1-

(b) 15 1

2-

(a) 10 1

3-

(b) 2 1

4-

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \quad \begin{matrix} C_2 - C_1 \\ C_3 - C_1 \end{matrix} \Rightarrow$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} \quad \boxed{1}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & (b-a) & (c-a) \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix} \quad \frac{1}{2}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix} \quad \frac{1}{2} \quad C_3 - C_2 \Rightarrow$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^2 & b+a & c-b \end{vmatrix} \quad \frac{1}{2}$$

$$= (b-a)(c-a)(c-b) \quad \frac{1}{2}$$

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5-

(d) π 1

6-

(c) $\tan\theta$ 1

7-

Let the term Contains x^4 be T_{r+1}

$T_{r+1} = {}^{12}C_r \left(\frac{-1}{x^2}\right)^r (x^2)^{12-r}$ $\frac{1}{2}$

$= {}^{12}C_r (-1)^r x^{24-4r}$ $\frac{1}{2}$

let $24 - 4r = 4 \Rightarrow r = 5$

$\therefore$ The term Contains x^4 is T_6 $\frac{1}{2}$

$T_6 = {}^{12}C_5 (-1)^5 (x^4) = -792x^4$

- The order of the middle term : $\frac{12}{2} + 1 = 7$

$T_7 = {}^{12}C_6 \times \left(\frac{-1}{x^2}\right)^6 \times (x^2)^6$ $\frac{1}{2}$ $= {}^{12}C_6 = 924$ $\frac{1}{2}$

$\therefore \frac{\text{The coeff. of } T_6}{\text{The coeff. of } T_7} = \frac{-792}{924} = \frac{-6}{7}$ $\frac{1}{2}$

or

$\frac{\text{The coeff. of } T_6}{\text{The coeff. of } T_7} = \frac{r}{n-r+1} \times \frac{\text{coeff of } 1^{st}}{\text{coeff of } 2^{nd}}$

$= \frac{6}{12-6+1} \times \frac{1}{-1}$ $\frac{1}{2}$

$= \frac{-6}{7}$ $\frac{1}{2}$

8-

The equation of the plane:

• $\overrightarrow{n} \cdot \overrightarrow{r} = \overrightarrow{n} \cdot \overrightarrow{A}$ $\frac{1}{2}$

$(4, 10, -7) \cdot \overrightarrow{r} = (4, 10, -7) \cdot (2, -1, 0)$

$(4, 10, -7) \cdot \overrightarrow{r} = -2$ $\frac{1}{2}$ The vector form

• $4(x-2) + 10(y+1) - 7z = 0$ $\frac{1}{2}$

The standard form

• $4x + 10y - 7z + 2 = 0$ $\frac{1}{2}$

The general form

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9-

(b) (1,1) $\triangle 1$

10-

$\because$ The st. line makes equal angles with the positive directions of the coordinated axes

$\therefore \cos\theta_x = \cos\theta_y = \cos\theta_z$

$\because \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\therefore \cos^2\theta_x = \cos^2\theta_y = \cos^2\theta_z = \frac{1}{3}$

$\therefore \cos\theta_x = \cos\theta_y = \cos\theta_z = \pm\frac{1}{\sqrt{3}}$ $\triangle \frac{1}{2}$

$\therefore$ The direction vector of the st. line $= \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
or $= (1,1,1)$

$\therefore \vec{r} = (3,2,-1) + t\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
or $\therefore \vec{r} = (3,2,-1) + t\,(1,1,1)$ $\triangle \frac{1}{2}$ [Vector form]

- $x = 3+t$, $y = 2+t$, $z = -1+t$
or $x = 3+\frac{1}{\sqrt{3}}t$, $y = 2+\frac{1}{\sqrt{3}}t$, $z = -1+\frac{1}{\sqrt{3}}t$ $\triangle \frac{1}{2}$ [Parametric form]

- $x-3 = y-2 = z+1$
or $\dfrac{x-3}{\left(\frac{1}{\sqrt{3}}\right)} = \dfrac{y-2}{\left(\frac{1}{\sqrt{3}}\right)} = \dfrac{z+1}{\left(\frac{1}{\sqrt{3}}\right)}$ $\triangle \frac{1}{2}$ [Cartesin form]

11-

$$\begin{pmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$$

$$AX = B$$

$$|A| = \begin{vmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{vmatrix} = 3(-5) + 2(-11) = -37 \neq 0 \qquad \frac{1}{2}$$

$$\text{Adj}(A) = \begin{pmatrix} -1 & 5 & -11 \\ -7 & -2 & -3 \\ -2 & 10 & 15 \end{pmatrix}^t \ \frac{1}{2} = \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix} \qquad \frac{1}{2}$$

$$A^{-1} = \frac{1}{|A|}\,\text{Adj}(A)$$

$$\therefore A^{-1} = \frac{1}{-37}\begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix} \qquad \frac{1}{2}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{37}\begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}\begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$$

$$\frac{1}{2}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{37}\begin{pmatrix} -37 \\ 37 \\ -74 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \qquad \frac{1}{2}$$

$$\therefore x = 1, \quad y = -1, \quad z = 2$$

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12-

(b) $^{3}C_{1} \times {}^{9}C_{3}$ 1

13-

(b) $2\cos\theta$ 1

14-

(d) $\vec{r} = (2,1,-3) + k(-1,1,-2)$ 1

15-

ⓐ $Z = \dfrac{8(\sqrt{3}+i)}{(\sqrt{3}-i)} \times \dfrac{\sqrt{3}+i}{\sqrt{3}+i}$

$Z = \dfrac{8(3+2\sqrt{3}\,i-1)}{4}$ $\frac{1}{2}$

$Z = 8\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$Z = 8\, e^{\frac{\pi}{3}i}$ $\frac{1}{2}$

$\sqrt[3]{Z} = 2\, e^{\frac{\frac{\pi}{3}+2\pi n}{3}i}$, $n = 0, 1, -1$ $\frac{1}{2}$

at $n=0$ $\therefore \sqrt[3]{Z} = 2\, e^{\frac{\pi}{9}i}$ $\frac{1}{2}$

at $n=1$ $\therefore \sqrt[3]{Z} = 2\, e^{\frac{7\pi}{9}i}$ $\frac{1}{2}$

at $n=-1$ $\therefore \sqrt[3]{Z} = 2\, e^{-\frac{5\pi}{9}i}$ $\frac{1}{2}$

(b) $(x+yi)(1-3i) = 37\left(\dfrac{7+4\omega^2+3-4\omega^2}{(3-4\omega^2)(7+4\omega^2)}\right)$ $\frac{1}{2}$

$= 37\left(\dfrac{10}{(3-4\omega^2)(7+4\omega^2)}\right)$

$= 37\left(\dfrac{10}{21-16\omega^2-16\omega}\right)$ $\frac{1}{2}$

$= 37\left(\dfrac{10}{21-16(\omega^2+\omega)}\right)$ $\frac{1}{2}$ $= \dfrac{370}{37} = 10$ $\frac{1}{2}$

$\therefore x+yi = \dfrac{10}{1-3i} \times \dfrac{1+3i}{1+3i} = \dfrac{10(1+3i)}{10}$ $\frac{1}{2}$

$x+yi = 1+3i \Rightarrow x = 1$ and $y = 3$ $\frac{1}{2}$

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16-

(b) 15 $\triangle 1$

17-

(c) $(x-2)^2+(y+3)^2+(z-4)^2=16$ $\triangle 1$

18-

(b) $z=5$ $\triangle 1$

19-

(a) $\overrightarrow{BA}=\overrightarrow{A}-\overrightarrow{B}=(-1,-2,-3)$

$\overrightarrow{BC}=\overrightarrow{C}-\overrightarrow{B}=(-1,4,0)$

(i) $\because \cos(\angle ABC)=\dfrac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{\|\overrightarrow{BA}\|\,\|\overrightarrow{BC}\|}$ $\triangle \frac{1}{2}$

$=\dfrac{(-1,-2,-3)\cdot(-1,4,0)}{\sqrt{1+4+9}\,\sqrt{1+16+0}}$

$=\dfrac{1-8+0}{\sqrt{14}\sqrt{17}}=\dfrac{-7}{\sqrt{14}\sqrt{17}}$ $\triangle \frac{1}{2}$

$\therefore m(\angle ABC)\simeq 117^\circ$

(ii) $\because \overrightarrow{BC}=\overrightarrow{C}-\overrightarrow{B} \Rightarrow \overrightarrow{C}=\overrightarrow{BC}+\overrightarrow{B}$

$\overrightarrow{C}=(-1,4,0)+(3,5,4)=(2,9,4)$

$\therefore \overrightarrow{AC}=\overrightarrow{C}-\overrightarrow{A}=(0,6,3)$

The direction Component $=\left(\dfrac{\overrightarrow{AC}\cdot\overrightarrow{AB}}{\|\overrightarrow{AB}\|^2}\right)\cdot\overrightarrow{AB}$ $\triangle \frac{1}{2}$

$=\dfrac{(0,6,3)\cdot(1,2,3)}{1+4+9}\cdot(1,2,3)$

$=\dfrac{0+12+9}{14}\cdot(1,2,3)=\left(\dfrac{3}{2},3,\dfrac{9}{2}\right)$ $\triangle \frac{1}{2}$

b (i) The volume of the parallelepiped

$= |\vec{A} \cdot \vec{B} \times \vec{C}|$ $\frac{1}{2}$

$= \begin{vmatrix} 1 & 4 & 2 \\ -3 & 2 & 1 \\ -1 & 1 & 4 \end{vmatrix}$

$= 1(7) - 4(-11) + 2(-1) = 49$ Volume unit $\frac{1}{2}$

(ii) The base area $= \|\vec{A} \times \vec{B}\|$

$\because \vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 2 \\ -3 & 2 & 1 \end{vmatrix} = -7\vec{j} + 14\vec{k}$

$\therefore \|\vec{A} \times \vec{B}\| = \sqrt{(-7)^2 + (14)^2} = 7\sqrt{5}$ $\frac{1}{2}$

$\therefore$ The base area $= 7\sqrt{5}$ area unit

The height of the parallelepiped $= \frac{\text{Volume}}{\text{base area}}$

$= \frac{49}{7\sqrt{5}} = \frac{7\sqrt{5}}{5}$ length unit.

$\simeq 3.13$ length unit. $\frac{1}{2}$

(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)