15 Parameter Design

15.1. Introduction

After a system is selected, a prototype is made for experimentation, or simulation without experimentation is conducted. If the prototype functions, or the results of the simulation, are satisfactory under certain conditions (normal or standard), the system design is complete. *Prototype functions* indicates that it hits the objective target value "under certain conditions."

The most important objective after the completion of system design is the determination of parameter midvalues (levels). This study is conducted by using the lowest-grade, least expensive raw materials and component parts. Tolerance design, which is to tighten tolerances to secure the function and better product quality, is accompanied by cost increase. Therefore, *tolerance design should be conducted after parameter design is complete.*

In many cases of traditional product design, drawings and specifications are made for production immediately after the prototype functions only under certain conditions. No further studies are made until problems are reported or complaints occur in the marketplace. The product is then reviewed, adjusted, or redesigned, which is called *firefighting*. Firefighting is needed when there has not been robust design.

Good product design engineers study robustness after system design. The prototype is tested under some customer's usage conditions. Suppose that there are several extreme usage conditions (noise factors). Using the traditional approach, an engineer tests a product by varying one of the noise conditions. If the output response deviates from the target, the engineer adjusts one of the design parameters to hit the target. Then the second noise factor is varied. The output deviates

 Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu again. The engineer then varies another or other control factors to adjust output to the target. Such procedures are repeated again and again until the target is hit under all extreme conditions. But this is not a product design; it is merely operational work, called *modification*, *adjusting*, or *tuning*. It is extremely tedious, since this approach is similar to trying to solve several simultaneous equations—not mathematically, but through hardware.

The foregoing approach is misguided because the engineer tries to hit the target first and reduce variability last. In parameter design, robustness must be improved first, and the target is adjusted last. This is called two-step optimization.

15.2. Noise

Variables that cause product functions are called *noise*. There are three types of noise:

1. *Outer noise:* variation caused by environmental conditions (e.g., temperature, humidity, dust, input voltage)
2. *Inner noise:* deterioration of elements or materials in the product
3. *Between-product noise:* piece-to-piece variation between products

Parameter design is used to select the best control-factor-level combination so that the effect of all of the noise above can be minimized.

❒ Example [1]

The objective of a TV set's power circuit is to convert 100-V ac input into 115-V dc output. If the power circuit maintains a 115-V output anytime, anywhere, the quality of this power circuit is perfect as far as voltage is concerned. For example, element resistor (A) and the h_{FE} value of a transistor (B) in a power circuit affect the output voltage as shown in Figure 15.1.

Suppose that a design engineer forecasts the midvalues of the elements in the circuit, assembles these to get a circuit, and puts 100 V of ac into it. If the output voltage so obtained is only 100 V instead of 115 V, he then changes resistance A from $A_1 = 200\ \Omega$ to $A' = 250\ \Omega$ to adjust the 15-V deviation from the target value. From the standpoint of quality control, this is very poor methodology. Assume that the resistance used in the circuit is the cheapest grade. It either varies or deteriorates to a maximum range of $\pm 10\%$ during its service period as the power source of a TV set. From Figure 15.1, we see that the output voltage varies within a range of ± 6 V. In other words, the output voltage varies by the influence of inner noise, such as the original variation of the resistance itself, or due to deterioration. If resistance A'', which has a midvalue of 350 Ω, is used instead of A', its influence on the output voltage is only ± 1 V, even if its variation is $\pm 10\%$ (35 Ω). However, the output voltage goes up about 10 V, as seen from the figure. Such a deviation can be adjusted by a factor such as B with an almost linear influence on the output voltage. In this case, 200 Ω is selected instead of 500 Ω for level B. A factor such

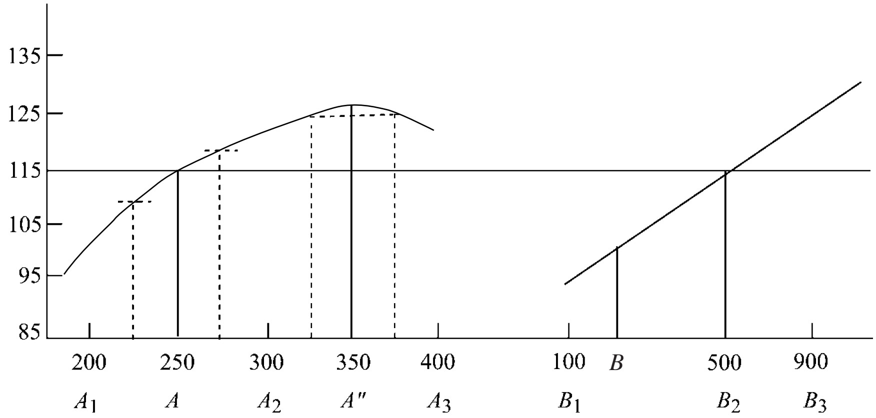

Figure 15.1
Relationship between factors *A* and *B* and the output voltage

as *B*, with a differential coefficient to the output voltage that is nearly constant no matter what level is selected, is not useful to reduce variability. It is used merely for the purpose of adjusting the deviation from a target value.

Parameter design is the most important step in developing stable and reliable products or manufacturing processes. With this technique, nonlinearity may be utilized positively. (Factor *A* has this property.) In this step we find a combination of parameter levels that are capable of damping the influences not only of inner noise, but also of all noise sources, while keeping the output voltage constant. At the heart of research lives a conflict: to design a product that is reliable within a wide range of performance conditions but at the lowest price. Naturally, elements or component parts with a short life and wide tolerance variation are used. The aim of the design of experiments is the utilization of nonlinearity.

15.3. Parameter Design of a Wheatstone Bridge [2]

❒ Example

The purpose of a Wheatstone bridge is to measure a resistance denoted by *y*. Figure 15.2 shows the circuit diagram for the Wheatstone bridge. The task is to select the midvalues of parameters *A*, *C*, *D*, *E*, and *F*.

To measure resistor *y*, adjustable resistor *B* is adjusted to bring the reading of the ammeter, *X*, to zero. *B* is therefore not a control factor, but factors *A*, *C*, *D*, *E*, and *F* are. The levels of these control factors are selected as shown in Table 15.1.

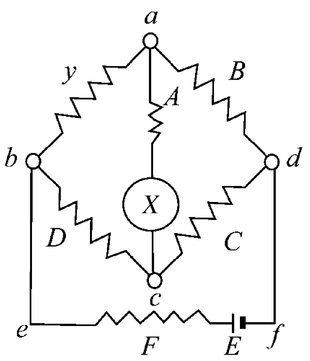

Figure 15.2
Wheatstone Bridge

The purpose of parameter design is to investigate the overall variation caused by inner and outer noise when the levels of the control factors are allowed to vary widely. The next step is to find a stable or robust design that is essentially unaffected by inner or outer noise. Therefore, the most likely types of inner and outer noise factors must be identified and their influence must be investigated.

Consider the variations of the elements of the bridge. First, the low-priced elements are used. The varying ranges of these characteristics are shown in the three levels of the seven noise factors in Table 15.2. Since there may be a reading error, the error factors B', adjustable resistor, and X', the ammeter, are cited in the table.

Since it is believed that the power source voltage has a minimal effect on the measurement error, the lowest-priced battery on the market is used, and the three levels of battery error are set at −5%, 0, and +5%. Regardless of whether the research concerns a measurement technique for minimizing error or a design for product stability, it is advisable first to consider the lower-priced component parts and materials.

In parameter design, wide intervals between factor levels are used to increase the possibility of finding a parameter level combination at which the variability of the product quality characteristic is reduced. Therefore, it would be wasteful to use high-priced components or materials at the early stage of product design.

Table 15.1
Three levels of control factors

Factor	Level 1	Level 2	Level 3
A (Ω)	20	100	500
C (Ω)	2	10	50
D (Ω)	2	10	50
E (V)	1.2	6	30
F (Ω)	2	10	50

Table 15.2
Three levels of noise factors

Factor	Level 1	Level 2	Level 3
A' (%)	−0.3	0	0.3
B' (%)	−0.3	0	0.3
C' (%)	−0.3	0	0.3
D' (%)	−0.3	0	0.3
E' (%)	−5.0	0	5.0
F' (%)	−0.3	0	0.3
X' (mA)	−0.2	0	0.2

Parameter design is the first priority in the improvement of measuring precision, stability, and/or reliability. When parameter design is completed, tolerance design is used to further reduce error factor influences.

In a Wheatstone bridge measurement, an unknown resistor, y, is connected between points a and b (Figure 15.2). Resistor B is adjusted to the point where no current, denoted by X, flows through the ammeter. The unknown resistance, y, is calculated by

$$y = \frac{BD}{C} \tag{15.1}$$

To investigate the error of measurement, assume that when the ammeter reads zero, there is a small amount of current actually flowing. In this case, the resistance, y, is not calculated by equation (15.1) but by

$$y = \frac{BD}{C} - \frac{X}{C^2E}[A(D + C) + D(B + C)][B(C + D) + F(B + C)] \tag{15.2}$$

Control factors A, C, D, E, and F, shown in Table 15.1, are assigned in columns 1, 3, 4, 5, and 6 of orthogonal array L_{36}. An orthogonal array where control factors are assigned is called an *inner orthogonal array*. Orthogonal array L_{36} is the most popular array for use in parameter design and tolerance design when there are equations for the research. This is true because, in most instances, three levels are used for control and noise factors and the interactions between control factors are not considered. Since interactions are not calculated, the interaction effects are treated as errors. It is advantageous to have the effects of these interactions distributed uniformly in all columns. Orthogonal array L_{36} is the one array with interactions between any columns distributed uniformly among all columns. Of course, there are situations where orthogonal arrays other than the L_{36} array are used to assign control factors. In actual experiments, the adoption of the L_{36} array requires an enormous amount of time and expense. Therefore, many other sophisticated layout techniques have been developed that can be used.

Layout and Data Analysis

In Table 15.3 there are 36 combinations of the control factors *A*, *C*, *D*, *E*, and *F*. These represent 36 combinations of the midvalues of these factors and form the inner array. Prepare another orthogonal array, L_{36}, the *outer array*, to assign the error factors of A', B', C', D', E', F', and X'. There are 36 × 36, or 1296, combinations, and the values of y are calculated from these 1296 combinations. Such a layout is called the *direct product layout* of the inner orthogonal array (L_{36}) and the outer orthogonal array (L_{36}). Table 15.3 displays the direct product layout.

There are nine combinations between a control factor and its noise factor. For example, actual levels of the nine combinations between control factor *A* and noise factor A' are given in Table 15.4. In the outer array, noise factors A', B', C', D', E', F', and X' are assigned to columns 1, 2, 3, 4, 5, 6, and 7, respectively. Table 15.5 shows the three levels of noise factors for level 2 configuration (condition) of control factors in the inner array. The combination of experiment 2 of the inner array is $A_2C_2D_2E_2F_2$. The actual levels are: resistors $A_2 = 100\ \Omega$, $C_2 = 10\ \Omega$, $D_2 = 10\ \Omega$, battery $E_2 = 6$ V, and resistor $F_2 = 10\ \Omega$. Around these control factor

Table 15.3
Direct product layout

									Outer Array								
									Col.	**No.**	**1**	**2**	**3**	**4**	⋯	**36**	
									A'	1	1	2	3	1	⋯	3	
									B'	2	1	2	3	1	⋯	2	
									C'	3	1	2	3	1	⋯	3	
									D'	4	1	2	3	1	⋯	1	
									E'	5	⋮	⋮	⋮	⋮	⋯	⋮	
									F'	6	⋮	⋮	⋮	⋮	⋯	⋮	
									X'	7	⋮	⋮	⋮	⋮	⋯	⋮	
										⋮	⋮	⋮	⋮	⋮	⋯	⋮	
Inner Array																	
Col	***A***	***e***	***C***	***D***	***E***	***F***	⋯	***e***									**η**
No.	**1**	**2**	**3**	**4**	**5**	**6**	⋯	**13**		**13**							
1	1	1	1	1	·	·	⋯	1			$y_{1.1}$	$y_{1.2}$	$y_{1.3}$	$y_{1.4}$	⋯	$y_{1.36}$	η_1
2	2	2	2	2	·	·	⋯	1			$y_{2.1}$	$y_{2.2}$	$y_{2.3}$	$y_{2.4}$	⋯	$y_{2.36}$	η_2
3	3	3	3	3	·	·	⋯	1			$y_{3.1}$	$y_{3.2}$	$y_{3.3}$	$y_{3.4}$	⋯	$y_{3.36}$	η_3
4	1	1	1	1	·	·	⋯	1			$y_{4.1}$	$y_{4.2}$	$y_{4.3}$	$y_{4.4}$	⋯	$y_{4.36}$	η_4
⋮	⋮	⋮	⋮	⋮	·	·	⋯	·			⋮	⋮	·	·	⋯	⋮	⋮
36	3	2	3	1	·	·	⋯	3			$y_{36.1}$	$y_{36.2}$	·	·	⋯	$y_{36.36}$	η_{36}

Table 15.4
Actual levels of resistor *A*

Control Factor *A*	Noise Factor *A′* Level 1 A'_1	Level 2 A'_2	Level 3 A'_3
Level 1, A_1	19.94	20.00	20.06
Level 2, A_2	99.70	100.00	100.30
Level 3, A_3	498.50	500.00	501.50

levels, three-level noise factors are prepared, that is, ±0.3% around the foregoing level of each resistor and ±5% around the 6 V for the battery. The midvalue of resistor *B* is always equal to 2 Ω, and the midvalue of the ammeter reading is always equal to zero. The three levels of these two factors of experiment 2 of the inner orthogonal array are shown in Table 15.5.

For example, the actual levels of noise factors of experiment 2 of the inner array and experiment 1 of the outer array are shown in the first column of Table 15.5, such as $A = 99.7\ \Omega$, B = 1.994 Ω, ... , $X = -0.0002$ A. Putting these figures in equation (15.2), *y* is calculated as

$$\begin{aligned} y &= \frac{(1.994)(9.97)}{9.97} - \frac{-0.0002}{(9.97^2)(5.7)} \\ &\quad \times [(99.7)(9.97 + 9.97) + (9.97)(1.994 + 9.97)] \\ &\quad \times [(1.994)(9.97 + 9.97) + (9.97)(1.994 + 9.97)] \\ &= 2.1123 \end{aligned} \tag{15.3}$$

Table 15.5
Three levels of noise factors of experiment 2 of inner array

Factor	1	2	3
A (Ω)	99.7	100.0	100.3
B (Ω)	1.994	2.0	2.006
C (Ω)	9.97	10.0	10.03
D (Ω)	9.97	10.0	10.03
E (V)	5.7	6.0	6.3
F (Ω)	9.97	10.0	10.03
X (A)	−0.0002	0	0.0002

Assume that the true value of y is 2 Ω. From this value the true value, 2 Ω, must be subtracted to obtain an error, which is shown in column 1 of the results of Table 15.6.

$$2.1123 - 2 = 0.1123 \tag{15.4}$$

Similar calculations are made for the other 35 configurations of the outer array to get the data of results column 1 of Table 15.6. Since the configuration of experiment 2 represents current levels (before parameter design), current variability is calculated from the experiment 2 data. The total variation of errors of these 36 pieces of data, denoted by S_T, is

$$S_T = 0.1123^2 + 0.0000^2 + \cdots + (-0.0120)^2 \tag{15.5}$$

$$= 0.31141292 \qquad (f = 36) \tag{15.6}$$

The number shown in equation (15.6) was calculated using a computer, with the control factors at the second level and with the varying noise factor ranges as given in Table 15.2, the total error variance, V_T, is

$$V_T = \frac{S_T}{36} = \frac{0.31141292}{36} = 0.00865036 \tag{15.7}$$

With a different combination of control factors, and the noise factors varying according to the levels in Table 15.2, the error variance of measurements changes. From experiment 1 through 36, combinations of the inner array, the sum of squares of errors, S_e, and the sum of squares of the general mean, S_m, are calculated:

$$S_e = \text{(total of the squares of 36 errors)}$$

$$- (\text{correction factor, } S_m) \qquad (f = 35) \tag{15.8}$$

$$V_e = \frac{S_e}{35} \tag{15.9}$$

$$S_m = \frac{[(2)(36) + (\text{total of 36 errors})]^2}{36} \tag{15.10}$$

The SN ratio, η, is

$$\eta = \frac{\frac{1}{36}(S_m - V_e)}{V_e} \tag{15.11}$$

For example, η of experiment 2 is calculated as

$$S_e = 0.1123^2 + 0.0000^2 + (-0.1023)^2 + \cdots + (-0.0120)^2 - S_m$$

$$= 0.31140718 - \frac{0.0006^2}{36} = 0.31140717 \tag{15.12}$$

$$V_e = \frac{0.31140717}{35} = 0.008897347 \tag{15.13}$$

Table 15.6
Layout of noise factors and the data of measurement error

No.	A′ 1	B′ 2	C′ 3	D′ 4	E′ 5	F′ 6	X′ 7	8	9	10	11	12	13	(1) Expt. 2	(2) Improved Condition
1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.1123	−0.0024
2	2	2	2	2	2	2	2	2	2	2	2	2	1	0.0000	0.0000
3	3	3	3	3	3	3	3	3	3	3	3	3	1	−0.1023	0.0027
4	1	1	1	1	2	2	2	2	3	3	3	3	1	−0.0060	−0.0060
5	2	2	2	2	3	3	3	3	1	1	1	1	1	−0.1079	−0.0033
6	3	3	3	3	1	1	1	1	2	2	2	2	1	0.1252	0.0097
7	1	1	2	3	1	2	3	3	1	2	2	3	1	−0.1188	−0.0036
8	2	2	3	1	2	3	1	1	2	3	3	1	1	0.1009	−0.0085
9	3	3	1	2	3	1	2	2	3	1	1	2	1	0.0120	0.0120
10	1	1	3	2	1	3	2	3	2	1	3	2	1	−0.0120	−0.0120
11	2	2	1	3	2	1	3	1	3	2	1	3	1	−0.1012	0.0086
12	3	3	2	1	3	2	1	2	1	3	2	1	1	0.1079	0.0033
13	1	2	3	1	3	2	1	3	3	2	1	2	2	0.0950	−0.0087
14	2	3	1	2	1	3	2	1	1	3	2	3	2	0.0120	0.0120
15	3	1	2	3	2	1	3	2	2	1	3	1	2	−0.1132	−0.0035
16	1	2	3	2	1	1	3	2	3	3	2	1	2	−0.1241	−0.0096
17	2	3	1	3	2	2	1	3	1	1	3	2	2	0.1317	0.0215
18	3	1	2	1	3	3	2	1	2	2	1	3	2	−0.0120	−0.0120
19	1	2	1	3	3	3	1	2	2	1	2	3	2	0.1201	0.0153
20	2	3	2	1	1	1	2	3	3	2	3	1	2	0.0000	0.0000
21	3	1	3	2	2	2	3	1	1	3	1	2	2	−0.1250	−0.0154
22	1	2	2	3	3	1	2	1	1	3	3	2	2	0.0060	0.0060
23	2	3	3	1	1	2	3	2	2	1	1	3	2	−0.1247	−0.0096
24	3	1	1	2	2	3	1	3	3	2	2	1	2	0.1138	0.0035
25	1	3	2	1	2	3	3	1	3	1	2	2	3	−0.1129	−0.0035
26	2	1	3	2	3	1	1	2	1	2	3	3	3	0.0951	−0.0087
27	3	2	1	3	1	2	2	3	2	3	1	1	3	0.0120	0.0120
28	1	3	2	2	2	1	1	3	2	3	1	3	3	0.1186	0.0095
29	2	1	3	3	3	2	2	1	3	1	2	1	3	−0.0060	−0.0060
30	3	2	1	1	1	3	3	2	1	2	3	2	3	−0.1197	−0.0036
31	1	3	3	3	2	3	2	2	1	2	1	1	3	0.0060	0.0060
32	2	1	1	1	3	1	3	3	2	3	2	2	3	−0.1133	−0.0093
33	3	2	2	2	1	2	1	1	3	1	3	3	3	0.1194	0.0036
34	1	3	1	2	3	2	3	1	2	2	3	1	3	−0.0957	0.0087
35	2	1	2	3	1	3	1	2	3	3	1	2	3	0.1194	0.0036
36	3	2	3	1	2	1	2	3	1	1	2	3	3	−0.0120	−0.0120

$$S_m = \frac{[(2)(36) + 0.0006)]^2}{36} = 144.00240000 \tag{15.14}$$

$$\eta = \frac{\frac{1}{36}(144.0024 - 0.008897347)}{0.008897347} = 449.552 \tag{15.15}$$

On the decibel scale,

$$\eta = 10 \log \frac{\frac{1}{36}(S_m - V_e)}{V_e} = 10 \log(449.552) = 26.7 \text{ dB} \qquad (15.16)$$

Analysis of SN Ratio

From the 36 data for each of the 36 conditions of the inner orthogonal array, the SN ratios are calculated using equation (15.11). Table 15.7 shows the SN ratios converted to decibels. The SN ratio is the reciprocal measurement of error variance. Accordingly, the optimum design is a combination of levels, which maximizes the decibel value. For this purpose, the decibel is used as the objective characteristic unit, and data analysis is made based on that value for the inner orthogonal array. The method is the same as the ordinary analysis of variance. For example, the main effect of *A* is (result from nontruncated data):

$$S_A = \frac{A_1^2 + A_2^2 + A_3^2}{12} - S_m$$

$$= \frac{378.7^2 + 225.4^2 + 80.9^2}{12} - \frac{685.0^2}{36} \qquad (15.17)$$

$$= 3700.21 \qquad (f = 2) \qquad (15.18)$$

Other factors—S_C, S_D, S_E, S_F, and S_T—are calculated similarly. Table 15.8 is the ANOVA table for the SN ratio.

$$S_T = 32.2^2 + 26.7^2 + \cdots + 8.0^2 - \frac{685.0^2}{36} = 11{,}397.42 \qquad (f = 35) \qquad (15.19)$$

When the SN ratio is the objective characteristic, it is important to investigate how it varies with respect to control-factor-level changes. Factor-level selection should be made based on the respective SN ratios, no matter how small differences between ratios may be. Cost differences between levels should always be taken into account.

Table 15.9 shows the average of each significant control factor level. These effects are also shown in Figure 15.3. From Figure 15.3, the combination that gives the largest SN ratio is A_1, C_3, D_2, E_3, *and* F_1. The forecasting of the gain from the midlevel combination $A_2C_2D_2E_2F_2$ is

$$\begin{aligned} \text{gain} &= (31.56 - 18.78) + (21.42 - 21.10) + (21.24 - 21.24) \\ &\quad + (32.89 - 18.52) + (27.58 - 19.68) \\ &= 12.78 + 0.32 + 0 + 14.37 + 7.90 \\ &= 35.37 - 14.54 = 20.83 \end{aligned} \qquad (15.20)$$

Table 15.7
Layout of control factors (inner array) and SN ratio

No.	*A* 1	*B* 2	*C* 3	*D* 4	*E* 5	*F* 6	*e* 7	*e* 8	*e* 9	*e* 10	*e* 11	*e* 12	*e* 13	Results (dB)
1	1	1	1	1	1	1	1	1	1	1	1	1	1	32.2
2	2	2	2	2	2	2	2	2	2	2	2	2	1	26.7[a]
3	3	3	3	3	3	3	3	3	3	3	3	3	1	15.9
4	1	1	1	1	2	2	2	2	3	3	3	3	1	36.4
5	2	2	2	2	3	3	3	3	1	1	1	1	1	28.6
6	3	3	3	3	1	1	1	1	2	2	2	2	1	7.2
7	1	1	2	3	1	2	3	3	1	2	2	3	1	16.5
8	2	2	3	1	2	3	1	1	2	3	3	1	1	13.0
9	3	3	1	2	3	1	2	2	3	1	1	2	1	28.0
10	1	1	3	2	1	3	2	3	2	1	3	2	1	15.0
11	2	2	1	3	2	1	3	1	3	2	1	3	1	16.4
12	3	3	2	1	3	2	1	2	1	3	2	1	1	25.5
13	1	2	3	1	3	2	1	3	3	2	1	2	2	43.8
14	2	3	1	2	1	3	2	1	1	3	2	3	2	−8.3
15	3	1	2	3	2	1	3	2	2	1	3	1	2	14.6
16	1	2	3	2	1	1	3	2	3	3	2	1	2	29.0
17	2	3	1	3	2	2	1	3	1	1	3	2	2	6.9
18	3	1	2	1	3	3	2	1	2	2	1	3	2	14.7
19	1	2	1	3	3	3	1	2	2	1	2	3	2	21.5
20	2	3	2	1	1	1	2	3	3	2	3	1	2	17.4
21	3	1	3	2	2	2	3	1	1	3	1	2	2	17.4
22	1	2	2	3	3	1	2	1	1	3	3	2	2	46.5
23	2	3	3	1	1	2	3	2	2	1	1	3	2	5.5
24	3	1	1	2	2	3	1	3	3	2	2	1	2	−8.2
25	1	3	2	1	2	3	3	1	3	1	2	2	3	27.3
26	2	1	3	2	3	1	1	2	1	2	3	3	3	43.4
27	3	2	1	3	1	2	2	3	2	3	1	1	3	−20.9
28	1	3	2	2	2	1	1	3	2	3	1	3	3	44.1
29	2	1	3	3	3	2	2	1	3	1	2	1	3	39.3
30	3	2	1	1	1	3	3	2	1	2	3	2	3	−17.0
31	1	3	3	3	2	3	2	2	1	2	1	1	3	23.0
32	2	1	1	1	3	1	3	3	2	3	2	2	3	44.2
33	3	2	2	2	1	2	1	1	3	1	3	3	3	−0.9
34	1	3	1	2	3	2	3	1	2	2	3	1	3	43.4
35	2	1	2	3	1	3	1	2	3	3	1	2	3	−7.7
36	3	2	3	1	2	1	2	3	1	1	2	3	3	8.0

[a] Example calculated.

Table 15.8
ANOVA table for the SN ratio

Source	*f*	*S*	*V*
A	2	3,700.21	1,850.10
C	2	359.94	179.97
D	2	302.40	151.20
E	2	4,453.31	2,226.65
F	2	1,901.56	950.97
e	25	680.00	27.20
Total	35	11,397.42	

The next step is to attempt to confirm the foregoing conclusion by an actual error calculation. In the neighborhood of the optimum condition, three levels of error factors A', B', C', D', E', F', and X' are prepared according to Table 15.2, and the errors are calculated.

Results are shown in the last column of Table 15.6. The sum of squares of error, S_T, is

$$S_T = (-0.0024)^2 + 0.0000^2 + \cdots + (-0.0120)^2$$
$$= 0.00289623 \qquad (15.21)$$

The error variance including the general mean is

$$V_T = \frac{0.00289623}{36} = 0.00008045 \qquad (15.22)$$

Compared with the result in equation (15.7), this error variance, under the optimum condition, is reduced by 1/107.5, a gain of 20.32 dB. Such an improvement is much greater than the improvement obtainable by tolerance design when the varying range of each element is reduced by $\frac{1}{10}$ (probably at a huge cost increase). In

Table 15.9
Estimate of significant factors: Average of each level (decibels)

Level	*A*	*C*	*D*	*E*	*F*
1	31.56	14.56	20.91	5.66	27.58
2	18.78	21.10	21.24	18.52	19.68
3	6.73	21.42	14.93	32.89	9.81

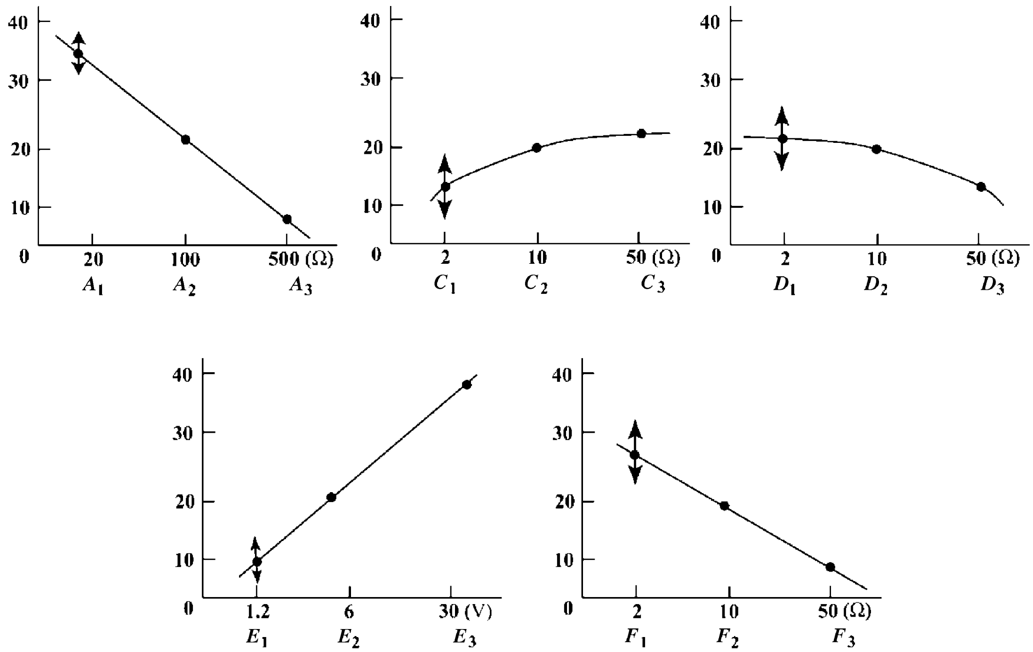

Figure 15.3
Effects of significant factors

parameter design, all component parts used in the circuit are low-priced, with wide varying ranges. In fact, the role of parameter design is to reduce measurement error, achieve higher stability, and create a significant improvement of quality by using components and/or materials that have a widely varying range and that are inexpensive. In the case of product or process design, a significant quality improvement is expected by adopting this same method.

15.4. Parameter Design of a Water Feeder Valve [3]

❒ Example

McDonnell & Miller Company produces hot-water boiler control systems. There was a problem from the field: chattering of water feeder valves. Chattering manifested itself when a feeder valve was slightly open or slightly closed. In such a case, a system harmonic was created, resulting in a loud, rolling noise through the heating pipes. The noise was considered by customers to be a significant nuisance that had to be minimized or eliminated. Figure 15.4 shows the configuration of the cartridge in the valve. A traditional one-factor-at-a-time approach was tried. The solutions, in particular the quad ring and the round poppet, caused ancillary difficulties and later had to be reversed.

Based on the Taguchi methods concept that "to get quality, don't measure quality," the engineers tried not to measure quality, that is, symptoms such as vibration and audible noise. Instead, the product function was discussed to define a generic function.

The generic function was defined as (Figure 15.5)

$$y = \frac{\beta M}{M^*} \tag{15.23}$$

where y is the flow rate, M the cross-sectional flow area, and M^* the square root of the inlet pressure. The idea was that if the relationship above was optimized, problems such as vibration or audible noise should be eliminated. For calculation of the SN ratio, zero-point proportional equation was used.

Experimental Layout

Figure 15.6 illustrates the parameter diagram that shows the control, noise, and signal factors. Table 15.10 explains the control, noise, and signal factors and their levels.

Orthogonal array L_{18} was used to assign control factors. Noise and signal factors were assigned to outside the array. Table 15.11 shows the layout, calculated SN ratio, and sensitivity. From Tables 15.11 and 15.12, response tables and response

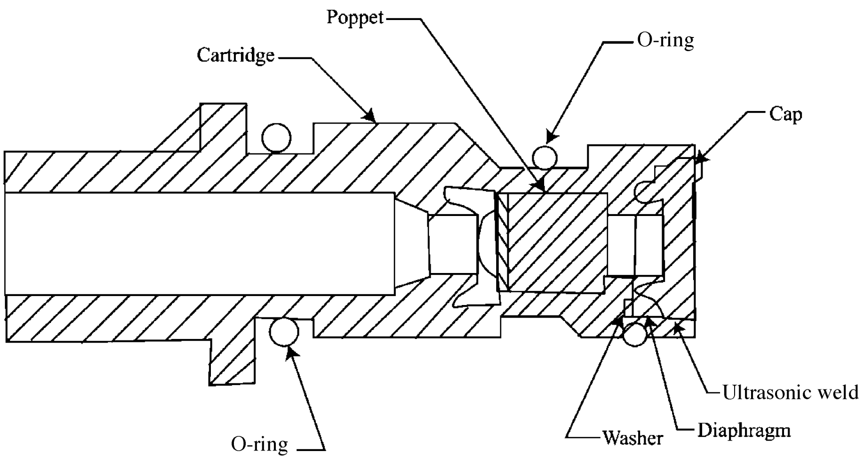

Figure 15.4
Cartridge configuration

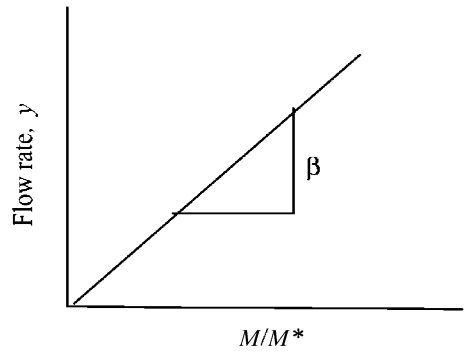

Figure 15.5
Ideal function

graphs for the SN ratio and sensitivity were prepared. Table 15.13 shows the response tables for the SN ratio and sensitivity, respectively. Figure 15.7 shows the response graph of the SN ratio.

Optimization and Confirmation

The optimum condition was selected as

$$A_2\ B_2\ C_3\ D_2\ E_2\ F_2$$

The optimum condition predicted for the SN ratio was calculated by adding together to larger effects: A, C, D, and E, as follows:

$$\begin{aligned}\eta_{\text{opt}} &= \overline{T} + (\overline{A}_2 - \overline{T}) + (\overline{C}_3 - \overline{T}) + (\overline{D}_2 - \overline{T}) + (\overline{E}_2 - \overline{T}) \\ &= \overline{A}_2 + \overline{C}_3 + \overline{D}_2 + \overline{E}_2 - 3\overline{T} \\ &= -5.12 - 4.94 - 5.24 - 4.87 - (3)(-5.59) \\ &= -3.40 \text{ dB} \end{aligned} \qquad (15.24)$$

The initial condition was purposely set as trial 1:

$$A_1\ B_1\ C_1\ D_1\ E_1\ F_1$$

Therefore, its SN ratio (actual or predicted) is found from Table 15.13 as

$$\eta_{\text{initial}} = -8.27 \text{ dB} \qquad (15.25)$$

The predicted optimum conditions of sensitivity was calculated using the larger effects on sensitivity, A, B, D, E, and F.

$$\begin{aligned}S_{\text{opt}} &= \overline{T} + (\overline{A}_2 - \overline{T}) + (\overline{B}_2 - \overline{T}) + (\overline{D}_2 - \overline{T}) + (\overline{E}_2 - \overline{T}) + (\overline{F}_2 - \overline{T}) \\ &= \overline{A}_2 + \overline{B}_2 + \overline{D}_2 + \overline{E}_2 + \overline{F}_2 - 4\overline{T} \\ &= -6.27 - 6.24 - 6.44 - 7.80 - 6.60 - (4)(-6.45) \\ &= -7.55 \text{ dB} \end{aligned} \qquad (15.26)$$

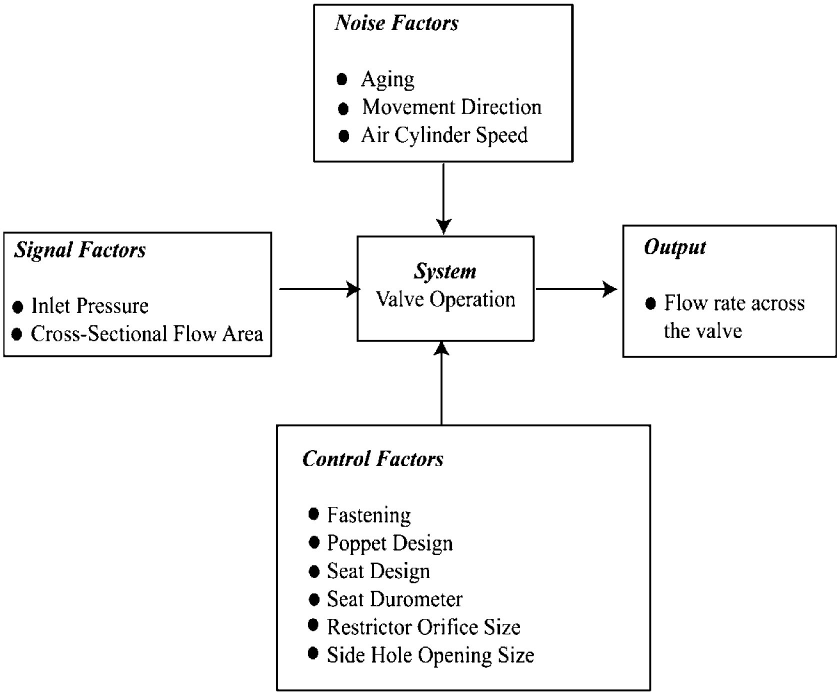

Figure 15.6
Parameter diagram

Table 15.10
Factors and levels

Factor	Description	Level 1	Level 2	Level 3
Control Factors				
A	Fastening	Not fastened	Fastened	
B	Poppet design	Hex	Square	
C	Seat design	Flat	Conical	Spherical
D	Durometer	80	60	90
E	Restrictor size (in.)	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{16}$
F	Cartridge side hole size (in.)	0.187	0.210	
Noise Factors				
N	Aging	New	Aged	
Flow rate	Air cylinder speed (in./sec)	0.002	0.001	
Drain rate	Air cylinder direction	Down (as valve closing)	Up (as valve opening)	
Signal factors				
M	Inlet pressure (psig)	40	150	80
*M**	Cross-sectional flow area	Flow area calculated as valve closing or opening		

Table 15.11
Layout of experiments[a]

	Inner Array							
Trial No.	*A* Fastening	*B* Poppet Design	*C* Seat Design	*D* Durometer	*e*	*E* Restrictor Size	*F* Cartridge Slide Hole Size	*e*
1	Not fastened	Hex	Flat	80		0.500	0.187	
2	Not fastened	Hex	Conical	60		0.125	0.210	
3	Not fastened	Hex	Spherical	90		0.063	*0.187	
4	Not fastened	Square	Flat	80		0.125	*0.187	
5	Not fastened	Square	Conical	60		0.063	0.187	
6	Not fastened	Square	Spherical	90		0.500	0.210	
7	Not fastened	*Hex	Flat	60		0.063	0.210	
8	Not fastened	*Hex	Conical	90		0.500	*0.187	
9	Not fastened	*Hex	Spherical	80		0.125	0.187	
10	Fastened	Hex	Flat	90		0.125	0.210	
11	Fastened	Hex	Conical	80		0.063	*0.187	
12	Fastened	Hex	Spherical	60		0.500	0.187	
13	Fastened	Square	Flat	60		0.500	*0.187	
14	Fastened	Square	Conical	90		0.125	0.187	
15	Fastened	Square	Spherical	80		0.063	0.210	
16	Fastened	*Hex	Flat	90		0.063	0.187	
17	Fastened	*Hex	Conical	80		0.500	0.210	
18	Fastened	*Hex	Spherical	60		0.125	*0.187	

[a] Cells marked with asterisks.

Again, the initial condition of sensitivity was trial 1:

$$S_{\text{initial}} = -1.12 \text{ dB} \tag{15.27}$$

The gain predicted for the SN ratio is calculated from equations (15.24) and (15.25) as

$$\text{gain}(\eta) = -3.40 - (-8.27) = 4.87 \text{ dB} \tag{15.28}$$

The predicted gain in sensitivity is calculated from equations (15.26) and (15.27) as

$$\text{gain}(S) = -7.55 - (-1.12) = -6.43 \text{ dB} \tag{15.29}$$

Table 15.12
SN ratio and sensitivity

Trial No.	SN Ratio	Sensitivity (S = 10 log β^2)
1	−8.27	−1.12
2	−4.73	−7.96
3	−3.97	−18.32
4	−6.86	−7.62
5	−4.33	−17.60
6	−5.85	−1.54
7	−5.86	−18.21
8	−9.74	−0.62
9	−4.83	−8.92
10	−3.96	−9.02
11	−4.26	−18.01
12	−6.15	−1.82
13	−5.86	−0.48
14	−4.29	−6.56
15	−4.30	−6.56
16	−4.56	−18.25
17	−8.21	−0.28
18	−4.54	−6.99

Table 15.14 compares the gains between the optimum and initial conditions.

Based on the new design, products were manufactured and shipped to the field. In the first year there were no problems of chattering. This success encouraged the company to start applying such robust design approaches to other projects.

Discussion of the Case

In quality engineering it is recommended that noise factors be compounded to simplify experimentation. But in this particular case, all combinations of noise factors were tested. Also, as a rule, one data point for each noise factor level should be enough for analysis. In this case, a multitude of data points were collected, and the number of data points for different noise factor levels were different. The conclusions were made based on these methods of data collection.

Table 15.13
Response tables of SN ratio and sensitivity

	SN Ratio	Sensitivity (S = 10 log β^2)
A_1	−6.05	−6.63
A_2	−5.12	−6.27
B_1	−5.76	−6.55
B_2	−5.25	−6.24
C_1	−5.90	−6.50
C_2	−5.93	−5.94
C_3	−4.94	−6.93
D_1	−6.12	−6.40
D_2	−5.24	−6.44
D_3	−5.39	−6.51
E_1	−7.35	−0.96
E_2	−4.87	−7.80
E_3	−4.55	−10.59
F_1	−5.64	−6.38
F_2	−5.48	−6.60
Average	−5.59	−6.45

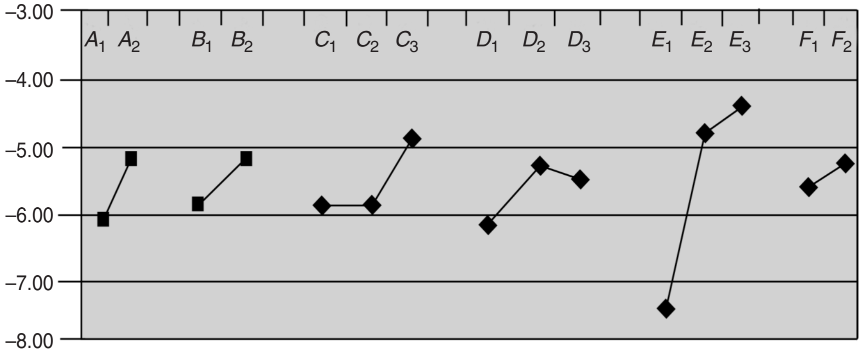

Figure 15.7
Response graph of SN ratio

Table 15.14
Results from the confirmation run

	SN Ratio (dB)		Sensitivity (dB)	
	Predicted	**Actual**	**Predicted**	**Actual**
Initial	−8.27	−8.27	−1.12	−1.12
Optimum	−3.40	−4.25	−7.55	−8.64
Gain	4.87	4.02		

After the success of using quality improvement methods, their methods were studied further to see the influence of reducing the number of data points for analysis. First, six data points for each noise factor condition were used for calculation. The results of optimization were the same. In other words, comparing with the case when all data points were used for calculation, there was no difference in the selection of the optimum level for an individual control factor. Next, two data points, maximum and minimum, of each noise factor condition were used for calculation. The results of optimization were again the same. These comparisons can be seen from Figures 15.8 and 15.9.

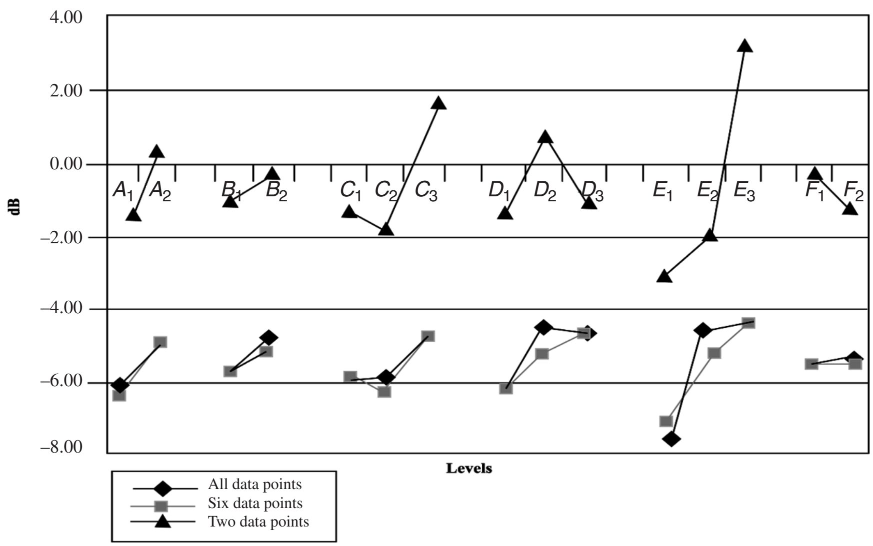

Figure 15.8
SN ratio comparison

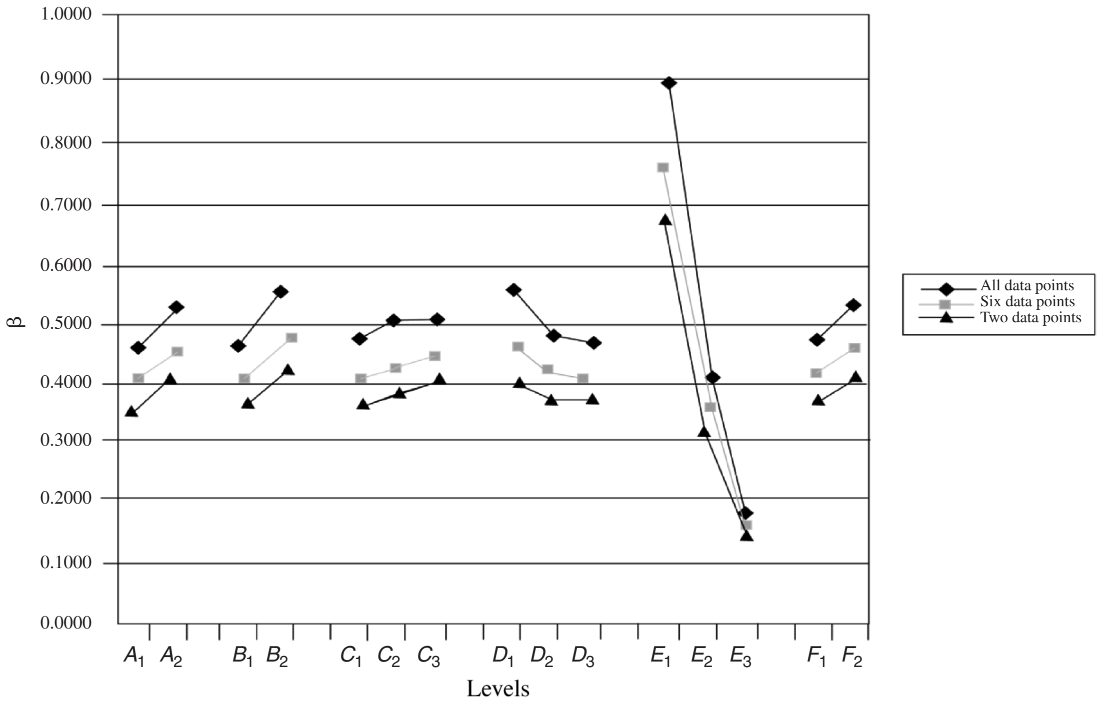

Figure 15.9
Sensitivity comparison

There are some important points in parameter design.

1. From the foregoing two examples, it is noted that cause detection and its removal were not made. Instead, the midvalues of parameters (control factors) were widely varied and the SN ratio was calculated from each of their combinations.
2. Parameter design can be performed by simulation, as shown in the first example of a Wheatstone bridge and also by experimentation in the second example.
3. In simulation, mathematical equations can be used. A more complicated system is recommended, as shown in the first example.
4. As shown in the second example, the generic function of the system was established for SN ratio calculation instead of studying the symptom, chattering. After optimizing the SN ratio, chattering disappeared without studying it.
5. In the second example, too many data were collected. However, the study showed that repetitions were unnecessary. It is important to provide proper noise factors rather than repetitions. The study indicated that noise factors could be compounded without affecting conclusions.

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3. Amjed Shafique, 2000. Reduction of chatter noise in 47-feeder valve. Presented at ASI's 18th Annual Taguchi Methods Symposium.