

## CASE 46

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# Development of Plastic Injection Molding Technology by Transformability

**Abstract:** Because the selection of molding conditions greatly influences the dimensional accuracy of a product in the process of plastic injection molding, we inject, then cool melted plastic material in the mold. We used the concept of transformability to develop the plastic injection molding technology to secure stable dimensional accuracy.

### 1. Introduction

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CFRP is material made of a mixture of superengineering plastics and carbon fibers. Compared with materials used for plastic injection molding, this has a much higher melting point and becomes carbonized and solidified at temperatures 50 to 60° higher than the melting point. Therefore, since a conventional plastic injection machine cannot mold this material, by adjusting the shapes of the nozzle and screw in the preliminary experiment, we confirmed factors and their ranges regarding possible molding conditions.

As the shape to be molded, we selected an easily measurable shape for both mold and product without considering actual product shapes to be manufactured in the future, because our objective was technological development of plastic injection molding. On the other hand, taking into account a shape with which we can analyze the proportionality (resemblance relationship), we prepared the shape shown in Figure 1 by using a changeable mold. As a reference, we note the die structure in Figure 2. The measurements are as follows:

*Bottom Stage:* ABCDE (Points)

*Third Stage:* FGHIJK (Points)

The signal factors are defined in Table 1.

Using a coordinate measuring machine as measurements, we read each set of  $x$ ,  $y$ , and  $z$  coordinates for both the mold and the product. Then we defined the linear distance for every pair of measurements in the mold as a signal factor.

As control factors, we selected from the factors confirmed in the preliminary experiment eight molding conditions believed to greatly affect moldability and dimensions. For a noise factor, we chose the number of shots whose levels are the second and fifth shots. Table 2 summarizes all factor levels of the control and noise factors.

### 2. SN Ratio of Transformability Calculated from the Entire Product Shape

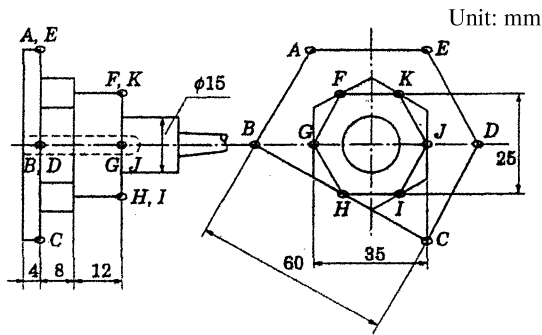
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After assigning the control factors selected to an  $L_{18}$  orthogonal array and the signal and error factors to the outer array, we performed 18 experiments to calculate the distance data shown in Table 3.

Total variation:

$$\begin{aligned} S_T &= 34.668^2 + \dots + 35.877^2 \\ &\quad + 34.672^2 + \dots + 35.885^2 \\ &= 179,318.477815 \quad (f = 110) \quad (1) \end{aligned}$$

Linear equation for the second shot:



**Figure 1**  
Shape of molded product and measurements

$$L_1 = (34.729)(34.668) + (60.114)(59.996) + \dots + (36.316)(35.877) = 90,229.763098 \quad (2)$$

Linear equation for the fifth shot:

$$L_2 = 90,265.630586$$

Effective divider:

$$r = (2)(34.729^2 + 60.114^2 + \dots + 36.316^2) = 181,682.363768 \quad (3)$$

Variation of proportional term:

$$S_{NB} = \frac{1}{181,682.363768 (90,229.763098 + 90,265.630586)^2} = 179,316.178332 \quad (f = 1) \quad (4)$$

Variation of proportional term due to differences between molding shots:

$$S_N = \frac{1}{181,682.363768 (90,229.763098 - 90,265.630586)^2} = 0.007108 \quad (f = 1) \quad (5)$$

Error variation:

$$S_e = 179,318.477815 - 179,316.178332 - 0.007108 = 2.292402 \quad (f = 108) \quad (6)$$

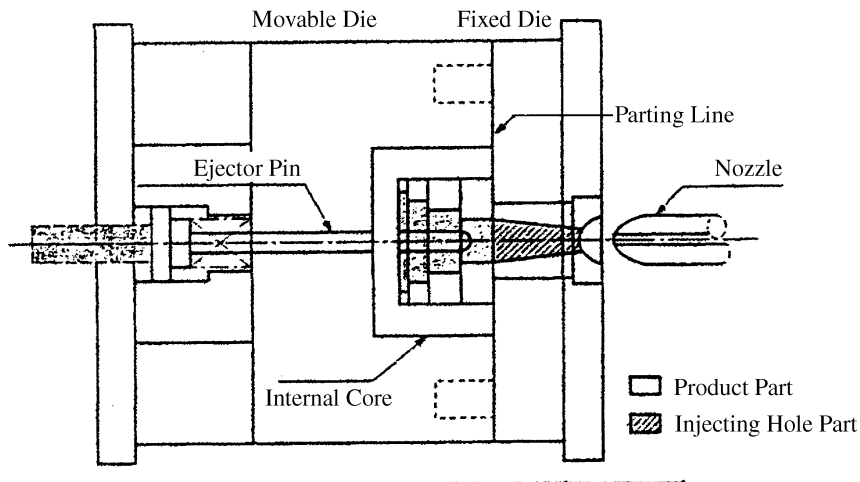
Error variance:

$$V_e = \frac{2.292402}{108} = 0.021226 \quad (7)$$

Total error variance:

$$V_N = \frac{2.292402 + 0.007108}{109} = 0.021096 \quad (8)$$

SN ratio:



**Figure 2**  
Model mold used for injection molding of CFRP

**Table 1**  
Signal factors

Planar Distance on Bottom Stage			Planar Distance on Third Stage				Distance Connecting Points on Bottom and Third Stages				
<i>A</i> – <i>B</i>	<i>A</i> – <i>C</i>	...	<i>D</i> – <i>E</i>	<i>F</i> – <i>G</i>	<i>F</i> – <i>H</i>	...	<i>J</i> – <i>K</i>	<i>A</i> – <i>F</i>	<i>A</i> – <i>G</i>	...	<i>E</i> – <i>K</i>
<i>M</i> <sub>1</sub>	<i>M</i> <sub>2</sub>	...	<i>M</i> <sub>10</sub>	<i>M</i> <sub>11</sub>	<i>M</i> <sub>12</sub>	...	<i>M</i> <sub>25</sub>	<i>M</i> <sub>31</sub>	<i>M</i> <sub>32</sub>	...	<i>M</i> <sub>60</sub>

$$\eta = 10 \log \frac{(1/181,682.363768)}{(179,316.178332 - 0.021226)} \frac{1}{0.021096}$$

$$= 16.70 \text{ dB} \quad (9)$$

Sensitivity:

$$S(\beta) = 10 \log \frac{1}{181,682.363768} \frac{1}{(179,316.178332 - 0.021226)}$$

$$= -0.0569 \text{ dB} \quad (10)$$

$F_3C_2H_1$ . Now if the difference between the initial and optimal conditions for control factor *C*, nozzle temperature, is small, we fix the factor level to level 2 because we wish to leave the temperature at the lower level. As a result, we have:

*Optimal configuration:*  $A_2B_1C_2D_1E_3F_3G_2H_1$

*Initial configuration:*  $A_1B_2C_2D_2E_2F_2G_2H_2$

### 3. Optimal Configuration and Prediction of Effects

Selecting levels with a high SN ratio from the response graphs shown in Figure 3, we can estimate the following optimal configuration:  $A_2B_1C_3D_1E_3$ –

When preparing a test piece for the confirmatory experiment, we set up the following factor levels after determining that we recheck the factor effects of factors *E* and *H*, both of which have a V-shaped effect, with the lowest value at level 2. Since for cooling time, *H*, we have better productivity at a shorter time, we selected two levels,  $H_1$  and  $H_2$ . For the switching position of the holding pressure, *E*, we reconfirmed all levels,  $E_1$ ,  $E_2$ , and  $E_3$ . By combining the levels of *E* and *H* in reference to each optimal level estimated before, we performed a

**Table 2**  
Control factors and levels<sup>a</sup>

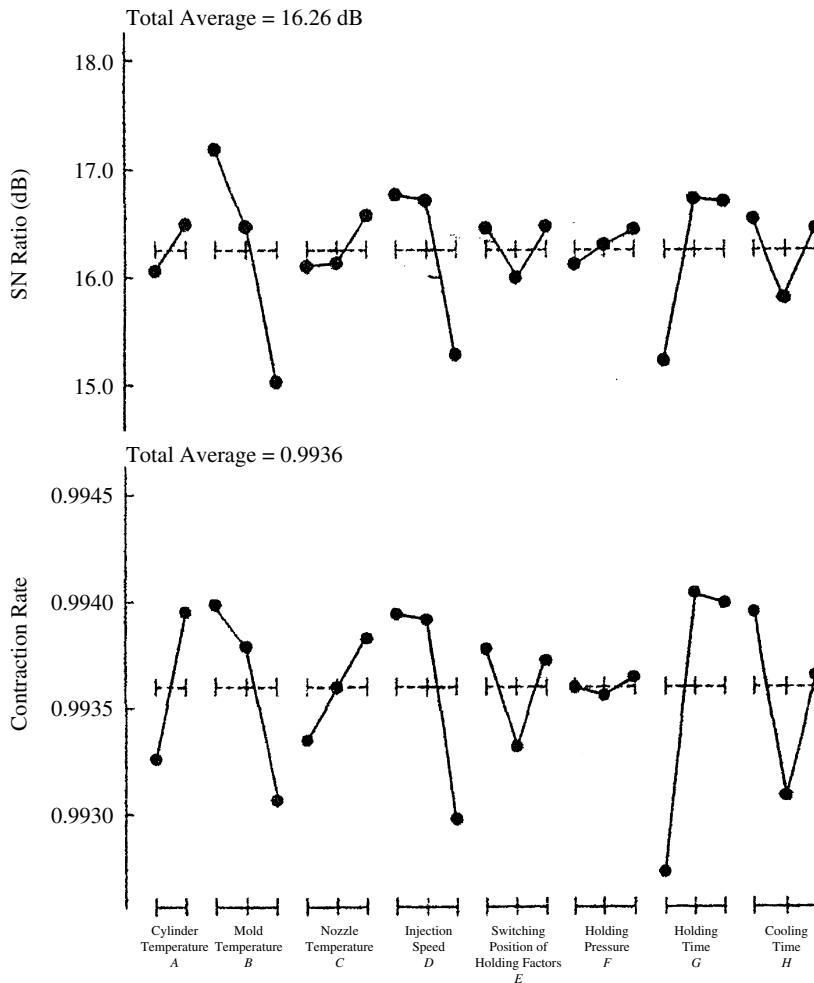
Factor	Level		
	1	2	3
Control Factors			
<i>A</i> : cylinder temperature (°C)	Low <sup>a</sup>	Mid	—
<i>B</i> : mold temperature (°C)	Low	Mid <sup>a</sup>	High
<i>C</i> : nozzle temperature (°C)	Low	Mid <sup>a</sup>	High
<i>D</i> : injection speed (%)	5	8 <sup>a</sup>	15
<i>E</i> : switching position of holding pressure (mm)	5	10 <sup>a</sup>	15
<i>F</i> : holding pressure (%)	70	85 <sup>a</sup>	99
<i>G</i> : holding time (s)	15	30 <sup>a</sup>	45
<i>H</i> : cooling time (s)	40	110 <sup>a</sup>	180
Noise factor			
<i>N</i> : number of shots	Second shot	Fifth shot	—

<sup>a</sup> Initial conditions.

**Table 3**

Distance data of molded product for transformability (experiment 1) (mm)

Dimensions of Mold (Signal Factor, Number of Levels: 55)									
Noise Factor	$M_1$	$M_2$	...	$M_{11}$	$M_{12}$	...	$M_{31}$	...	$M_{60}$
	34.729	60.114	...	14.447	25.018	...	28.541	...	36.316
$N_1$	34.668	59.966	...	14.294	24.769	...	28.104	...	35.877
$N_2$	34.672	59.970	...	14.302	24.780	...	28.137	...	35.885
Total	79.340	119.936	...	28.596	49.549	...	56.241	...	71.762



**Figure 3**  
Response graphs

confirmatory experiment and summarize the analyzed results in Table 4.

Based on these results, we selected the following as the optimal configuration:  $A_2B_1C_2D_1E_3F_3G_2H_1$ . The confirmations under the optimal and initial configurations are tabulated in Table 5. These results demonstrate that the SN ratio and its gain are about the same as were estimated; that is, both have high reproducibility. Even so, their absolute values did not satisfy our targets completely. We could not obtain any remarkable improvement for the gain, possibly because it already approached the optimal value in the preliminary experiment.

The reason for the small SN ratio was probably related to the fact that shrinkage has a direction because we mixed plastic material with carbon fibers. By studying each portion or direction of a test piece under the optimal configuration, we computed an SN ratio and shrinkage for each of the following three analyses:

1. *Planar bottom stage*: analysis using planar distance dimensions on the bottom stage
2. *Planar top stage*: analysis using planar distance dimensions on the third stage
3. *Vertical direction*: analysis using distance dimensions between the bottom and third stages

The results summarized in Table 6 imply that the differences between shrinkage generated in each pair of portions or directions lower the SN ratio of the whole. Next, we computed the SN ratio when changing the shrinkage for each portion.

#### 4. Analysis Procedure

Essentially, the reason that we applied the concept of transformability to injection molding was to de-

**Table 4**  
Results of confirmatory experiment (dB)

	SN Ratio, $\eta$		Sensitivity, $S(\beta)$	
	$H_1$	$H_2$	$H_1$	$H_2$
$E_1$	17.16	18.32	-0.0478	-0.0305
$E_2$	18.76	17.88	-0.0345	-0.0340
$E_3$	19.19	18.63	-0.0320	-0.0346

termine an optimal configuration where the shrinkage remains constant, thereby designing a mold that allows us to manufacture a product with high accuracy. Nevertheless, in actuality, we modified the dimensions of the mold through tuning processes after calculating the shrinkage for each portion.

Table 7 shows the decomposition of  $S_{\beta \times \text{portion}}$ , variation caused by different portions. Only  $S_e$  is treated as the error. For the calculation procedure, we rewrote the distance data obtained from the test piece prepared for the optimal configuration in Table 8. Based on them, we proceeded with the calculation as follows.

#### Analysis of Planar Bottom Stage

Total variation:

$$\begin{aligned} S_{T1} &= 34.705^2 + 34.697^2 + \dots + 34.690^2 \\ &= 57,884.917281 \quad (f = 20) \end{aligned} \quad (11)$$

Linear equation:

$$\begin{aligned} L_1 &= (34.729)(69.402) + (60.114)(120.149) \\ &\quad + \dots + (34.729)(69.383) \\ &= 57,931.241963 \end{aligned} \quad (12)$$

Effective divider:

$$\begin{aligned} r_1 &= (2)(34.729^2 + 60.114^2 + \dots + 34.729^2) \\ &= 57,977.605121 \end{aligned} \quad (13)$$

Variation of proportional term:

$$\begin{aligned} S_{\beta 1} &= \frac{L_1^2}{r_1} = \frac{57,931.241963^2}{57,977.605121} \\ &= 57,884.915880 \quad (f = 1) \end{aligned} \quad (14)$$

Error variation:

$$\begin{aligned} S_{d1} &= S_{T1} - S_{\beta 1} = 57,884.917281 \\ &\quad - 57,884.915880 \\ &= 0.001401 \quad (f = 19) \end{aligned} \quad (15)$$

Error variance:

$$V_{d1} = \frac{S_{d1}}{19} = \frac{0.001401}{19} = 0.0000738 \quad (16)$$

Shrinkage:

**Table 5**  
Results of estimation and confirmatory experiment (dB)

Configuration	SN Ratio		Shrinkage	
	Estimation	Confirmation	Estimation	Confirmation
Optimal	18.52	19.19	0.995	0.996
Initial	16.39	16.44	0.993	0.994
Gain	2.13	2.75		

$$\begin{aligned} \beta_1 &= \sqrt{\frac{1}{r_1} (S_{\beta_1} - V_r)} \\ &= \sqrt{\frac{1}{57,977.605121} (57,884.915880 - 0.0000738)} \\ &= 0.999 \end{aligned} \tag{17}$$

Similar analyses are made for planar top stage and vertical direction.

**Overall Analysis**

Total variation:

$$\begin{aligned} S_T &= S_{T1} + S_{T2} + S_{T3} \\ &= 57,884.917281 + 14,868.183649 \\ &\quad + 10,8094.440286 \\ &= 180,847.541216 \quad (f = 110) \end{aligned} \tag{18}$$

**Table 6**  
SN ratio and shrinkage for each portion of product

	SN Ratio (dB)	Shrinkage
Planar bottom stage	41.31	0.999
Planar top stage	38.83	0.995
Vertical direction	31.01	0.995

Linear equation:

$$\begin{aligned} L &= L_1 + L_2 + L_3 \\ &= 57,931.241963 + 14,944.716946 \\ &\quad + 108,637.586755 \\ &= 181,513.545664 \end{aligned} \tag{19}$$

Effective divider:

$$\begin{aligned} r &= r_1 + r_2 + r_3 \\ &= 57,977.605121 + 15,021.647989 \\ &\quad + 109,183.504276 \\ &= 182,182.757386 \end{aligned} \tag{20}$$

Variation of proportional term:

$$\begin{aligned} S_{\beta} &= \frac{1}{r} (L_1 + L_2 + L_3)^2 \\ &= \frac{1}{182,182.757386} \left( \begin{matrix} 57,931.241962 \\ + 14,944.716946 \\ + 108,637.586755 \end{matrix} \right)^2 \\ &= 180,846.792157 \end{aligned} \tag{21}$$

Variation of proportional terms between portions:

$$\begin{aligned} S_{\beta \times \text{proportion}} &= \frac{L_1^2}{r_1} + \frac{L_2^2}{r_2} + \frac{L_3^2}{r_3} - S_{\beta} \\ &= \frac{57,931.241963^2}{57,977.605121} + \frac{14,944.716946^2}{15,021.647989} \\ &\quad + \frac{108,637.586755^2}{109,183.504276} - 180,846.792157 \\ &= 0.702439 \quad (f = 2) \end{aligned} \tag{22}$$

**Table 7**  
ANOVA table separating portion-by-portion effects

Factor	<i>f</i>	S
$\beta$	1	$S_{\beta}$
$\beta_1, \beta_2, \beta_3$	2	$S_{\beta \times \text{portion}}$
<i>e</i>	$n - 3$	$S_e$
Total	<i>n</i>	$S_T$

Error variation:

$$\begin{aligned}
 S_e &= S_T - S_{\beta} - S_{\beta \times \text{portion}} \\
 &= 180,847.541216 - 180,846.792157 - 0.702439 \\
 &= 0.046620 \quad (f = 107)
 \end{aligned} \quad (23)$$

Error variance:

$$V_e = \frac{S_e}{107} = \frac{0.046620}{107} = 0.0004357 \quad (24)$$

SN ratio:

$$\begin{aligned}
 \eta &= 10 \log \frac{(1/r)(S_{\beta} - V_e)}{V_e} \\
 &= 10 \log \frac{(1/182,182.757386)(180,846.792157 - 0.0004357)}{0.0004357} \\
 &= 33.58 \text{ dB} \quad (25)
 \end{aligned}$$

In Table 9, we put together the data calculated for the SN ratio and shrinkage and those computed for a test piece under the initial configuration. According to these results, we can observe that differences in shrinkage among different portions of a mold contribute greatly to the total SN ratio.

To date, we have used a constant shrinkage figure of 0.998 based on technical information from material suppliers. However, by applying the concept of transformability, we have clarified that there exists a difference in shrinkage among different portions. To reflect this result, we should design a mold and manufacture products to meet dimensional requirements by using a different shrinkage value for each portion of a mold.

**Table 8**  
Distance data under optimal configuration

		Signal Factor			
		$M_1$ 34.729	$M_2$ 60.114	...	$M_{10}$ 34.729
Planar bottom stage	Noise Factor				
	$N_1$	34.705	60.075	...	34.693
	$N_2$	34.697	60.074	...	34.690
	Total	69.402	120.149	...	69.383
Planar top stage		$M_{11}$ 14.447	$M_{12}$ 25.016	...	$M_{25}$ 14.450
	$N_1$	14.347	24.873	...	14.368
	$N_2$	14.357	24.887	...	14.364
	Total	28.704	49.760	...	28.732
Vertical direction		$M_{31}$ 28.541	$M_{32}$ 36.277	...	$M_{60}$ 36.316
	$N_1$	28.313	36.032	...	36.082
	$N_2$	28.315	36.024	...	36.122
	Total	56.628	72.056	...	72.204

**Table 9**

Results of confirmatory experiment (dB)

Configuration	SN Ratio (dB)	Shrinkage		
		Planar Bottom Stage	Planar Top Stage	Vertical Direction
Optimal	33.58	0.999	0.995	0.995
Initial	28.19	0.999	0.993	0.993
Gain	5.39			

In addition, although several production trials have been needed to obtain even a target level under the initial configuration in the conventional trial-and-error method of selecting molding conditions to eliminate internal defects, by taking advantage of our developed process base on the concept of transformability, we can confirm a better molding condition, including internal uniformity, in just a single experiment, and at the same time, earn a gain of approximately 5 dB.

## Reference

Tamkai Asakawa and Kenzo Ueno, 1992. *Technology Development for Transformability*. Quality Engineering Application Series. Tokyo: Japanese Standards Association, pp. 61–82.

*This case study is contributed by Tamaki Asakawa and Kenzo Ueno.*