

13 SN Ratios for Classified Attributes

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13.1. Introduction

Classified data consist of 0 and 1. For example, the number of defects can be expressed by 1, and nondefects can be expressed by 0. The yield calculated from a mixture of defects and nondefects looks like a continuous variable, but actually, it is calculated from 0 and 1 data. Even in the case of chemical reactions, yield is the fraction of substance reacted within a whole substance in units of molecules or atoms.

Classified attributes are used very widely for data collection and analysis, but they are not good-quality characteristics from a quality engineering viewpoint. Using such characteristics only, a lot of information cannot be picked up, so the study would be inefficient and there would be interactions between control factors.

There are strategies to avoid such interactions. First, avoid 0–1 data. Second, when continuous-variable data cannot be measured due to lack of technology or for other reasons, try to provide more than two classes, such as good, normal, and bad, or large, medium, and small. The more levels that are classified, the more information that is revealed. In this way, the trend toward control factor change can be observed and incorrect conclusions can be avoided to some extent. The third and best strategy is to use an SN ratio based on the function. This is discussed in Chapter 14.

When 0 and 1 are used to classify attribute data, there are only two classes. In this case, there are two situations: one type of mistake and two types of mistakes. Both types of situation are discussed below.

13.2. Cases with Two Classes, One Type of Mistake

When using a vending machine, for example, we put coins into the machine and expect to get a drink. If no drink is delivered, there is one type of mistake. Another type of mistake is that a drink comes out of the machine when no coins were put in. Normally, vending machines are designed in such a way that the second type of mistake rarely occurs. In a reliability study of vending machine, only the first type of mistake is studied.

As we said earlier, the use of 0–1 data will reveal interactions between control factors. Interaction is synonymous with nonadditivity, inconsistency, and nonreproducibility. In other words, good plus good cannot be better. For example, in a manufacturing process, the fraction defective of producing a component part, denoted by p , is 0.10. Three countermeasures were taken separately to obtain the following results:

A : material

A_1 : current material (B_1C_1); $p = 0.10$

A_2 : new material (B_1C_1); $p = 0.02$

B : machine

B_1 : current machine (A_1C_1); $p = 0.10$

B_2 : new machine (A_1C_1); $p = 0.04$

C : method

C_1 : current method (A_1B_1); $p = 0.10$

C_2 : new method (A_1B_1); $p = 0.02$

Such a method is called the *one-factor-at-a-time approach*. If arithmetic additivity is assumed, the fraction defective under $A_2B_2C_2$ is calculated as

$$\begin{aligned} p &= 0.10 + (0.02 - 0.10) + (0.04 - 0.10) + (0.02 - 0.10) \\ &= -0.12 \end{aligned} \quad (13.1)$$

A negative fraction is unrealistic. To avoid such a problem, it is recommended that one use the omega transformation. Following are the results of a test: y_1, y_2, \dots, y_n . y values are either 0 or 1. Assume that 0 and 1 are good and defective products, respectively.

Letting the fraction defective be p , we have

$$p = \frac{y_1 + y_2 + \dots + y_n}{n} \quad (13.2)$$

To calculate the SN ratio, data are decomposed as follows. The total variation is given by

$$S_T = y_1^2 + y_2^2 + \dots + y_n^2 = np \quad (13.3)$$

The effect of the signal is calculated as the variation of p . Since equation (13.2) is a linear equation, denoted by L , its variation, denoted by S_p , is given by

$$S_p = \frac{L^2}{D} \quad (13.4)$$

where D is the number of units of equation (13.2):

$$D = \left(\frac{1}{n}\right)^2 + \left(\frac{1}{n}\right)^2 + \dots + \left(\frac{1}{n}\right)^2 = \left(\frac{1}{n}\right)^2 (n) = \frac{1}{n} \quad (13.5)$$

Putting (13.2) and (13.5) into equation (13.4), we get

$$S_p = \frac{p^2}{1/n} = np^2 = S_m \quad (13.6)$$

The error variation, S_e , is calculated by subtracting S_p from S_T :

$$\begin{aligned} S_e &= S_T - S_p \\ &= np - np^2 = np(1 - p) \end{aligned} \quad (13.7)$$

The SN ratio of digital data is calculated as the ratio of the variation of the signal to the variation of error in decibel (dB) units:

$$\begin{aligned} \eta &= 10 \log \frac{S_p}{S_e} = 10 \log \frac{np^2}{np(1 - p)} \\ &= 10 \log \frac{p}{1 - p} = -10 \log \left(\frac{1}{p} - 1 \right) \end{aligned} \quad (13.8)$$

This is the same equation as that used in the omega transformation.

Using the omega transformation, the omega value of each condition is calculated:

$$A_1B_1C_1(p = 0.10) = -10 \log \left(\frac{1}{p} - 1 \right) = -10 \log \left(\frac{1}{0.10} - 1 \right) = -9.54 \quad (13.9)$$

$$A_2B_1C_1(p = 0.02) = -10 \log \left(\frac{1}{0.02} - 1 \right) = -16.90 \quad (13.10)$$

$$A_1B_2C_1(p = 0.04) = -10 \log \left(\frac{1}{0.04} - 1 \right) = -13.80 \quad (13.11)$$

$$A_1B_1C_2(p = 0.02) = -10 \log \left(\frac{1}{0.02} - 1 \right) = -16.90 \quad (13.12)$$

Letting current condition be T , condition $A_2B_2C_2$ is estimated by

$$\begin{aligned} -10 \log \left(\frac{1}{p} - 1 \right) &= T + (A_2 - T) + (B_2 - T) + (C_2 - T) \\ &= A_2 + B_2 + C_2 - 2T \\ &= -16.90 - 13.80 - 16.90 - (2)(-9.54) \\ &= -28.52 \end{aligned} \quad (13.13)$$

p in the fraction is

$$p = 0.0014 \quad (13.14)$$

13.3. Case with Two Classes, Two Types of Mistake

To explain this case, a copper smelting process is used as an example. From copper ore (copper sulfide), metal copper is extracted to produce crude copper. It is desirable that all copper in the ore be included in the crude copper and that all noncopper materials be included in the slag (waste). Actually, some impurities are included in the crude copper, and some copper is included in the slag.

Table 13.1 shows the input/output in the copper smelting process. In the table, p^* is the fraction of copper included in slag, and q^* is the fraction of impurities in the product. See also Table 13.2 There are two types of mistakes:

p : fraction of copper included in the slag

q : the fraction of impurities included in crude copper ingot

From the table, the two types of mistakes are calculated as follows:

$$p = \frac{Bp^*}{A(1 - q^*) + Bp^*} \quad (13.15)$$

$$q = \frac{Aq^*}{Aq^* + B(1 - p^*)} \quad (13.16)$$

p is the fraction of copper mistakenly included in the slag, and $(1 - p)$ is called *yield*. q is the fraction of noncopper materials included in the product. Both fractions are calculated by the fraction in weight.

To compare p and q , when the furnace temperature is high, more copper melts to be the product; therefore, p becomes smaller. But at the same time, more

Table 13.1

Input/output of the copper smelting process

Input	Output		Total
	Product	Slag	
Copper	$A(1 - q^*)$	Bp^*	$A(1 - q^*) + Bp^*$
Noncopper	Aq^*	$B(1 - p^*)$	$Aq^* + B(1 - p^*)$
Total	A	B	$A + B$

Table 13.2
Input/output using p and q

Input	Output		Total
	Product	Slag	
Copper	$1 - p$	p	1
Noncopper	q	$1 - q$	1
Total	$1 - p + q$	$1 + p - 1$	$\frac{1}{2}$

noncopper materials melt into the product, which increases q . The factor, which reduces p but increases q , is called a *tuning factor*. The tuning factor does not improve the separating function of the process. A good separating function is to reduce both p and q at the same time.

To find the control factors that can reduce p and q together, it is important to evaluate the function after tuning p and q on the same basis. For example, we want to compare which level of control factor, A_1 and A_2 , has a better separating function. Assume that the p and q values for A_1 and A_2 are p_1, q_1 and p_2, q_2 , respectively. As described before, we cannot compare $(p_1 + q_1)$ with $(p_2 + q_2)$. We need to transform the data so that the comparison can be made on the same basis.

For this purpose, the standard SN ratio is used. The SN ratio is calculated after tuning to reach the point when $p = q$. This fraction is denoted by p_0 , calculated as

$$p_0 = \frac{1}{1 + \sqrt{[(1/p) - 1][(1/q) - 1]}} \quad (13.17)$$

The standard SN ratio is calculated by

$$\eta = 10 \log \frac{(1 - 2p_0)^2}{4p_0(1 - p_0)} \quad (13.18)$$

For a derivation, see Reference 1.

□ Example [1]

In a uranium concentrating process, the aim was to separate ^{235}U from the mixture of ^{235}U and ^{238}U . The separating functions of two conditions were compared. Table 13.3 shows hypothetical results after separation. To calculate the standard SN ratio, the results in the table are converted into fractions to obtain Table 13.4.

Condition A_1 :

$$p = \frac{3975}{5000} = 0.79500 \quad (13.19)$$

Table 13.3

Results of experiment

	Input	Output		Total
		Product	Slag	
A_1	^{235}U	1,025	3,975	5,000
	^{238}U	38,975	156,025	195,000
	Total	40,000	160,000	200,000
A_2	^{235}U	1,018	3,782	4,800
	^{238}U	38,982	256,218	195,200
	Total	40,000	160,000	200,000

$$q = \frac{38,975}{195,000} = 0.19987 \quad (13.20)$$

$$\begin{aligned} p_0 &= \frac{1}{1 + \sqrt{[(1/p) - 1][(1/q) - 1]}} \\ &= \frac{1}{1 + \sqrt{[(1/0.79500) - 1][(1/0.19987) - 1]}} \\ &= 0.49602 \end{aligned} \quad (13.21)$$

$$\begin{aligned} \eta &= 10 \log \frac{(1 - 2p_0)^2}{4p_0(1 - p_0)} \\ &= 10 \log \frac{[1 - (2)(0.49602)]^2}{(4)(0.49602)(1 - 0.49602)} \\ &= -41.981 \text{ dB} \end{aligned} \quad (13.22)$$

Table 13.4

Results in fraction

	Input	Output		Total
		Product	Slag	
A_1	^{235}U	0.20500	0.79500	1.00000
	^{238}U	0.19987	0.80013	1.00000
	Total	0.40487	1.59513	2.00000
A_2	^{235}U	0.21208	0.78792	1.00000
	^{238}U	0.19970	0.80030	1.00000
	Total	0.41178	1.58822	2.00000

Condition A_2 :

$$\eta = -34.449 \text{ dB} \quad (13.23)$$

Therefore, A_2 is better than A_1 by $(-34.449) - (-41.981) = 7.532 \text{ dB}$

13.4. Chemical Reactions without Side Reactions

Nature of Chemical Reactions

The function of a chemical reaction is to change the combining condition of molecules or atoms. In most cases, it is impossible to measure the behavior of individual molecules or atoms. Instead, only the unit, such as yield, can measure the behavior of an entire group. The yield used in chemical reactions sounds like a continuous variable, but it is a fraction of the numbers of molecules or atoms. Therefore, it should be classified as two classified attributes (i.e., the data with 0 and 1).

Basic Function of Chemical Reactions

Conducting a chemical reaction is to change the combining state of molecules or atoms. When substances A and B react to form substance C , A and B collide and the molecules of A gradually decrease. In this case it is ideal that "reaction speed is proportional to the concentration of A ."



Letting the initial amount of main raw material A be Y_0 and the amount at time T be Y , the fraction of the amount of change by time T is Y/Y_0 . The reaction speed is the fraction of the reduced amount differentiated by time:

$$\frac{d(Y/Y_0)}{dT} = \beta \left(\frac{1 - Y}{Y_0} \right) \quad (13.25)$$

The amount shown in the parentheses on the right side of (13.25) is the concentration of A remaining, denoted by p_0 :

$$p_0 = 1 - \frac{Y}{Y_0} \quad (13.26)$$

From equation (13.25),

$$\frac{Y}{Y_0} = 1 - e^{-\beta T} \quad (13.27)$$

The concentration of A remaining is then

$$p_0 = e^{-\beta T} \quad (13.28)$$

$$-\ln p_0 = \beta T \quad (13.29)$$

Setting

$$-\ln p_0 = y \quad (13.30)$$

in Equation (13.29), the ideal function is expressed by zero-point proportion equation.

Letting time, T , be the signal factor, the fractions remaining at different times, p_1, p_2, \dots, p_k are shown in Table 13.5.

The ideal relationship between the fraction remaining and time is shown in Figure 13.1.

From the results in Table 13.5, the variation of y_1, y_2, \dots, y_k is decomposed as follows.

Total variation:

$$S_T = y_1^2 + y_2^2 + \dots + y_k^2 \quad (f = k) \quad (13.31)$$

The total variation is decomposed into the variation due to the generic function and the variation caused by the deviation from the generic function.

Variation of proportional term:

$$S_\beta = \frac{L^2}{\gamma} \quad (f = 1) \quad (13.32)$$

Linear equation:

$$L = y_1 T_1 + y_2 T_2 + \dots + y_k T_k \quad (13.33)$$

Effective divider:

$$\gamma = T_1^2 + T_2^2 + \dots + T_k^2 \quad (13.34)$$

Error variation:

$$S_e = S_T - S_\beta \quad (f = k - 1) \quad (13.35)$$

Table 13.6 shows the ANOVA table.

SN ratio:

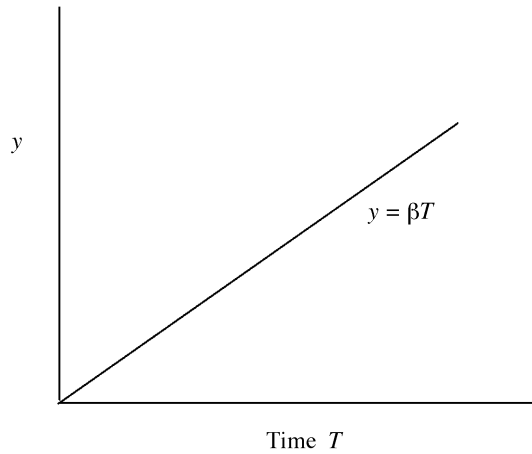
$$\eta = 10 \log \frac{(1/\gamma)(S_\beta - V_e)}{V_e} \quad (13.36)$$

Table 13.5

Fraction remaining at different times

Time	T_1	T_2	...	T_k
Fraction of Remaining Main Raw Material	p_1	p_2	...	p_k
$\ln \frac{1}{p_0}$	y_1	y_2	...	y_k

Figure 13.1
Fraction of remained
material and time



η is calculated to estimate the following:

$$\eta \rightarrow 10 \log \frac{\beta^2}{\sigma^2} \quad (13.37)$$

Sensitivity:

$$S = 10 \log \frac{1}{\gamma} (S_B - V_e) \quad (13.38)$$

S is calculated to estimate the following:

$$S \rightarrow 10 \log \beta^2 \quad (13.39)$$

13.5. Chemical Reactions with Side Reactions: Dynamic Operating Window

Reaction Speed Difference Method

When substances A and B react to form the objective substance C and side-reacted substance D , it is desirable to maximize the reaction speed for producing C and minimize the one for D :



Table 13.6

ANOVA table for chemical reactions

Source	f	S	V	$E(V)$
β	1	S_β	—	$\sigma^2 + \gamma\beta^2$
e	$k - 1$	S_e	V_e	σ^2
Total	k			

To do this, the following fractions are measured at time T as shown in Table 13.7.

p_0 : fraction of unreacted raw material, A

p_1 : fraction of objective product, C

p_2 : fraction of side-reacted substance, D

From p_0 , p_1 , and p_2 , y_1 and y_2 are calculated to show two situations, M_1 and M_2 , respectively. For total reaction M_1 and reaction speed β_1 ,

$$y_1 = \beta_1 T \quad (13.41)$$

$$y_1 = \ln \frac{1}{p_0} \quad (13.42)$$

For side reaction M_2 and reaction speed β_2 ,

$$y_2 = \beta_2 T \quad (13.43)$$

$$y_2 = \ln \frac{1}{p_0 + p_1} \quad (13.44)$$

Since the objectives are to maximize total reaction M_1 and minimize side reaction M_2 , the operating window is shown in Figures 13.2 and 13.3. This is called the *speed difference method*.

The SN ratio is calculated from the following decomposition.

Total variation:

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{2k}^2 \quad (f = 2k) \quad (13.45)$$

Variation of proportional terms:

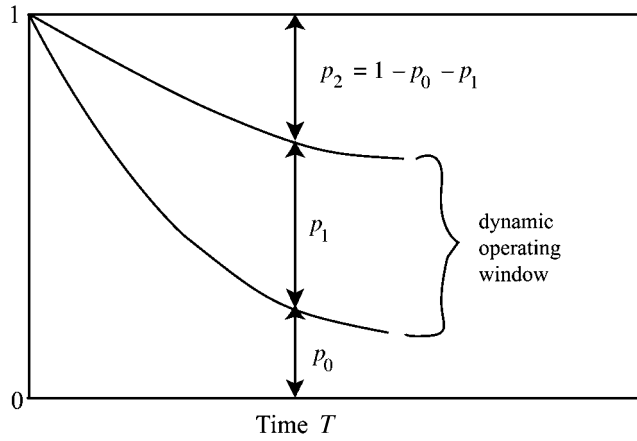
$$S_\beta = \frac{(L_1 + L_2)^2}{2\gamma} \quad (f = 1) \quad (13.46)$$

Table 13.7

Data collection for dynamic operating window

Time	T_1	T_2	...	T_k
Unreacted Raw Material	p_{01}	p_{02}	...	p_{0k}
Objective Product	p_{11}	p_{12}	...	p_{1k}
Side-Reacted Substance	p_{21}	p_{22}	...	p_{2k}
$M_1: \ln \frac{1}{p_0}$	y_{11}	y_{12}	...	y_{1k}
$M_2: \ln \frac{1}{p_0 + p_1}$	y_{21}	y_{22}	...	y_{2k}

Figure 13.2
Dynamic operating window (shown in a fraction)



Effective divider:

$$\gamma = T_1^2 + T_2^2 + \dots + T_k^2 \tag{13.47}$$

Linear equations:

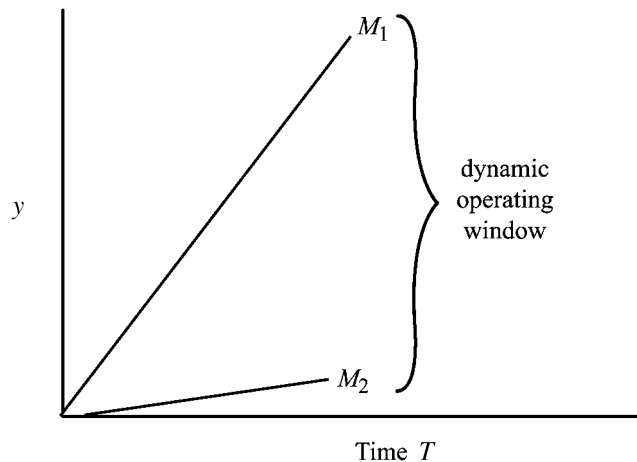
$$L_1 = y_{11} T_1 + y_{12} T_2 + \dots + y_{1k} T_k \tag{13.48}$$

$$L_2 = y_{21} T_1 + y_{22} T_2 + \dots + y_{2k} T_k \tag{13.49}$$

Variation of the difference between proportional terms:

$$S_{M\beta} = \frac{(L_1 - L_2)^2}{2\gamma} \quad (f = 1) \tag{13.50}$$

Figure 13.3
Dynamic operating window (after data transformation)



Error variation:

$$S_e = S_T - S_\beta - S_{M\beta} \quad (f = 2k - 2) \quad (13.51)$$

Table 13.8 shows the ANOVA table for a dynamic operating window.

SN ratio of the speed difference method:

$$\eta = 10 \log \frac{(1/2\gamma)(S_{M\beta} - V_e)}{V_e} \quad (13.52)$$

Sensitivity of the speed difference method:

$$S = 10 \log \frac{1}{2\gamma} (S_{M\beta} - V_e) \quad (13.53)$$

Sensitivity of total reaction:

$$S^* = 10 \log \frac{1}{2\gamma} (S_\beta - V_e) \quad (13.54)$$

In chemical reactions, noise factors are not included in the experiment because of the cost increase. When there are side reactions, the reaction speed of side-reacted products, β_2 , may be considered as noise. Therefore, its SN ratio is written as

Reaction Speed Ratio Method

$$\eta = \frac{\beta_1^2}{\beta_2^2} \quad (13.55)$$

Since we want to maximize the total reaction speed, β_1 , denoted by η_1 , it is calculated using a larger-the-better characteristic. We also want to minimize the side-reacting speed, β_2 , denoted by η_2 . The SN ratio of β_2 is calculated using a smaller-the-better characteristic. The two SN ratios are added together to obtain the overall SN ratio, denoted by η :

$$\eta = \eta_1 + \eta_2 \quad (13.56)$$

In the decibel scale,

$$\eta = 10 \log \frac{\beta_1^2}{\beta_2^2} \quad (13.57)$$

Table 13.8

ANOVA table for a dynamic operating window

Source	f	S	V	$E(V)$
β	1	S_β		$\sigma^2 + 2\gamma\beta^2$
$M\beta$	1	$S_{M\beta}$		$\sigma^2 + 2\gamma\sigma_{M\beta}^2$
e	$2k - 2$	S_e	V_e	σ^2
Total	$2k$	S_T		

This is called the *speed ratio method*. A sample of the data collection is shown in Table 13.9.

$$\begin{aligned}\beta_{11} &= \frac{y_{11}}{T_1} \\ \beta_{12} &= \frac{y_{12}}{T_2} \\ &\vdots \\ \beta_{1k} &= \frac{y_{1k}}{T_k}\end{aligned}\quad (13.58)$$

$$\begin{aligned}\beta_{21} &= \frac{y_{21}}{T_1} \\ \beta_{22} &= \frac{y_{22}}{T_2} \\ &\vdots \\ \beta_{2k} &= \frac{y_{2k}}{T_k}\end{aligned}\quad (13.59)$$

SN ratio for M_1 :

$$\eta_1 = -10 \log \frac{1}{k} \left(\frac{1}{\beta_{11}^2} + \frac{1}{\beta_{12}^2} + \dots + \frac{1}{\beta_{1k}^2} \right) \quad (13.60)$$

SN ratio for M_2 :

$$\eta_2 = -10 \log \frac{1}{k} (\beta_{21}^2 + \beta_{22}^2 + \dots + \beta_{2k}^2) \quad (13.61)$$

SN ratio of the speed ratio method:

$$\eta = \eta_1 + \eta_2 \quad (13.62)$$

Table 13.9

Data collection for the speed ratio method

	T_1	T_2	...	T_k
M_1	y_{11}	y_{12}	...	y_{1k}
M_2	y_{21}	y_{22}	...	y_{2k}
M_1	β_{11}	β_{12}	...	β_{1k}
M_2	β_{21}	β_{22}	...	β_{2k}

□ Example [2]

In a chemical reaction, the primary raw material, F_0 , reacts with another raw material to produce F_3 after forming intermediate products F_1 and F_2 :



In the reaction, side-reacted product was also produced. The fraction of side-reacted product is calculated by

$$\text{fraction of side-reacted product} = 1 - (F_0 + F_1 + F_2 + F_3) \quad (13.64)$$

Table 13.10 shows the fraction changes of constituents at different time segments.

SN Ratio Using the Speed Difference Method

The function of unreacted raw material, denoted by ρ , at time T is given by

$$\rho = e^{-\beta T} \quad (13.65)$$

Equation (13.65) is rewritten as

$$\ln \frac{1}{\rho} = \beta T \quad (13.66)$$

The fraction of unreacted raw material, ρ , is transformed to y as follows:

$$y = \ln \frac{1}{\rho} \quad (13.67)$$

From equations (13.66) and (13.67), the ideal function is written as a zero-point proportional equation:

$$y = \beta T \quad (13.68)$$

The objectives are to: maximize the reaction speed of the objective product (or maximize the reducing speed of raw material) and to minimize the reaction speed of the side-reacted product.

In Table 13.10, F_0 is raw material, F_1 and F_2 are intermediate products, and F_3 is the objective product. Therefore, the fraction of unreacted raw material is $F_0 + F_1 + F_2$. Let the total of F_0 , F_1 , and F_2 be ρ . Also, the fraction of the objective product is q . The fraction of side-reacted product is then equal to $1 - \rho - q$. Table 13.11 is calculated from Table 13.10.

The values of y to be used for the SN ratio are calculated as follows. At time T_3 , for example, the total reaction M_1 is

$$y_{13} = \ln \frac{1}{\rho_3} = \ln \frac{1}{0.786} \quad (13.69)$$

Table 13.11

Fractions at time levels

	Time (h)									
	T_1 1.0	T_2 2.0	T_3 3.0	T_4 4.0	T_5 5.0	T_6 6.0	T_7 7.0	T_8 8.0	T_9 9.0	T_{10} 10.0
$M_1: p$	0.973	0.938	0.786	0.618	0.421	0.286	0.209	0.160	0.122	0.084
q	0.027	0.06	0.204	0.362	0.537	0.652	0.719	0.768	0.798	0.834
$1 - p - q$	0.000	0.004	0.010	0.020	0.042	0.062	0.072	0.078	0.081	0.082
$M_2: p + q$	1.000	0.944	0.990	0.980	0.958	0.938	0.928	0.922	0.920	0.918

and the side reaction M_2 is

$$y_{23} = \ln \frac{1}{p_3 + q_3} = \ln \frac{1}{0.786 + 0.204} = \ln \frac{1}{0.990} \quad (13.70)$$

Table 13.12 shows the results of calculation. The dynamic operating window is shown in Figure 13.4.

The SN ratio is calculated from Table 13.11. There are 10 time levels. Two linear equations, L_1 and L_2 , are calculated.

$$L_1 = (0.0275)(1.0) + (0.0636)(2.0) + \dots + (2.477)(10.0) = 83.990 \quad (13.71)$$

$$L_2 = (0.000)(1.0) + (0.0036)(2.0) + \dots + (0.0857)(10.0) = 3.497 \quad (13.72)$$

Effective divider:

$$\gamma = 1^2 + 2^2 + \dots + 10^2 = 385 \quad (13.73)$$

Table 13.12

Results after conversion

	Time (h)									
	T_1 1.0	T_2 2.0	T_3 3.0	T_4 4.0	T_5 5.0	T_6 6.0	T_7 7.0	T_8 8.0	T_9 9.0	T_{10} 10.0
$M_1: y_1$	0.0275	0.0636	0.2410	0.4815	0.8654	1.2510	1.5661	1.8313	2.1078	2.4770
$M_2: y_2$	0.000	0.0036	0.0100	0.0203	0.0429	0.0636	0.0746	0.0811	0.0839	0.0857

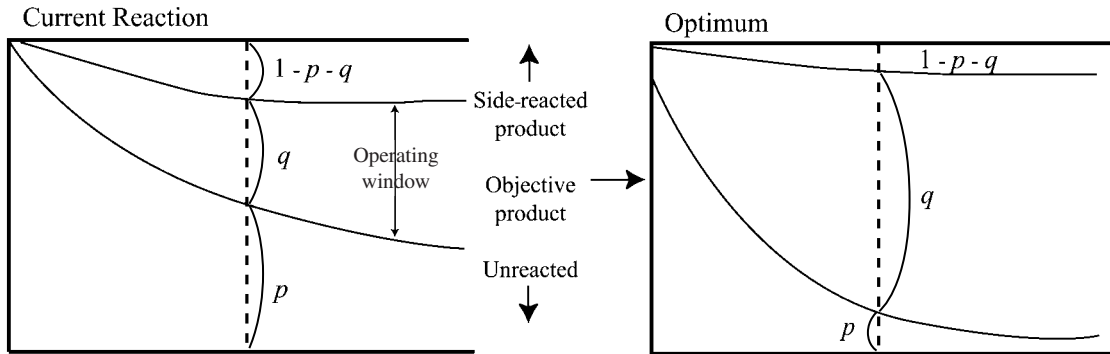


Figure 13.4
Dynamic operating window

Variation of linear term:

$$S_{\beta} = \frac{(L_1 + L_2)^2}{2\gamma} = \frac{(83.990 + 3.497)^2}{(2)(385)} = 9.940 \quad (f = 1) \quad (13.74)$$

Variation of the difference between M_1 and M_2 :

$$S_{M_{\beta}} = \frac{(L_1 - L_2)^2}{2\gamma} = \frac{(83.990 - 3.497)^2}{(2)(385)} = 8.4144 \quad (f = 1) \quad (13.75)$$

Total variation:

$$\begin{aligned} S_T &= 0.0275^2 + 0.0636^2 + \dots + 2.4770 \\ &\quad + 0.000^2 + 0.0036^2 + \dots + 0.0857^2 \\ &= 19.026 \quad (f = 20) \end{aligned} \quad (13.76)$$

Error variation:

$$\begin{aligned} S_e &= S_T - S_{\beta} - S_{M_{\beta}} \\ &= 19.0262 - 4.940 - 8.4144 \\ &= 5.6715 \quad (f = 18) \end{aligned} \quad (13.77)$$

Error variance:

$$V_e = \frac{S_e}{18} = \frac{5.6715}{18} = 0.0373 \quad (13.78)$$

Table 13.13 shows the ANOVA table.

Table 13.13
ANOVA table

Source	<i>f</i>	<i>S</i>	<i>V</i>
β	1	9.9403	9.9403
$M\beta$	1	8.4144	8.4144
<i>e</i>	18	0.6715	0.0373
Total	20	19.0262	

The magnitude of the objective reaction is shown by $M\beta$, which corresponds to the difference between β_1 and β_2 . Therefore, the SN ratio is calculated by the following equation:

$$\begin{aligned}\eta &= 10 \log \frac{(1/2\gamma)(S_{M\beta} - V_e)}{V_e} \\ &= 10 \log \frac{[1/(2)(385)](8.4144 - 0.0373)}{0.0373} \\ &= -5.35 \text{ dB} \quad (13.79)\end{aligned}$$

The sensitivity corresponding the difference between β_1 and β_2 , denoted by S , is calculated by

$$\begin{aligned}S &= 10 \log \frac{1}{2r} (S_{M\beta} - V_e) = 10 \log \frac{1}{(2)(385)} (8.4144 - 0.0373) \\ &= -19.6 \quad (13.80)\end{aligned}$$

The sensitivity corresponding to the total reaction, or corresponding to the average of β_1 , and β_2 , denoted by S^* , is calculated by

$$\begin{aligned}S^* &= 10 \log \frac{1}{2\gamma} (S_\beta - V_e) \\ &= 10 \log \frac{1}{(2)(385)} (9.940 - 0.0373) \\ &= -17.6 \text{ dB} \quad (13.81)\end{aligned}$$

SN Ratio Using the Speed Ratio Method

Since the generic function of chemical reactions is defined as

$$y = \beta T \quad (13.82)$$

where

$$y = \ln \frac{1}{\rho} \quad (13.83)$$

β is estimated from the responses of M_1 and M_2 in Table 13.12.

$$\beta = \frac{y}{T} \quad (13.84)$$

The values of β estimated under M_1 are larger-the-better, and the ones under M_2 are smaller-the-better. Therefore, the larger-the-better SN ratio, η_1 , is calculated from the former β_1 values, and the smaller-the-better SN ratio, η_2 , is calculated from the latter β_2 values. The SN ratio of the operating window using the speed ratio method is calculated by

$$\eta = \eta_1 + \eta_2 \quad (13.85)$$

Table 13.14 shows the estimates of β_1 (under M_1) and β_2 (under M_2).

From Table 13.14, η_1 and η_2 are calculated:

$$\begin{aligned} \eta_1 &= -10 \log \frac{1}{10} \left(\frac{1}{\beta_{11}^2} + \frac{1}{\beta_{12}^2} + \dots + \frac{1}{\beta_{1-10}^2} \right) \\ &= -10 \log \frac{1}{10} \left(\frac{1}{0.275^2} + \frac{1}{0.0318^2} + \dots + \frac{1}{0.2477^2} \right) \\ &= -25.17 \text{ dB} \end{aligned} \quad (13.86)$$

$$\begin{aligned} \eta_2 &= -10 \log \frac{1}{10} (\beta_{21}^2 + \beta_{22}^2 + \dots + \beta_{2-10}^2) \\ &= -10 \log \frac{1}{10} (0^2 + 0.0018^2 + \dots + 0.0086^2) \\ &= 42.57 \text{ dB} \end{aligned} \quad (13.87)$$

Therefore,

$$\begin{aligned} \eta &= \eta_1 + \eta_2 \\ &= -25.17 + 42.57 = 17.40 \end{aligned} \quad (13.88)$$

Table 13.14

Estimates of proportional constant

	Time (h)									
	T_1 1.0	T_2 2.0	T_3 3.0	T_4 4.0	T_5 5.0	T_6 6.0	T_7 7.0	T_8 8.0	T_9 9.0	T_{10} 10.0
$M_1: \beta_1$	0.2750	0.0318	0.0803	0.1204	0.1731	0.2085	0.2237	0.2289	0.2342	0.2477
$M_2: \beta_2$	0.0000	0.0018	0.0033	0.0051	0.0086	0.0106	0.0107	0.0101	0.0093	0.0086

References

1. Genichi Taguchi, 1987. *System of Experimental Design*. Livonia, Michigan: Unipub/American Supplier Institute.
2. Genichi Taguchi et al., 1999. *Technology Development for Chemical, Pharmaceutical and Biological Industries*. Tokyo: Japanese Standards Association.