = -4x^4 - 2x^3 - 12x^2 - x - 23$
3.8 Rational Zero Theorem	
Find all the zeros of each function	
$f(x) = x^3 + 4x^2 + 3x - 2$	$f(x) = 4x^3 + 4x^2 - x - 1$