CASE 70

Application of the MTS to Thermal Ink Jet Image Quality Inspection

Abstract: Efficient conversion of electrical and thermal energy into robust ink drop generation (and subsequent delivery) to a wide variety of substrates requires target directionality, drop velocity, drop volume, image darkness, and so on, over a wide range of signal and noise space. One hundred percent inspection is justified based on loss function estimates, considering inspection cost, defective loss, and quality loss after shipment. The Mahalanobis–Taguchi system (MTS) was applied to the final inspection of image quality characteristics for a number of reasons: (1) to summarize multivariate results with a continuous number expressing distance from the centroid of "good" ones; (2) to reduce measurement cost through identification and removal of measurements with low discrimination power; (3) to balance misclassification cost for both type I and type II errors; and (4) to develop a case study from which practitioners and subject-matter experts in training could learn the MTS.

1. Introduction

Classification of thermal ink jet printheads as either acceptable (for shipment customers) or rejectable (for scrap) is a decision with two possible errors. Type I error occurs when a printhead is declared conforming when it is okay. Type II error occurs when a printhead is declared conforming when actually it is not. Type I error means that scrap costs may be high or orders cannot be filled due to low manufacturing process yield. Type II error means that the quality loss after shipment may be high when customers discover they cannot obtain promised print quality, even with printhead priming and other driver countermeasures. Over time, type II errors may be substantially more costly to Xerox than type I errors. Consequently, engineers are forced to consider a balance between shipping bad printheads and scrapping good ones.

A Mahalanobis space was constructed using multivariate image quality data from 144 good-print-quality printheads. The data were used to create a Mahalanobis space. Nonsequential serial numbers were used from several months of production. In addition, the Mahalanobis distance, D_2, of 45 nonconforming printheads were calculated using the aforementioned Mahalanobis space. The feasibility of the technique to better discriminate good from bad printheads was demonstrated. In addition, from these results, the contribution rate of each measurement characteristic to the Mahalanobis distance was calculated using L_{64} and analysis of variance (ANOVA) methods. The minimum set of measurement characteristics was then selected to reduce measurement costs and to speed up the analysis process. The accelerating pace of printhead fabrication, pulled by customer demand, has put considerable stress on the manufacturing system (which worked quite well for lower quantities of printheads). Reduction of these inspection data to one continuous Mahalanobis number and reducing measures, which have very low discrimination

 Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

power, will speed up the inspection and disposition process.

2. Camera Inspection System

Assembled printheads were print-tested using 100% inspection. During the test, power was applied to the device, ink nozzles were primed, and a variety of patterns of drops were ejected from the linear array of nozzles. A standard Xerox paper was the medium, which received the various patterns of drops. A tricolor line scan camera captured the images of the print pattern automatically and sent the image data to a machine vision processor. The vision processor controlled the camera and used video data to place templates and gather statistics on the printed patterns. A host PC initiated inspection and configuration processes, created reports, and provided a user interface for the operators.

Certain camera calibration steps were done before testing, including white balance (to compensate for uneven light distribution over the image plane and the spatial nonuniformity of the sensitivity of the camera's sensors). Other calibrations included printed pattern measurement and alignment (to center the image in the camera's field of view), focusing and camera pixel-size calibration, focusing the light guides so that the maximum amount of light was in the camera's field of view, and calibration of color dot size so that dots of different colors were considered to be of the same size.

Dot aspect ratio, the ratio of a dot's height to its width, was one of the many characteristics captured by the camera system. It is shown in Figure 1 for magenta ink. A maximum aspect ratio is allowed, to control certain image quality problems.

A second characteristic captured by the camera system was dot diameter. The diameter is an equivalent diameter calculated for noncircular dots. For each color, the mean and standard deviation of both repeated dots and same jets were calculated. Upper and lower tolerances were applied to the dot size. Missing dot counts (or missing jets counts) were tabulated. Dot misallocation (directionality) results were summarized as both x and y error variances from least squares fitting routines of multiple dot patterns. If any single criteria limit is violated, the printhead is scrapped. Inspection results are generally reported in terms of pass/fail percentages considering decomposition by dot, by jet, and by color for each printhead. All of this existing inspection process does not consider any correlation among response characteristics in the decision making.

Figure 2 shows a hypothetical plot of aspect ratio versus dot diameter. With correlation, a printhead is classified as abnormal; with no correlation, a printhead is classified as normal. It is clear that a point labeled as abnormal (because it is outside the ellipse) would be considered normal if each characteristic were considered in isolation. It is at +1.4 from the mean of the spot diameter and 0.7 from the mean of the aspect ratio. Because of the correlation between the characteristics measured, one can say that this printhead has too large a spot diameter for its aspect ratio, and therefore it should be classified as abnormal.

3. Mahalanobis Distance

The quantity D^2 is called the Mahalanobis distance from the measured featured vector x to the mean vector mx, where Cov x is the covariance matrix x. The future vector represents the vector-measured characteristics for a given printhead in this study. The question is: How far is a given printhead from the database of good ones? If the distance is greater than some threshold, it is classified as defective. If the distance is less than some threshold, it is classified as no different from the population of good ones. It can be shown that the surfaces on which D^2 is constant are ellipsoids that are centered about

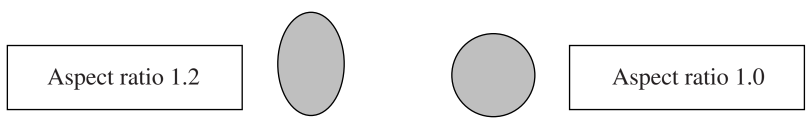

Figure 1
Dot height-to-width ratio for magenta

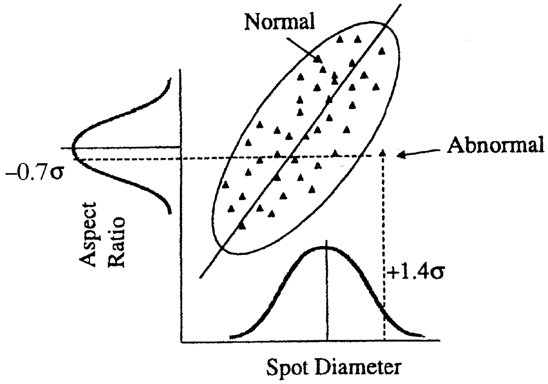

Figure 2
Hypothetical plot of aspect ratio versus dot diameter

the mean *mx*. In the special case where the measurements are correlated and the variances in all directions are the same, these spheres and the Mahalanobis distance become equivalent to the Euclidean distance.

The Mahalanobis distance, D^2, was calculated as:

$$D^2 = (x - \mu_x) C_x^{-1} (x - \mu_x)^{\mathrm{T}} \qquad (1)$$

where x is an n-dimensional vector of measured characteristics for a given printhead. The population of good printheads was characterized by a mean feature vector $m = (m_1, m_2, \ldots, m_{144})$. C^1 is the inverse of the variance–covariance matrix, σ_{ij}, for the measured characteristics x, where

$$\mathrm{Cov}(x_i x_j) = \frac{1}{m} \sum_1^m (x_{i\alpha} - m_i)(x_{j\varepsilon} - m_j) \qquad (2)$$

For $i = j$ this becomes the usual equation for variance. D^2 is the generalized distance between the αth product and the mean feature vector for the population of good printheads.

The covariance of two measures captures their tendency to vary together (i.e., to co-vary). Where the variance is the average of squared deviation of a measure from its mean, the covariance is the average of the products of the deviations of measured values from their means. To be more precise, consider measure I and measure j. Let $\{x(1,I), x(2,I), \ldots, x(n,I\}$ be a set of n examples of characteristic I, and le$\{x(1,j), x(2,j), \ldots, x(n,j)\}$ be a corresponding set of n examples of characteristic j. [That is, $x(k,I)$ and $x(k,j)$ are measures of the same pattern, pattern k.] Similarly, let $m(i)$ be the mean of feature I and $m(j)$ be the mean of feature j. Then the covariance of features I and j is defined by $c(i,j) = \{[x(1,i) - m(i)]\,[x(1,j) - m(j)] + \cdots + [x(n,i) - m(i)]\,[x(n,j) - m(j)]\}/(n - 1)$.

The covariance has several important properties:

- If measure i and measure j tend to increase together, then $c(i,j) > 0$
- If measure i tends to decrease when feature j increases, then $c(i,j) < 0$.
- If features i and j are independent, then $c(i,j) = 0$.
- $|c(i,j)| \leqslant s(i)s(j)$, where $s(i)$ is the standard deviation of feature i.
- $c(i,i) = s(i)^2 = v(i)$.

Thus, the covariance $C(i,j)$ is a number between $-s(i)s(j)$ and $+s(i)s(j)$ that means dependence between measures i and j, with $c(i,j) = 0$ if there is no dependence. All of the covariances $c(i,j)$ can be collected together into a covariance matrix as shown below:

$$\begin{matrix} c(1,1) & c(1,2) & \cdots & c(1,d) \\ c(2,1) & c(2,2) & \cdots & c(2,d) \\ \vdots & \vdots & & \vdots \\ c(d,1) & c(d,2) & \cdots & c(d,d) \end{matrix}$$

This matrix provides us with a way to measure distance that is invariant to linear transformations of the data. The covariance among the responses is a valuable additional piece of information characterizing the se of good printheads. For this study, the covariance matrix (among all of the responses for the good printhead data) was calculated. The inspection and rationalization of these results can provide very important insights into the performance of the printheads. For example, the spot size performance for the cyan ink is highly correlated with the spot size performance for the magenta ink, and moderately correlated with the spot size for the cyan ink. None of these three measures is correlated with the black ink spot size performance. This should make sense to the technologist.

Mahalanobis distance calculations were performed using SAS/I ML with custom macros. Fifty-eight different image-quality summary measures were considered, approximately 15 for each ink color. All 144 good printhead data were standardized according to

$$Y_{ij} = (X_{ij} - m_i)\sigma_i$$

This scaling creates a standardized distance of characteristic X to mean, m_x, relative to the standard deviation, σ_x. This scaling removes the effects of different units for the various characteristics. All measures were continuous data in this case, but different units for the various characteristics. All measures were continuous data in this case, although other types of data could be considered. Scaling is a particular example of a linear transformation. This standardization is also useful since it reduces the variance–covariance matrix to the better-known correlation matrix $R = \rho_{ij}$. It can be shown that the Mahalanobis distance for the xth product is given by

$$D^2_\alpha = XR^{-1}X^{\mathrm{T}} \tag{3}$$

This is the form used to calculate D^2 by the software code.

The use of the Mahalanobis metric removes several of the limitations of the Euclidean metric for calculating the distance between the printhead and the population of good ones:

1. It accounts automatically for the scaling of the coordinate axes.
2. It corrects for correlation between the different measures.
3. It can provide curved as well as linear decision boundaries.

4. Mahalanobis Distance Results

The Mahalanobis distance was calculated for each good printhead using the inverse correlation matrix, and values for D^2 ranged from 0.5 to 2.5. Singular matrix problems on inversion indicated very high correlation between several of the measured characteristics, and the elimination of one of them was required.

The average D^2 value was 1.000 for the good ones, as expected. (These distances are in units of standard deviations from the centroid of the multivariate good printhead data.) From these results an origin and unit amount were established. The inverse correlation matrix from the good ones, in turn, was used to calculate the Mahalanobis distance D^2 for each of the 44 bad printheads. It was expected that the D^2 values would be considerably larger than D^2 values for the good printheads. As a rule of thumb, a value below 4.0 for the bad ones usually indicated a misclassification with type 1 error. These numbers ranged from 2.6 to 179.4.

The Mahalanobis distance identifies the range of variation of the bad ones, which is much more informative than a simple good or bad binary classification system. It is a simple matter to go back through the individual printhead data to see why the really bad ones have much higher D^2 values. Knowing the level of badness enables continuous improvement activities.

If the distribution of the many measured characteristics is multivariate Gaussian, the D^2 follow a chi-square distribution with k degrees of freedom. Probabilities of membership in a defective classification can be determined. Again, as a rule of thumb, those printheads with D^2 values greater than about 3.0 to 4.0 can be classified as not belonging to the good group of printheads with fairly high confidence.

The Mahalanobis–Taguchi system allows you to distinguish normal and abnormal printhead conditions as long as you have data for the normal condition. Only the normal condition produces a uniform state, and infinite abnormal conditions exist. To collect data for all those abnormal conditions produced the Mahalanobis space. For new data we calculated the Mahalanobis distance, and by judging whether it exceeds the normal range, we can distinguish between normal and abnormal printhead conditions.

The traditional way is to discriminate which of multiple preexisting categories the actual device belongs to. For printheads, where one has to decide if it is good or not, a large volume of defective condition data and normal condition data are measured beforehand, and a database is constructed with both. For newly measured data (for which you do not know whether it is a good one or bad one), you calculate which group the data belongs to.

However, in reality, there are a multitude of image-quality problems. So if information about various types of images is not obtained beforehand, printheads cannot be distinguished. Because the traditional discrimination method requires this information, it is not practical.

5. Results

A $d \times d$ correlation matrix of the 58 observed characteristics is partially shown in Table 1. These correlations make looking at individual measured characteristics problematic for classifying printheads. When there is correlation between characteristics, diagnosis cannot be made from single characteristics. Table 2 shows the calculated D^2 values for the 145 good printheads. Table 3 shows the Mahalanobis distance for the 44 bad printheads, using the inverse correlation matrix from the good printheads.

Figure 3 shows the Mahalanobis distances D^2 for both the good and bad printheads. A wide range of D^2 values were observed for the nonconformant printheads. Note that the good ones are quire uniform. This is usually the case. There are one or two cases of printheads classified as bad but more likely part of the good ones. The threshold value for D^2 was determined by economic considerations. Misclassification countermeasure costs are usually considered.

6. Measurement System Cost Reduction

Among the 58 different measured characteristics in this example, there were a number that were doing most of the discrimination work. An L_{64} orthogonal array was used to identify those characteristics. In this case, the orthogonal array was used as a decision tool. One measured characteristic was assigned to each column, using 58 of the 63 available columns. L_{64} is a two-level design with 63 available degrees of freedom for assignment. A value of $+1$ in any cell of the standard array layout indicated that the measured characteristic was considered, while a value of -1 in any given cell meant that the measured characteristic was not considered. In row 1, for example, 27 of the 58 measures were considered since there were 27 positive ones. Using just those 27 columns, the analysis was run to generate a Mahalanobis space with the good printheads, and Mahalanobis distances D^2 for each of the 44 bad printheads was calculated. From these 27 responses, and 44 bad printheads, a single SN ratio (larger-the-better characteristic) was calculated according to

$$\eta = -10 \log \frac{1}{m}\left(\frac{1}{D_1^2} + \frac{1}{D_2^2} + \cdots + \frac{1}{D_m^2}\right) \qquad (4)$$

where summation is from $m = 1$ to $m = 44$.

This same procedure was followed for the remaining L_{64} rows. A larger-the-better form was used, since the intent was to amplify the discrimination power by selecting a subset of measures. This in turn would reduce cost and time to process inspection data. The results are shown in Table 4 for each of the L_{64} rows. The SN ratios ranged from -16.6 dB to -4.7 dB. Row 64 of the L_{64} included use of each of the 58 measures, and consequently had the highest positive SN ratio.

An ANOVA procedure was conducted on these 64 SN ratios to decompose the total variation into constituent parts. Table 5 shows the large effects of a handful of measures (e.g., measure C_2, C_8, C_{15}, C_{17}, C_{18}, C_{19}, C_{29}, C_{43}, C_{57}), and the small effects of many of the others. This suggests that not all these measures are adding value to the decision to scrap or ship printheads. Much of the time, the measurements are adding little or nothing to the decision. Reducing the number of measures substantially

Table 1
Print quality inspection correlation matrix (partial for 58 variables)

1.00	−0.24	0.24	−0.25	0.12	−0.15	−0.03	−0.24	0.34	0.24	0.37	0.44	0.42
−0.20	−0.10	0.18	0.18	−0.29	0.30	−0.27	0.05	0.07	0.03	−0.08	0.29	0.38
0.25	−0.05	−0.15	0.21	0.45	−0.17	−0.05	−0.01	0.01	−0.00	0.02	0.27	0.24
0.7	0.25	0.04	−0.13	0.24	0.03	−0.19	0.07	−0.18	−0.03	0.04	−0.02	0.06
−0.04	0.20	0.16	0.06	−0.16	0.27							
−0.24	1.00	0.08	0.03	0.19	0.13	0.25	−0.18	−0.42	0.18	0.12	−0.40	−0.32
0.03	−0.24	−0.15	−0.31	0.97	0.06	0.14	−0.26	0.16	−0.25	0.11	0.02	−0.41
−0.28	0.05	−0.24	−0.21	−0.11	0.94	0.01	0.06	0.02	0.06	0.01	−0.06	0.07
−0.35	−0.29	0.27	−0.28	−0.20	−0.29	0.83	−0.15	0.14	−0.19	0.23	−0.31	0.07
0.08	−0.37	−0.28	−0.02	−0.25	−0.22							
0.24	0.08	1.00	−0.53	0.44	−0.21	−0.11	−0.35	−0.02	0.21	0.22	0.02	−0.01
0.16	−0.10	−0.06	−0.18	0.08	0.29	−0.35	−0.09	0.07	−0.15	−0.30	0.05	−0.03
−0.05	0.14	−0.16	−0.05	0.13	0.15	0.08	0.08	−0.22	0.08	−0.12	0.26	0.09
−0.07	−0.11	0.29	−0.15	−0.02	−0.17	0.04	0.21	−0.32	−0.18	0.28	−0.18	0.21
0.06	−0.18	−0.07	−0.03	−0.19	−0.02							
−0.25	0.03	−0.53	1.00	−0.20	−0.02	−0.23	0.29	−0.10	−0.34	−0.33	−0.26	−0.16
0.04	0.24	−0.09	−0.03	0.05	−0.28	0.82	0.00	−0.09	−0.13	0.14	−0.11	−0.20
−0.09	−0.01	0.28	−0.10	−0.13	0.00	−0.15	0.29	0.37	−0.17	−0.04	−0.34	−0.05
0.11	−0.02	−0.10	0.24	−0.12	0.03	0.11	−0.28	0.72	0.04	−0.23	0.06	−0.27
−0.01	0.02	0.01	0.09	0.31	−0.12							
0.12	0.19	0.44	−0.20	1.00	−0.26	−0.05	−0.08	−0.21	0.31	0.17	−0.13	0.02
−0.01	−0.06	−0.09	−0.14	0.15	0.28	−0.08	−0.05	−0.05	−0.15	−0.05	0.11	−0.22
−0.16	0.05	−0.07	−0.12	0.13	0.25	−0.06	0.13	0.03	−0.13	0.08	0.17	0.11
−0.08	−0.01	0.19	−0.06	−0.12	−0.10	0.22	−0.07	−0.06	−0.12	0.07	−0.08	0.12
0.05	−0.12	−0.10	0.07	−0.08	−0.12							
−0.15	0.13	−0.21	−0.02	−0.26	1.00	0.48	0.20	−0.16	−0.07	0.01	−0.25	−0.23
−0.04	0.07	−0.02	−0.01	0.11	0.04	−0.08	0.03	0.67	0.38	0.19	0.10	−0.18
−0.09	0.06	0.11	−0.05	−0.25	0.10	0.04	−0.50	−0.02	0.53	0.14	0.01	0.03
−0.20	−0.08	−0.09	0.11	−0.07	0.10	0.01	0.12	−0.28	0.12	0.14	0.12	−0.05
−0.08	−0.10	−0.01	0.01	0.12	−0.08							
−0.03	0.25	−0.11	−0.23	−0.05	0.48	1.00	0.11	−0.10	0.31	0.26	−0.11	−0.10
−0.12	−0.10	−0.01	−0.11	0.22	0.08	−0.11	−0.12	0.14	0.17	0.28	0.02	−0.11
−0.04	0.03	−0.06	−0.06	−0.09	0.23	0.00	−0.14	−0.14	0.10	0.25	0.07	0.03
−0.07	−0.04	0.05	−0.07	−0.04	−0.05	0.28	−0.18	−0.15	−0.04	−0.04	0.07	0.13
−0.01	0.02	0.02	0.03	−0.07	−0.05							

reduces the time and cost of inspection and dispositioning.

Using equation (4), a larger-the-better type of SN ratio was calculated from each of the L_{64} combinations, since we were looking for ways to amplify Mahalanobis distance by selecting just a few measures out of the total of 58 (Tables 6 and 7).

7. Conclusions

The Mahalanobis–Taguchi system was used to analyze image-quality data collected during final inspection of thermal ink jet printheads. A Mahalanobis space was constructed using results from a variety of printheads classified as good by all criteria,

Table 2
Print quality inspection Mahalanobis distances for 145 good printheads

0.9788539	0.7648473	0.8365425	1.1541308	1.5237813	0.6360767	1.9261662
0.6253195	1.8759381	0.6151459	0.8409623	2.3004483	0.8254882	0.9997392
0.8141697	1.0554864	1.3502383	1.4477616	1.575254	0.6408639	0.9636183
0.6377904	1.1194686	0.7462556	1.0316818	1.8069662	1.9241475	1.0319332
0.9899644	1.2182822	1.2182822	1.1331171	0.8370348	2.2826932	0.9207974
1.1467803	1.0811343	1.005008	0.9170464	1.2295005	0.5892113	0.4927686
0.8277777	0.9806735	0.9319074	0.9434869	0.8671445	0.8750086	0.8219719
1.2870564	0.6186984	1.5087154	1.0080104	1.343378	1.3982634	1.338509
0.5055908	0.9222672	1.2616672	2.0557088	0.4774843	0.7116405	0.4109008
0.9073186	1.0098421	0.610628	1.8020641	0.7664054	0.5304222	1.22455
0.6284057	1.407896	0.9961997	0.6662852	0.8032502	1.2031408	0.9997437
1.2198565	1.0614666	0.5896821	0.4663961	1.63111	0.7763159	1.0746786
0.744278	0.5047632	1.0467326	1.0163124	0.9345957	0.6408224	0.5880229
1.3450012	0.7046379	1.4418307	0.7789877	1.0798978	0.6764585	0.8649927
0.4805313	0.4805313	1.0123116	0.7207266	1.3288415	0.6955458	0.9248074
1.2550094	0.9059842	1.1129645	0.7871329	0.6158813	0.762891	0.7765336
0.5245411	0.5759779	1.0465223	1.2335352	1.6694431	0.9200819	1.4373301
0.8082051	0.9922023	0.9889715	0.5822106	1.5744132	0.5348402	1.0790753
0.6256058	0.8307524	0.8303376	1.3077433	0.6317183	1.2935359	0.509095
1.2105074	1.1197155	0.7580503	0.9266635	0.6304593		

Self-check: 58.00000

Table 3
Mahalanobis distances for 44 bad printheads

22.267015	6.3039776	47.516228	86.911572	15.757659	47.675354	9.2641579
2.5930287	14.052908	11.691446	15.090696	13.894336	8.6390361	179.43874
20.041785	10.695167	29.33736	13.926274	34.2429	9.3282624	44.442126
35.30109	18.388437	55.787193	6.0059889	63.12819	16.813476	45.355212
78.914301	29.200082	42.105292	28.056818	46.802471	121.26875	8.1542113
33.730109	9.3395192	6.777362	22.493329	16.492573	4.6645559	26.666635
22.46681	153.76889					

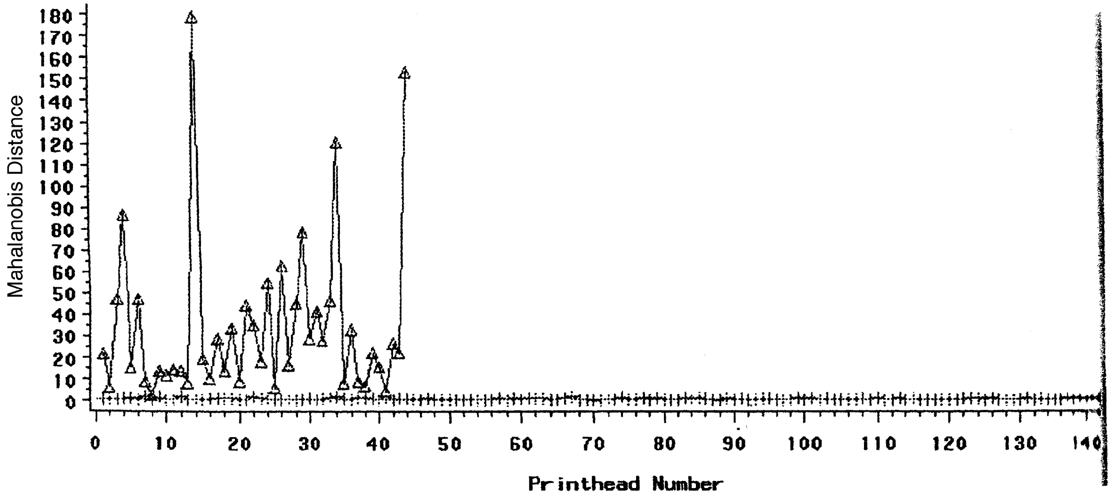

Figure 3
Mahalanobis distance versus printhead number for both passing and failing printheads

establishing an origin and unit number. Mahalanobis distance for nonconforming printheads was calculated, demonstrating the ability to discriminate normal and abnormal printheads. Among the 58 product image quality attributes investigated, only a fraction of the total were doing the work of discriminating good from bad for actual production, identifying the opportunity to reduce measurement and overhead cost for the factories. Misclassification of printheads was checked, and as expected, scrapping good printheads was more prevalent than shipping bad printheads. The scrap cost was approximately one-tenth the cost for type II error, so naturally one would be more likely to scrap good ones.

Other opportunities to apply the Mahalanobis–Taguchi system are being explored for asset recovery. For example, when copiers and printers are returned from customer accounts, recovery of motors, solenoids, clutches, lamps, and so on, that are still classified as good can be reused in new or refurbished machines. Other opportunities include image segmentation algorithm development, pattern recognition to prevent counterfeit copying of currencies, outlier detection in data analysis, system test improvement, diagnosis and control of manufacturing processes, chemical spectral analysis, specification-free disposition, assembly process diagnosis, Kanji symbol recognition, magnetic ink check reading, and others.

The Mahalanobis–Taguchi system considers the following issues:

- ❑ Whether or not a newly manufactured or remanufactured product (or subsystem or component) is a member of the population of previously made good products (scrap/rework/disposition decision)
- ❑ How to characterize the degree of badness of recently made product with one scalar number derived from a multivariate set of measures
- ❑ How to define the contribution of each feature measured to the overall distance from the good population (cause detection)
- ❑ How to eliminate measures that do not help discriminate good from bad products (cost reduction due to misclassification)
- ❑ Use of loss function minimization for balancing false positive and false negative errors
- ❑ Optimization of multivariate data to move all output to the good classification without adding cost

Table 4
SN ratios for each L_{64} combination

Obs.	Expt.	η	Obs.	Expt.	η
1	1	−16.1455	33	33	−137804
2	2	−15.1823	34	34	−15.8187
3	3	−13.4513	35	35	−11.5528
4	4	−14.7418	36	36	−14.3503
5	5	−13.0265	37	37	−14.3373
6	6	−13.9980	38	38	−14.3891
7	7	−14.8928	39	39	−14.0671
8	8	−15.1808	40	40	−14.8759
9	9	−14.3932	41	41	−15.3622
10	10	−15.7381	42	42	−15.4632
11	11	−16.0573	43	43	−12.5654
12	12	−13.2949	44	44	−13.9813
13	13	−11.3572	45	45	−16.3452
14	14	−12.5372	46	46	−13.0218
15	15	−14.0662	47	47	−15.3362
16	16	−15.9616	48	48	−15.8292
17	17	−10.4232	49	49	−12.2162
18	18	−12.4296	50	50	−14.5625
19	19	−14.1015	51	51	−15.3548
20	20	−11.5102	52	52	−16.6430
21	21	−15.1508	53	53	−13.9844
22	22	−14.3844	54	54	−8.9751
23	23	−12.2187	55	55	−8.3045
24	24	−15.3981	56	56	−13.8523
25	25	−13.3026	57	57	−11.0553
26	26	−9.4273	58	58	−14.9251
27	27	−12.2470	59	59	−15.4196
28	28	−15.1932	60	60	−11.7635
29	29	−14.4695	61	61	−9.8751
30	30	−14.4875	62	62	−14.5514
31	31	−14.8804	63	63	−13.5409
32	32	−14.6857	64	64	−4.7039

Table 5
Mahalanobis distances for a sample of L_{64}

			Row 1 of L_{64}			
3.3948335	0.7672923	3.7212996	7.5160489	2.1628283	0.4735541	0.9902953
0.8921714	0.8890575	0.4336029	0.7916304	3.1689748	0.3589341	15.351063
1.6113871	0.8156152	5.1049158	3.53441	0.4765632	0.6596119	0.5178774
7.8548821	0.6789542	2.3212435	1.1780486	0.620615	4.3913711	30.419276
1.3079055	1.5259677	0.5979884	0.9019846	4.0599383	6.7014764	0.35931
0.6227532	3.3683819	1.3396863	5.3121466	0.7085592	0.7498931	11.770036
1.7869636	2.6817723					
			Row 2 of L_{64}			
2.1538859	0.5691297	5.0493192	6.0833953	2.26442	1.1433147	1.1618087
0.8776536	0.743007	0.861313	0.5233098	2.0991015	1.2277685	14.323771
3.5200479	0.2022022	4.8885268	2.3330918	0.8417254	1.1887293	0.7514819
4.1954716	6.0096008	1.0382971	3.080755	1.5613432	2.0078949	3.9968699
0.9804545	2.4407172	0.5850997	1.1164188	1.5777033	7.4701672	0.5262845
0.7533587	1.1707115	1.3342356	2.7278281	1.0099527	0.8908078	12.579455
2.772986	7.0534854					
			Row 3 of L_{64}			
2.432585	1.8731996	6.2922649	8.802438	2.2404416	1.631539	1.9070269
1.0529453	1.2410093	1.0671635	4.211971	2.7976614	0.7359613	6.9516176
1.2869917	3.3668795	4.4021798	5.6621817	1.9823413	2.2946022	0.7033499
14.94987	1.6268848	2.0984149	1.7826083	1.5005829	7.4753245	25.570186
0.9018103	7.125921	2.3458211	1.0736915	6.0013726	31.945727	3.6939773
3.2325816	2.7765327	0.5454912	3.371364	1.5793315	1.2316633	10.05149
2.5569832	2.9596748					
			⋮			
			Row 64 of L_{64}			
22.267015	6.3039776	47.516228	86.911572	15.757659	47.675354	9.2641579
2.5930287	14.052908	11.691446	15.090696	13.894336	8.6390361	179.43874
20.041785	10.695767	29.33736	13.926274	34.2429	9.3282624	44.442126
35.30109	18.388437	55.787193	6.0059889	63.12819	16.813476	45.355212
78.914301	29.200082	42.105292	28.056818	46.802471	121.26875	8.1542113
33.730109	9.3395192	6.777362	22.493329	16.492573	4.6645559	26.666635
22.46681	153.76889					

Table 6
ANOVA procedure for L_{64} results

Dependent variable: η, larger-the-better form

Source	d.f.	Sum of Squares	Mean Square	*F* Value	Pr > *F*
Model	58	275.93384768	4.75748013	1.06	0.5408
Error	5	22.47309837	4.49461967		
Corrected total	63	298.40694604			
	R^2	C.V.	Root MSE		η Mean
	0.924690	−15.50422	2.12005181		−13.67403198

Table 7
ANOVA table

Source	d.f.	ANOVA SS	Mean Square	*F* Value	Pr > *F*
C_1	1	2.86055676	2.86055676	0.64	0.4612
C_2	1	34.60858396	34.60858396	7.70	0.0391
C_3	1	0.187724428	0.18724428	0.04	0.8463
C_4	1	1.49095175	1.49095175	0.33	0.5896
C_5	1	0.37607339	0.37607339	0.08	0.7840
C_6	1	1.14916741	1.14916741	0.26	0.6346
C_7	1	0.16314465	0.16314465	0.04	0.8564
C_8	1	29.49021893	29.49021893	6.56	0.0506
C_9	1	0.82834518	0.82834518	0.18	0.6856
C_{10}	1	8.20578845	8.20578845	1.83	0.2346
C_{11}	1	0.19192667	0.19192667	0.04	0.8444
C_{12}	1	0.11836547	0.11836547	0.03	0.8774
C_{13}	1	0.34995540	0.3499540	0.08	0.7914
C_{14}	1	2.07436008	2.07436008	0.46	0.5271
C_{15}	1	33.60580077	33.630580077	7.48	0.0411
C_{16}	1	7.51486220	7.51486220	1.67	0.2525
C_{17}	1	13.34268884	13.34268884	2.97	0.1455
C_{18}	1	9.47454201	9.47454201	2.11	0.2062
C_{19}	1	10.53510713	10.53510713	2.34	0.1863
C_{20}	1	1.69292927	1.69292927	0.38	0.5662
C_{21}	1	1.27346455	1.27346455	0.28	0.6173
C_{22}	1	0.00896986	0.00896986	0.00	0.9661
C_{23}	1	0.37507340	0.37507340	0.08	0.7843
C_{24}	1	0.70398097	0.70398097	0.16	0.7086
C_{25}	1	0.13738739	0.13738739	0.03	0.8681
C_{26}	1	0.00175670	0.00175670	0.00	0.9850
C_{27}	1	0.09980698	0.9980698	0.02	0.8874
C_{28}	1	0.01056163	0.01056163	0.00	0.9632
C_{29}	1	9.74085337	9.74085337	2.17	0.2010
C_{30}	1	0.68426748	0.68426748	0.15	0.7125
C_{31}	1	0.05612707	0.05612707	0.01	0.9154
C_{32}	1	0.26460629	0.26460629	0.06	0.8179
C_{33}	1	2.35115887	2.35115887	0.52	0.5019
C_{34}	1	5.05833213	5.05833213	1.13	0.3373

Table 7 (*Continued*)

Source	d.f.	ANOVA SS	Mean Square	F Value	Pr > F
C_{35}	1	0.04174250	0.04174250	0.01	0.9270
C_{36}	1	0.69712660	0.69712660	0.16	0.7099
C_{37}	1	0.00644011	0.00644011	0.00	0.9713
C_{38}	1	0.17791141	0.17791141	0.04	0.8501
C_{39}	1	0.21159149	0.21159149	0.05	0.8368
C_{40}	1	0.00395108	0.00395108	0.00	0.9775
C_{41}	1	0.06935294	0.06935294	0.02	0.9060
C_{42}	1	0.03615277	0.03615277	0.01	0.9320
C_{43}	1	32.02214448	32.02214448	7.12	0.0444
C_{44}	1	0.93469126	0.93469126	0.21	0.6675
C_{45}	1	0.01268770	0.01268770	0.00	0.9597
C_{46}	1	1.39043263	1.39043263	0.31	0.6020
C_{47}	1	2.01474985	2.01474985	0.45	0.5328
C_{48}	1	5.24140378	5.24140378	1.17	0.3295
C_{49}	1	0.03621724	0.03621724	0.01	0.9320
C_{50}	1	0.31732921	0.31732921	0.07	0.8011
C_{51}	1	0.61788448	0.61788448	0.14	0.7260
C_{52}	1	5.03501292	5.03501292	1.12	0.3383
C_{53}	1	0.00618522	0.00618522	0.00	0.9718
C_{54}	1	0.38732998	0.38732998	0.09	0.7809
C_{55}	1	0.91071239	0.91071239	0.20	0.6715
C_{56}	1	0.00739509	0.00739509	0.00	0.9692
C_{57}	1	46.41832533	46.41832533	10.33	0.0236
C_{58}	1	0.31011793	0.31011793	0.07	0.8033

- ❑ Definition of the probability of membership in a certain classification category
- ❑ Temporal change and control of a manufacturing process
- ❑ Getting practical use from advanced statistical concepts combined with robust design ideas

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