24 Feedback Control of a Process Condition

24.1. Introduction

In this chapter the method for designing a feedback control system for changes in process conditions is explained. A case where changes in the process condition affect product quality is given. A case where deterioration of the process condition leads to stoppage of the process is discussed in later chapters.

24.2. Process Condition Control

Process condition control deals not only with product quality but also with process failure. This chapter deals with issues such as how long the checking interval should be and what the adjustment limit would be if a process condition such as temperature affects product quality.

The most important aspect of process condition control is the derivation of tolerance of the process condition *D*, that is, the functional limit for producing defectives. Since a change in the process condition by *D* causes defectives, the following equation is suggested for calculation of the proportion coefficient of the quality level:

$$k = \frac{\text{loss } A \text{ when the product is defective}}{\Delta^2} \tag{24.1}$$

Loss *A* in the numerator is the loss when a change in process condition creates defectives. A process condition is irrelevant to the concept of defect.

Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

24.3. Feedback Control System Design: Viscosity Control

Consider a problem where process parameter x is controlled through feedback so as to achieve a certain level of a quality characteristic, y. Suppose that the tolerance of the objective characteristic, y, is $\pm\Delta_0$, and the loss when the product falls off the tolerance is A dollars. It is assumed that when process condition x changes by Δ, the objective characteristic y becomes unacceptable.

The parameters of process condition x is defined as follows:

- *Tolerance of x:* Δ (the tolerance of x that determines the pass/fail of objective characteristic y)
- *Measuring costs of x: B* (dollars)
- *Measuring time lag: l* (units)
- *Current adjustment limit of x: C* (dollars)
- *Current checking interval of x:* n_0 (units)
- *Current mean adjustment interval of x:* u_0 (units)

The optimal checking interval, n, and optimal adjustment limit, D, of process condition x are given by the following equations:

$$L_0 = \frac{B}{n_0} + \frac{C}{u_0} + \frac{A}{\Delta^2}\left[\frac{D_0^2}{3} + \left(\frac{n_0 + 1}{2} + l\right)\frac{D_0^2}{u_0}\right] \tag{24.2}$$

$$L = \frac{B}{n} + \frac{C}{u} + \frac{A}{\Delta^2}\left[\frac{D_0^2}{3} + \left(\frac{n + 1}{2} + l\right)\frac{D^2}{u}\right] \tag{24.3}$$

The estimated mean adjustment interval, u, is given by

$$u = u_0 \frac{D^2}{D_0^2} \tag{24.4}$$

❒ Example

Consider a process of continuous emulsion coating. The object of control is the viscosity of the emulsion. The standard of coating thickness is $m_0 + 8$ μm and the loss caused by failing to meet the standard is \$3/m^2. When the viscosity of the emulsion changes by 5.3 P, the coated film fails to meet the standard. The current checking interval, n_0, is 6000m^2 (once every 2 hours), the adjustment limit is 0.9 P, and the mean adjustment interval is 12,000m^2 (twice a day). Cost B of measuring x is \$2, and the adjustment cost of viscosity, C, is \$10. The time lag, l, of the measurement is 30m^2. Derive the optimal checking interval, n, optimal adjustment limit, D, and the profit per m^2 and per year (assume 2000-hour/year operation).

According to the assumptions, the tolerance Δ of viscosity x is 5.3. The parameters can be summarized as follows.

- *Tolerance:* $D = 5.3$ P
- *Loss due to defective:* $A = \$3$
- *Measurement cost:* $B = \$2$
- *Time lag for measurement:* $l = 30m^2$
- *Adjustment cost:* $C = \$10$
- *Current checking interval:* $n_0 = 6000m^2$
- *Current adjustment limit:* $\pm D_0 = 0.9$ P
- *Current mean adjustment interval:* $u_0 = 12{,}000m^2$

The current loss function, L_0, is

$$\begin{aligned} L_0 &= \frac{B}{n_0} + \frac{C}{u_0} + \frac{A}{\Delta^2}\left[\frac{D_0^2}{3} + \left(\frac{n_0 + 1}{2} + 1\right)\frac{D_0^2}{u_0}\right] \\ &= \frac{2}{6000} + \frac{10}{12{,}000} + \frac{3}{5.3^2}\left[\frac{0.9^2}{3} + \left(\frac{6001}{2} + 30\right)\left(\frac{0.9^2}{12{,}000}\right)\right] \\ &= 0.00033 + 0.00083 + 0.02884 + 0.02185 \\ &= \$0.05185 \end{aligned} \tag{24.5}$$

Optimal checking interval, n, and optimal adjustment limit, D, are

$$\begin{aligned} n &= \sqrt{\frac{2u_0B}{A}}\,\frac{\Delta}{D_0} = \sqrt{\frac{(2)(12{,}000)(2)}{3}}\left(\frac{5.3}{0.9}\right) \\ &= 745 \rightarrow 1000m^2 \end{aligned} \tag{24.6}$$

$$\begin{aligned} D &= \left(\frac{3C}{A}\,\frac{D_0^2}{u_0}\,\Delta^2\right)^{1/4} = \left[\frac{(3)(10)}{3}\left(\frac{0.9^2}{12{,}000}\right)(5.3^2)\right]^{1/4} \\ &= 0.37 \rightarrow 0.4 \text{ P} \end{aligned} \tag{24.7}$$

The expected mean adjustment interval, u, is

$$u = u_0\frac{D^2}{D_0^2} = (12{,}000)\left(\frac{0.4^2}{0.9^2}\right) = 2400m^2 \tag{24.8}$$

and the loss function, L, is

$$\begin{aligned} L &= \frac{B}{n} + \frac{C}{u} + \frac{A}{\Delta^2}\left[\frac{D^2}{3} + \left(\frac{n + 1}{2} + 1\right)\frac{D^2}{u}\right] \\ &= \frac{2}{1000} + \frac{10}{2{,}400} + \frac{3}{5.3^2}\left[\frac{0.4^2}{3} + \left(\frac{1001}{2} + 30\right)\left(\frac{0.4^2}{2400}\right)\right] \\ &= 0.002 + 0.004 + 0.006 + 0.378 = 1.564 \text{ cents} \end{aligned} \tag{24.9}$$

Thus, the yearly improvement is

$$(0.05 - 0.016)(3000)(2000) = \$204{,}000 \qquad (24.10)$$

24.4. Feedback Control System: Feedback Control of Temperature

❒ Example

Consider a heat process where objects are fed continuously under 24-hour operation. When the temperature changes by 40°C, the product becomes defective and the unit loss is \$3. Although the temperature is controlled automatically at all times, a worker checks the temperature with a standard thermometer once every 4 hours. The cost of checking, B, is \$5; the current adjustment limit, D_0, is 5°C; the mean adjustment interval is 6 hours; the adjustment cost is \$5; the current adjustment limit D_0 is 5°C; the mean adjustment interval is 6 hours; the adjustment cost is \$8; the number of units heat-treated in 1 hour is 300; the time lag of measurement is 20 units; and the measurement error (standard deviation) of the standard thermometer is about 1.0°C.

(a) Design an optimal temperature control system and calculate the yearly improvement over the current system. Assume that the number of operation-days per year is 250.

(b) If a new temperature control system is introduced, the annual cost will increase by \$12,000, but the degree of temperature drift will be reduced to one-fifth of the existing level. Calculate the loss/gain of the new system.

Parameters are summarized as follows:

$$D = 40°\text{C}$$

$$A = \$3$$

$$B = \$5$$

$$C = \$18$$

$$n_0 = (3)(4) = 12$$

$$D_0 = 5°\text{C}$$

$$u_0 = (3)(6) = 18 \text{ units}$$

$$l = 20 \text{ units}$$

$$\sigma_s = 1.0°\text{C}$$

(a)] The loss function, L_0, of the current system is

$$L_0 = \frac{B}{n_0} + \frac{C}{u_0} + \frac{A}{\Delta^2}\left[\frac{D_0^2}{3} + \left(\frac{n_0 + 1}{2} + 1\right)\frac{D_0^2}{u_0} + \sigma_s^2\right]$$

$$= \frac{5}{1200} + \frac{18}{1800} + \frac{3}{40^2}\left[\frac{5^2}{3} + \left(\frac{1201}{2} + 20\right)\left(\frac{5^2}{1800}\right) + 1.0^2\right]$$

$$= 0.0042 + 0.0100 + 0.0156 + 0.0162 + 0.0478$$

$$= \$0.0478 \qquad (24.11)$$

The optimal checking interval, n, is

$$n = \sqrt{\frac{2u_0B}{A}}\,\frac{\Delta}{D_0} = \sqrt{\frac{(2)(1800)(5)}{3}}\left(\frac{40}{5}\right)$$

$$= 620 \rightarrow 600 \text{ units} \quad \text{(once in 2 hours)} \qquad (24.12)$$

The optimal adjustment interval, D, is

$$D = \left(\frac{3C}{A}\,\frac{D_0^2}{u_0}\,\Delta^2\right)^{1/4} = \left(\frac{(3)(18)}{3}\,\frac{5^2}{1800}\,(40^2)\right)^{1/4}$$

$$= 4.5 \rightarrow 5.0°\text{C (The current level is acceptable)} \qquad (24.13)$$

Accordingly, the mean adjustment interval remains the same, and the new loss function is

$$L = \frac{5}{600} + \frac{18}{1800} + \frac{3}{40^2}\left[\frac{5^2}{3} + \left(\frac{601}{2} + 20\right)\left(\frac{5^2}{1800}\right) + 1.0^2\right]$$

$$= 0.0083 + 0.0100 + 0.0156 + 0.0083 + 0.0019$$

$$= \$0.0442 \qquad (24.14)$$

The yearly improvement is

$$(0.0479 - 0.0441)(300)(24)(250) = \$6840 \qquad (24.15)$$

(b) The total production in one year is (300)(24)(250) = 1,800,000. Thus, the increase of cost per unit of production by introduction new equipment is

$$\frac{120}{180} = \$0.0067 \qquad (24.16)$$

Reduction of drift to one-fifth of the current level can be interpreted as meaning that the mean adjustment interval becomes five times the current adjustment interval, or the standard deviation per unit of output is reduced to one-fifth.

Assuming that $u_0 = (1800)(5) = 9000$, optimal checking interval, n, and optimal adjustment limit, D, are

$$n = \sqrt{\frac{(2)(9000)(5)}{3}}\left(\frac{40}{5}\right)$$

$$= 1386 \rightarrow 1200 \text{ (once every 4 hours)}$$

$$D = \left[\frac{(3)(18)}{3}\left(\frac{5^2}{9000}\right)(40^2)\right]^{1/4}$$

$$= 3.0^\circ\text{C} \qquad (24.17)$$

Furthermore, the mean adjustment interval is estimated as

$$u = u_0 \times \frac{D^2}{D_0^2} = (9000)\left(\frac{3.0^2}{5^2}\right)$$

$$= 3240 \text{ units} \qquad (24.18)$$

Thus,

$$L = \frac{5}{1200} + \frac{18}{3240} + \frac{3}{40^2}\left[\frac{3^2}{3} + \left(\frac{1201}{2} + 20\right)\left(\frac{3^2}{3240}\right) + 1.0^2\right]$$

$$= 0.0042 + 0.0056 + 0.0056 + 0.0032 + 0.0019$$

$$= \$0.0205 \qquad (24.19)$$

Even if the increase in the cost given by equation (24.16) (i.e., \$0.0067) is added, the loss is \$0.0272. Therefore, compared to equation (24.14), a yearly improvement of \$30,420 can be expected:

$$(0.0441 - 0.0272)(300)(24)(250) \approx \$30{,}420) \qquad (24.20)$$