CASE 26

Transformability of Plastic Injection-Molded Gear

Abstract: To improve the process accuracy of small, precise plastic parts, by selecting after-molding part placement, die temperature, injection speed, and injection pressure as control factors, and die dimensions as a signal factor of transformability, we conducted an experiment. Since we usually adjust product size using molding conditions, we added holding pressure as an adjusting signal factor as well as the number of molding shots as a noise factor. In sum, we chose two signal factors. Furthermore, we proved that there are systematic differences in shrinkage rate due to positions of mold dimensions caused by product design. By separating the effect of shrinkage difference from noise terms through respective tuning, we improved the process accuracy.

1. Introduction

Small, precise plastic gears require an extremely rigorous tolerance between 5 and 20 μm. Furthermore, as the performance of a product equipped with gears improves, the tolerance also becomes more severe. Traditionally, in developing a plastic-molded part, we have repeated the following processes to determine mold dimensions:

1. Decide the plastic molding conditions based on previous technical experience.
2. Mold and machine the products.
3. Inspect the dimensions of the products.
4. Modify and adjust the mold.

For certain product shapes, we currently struggle to modify and adjust a die due to different shrinkage among different portions of a mold. To solve this, we need to clarify such technical relationships, thereby reducing product development cycle time, eliminating waste of resources, and improving product accuracy. In this experiment we applied the idea of transformability to assess our conventional, empirical method of determining molding conditions and investigated the feasibility of improving molding accuracy.

2. Transformability Process

As signal factors, we chose mold dimensions to evaluate transformability and a parameter in the production process for adjusting. Transformability corresponds to the dimensions of a model gear (Figure 1). More specifically, M_1, M_2, M_3, M_4, M_5 and M_6 were selected, and in particular, each of M_1, M_2, M_3 and M_4 contains two directions of X and Y. In sum, since one model has six signal factor levels and one mold produces two pieces, $2 \times 6 = 12$ signal factors were set up in total. We chose holding pressure as a three-level adjusting signal factor.

On the other hand, for all control factors, we set the current factor levels to level 2. As a noise factor, we selected the number of plastic molding shots completed and set the third and twentieth shots to levels 1 and 2, respectively. They represented the noise at the initial and stable stages of the molding process. Table 1 summarizes signal and noise

Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

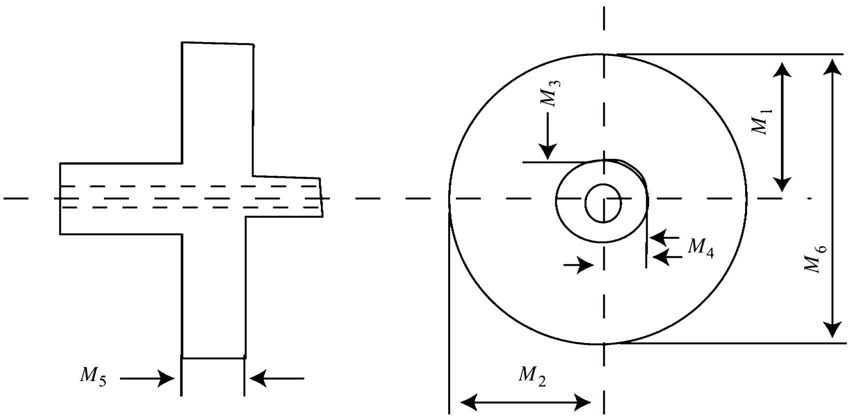

Figure 1
Section of model gear

factors. Dimensions corresponding to signals were chosen as measurement characteristics.

3. SN Ratio

We show some of the experimental data of the L_{18} orthogonal array in Table 2. Based on these, we proceeded with an analysis.

Total variation:

$$S_T = 9.782^2 + 9.786^2 + \cdots + 19.666^2 + 9.924^2 + 9.921^2 + \cdots + 19.931^2 = 7049.764914 \qquad (f = 72) \tag{1}$$

Table 1
Signal and noise factors

Factor	Level
Signal	
Transformability (mold dimension)	M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8, M_9, M_{10}, M_{11}, M_{12}
Adjusting (holding pressure)	$M_1^* = 300$, $M_2^* = 550$, $M_3^* = 800$ kgf/cm^2
Noise	
Number of shots	N_1, third shot; N_2, twentieth shot

Linear equations holding pressure corresponding to mold dimension and noise factor:

$$L_1 = (9.996)(9.782) + \cdots + (0.925)(0.900) + (20.298)(19.818) = 1195.315475 \tag{2}$$

$$L_2 = (9.996)(9.786) + \cdots + (0.925)(0.890) + (20.298)(19.820) = 1195.845021 \tag{3}$$

$$L_3 = (9.996)(9.819) + \cdots + (0.925)(0.900) + (20.298)(19.894) = 1199.881184 \tag{4}$$

$$L_4 = (9.996)(9.808) + \cdots + (0.925)(0.896) + (20.298)(19.892) = 1199.350249 \tag{5}$$

$$L_5 = (9.996)(9.832) + \cdots + (0.925)(0.910) + (20.298)(19.941) = 1202.810795 \tag{6}$$

$$L_6 = (9.996)(9.829) + \cdots + (0.925)(0.906) + (20.298)(19.931) = 1202.271255 \tag{7}$$

Effective divider:

Table 2
Example of one run of an L_{18} orthogonal array (mm)

Adjusting Factor	Noise Factor	Signal M_1 9.996	M_2 9.989	M_3 1.018	M_4 1.026	M_5 0.978	M_6 20.004	
M_1^*	N_1	9.782	9.768	0.903	0.905	0.955	19.545	
	N_2	9.786	9.770	0.901	0.894	0.954	19.565	
M_2^*	N_1	9.819	9.805	0.900	0.900	0.928	19.630	
	N_2	9.808	9.808	0.896	0.900	0.959	19.616	
M_3^*	N_1	9.832	9.836	0.908	0.892	0.977	19.680	
	N_2	9.829	9.833	0.905	0.904	0.970	19.666	
		M_7 10.164	**M_8 10.142**	**M_9 1.041**	**M_{10} 1.043**	**M_{11} 0.925**	**M_{12} 20.298**	**Linear Equation**
M_1^*	N_1	9.924	9.901	0.883	0.864	0.900	19.818	L_1
	N_2	9.921	9.910	0.875	0.864	0.890	19.820	L_2
M_2^*	N_1	9.950	9.9934	0.880	0.857	0.900	19.894	L_3
	N_2	9.950	9.921	0.879	0.866	0.896	19.892	L_4
M_3^*	N_1	9.979	9.954	0.878	0.869	0.910	19.941	L_5
	N_2	9.970	9.964	0.873	0.869	0.906	19.931	L_6

$$r = 9.996^2 + \cdots + 20.004^2 + \cdots + 20.298^2 = 1224.108656 \qquad (8)$$

Variation of proportional term of transformability:

$$S_\beta = \frac{1}{6r}[(9.996)(58.856) + \cdots + (20.004)(117.70) + (10.164)(59.694) + \cdots + (20.298)(119.296)]^2 = 7049.325990 \qquad (f = 1) \qquad (9)$$

Variation of the first-order term of adjustability, $S_{\beta*}$, can be calculated in a three-level orthogonal polynomial equation because $M*$ is orthogonal to M around M_2^*.

Variation of proportional term of adjustability:

$$S_{\beta*} = \frac{(-L_1 - L_2 + L_5 + L_6)^2}{(2)(2r)} = 0.039582 \qquad (f = 1) \qquad (10)$$

Variation of individual proportional term of transformability:

$$S_L = \frac{L_1^2 + L_2^2 + \cdots + L_5^2 + L_6^2}{r} = 7049.366256 \qquad (f = 6) \qquad (11)$$

Variation of proportional term due to noise:

$$S_{\beta N} = \frac{\begin{bmatrix}(9.996)(29.433) + \cdots + (20.004)(58.855) \\ + (10.164)(29.853) + \cdots + (20.298)(59.653)\end{bmatrix}^2}{(3)(9.996^2 + \cdots + 20.004^2 + 10.164^2 + \cdots + 20.298^2)} + \frac{\begin{bmatrix}(9.996)(29.423) + \cdots + (20.004)(58.847) \\ + (10.164)(29.841) + \cdots + (20.298)(59.643)\end{bmatrix}^2}{(3)(9.996^2 + \cdots + 20.004^2 + 10.164^2 + \cdots + 20.298^2)} - S_\beta = 0.000039 \qquad (12)$$

Residual variation due to individual proportional term:

$$S_{res} = S_L - S_\beta - S_{\beta*} - S_{\beta N} = 0.000645 \qquad (f = 3) \qquad (13)$$

Table 3
ANOVA table of one run of the L_{18} orthogonal array

Level		f	S	V
β:	proportional term	1	7049.325990	7049.325990
$\beta M'$:	positional noise	1	0.377766	0.377766
βN:	compounded noise	1	0.000039	0.000039[a]
β*:	proportional term	1	0.039582	0.039582
res:	residual	3	0.000645	0.000215[a]
e:	error	65	0.020853	0.000321[a]
e′:	error (after pooling factors indicated by [a])	69	0.021537	0.000312
Total		72	7049.764914	

[a] Factors to be pooled.

Now, looking at these experimental data in detail, we notice that contraction rates for die dimensions corresponding to signal factors M_3, M_4, M_9, and M_{10} have a tendentious difference compared to other dimensions, because of mold structure, including gate position or thickness. Since we have a similar tendency for other experiments of the L_{18} orthogonal array, by substituting M_1' for M_1, M_2, M_5, M_6, M_7, M_8, M_{11}, and M_{12}, and M_2' for M_3, M_4, M_9, and M_{10}, we calculated the variation of interaction between M's and β and removed this from error variation.

$$S_{\beta M'} = \text{(variation of proportional term for factors other than } M_3, M_4, M_9, \text{ and } M_{10}) + \text{(variation of proportional term for } M_3, M_4, M_9, \text{ and } M_{10}) - S_\beta$$

$$= \frac{[(9.996)(58.856) + \cdots + (20.298)(119.296)]^2}{(6)(9.996^2 + \cdots + 20.298^2)} + \frac{[(1.018)(5.413) + \cdots + (1.043)(5.189)]^2}{(6)(1.018^2 + \cdots + 1.043^2)} - S_\beta$$

$$= \frac{(7173.532160)^2}{(6)(1219.848126)} + \frac{(21.941819)^2}{(6)(4.26053)} - S_\beta$$

$$= 7030.870285 + 18.833471 - S_\beta$$

$$= 0.377766 \qquad (f = 1) \tag{14}$$

Error variation:

$$S_e = S_T - S_\beta - S_{\beta M'} - S_{\beta N} - S_{\beta *} - S_{\text{res}}$$

$$= 0.020853 \qquad (f = 65) \tag{15}$$

To summarize the above, we show Table 3 for ANOVA (analysis of variance). Based on this result, we compute the SN ratio and sensitivity.

SN ratio of transformability:

$$\eta = \frac{[1/(6)(1224.108656)](7049.325990 - 0.000312)}{0.000312}$$

$$= 3076.25$$

$$10 \log \eta = 34.88 \text{ dB} \tag{16}$$

Since the range of level of holding pressure, 250 kgf/cm^2, does not have any significance as an absolute value when its SN ratio is calculated, we set $h = 1$.

SN ratio of adjustability:

$$\eta^* = \frac{[1/(2)(2)(1224.108656)(1^2)](0.039582 - 0.000312)}{0.000312}$$

$$= 0.02571$$

$$10 \log \eta^* = -15.90 \text{ dB} \tag{17}$$

For the sensitivity of transformability, S, we calculated the sensitivity, S_2, of signal factors M_3, M_4,

M_9, and M_{10}, and the S_1 values of other signal factor levels, respectively, because we found a different shrinkage rate for a different portion of a model gear. In short, for the data in Table 3, we can calculate each sensitivity according to each variation of proportional term in equation (14), as follows.

Sensitivity of dimensions M'_1:

$$S_1 = \frac{1}{(6)(1219.848126)}(7030.870285 - 0.000154) = 0.960621$$

$$10 \log S_1 = -0.17 \text{ dB} \tag{18}$$

Sensitivity of dimensions M'_2:

$$S_2 = \frac{1}{(6)(4.26053)}(18.833471 - 0.000787) = 0.736711$$

$$10 \log S_2 = -1.33 \text{ dB} \tag{19}$$

4. Factors and Levels

We designed a model gear for our molding experiment. Table 4 shows control factor and levels. As control factors, we selected mold temperature (for both fixed and movable), cylinder temperature, injection speed, injection pressure, and cooling time from molding conditions. In addition, after-molding part placement, which is believed to affect the dimensional accuracy of small, precise parts, was also chosen as one of the control factors. According to the sliding-level method, we related the low level of the fixed mold to the low level of the movable mold as level 2, which is the same temperature as that of the fixed mold, and set the low level ±5°C to levels 1 and 3, respectively. The control factors were assigned to an L_{18} as the inner array. The signal and noise factors were assigned to the outside.

Following the foregoing procedure, we can compute each SN ratio of each experiment in the L_{18} orthogonal array as shown in Table 5. Table 6 shows the level-by-level SN ratios, and Figure 2 plots the factor effects.

5. Estimation of Molding Conditions and Confirmatory Experiment

According to the results obtained thus far, the control factors affecting transformability to a large degree are B, D, E, F, and G. However, B, E, and G are either peaked or V-shaped. Since control factor B represents a difference between the upper and lower mold temperatures, B_2 was set to the condition with no temperature difference, and others to a configuration with a certain difference. Judging from Figure 3, showing A's factor effect for each of B's levels, we can see that all B factors except B_2 are regarded as unstable because their effects on a

Table 4
Control and noise factors and levels

Factor		Level 1	Level 2	Level 3
A:	mold temperature (fixed die)	Low (A_1)	High (A_2)	—
B:	mold temperature (movable die)	$A_1 - 5$ $A_2 - 5$	A_1 A_2	$A_1 + 5$ $A_2 + 5$
C:	cylinder temperature	Low	Current	High
D:	injection speed	Slow	Current	Fast
E:	injection pressure	Low	Current	High
F:	cooling time	Short	Current	Long
G:	part placement	I	II	III

Table 5
SN ratios of the L_{18} orthogonal array (dB)

No.	Transformability	Adjustability	No.	Transformability	Adjustability
1	34.88	−15.90	10	33.53	−24.26
2	34.38	−17.82	11	36.07	−14.38
3	32.73	−13.78	12	30.07	−21.60
4	34.96	−15.87	13	35.46	−14.81
5	34.69	−16.41	14	33.39	−22.37
6	35.57	−21.92	15	36.29	−14.55
7	35.38	−14.82	16	33.17	−22.37
8	28.97	−31.51	17	35.49	−16.22
9	36.54	−14.91	18	31.14	−32.88

model gear are not constant due to a temperature difference. This is the reason that B has a peak in the plot. Although we believed that control factor E assumes no peaked shape, we suggest that this may be caused by certain interactions. We should reexamine this phenomenon in the future. Since control factor G is associated with part placement and is not continuous, it can become peaked. Thus, the best level of G, G_2, demonstrates that our current part placement is best. The adjustability SN ratio shows a tendency similar to that of transformability. Finally, as the optimal configuration, we selected the combination of $A_1B_2C_1D_1E_3F_3G_2$ because we judged that B and E should be excluded in calculating the SN ratio, due to their instability. The SN ratio at the optimal configuration was calculated as follows.

Transformability:

$$\mu = 35.70 + 34.72 + 35.11 - (2)(34.06) = 37.41 \text{ dB} \quad (20)$$

Adjustability:

$$\mu^* = -15.42 - 16.05 - 18.26 + (2)(19.28) = -11.17 \text{ dB} \quad (21)$$

The SN ratio at the current configuration of $A_1B_2C_2D_2E_2F_2G_2$ was calculated as follows.

Table 6
Average SN ratios by level (dB)

Factor	Transformability 1	Transformability 2	Transformability 3	Adjustability 1	Adjustability 2	Adjustability 3
A: mold temperature (fixed die)	34.23	33.87	—	−18.18	−20.38	—
B: mold temperature (movable die)	33.66	35.06	33.45	−18.07	−17.66	−22.12
C: cylinder temperature	34.56	33.83	33.77	−18.12	−19.79	−19.94
D: injection speed	35.70	33.57	32.89	−15.42	−19.72	−22.70
E: injection pressure	34.40	33.02	34.74	−20.50	−20.62	−16.73
F: cooling time	33.46	33.99	34.72	−20.44	−21.35	−16.05
G: part placement	33.84	35.11	33.22	−19.04	−18.26	−20.54

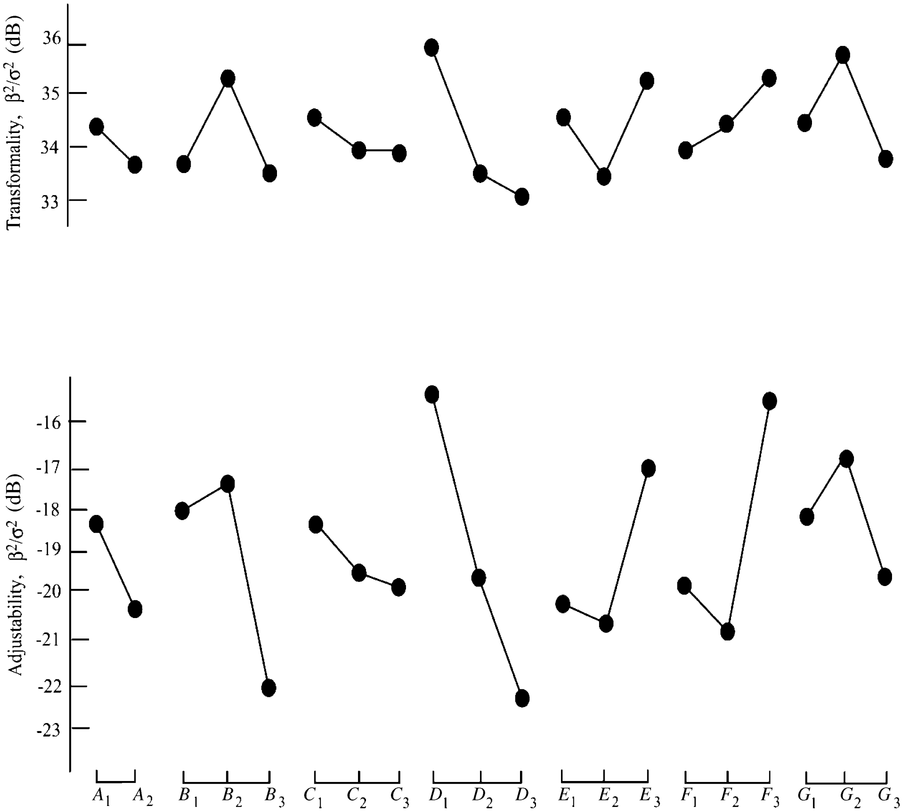

Figure 2
Response graphs

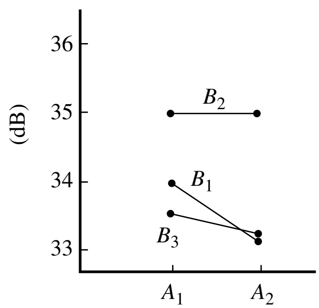

Figure 3
Interaction between *A* and *B*

Transformability:

$$\mu = 33.57 + 33.99 + 35.11 - (2)(34.06)$$
$$= 34.55 \text{ dB} \quad (22)$$

Adjustability:

$$\mu^* = -19.72 - 21.35 - 18.26 - (2)(19.28)$$
$$= -20.77 \text{ dB} \quad (23)$$

As a result, we can obtain (37.41 − 34.55) = 2.86 dB and (−11.17 + 20.77) = 9.60 dB as the gains of transformability and adjustability.

Although we chose the third and twentieth shots as error factor levels, we found that there was only a small fluctuation between them because our gear model quickly became stable after being molded, due to its small dimension. Indeed, our original setup of signal factor levels were not wide enough;

Table 7
ANOVA table of confirmatory experiment (optimal configuration)

Level		f	S	V
β:	proportional term	1	7075.810398	7075.810398
β*M′*:	positional noise	1	0.318380	0.318380
β*N*:	compounded noise	1	0.000202	0.000202[a]
β*:	first-order term	1	0.038291	0.038291
res:	residual	4	0.000442	0.000111[a]
e:	error	64	0.013529	0.000211[a]
e′:	error (after pooling factors indicated by [a])	69	0.014173	0.000205
Total		72	7076.181243	

[a] Factors to be pooled.

Table 8
ANOVA table of confirmatory experiment (current configuration)

Level		f	S	V
β:	proportional term	1	7065.759455	7065.759455
β*M′*:	positional noise	1	0.311792	0.311792
β*N*:	compounded noise	1	0.000128	0.000128[a]
β*:	first-order term	1	0.044723	0.044723
res:	residual	4	0.002015	0.000504[a]
e:	error	64	0.017974	0.000281[a]
e′:	error (after pooling factors indicated by [a])	69	0.020117	0.000292
Total		72	7066.136083	

[a] Factors to be pooled.

Table 9
Estimation and confirmation (dB)

	Configuration		
	Optimal	Current	Gain
Transformability			
Estimation	37.41	34.55	2.86
Confirmation	36.72	35.18	1.54
Adjustability			
Estimation	−11.17	−20.77	9.6
Confirmation	−16.08	−16.84	0.76

Table 10
Sensitivity *S* for confirmatory experiment

	Configuration	
	Current	Optimal
Transformability S_1	0.962783	0.964159
S_2	0.757898	0.757105
Adjustability S_1	0.044507	0.031529
S_2	0.000166	0.000152

however, by prioritizing transformability, we performed a confirmatory experiment based on the optimal configuration $A_1B_2C_1D_1E_3F_3G_2$ and current configuration $A_1B_2C_2D_2E_2F_2G_2$. Tables 7 and 8 show the ANOVA tables, and we summarize the SN ratios in Table 9.

Although we obtained fairly good reproducibility of transformability, for adjustability we concluded that we should examine the reason that its reproducibility was not satisfactory. Since the shrinkage of different dimensions in a piece is different, sensitivity from M_3, M_4, M_9 and M_{10}, and also that from other dimensions, were calculated from the confirmatory experiment, as shown in Table 10. No significant differences were found.

Reference

Takayoshi Matsunaga and Shinji Hanada, 1992. *Technology Development of Transformability*. Quality Engineering Application Series. Tokyo: Japanese Standards Association, pp. 83–95.

This case study is contributed by Takayoshi Matsunaga and Shinji Hanada.