Part IX

Experimental Regression

26 Parameter Estimation in Regression Equations

26.1. Introduction

Quite often we wish to express results or objectives by functions of causes or means in various specialized fields, such as natural science, social science, or engineering. If we cannot perform experiments to obtain results or objectives after arranging causes or means, that is, factors in a particular manner, we often estimate parameters of a regression function predicted by data obtained from research or observations. Although the least squares method is frequently used to estimate unknown parameters after a linear regression model is selected as a regression function, in most cases this procedure is not used appropriately and predicted regression equations do not function well from the standpoint of specialized engineering fields. Among several methods of avoiding this situation is experimental regression analysis.

26.2. Estimation of Operation Time and Least Squares Method

As a simple example, let's examine a problem of estimating standard operation times for office work. Since office work does not consist of several clear-cut portions and its transaction time has large variability, we need a considerable number of observations to estimate a standard operation time accurately after breaking total time down into many work elements. Now we define k types of major transaction times among various office works as $a_1, a_2, \ldots, a_k$ and the corresponding number of jobs to be completed as $x_1, x_2, \ldots, x_k$. The total operation time y_0 is expressed as follows:

$$y_0 = a_1x_1 + a_2x_2 + \cdots + a_kx_k \tag{26.1}$$

In actuality, since there are jobs other than the k types of work or slack times, we should add a constant a_0 to equation (26.1). Therefore, the total operation time y is

$$y = a_0 + a_1x_1 + a_2x_2 + \cdots + a_kx_k \tag{26.2}$$

Table 26.1 shows the incidence of seven different works ($x_1, x_2, \ldots, x_7$) and corresponding operation time y (minutes) for each business office. For the sake of convenience, we take data for a certain day. However, to improve the adequacy of analysis, we sometimes sum up all data for one month. To estimate $a_0, a_1, a_2, \ldots, a_7$ of the following equation, using these data is our objective:

$$y = a_0 + a_1x_1 + a_2x_2 + \cdots + a_7x_7 \tag{26.3}$$

This process is called *estimation of unknown parameters* $a_0, a_1, a_2, \ldots, a_7$.

An equation such as equation (26.2), termed a *single regression equation* or *linear regression equation*, takes y as the objective variable and $x_1, x_2, \ldots, x_7$ as the explanation variables with respect to unknowns $a_0, a_1, a_2, \ldots, a_7$.

As one of the estimation methods of unknowns in a linear regression equation, the multivariate analysis is known, which is based on the least squares method. By applying the least squares method to the data in Table 26.1, we obtain the following result:

$$\begin{aligned} y &= 551.59 - 72.51x_1 + 25.92x_2 - 44.19x_3 + 65.55x_4 \\ &\quad + 6.58x_5 + 24.02x_6 + 11.35x_7 \end{aligned} \tag{26.4}$$

$$\text{error variance} = V_e = 233{,}468.19$$

Although the true values of $a_0, a_1, a_2, \ldots, a_7$ are unknown, the fact that a_1 are estimated as -72.51 or a_3 is estimated as -44.19 is obviously unreasonable because each of the values indicates the operation time for one job.

In the least squares method, the coefficients $a_1, a_2, \ldots, a_7$ are estimated in such a way that the coefficients may take a range of $(-\infty, \infty)$. It is well known that the estimation accuracy of $a_0, a_1, a_2, \ldots, a_7$ depends significantly on the correlation among $x_1, x_2, \ldots, x_7$. If a certain variable x_1 correlates strongly with other variables, the estimation accuracy of a_1 is decreased; otherwise, it is improved.

Actual data observed always correlate with each other. In this case, since the volume of all jobs is supposed to increase on a busy day, the data $x_1, x_2, \ldots, x_7$ tend to correlate with each other, reflecting actual situations. When the least squares method is applied to this case, the estimation obtained quite often digresses from the true standard time. In calculating appropriate coefficients, the least squares method is not effective.

26.3. Experimental Regression Analysis

To eliminate the drawbacks of the least squares method we need to use a new method that reflects on the following:

Table 26.1
Incidence of works and total operation time

No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y
1	4	6	18	6	58	120	22	3667
2	2	2	16	12	74	85	31	3683
3	2	12	18	10	58	127	37	3681
4	0	2	3	4	82	39	14	2477
5	2	2	2	0	28	31	0	1944
6	2	0	0	0	26	27	12	414
7	0	2	6	8	52	31	14	1847
8	0	0	2	2	110	15	5	1689
9	0	2	6	6	86	31	11	2459
10	0	0	2	8	64	20	4	1934
11	0	0	2	6	84	15	1	2448
12	0	0	4	2	50	13	3	1228
13	0	0	2	4	72	12	2	1234
14	0	0	4	4	32	12	5	1240
15	0	0	2	0	40	7	0	648
16	0	0	2	8	70	13	2	1252
17	0	0	0	0	42	6	1	715
18	0	0	6	2	46	15	4	1309
19	0	2	4	4	84	62	22	3023
20	2	2	12	4	72	57	24	2433
21	2	2	6	6	86	45	0	2447
22	2	0	4	4	74	76	21	3210
23	0	2	2	4	58	24	2	1239
24	0	4	2	2	52	28	13	3635
25	0	2	4	4	94	22	6	1341
26	0	2	4	4	56	24	11	1930
27	0	0	4	8	56	18	4	1940
28	0	0	6	0	56	18	6	7565
29	0	0	2	0	12	3	0	639
30	0	0	2	0	36	4	2	1229
31	0	0	2	4	52	9	2	1239
32	0	0	2	4	94	22	6	1341
33	0	0	2	0	10	4	2	1245

Table 26.1
(*Continued*)

No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y
34	0	0	0	0	48	4	0	748
35	0	0	0	4	36	4	0	1218
36	0	0	2	2	20	5	2	518
37	0	0	0	0	56	5	1	636
38	0	0	2	0	26	5	1	631
39	0	0	0	0	48	2	0	650
40	0	0	0	0	16	1	1	649
41	0	0	0	0	6	1	1	653
42	0	0	0	0	18	1	0	653
43	0	0	0	0	16	2	0	640

1. For unknowns (or coefficients), we set up a practically reasonable range and seek a suitable equation within the range. This is called *quadratic planning*.
2. To rationally seek reasonable coefficients, we narrow down the range of each coefficient using an orthogonal array.
3. When we determine coefficients based on narrowed-down results, we emphasize ideas of users of the equation or experts in specialized engineering fields.

As one of the methods satisfying these requirements, we describe the experimental regression analysis below. This is called *quadratic planning* in the Taguchi method.

Setting of Initial Values

First, by assuming how much time a_0, a_1, a_2, ... , a_7 affects the total operation time y when each piece of work increases by 1 unit (number of jobs or papers), or a rough range of the standard operation time required for each piece of work, we set up three levels within the range. More specifically:

1. After setting the minimum to level 1 and the maximum to level 3, we select the middle of the two values as level 2.
2. After setting the forecasting value to level 2, by creating a certain range before and after level 1 based on level 2's confidence, we establish level 2 − the range as levels 1 and 2 + the range as level 3.

Both of the methods above can be used. For example, we assume that work 1 takes approximately 2 hours on average and has a range of 30 minutes.

- *Level 1 of* a_1: $a_{11} = 90$ minutes
- *Level 2 of* a_1: $a_{12} = 120$ minutes
- *Level 3 of* a_1: $a_{13} = 150$ minutes

These are called *initial values of* a_1. Although the range should be as precise as possible, we do not need to be too sensitive to it. Table 26.2 shows the initial values

Table 26.2
Initial values of coefficients (minutes)

Coefficient	Level 1	2	3
a_1	90	120	150
a_2	0	60	120
a_3	0	30	60
a_4	0	30	60
a_5	0	10	20
a_6	0	20	40
a_7	0	20	40

for other work. Now, we do not set up a variable range of a_1 but calculate it using the following relationship:

$$a_0 = \overline{y} - a_1\overline{x}_1 - a_2\overline{x}_2 - \cdots - a_7\overline{x}_7 \tag{26.5}$$

Selection of Orthogonal Array and Calculation

If, as for level 2, all values of a_0, a_1, a_2, ... , a_7 follow the data in Table 26.1, the total operation time of business office 1, y, is calculated as follows:

$$\begin{aligned} y_1 &= a_0 + a_{12}x_1 + a_{22}x_2 + \cdots + a_{72}x_7 \\ &= a_0 + (120)(4) + (60)(6) + \cdots + (20)(22) \\ &= a_0 + 4980 \end{aligned} \tag{26.6}$$

However, since the actual time is 3667 minutes, $\overline{y_1} - y_1$ indicates the difference between the actual time and the estimate assuming $a_1 = 120$, $a_2 = 60$, ... , $a_7 = 20$ as a unit operation time. This difference involves assumptive errors. On the other hand, if each of the assumed coefficients a has little error, $\overline{y_1} - y_1$ for each business office is not supposed to vary greatly. The magnitude of change can be evaluated by variation (a sum of squares of residuals, S_e). Therefore, we need to do the same calculation and comparison for other business offices. Now, we exclude a_0 in equation (26.3). In this case, $\overline{y}_1 - y_1$ involves not merely assumptive errors regarding $a_1 = 120$, $a_2 = 60$, ... , $a_7 = 20$ but also operation times other than x_1, x_2, ... , x_7.

When we have three assumed values for each of the unknowns a_1, a_2, ... , a_7, the eventual total number of combinations amounts to $3^7 = 2187$. However, we select a certain number of combinations from them. Which combination should be chosen depends on an orthogonal array.

By taking into account the number of unknown parameters, we select the size of an orthogonal array according to Table 26.3. The orthogonal arrays shown in Table 26.3 have no particular columns where inter-column interactions emerge. Since we can disperse interaction effects between parameters for the sum of squares of residuals S_e to various columns by using these arrays, we can perform sequential approximation even if there are no interactions.

Table 26.3
Selection of orthogonal array

Number of Unknown Parameters	Orthogonal Array to Be Used
2	3 × 3 two-dimensional layout
3–6	Three-level portion of L_{18} orthogonal array
7–13	Three-level portion of L_{36} orthogonal array
14–49	Three-level portion of L_{108} orthogonal array

Because the L_{18} orthogonal array has seven three-level columns, we can allocate up to seven parameters. However, if the actual calculation does not take long, we recommend that the L_{36} array be used in cases of seven parameters or more. For this case, we utilize the L_{36} array, where there are only seven unknown parameters. Table 26.4 illustrates the L_{36} array. In the table, each number from 1 to 13 in the top horizontal row shows a type of unknown, and each digit 1, 2, and 3 in the table itself indicates the unknown's level. Each row in the table represents a combination of levels: 36 combinations in case of an L_{36} orthogonal array.

Now, we assign each of the seven unknowns $a_1, a_2, \ldots, a_7$ to each column from 1 to 7. This process is called *assignment* (*layout*) *to an orthogonal array*. As a consequence, the numbers in rows 1 to 36 represent the following 36 equations:

$$\begin{aligned} &\textit{Row 1:} \quad y = 90x_1 + 0x_2 + \cdots + 0x_7 \\ &\textit{Row 2:} \quad y = 120x_1 + 60x_2 + \cdots + 20x_7 \\ &\qquad \vdots \\ &\textit{Row 36:} \quad y = 150x_1 + 60x_2 + \cdots + 20x_7 \end{aligned} \tag{26.7}$$

As a next step, we plug $x_1, x_2, \ldots, x_7$ of 43 business offices into these equations and calculate the following differences between the result calculated and its corresponding actual operation time:

$$e_{ij} = y_{ij} - y_j \qquad (i = 1,2, \ldots, 36; j = 1,2, \ldots, 43) \tag{26.8}$$

Then we compute a sum of all of the differences and variation as follows:

$$T_1 = e_{11} + e_{12} + \cdots + e_{143} \tag{26.9}$$

$$S_1 = e_{11}^2 + e_{12}^2 + \cdots + e_{143}^2 - \frac{T_1^2}{43} \tag{26.10}$$

Table 26.5 summarizes the result for the first data in Table 26.2. However, the data in the first column of Table 26.5 do not indicate T itself but the averaged T, which is equivalent to the constant term a_0 in each equation.

Sequential Approximation

Using Table 26.5, we compute a level-by-level sum of S values for each of the unknown coefficients $a_1, a_2, \ldots, a_7$, as shown in Table 26.6. For each unknown we compare three different level-by-level sums of S. If all of them are almost the same, any of the three levels assumed for each coefficient leads to the same amount of

Table 26.4
L_{36} orthogonal array

	Factor												
No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	1
3	3	3	3	3	3	3	3	3	3	3	3	3	1
4	1	1	1	1	2	2	2	2	3	3	3	3	1
5	2	2	2	2	3	3	3	3	1	1	1	1	1
6	3	3	3	3	1	1	1	1	2	2	2	2	1
7	1	1	2	3	1	2	3	3	1	2	2	3	1
8	2	2	3	1	2	3	1	1	2	3	3	1	1
9	3	3	1	2	3	1	2	2	3	1	1	2	1
10	1	1	3	2	1	3	2	3	2	1	3	2	1
11	2	2	1	3	2	1	3	1	3	2	1	3	1
12	3	3	2	1	3	2	1	2	1	3	2	1	1
13	1	2	3	1	3	2	1	3	3	2	1	2	2
14	2	3	1	2	1	3	2	1	1	3	2	3	2
15	3	1	2	3	2	1	3	2	2	1	3	1	2
16	1	2	3	2	1	1	3	2	3	3	2	1	2
17	2	3	1	3	2	2	1	3	1	1	3	2	2
18	3	1	2	1	3	3	2	1	2	2	1	3	2
19	1	2	1	3	3	3	1	2	2	1	2	3	2
20	2	3	2	1	1	1	2	3	3	2	3	1	2
21	3	1	3	2	2	2	3	1	1	3	1	2	2
22	1	2	2	3	3	1	2	1	1	3	3	2	2
23	2	3	3	1	1	2	3	2	2	1	1	3	2
24	3	1	1	2	2	3	1	3	3	2	2	1	2
25	1	3	2	1	2	3	3	1	3	1	2	2	3
26	2	1	3	2	3	1	1	2	1	2	3	3	3
27	3	2	1	3	1	2	2	3	2	3	1	1	3
28	1	3	2	2	2	1	1	3	2	3	1	3	3
29	2	1	3	3	3	2	2	1	3	1	2	1	3
30	3	2	1	1	1	3	3	2	1	2	3	2	3
31	1	3	3	3	2	3	2	2	1	2	1	1	3
32	2	1	1	1	3	1	3	3	2	3	2	2	3

Table 26.4
(Continued)

	Factor												
No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
33	3	2	2	2	1	2	1	1	3	1	3	3	3
34	1	3	1	2	3	2	3	1	2	2	3	1	3
35	2	1	2	3	1	3	1	2	3	3	1	2	3
36	3	2	3	1	2	1	2	3	1	1	2	3	3

error. However, if S for level 2 is small while S values for levels 1 and 3 are large, the true value of the coefficient a_i will lie close to level 2. In this case, by resetting three levels in the proximity to level 2, we repeat the calculation. For practical calculation, we determine new level values using level-by-level sums of S values in Table 26.6 according to Table 26.7.

Now, for V_e shown in the determinant column of Table 26.7, we use error variance obtained by the least squares method. Indeed, coefficients calculated by the least squares method are unreliable; however, error variances can be considered trustworthy. For the sake of convenience, we recommend using 3 as the F-test criterion because a value of 2 to 4 has been regarded as the most appropriate in our experience.

For instance, coefficient a_1 has three levels 90, 120, and 150. Table 26.6 shows the following:

$$S(a_{11}) = 0.39111173 \times 10^9$$

$$S(a_{12}) = 0.34771989 \times 10^9$$

$$S(a_{13}) = 0.36822313 \times 10^9$$

The fact that the value of S at level 1 is the largest and the value at level 2 is the smallest indicates a V-shaped type (type V), in particular, pattern 3.

$$\frac{(1/r)\ (S_3 - S_2)}{V_e} = \frac{\frac{1}{12}\ (0.36822313 \times 10^9 - 0.34771989 \times 10^9)}{23{,}346.19}$$

$$= 87.8 > 3 \qquad (26.11)$$

In this equation, we set r to 12 because both S_3 and S_2 are sums of 12 data.

Accordingly, the new levels for coefficient a_1 in the second round are established in the proximity to level 2. By proceeding with this process, we obtain the new level setting, as illustrated in Tables 26.7 and 26.8.

Selection of Estimated Values

Based on the newly selected levels, we repeat the same calculation as in the first round. In addition, after making the same judgment as that shown in Table 26.7, for coefficients not converging sufficiently, we set up the three new levels and move on to the third round. In this case, all coefficients have converged in the fifth round, as summarized Table 26.9.

Table 26.5
T and *S* in first round

No.	Averaged *T*	Variation *S*	Combination												
1	1,553.58140	36,740,548.5	1	1	1	1	1	1	1	1	1	1	1	1	1
2	143.581395	16,829,328.5	2	2	2	2	2	2	2	2	2	2	2	2	1
3	−1,266.41860	11,911,050.8	3	3	3	3	3	3	3	3	3	3	3	3	1
4	4,221.953488	10,251,155.9	1	1	1	1	2	2	2	2	3	3	3	3	1
5	−988.046512	81,015,535.9	2	2	2	2	3	3	3	3	1	1	1	1	1
6	996.837209	14,439,209.9	3	3	3	3	1	1	1	1	2	2	2	2	1
7	497.069767	18,175,430.8	1	1	2	3	1	2	3	3	1	2	2	3	1
8	−225.023256	45,653,675.0	2	2	3	1	2	3	1	1	2	3	3	1	1
9	158.697674	11,285,447.1	3	3	1	2	3	1	2	2	3	1	1	2	1
10	128.465116	46,156,958.7	1	1	3	2	1	3	2	3	2	1	3	2	1
11	512.186047	9,359,614.51	2	2	1	3	2	1	3	1	3	2	1	3	1
12	−209.906977	19,597,979.6	3	3	2	1	3	2	1	2	1	3	2	1	1
13	−232.930233	18,923,170.8	1	2	3	1	3	2	1	3	3	2	1	2	2
14	212.186047	45,252,574.5	2	3	1	2	1	3	2	1	1	3	2	3	2
15	451.488372	10,126,044.7	3	1	2	3	2	1	3	2	2	1	3	1	2
16	909.395349	13,132,844.3	1	2	3	2	1	1	3	2	3	3	2	1	2
17	231.255814	13,060,792.2	2	3	1	3	2	2	1	3	1	1	3	2	2
18	−709.906977	52,139,539.6	3	1	2	1	3	3	2	1	2	2	1	3	2
19	−692.00000	43,558,928.0	1	2	1	3	3	3	1	2	2	1	2	3	2
20	1,174.27907	16,310,512.7	2	3	2	1	1	1	2	3	3	2	3	1	2
21	−51.5348837	26,925,158.7	3	1	3	2	2	2	3	1	1	3	1	2	2
22	40.7906977	12,195,333.1	1	2	2	3	3	1	2	1	1	3	3	2	2
23	441.953488	31,986,195.9	2	3	3	1	1	2	3	2	2	1	1	3	2
24	−52.000000	27,263,368.0	3	1	1	2	2	3	1	3	3	2	2	1	2
25	−433.162791	71,126,909.9	1	3	2	1	2	3	3	1	3	1	2	2	3
26	209.395349	15,054,924.3	2	1	3	2	3	1	1	2	1	2	3	3	3
27	654.511628	13,870,504.7	3	2	1	3	1	2	2	3	2	3	1	1	3
28	717.534884	13,035,738.7	1	3	2	2	2	1	1	3	2	3	1	3	3
29	−508.279070	28,202,412.7	2	1	3	3	3	2	2	1	3	1	2	1	3
30	221.488372	48,144,624.7	3	2	1	1	1	3	3	2	1	2	3	2	3
31	−598.046512	78,524,235.9	1	3	3	3	2	3	2	2	1	2	1	1	3
32	254.511628	13,377,984.7	2	1	1	1	3	1	3	3	2	3	2	2	3

Table 26.5
(*Continued*)

No.	Averaged *T*	Variation *S*	Combination												
33	774.279070	13,789,392.7	3	2	2	2	1	2	1	1	3	1	3	3	3
34	−438.976744	29,290,475.0	1	3	1	2	3	2	3	1	2	2	3	1	3
35	264.976744	31,616,335.0	2	1	2	3	1	3	1	2	3	3	1	2	3
36	604.744186	11,531,352.2	3	2	3	1	2	1	2	3	1	1	2	3	3

As a reference, see Tables 26.10 and 26.11, showing averaged T and S values and level-by-level sums of S', respectively. When all coefficients are converged, we can use any of the three levels. However, without any special reasons to select a specific level, we should adopt level 2. Selecting level 2 for all of a_1, a_2, ... , a_7, we can express them as follows:

$$
\begin{aligned}
a_1 &= 127.5 \pm 7.5 \text{ minutes} \\
a_2 &= 30.0 \pm 15.0 \text{ minutes} \\
&\vdots \\
a_7 &= 15.0 \pm 5.0 \text{ minutes}
\end{aligned}
\tag{26.12}
$$

In addition, the constant term a_0 is calculated as

$$
\begin{aligned}
a_0 &= \bar{y} - a_1\bar{x}_1 - a_2\bar{x}_2 - \cdots - a_7\bar{x}_7 \\
&= 1591.26 - (127.5)(0.4183) - (30.0)(1.02) \cdots - (15.0)(6.72) \\
&= 573.52
\end{aligned}
\tag{26.13}
$$

This value is equal to the value of averaged T in row 2 of Table 26.10 because all levels in the row are set to level 2.

Table 26.6
Level-by-level sum of S values[a]

Coefficient	Level 1	Level 2	Level 3
a_1	0.39111173E + 09	0.34771989E + 09	0.36822313E + 09
a_2	0.316029886E + 09	0.32800430E + 09	0.46302058E + 09
a_3	0.30145602E + 09	0.35595808E + 09	0.44964065E + 09
a_4	0.37578365E + 09	0.33903175E + 09	0.39223935E + 09
a_5	0.32961513E + 09	0.33368737E + 09	0.44375224E + 09
a_6	0.17658955E + 09	0.24090200E + 09	0.68956319E + 09
a_7	0.29273406E + 09	0.34254936E + 09	0.47177133E + 09

[a] 0.39111173E + 09 = 0.39111173×10^9.

Table 26.7
New setting of levels[a]

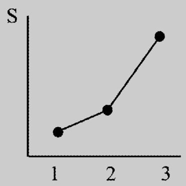
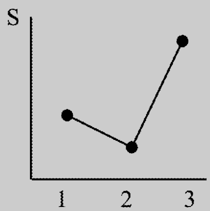
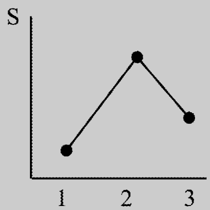
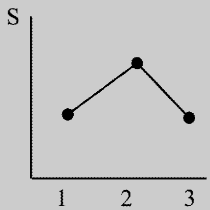
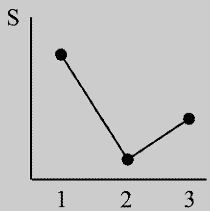
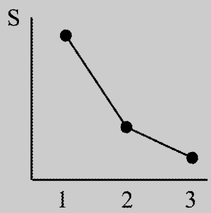
Plot of Variation S	Determinant	New Level		
		1′	2′	3′
I 	(1) $\frac{(1/r)(S_3 - S_1)}{V_e} < F$	1	2	3
	(2) $\frac{(1/r)(S_2 - S_1)}{V_e} < F$	1	1.5	2
	(3) $\frac{(1/r)(S_2 - S_1)}{V_e} \geq F$	0.5 (1)	1 (1.5)	1.5 (2)
II 	(1) $\frac{(1/r)(S_3 - S_2)}{V_e} < F$	1	2	3
	(2) $\frac{(1/r)(S_1 - S_2)}{V_e} < F$	1	1.5	2
	(3) $\frac{(1/r)(S_1 - S_2)}{V_e} \geq F$	1.5	2	2.5
III 	Omitted	1	2	3
IV 	Omitted	1	2	3
V 	(1) $\frac{(1/r)(S_1 - S_3)}{V_e} < F$	1	2	3
	(2) $\frac{(1/r)(S_1 - S_2)}{V_e} < F$	2	2.5	3
	(3) $\frac{(1/r)(S_3 - S_2)}{V_e} \geq F$	1.5	2	2.5

Table 26.7
(Continued)

Plot of Variation S	Determinant	New Level 1′	2′	3′
VI 	(1) $\frac{(1/r)(S_1 - S_3)}{V_e} < F$	1	2	3
	(2) $\frac{(1/r)(S_2 - S_3)}{V_e} < F$	2	2.5	3
	(3) $\frac{(1/r)(S_2 - S_3)}{V_e} \geq F$	2.5 (2)	3 (2.5)	3.5 (3)

[a] $r = N/3$, where N is the size of an orthogonal array; Level 1.5 means to create an intermediate level between levels 1 and 2; values in parentheses are used when newly determined values exceed the initial variable range.

Finally, we obtain the following regression equation:

$$y = 573.52 + 127.5x_1 + 30.0x_2 + \cdots + 15.0x_7 \qquad (S_e = 9279244.98) \qquad (26.14)$$

At the same time, $S_e = 9279244.98$ in (26.14) can be seen in variation S in row 2 of Table 26.10. Indeed, this value is somewhat larger than the variation of $S_e = 8171386.56$ obtained by the least squares method; however, this can be regarded as quite practical because the coefficients determined here do not involve any contradiction.

The regression equation obtained here is normally utilized for data other than those used for this regression equation: for example, for predicting future values or making a decision for other groups. Then it is important to evaluate the magnitude of errors by using different data. For instance, we can make the comparison

Table 26.8
Three levels for second round

Coefficient	Level 1	2	3
a_1	105	120	135
a_2	0	30	60
a_3	0	15	30
a_4	15	30	45
a_5	0	5	10
a_6	0	10	20
a_7	0	10	20

Table 26.9
Three levels for fifth round

Coefficient	Level 1	Level 2	Level 3
a_1	120.0	127.5	135.0
a_2	15.0	30.0	45.0
a_3	0.0	7.5	15.0
a_4	30.0	37.5	45.0
a_5	5.0	7.5	10.0
a_6	10.0	12.5	15.0
a_7	10.0	15.0	20.0

Table 26.10
T and *S* in fifth round

No.	Averaged *T*	Variation *S*	Combination												
1	864.976744	11,740,655.0	1	1	1	1	1	1	1	1	1	1	1	1	1
2	573.523256	9,279,244.98	2	2	2	2	2	2	2	2	2	2	2	2	1
3	282.069767	11,255,440.8	3	3	3	3	3	3	3	3	3	3	3	3	1
4	643.116279	9,535,102.42	1	1	1	1	2	2	2	2	3	3	3	3	1
5	351.662791	10,016,397.4	2	2	2	2	3	3	3	3	1	1	1	1	1
6	725.790698	9,962,633.12	3	3	3	3	1	1	1	1	2	2	2	2	1
7	661.895349	9,489,529.28	1	1	2	3	1	2	3	3	1	2	2	3	1
8	542.418605	9,702,168.47	2	2	3	1	2	3	1	1	2	3	3	1	1
9	516.255814	9,133,562.19	3	3	1	2	3	1	2	2	3	1	1	2	1
10	630.790698	9,877,123.12	1	1	3	2	1	3	2	3	2	1	3	2	1
11	604.627907	9,122,004.05	2	2	1	3	2	1	3	1	3	2	1	3	1
12	485.151163	9,326,132.77	3	3	2	1	3	2	1	2	1	3	2	1	1
13	479.395349	9,390,239.28	1	2	3	1	3	2	1	3	3	2	1	2	2
14	651.720930	9,654,002.65	2	3	1	2	1	3	2	1	1	3	2	3	2
15	589.453488	9,293,008.41	3	1	2	3	2	1	3	2	2	1	3	1	2
16	703.930233	9,779,070.79	1	2	3	2	1	1	3	2	3	3	2	1	2
17	595.441860	9,142,876.60	2	3	1	3	2	2	1	3	1	1	3	2	2
18	421.197674	9,549,964.57	3	1	2	1	3	3	2	1	2	2	1	3	2

Table 26.10
(*Continued*)

No.	Averaged *T*	Variation *S*	Combination												
19	425.674419	9,087,415.44	1	2	1	3	3	3	1	2	2	1	2	3	2
20	770.151163	10,164,835.3	2	3	2	1	1	1	2	3	3	2	3	1	2
21	524.744186	9,579,907.19	3	1	3	2	2	2	3	1	1	3	1	2	2
22	486.779070	9,120,635.15	1	2	2	3	3	1	2	1	1	3	3	2	2
23	648.116279	10,081,987.4	2	3	3	1	1	2	3	2	2	1	1	3	2
24	585.674419	9,342,025.44	3	1	1	2	2	3	1	3	3	2	2	1	2
25	490.383721	9,972,519.92	1	3	2	1	2	3	3	1	3	1	2	2	3
26	528.930233	9,688,640.79	2	1	3	2	3	1	1	2	1	2	3	3	3
27	701.255814	9,511,572.19	3	2	1	3	1	2	2	3	2	3	1	1	3
28	655.965116	9,647,941.20	1	3	2	2	2	1	1	3	2	3	1	3	3
29	410.558140	9,381,541.60	2	1	3	3	3	2	2	1	3	1	2	1	3
30	654.046512	9,862,355.91	3	2	1	1	1	3	3	2	1	2	3	2	3
31	449.162791	10,108,859.9	1	3	3	3	2	3	2	2	1	2	1	1	3
32	540.209302	9,418,093.12	2	1	1	1	3	1	3	3	2	3	2	2	3
33	731.197674	9,944,122.07	3	2	2	2	1	2	1	1	3	1	3	3	3
34	427.883721	9,123,062.42	1	3	1	2	3	2	3	1	2	2	3	1	3
35	664.918605	9,628,645.97	2	1	2	3	1	3	1	2	3	3	1	2	3
36	627.767442	9,686,485.67	3	2	3	1	2	1	2	3	1	1	2	3	3

Table 26.11
Level-by-level sum of *S* values

Coefficient	Level 1	Level 2	Level 3
a_1	0.11687215E + 09	0.11528044E + 09	0.11644715E + 09
a_2	0.11652424E + 09	0.11450165E + 09	0.11757385E + 09
a_3	0.11467273E + 09	0.11543298E + 09	0.11849404E + 09
a_4	0.11843048E + 09	0.11506510E + 09	0.11510416E + 09
a_5	0.11969653E + 09	0.11441208E + 09	0.11449113E + 09
a_6	0.11675750E + 09	0.11378532E + 09	0.11805692E + 09
a_7	0.11660350E + 09	0.11500287E + 09	0.11699338E + 09

Table 26.12
Comparison between least squares method and experimental regression analysis with different business office data[a]

No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y	Least Squares Method	Experimental Regression
1	2	4	10	4	144	85	21	2980	596.37	651.02
2	0	0	4	10	120	28	5	2431	140.17	−127.48
3	0	2	2	4	42	19	1	1254	287.43	112.02
4	0	4	6	0	78	38	0	1815	19.38	−16.48
5	2	0	2	4	118	27	6	2452	−356.60	−145.98
6	0	0	4	4	56	14	5	1223	197.47	200.52
7	0	0	2	6	70	14	0	1234	441.32	279.52
								Sum of squares of residual:	819,226.319	592,554.321
								Variance:	117,032.331	84,650.617

[a] Variance = sum of square of residuals/7.

shown in Table 26.12. This table reveals that the mean sum of squares of deviations between the estimation and actual value (variance) calculated based on the least squares method is 1.38 times as large as that by the experimental regression analysis. This fact demonstrates that we cannot necessarily minimize the sum of squares of residuals for different business office data, whereas we can do so for the data to be used for coefficient estimation. This is considered to be caused the by uncertainty of parameters estimated using the least squares method. Then if volume of jobs such as x_1 and x_3 is increased in the future, the equation based on the least squares method will lose its practicality.