

نموذج إجابة مادة الجبر والهندسة الفراغية (باللغة الإنجليزية) لشهادة إتمام الدراسة الثانوية العامة - الدور الأول - العام الدراسي ٢٠١٧/٢٠١٨  
النموذج (ج)

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1-

(b) 15



2-

(a) 10



3-

(b) 2



4-

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \begin{matrix} C_2 - C_1 \\ C_3 - C_1 \end{matrix} \Rightarrow$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} \begin{matrix} 1 \\ \\ \end{matrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & (b-a) & (c-a) \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix} \begin{matrix} 1 \\ 2 \\ \end{matrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix} \begin{matrix} 1 \\ 2 \\ \end{matrix} \begin{matrix} C_3 - C_2 \\ \Rightarrow \end{matrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^2 & b+a & c-b \end{vmatrix} \begin{matrix} 1 \\ 2 \\ \end{matrix}$$

$$= (b-a)(c-a)(c-b) \begin{matrix} 1 \\ 2 \\ \end{matrix}$$

(تراجعى الحلول الأخرى)

5-

(d)  $\pi$



6-

(c)  $\tan \theta$



7-

Let the term Contains  $x^4$  be  $T_{r+1}$

$$T_{r+1} = {}^{12}C_r \left(\frac{-1}{x^2}\right)^r (x^2)^{12-r}$$



$$= {}^{12}C_r (-1)^r x^{24-4r}$$



let  $24 - 4r = 4 \Rightarrow r = 5$

$\therefore$  The term Contains  $x^4$  is  $T_6$



$$T_6 = {}^{12}C_5 (-1)^5 (x^4) = -792x^4$$

• The order of the middle term :  $\frac{12}{2} + 1 = 7$

$$T_7 = {}^{12}C_6 x \left(\frac{-1}{x^2}\right)^6 x (x^2)^6 = {}^{12}C_6 = 924$$

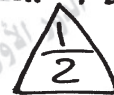


$$\therefore \frac{\text{The coeff. of } T_6}{\text{The coeff. of } T_7} = \frac{-792}{924} = \frac{-6}{7}$$



or  $\frac{\text{The coeff. of } T_6}{\text{The coeff. of } T_7} = \frac{r}{n-r+1} \times \frac{\text{Coeff of 1st}}{\text{Coeff of 2nd}}$

$$= \frac{6}{12-6+1} \times \frac{1}{-1}$$



$$= \frac{-6}{7}$$

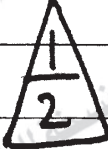



8-

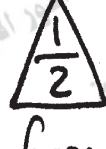
The equation of the plane:

$$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$$

$$(4, 10, -7) \cdot \vec{r} = (4, 10, -7) \cdot (2, -1, 0)$$

$$(4, 10, -7) \cdot \vec{r} = -2$$
  The vector form

$$4(x-2) + 10(y+1) - 7z = 0$$
  The standard form

$$4x + 10y - 7z + 2 = 0$$
  The general form

(تراعى الحلول الأخرى)

9-

(b)  $(1, 1)$  

10-

$\therefore$  The st. line makes equal angles with the positive directions of the Coordinated axes

$$\therefore \cos \theta_x = \cos \theta_y = \cos \theta_z$$


$$\therefore \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\therefore \cos^2 \theta_x = \cos^2 \theta_y = \cos^2 \theta_z = \frac{1}{3}$$


$$\therefore \cos \theta_x = \cos \theta_y = \cos \theta_z = \pm \frac{1}{\sqrt{3}}$$

$\therefore$  The direction vector of the st. line  $= \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$   
or  $= (1, 1, 1)$


$$\therefore \vec{r} = (3, 2, -1) + t \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

or  $\therefore r = (3, 2, -1) + t (1, 1, 1)$   [Vector form]

•  $x = 3 + t, y = 2 + t, z = -1 + t$  [Parametric form]

or  $x = 3 + \frac{1}{\sqrt{3}}t, y = 2 + \frac{1}{\sqrt{3}}t, z = -1 + \frac{1}{\sqrt{3}}t$   [form]

•  $x - 3 = y - 2 = z + 1$

or  $\frac{x-3}{\left(\frac{1}{\sqrt{3}}\right)} = \frac{y-2}{\left(\frac{1}{\sqrt{3}}\right)} = \frac{z+1}{\left(\frac{1}{\sqrt{3}}\right)}$   [Cartesian form]

11-

$$\begin{pmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$$

$$AX = B$$

$$|A| = \begin{vmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{vmatrix} = 3(-5) + 2(-11) = -37 \neq 0$$

$$\text{Adj}(A) = \begin{pmatrix} -1 & 5 & -11 \\ -7 & -2 & -3 \\ -2 & 10 & 15 \end{pmatrix}^t = \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

$$\therefore A^{-1} = \frac{1}{-37} \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-37} \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix} \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{37} \begin{pmatrix} -37 \\ 37 \\ -74 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\therefore x = 1, y = -1, z = 2$$

(تراجعى الحلول الأخرى)

النموذج (ج)

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12-

$$(b) {}^3C_1 \times {}^9C_3 \quad \triangle 1$$

13-

$$(b) 2 \cos \theta \quad \triangle 1$$

14-

$$(d) \vec{r} = (2, 1, -3) + k(-1, 1, -2) \quad \triangle 1$$

15-

$$(a) Z = \frac{8(\sqrt{3}+i)}{(\sqrt{3}-i)} \times \frac{\sqrt{3}+i}{\sqrt{3}+i}$$

$$Z = \frac{8(3+2\sqrt{3}i-1)}{4} \quad \triangle \frac{1}{2}$$

$$Z = 8 \left( \frac{1}{2} + \frac{\sqrt{3}}{2} i \right) = 8 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$Z = 8 e^{\frac{\pi}{3}i} \quad \triangle \frac{1}{2}$$

$$\sqrt[3]{Z} = 2 e^{\frac{\pi+2\pi n}{3}i}, \quad n = 0, 1, -1 \quad \triangle \frac{1}{2}$$

$$\text{at } n=0 \quad \therefore \sqrt[3]{Z} = 2 e^{\frac{\pi}{9}i} \quad \triangle \frac{1}{2}$$

$$\text{at } n=1 \quad \therefore \sqrt[3]{Z} = 2 e^{\frac{7\pi}{9}i} \quad \triangle \frac{1}{2}$$

$$\text{at } n=-1 \quad \therefore \sqrt[3]{Z} = 2 e^{-\frac{5\pi}{9}i} \quad \triangle \frac{1}{2}$$

$$\textcircled{b} (x+yi)(1-3i) = 37 \left( \frac{7+4\omega^2+3-4\omega^2}{(3-4\omega^2)(7+4\omega^2)} \right) \triangle \frac{1}{2}$$

$$= 37 \left( \frac{10}{(3-4\omega^2)(7+4\omega^2)} \right)$$

$$= 37 \left( \frac{10}{21-16\omega^2-16\omega} \right) \triangle \frac{1}{2}$$

$$= 37 \left( \frac{10}{21-16(\omega^2+\omega)} \right) \triangle \frac{1}{2} = \frac{370}{37} = 10 \triangle \frac{1}{2}$$

$$\therefore x+yi = \frac{10}{1-3i} \times \frac{1+3i}{1+3i} = \frac{10(1+3i)}{10} \triangle \frac{1}{2}$$

$$x+yi = 1+3i \Rightarrow x=1 \text{ and } y=3 \triangle \frac{1}{2}$$

(تراعى الحلول الأخرى)



16-

(b) 15



17-

$$(c) (x-2)^2 + (y+3)^2 + (z-4)^2 = 16$$



18-

$$(b) z = 5$$



19-

$$\text{a) } \vec{BA} = \vec{A} - \vec{B} = (-1, -2, -3)$$

$$\vec{BC} = \vec{C} - \vec{B} = (-1, 4, 0)$$

$$(i) \cos(\angle ABC) = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \|\vec{BC}\|}$$



$$= \frac{(-1, -2, -3) \cdot (-1, 4, 0)}{\sqrt{1+4+9} \sqrt{1+16+0}}$$

$$= \frac{1-8+0}{\sqrt{14} \sqrt{17}} = \frac{-7}{\sqrt{14} \sqrt{17}}$$



$$\therefore m(\angle ABC) \approx 117^\circ$$

$$(ii) \therefore \vec{BC} = \vec{C} - \vec{B} \Rightarrow \vec{C} = \vec{BC} + \vec{B}$$

$$\vec{C} = (-1, 4, 0) + (3, 5, 4) = (2, 9, 4)$$

$$\therefore \vec{AC} = \vec{C} - \vec{A} = (0, 6, 3)$$

$$\text{The direction Component} = \left( \frac{\vec{AC} \cdot \vec{AB}}{\|\vec{AB}\|^2} \right) \cdot \vec{AB}$$



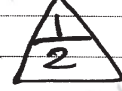
$$= \frac{(0, 6, 3) \cdot (1, 2, 3)}{1+4+9} \cdot (1, 2, 3)$$

$$= \frac{0+12+9}{14} \cdot (1, 2, 3) = \left( \frac{3}{2}, 3, \frac{9}{2} \right)$$



(b) (i) The volume of the parallelepiped

$$= | \vec{A} \cdot \vec{B} \times \vec{C} |$$



$$= \begin{vmatrix} 1 & 4 & 2 \\ -3 & 2 & 1 \\ -1 & 1 & 4 \end{vmatrix}$$

$$= 1(7) - 4(-11) + 2(-1) = 49$$



(ii) The base area =  $|| \vec{A} \times \vec{B} ||$

$$\therefore \vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 2 \\ -3 & 2 & 1 \end{vmatrix} = -7\vec{j} + 14\vec{k}$$

$$\therefore || \vec{A} \times \vec{B} || = \sqrt{(-7)^2 + (14)^2} = 7\sqrt{5}$$



$\therefore$  The base area =  $7\sqrt{5}$  area unit

The height of the parallelepiped =  $\frac{\text{Volume}}{\text{base area}}$

$$= \frac{49}{7\sqrt{5}} = \frac{7\sqrt{5}}{5} \text{ length unit.}$$

$$\approx 3.13 \text{ length unit.}$$



(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)