Part VIII

On-Line Quality Engineering

22 Tolerancing and Quality Level

22.1. Introduction

The difference between parameter design and on-line quality engineering is that in parameter design, the parameters of the process are fixed at certain levels so that variability is reduced. Parameter design is used during manufacturing to vary manufacturing conditions and reduce variability. But in the on-line approach, a certain parameter level is varied from time to time based on the manufacturing situation to adjust the quality level to hit the target. In the case of a watch, for example, its error can be reduced by designing (purchasing) a more accurate watch—this is parameter design. Using the on-line approach, the inaccurate watch is used and errors are reduced by checking more frequently with the time signal and making adjustments to it.

Although it is important to improve working conditions and the method of operating machines and equipment on the job, such issues are off-line improvements in terms of production technology. Instead of such process improvement, here we deal with process control through checking, as well as process maintenance through prevention. In other words, general issues related to the design of process control of on-line real-time processing and production are covered in this chapter. It is important to control or automate an on-line production process based on this theory, using economic calculations.

Design Quality and Production Quality

It is important to discuss two types of quality: design and production. *Design quality* is represented by the properties contained in a product. Such properties are a part of the product and can be described in catalogs. The marketing department determines which quality items should be in the catalogs. Design quality includes the following four aspects, along with usage conditions and problem occurrence:

1. *Natural conditions:* temperature, humidity, atmospheric pressure, sunlight, hydraulic pressure, rain, snow, chemical durability, etc.
2. *Artificial conditions:* impact, centrifugal force, excess current, air pollution, power failure, chemical durability, etc.
3. *Individual differences or incorrect applications:* differences in individual ability, taste, physical conditions, etc.
4. *Indication of problems:* ease or difficulty of repair, disposability, trade in, etc.

Production quality represents deviations from design quality, such as variations due to raw materials, heat treatment, machine problems, or human error. Production quality is the responsibility of the production department.

Quality Problems before and after Production

The quality target must be selected by the R&D department based on feedback from the marketing department about customer desires and competitors products. After the development of a product, the marketing department evaluates the design quality, and the production department evaluates the manufacturing cost.

Generally, production quality problems are caused either by variability or mistakes, such as the variations in raw materials or heat treatment or problems with a machine. The production department is responsible not for deviation from the drawings but for deviation from design quality.

Since there is responsibility, there is freedom. It is all right for the production department to produce products that do not follow the drawings, but it must then be responsible for the claims received due to such deviations. Production departments are also responsible for claims received due to variations, even if the product was made within specifications.

Design quality also includes functions not included in catalogs because they are not advantageous for marketing. However, claims due to items included in catalogs but not in the design must be the responsibility of the marketing department. To summarize: Design quality is the responsibility of the marketing department, and production quality is the responsibility of the production department.

On-Line Approaches

Various concerns of on-line production are discussed below.

QUANTITATIVE EVALUATION OF PRODUCT QUALITY

Since defective products found on the line are not shipped, the problem of these defective products does not affect customers directly, but production costs increase. Product design engineers do not deal with process capability but work to design a product that works under various customers' conditions for a certain number of years. This is called *parameter design*. The point of parameter design for production is to provide a wide tolerance for manufacturing or to make it easy for production personnel to make the product.

In addition, production engineering should design a stable manufacturing process, to balance loss due to variations in cost. When a process is very stable, engineers should try to increase the production speed to reduce the manufacturing cost. When production speed increases, variation normally increases. An engineer should adjust the production speed to a point where the manufacturing cost and the loss due to variation balance. This is important because a product is normally sold at three or four times unit manufacturing cost (UMC). Therefore, any reduction in the UMC yields a tremendous benefit to the manufacturer in terms of profit.

DETERMINATION OF SPECIFICATIONS FOR MANUFACTURED PRODUCTS: TOLERANCING

Tolerancing is a method to quantitatively determine the specifications for transactions or a contract. A new and unique approach to tolerancing utilizes the safety factor, previously vague.

FEEDBACK CONTROL

In *feedback control*, product quality or manufacturing conditions are checked at intervals. Production is continued if the result is within a certain limit; otherwise, feedback control is conducted. The optimum design for such a feedback control system is explained below.

PROCESS DIAGNOSIS, ADJUSTMENT, AND MAINTENANCE DESIGN

We discuss the design of several preventive maintenance systems. In the case of soldering, for example, its apparent quality cannot be measured by using continuous variables. In the same way, it is difficult to design a gauge for a product with a complicated shape. The design of such process control systems using basic equations and various applications is illustrated later in the chapter.

FEEDFORWARD CONTROL

Feedback control involves measurement of a product's quality characteristics, then making changes to the process as required. Feedforward control involves process change *before* production so that the quality level can meet the target. In the manufacture of photographic film or iron, for example, the quality of incoming gelatin or iron ore is measured so that the mixing ratios or reacting conditions may be adjusted.

PRODUCTION CONTROL FOR THE QUALITY OF INDIVIDUAL PRODUCTS: INSPECTION

Using inspection, product quality characteristics are measured and compared with the target. A product whose quality level deviates from the target is either adjusted or discarded. This is inspection in a broad sense, where individual products are inspected.

22.2. Product Cost Analysis

Production costs can be shown by the following equation:

$$\text{(production costs)} = \text{(material costs)} + \text{(processing costs)} + \text{(control costs)} + \text{(pollution control costs)} \quad (22.1)$$

Control costs consist of both production control and quality control costs. The product design division is responsible for all the costs seen on the right-hand side of equation (22.1). When we say that a product designer is responsible for the four terms in equation (22.1), we mean that he or she is responsible for the design of a product so that it functions under various conditions for the duration of its design life (the length of time it is designed to last). In particular, the method of parameter design used in product design enables the stability of the product function to be improved while providing a wide range of tolerance. For example, doubling the stability means that the variability of an objective characteristic does not change even if the level of all sources of variability is doubled.

The production engineering division seeks to produce the initial characteristics of the product as designed, with minimum cost and uniform quality. The production engineering division is responsible for the sum of the second to fourth terms on the right-hand side of equation (22.1). In other words, they are responsible for designing stable production processes without increasing cost and within the tolerance of the initial characteristics given by the product design division.

However, no matter how stable a production process is, it will still generate defectives if process control is not conducted. Additionally, if production speed is increased to reduce cost by producing more product within tolerance limits, more loss will occur, due to increased variability. Basically, it is difficult to balance production costs and loss due to variability in quality, so an appropriate level of process control must be employed.

22.3. Responsibilities of the Production Division

Production line workers are responsible for the daily production of a product by using a given production process and specifications, that is, the quality characteristics that should be controlled at each step, as well as the major target values and control limits from technical and manufacturing standards. However, some process conditions are often overlooked, and therefore control limits may need to be adjusted through calibration.

Initially, the cost of a product is more important than its quality, since the price of a product results in the first loss to the consumer (customer). In this sense the balance between cost and quality is the balance between price and quality. Quality includes the following items: (1) operating cost, (2) loss due to functional variability, and (3) loss due to harmful quality items. The first and third items are estimated in the design stage, whereas item 2, operating cost, is usually evaluated under standard usage conditions. The second and third cost items are evaluated using a loss function, which is derived from the mean square of the difference from the ideal value. In daily production activities, on the other hand, economic loss due to the second and third items is evaluated using the mean square of the difference from the target value. The quality level is evaluated using the equation

$$L = \frac{A}{\Delta^2}\sigma^2 \tag{22.2}$$

where A (dollars) is the loss when the initial characteristic value of a product does not satisfy the tolerance, Δ is the tolerance of the characteristic, and σ^2 is the mean square of the difference from the objective value of the initial characteristic.

If the cost is far more important than the quality and inventory losses, one has to use a number several times that of the real cost as the cost figure in the following calculations. According to Professor Trivus, the director of MIT's Advanced Technology Research Institute, Xerox Corporation used to set the product price at four times the UMC. Since the UMC does not include the costs of development, business operations, head-office overhead, and so on, corporations cannot generate profit if their product is not sold at a price several times higher than the UMC.

Production attempts to minimize the sum of process control costs and quality-related loss through process and product control. Process control costs are evaluated using real cost figures, although multiples of the figure are often used. If the cost is several times more important than the quality-related loss, one can take the cost figure times some number to use as the real cost. To balance the two types of unit costs necessary for control and the quality-related loss, we consider the following cost terms:

B: checking (measurement) cost
C: adjustment cost, to bring the process under control or to adjust the product

If the amount of control cost is far greater than the amount of quality-related loss, excessive control is indicated; alternatively, if quality-related losses dominate, a lack of control is indicated.

The main task of people on the production line is to control the process and product while machines do the actual processing. Therefore, we can also use quality engineering methods to determine the optimal number of workers on the production line.

22.4. Role of Production in Quality Evaluation

The manufacturing process is composed of the following six stages:

1. Product planning (to decide product function, price, and design life)
2. Product design (to design a product with functions determined by product planning)
3. Production process design (to design the production process necessary to produce a product)
4. Production
5. Sales
6. After-service

The production division is involved directly in stages 3 and 4. Its task is to produce products efficiently as specified by drawings and specifications, within quality standards.

Whereas production cost is obvious, a definition of product quality is not always clear. Many firms evaluate product quality using fraction defective; however, defectives do not cause problems for consumers because defectives are generally not delivered to them, Therefore, the production of defectives is a cost issue, not a quality issue. Quality is related to the loss incurred by consumers due to function variability, contamination, and so on, of the product as shipped. Therefore, the idea is to find a way to evaluate quality levels of accepted products. To conduct such an evaluation, it is necessary to determine product standards, especially tolerance.

Quality can be defined as "the loss that a product costs society after its shipment, not including the loss due to the function itself." Under such a definition, quality is divided roughly into two components: (1) loss due to function variability and (2) loss that is unrelated to function, such as loss due to side effects or poor fuel efficiency. Here we focus primarily on loss due to function variability.

Engineers in the United States and other countries have used the following two methods for tolerance design: (1) a method that uses the safety factor derived from past experience, and (2) a method that allocates tolerance based on reliability design. The Monte Carlo method, which obtains tolerance by generating a random vector, is used in the area of reliability design because other factors of variability also influence the function. In the following discussion, these methods are explained and some examples are given that compare them to using the loss function.

22.5. Determination of Tolerance

When it is desirable that an objective characteristic, y, be as close to a target value as possible, we call it a *nominal-the-best characteristic*.

❐ Example

In the power circuit of a TV set, 100-V ac input in a home should be converted into 115-V dc output. If the circuit generates 115-V of output constantly during its designed life, the power circuit has perfect functional quality. Thus, this power circuit is a subsystem with a nominal-the-best characteristic whose target value is 115-V.

Suppose that the target value is m and the actual output voltage is y. Variability of component parts in the circuit, deterioration, variability of input power source, and so on, create the difference $(y - m)$. Tolerance for the difference $(y - m)$ is determined by dividing the difference by the safety factor ϕ, derived from past experience. Suppose that the safety factor in this case is 10. When a 25% change in output voltage causes trouble, the tolerance is derived as follows:

$$\text{tolerance} = \frac{\text{functional limit}}{\text{safety factor}} = \frac{25}{10} = 2.5\% \qquad (22.3)$$

If the target value of output voltage is 115 V, the tolerance is

$$\Delta = (115)(0.025) = 2.9 \text{ V} \qquad (22.4)$$

Therefore, the factory standard of output voltage is determined as follows after rounding:

$$115 \pm 3 \text{ V} \qquad (22.5)$$

Let Δ_0 denote the functional limit, that is, deviation from the target value when the product ceases to function. At Δ_0, product function breaks down. In this case, other characteristics and usage conditions are assumed to be normal, and Δ_0 is derived as the deviation of y from m when the product stops functioning.

The assumption of normality for other conditions in the derivation of functional limits implies that standard conditions are assumed for the other conditions. The exact middle condition is where the standard value cuts the distribution in half, for example, median and mode, and it is often abbreviated as LD_{50} (lethal dosage), that is, there is a 50–50 chance of life (L) or death (D)——50% of people die if the quantity is less than the middle value and the other 50% die if the quantity is greater. Consequently, one test is usually sufficient to obtain the function limit.

The value of Δ_0 is one of the easiest values to obtain. For example, if the central value of engine ignition voltage is 20 kV, LD_{50} is the value when ignition fails after changing the voltage level. (The central value is determined by parameter design and not by tolerance design. It is better to have a function limit as large as possible in parameter design, but that is a product design problem and is not discussed here.) After reducing voltage, if ignition fails at 8 kV, −12 kV is the lower functional limit. On the other hand, the upper functional limit is obtained by raising the voltage until a problem develops. A high voltage generates problems such as corona discharge; let its limit be +18 kV. If the upper- and lower-side functional limits are different, different tolerances may be given using different safety factors. In such a case, the tolerance is indicated as follows:

$$+\Delta_1 \; m \quad -\Delta_2 \qquad (22.6)$$

Upper and lower tolerance, Δ_1 and Δ_2, are obtained separately from respective functional limit Δ_{01} and Δ_{02} and safety factor ϕ_1 and ϕ_2:

$$\Delta_i = \frac{\Delta_{0i}}{\phi_i} \qquad (i = 1,2) \qquad (22.7)$$

The functional limit Δ_0 is usually derived through experiment and design calculations. However, derivation of the safety factor is not clear. Recently, some people have started using probability theory to determine tolerance [1]. The authors of this book firmly believe that the probability approach is inefficient and incorrect. The authors recommend the derivation of safety factors without using probability statistics. We suggest that safety factor ϕ be derived according to the following equation, which is becoming popular:

$$\phi = \sqrt{\frac{\text{average loss when falling outside the functional limit}}{\text{loss to the factory when falling outside the factory standard}}} \tag{22.8}$$

Letting A_0 be the average loss when the characteristic value falls outside the functional limit, and A be the loss to the factory when the characteristic value falls outside the factory standard:

$$\phi = \sqrt{\frac{A_0}{A}} \tag{22.9}$$

The reason for using this formula is as follows. Let y denote the characteristic value and m the target value. Consider the loss when y differs from m. Let $L(y)$ be the economic net loss when a product with characteristic value y is delivered to the market and used under various conditions. Assume that the total market size of the product is N and that $L_i(t,y)$ is the actual loss occurring on location. In the tth year after the product stops functioning, the loss jumps from zero to a substantial amount. With a given design life T, $L(y)$ gives the average loss over the entire usage period in the entire usage location; that is,

$$L(y) = \frac{1}{N}\sum_{i=1}^{N}\int_0^T L_i(t,y)\ dt \tag{22.10}$$

Even if individual $L_i(t,y)$ is a discontinuous function, we can take the average usage condition over many individuals, and thus approximate it as a continuous function. We can conduct a Taylor expansion of $L(y)$ around target value m:

$$\begin{aligned} L(y) &= L(m + y - m) \\ &= L(m) + \frac{L'(m)}{1!}(y - m) + \frac{L'(m)}{2!}(y - m)^2 + \cdots \end{aligned} \tag{22.11}$$

In equation (22.11), the loss is zero when y is equal to m and $L'(m)$ is zero, since the loss is minimum at the target value. Thus, the first and second terms drop out and the third term becomes the first term. The cubic term should not be considered. By omitting the quartic term and above, $L(y)$ is approximated as follows:

$$L(y) = k(y - m)^2 \tag{22.12}$$

The right-hand side of equation (22.12) is loss due to quality when the characteristic value deviates from the target value. In this equation, the only unknown is k, which can be obtained if we know the level of loss at one point where y is different from m. For that point we use the function limit Δ_0 (see Figure 22.1). Assuming that A dollars is the average loss when the product does not function in

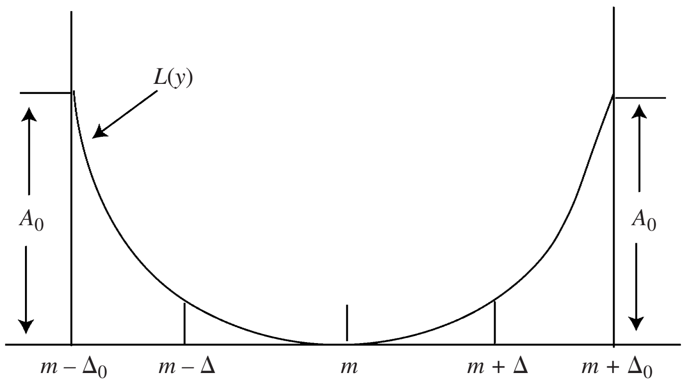

Figure 22.1
Loss function

the market, we substitute A_0 on the left-hand side of equation (22.12) and Δ_0 for $y - m$ on the right-hand side. Thus,

$$A_0 = k\Delta_0^2 \tag{22.13}$$

Solving this for k gives us

$$k = \frac{A_0}{\Delta_0^2} \tag{22.14}$$

Therefore, the loss function, $L(y)$, due to quality variability in the case of a nominal-the-best characteristic becomes

$$L(y) = \frac{A_0}{\Delta_0^2}(y - m)^2 \tag{22.15}$$

As we can see from equation (22.10), the value of quality evaluation given by equation (22.15) is the average loss when the product with characteristic value y is used under various usage conditions, including a major problem such as an accident. Since it is not known under which condition the product is used, it is important to evaluate the average loss by assuming that the product is used under the entire range of usage conditions. Tolerance should be decided by taking the balance between average loss and cost.

To determine the safety factor, ϕ, we need to evaluate one more economic constant, the cost, A, when the product fails to meet factory standards. A product that does not meet these standards is called *defective*. A defective item needs to be repaired or scrapped. If it can be modified, the cost of modification is used for A. But in some cases the product may not even pass after modification. If the rate of acceptance is close to 10%, the product price is used as the cost of modification, A. If the rate of acceptance is lower, say q, A is defined as

$$A = \frac{\text{factory pine}}{q} \tag{22.16}$$

Since the rate of acceptance cannot be determined before establishing the factor standards, the equation obtained by substituting the value of equation

(22.16) into the left-hand side of equation (22.15) becomes nonlinear and thus is difficult to solve. Generally, the method of sequential approximation is used to solve the problem.

❐ Example

Continuing the example introduced at the beginning of this section, the functional limit of the power circuit is assumed to be 25%, and the loss to the company when the product falls outside the function limit in the market, A_0, is assumed to be $300 (this includes repair cost and replacement cost during repair). We assume further that the power circuit, which does not satisfy factory standards, can be fixed by changing the level of resistance of the resistor within the circuit. Let the sum of labor and material cost for repair, A, be $1. The safety factor, ϕ, is determined as follows using equation (22.9).

$$\phi = \sqrt{\frac{A_0}{A}} = \sqrt{\frac{300}{1}} = 17.3 \tag{22.17}$$

Since the safety factor is 17.3, the factory standard is

$$\Delta = \frac{\Delta_0}{\phi} = \frac{25}{17.3} = 1.45\% \tag{22.18}$$

Therefore, with a target value of 115 V, the factory standard, Δ, is just 1.7 V:

$$115 \pm (115)(0.0145) \approx 115 \pm 1.7 \text{ V} \tag{22.19}$$

Even if the functional limit in the market is 115 ± 29 V, the standard for the production division of the factory is 115 ± 1.7 V, and those products that do not satisfy this standard need to be repaired.

A characteristic value that is nonnegative and as small as possible is called a *smaller-the-better characteristic*. The safety factor for a smaller-the-better characteristic is found in equation (22.9). For example, if the fatal dose of a poisonous element (LD_{50}) is 8000 ppm, the loss, A_0, when someone has died due to negligence, for example, is obtained using the following equation:

$$\begin{aligned} A_0 &= (\text{national income}) \times \text{average life (or average life expectancy)} \\ &= \$20{,}000 \times 77.5 \text{ years} \\ &= \$1{,}550{,}000 \end{aligned} \tag{22.20}$$

The safety factor, ϕ, and factory tolerance, Δ, are determined from equation (22.9), assuming that the price of the product, A, is $3.

$$\phi = \sqrt{\frac{A_0}{A}} = \sqrt{\frac{1{,}550{,}000}{3}} = 719 \tag{22.21}$$

Since the safety factor is 719, the factory standard is

$$\Delta = \frac{\Delta_0}{\phi} = \frac{8000}{719} = 11 \text{ ppm} \tag{22.22}$$

For a characteristic that is nonnegative and should be as large as possible, that is, the *larger-the-better characteristic*, the safety factor, ϕ, is again given by equation (22.9). For example, a pipe made of a certain resin breaks with 5000 kg_f of external force, and the resulting loss, A_0, is \$300,000. Assume that the strength and cost of the pipe are proportional to the cross-sectional area and that their coefficients of proportion $b = 80$ (kg_f/mm^2) and $a =$ \$40 per/mm^2. First, let us derive the optimal cross-sectional area.

The loss due to quality is handled as a smaller-the-better characteristic after taking the inverse. The sum, L, of the loss due to price and quality is

$$L = \text{price} + \text{quality} = ax + \frac{A_0 \Delta_0^2}{(bx)^2} \tag{22.23}$$

where x is the cross-sectional area.

To minimize the above, differentiate equation (22.23) and solve it by equating the derivative to zero. The following formula is derived:

$$x = \left(\frac{2A_0\,\Delta_0^2}{ab^2}\right)^{1/3} \tag{22.24}$$

By substituting the parameter values, that is, $a =$ \$40, $b = 80$ kg_f, $A_0 =$ \$300,000, and $\Delta_0 = 5000$ kg_f, the optimal cross-sectional area x is derived as follows:

$$x = \left(\frac{(2)(300{,}000)(5000)^2}{(40)(80^2)}\right)^{1/3} = 388 \tag{22.25}$$

Therefore, the price is determined by multiplying the result for x in (22.25) by $a =$ \$40:

$$A = (40)(388) = \$15{,}520 \tag{22.26}$$

The safety factor is derived by substituting $A =$ \$15,520 and $A_0 =$ \$300,000 into equation (22.9):

$$\phi = \sqrt{\frac{30}{1.55}} = 4.4 \tag{22.27}$$

Thus, the lower limit of the standard is 22 $tons_f$, that is, 4.4 times the strength corresponding to the function limit $\Delta_0 = 5$ $tons_f$.

The safety factor method can be applied to any component part and material tolerance in the same way. In this case you would use the point of LD_{50}, that is, the characteristic value when the objective characteristic stops functioning (the value of functional limit when the characteristic of other parts and usage conditions are at the standard levels), Δ_0 and A_0, as the loss incurred to the society. A is the loss incurred to the factory when the component part or raw material does not satisfy the delivery standard of the factory.

Therefore, the safety factor, ϕ, is given by the following equation in all cases of nominal-the-best, larger-the-better, and smaller-the-better characteristics:

$$\phi = \sqrt{\frac{A_0}{A}} \tag{22.28}$$

Further, the factory standard, Δ, is determined by the following:

$$\Delta = \begin{cases} \dfrac{\Delta_0}{\phi} & \text{nominal-the-best and smaller-the-better} \quad (22.29) \\ \phi\Delta_0 & \text{larger-the better} \quad (22.30) \end{cases}$$

22.6. Quality Level at Factories for Nominal-the-Best Characteristics

Production will accept the tolerances given by specifications and drawings and use it as the basis for quality-level evaluation. It is important to use the loss function for quality-level evaluation instead of fraction defective or rate of acceptance. In the case of nominal-the-best characteristics, the following equation is used:

$$L = \frac{A_0}{\Delta_0^2}\sigma^2 = \frac{A}{\Delta^2}\sigma^2 \tag{22.31}$$

where σ^2 is the mean square of the difference from the target value m of n data, $y_1, y_2, \ldots, y_n$, or the mean-squared error, or variance.

$$\sigma^2 = \frac{1}{n}[(y_1 - m)^2 + (y_2 - m)^2 + \cdots + (y_n - m)^2] \tag{22.32}$$

❐ Example

Consider a plate glass product having a shipment cost of \$3 per unit, with a dimensional tolerance, Δ, of 2.0 mm. Dimensional data for glass plates delivered from a factory minus the target value, m, are

0.3	0.6	−0.5	−0.2	0.0	1.0	1.2	0.8	−0.6	0.9	0.0
0.2	0.8	1.1	−0.5	−0.2	0.0	0.3	0.8	1.3		

To obtain the loss due to variability, we must calculate the mean-squared error from the target value, Δ^2, and substitute it into the loss function. We refer to this quantity as the mean-squared error, σ^2 (variance):

$$\sigma^2 = \frac{1}{20}(0.3^2 + 0.6^2 + \cdots + 1.3^2) = 0.4795 \text{ mm}^2 \tag{22.33}$$

Substituting this into the loss function, equation (22.31), yields

$$L = \frac{A}{\Delta^2}\sigma^2 = \frac{3}{2^2}(0.4795) = 36 \text{ cents} \qquad (22.34)$$

The mean of the deviations of glass plate size from the target value is 0.365, indicating that the glass plates are a bit oversized on average. One can create an ANOVA table to observe this situation (see Chapter 29).

$$S_T = 0.3^2 + 0.6^2 + \cdots + 1.3^2 = 9.59 \qquad (f = 20) \qquad (22.35)$$

$$S_m = \frac{(0.3 + 0.6 + \cdots + 1.3)^2}{20} = \frac{7.3^2}{20} = 2.66 \qquad (f = 1) \qquad (22.36)$$

$$S_e = S_T - S_m = 9.59 - 2.66 = 6.93 \qquad (f = 19) \qquad (22.37)$$

Variance, V, is obtained by dividing variation, S_e, by its degrees of freedom, f. Net variation, S', is derived by subtracting from variation, S, its error variance times the degrees of freedom. The rate of contribution, ρ, is derived by dividing net variation, S', by total variation, S_T. The calculations are illustrated in Table 22.1.

We can observe from Table 22.1 that the mean size is greater than the central value of the standard, thus increasing the mean-squared error and the loss due to variability. It is usually easy to adjust the mean to the target value in factories. Accordingly, if the mean is adjusted to the target value, the variance, σ^2, should be reduced to that of the error variance, V_e, in Table 22.1. In this case, the value of the loss function will be

$$L = \frac{3}{2^2}(0.365) = (36.0)(0.761) = 27.4 \text{ cents} \qquad (22.38)$$

Compared to equation (22.34), this means a quality improvement of

$$36.0 - 27.4 = 8.6 \text{ cents} \qquad (22.39)$$

per sheet of glass plate. If the monthly output is 100,000 sheets, quality improvement will be $8600 per month.

No special tool is required to adjust the mean to the target value. One needs to compare the size with the target value occasionally and cut the edge of the plate

Table 22.1
ANOVA table

Source	f	S	V	S'	ρ (%)
m	1	2.66	2.66	2.295	23.9
e	19	6.93	0.365	7.295	76.1
Total	20	9.59	0.4795	9.590	100.0

if it is oversized. By varying the interval of comparison with the measure, not only the mean but also the variance will change. This is the determination of calibration cycle in quality engineering.

22.7. Quality Level at Factories for Smaller-the-Better Characteristics

A smaller-the-better characteristic is nonnegative and its optimal value is zero. The following loss function, L, gives the quality level for smaller-the-better characteristics:

$$L = \frac{A_0}{\Delta_0^2}\sigma^2 \tag{22.40}$$

where A_0 is the loss incurred by the company if the functional limit is exceeded, Δ_0 the functional limit, and σ^2 the mean-squared error, or variance.

In production factories, the following Δ and A are used instead of Δ_0 and A_0, respectively.

Δ: factory standard

A: loss incurred by the factory if the factory standard is not fulfilled

This is because Δ is obtained from

$$\Delta = \frac{\Delta_0}{\phi} \tag{22.41}$$

and the following equality holds, as in equation (22.31):

$$\frac{A}{\Delta^2} = \frac{A_0}{\Delta_0^2} \tag{22.42}$$

❒ Example

Suppose that the specification of roundness is less than 12 μm. The loss when the product is not acceptable, A, is 80 cents. Two machines, A_1 and A_2, are used in production, and the data for roundness of samples taken two per day for two weeks is summarized in Table 22.2. The unit is micrometers.

Table 22.2
Roundness of samples from two machines

A_1	0	5	4	2	3	1	7	6	8	4
	6	0	3	10	4	5	3	2	0	7
A_2	5	4	0	4	2	1	0	2	5	3
	2	1	3	0	2	4	1	6	2	1

We compare the quality level of two machines, A_1 and A_2. The characteristic value of this example is a smaller-the-better characteristic with an upper standard limit. Thus, equation (22.40) is the formula to use:

$$\Delta = 12\ \mu\text{m} \tag{22.43}$$

$$A = 80 \text{ cents} \tag{22.44}$$

Variance, σ^2, is calculated separately for A_1 and A_2. For A_1,

$$\sigma^2 = \tfrac{1}{20}(y_1^2 + y_2^2 + \cdots + y_{20}^2) = \tfrac{1}{20}(0^2 + 5^2 + 4^2 + \cdots + 7^2)$$

$$= 23.4\ \mu\text{m}^2 \tag{22.45}$$

$$L = \frac{A}{\Delta^2}\sigma^2 = \frac{0.80}{12^2}(23.4) = 13 \text{ cents} \tag{22.46}$$

For A_2,

$$\sigma^2 = \tfrac{1}{20}(5^2 + 4^2 + 0^2 + \cdots + 1^2) = 8.8\ \mu\text{m}^2 \tag{22.47}$$

$$L = \frac{0.80}{12^2}(8.8) = 4.9 \text{ cents} \tag{22.48}$$

Thus, compared to A_1, A_2 has a better quality level.

$$13.0 - 4.9 = 8.1 \text{ cents} \tag{22.49}$$

or A_2 is

$$\frac{13.0}{4.9} = 2.7 \tag{22.50}$$

times better than A_1. If production is 2000 per day, the difference between the two machines will amount to $40,500 mil per year, assuming that the number of working days per year is 250.

22.8. Quality Level at Factories for Larger-the-Better Characteristics

A larger-the-better characteristic is a characteristic value that is nonnegative and whose most desirable value is infinity. For thermal efficiency, yield, and fraction acceptance, the maximum value is 1 (100%) even though the value never takes on a negative value and the larger value is better. Thus, characteristics that do not have a target value and whose number should be as large as possible (e.g., amplification factor, power, strength, harvest) are larger-the-better characteristics.

Suppose that y is a larger-the-better characteristic and that its loss function is

$$L(y) \qquad (0 \le y \le \infty) \tag{22.51}$$

$L(y)$ is expanded around $y = \infty$ using Maclaurin's expansion.

$$L(y) = L(\infty) + \frac{L'(\infty)}{1!}\frac{1}{y} + \frac{L''(\infty)}{2!}\frac{1}{y^2} + \cdots \tag{22.52}$$

Loss is zero at $y = \infty$, and the following is immediate:

$$L(\infty) = 0 \tag{22.53}$$

$$L'(\infty) = 0 \tag{22.54}$$

Thus, by substituting the above into equation (22.52) and omitting the cubic terms and beyond, we get the following approximation:

$$L(y) = k\frac{1}{y^2} \tag{22.55}$$

The proportion coefficient, k, can be determined by the following equation once the loss at a certain point, $y = y_0$, that is, $L(y_0)$, is known:

$$k = y_0^2\, L(y_0) \tag{22.56}$$

For this point we can use the point when a problem actually occurs in the market, Δ_0, and derive k with loss A_0 at that point:

$$k = \Delta_0^2 A_0 \tag{22.57}$$

Thus, the loss function becomes

$$L = \frac{\Delta_0^2 A_0}{y^2} \tag{22.58}$$

Let $y_1, y_2, \ldots, y_n$ denote n observations. We can calculate the quality level per product by substituting the data into equation (22.58). Let L be the quality level per product. Then

$$L = A_0 \Delta_0^2 \sigma^2 \tag{22.59}$$

where

$$\sigma^2 = \frac{1}{n}\left(\frac{1}{y_1^2} + \frac{1}{y_2^2} + \cdots + \frac{1}{y_n^2}\right) \tag{22.60}$$

In other words, the inverse of a larger-the-better characteristic is approximated by a smaller-the-better characteristic.

❒ Example

For three-layered reinforced rubber hose, the adhesive strength between layers—K_1, the adhesion of rubber tube and reinforcing fiber, and K_2, the adhesion of reinforcing fiber and cover rubber—is very important. For both adhesive strengths, a lower standard, Δ, is set as 5.0 kg_f. If a defective product is scrapped, its loss is \$5 per unit. Annual production is 120,000 units. Two different types of adhesive, A_1 (50 cents) and A_2 (60 cents), were used to produce eight test samples each,

Table 22.3
Adhesive strength

A_1	K_1	10.2	5.8	4.9	16.1	15.0	9.4	4.8	10.1
	K_2	14.6	19.7	5.0	4.7	16.8	4.5	4.0	16.5
A_2	K_1	7.6	13.7	7.0	12.8	11.8	13.7	14.8	10.4
	K_2	7.0	10.1	6.8	10.0	8.6	11.2	8.3	10.6

and the adhesive strength of K_1 and K_2 was checked. The data are given in Table 22.3.

Let us compare the quality level of A_1 and A_2. Since it is a larger-the-better characteristic, we want the mean to be large and the variability to be small. A measurement that takes both of these requirements into account is the *variance*, that is, the mean square of the inverse, given by equation (22.60). Variances for A_1 and A_2 are derived from equation (22.60) separately, and the loss function is obtained from equation (22.59). For A_1,

$$\sigma_1^2 = \frac{1}{16}\left(\frac{1}{0.2^2} + \frac{1}{5.8^2} + \cdots + \frac{1}{16.5^2}\right)$$
$$= 0.02284 \qquad (22.61)$$
$$L_1 = A_0\Delta_0^2\sigma_1^2 = (5)(5^2)(0.02284)$$
$$= \$2.85 \qquad (22.62)$$

For A_2,

$$\sigma_2^2 = \frac{1}{16}\left(\frac{1}{7.6^2} + \frac{1}{13.7^2} + \cdots + \frac{1}{10.6^2}\right)$$
$$= 0.01139 \qquad (22.63)$$
$$L_2 = (5)(5^2)(0.01139) = \$1.42 \qquad (22.64)$$

Therefore, even if the adhesion cost (the sum of adhesive and labor) is 10 cents higher in the case of A_2, the use of A_2 is better than the use of A_1.

The amount of savings per unit is

$$(0.50 + 2.85) - (0.60 + 1.42) = \$1.33 \qquad (22.65)$$

In one year, this amounts to a savings of \$159,700.

Reference

1. Genichi Taguchi et al., 1994. *Quality Engineering Series 2: On-Line Production*. Tokyo: Japanese Standards Association and ASI Press.