30 One-Way Layout

30.1. Introduction

One-way layout is a type of experiment with one factor with k conditions (or k levels). The number of repetitions of each level may be the same or may be different; they are shown as $n_1, n_2, \ldots, n_k$.

The experiments based on a k value of 2 were described in Chapter 29. In this chapter, experiments with k equal to or larger than 3 are described. For a more precise comparison, it is desirable that the number of repetitions in each level be the same. However, if this is not possible, the number of repetitions in each level can vary. This chapter is based on Genichi Taguchi et al., *Design of Experiments*. Tokyo: Japanese Standards Association, 1973.

30.2. Equal Number of Repetitions

In pain-killer experiments, if the dosage of a new pain killer were changed in three ways, such as

$$A_2 = 1 \text{ g/day}$$

$$A_3 = 2 \text{ g/day}$$

$$A_4 = 4 \text{ g/day}$$

and the result compared with the existing pain killer A_1 (the standard dosage), there would be four levels for factor A.

Suppose that an experiment for factor A were carried out with the same number of repetitions (n) for each of the a levels: $A_1, A_2, \ldots, A_a$. The data would be tabulated as shown in Table 30.1.

Table 30.1
Data with *n* repetitions

Level of *A*	Data	Total
A_1	$y_{11}\ y_{12} \cdots y_{1n}$	A_1
A_2	$y_{21}\ y_{22} \cdots y_{2n}$	A_2
⋮	⋮	⋮
Total	$y_{a1}\ y_{a2}\ y_{an}$	A_a

Suppose that datum y_{ij} denotes the deviation from the objective value, y_0. Variations are calculated from $A_1, A_2, \ldots, A_a$.

$$S_m = \frac{(A_1 + A_2 + \cdots + A_a)^2}{an} \qquad (f = 1) \tag{30.1}$$

$$S_A = \frac{A_1^2 + A_2^2 + \cdots + A_a^2}{} - S_m \qquad (f = a - 1) \tag{30.2}$$

$$S_T = y_{11}^2 + y_{12}^2 + \cdots + y_{an}^2 \qquad (f = an) \tag{30.3}$$

$$S_e = S_T - S_m - S_A \qquad [f = a(n-1)] \tag{30.4}$$

The ANOVA table is shown in Table 30.2. The symbols in the table signify the following:

$$V_m = \frac{S_m}{1} = S_m \tag{30.5}$$

$$V_A = \frac{S_A}{a - 1} \tag{30.6}$$

$$V_e = \frac{S_e}{a(n-1)} \tag{30.7}$$

$$S'_m = S_m - V_e \tag{30.8}$$

$$S'_A = S_A - (a - 1)V_e \tag{30.9}$$

$$S'_e = S_e + aV_e \tag{30.10}$$

$$\rho_m = \frac{S'_m}{S_T} \tag{30.11}$$

$$\rho_A = \frac{S'_A}{S_T} \tag{30.12}$$

$$\rho_e = \frac{S'_e}{S_T} \tag{30.13}$$

Table 30.2
ANOVA table

Source	f	S	V	S'	ρ (%)
m	1	S_m	V_m	S'_m	ρ_m
A	$a - 1$	S_A	V_A	S'_A	ρ_A
e	$a(n - 1)$	S_e	V_e	S'_e	ρ_e
Total	an	S_T		S_T	100.0

Estimation is made as follows: When A is insignificant, only the general mean is estimated in most cases. That is to determine that the effect of A is inconsequential; accordingly, S_A is pooled with S_e to get a new pooled error variance (using the same symbol, V_e):

$$V_e = \frac{S_A + S_e}{(a - 1) + a(n - 1)} = \frac{S_A + S_e}{an - 1} \tag{30.14}$$

❒ Example

In a pinhole manufacturing process, the order of processing was changed in three levels. The experiment was carried out with 10 repetitions for each level. The roundness of pinholes were measured, as shown in Table 30.3.

A_1: (penetrate the lower hole) – (ream the lower hole) – (ream) – (process the leading hole)

A_2: (process the lower hole to half-depth) – (ream the lower hole) – (ream) – (process the leading hole)

A_3: (process the leading hole) – (process the lower hole) – (ream the lower hole) – (ream)

Table 30.3
Data for roundness (μm)

Level	Data	Total
A_1	10, 15, 3, 18, 8, 4, 6, 10, 0, 13	87
A_2	12, 14, 5, 6, 4, 1, 11, 15, 7, 10	85
A_3	8, 2, 0, 4, 1, 6, 5, 3, 2, 4	35
Total		207

There is an objective value (i.e., zero), in this case, so the general mean, *m*, is tested:

$$S_m = \frac{207^2}{30} = 1428 \tag{30.15}$$

$$S_A = \frac{A_1^2 + A_2^2 + A_3^2}{10} - S_m$$

$$= \frac{87^2 + 85^2 + 35^2}{10} - 1428 = \frac{16{,}019}{10} - 1428$$

$$= 1602 - 1428 = 174 \tag{30.16}$$

$$S_T = 10^2 + 15^2 + 3^2 + \cdots + 4^2 = 2131 \tag{30.17}$$

$$S'_m = 1428 - 19.6 = 1408.4 \tag{30.18}$$

$$S'_A = 174 - (2)(19.6) = 134.8 \tag{30.19}$$

$$S'_e = 529 + (3)(19.6) = 587.8 \tag{30.20}$$

$$\rho_m = \frac{S'_m}{S_T} = \frac{1408.4}{2131} = 0.661 \tag{30.21}$$

$$\rho_A = \frac{134.8}{2131} = 0.063 \tag{30.22}$$

$$\rho_e = \frac{587.8}{2131} = 0.276 \tag{30.23}$$

The analysis of variance is summarized in Table 30.4.

Since the general mean, *m*, and *A* are significant, estimation is made for each level of *A*.

$$\overline{A}_1 = \frac{A_1}{n} = \frac{87}{10} = 8.7 \tag{30.24}$$

Table 30.4
ANOVA table

Source	*f*	*S*	*V*	*S′*	ρ (%)
m	1	1428	1428	1408.4	66.1
A	2	174	87	134.8	6.3
e	27	529	19.6	587.8	27.6
Total	30	2131		2131.0	100.0

Table 30.5
ANOVA table

Source	f	S	V	S'	ρ (%)
m	1	S_m	V_m	S'_m	ρ_m
A	$a - 1$	S_A	V_A	S'_A	ρ_A
e	$n - a$	S_e	V_e	S'_e	ρ_e
Total	n	S_T		S_T	100.0

Similarly,

$$\bar{A}_2 = \frac{85}{10} = 8.5 \qquad (30.25)$$

$$\bar{A}_3 = \frac{35}{10} = 3.5 \qquad (30.26)$$

30.3. Number of Repetitions Is Not Equal

When the numbers or repetitions, $n_1, n_2, \ldots, n_1$, for an a-level factor A, $A_1, A_2, \ldots, A_a$ are not equal, the following calculation is made:

$$S_T = \text{sum of } (n_1 + n_2 + \cdots + n_a) \text{ individual values squared}$$

$$(f = n_1 + n_2 + \cdots + n_a) \qquad (30.27)$$

$$S_m = \frac{(\text{total})^2}{n_1 + n_2 + \cdots + n_a} \qquad (f = 1) \qquad (30.28)$$

$$S_A = \frac{A_1^2}{n_1} + \frac{A_2^2}{n_2} + \cdots + \frac{A_a^2}{n_a} - S_m \qquad (f = a - 1) \qquad (38.29)$$

$$S_e = S_T - S_m - S_A \qquad (f = n_1 + n_2 + \cdots + n_a - a) \qquad (30.30)$$

The analysis of variance is shown in Table 30.5. In the table,

$$n = n_1 + n_2 + \cdots + n_a$$