## CASE 15

# Robust Design for Frequency-Modulation Circuits

**Abstract:** In our research we verified highly repeatable experimentation (measurement characteristics) in regard to resonant circuits such as oscillator and modulation circuits, typical in the electrical engineering field.

## 1. Introduction

The majority of tasks requisite for electrical circuit design involve adjustment of quality characteristics to target values. More specifically, by implementing product confirmatory experiments called environmental or reliability tests, we have made a strong effort to match the quality characteristics to final specifications through the modification of part characteristics if the quality characteristics go beyond the standards. However, since this procedure is regarded only as "adjustment" and robustness cannot be built in, complaints in the marketplace regarding products have not been reduced significantly. This is the reason that parameter design in quality engineering is needed desperately. On the other hand, most conventional examples to which parameter design has been applied have not had sufficient repeatability because their designs are implemented based on measurement of powers such as output voltage. We assume that one of the most important characteristics related to experimentation is reproducibility. Even if a certain function is stabilized in a laboratory, if it cannot be reproduced under mass-production conditions, the experiments will be wasted.

## 2. Generic Function

As shown in Figure 1, the experimental circuit is designed based on a combination of a Corvit oscillator and a variable capacitor (voltage-variable capacitor). It consists of an inverter IC, ceramic oscillator ($C_3$), two resistors ($R_1$ and $R_2$), two capacitors ($C_1$ and $C_2$), and two variable capacitors ($C_{v1}$ and $C_{v2}$), whose capacitance varies according to the voltage.

As illustrated in Figure 2, in actual modulation, when an alternating current such as a human voice is given on a direct bias voltage, the output voltage alternates between the lower and upper voltages around the bias voltage. This alternating signal makes the terminal capacitance, and consequently the frequency, change.

Therefore, the proportional relationship between voltages imposed on the variable capacitors and the frequency is the function of the modulation circuit. We selected the voltage, $V_t$, of the variable capacitor as a signal factor, set the dc bias voltage to 2 V, and altered this voltage within the range of $\pm 1.0$ V at an interval of 0.5 V equivalent to one level. In sum, as shown in Table 1, we laid out five levels.

The generic function of a frequency-modulation circuit is to maintain a relationship between a signal voltage, $V_t$, and a frequency deviation (difference between an initial and a deviated frequency; Figure 3). Obviously, the frequency deviation should not fluctuate in accordance with usage and environmental conditions. In our study, noise factors such as environmental temperature or time drift can be regarded as contained in the fluctuation of the power supply's voltage. After all, we chose only the power source's voltage as a noise factor, which ranges between $\pm 0.5$ V, $N_1$ and $N_2$, around the power source's initial voltage.

Based on the design of experiments mentioned above, we collected measured data. As an example,

*Taguchi's Quality Engineering Handbook.* Genichi Taguchi, Subir Chowdhury and Yuin Wu

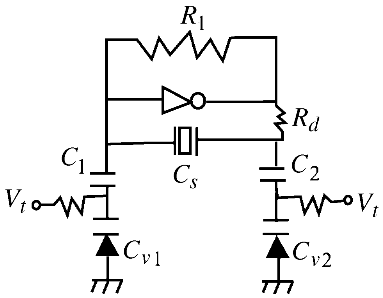

**Figure 1**
Experimental circuit

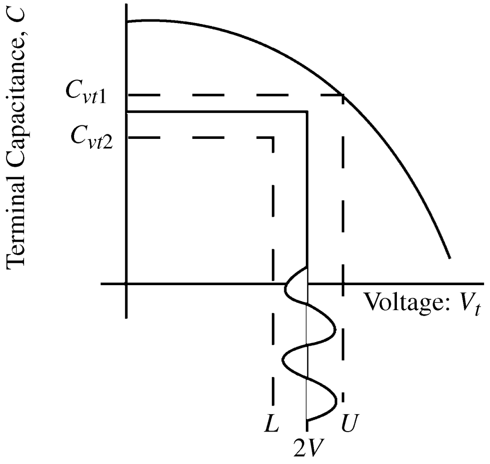

**Figure 2**
Frequency modulation voltage $V_t$ and terminal capacitance

**Table 1**
Signal factors and levels (V)

| | Signal | | | | |
|---|---|---|---|---|---|
| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
| Absolute voltage | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| Signal voltage | −1.0 | −0.5 | ±0 | +0.5 | +1.0 |

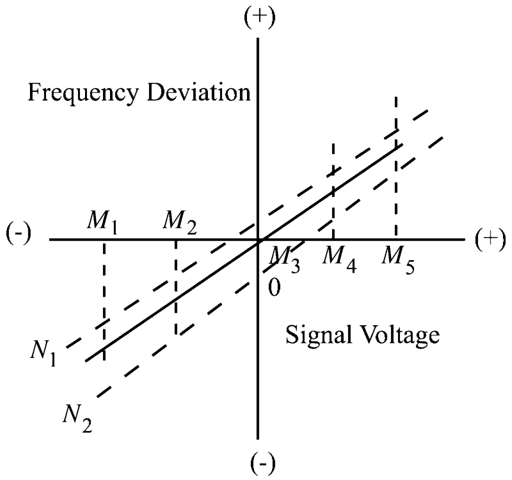

**Figure 3**
Generic function

raw and converted data of experiment 1 are shown in Table 2. Since under the initial state (initial power source's and signal voltages were 0 V) in experiment 1, the frequency ($M_N$) was 453,286; the converted frequency deviation was equal to the corresponding raw frequency minus the initial frequency, $M_N$. Next, using these data, we decomposed the total variation.

Total variation:

$$S_T = y_{11}^2 + y_{21}^2 + y_{12}^2 + y_{22}^2 + \cdots + y_{15}^2 + y_{25}^2$$
$$= (-558)^2 + (-542)^2 + \cdots + 661^2$$
$$= 1{,}753{,}035 \qquad (1)$$

Linear equations:

$$L_1 = M_1y_{11} + M_2y_{12} + \cdots + M_4y_{14} + M_5y_{15}$$
$$= (-1.0)(-588) + \cdots + (1.0)(581) = 1453.5$$
$$L_2 = M_1y_{21} + M_2y_{22} + \cdots + M_4y_{24} + M_5y_{25}$$
$$= (-1.0)(-542) + \cdots + (1.0)(661) = 1496.0$$
$$(2)$$

Effective divider:

$$r = M_1^2 + M_2^2 + M_3^2 + M_4^2 + M_5^2$$
$$= (1.0)^2 + (-0.5)^2 + 0.5^2 + 1.0^2 = 2.5 \qquad (3)$$

**Table 2**
Results of experiment 1

| Noise | Signal | | | | |
|---|---|---|---|---|---|
| | $M_1$ (−1.0) | $M_2$ (−0.5) | $M_3$ (0.0) | $M_4$ (+0.5) | $M_5$ (+1.0) |
| Raw data | | | | | |
| $N_1$ | 452,698 | 452,971 | 453,246 | 453,540 | 453,867 |
| $N_2$ | 452,744 | 453,022 | 453,304 | 453,608 | 453,947 |
| Converted data | | | | | |
| $N_1$ | −588 | −315 | −40 | 254 | 581 |
| $N_2$ | −542 | −264 | 18 | 322 | 661 |

Variation of proportional term:

$$S_\beta = \frac{(L_1 + L_2)^2}{2\gamma}$$

$$= \frac{(1453.5 + 1496.0)^2}{(2)(2.5)(2.5)}\ 1{,}739{,}910.05 \qquad (4)$$

Interaction between proportional term and noise:

$$S_{N\beta} = \frac{L_1^2 + L_2^2}{\gamma - S_\beta} = \frac{(1453.5 + 1496.0)^2}{2.5 - 1{,}739{,}910.05}$$

$$= 361.25 \qquad (5)$$

Error variation:

$$S_e = S_T - (S_\beta + S_{N\beta})$$

$$= 1{,}753{,}035 - (1{,}739{,}910.05 + 361.25)$$

$$= 12{,}763.70 \qquad (6)$$

Error variance:

$$\frac{V_e = S_e}{8} = 1595.46 \qquad (7)$$

Total error variance:

**Table 3**
Control factors and levels

| Control Factor | | Level | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| *A*: | ceramic oscillation element type | CSB 455 | B456 F15 | — |
| *B*: | resistance | 680 | 820 | 1000 |
| *C*: | resistance | 3.3 | 4.7 | 5.6 |
| *D*: | capacitance | 560 | 680 | 820 |
| *E*: | capacitance | Two levels below $C_1$ | One level below $C_1$ | Same as $C_1$ |
| *F*: | capacitor type | Ceramic | Chip | Film |
| *G*: | capacitor type | Ceramic | Chip | Film |
| *H*: | power source voltage | 3 | 5 | 7 |

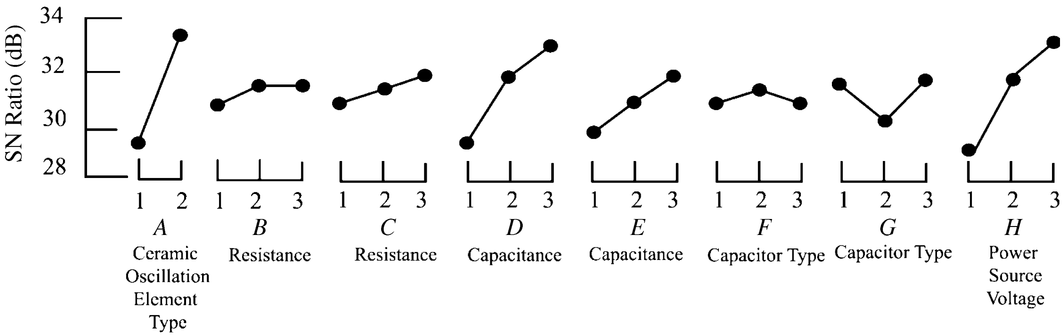

**Figure 4**
Response graph of SN ratio

$$V_N = \frac{S_e + S_{N\beta}}{9} = 1458.33 \qquad (8)$$

SN ratio:

$$\eta = 10 \log \frac{(1/2\gamma)(S_\beta - V_e)}{V_N} = 23.77 \text{ dB} \qquad (9)$$

Sensitivity:

$$S = 10 \log \frac{S_\beta - V_e}{2\gamma} = 55.41 \text{ dB} \qquad (10)$$

## 3. Results of Experiment

As control factors we selected seven constant values and types of parts, shown in Figure 1. Additionally, the voltage of the power voltage supplied to the inverter, IC, was chosen as another control factor. All the control factors were assigned to an $L_{18}$ orthogonal array. Table 3 lists the control factors and levels.

Using the aforementioned calculation formulas, we analyzed all data from experiments 1 to 18 and created the response graph of SN ratio and sensitivity as Figure 4. The fact that the trend of factor *G* is V-shaped does not indicate the interaction between control factors because *G*, which represents a capacitor type, is not continuous.

## 4. Confirmatory Experiment and Analysis

Based on the factor effect plot, we defined optimal and current configurations and conducted confirmatory experiments. Although we selected the highest level for each control factor, we chose a chip capacitor as most practical for surfacing because there was no significant difference among all types of capacitors. In sum, the configurations are:

*Optimal:* $A_2B_3C_3D_3E_3F_2G_2H_3$

*Current:* $A_1B_2C_2D_2E_2F_2G_2H_2$

**Table 4**
Confirmatory experiment

| | SN Ratio | | Sensitivity | |
|---|---|---|---|---|
| **Configuration** | **Estimation** | **Confirmation** | **Estimation** | **Confirmation** |
| Optimal | 37.53 | 37.27 | 60.25 | 60.32 |
| Current | 29.78 | 29.77 | 57.32 | 57.24 |
| Gain | 7.74 | 7.5 | 2.94 | 3.08 |

Table 4 shows the estimated values and results of confirmatory experiments. Both SN ratios and sensitivities have quite good repeatability. This fact proves that to measure frequencies as characteristics or response variables is effective enough to obtain good repeatability. On the other hand, the optimal configuration is 7.50 dB better than the current one and halves the functional variability, due to the environmental temperature and fluctuation of power source's voltage.

The fact that we obtained highly repeatable results can be regarded as epoch-making in that the results can certainly be reproduced in the marketplace in the mass-production phase. For instance, we have designed the stability of a filter by setting the output voltage as a characteristic value (normally the logarithm of the ratio of input to output) at several frequencies around the cutoff frequency. However, because there is a radical change near the boundary between the pass and attenuation bands, the output voltage changes from 0 to 1 under a certain use and consequently does not function well as an addable characteristic. Our procedure can be applied to a wide range of devices, including digital circuits.

## Reference

Yoshishige Kanemoto, 1998. Robust design for frequency modulation circuit. *Quality Engineering*, Vol. 6, No. 5, pp. 33–37.

*This case study is contributed by Yoshishige Kanemoto.*