

(١)

1-

(b) 3 oder 2



2-

(b)

Die Funktion f hat einen lokalen Minimalwert bei $x=3$



3-

$$\therefore y = ax^b \quad \text{durch Differenzierung in Bezug auf } t$$

$$\therefore \frac{dy}{dt} = abx^{b-1} \frac{dx}{dt}$$



$$\frac{dy}{dt} = \frac{abx^b}{x} \cdot \frac{dx}{dt} \quad \text{Durch Multiplizieren mit } \frac{1}{y}$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dt} = \frac{1}{ax^b} \cdot \frac{abx^b}{x} \cdot \frac{dx}{dt}$$



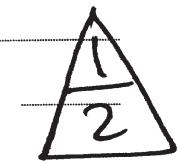
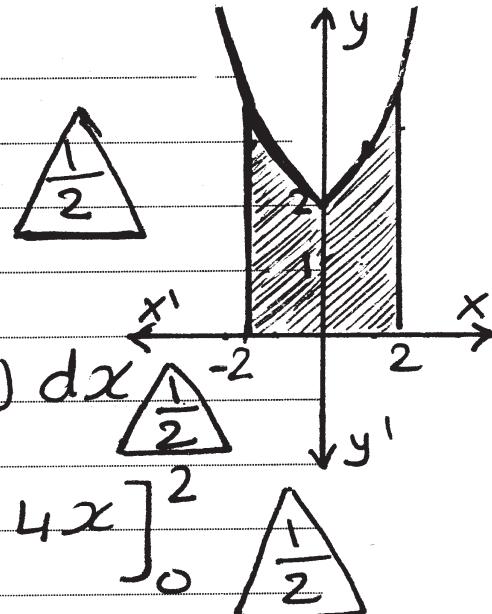
$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{b}{x} \cdot \frac{dx}{dt}$$



(٢)

4-

$$\begin{aligned}
 V &= \pi \int_{-2}^2 y^2 dx \\
 &= 2\pi \int_0^2 (x^2 + 2)^2 dx \\
 &= 2\pi \int_0^2 (x^4 + 4x^2 + 4) dx \\
 &= 2\pi \left[\frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x \right]_0^2 \\
 &= 2\pi \left[\frac{32}{5} + \frac{32}{3} + 8 \right] \\
 &= \frac{752}{15} \pi \text{ Kubische Einheit}
 \end{aligned}$$



(تراعى الحلول الأخرى)

(٣)

5-

(b) $\frac{1}{3} \ln 2$

6-

(d) 2

7-

Die Fläche des Kreissektors = L

$$\therefore \frac{1}{2} r L = 4 \Rightarrow L = \frac{8}{r}$$

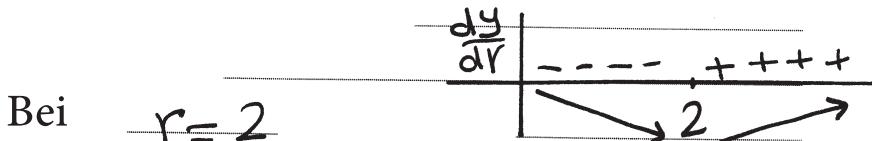
Angenommen, dass der Umfang des Kreissektors = y

$$\therefore y = 2r + L \Rightarrow y = 2r + \frac{8}{r}$$

$$\therefore \frac{dy}{dr} = 2 - \frac{8}{r^2}$$

Gesetzt, dass $\frac{dy}{dr} = 0 \Rightarrow \frac{8}{r^2} = 2 \Rightarrow r^2 = 4 \Rightarrow r = 2$ cm

Bei der Vorzeichensuche für die Fläche



ist der Umfang minimal.

$$\therefore L = \frac{8}{2} = 4, \theta^{\text{rad}} = \frac{L}{r} = \frac{4}{2} = 2^{\text{rad.}}$$

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8-

Um die Schnittpunkte zu ermitteln, setzen wir

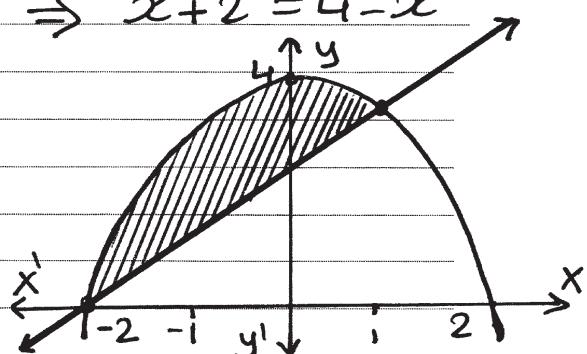
Gesetzt, dass $y_2 = y_1 \Rightarrow x+2 = 4-x^2$

$$\therefore x^2 + x - 2 = 0$$

$$(x+2)(x-1)=0$$

$$x = -2$$

$$x = 1$$



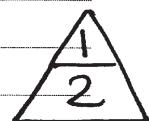
$$= \int_{-2}^1 [(4-x^2) - (x+2)] dx$$



$$= \int_{-2}^1 [4 - x^2 - x - 2] dx$$

$$= \int_{-2}^1 [-x^2 - x + 2] dx$$

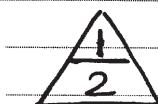
$$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \right]_{-2}^1$$



$$= \left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(\frac{8}{3} - 2 - 4 \right)$$

$$= \frac{9}{2}$$

Flächeneinheit



(تراعي الحلول الأخرى)

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٩-

(b) $\log_a b$



١٠-

(c) ٤



٦

11-

$$\text{ئ} \int x^3 (x^2 + 1)^6 dx$$

Gesetzt, dass $y = x^2 + 1 \Rightarrow x^2 = y - 1$

$$\therefore 2x dx = dy \Rightarrow dx = \frac{dy}{2x} \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$\therefore \int x^2 \cdot x (x^2 + 1)^6 dx \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$= \int (y-1) \cdot x \cdot y^6 \cdot \frac{dy}{2x} \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$= \int \left(\frac{1}{2} y^7 - \frac{1}{2} y^6 \right) dy \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$= \frac{1}{16} y^8 - \frac{1}{14} y^7 + C \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$= \frac{1}{16} (x^2 + 1)^8 - \frac{1}{14} (x^2 + 1)^7 + C \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$\text{ئ} \int (x-3) e^{2x} dx$$

Gesetzt, dass $u = x-3, dv = e^{2x} dx$

$$= (x-3) \left(\frac{1}{2} e^{2x} \right) - \int \frac{1}{2} e^{2x} dx \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$= (x-3) \left(\frac{1}{2} e^{2x} \right) - \left[\frac{1}{2} \times \frac{1}{2} e^{2x} \right] + C \quad \begin{array}{c} 1 \\ 2 \end{array}$$

$$= \frac{(x-3)}{2} e^{2x} - \frac{1}{4} e^{2x} + C \quad \begin{array}{c} 1 \\ 2 \end{array}$$

(تراعى الحلول الأخرى)

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12-

(a) $-\ln|\cos\varphi| + C$ 

13-

(b) null 

(٨)

14-

a) $f(x) = x^3 - 3x - 2$

$$f'(x) = 3x^2 - 3, \quad f''(x) = 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0$$

$$3(x-1)(x+1) = 0$$

$$\therefore x = 1$$

$$x = -1$$



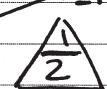
Bei der Vorzeichensuche



Es gibt einen lokalen Maximalwert bei

$$x = -1$$

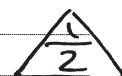
$$f(-1) = \text{null}$$



Es gibt einen lokalen Minimalwert bei

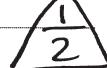
$$f(1) = -4$$

$$x = 1$$



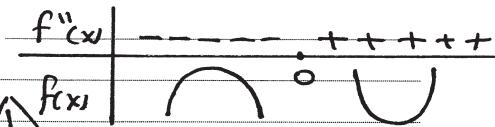
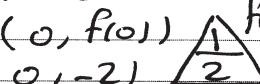
Gesetzt, dass $f''(x) = 0 \Rightarrow 6x = 0 \Rightarrow x = 0$

Bei der Vorzeichensuche



Bei $x = 0$

gibt es einen Wendepunkt.



b) $f(x) = x(x^2 - 12) = x^3 - 12x$

$$\therefore f'(x) = 3x^2 - 12$$

$$f'(x) = 0 \Rightarrow 3(x^2 - 4) = 0 \Rightarrow x = \pm 2$$

$$2 \in [-1, 4] \quad -2 \notin [-1, 4]$$

$$f(-1) = 11, \quad f(2) = -16, \quad f(4) = 16$$



ein absoluter Minimalwert



(تراعي الحلول الأخرى)

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15-

(a) -50



16-

(b) $]-\infty, 0 [$

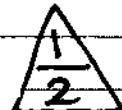


17-

$$x = \sec \theta, \quad y = \tan \theta$$

Bei $\theta = \frac{\pi}{6} \Rightarrow x = \sec \frac{\pi}{6} \Rightarrow x = \frac{2\sqrt{3}}{3}$
 $\Rightarrow y = \tan \frac{\pi}{6} \Rightarrow y = \frac{\sqrt{3}}{3}$

Der Punkt $(\frac{2\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$ \in der Kurve



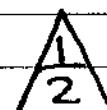
$$\therefore \frac{dx}{d\theta} = \sec \theta \cdot \tan \theta, \quad \frac{dy}{d\theta} = \sec^2 \theta$$



$$\therefore \frac{dy}{dx} = \frac{\sec^2 \theta}{\sec \theta \cdot \tan \theta} = \frac{\sec \theta}{\tan \theta} = \csc \theta$$

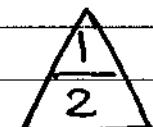


$$\theta = \frac{\pi}{6} \quad \text{Die Steigung ist } = 2$$



Die Gleichung der Tangente lautet:

$$y - \frac{\sqrt{3}}{3} = 2 \left(x - \frac{2\sqrt{3}}{3} \right)$$



Die Gleichung der Normalen lautet:

$$y - \frac{\sqrt{3}}{3} = -\frac{1}{2} \left(x - \frac{2\sqrt{3}}{3} \right)$$



(١٠)

١٨-

$$\sin y + \cos 2x = 0$$

$$\cos y \frac{dy}{dx} - 2 \sin 2x = 0$$

$$\cos y \frac{d^2y}{dx^2} + (-\sin y \frac{dy}{dx})(\frac{dy}{dx}) - 4 \cos 2x = 0$$

$$\cos y \frac{d^2y}{dx^2} - \sin y \left(\frac{dy}{dx}\right)^2 = 4 \cos 2x \quad \div (\cos y)$$

$$\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 \tan y = 4 \cos 2x \sec y .$$

(تراعى الحلول الأخرى)

(انتهت الإجابة وتراعى الحلول الأخرى)