

# 12 SN Ratios for Continuous Variables

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## 12.1. Introduction

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From a quality engineering viewpoint, continuous variables are far more important than classified attributes, as described in Chapter 11. The SN ratios of continuous variables are used much more frequently than those of classified attributes. SN ratios for continuous variables are classified as follows:

1. When the true values of the signal factor are known
  - a. Zero-point proportional equation
  - b. Reference-point proportional equation
  - c. Linear equation
2. When the level interval values are known
3. When the ratio of the signal factor level values is known
4. When the true values of the signal factor levels are unknown
5. When there is no signal factor (nondynamic characteristics)
  - a. Nominal-the-best characteristic
  - b. Smaller-the-better characteristic
  - c. Larger-the-better characteristic
  - d. Nondynamic operating window

The following explanation is based on an experiment done under the assumption that at each level of the control factor, there is one signal factor and the remaining factors are noise factors. It is assumed that the data shown in Table 12.1 are given. That is, for true signal value,  $M_i$  ( $i = 1, \dots, k$ ),  $r_0$  experiments are repeated and data  $y_{ij}$  ( $j = 1, \dots, r_0$ ) are obtained.  $y_i$  is the sum of data for signal  $M_i$ .

The value of the SN ratio obtained here is given in terms of the antilog value, and to apply it as a characteristic value for analysis of the effect of control factors, it is transformed into a decibel value, which is equal to 10 times the common log of the SN ratio in terms of the antilog value:  $10 \log \eta$  (dB). Selection of the correct SN ratio for a study is based on engineering objectives and viewpoints. It is important for an engineer to understand the function of a product or process and his or her objectives for the study. Once a proper SN ratio is selected, the calculation can be made by using the equations in this chapter or by inputting the data to the software. To simplify the explanation, only one control factor, and usually, one level of the factor, is used in the examples.

When the true values of a signal factor are known, there are basically three types: zero-point proportional equation, reference-point proportional equation, and linear equation. In such cases, the linear effect of the signal factor can be calculated accurately. *The error variance is estimated by including the deviation from the linearity (or nonlinear portion) into the error term.* Different equations are used depending on the ideal functional relationship considered to exist between the input

**Table 12.1**  
Signal factor and data

Signal	$M_1$	$M_2$	$M_3$	...	$M_k$
Data	$y_{11}$	$y_{21}$	$y_{31}$	...	$y_{k1}$
	$y_{12}$	$y_{22}$	$y_{32}$	...	$y_{k2}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\frac{y_{1r_0}}$	$\frac{y_{2r_0}}$	$\frac{y_{3r_0}}$	...	$\frac{y_{kr_0}}$
Total	$y_1$	$y_2$	$y_3$	...	$y_k$

(signal factor) and the output (response data). Following are typical functional relations.

1. *Zero-point proportional equation.* In this case,  $y = 0$  when  $M = 0$ :

$$y = \beta M$$

2. *Reference-point proportional equation.* In this case, the ideal relationship is expressed by the proportional equation that goes through a particular point,  $M = M_s$ :

$$y - \bar{y}_s = \beta(M - M_s)$$

3. *Linear equation.* In this case, the ideal relationship is expressed by the linear equation between  $M$  and  $y$ :

$$y = m + \beta(M - \bar{M})$$

## 12.2. Zero-Point Proportional Equation

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A zero-point proportional equation is used most frequently. In this case the input is equal to zero and the output “must” pass through the origin based on the situation considered by the engineer. It must be noted that the selection of an SN equation is not based on “how the response looks.”

For example, Figure 12.1a is an example of the voltage–current relationship of a dc motor study in a company. Since the regression line does not pass through the origin, as can be seen from Figure 12.1b, to improve quality the engineers tried to use a linear equation. The engineers said that based on their design, there is no current until the voltage reaches 1.5 V. However, this conclusion is misleading because, ideally, even when voltage is as low as 0.1 V, current is flowing.

Therefore, the zero-point proportional equation must be used for this case, as shown in Figure 12.1c. Any deviation from the regression line in the figure then becomes an error.

Table 12.2 shows the inputs and outputs of a measurement experiment. The inputs are the samples whose true values are known; these are the levels of the signal factors. The outputs are the results measured.

An explanation of symbols used is as follows:

$M_1, M_2, \dots, M_k$ : true values of samples

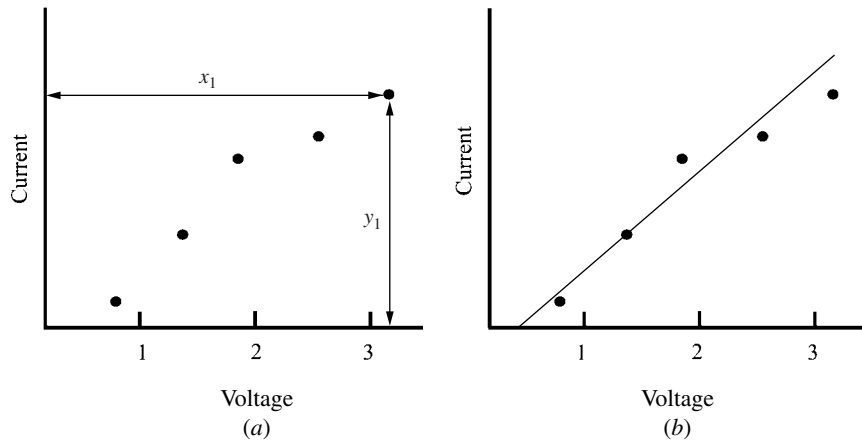
$y_{11}, y_{12}, \dots, y_{1r_0}$ : results of measuring  $M_1$

$y_{21}, y_{22}, \dots, y_{2r_0}$ : results of measuring  $M_2$

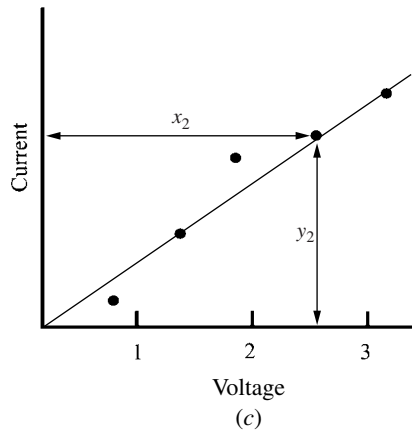
$y_1, y_2, \dots, y_k$ : total of measurements at  $M_i$

where  $r_0$  is the number of data points in each signal level.

There are  $kr_0$  observations in Table 12.2. Reading from  $y_{11}$  to  $y_{1r_0}$  shows the results of measuring sample  $M_1$ . Variation among  $kr_0$  pieces of data is caused by different samples and repetitions. Variation caused by different samples includes the following components: a linear portion, a quadratic portion, and a higher-order portion.



**Figure 12.1**  
Voltage-current relationship



Variation caused by repetitions is a type of noise factor. In many studies, a compounded noise factor is set and observed rather than just observing simple repetitions. Since an SN ratio is determined from two parts, a desirable portion and a harmful portion, the data must be decomposed as follows:

$$\text{total variation} = \text{variation caused by linear portion} + \text{variations caused by other portions}$$

**Table 12.2**  
Reading from a measurement system

Signal Factor	$M_1$	$M_2$	...	$M_k$
Reading	$y_{11}$	$y_{21}$	...	$y_{k1}$
	$y_{12}$	$y_{22}$	...	$y_{k2}$
	$\frac{y_{1\gamma_0}}$	$\frac{y_{2\gamma_0}}$	...	$\frac{y_{k\gamma_0}}$
Total	$y_1$	$y_2$	...	$y_k$

In the case of a zero-point proportional equation, when the signal is zero, the output must be zero. In a measurement system, if a sample to be measured contains no ingredient, the measured result must read zero. Here the ideal function is expressed as

$$y = \beta M \quad (12.1)$$

where  $y$  is the output,  $\beta$  the slope of the response line or the regression coefficient, and  $M$  the signal.

In reality, there are errors in observations, and the data in Table 12.2 are expressed by

$$y_{ij} = \beta M_i + e_{ij} \quad (12.1a)$$

where  $i = 1, 2, \dots, k$  and  $j = 1, 2, \dots, r_0$ .

Equation (12.1a) is expressed as a series of simultaneous equations with one unknown,  $\beta$ . The value of  $\beta$  is selected so as to minimize the total of the squares of differences between the right and left sides of the equation (the least squares method):

$$S_e = (y_{11} - \beta M_1)^2 + (y_{12} - \beta M_1)^2 + \dots + (y_{kr_0} - \beta M_k)^2 \quad (12.2)$$

The value of  $\beta$  that minimizes  $S_e$  is given by

$$\beta = \frac{1}{r_0 r} (M_1 y_1 + M_2 y_2 + \dots + M_k y_k) \quad (12.3)$$

where

$$r = M_1^2 + M_2^2 + \dots + M_k^2 \quad (12.4)$$

and  $r_0$  is the total number of measurements under  $M_i$ .

To decompose the data in Table 12.2,  $S_T$ , the total variation, is first calculated from the squares of  $kr_0$  pieces of data. The symbol  $f$  denotes the degrees of freedom:

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{kr_0}^2 \quad \text{and} \quad f = kr_0 \quad (12.5)$$

The variation caused by the linear effect, denoted by  $S_\beta$ , is given by

$$S_\beta = \frac{1}{r_0 r} (M_1 y_1 + M_2 y_2 + \dots + M_k y_k)^2 \quad \text{and} \quad f = 1 \quad (12.6)$$

The error variation  $S_e$ , including the deviation from linearity,  $V_e$ , is

$$S_e = S_T - S_\beta \quad \text{and} \quad f = kr_0 - 1 \quad (12.7)$$

The error variance  $V_e$  is the error variation divided by its degrees of freedom:

$$V_e = \frac{S_e}{kr_0 - 1} \quad (12.8)$$

The definition of this SN ratio is

$$\eta \rightarrow 10 \log \frac{\beta^2}{\sigma^2} \quad (12.9)$$

To calculate the SN ratio of equation (12.9), the error variance  $V_e$  calculated in equation (12.8) is used to estimate  $\sigma^2$  in equation (12.9). To estimate  $\beta^2$ , a correction is necessary. Statistically,  $S_\beta$ , the variation caused by the linear effect, has the following content:

$$E(S_\beta) = r_0 r \beta^2 + \sigma^2 \quad (12.10)$$

where  $E$  is the expected value as defined in statistics.

When data are repeatedly collected in the same way for an infinite number of times and their  $S_\beta$  value is calculated each time, the average of an infinite number of  $S_\beta$  values is denoted by  $E(S_\beta)$ . The term  $\sigma^2$  designates the error contained in the data, such as experimental error or measurement error. What is needed to calculate the SN ratio is the use of  $\beta^2$  as the numerator. Therefore, the SN ratio in decibel units (dB) is calculated as

$$\eta = 10 \log \frac{(1/r_0 r) (S_\beta - V_e)}{V_e} \quad \text{dB} \quad (12.11)$$

The sensitivity in decibel units is calculated as

$$S = 10 \log \frac{1}{r_0 r} (S_\beta - V_e) \quad (12.12)$$

### □ Example 1 [1]

In this example, the SN ratio of a newly developed displacement gauge is calculated. The value of displacement that was used as the signal is measured with another displacement gauge, which is believed to be accurate. The readings given in Table 12.3 were obtained from the measurement calculated twice by different testing personnel. A zero-point proportional equation was assumed, since the output,  $y$ , of the displacement gauge is zero when the displacement,  $M$ , is zero.

The ANOVA is done as follows:

$$S_T = 65^2 + 74^2 + \dots + 197^2 = 131,879 \quad (f = 6) \quad (12.13)$$

$$\gamma_0 \gamma = (2)(30^2 + 60^2 + 90^2) = 25,200 \quad (12.14)$$

$$\begin{aligned} S_\beta &= \frac{[(30)(139) + (60)(283) + (90) \times (405)]^2}{\gamma_0 r} \\ &= \frac{57,600^2}{25,200} = 131,657.14 \quad (f = 1) \quad (12.15) \end{aligned}$$

$$S_e = S_T - S_\beta = 131,879 - 131,657.14 = 221.86 \quad (f = 5) \quad (12.16)$$

$$V_e = \frac{S_e}{5} = \frac{221.86}{5} = 44.37 \quad (12.17)$$

The results are summarized in Table 12.4

**Table 12.3**

Displacement and displacement gauge data

Reading (mV)	Displacement $M$ ( $\mu\text{m}$ )		
	30	60	90
$R_1$	65	136	208
$R_2$	74	147	197
Total	139	283	405

The SN ratio is

$$\eta = \frac{(1/25,220) (131,657.14 - 44.37)}{44.37} = 0.1177 \quad (12.18)$$

This was transformed into a decibel value:

$$\eta = 10 \log(0.1177) = -9.29 \text{ dB} \quad (12.19)$$

The estimated value of the sensitivity coefficient was

$$\hat{\beta} = \frac{57,600}{25,200} = 2.285 \quad (12.20)$$

and the relationship between  $M$  and  $y$  was

$$y = 2.285M \quad (12.21)$$

### □ Example 2 [2]

In Example 1, the SN ratio of one product (or one condition) is calculated. When there is a control factor with two levels (two conditions), two SN ratios are calculated for comparison. In the present example two quantitative cadmium micro-analysis methods, dithizone extraction–atomic absorption spectrophotometry and

**Table 12.4**

ANOVA table for zero-point proportional equation: Displacement gauge

Factor	$F$	$S$	$V$	$E(V)$
$\beta$	1	131,657.14	131,657.14	$\sigma^2 + \gamma_0\gamma\beta^2$
$e$	5	221.86	44.37	$\sigma^2$
Total	6	131,879.00		

DDTC extraction–atomic absorption spectrophotometry, were compared. Control factor levels  $A_1$  and  $A_2$  denote the two methods, respectively.

□ *Signal factor:* cadmium content,  $M$

- $M_1$ : 1 ppm
- $M_2$ : 2 ppm
- $M_3$ : 3 ppm
- $M_4$ : 4 ppm
- $M_5$ : 5 ppm

□ *Noise factor:* measuring personnel,  $R_1$  and  $R_2$

Table 12.5 shows the results of measurement.

1. *SN ratio of  $A_1$ :*

$$S_T = 12.0^2 + 11.0^2 + \dots + 54.0^2 = 12,907.50 \quad (f = 10) \quad (12.22)$$

$$S_\beta = \frac{[(1)(23.0) + (2)(45.5) + (3)(67.0) + (4)(86.5) + (5)(106.0)]^2}{(2)(1^2 + 2^2 + 3^2 + 4^2 + 5^2)}$$

$$= \frac{1191.0^2}{110} = 12,895.28 \quad (f = 1) \quad (12.23)$$

$$S_e = 12,907.50 - 12,895.28 = 12.22 \quad (f = 9) \quad (12.24)$$

From this result, Table 12.6 is obtained.

$$\eta = 10 \log \frac{\frac{1}{10}(12,895.28 - 1.36)}{1.36} = 10 \log(86.2) = 19.36 \text{ dB} \quad (12.25)$$

2. *SN ratio of  $A_2$ :* A similar calculation is done and Table 12.7 is obtained.

$$\eta = 10 \log \frac{\frac{1}{10}(1,539.38 - 0.35)}{0.35} = 10 \log(39.97) = 16.02 \text{ dB} \quad (12.26)$$

Compared to the DDTC extraction method ( $A_2$ ), the dithizone extraction method ( $A_1$ ) is better by 3.34 dB.

**Table 12.5**  
Quantitative cadmium microanalysis data

		$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	Total
$A_1$	$R_1$	12.0	23.0	33.0	42.5	52.0	
	$R_2$	11.0	22.5	34.0	44.0	54.0	
	Total	23.0	45.5	67.0	86.5	106.0	328.0
$A_2$	$R_1$	3.5	7.0	10.5	14.5	18.0	
	$R_2$	4.0	7.5	12.0	15.5	19.5	
	Total	7.5	14.5	22.5	30.0	37.5	112.0



**Table 12.6**ANOVA table of quantitative cadmium microanalysis data of  $A_1$ 

Factor	$f$	S	V	$E(V)$
$\beta$	1	12,895.28	12,895.28	$\sigma^2 + (2)(1^2 + 2^2 + \dots + 5^2)$
e	9	12.22	1.36	$\sigma^2$
Total	10	12,907.50		

### 12.3. Reference-Point Proportional Equation

In a measuring system, there are two types of calibration: point calibration and slope calibration. For example, a watch measures time. We correct our watches by the standard time signal. Another example is a bathroom scale, on which an adjusting knob can be used to calibrate the reading to zero when nobody steps on the scale. These are examples of *point calibration*.

In old-style (mechanical and nonquartz) watches, there was a pace-adjusting slide inside the watch. The slide was used to adjust the pace. This is called *slope calibration* (adjustment).

In point calibration, the reference point could be either the zero point or some other point. However, it often happens that a small section in the entire range of measurement is used for a particular application and that range is quite far from the zero point. In such a case, it would be better to select a reference point within the range for calibration rather than using the zero point, because by doing so, the error after calibration becomes smaller.

In reference-point calibration, a standard,  $M_s$ , is used as the reference point. Measurements are conducted and the average is calculated to be  $\bar{y}_s$ . The ideal relationship between the readings and the signals is

$$y - \bar{y}_s = \beta(M - M_s) \quad (12.27)$$

Table 12.8 shows the data after reference-point calibration. As noted above, first the average of the  $r_0$  readings at the reference point,  $y_{s1}, y_{s2}, \dots, y_{s\gamma_0}$  is calculated:

$$\bar{y}_s = \frac{y_{s1} + y_{s2} + \dots + y_{s\gamma_0}}{r_0} \quad (12.28)$$

**Table 12.7**ANOVA table of quantitative cadmium microanalysis data of  $A_2$ 

Factor	$f$	S	V	$E(V)$
$B$	1	1539.38	1539.38	$\sigma^2 + 100\beta^2$
e	9	3.12	0.35	$\sigma^2$
Total	10	1542.50		

**Table 12.8**  
Data after reference-point calibration

Signal	$M_1 - M_s$	$M_2 - M_s$	...	$M_k - M_s$
Reading	$y_{11} - \bar{y}_s$	$y_{21} - \bar{y}_s$	...	$y_{k1} - \bar{y}_s$
	$y_{12} - \bar{y}_s$	$y_{22} - \bar{y}_s$	...	$y_{k2} - \bar{y}_s$
	...	...	...	...
	$y_{1\gamma_0} - \bar{y}_s$	$y_{2\gamma_0} - \bar{y}_s$	...	$y_{k\gamma_0} - \bar{y}_s$
Total	$y_1$	$y_2$	...	$y_k$

The slope or the linear coefficient,  $\beta$ , is also called the *sensitivity coefficient*. It is given as:

$$\beta = \frac{1}{r_0 r} [y_1(M_1 - M_s) + y_2(M_2 - M_s) + \dots + y_k(M_k - M_s)] \quad (12.29)$$

where

$$r = (M_1 - M_s)^2 + (M_2 - M_s)^2 + \dots + (M_k - M_s)^2 \quad (12.30)$$

The proportional equation is

$$y - \bar{y}_s = \beta(M - M_s) \quad (12.31)$$

For calculation of the SN ratio, the total variation,  $S_T$ , is

$$S_T = \text{total of square of } y_{ij} - \bar{y}_s = \sum_{i=1}^k \sum_{j=1}^{r_0} (y_{ij} - \bar{y}_s)^2 \quad (f = kr_0) \quad (12.32)$$

The linear effect of the signal,  $S_\beta$ , is

$$S_\beta = \frac{1}{r_0 r} [y_1(M_1 - M_s) + y_2(M_2 - M_s) + \dots + y_k(M_k - M_s)]^2 \quad (f = 1) \quad (12.33)$$

The error variation,  $S_e$ , is

$$S_e = S_T - S_\beta \quad (f = kr_0 - 1) \quad (12.34)$$

### □ Example [1]

The SN ratio of an analyzer that measures olefin density was calculated. Standard solutions of four different densities were used. The analyzer was used after the reference-point proportional equation, with 5% standard solution as the reference point. The data in Table 12.9 were obtained after measurement was conducted by two different testing personnel.

**Table 12.9**

Measured olefin density data

Reading	Density $M(\%)$			
	5	10	15	20
$R_1$	5.2	10.3	15.4	20.1
$R_2$	5.0	10.1	15.5	20.3

First, preparatory data processing was performed. The average of data at  $M_s = 5\%$  is

$$\bar{y}_s = \frac{5.2 + 5.0}{2} = 5.1 \quad (12.35)$$

and after subtracting this value from each datum, reference-point calibration was done. The result is shown in Table 12.10.

The ANOVA is done as follows:

$$S_T = 0.1^2 + (-0.1)^2 + \dots + 15.2^2 = 722.35 \quad (f = 8) \quad (12.36)$$

$$\gamma_0\gamma = (2)(0^2 + 5^2 + 10^2 + 15^2) = 700 \quad (12.37)$$

$$S_\beta = \frac{[(0)(0) + (5)(10.2) + (10)(20.7) + (15)(30.2)]^2}{700}$$

$$= \frac{711^2}{700} = 722.1729 \quad (f = 1) \quad (12.38)$$

$$S_e = S_T - S_\beta = 722.35 - 722.1729 = 0.1771 \quad (f = 7) \quad (12.39)$$

$$V_e = \frac{S_e}{7} = 0.1771/7 = 0.0253 \quad (12.40)$$

**Table 12.10**

Olefin density data after reference-point calibration

	Density $M(\%)$			
	5	10	15	20
$M - M_s$	0	5	10	15
Reading after calibration				
$R_1$	0.1	5.2	10.3	15.0
$R_2$	-0.1	5.0	10.4	15.2
Total	0.0	10.2	20.7	30.2

The result is summarized in Table 12.11.

The SN ratio is

$$\eta = \frac{(1/700) (722.1729 - 0.0523)}{0.0253} = 40.76 \quad (12.41)$$

The decibel transformation of the SN ratio is

$$\eta = 10 \log(40.76) = 16.10 \text{ dB} \quad (12.42)$$

The sensitivity,  $S_{\beta}$ , is

$$\eta = 10 \log \frac{1}{700} (722.1729 - 0.0523) = 10 \log(1.0316) \quad (12.43)$$

The estimate of the linear coefficient is

$$\hat{\beta} = \sqrt{1.0316} = 1.015 \quad (12.44)$$

The function of  $M$  and  $y$  is determined as

$$y - 5.1 = 1.015(M - 5) \quad (12.45)$$

## 12.4. Linear Equation

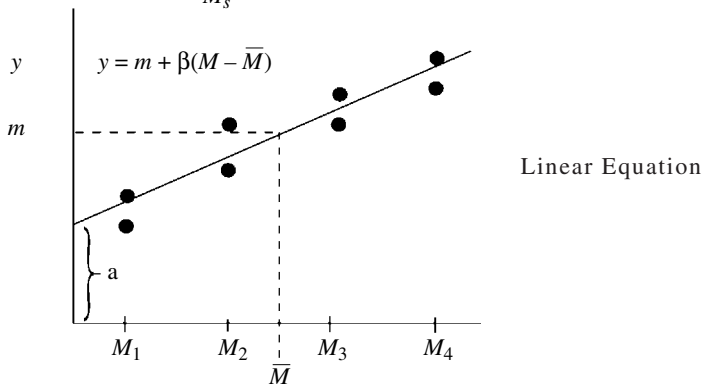
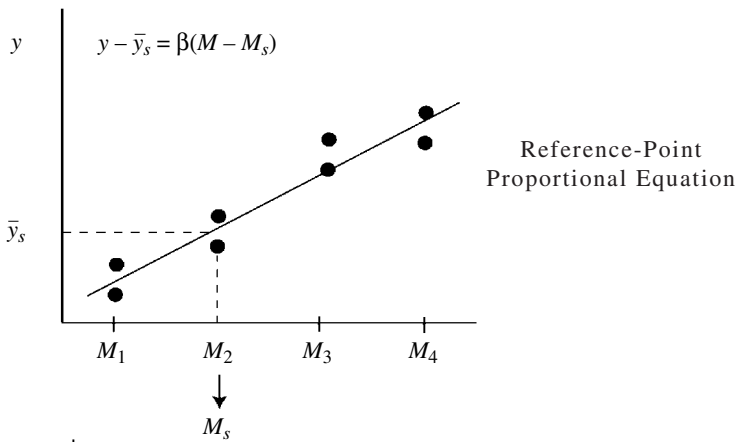
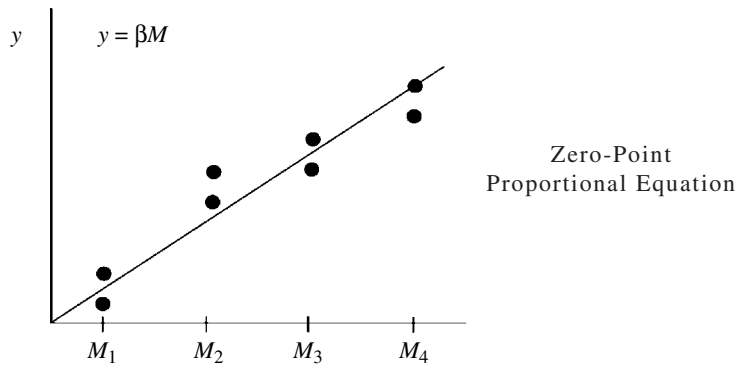
This is the case in which a linear relationship without a particular restriction is expected between the signal factor and the output responses. In zero-point proportional equations, the regression line must pass through the origin. In reference-point proportional equations, the line passes through the reference point and the reference data, as shown in Figure 12.2.

Which case should be used depends on the objective of the study or on the ideal function of the product or process to be studied. When there is no specific restriction on the input/output relationship shown by a regression line, a linear equation may be used. In a study of an injection molding process, we are not interested in the area in which the injection pressure is equal to zero, and whether the regression line passes through the origin is not our concern. In such a case, a linear equation may be used.

**Table 12.11**

ANOVA table of the reference-point proportional equation of olefin density

Factor	$f$	$S$	$V$	$E(V)$
$\beta$	1	722.1729	722.1729	$\sigma^2 + \gamma_0\gamma\beta^2$
$e$	7	0.1771	0.0253	$\sigma$
Total	8	722.3500		



**Figure 12.2**  
Three different cases of  
equations

From the results in Table 12.2, a linear equation is given as

$$y = m + \beta(M - \bar{M}) + e \quad (12.46)$$

where  $m$  is the average and  $e$  is the error. Parameters  $m$  and  $\beta$  are estimated by

$$m = \bar{y} \quad (12.47)$$

$$\beta = \frac{1}{r_0 r} [y_1(M_1 - \bar{M}) + y_2(M_2 - \bar{M}) + \dots + y_k(M_k - \bar{M})] \quad (12.48)$$

where

$$\bar{M} = \frac{M_1 + M_2 + \dots + M_k}{k} \quad (12.49)$$

$$r = (M_1 - \bar{M})^2 + (M_2 - \bar{M})^2 + \dots + (M_k - \bar{M})^2 \quad (12.50)$$

and  $r_0$  is the number of repetitions under each signal factor level.

It must be noted that in the case of the linear equation, the total variation,  $S_T$ , is calculated by subtracting  $S_m$  from the total of the squares of individual data. In this case,  $S_T$  shows the variation around the mean:

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{kr_0}^2 - S_m \quad (f = kr_0 - 1) \quad (12.51)$$

$$S_m = \frac{(\sum y_{ij})^2}{kr_0} \quad (12.52)$$

$$S_\beta = \frac{1}{r_0 r} [y_1(M_1 - \bar{M}) + y_2(M_2 - \bar{M}) + \dots + y_k(M_k - \bar{M})]^2 \quad (f = 1) \quad (12.53)$$

$$S_e = S_T - S_\beta \quad (f = kr_0 - 2) \quad (12.54)$$

$$V_e = \frac{1}{kr_0 - 2} S_e \quad (12.55)$$

The SN ratio is

$$\eta = 10 \log \frac{(1/r_0 r)(S_\beta - V_e)}{V_e} \quad (12.56)$$

### □ Example [1]

In an injection molding process of a plastic product, using the injection pressure as the signal factor controls the dimensions of the plastic product. To obtain the SN ratio for such a system, the molding injection pressure was altered, and the data given in Table 12.12 were obtained by measuring the dimensions of two molded products. In this case, a linear relationship with no particular restriction was assumed.

**Table 12.12**

Dimension measurement data for plastic injection molding

	Pressure (kg <sub>f</sub> /cm <sup>2</sup> )			
	30	40	50	60
$M - \bar{M}$	-15	-5	5	15
Data (mm)				
$R_1$	4.608	4.640	4.682	4.718
$R_2$	4.590	4.650	4.670	4.702
Total	9.198	9.290	9.352	9.420

Table 12.12 also shows the value of  $M_i - \bar{M}$  that was used in the ANOVA. Here  $\bar{M} = 45$ . ANOVA was performed as follows:

$$S_T = 4.608^2 + 4.590^2 + \dots + 4.702^2 = 173.552216 \quad (f = 8) \quad (12.57)$$

$$S_m = \left( \frac{9.198 + \dots + 9.420}{8} \right)^2 = \frac{37.26^2}{8}$$

$$= 173.538450 \quad (f = 1) \quad (12.58)$$

$$\gamma_0\gamma = (2)[(-15)^2 + (-5)^2 + 5^2 + 15^2] = 1000 \quad (12.59)$$

$$S_\beta = \frac{[(-15)(9.198) + \dots + (15)(9.420)]^2}{1000}$$

$$= \frac{3.64^2}{1000} = 0.0132496 \quad (f = 1) \quad (12.60)$$

$$S_e = S_T - S_m - S_\beta = 173.552216 - 173.538450$$

$$- 0.0132496 = 0.00051564 \quad (f = 6) \quad (12.61)$$

$$V_e = \frac{S_e}{6} = \frac{0.0005164}{6} = 0.0000861 \quad (12.62)$$

The calculation results are summarized in Table 12.13.

The SN ratio and the results of decibel transformation are

$$\eta = \frac{(1/1000)(0.0132496 - 0.0000861)}{0.0000861} = 0.1529 \quad (12.63)$$

$$= 10 \log(0.1529) = -8.155 \text{ dB} \quad (12.64)$$

**Table 12.13**

ANOVA table: Linear equation for plastic injection molding

Factor	<i>f</i>	<i>S</i>	<i>V</i>	<i>E(V)</i>
<i>m</i>	1	173.538450	173.538450	$\sigma^2 + (4)(2\sigma_m^2)$
$\beta$	1	0.0132496	0.0132496	$\sigma^2 + \gamma_0\gamma\beta^2$
<i>e</i>	6	0.0005164	0.0000861	$\sigma^2$
Total	8	173.552216		

The estimates of the mean value, *m*, and sensitivity coefficient,  $\beta$ , are

$$\hat{m} = \frac{37.26}{8} = 4.6575 \quad (12.65)$$

$$\hat{\beta} = \frac{3.64}{1000} = 0.00364 \quad (12.66)$$

The equation relating *M* and *y* is

$$y = 4.6575 + 0.00364 (M - 45) \quad (12.67)$$

## 12.5. Linear Equation Using a Tabular Display of the Orthogonal Polynomial Equation

When the intervals for signal factor level are set equal, the SN ratios can easily be calculated. Table 12.14 is part of a tabular display of Chebyshev's orthogonal polynomial equation. Although the table can be used to calculate linear, quadratic, or higher-order terms, only the linear term is used in the SN ratio calculation. (*Note:* This table is used only when level intervals are equal.) In the table,

$b_1$  = linear term

$b_2$  = quadratic term

$$b_i = \frac{W_1A_1 + \dots + W_kA_k}{r_0\lambda Sh^i} \quad (12.68)$$

$h$  = level interval

$$n_e \text{ of } b_i = r_0Sh^{2i} \quad (12.69)$$

$$S_{bi} = \frac{(W_1A_1 + \dots + W_kA_k)^2}{r_0\lambda^2S} \quad (12.70)$$



**Table 12.14**Orthogonal polynomial equation<sup>a</sup>

No. of Levels:	<i>k</i> = 2		<i>k</i> = 3		<i>k</i> = 4			<i>k</i> = 5			
Coefficients:	<i>b</i> <sub>1</sub>	<i>b</i> <sub>1</sub>	<i>b</i> <sub>2</sub>	<i>b</i> <sub>1</sub>	<i>b</i> <sub>2</sub>	<i>b</i> <sub>3</sub>	<i>b</i> <sub>1</sub>	<i>b</i> <sub>2</sub>	<i>b</i> <sub>3</sub>	<i>b</i> <sub>4</sub>	
<i>W</i> <sub>1</sub>	-1	-1	-1	-3	1	-1	-2	2	-1	1	
<i>W</i> <sub>2</sub>	1	0	-2	-1	-1	3	-1	-1	2	-4	
<i>W</i> <sub>3</sub>		1	1	1	-1	-3	0	-2	0	6	
<i>W</i> <sub>4</sub>				3	1	1	1	-1	-2	-4	
<i>W</i> <sub>5</sub>							2	2	1	1	
λ <sup>2</sup> <i>S</i>	2	2	6	20	4	20	10	14	10	70	
λ <i>S</i>	1	2	2	10	4	6	10	14	12	24	
<i>S</i>	1/2	2	2/3	5	4	9/5	10	14	72/5	288/35	
λ	2	1	3	2	1	10/3	1	1	5/6	35/12	

<sup>a</sup>Level intervals must be equal.

From Table 12.2 the SN ratio and the linear coefficient are calculated as follows:

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{kr_0}^2 - S_m \quad (f = kr_0 - 1) \quad (12.71)$$

$$S_m = \frac{(y_{11} + y_{12} + \dots + y_{kr_0})^2}{kr_0} \quad (f = 1) \quad (12.72)$$

$$S_B = \frac{(W_1 y_1 + W_2 y_2 + \dots + W_k y_k)^2}{r_0 \lambda^2 S} \quad (f = 1) \quad (12.73)$$

$$\beta = \frac{W_1 y_1 + W_2 y_2 + \dots + W_k y_k}{r_0 \lambda S h} \quad (12.74)$$

$$S_e = S_T - S_B \quad (f = kr_0 - 2) \quad (12.75)$$

$$V_e = \frac{S_e}{kr_0 - 2} \quad (12.76)$$

$$\eta = 10 \log \frac{(1/r_0 \lambda^2 S h^2)(S_B - V_e)}{V_e} \quad (12.77)$$

To calculate  $S_B$  and  $\beta$ , the values of  $W_1, W_2, \dots, W_k$  are found from Table 12.6 under column  $b_1$  corresponding to the signal factor level,  $k$ .  $h$  is the level interval of the signal factor.  $A_1, A_2, \dots, A_k$  in Table 12.14 correspond to  $y_1, y_2, \dots, y_k$  in Table 12.2.

### □ Example

The example in Table 12.12 can be calculated easily by use of Table 12.14. From the table,

$$\begin{aligned}
 k &= 4 \\
 r_0 &= 2 \\
 h &= 5 \\
 W_1 &= -3 \\
 W_2 &= -1 \\
 W_3 &= 1 \\
 W_4 &= 3 \\
 \lambda^2 S &= 20
 \end{aligned} \tag{12.78}$$

$S_\beta$  is calculated by

$$\begin{aligned}
 S_\beta &= \frac{(-3y_1 - y_2 + y_3 + 3y_4)^2}{20\gamma_0} \\
 &= \frac{[(3)(9.198) - 9.290 + 9.352 + (3)(9.420)]^2}{(20)(2)} \\
 &= 0.0132496
 \end{aligned} \tag{12.79}$$

and

$$\begin{aligned}
 \eta &= 10 \log \frac{(1/r_0)(\lambda^2 S)h^2 (S_\beta - V_e)}{V_e} \\
 &= 10 \log \frac{[1/(2)(20)(5^2)](0.0132496 - 0.0000861)}{0.0000861} \\
 &= 10 \log(0.1529) = -8.155 \text{ dB}
 \end{aligned} \tag{12.80}$$

## 12.6. When the Signal Factor Level Interval Is Known

If the signal factor level interval is known, the linear effect can be calculated without knowing the signal factor's true values. In a chemical analysis, we try to set the levels of the content of a certain ingredient.

Suppose that there is a sample containing a certain ingredient, such as the alcohol content in soybean sauce, but its true value is unknown. To set up levels as signal, the following samples are prepared:  $M_1$ : unknown sample containing  $x$

percent alcohol,  $M_2$ :  $M_1$  plus 3% alcohol,  $M_3$ :  $M_1$  plus 6% alcohol, and  $M_4$ :  $M_1$  plus 9% alcohol. Thus, the signal-factor levels  $M_1$ ,  $M_2$ ,  $M_3$ , and  $M_4$  have equal level intervals. Chebyshev's table of the orthogonal polynomial equation can be used to calculate the linearity and thus the SN ratios.

From Table 12.15, the SN ratio of this case is calculated as follows:

$$S_T = y_{11}^2 + y_{12}^2 + \cdots + y_{34}^2 - S_m \quad (f = 11) \quad (12.81)$$

$$S_m = \frac{(y_{11} + y_{12} + \cdots + y_{34})^2}{12} \quad (12.82)$$

and the following definitions apply:

- Level interval:  $h = 3$
- Number of levels:  $k = 4$
- Number of repetitions:  $r_0 = 3$

From Table 12.14,

$$\begin{aligned} W_1 &= -3 & W_2 &= -1 & W_3 &= 1 & W_4 &= 3 \\ A_1 &= y_1 & A_2 &= y_2 & A_3 &= y_3 & A_4 &= y_4 \\ \lambda^2 S &= 20 \end{aligned}$$

so

$$\begin{aligned} S_\beta &= \frac{(W_1 A_1 + W_2 A_2 + W_3 A_3 + W_4 A_4)^2}{r_0 (\lambda^2 S)} \\ &= \frac{(-3y_1 - y_2 + y_3 + 3y_4)^2}{(3)(20)} \quad (f = 1) \end{aligned} \quad (12.83)$$

$$S_e = S_T - S_\beta \quad (f = 10) \quad (12.84)$$

$$V_e = \frac{S_e}{10} \quad (12.85)$$

$$\eta = 10 \log \frac{(1/r_0 \lambda^2 S h^2)(S_\beta - V_e)}{V_e} \quad (12.86)$$

**Table 12.15**  
Equal level interval

Sample	$M_1 = x$	$M_2 = x + 3\%$	$M_3 = x + 6\%$	$M_4 = x + 9\%$
Reading	$y_{11}$	$y_{12}$	$y_{13}$	$y_{14}$
	$y_{21}$	$y_{22}$	$y_{23}$	$y_{24}$
	$y_{31}$	$y_{32}$	$y_{33}$	$y_{34}$
Total	$y_1$	$y_2$	$y_3$	$y_4$

### □ Example [3]

Table 12.16 shows the measuring results of an electronic balance for chemical analysis. Each raw data point is multiplied by 1000 and  $M_1$  is used as a reference point. Signal factor levels are set as follows:

$$\begin{aligned} M_1 &= x & M_1 - M_1 &= 0 \\ M_2 &= x + h & M_2 - M_1 &= h \\ &\vdots & &\vdots \\ M_k &= x + (k - 1)h & M_k - M_1 &= (k - 1)h \end{aligned} \quad (12.87)$$

Since  $M_1$  was used as a reference point ( $M_s$ ),  $M_1$  is subtracted from  $M_1, M_2, \dots, M_k$  to obtain the signal factor levels in Table 12.17b. The average of reading corresponding  $M_1$ , denoted by  $\bar{y}_s$ , is calculated as

$$\bar{y}_s = \frac{120,585.7 + 120,584.6 + 120,585.9}{3} = 120,585.4 \quad (12.88)$$

The following calculations are made for the zero-point proportional equation.

Total variation:

$$S_T = 0.3^2 + (-0.8)^2 + \dots + 38.4^2 = 7.132.19 \quad (f = 15) \quad (12.89)$$

**Table 12.16**

Measuring results of an electronic balance

(a) Raw Data						
Noise	Signal					
	$M_1$ $x$	$M_2$ $x + h$	$M_3$ $x + 2h$	$M_4$ $x + 3h$	$M_5$ $x + 4h$	
$N_1$	120.5857	120.5938	120.6018	120.6132	120.6206	
$N_2$	120.5846	120.5933	120.6026	120.6117	120.6213	
$N_3$	120.5859	120.5914	120.6041	120.6095	120.6238	
(b) Transformed Data						
Noise	Signal					Linear Equation
	0 $M_1 - M_s$	$h$ $M_2 - M_s$	$2h$ $M_3 - M_s$	$3h$ $M_4 - M_s$	$4h$ $M_5 - M_s$	
$N_1$	0.3	8.4	16.4	27.8	35.2	$L_1 = 265.4h$
$N_2$	-0.8	7.9	17.2	26.3	35.9	$L_2 = 264.8h$
$N_3$	0.5	6.0	18.7	24.1	38.4	$L_3 = 269.3h$
Total	$y_1 = 0.0$	$y_2 = 22.3$	$y_3 = 52.3$	$y_4 = 78.2$	$y_5 = 108.9$	

Note:  $L_1 = (0)(0.3) + h(8.4) + 2h(16.4) + 3h(27.8) + 4h(35.2) = 265.4h$ .

**Table 12.17**

COD values of wastewater

(a) Raw Data					
Sample	Signal				
	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$
$R_1$	9.3	25.0	43.3	61.9	81.5
$R_2$	11.3	27.0	44.5	63.1	80.6
$R_3$	10.4	27.7	45.2	62.0	79.4
(b) Transformed Data					
Sample	Signal				
	$M_1$ $-0.5(\beta - \alpha)$	$M_2$ $-0.25(\beta - \alpha)$	$M_3$ $0$	$M_4$ $0.25(\beta - \alpha)$	$M_5$ $0.5(\beta - \alpha)$
$R_1$	-35.0	-19.3	-1.0	17.6	37.2
$R_2$	-33.0	-17.3	0.2	18.8	36.3
$R_3$	-33.9	-16.6	0.9	17.7	35.1
Total	-101.9	-53.20	0.1	54.1	108.6

Effective divider:

$$r = 0^2 + h^2 + (2h)^2 + (3h)^2 + (4h)^2 = 30h^2 \quad (12.90)$$

Variation of linear term:

$$\begin{aligned} S_{\beta} &= \frac{1}{\gamma_0 \gamma} [y_1(M_1 - \bar{M}_s) + \dots + y_5(M_5 - \bar{M}_s)]^2 \\ &= \frac{(L_1 + L_2 + L_3)^2}{(3)(30h^2)} = \frac{(799.5h)^2}{(3)(30h^2)} = 7102.225 \quad (f = 1) \quad (12.91) \end{aligned}$$

Variation of linear term due to noise:

$$\begin{aligned} S_{N\beta} &= \frac{L_1^2 + L_2^2 + L_3^2}{30h^2} - S_{\beta} \\ &= \frac{(264.4h)^2 + (265.8h)^2 + (269.3h)^2}{30h^2} - S_{\beta} \\ &= 7102.623 - S_{\beta} = 0.398 \quad (f = 2) \quad (12.92) \end{aligned}$$

For the calculation of  $S'_{N\beta}$ , see Section 12.11.)

Error variation:

$$S_e = S_T - S_{\beta} - S_{N\beta} = 29.567 \quad (f = 12) \quad (12.93)$$

Error variance:

$$V_e = \frac{29.567}{12} = 2.464 \quad (12.94)$$

Pooled total error variance:

$$V_N = \frac{0.398 + 29.567}{14} = 2.140 \quad (12.95)$$

The linear coefficient is calculated from equation (12.29):

$$\begin{aligned} \beta &= \frac{1}{\gamma_0 \gamma} [y_1(M_1 - \bar{M}_s) + y_2(M_2 - \bar{M}_s) + \dots + y_5(M_5 - \bar{M}_s)] \\ &= \frac{(L_1 + L_2 + L_3)h}{(3)(30h^2)} = \frac{8.883}{h} \\ &\approx \frac{8.883}{10} = 0.8883 \end{aligned} \quad (12.96)$$

where  $h = 0.01$  (unit: grams) is used. The following SN ratio is calculated using  $h = 10$  (unit: milligrams):

$$\begin{aligned} \eta &= 10 \log \frac{[1/(3)(30)(10^2)](7102.225 - 2.140)}{2.140} \\ &= 10 \log(0.369) = -4.33 \text{ dB} \end{aligned} \quad (12.97)$$

Sensitivity is given by

$$\begin{aligned} S &= 10 \log \frac{1}{(3)(30)(10)^2} (7102.225 - 2.140) \\ &= 10 \log(0.789) = -1.03 \text{ dB} \end{aligned} \quad (12.98)$$

In this example, pooled error variance ( $V_N$ ) is used as the dominator in the calculation of SN ratio instead of using unpooled error variance. (This is discussed further in Section 12.11.)

## 12.7. When Signal Factor Levels Can Be Set Up

Although the true values of signal factor levels are unknown, their intervals can be set up correctly. Consider creating a level with a particular chemical content. Two samples,  $A$  and  $B$ , of content  $\alpha$  and  $\beta$  percent, respectively, are prepared, and samples with different content ratios are prepared by mixing these two samples. If the levels are taken by setting  $A:B = 0.2:0.8$  for  $M_1$ ,  $A:B = 0.4:0.6$  for  $M_2$ , and so on, the following will be obtained:

$$\begin{aligned}
 M_1 &= 0.2\alpha + 0.8\beta = (1 - 0.8)\alpha + 0.8\beta = \alpha + 0.8(\beta - \alpha) \\
 M_2 &= \alpha + 0.6(\beta - \alpha) \\
 M_3 &= \alpha + 0.4(\beta - \alpha) \\
 M_4 &= \alpha + 0.2(\beta - \alpha) \\
 M_5 &= \alpha
 \end{aligned}
 \tag{12.99}$$

The levels of such a signal factor have the same interval,  $0.2(\beta - \alpha)$ . Even if the values of  $\alpha$  and  $\beta$  are not known accurately, the equal-interval relationship between the adjacent levels is maintained. A level setting of this type is effective in experiments that check sample measurement errors.

### □ Example [3]

Table 12.17 shows the measuring results of chemical oxygen demand (COD) values from the wastewater of a factory. Two different water samples were mixed to set the following signed factor levels:

$$\begin{aligned}
 M_1 &= \alpha \\
 M_2 &= 0.75\alpha + 0.25\beta \\
 M_3 &= 0.50\alpha + 0.50\beta \\
 M_4 &= 0.25\alpha + 0.75\beta \\
 M_5 &= \beta
 \end{aligned}
 \tag{12.100}$$

The results of the table were transformed using  $M_3$  as the reference point. The average at  $M_3$  is calculated as

$$\bar{y}_s = \frac{43.3 + 44.5 + 45.2}{3} = 44.3
 \tag{12.101}$$

Total variation:

$$S_7 = (-35.0)^2 + (-33.0)^2 + \dots + 35.1^2 = 9322.43 \quad (f = 15)
 \tag{12.102}$$

Signal factor level interval:

$$h = 0.25(\beta - \alpha)
 \tag{12.103}$$

[The table of orthogonal polynomial equation (Table 12.14) is needed for calculation.]

Effect divider:

$$r = (-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2 = 10
 \tag{12.104}$$

Variation of linear effect:

$$S_{\beta} = \frac{[(-2)(-101.9) - (-53.20) + 54.1 + (2)(108.6)]^2}{3r}$$

$$= \frac{528.3^2}{(3)(10)} = 9303.3630 \quad (f = 1) \quad (12.105)$$

Error variation:

$$S_e = S_T - S_{\beta} = 9322.43 - 9303.3630 = 19.067 \quad (f = 14) \quad (12.106)$$

Error variance:

$$V_e = \frac{19.067}{14} = 1.3619 \quad (12.107)$$

SN ratio:

$$\eta = 10 \log \frac{(1/3r)(S_{\beta} - V_e)}{V_e} = \frac{[1/(3)(10)](9303.3630 - 1.3619)}{1.3619}$$

$$= 10 \log(227.67) = 23.57 \text{ dB} \quad (12.108)$$

Sensitivity:

$$S = 10 \log \frac{1}{(3)(10)} (9303.3630 - 1.3619)$$

$$= 10 \log(310.07) = 24.91 \text{ dB} \quad (12.109)$$

## 12.8. When the Signal Factor Level Ratio Is Known

There are times when the absolute value of signal factor levels is not accurately known, but the ratio of the level values can be established accurately. In such a case, the base value of the comparison does not need to be known as long as the proportions to the base value are known (e.g., a geometrically progressing level case or a case where  $M_1$  is a certain times larger than  $M_2$ ). This is because the linear effect of the signal factor can be calculated accurately as long as the relation among signal factor levels is known.

For example, when the SN ratio is calculated for a method measuring the content ratio of a chemical compound, diluting the sample of certain content by  $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$  can create levels with a common ratio of  $\frac{1}{2}$ . Further, if a scale is used, it is possible to divide a mass into  $\frac{1}{2}$ , and by repeating this, the levels with a common ratio of  $\frac{1}{2}$  can be arranged.

Although the absolute values of levels are also unknown in this case, it is not practical to assume a reference-point proportional equation between  $M$  and  $y$ , since the distance between the zero point and the maximum level point,  $M_2$ , is divided into ratio intervals to create the levels. The following two cases are appli-



cable in practice: (1) a zero-point proportional equation or (2) a linear equation. In either case, the signal-factor-level values are transformed into  $x$  times the value  $M_s$ , which is used as the base:

$$M'_i = \frac{M_i}{M_s}$$

Specifically, the transformation is done as follows:

$$M_1 \rightarrow M'_1 M_s$$

$$M_2 \rightarrow M'_2 M_s$$

⋮

$$M_k \rightarrow M'_k M_s$$

The value  $M_s$ , which is used as the base, can be chosen arbitrarily. The maximum level value is generally used. The transformed level value,  $M'$ , is used for the remaining calculation.

In the case of a zero-point proportional equation, it is known that data  $y = 0$  if signal  $M = 0$ . It is assumed that the signal factor,  $M$ , and data,  $y$ , are proportional. In other words, the relationship between  $M$  and  $y$  is assumed to be

$$y = \beta M \quad (12.110)$$

The distance between levels is not equal, and the calculation formulations are the same as the case of a zero-point proportional equation. The linear effect of the signal factor is calculated using the transformed signal level value,  $M'$ . For example, if the geometrically progressive levels have a common ratio of  $\frac{1}{2}$  and the number of levels  $k = 4$ , the following will result:

$$M'_1 = \frac{1}{8} \quad M'_2 = \frac{1}{4} \quad M'_3 = \frac{1}{2} \quad M'_4 = 1 \quad (12.111)$$

$$S_\beta = \frac{1}{\gamma_0 \gamma} \left( \frac{1}{8} y_1 + \frac{1}{4} y_2 + \frac{1}{2} y_3 + y_4 \right)^2 \quad (12.112)$$

$$r = \left( \frac{1}{8} \right)^2 + \left( \frac{1}{4} \right)^2 + \left( \frac{1}{2} \right)^2 + 1^2 \quad (12.113)$$

The alternative SN ratio,  $\eta'$ , which is based on  $M_s$ , is given by

$$\eta' = \frac{(1/\gamma_0 \gamma)(S_\beta - V_e)}{V_e} \quad (12.114)$$

This SN ratio, which is calculated assuming that  $M_s = 1$ , is not the minus square of the unit of the signal factor. However, this alternative SN ratio is sufficient if it is used only for comparison.

To obtain the SN ratio corresponding to the unit of the signal factor, we substitute  $M_i$  with the value expressed in terms of the signal factor unit and transform it according to the equation

$$\eta = \frac{\eta'}{M_s^2} \quad (12.115)$$

### 12.9. When the True Values of Signal Factor Levels Are Unknown

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When the signal factor level values are not known accurately, the variation of samples with unknown values is used as the signal. Signal factor variation  $S_M$  is calculated, and the residual of the total variation,  $S_T$ , after subtracting  $S_{M_p}$  is considered the error term. This is an expedient measure which assumes that a linear relationship holds between the signal  $M$  and the data  $y$ .

The method of decomposing the total variation is different depending on what is expected with the linear relationship, as in the following two cases:

1. *Proportional equation:*  $S_T = S_M + S_e$

2. *Linear equation:*  $S_T = S_m + S_M + S_e$

However, these two cases share a basic concept: Calculation of the effect,  $S_M$ , of the signal factor,  $M$ , and the general mean,  $S_m$ , as well as the degrees of freedom is different. The proportional equation is explained below.

The total variation is calculated similarly to the case in which the true value is known. The linear effect cannot be calculated since the signal factor true value is unknown. Therefore, the total effect,  $S_{M_p}$ , of  $M$  is considered the signal variation and is obtained as follows:

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{kr_0}^2 \quad (f = kr_0) \quad (12.116)$$

$$S_M = \frac{y_1^2 + y_2^2 + \dots + y_k^2}{r_0} \quad (f = k) \quad (12.117)$$

In the case of a proportional equation, the degrees of freedom of the signal variation,  $S_M$ , is  $k$ . The error variation is calculated as follows:

$$S_e = S_T - S_M \quad (12.118)$$

Since the degrees-of-freedom expression of the error term is

$$f_e = kr_0 - k \quad (12.119)$$

the error variance becomes

$$V_e = \frac{1}{kr_0 - k} S_e \quad (12.120)$$

The variance of signal factor is obtained as

$$V_M = \frac{1}{k} S_M \quad (12.121)$$

The SN ratio is determined by

$$\eta = 10 \log \frac{(1/r_0)(V_M - V_e)}{V_e} \quad (12.122)$$

### 12.10. When There Is No Signal Factor (Nondynamic SN Ratio)

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One of the most important applications of the dynamic SN ratio is to improve the robustness of product or process functions within a certain output range. In a

group of products, each product has a different output. A process can produce different outputs for different products or components. It is therefore important to use dynamic SN ratios to develop a group of products or a process for manufacturing different components so that different outputs can easily be obtained.

Traditionally, only one output of a product or only one output in a process has been studied in research and development. Even today, most of the product or process development is being performed this way. In such a case, a nondynamic SN ratio may be used to improve robustness for only a specific or fixed target instead of the entire output range.

For example, a study was conducted to plate gold on the contact point of an electronic component. The target was to plate to a thickness of 50  $\mu\text{in.}$  on the surface, also to reduce variability around 50  $\mu\text{in.}$  using a parameter design approach. This is a nondynamic approach. The target may be 65  $\mu\text{in.}$  for another component. Then another parameter design has to be conducted. But using a dynamic SN ratio, this plating process can be optimized within a certain range of thickness. After the completion of a dynamic approach, any plate thickness within that range can be manufactured without repeating the study for a different thickness every time. Thus, it is easy to understand that the dynamic approach is far more powerful and efficient than the nondynamic approach.

In a nondynamic SN ratio, there are two issues: (1) to reduce variability and (2) to adjust the average to the target. Once variability is reduced and adjusted to the target, there will be no more issues of moving or adjusting the target. It is seen that a nondynamic approach is useful for a specific product with a fixed output, in contrast to a dynamic approach, where the output can easily be adjusted within a range.

There are four types of nondynamic output response applications: nominal-the-best, smaller-the-better, larger-the-better, and nondynamic operating window. In nominal-the-best, there are two types, making a total of five.

Recall that in nominal-the-best, there is a fixed target: for example, the plating thickness of a component, the resistance of a resistor, or various dimensions of a product. There are typically upper and lower specification limits. Type I is the case where data are all nonnegative. In type II, data are a mixture of positive and negative values.

In two-step optimization for a nominal-the-best application, the first step is to maximize the SN ratio and thereby minimize variability around the average. The second step is to adjust the average to the target.

The nominal-the-best SN ratio is described as

$$\begin{aligned} \text{SN} &= \frac{\text{desired output}}{\text{undesired output}} \\ &= \frac{\text{effect of the average}}{\text{variability around the average}} \end{aligned}$$

In nominal-the-best applications, the data are not negative. This is especially true if the output response is energy related, where zero input to the system should produce zero output; thus, there are no negative data.

**Nominal-the-Best,  
Type I**

Calculations for a nominal-the-best SN ratio are as follows. There are  $n$  pieces of data:

$$y_1, y_2, \dots, y_n \quad (f = n) \quad (12.123)$$

The average variation,  $S_m$ , is

$$S_m = \frac{(y_1 + y_2 + \dots + y_n)^2}{n} \quad (f = 1) \quad (12.124)$$

The error variation,  $S_e$ , is

$$S_e = S_T - S_m \quad (f = n - 1) \quad (12.125)$$

The error variance,  $V_e$ , is the error variation divided by its degrees of freedom:

$$V_e = \frac{S_e}{n - 1} \quad (12.126)$$

The SN ratio, denoted by  $\eta$ , is given by

$$\eta = 10 \log \frac{(1/n) (S_m - V_e)}{V_e} \quad (12.127)$$

The sensitivity, denoted by  $S$ , is given by

$$S = 10 \log \frac{1}{n} (S_m - V_e) \quad (12.128)$$

### □ Example

In a tile manufacturing experiment, samples were taken from different locations in the tunnel kiln, with the following results: 10.18 . . . 10.18 . . . 10.12 . . . 10.06 . . . 10.02 . . . 9.98 . . . 10.20

Total variation:

$$\begin{aligned} S_T &= y_1^2 + y_2^2 + \dots + y_7^2 \\ &= 10.18^2 + 10.18^2 + \dots + 10.20^2 \\ &= 714.9326 \quad (f = 7) \end{aligned} \quad (12.129)$$

Average variation:

$$\begin{aligned} S_m &= \frac{(y_1 + y_2 + \dots + y_7)^2}{7} \\ &= \frac{(10.18 + 10.18 + \dots + 10.20)^2}{7} \\ &= 714.8782 \quad (f = 1) \end{aligned} \quad (12.130)$$

Error variation:

$$\begin{aligned} S_e &= S_r - S_m \\ &= 714.9326 - 714.8782 \\ &= 0.05437 \quad (f = 6) \end{aligned} \quad (12.131)$$

Error variance:

$$V_e = \frac{S_e}{6} = \frac{0.05437}{6} = 0.007562 \quad (12.132)$$

SN ratio:

$$\begin{aligned} \eta &= 10 \log \frac{(1/n)(S_m - V_e)}{V_e} \\ &= 10 \log \frac{(1/7)(714.8782 - 0.007562)}{0.007562} \\ &= 10 \log(13504.9427) = 41.37 \text{ dB} \end{aligned} \quad (12.133)$$

Sensitivity:

$$\begin{aligned} S &= 10 \log \frac{1}{n} (S_m - V_e) \\ &= 10 \log \frac{1}{7} (714.8782 - 0.007562) \\ &= 10 \log(102.1244) = 20.09 \text{ dB} \end{aligned} \quad (12.134)$$

When the results of an experiment include negative values, the positive and negative values cancel each other, which makes the calculation of  $S_m$  meaningless. Therefore, the information regarding sensitivity, or average, cannot be obtained. The SN ratio calculated for this case shows variability only. It is less informative than type I.

**Nominal-the-Best,  
Type II**

Conceptually, the SN ratio gives the following information:

$$\frac{\text{SN} = 1}{\text{variability around the average}}$$

The SN ratio is calculated as follows where there are  $n$  pieces of data:  $y_1, y_2, \dots, y_n$ . The error variation is

$$S_e = y_1^2 + y_2^2 + \dots + y_n^2 - \frac{(y_1 + y_2 + \dots + y_n)^2}{n} \quad (f = m - 1) \quad (12.135)$$

The error variance is

$$V_e = \frac{S_e}{n - 1} \quad (12.136)$$

The SN ratio is denoted by

$$\begin{aligned}\eta &= 10 \log \frac{1}{V_e} \\ &= -10 \log V_e\end{aligned}\quad (12.137)$$

### □ Example

For data points 1.25, -1.48, -2.70, and 0.19, the SN ratio is calculated as follows:

$$\begin{aligned}S_e &= 1.25^2 + (-1.48)^2 + (-2.70)^2 + 0.19^2 \\ &\quad - \frac{(1.25 - 1.48 - 2.70 + 0.19)^2}{4} \\ &= 9.2021 \quad (f = 3)\end{aligned}\quad (12.138)$$

$$V_e = \frac{S_e}{n - 1} = \frac{9.2021}{3} = 3.0674 \quad (12.139)$$

$$\begin{aligned}\eta &= -10 \log(3.0674) \\ &= -4.87 \text{ dB}\end{aligned}\quad (12.140)$$

**Smaller-the-Better** Smaller-the-better obtains when the objective is to minimize the output value and there are no negative data. In such a case, the target is zero. For example, we want to minimize the audible engine noise level or to minimize the number of impurities. These examples are easy to understand; however, they are used without success in many cases because such outputs are “symptoms.” They are called *downstream quality characteristics*. As explained in Chapter 14, using symptoms as the output quality characteristics in a study is not recommended. It is better, instead, to use this type of SN ratio together with a larger-the-better SN ratio to construct an SN ratio for a nondynamic operating window.

A smaller-the-better SN ratio is calculated as

$$\eta = -10 \log \frac{1}{n} (y_1^2 + y_2^2 + \dots + y_n^2) \quad (12.141)$$

### □ Example

For data points 0.25, 0.19, and 0.22, the SN ratio is calculated as

$$\begin{aligned}\eta &= -10 \log \frac{1}{3}(0.25^2 + 0.19^2 + 0.22^2) \\ &= 13.10 \text{ dB}\end{aligned}\quad (12.142)$$

**Larger-the-Better**

Larger-the-better obtains when the intention is to maximize the output and there are no negative data. In such a case, the target is infinity. For example, we use this if we want to maximize the power of an engine or the strength of material. Similar to smaller-the-better, such quality characteristics are symptoms: downstream quality characteristics. A study of downstream quality characteristics will exhibit interactions between control factors; therefore, conclusions from the study cannot be reproduced in many cases. It is recommended that this type of SN ratio be used together with the smaller-the-better type to construct an SN ratio of a nondynamic operating window.

The calculation of a larger-the-better SN ratio is given by

$$\eta = -10 \log \frac{1}{n} \left( \frac{1}{y_1^2} + \frac{1}{y_2^2} + \dots + \frac{1}{y_n^2} \right) \quad (12.143)$$

**Example**

For data points 23.5, 43.1, and 20.8, the SN ratio is calculated as

$$\begin{aligned} \eta &= -10 \log \frac{1}{3} \left( \frac{1}{23.5^2} + \frac{1}{43.1^2} + \frac{1}{20.8^2} \right) \\ &= 28.09 \text{ dB} \end{aligned} \quad (12.144)$$

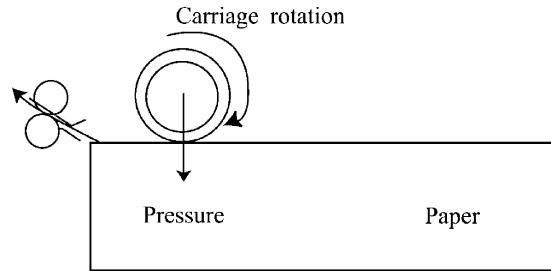
As described for smaller-the-better and larger-the-better, the approaches tend to be unsuccessful if the output quality characteristic is a symptom. However, if we can express a function using either smaller-the-better or larger-the-better characteristics, the results have a better chance of being reproduced. A nondynamic operating window combines smaller-the-better and larger-the-better SN ratios to express a function that moves in two opposite directions.

For example, the carriage of a copy machine needs to send the top sheet of paper on a pile to the next process. When the carriage fails to feed a sheet of paper, it is called a *misfeed*; when it feeds two or more sheets of paper, it is called a *multifeed*. The ideal function is that there be no misfeed or multifeed under any conditions of paper quality or use environment. The authors of this book have been promoting a method for checking the effect of noise using the signal if the input is a continuous variable.

The evaluation of carriage function can be done as follows: When friction between the paper and the carriage roller, which is pressed by a spring toward the paper, is used to feed paper, different spring strengths create different levels of pressure and thus different levels of paper-feeding force. If the pressure is zero, paper cannot be fed (Figure 12.3). As the pressure level is increased, the carriage starts feeding paper. If only one sheet of paper is fed, the pressure level is the threshold value of misfeed. Let  $x$  denote that value. A further increase in the pressure makes the carriage feed two or more sheets of paper. That pressure level, which is denoted as  $y$ , is the threshold value of multifeed. If two sheets of paper are fed as soon as the carriage starts feeding paper,  $x = y$ . Such a case is called a *zero operating window*. It suggested that a design that gives a large operating window

**Nondynamic  
Operating Window**

**Figure 12.3**  
Paper carriage function



is a good design and has developed a method of stability design called the *operating window method*.

It is possible to have  $x = 0$  or  $y = \infty$ , but these are not a problem. The important point is that the values of  $x$  and  $y$  change depending on the paper and environmental conditions. Based on noise factors such as paper size, weight, static electricity, water content, wrinkle size, room temperature, humidity, and power voltage, the following compounded noise factor is created using technical know-how and preparatory experiments (which are done under only one design condition).

$N_1$ : standard condition

$N_2$ : condition that is most likely to create misfeed

$N_3$ : condition that is most likely to create multifeed

Under these three conditions, misfeeding threshold, value  $x$ , and double-feeding threshold, value  $y$ , are obtained. According to the data in Table 12.18, the following two SN ratios are obtained by treating  $x$  as a smaller-the-better characteristic and  $y$  as a larger-the-better characteristic.

$$\eta_x = -10 \log \frac{1}{3}(x_1^2 + x_2^2 + x_3^2) \quad (12.145)$$

$$\eta_y = -10 \log \frac{1}{3} \left( \frac{1}{y_1^2} + \frac{1}{y_2^2} + \frac{1}{y_3^2} \right) \quad (12.146)$$

Two SN ratios are obtained for each design (e.g., every condition of an inner orthogonal array to which control factors are assigned). By considering  $\eta_x$  and  $\eta_y$  as outside data at  $K_1$  and  $K_2$ , in contrast to the design condition, the main effect of the control factor is obtained from the sum of  $K_1$  and  $K_2$ . The key point is to maximize the sum,  $\eta_x + \eta_y$ .

**Table 12.18**  
Two threshold values  $x$  and  $y$

Characteristic Noise	Misfeed	Multifeed
$N_1$	$x_1$	$y_1$
$N_2$	$x_2$	$y_2$
$N_3$	$x_3$	$y_3$



### □ Example

In a paper-feeding experiment, the responses selected were:

- x: spring force required to begin feeding one piece of paper
- y: spring force required to begin feeding two or more pieces of paper

The experimental objective was defined as:

- *Minimize x*: x was treated as a smaller-the-better response.
- *Maximize y*: y was treated as a larger-the-better response.

Table 12.19 shows the results of one run in the  $L_{18}$  experiment. The SN ratio is calculated as

$$\begin{aligned}
 \eta &= \eta_x + \eta_y \\
 &= -10 \log \frac{1}{3}(x_1^2 + x_2^2 + x_3^2) \\
 &\quad -10 \log \frac{1}{3} \left( \frac{1}{y_1^2} + \frac{1}{y_2^2} + \frac{1}{y_3^2} \right) \\
 &= -10 \log \frac{1}{3} (30^2 + 50^2 + 50^2) - 10 \log \frac{1}{3} \left( \frac{1}{50^2} + \frac{1}{80^2} + \frac{1}{100^2} \right) \\
 &= 3.66 \text{ dB} \qquad (12.147)
 \end{aligned}$$

## 12.11. Estimation of Error

Table 12.20 shows the results of a dynamic study. For the zero-point proportional equation, the SN ratio is given by

$$\eta = 10 \log \frac{(1/r_0 r)(S_p - V_e)}{V_e} \qquad (12.148)$$

**Table 12.19**  
Results of experiment

	x	y
$N_1$	30	50
$N_2$	50	80
$N_3$	50	100
	$N_x$	$N_y$

**Table 12.20**  
Inputs and outputs

	$M_1$	$M_2$	...	$M_k$	$L$
$N_1$	$y_{11}$	$y_{21}$	...	$y_{k1}$	$L_1$
$N_2$	$y_{12}$	$y_{22}$	...	$y_{k2}$	$L_2$
...	...	...	...	...	...
$N_l$	$y_{1l}$	$y_{2l}$	...	$y_{kl}$	$L_l$
Total	$y_1$	$y_2$	...	$y_k$	

Based on the concept of SN ratio, the total variation is decomposed as

$$\begin{aligned} \text{total variation } (S_T) &= \text{useful part} + \text{harmful part} \\ &= \text{proportional part } (S_\beta) + \text{remaining part } (S_e) \end{aligned} \quad (12.149)$$

$V_e$  in equation (9.148) is calculated from  $S_e$  in equation (12.149).

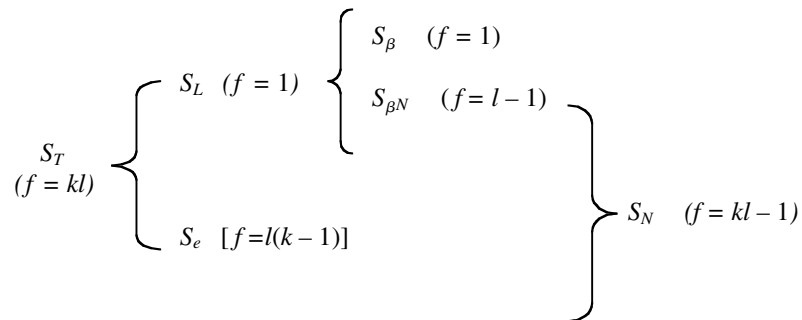
Table 12.20 includes various sources of variation. In addition to the proportional part of  $M$ , there are high-order terms of  $M$ , the main effect of  $N$ , and interactions between  $M$  and  $N$ . It is obvious that using the remaining part in equations (12.149) to calculate  $V_e$  is an overestimate of  $V_e$  to be used in the numerator in equation (12.148). Also, the  $V_e$  in the denominator should have a different notation to distinguish the  $V_e$  in the numerator.

In Table 12.20,  $L_1, L_2, \dots, L_l$  denote the linear effects at noise factor conditions  $N_1, N_2, \dots, N_l$  respectively. The difference between  $L$ 's is the variation caused by different noise factor conditions, or the interaction between  $L$  and  $N$ . The main interaction, the variation of linear effect caused by noise, is denoted by  $\beta N$ .

To avoid overestimation of  $V_e$ , the total variation in Table 12.20 is decomposed as shown in Figure 12.4. Following is the information for the decomposition of total variation,  $S_T$ , into  $S_\beta, S_N$ , and  $S_e$  from the results in Table 12.19.

Total variation:

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{kl}^2 \quad (f = kl) \quad (12.150)$$



**Figure 12.4**  
Decomposition of variation

Linear effect:

$$\begin{aligned}
 L_1 &= M_1 y_{11} + M_2 y_{21} + \dots + M_k y_{k1} \\
 L_2 &= M_1 y_{12} + M_2 y_{22} + \dots + M_k y_{k2} \\
 &\vdots \\
 L_l &= M_1 y_{1l} + M_2 y_{2l} + \dots + M_k y_{kl}
 \end{aligned} \tag{12.151}$$

Variation of linear effect:

$$\begin{aligned}
 S_\beta &= \frac{(M_1 y_1 + M_2 y_2 + \dots + M_k y_k)^2}{\gamma_0 \gamma} \\
 &= \frac{(L_1 + L_2 + \dots + L_l)^2}{\gamma_0 \gamma} - S_\beta \quad (f = 1)
 \end{aligned} \tag{12.152}$$

where

$$\gamma_0 = l \tag{12.153}$$

$$\gamma = M_1^2 + M_2^2 + \dots + M_k^2 \tag{12.154}$$

Variation of  $L$ :

$$S_L = \frac{L_1^2 + L_2^2 + \dots + L_l^2}{\gamma} \quad (f = l) \tag{12.155}$$

Variation of  $\beta$  due to  $N$ :

$$S_{\beta N} = \frac{L_1^2 + L_2^2 + \dots + L_l^2}{\gamma} - S_\beta \quad (f = l - 1) \tag{12.156}$$

Variation of error:

$$S_e = S_T - S_\beta - S_{\beta N} \quad [f = l(k - 1)] \tag{12.157}$$

Error variance:

$$V_e = \frac{S_e}{l(k - 1)} \tag{12.158}$$

Variation of the harmful part:

$$S_N = S_T - S_\beta \quad (f = kl - 1) \tag{12.159}$$

Variance of the harmful part:

$$V_N = \frac{S_N}{kl - 1} \tag{12.160}$$

SN ratio:

$$\eta = 10 \log \frac{(1/\gamma_0 \gamma)(S_\beta - V_e)}{V_N} \tag{12.161}$$

### □ Example [4]

Table 12.21 shows the results of an experiment for a car brake. It is ideal that the input signal, the line pressure, is proportional to the output, the braking torque.

For this case, two noise conditions, labeled  $N_1$  and  $N_2$ , are determined by compounding a few important noise factors, so that

$N_1$ : noise condition when the response tends to be lower

$N_2$ : noise condition when the response tends to be higher

The noise factors are compounded into two levels:

$N_1$ : 60°F, wet, 80% worn pad

$N_2$ : 360°F, dry, 10% worn pad

Another noise factor,  $Q$ , is torque variation:

$Q_1$ : maximum torque during braking

$Q_2$ : minimum torque during braking

$$S_T = 4.8^2 + 0.9^2 + 5.8^2 + 0.8^2 + 8.5^2 + \dots + 34.5^2 = 7342.36 \quad (12.162)$$

$$\begin{aligned} L_1 &= (0.008)(4.8) + (0.016)(8.5) + (0.032)(20.4) + (0.064)(36.9) \\ &= 3.1888 \end{aligned}$$

$$\begin{aligned} L_2 &= (0.008)(0.9) + (0.016)(6.5) + (0.032)(13.2) + (0.064)(32.7) \\ &= 2.6264 \end{aligned}$$

$$\begin{aligned} L_3 &= (0.008)(5.8) + (0.016)(11.5) + (0.032)(25.0) + (0.064)(43.5) \\ &= 3.8144 \end{aligned}$$

$$\begin{aligned} L_4 &= (0.008)(0.8) + (0.016)(6.8) + (0.032)(16.2) + (0.064)(34.5) \\ &= 2.8416 \end{aligned} \quad (12.163)$$

**Table 12.21**  
Results of experiment

	$M_1 = 0.008$	$M_2 = 0.016$	$M_3 = 0.032$	$M_4 = 0.064$	$L$
$N_1Q_1$	4.8	8.5	20.4	36.9	$L_1$
$N_1Q_2$	0.9	6.5	13.2	32.7	$L_2$
$N_2Q_1$	5.8	11.5	25.0	43.5	$L_3$
$N_2Q_2$	0.8	6.8	16.2	34.5	$L_4$
Total	12.3	33.3	74.8	147.6	

$$\begin{aligned}
 S_{\beta} &= \frac{(L_1 + L_2 + L_3 + L_4)^2}{\gamma_0 \gamma} \\
 &= \frac{(3.1888 + 2.6264 + 3.8144 + 2.8416)^2}{(4)(0.00544)} \\
 &= 7147.5565 \quad (f = 16) \qquad (12.164)
 \end{aligned}$$

$$\begin{aligned}
 \gamma_0 &= 4 \\
 \gamma &= 0.008^2 + 0.016^2 + 0.032^2 + 0.064^2 = 0.00544 \quad (12.165)
 \end{aligned}$$

$$\begin{aligned}
 S_{\beta N} &= \frac{L_1^2 + L_2^2 + L_3^2 + L_4^2}{\gamma} - S_{\beta} \\
 &= \frac{3.1888^2 + 2.6264^2 + 3.8144^2 + 2.8416^2}{0.00544} - S_{\beta} \\
 &= 148.5392 \quad (f = 3) \qquad (12.166)
 \end{aligned}$$

$$\begin{aligned}
 S_e &= S_T - S_{\beta} - S_{\beta N} \\
 &= 7342.36 - (7147.5565 + 148.5392) \\
 &= 46.2643 \quad (f = 12) \qquad (12.167)
 \end{aligned}$$

$$V_e = \frac{S_e}{(3)(4)} = \frac{46.2643}{12} = 3.8544 \quad (12.168)$$

$$\begin{aligned}
 S_N &= S_T - S_{\beta} = 7342.36 - 7147.5565 \\
 &= 194.8035 \quad (f = 15) \qquad (12.169)
 \end{aligned}$$

$$V_N = \frac{S_N}{(4)(4) - 1} = \frac{194.8035}{15} = 12.9869 \quad (12.170)$$

$$\begin{aligned}
 \eta &= 10 \log \frac{(1/\gamma_0 \gamma)(S_{\beta} - V_e)}{V_N} \\
 &= 10 \log \frac{[1/(4)(0.00544)](7147.5565 - 3.8554)}{12.9869} \\
 &= 44.03 \text{ dB} \qquad (12.171)
 \end{aligned}$$

## 12.12. Double Signals

In the dynamic SN ratio study, the number of signal factors may not always be equal to 1. For example, in a study of fine patterning for integrated-circuit fabrication, the ideal function is that current be proportional to voltage. It is also ideal that current be proportional to line width. The former is a product-related function, whereas the latter is manufacturing related. The study would become efficient if these two ideal functions were put together for study at the same time. This is

called *concurrent engineering* or *simultaneous engineering*: to develop product and process at the same time. The use of double signals makes it possible.

The ideal function for double signals would be

$$y = \beta MM^* \quad (12.172)$$

where  $y$  is the current,  $\beta$  the regression coefficient,  $M$  the voltage, and  $M^*$  the line width.

Calculation of the SN ratio is done similarly by considering the combinations of  $M$  and  $M^*$  as one signal. Table 12.22 shows the layout of input/output for double signals with noise. By considering the product of  $M$  and  $M^*$  as one signal factor,  $L_1$  and  $L_2$ , the linear equations of the proportional term at  $N_1$  and  $N_2$ , respectively, are calculated as follows:

$$L_1 = M_1^* M_1 y_{11} + M_1^* M_2 y_{12} + \dots + M_3^* M_k y_{5k} \quad (12.173)$$

$$L_2 = M_1^* M_1 y_{21} + M_1^* M_2 y_{22} + \dots + M_3^* M_k y_{6k}$$

Total variation,  $S_T$ , is decomposed as shown in Figure 12.5

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{6k}^2 \quad (f = 6k) \quad (12.174)$$

$$S_\beta = \frac{(L_1 + L_2)^2}{2r} \quad (f = 1) \quad (12.175)$$

$$r = (M_1^* M_1)^2 + (M_1^* M_2)^2 + \dots + (M_3^* M_k)^2 \quad (12.176)$$

$$S_{\beta,N} = \frac{L_1^2 + L_2^2}{\gamma} - S_\beta \quad (f = 1) \quad (12.177)$$

$$S_e = S_T - S_\beta - S_{\beta,N} \quad (f = 6k - 2) \quad (12.178)$$

$$V_e = \frac{S_e}{6k - 2} \quad (12.179)$$

$$V_N = \frac{S_{\beta,N}}{6k - 1} \quad (12.180)$$

$$\eta = 10 \log \frac{(1/2r)(S_\beta - V_e)}{V_N} \quad (12.181)$$

$$S = 10 \log \frac{1}{2r} (S_\beta - V_e) \quad (12.182)$$

### □ Example [5]

In a laser welding experiment, the ideal function was defined as follows:

$$y = \beta MM^*$$

**Table 12.22**

Input/output table for double signals

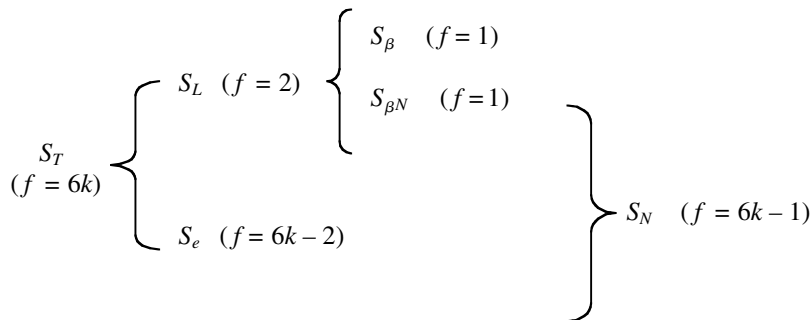
		$M_1$	$M_2$	...	$M_k$
$M_1^*$	$N_1$	$y_{11}$	$y_{12}$	...	$y_{1k}$
	$N_2$	$y_{21}$	$y_{22}$	...	$y_{2k}$
$M_2^*$	$N_1$	$y_{31}$	$y_{32}$	...	$y_{3k}$
	$N_2$	$y_{41}$	$y_{42}$	...	$y_{4k}$
$M_3^*$	$N_1$	$y_{51}$	$y_{52}$	...	$y_{5k}$
	$N_2$	$y_{61}$	$y_{62}$	...	$y_{6k}$

where  $y$  is the force,  $M$  the deformation, and  $M^*$  the welding length. The force at each deformation level was measured for each test piece of different welding length. Table 12.23 shows the results of one of the runs.

$$\begin{aligned}
 L_1 &= M_1^*M_1y_{11} + M_1^*M_2y_{12} + \dots + M_3^*M_2y_{52} \\
 &= (2)(20)(149.0) + (2)(40)(146.8) + \dots + (6)(40)(713.2) \\
 &= 338840 \tag{12.183}
 \end{aligned}$$

$$\begin{aligned}
 L_2 &= M_1^*M_1y_{21} + M_1^*M_2y_{22} + \dots + M_3^*M_2y_{62} \\
 &= (2)(20)(88.6) + (2)(40)(87.0) + \dots + (6)(40)(304.4) \\
 &= 152196 \tag{12.184}
 \end{aligned}$$

$$\begin{aligned}
 S_T &= y_{11}^2 + y_{12}^2 + \dots + y_{62}^2 \\
 &= 149.0^2 + 146.8^2 + \dots + 304.4^2 \\
 &= 1,252,346.16 \tag{12.185} \quad (f = 12)
 \end{aligned}$$



**Figure 12.5**  
Decomposition of variation

**Table 12.23**  
Results of experiment

		$M_1$ (20)	$M_2$ (40)
$M_1^*$ (2 mm)	$N_1$	149.0	146.8
	$N_2$	88.6	87.0
$M_2^*$ (4 mm)	$N_1$	300.4	481.1
	$N_2$	135.7	205.5
$M_3^*$ (6 mm)	$N_1$	408.0	713.2
	$N_2$	207.5	304.4

$$S_\beta = \frac{(L_1 + L_2)^2}{2r}$$

$$= \frac{(338,840 + 152,196)^2}{(2)(112,000)} = 1,076,412.292 \quad (f = 1) \quad (12.186)$$

$$\gamma_0 = 2 \quad (12.187)$$

$$\gamma = (M_1^*M_1)^2 + (M_1^*M_2)^2 + \dots + (M_3^*M_2)^2$$

$$= [(2)(20)]^2 + [(2)(40)]^2 + \dots + [(6)(40)]^2$$

$$= 112,000 \quad (12.188)$$

$$S_{\beta N} = \frac{L_1^2 + L_2^2}{\gamma} - S_\beta$$

$$= \frac{338,840^2 + 152,196^2}{112,000} - 1,076,412.292$$

$$= 155,517.78 \quad (f = 1) \quad (12.189)$$

$$S_e = S_T - S_\beta - S_{\beta N}$$

$$= 1,252,436.16 - 1,076,412.292 - 155,517.78$$

$$= 20,506.088 \quad (f = 10) \quad (12.190)$$

$$S_N = S_T - S_\beta$$

$$= 1,252,436.16 - 1,076,412.292$$

$$= 176,023.868 \quad (f = 11) \quad (12.191)$$

$$V_e = \frac{S_e}{10} = \frac{20,506.088}{10} = 2050.6088 \quad (12.192)$$



$$V_N = \frac{S_N}{11} = \frac{176,023.868}{11} = 16,002.1698 \quad (12.193)$$

$$\begin{aligned} \eta &= 10 \log \frac{(1/2r)(S_\beta - V_e)}{V_N} \\ &= 10 \log \frac{[1/(2)(112,000)](1,076,412.295 - 2050.6088)}{16,002.1698} \\ &= -35.23 \text{ dB} \end{aligned} \quad (12.194)$$

$$\begin{aligned} S &= 10 \log \frac{1}{2r} (S_\beta - V_e) \\ &= 10 \log \frac{1}{(2)(112,000)} (1,076,412.292 - 2050.6088) \\ &= 0.68 \text{ dB} \end{aligned} \quad (12.195)$$

When there are two signals, the ideal function could be as follows:

$$y = \beta \frac{M}{M^*} \quad (12.196)$$

where  $y$  is the deformation,  $M$  the load, and  $M^*$  the cross-sectional area. In a material development, for example, the deformation of test pieces must be linearly proportional to the load added and also inversely proportional to the cross-sectional area. Table 12.24 shows the deformation of test pieces at each load and cross-sectional area under noise conditions  $N_1$  and  $N_2$ .

**Table 12.24**

Load, cross section, and deformation

		$M_1^*$	$M_2^*$	...	$M_k^*$
$M_1$	$N_1$	$y_{11}$	$y_{12}$	...	$y_{1k}$
	$N_2$	$y_{21}$	$y_{22}$	...	$y_{2k}$
$M_2$	$N_1$	$y_{31}$	$y_{32}$	...	$y_{3k}$
	$N_2$	$y_{41}$	$y_{42}$	...	$y_{4k}$
$M_3$	$N_1$	$y_{51}$	$y_{52}$	...	$y_{5k}$
	$N_2$	$y_{61}$	$y_{62}$	...	$y_{6k}$

The linear equations at different noise factor levels are calculated by

$$\begin{aligned}
 L_1 &= \frac{M_1}{M_1^*} y_{11} + \frac{M_1}{M_2^*} y_{12} + \dots + \frac{M_3}{M_k^*} y_{5k} \\
 L_2 &= \frac{M_1}{M_1^*} y_{21} + \frac{M_1}{M_2^*} y_{22} + \dots + \frac{M_3}{M_k^*} y_{6k}
 \end{aligned}
 \tag{12.197}$$

The total variation is

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{6k}^2 \quad (f = 6k)
 \tag{12.198}$$

The total variation is decomposed as shown in Figure 12.6.

Variation of  $L$ :

$$S_L = \frac{L_1^2 + L_2^2}{r} \quad (f = 2)
 \tag{12.199}$$

Variation of the proportional term:

$$S_\beta = \frac{(L_1 + L_2)^2}{2r} \quad (f = 1)
 \tag{12.200}$$

where

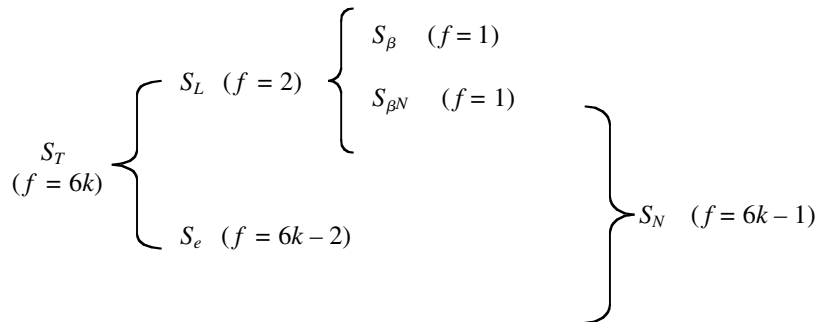
$$r = \left(\frac{M_1}{M_1^*}\right)^2 + \left(\frac{M_1}{M_2^*}\right)^2 + \dots + \left(\frac{M_3}{M_k^*}\right)^2
 \tag{12.201}$$

Variation of  $L$  due to noise:

$$\begin{aligned}
 S_{\beta N} &= S_L - S_\beta \\
 &= \frac{L_1^2 + L_2^2}{r} - \frac{(L_1 + L_2)^2}{2r} = \frac{(L_1 - L_2)^2}{2r} \quad (f = 1)
 \end{aligned}
 \tag{12.202}$$

Variation of error:

$$S_e = S_T - S_L = S_T - S_\beta - S_{\beta N} \quad (f = 6k - 2)
 \tag{12.203}$$



**Figure 12.6**  
Decomposition of variation

Variation of noise:

$$S_N = S_T - S_B \quad (f = 6k - 1) \quad (12.204)$$

Variance of error:

$$V_e = \frac{S_e}{6k - 2} \quad (12.205)$$

Variance of noise:

$$V_N = \frac{S_N}{6k - 1} \quad (12.206)$$

SN ratio:

$$\eta = 10 \log \frac{(1/2r)(S_B - V_e)}{V_N} \quad (12.207)$$

Sensitivity:

$$S = 10 \log \frac{1}{2r} (S_B - V_e) \quad (12.208)$$

### □ Example [6]

A process used to apply an encapsulant, which provides electrical isolation between elements on the outer surface of a night-vision image intensifier tube, was optimized using double signal factor SN ratio. The ideal function was identified as

$$y = \beta \frac{M}{M^*} \quad (12.196)$$

where  $y$  is the current leakage,  $M$  the applied voltage ( $M_1 = 2$ ,  $M_2 = 4$ ,  $M_3 = 6$ ,  $M_4 = 8$ ,  $M_5 = 10$ ), and  $M^*$  the electrode spacing ( $M_1^* = 0.016$ ,  $M_2^* = 0.030$ ,  $M_3^* = 0.062$ ), Table 12.25 shows the input/output relationship of this study, and Table 12.26 shows the results of one run of the experiment.

Calculations for SN ratio and sensitivity are made as follows:

$$\begin{aligned} L_1 &= \frac{M_1}{M_1^*} y_{11} + \frac{M_1}{M_2^*} y_{12} + \dots + \frac{M_5}{M_3^*} y_{53} \\ &= \frac{2}{0.016} (9) + \frac{2}{0.030} (5) + \dots + \frac{10}{0.062} (34) \\ &= 217,781.7 \quad (12.209) \\ L_2 &= 266,009.9 \\ L_3 &= 3,687,281 \\ L_4 &= 3,131,678 \end{aligned}$$

**Table 12.25**  
Input/output table

		$M_1$	$M_2$	$M_3$	$M_4$	$M_5$
$N_1$	$M_1^*$	$y_{11}$	$y_{12}$	$y_{13}$	$y_{14}$	$y_{15}$
	$M_2^*$	$y_{21}$	$y_{22}$	—	—	$y_{25}$
	$M_3^*$	$y_{31}$	$y_{32}$	—	—	$y_{35}$
$N_2$	$M_1^*$	$y_{41}$	$y_{42}$	—	—	$y_{45}$
	$M_2^*$	$y_{51}$	$y_{52}$	—	—	$y_{55}$
	$M_3^*$	$y_{61}$	$y_{62}$	—	—	$y_{65}$
$N_3$	$M_1^*$	$y_{71}$	$y_{72}$	—	—	$y_{75}$
	$M_2^*$	$y_{81}$	$y_{82}$	—	—	$y_{85}$
	$M_3^*$	$y_{91}$	$y_{92}$	—	—	$y_{95}$
$N_4$	$M_1^*$	$y_{10-1}$	$y_{10-2}$	—	—	$y_{10-5}$
	$M_2^*$	$y_{11-1}$	$y_{11-2}$	—	—	$y_{11-5}$
	$M_3^*$	$y_{12-1}$	$y_{12-2}$	—	—	$y_{12-5}$

$$\begin{aligned}
 S_r &= y_{11}^2 + y_{12}^2 + \dots + y_{12-5}^2 \\
 &= 9^2 + 26^2 + \dots + 361^2 \\
 &= 20,974,160 \quad (f = 60)
 \end{aligned} \tag{12.210}$$

$$\begin{aligned}
 S_l &= \frac{L_1^2 + L_2^2 + L_3^2 + L_4^2}{\gamma} \\
 &= \frac{217,781.7^2 + 266,009.9^2 + 3,687,281^2 + 3,131,678^2}{1,161,051} \\
 &= 20,258,919
 \end{aligned} \tag{12.211}$$

$$\gamma_0 = 4 \tag{12.212}$$

$$\begin{aligned}
 \gamma &= \left(\frac{M_1}{M_1^*}\right)^2 + \left(\frac{M_1}{M_2^*}\right)^2 + \dots + \left(\frac{M_5}{M_3^*}\right)^2 \\
 &= \left(\frac{2}{0.016}\right)^2 + \left(\frac{2}{0.030}\right)^2 + \dots + \left(\frac{10}{0.062}\right)^2 = 1,161,051
 \end{aligned} \tag{12.213}$$

$$\begin{aligned}
 S_\beta &= \frac{(L_1 + L_2 + L_3 + L_4)^2}{4\gamma} \\
 &= \frac{(217,781.7 + 266,009.9 + 3,687,281 + 3,131,678)^2}{(4)(1,161,051)} \\
 &= 11,483,166 \quad (f = 1)
 \end{aligned} \tag{12.214}$$

**Table 12.26**

Results of experiment

		$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$L$
$N_1$	$M_1^*$	9	26	53	92	152	$L_1$
	$M_2^*$	5	13	24	42	62	
	$M_3^*$	5	9	14	23	34	
$N_2$	$M_1^*$	13	36	66	100	170	$L_2$
	$M_2^*$	8	23	44	62	90	
	$M_3^*$	6	14	24	37	45	
$N_3$	$M_1^*$	209	517	915	1452	2266	$L_3$
	$M_2^*$	165	374	622	892	1186	
	$M_3^*$	110	235	385	515	655	
$N_4$	$M_1^*$	177	457	897	1321	2113	$L_4$
	$M_2^*$	100	234	408	621	895	
	$M_3^*$	50	104	171	262	361	

$$\begin{aligned}
 S_{\beta N} &= S_L - S_\beta \\
 &= 20,258,919 - 11,483,166 \\
 &= 8,775,753 \qquad (f = 3) \quad (12.215)
 \end{aligned}$$

$$\begin{aligned}
 S_e &= S_T - S_L = S_T - S_\beta - S_{\beta N} \\
 &= 20,974,160 - 20,258,919 \\
 &= 715,241 \qquad (f = 56) \quad (12.216)
 \end{aligned}$$

$$\begin{aligned}
 S_N &= S_T - S_\beta = 20,974,160 - 11,483,166 \\
 &= 9,490,994 \qquad (f = 59) \quad (12.217)
 \end{aligned}$$

$$V_e = \frac{S_e}{56} = \frac{715,241}{56} = 12,772 \quad (12.218)$$

$$V_N = \frac{S_N}{59} = \frac{9,490,994}{59} = 160,864 \quad (12.219)$$

$$\begin{aligned}
 \eta &= 10 \log \frac{(1/\gamma_0\gamma)(S_\beta - V_e)}{V_N} \\
 &= 10 \log \frac{1}{(4)(1,161,051)} \frac{(11,483,166 - 12,772)}{160,864} \\
 &= -48.138 \text{ dB} \qquad (12.220)
 \end{aligned}$$

$$\begin{aligned}
 S &= 10 \log \frac{1}{\gamma_0 \gamma} (S_\beta - V_e) \\
 &= 10 \log \frac{1}{(4)(1,161,051)} (11,483,166 - 12,772) \\
 &= 3.93 \text{ dB} \qquad (12.221)
 \end{aligned}$$

### 12.13. Split Analysis: When There Is No Noise

The objective of parameter design is to reduce the effect of noise factors by evaluating the magnitude of their SN ratio. Therefore, noise factors must be assigned in an experiment. But sometimes it is difficult to assign noise factors. In experiments with chemical reactions, for example, taking extra data points means conducting one more experiment for a run, which doubles the time and cost of the entire experiment. Therefore, only one experiment is conducted for one run of the orthogonal array in many cases.

In such cases it is necessary to provide some type of noise for SN ratio calculation. When dynamic characteristics are used, there are signal factors. In such cases we want the input signal and the output response to be proportional. Therefore, the deviation from linearity can be used as the noise. But the number of signal factor levels is limited and the degrees of freedom for noise are limited. Generally, it is better to have more degrees of freedom for noise. Split design is used for this purpose. It is a method of calculating the SN ratio from the results obtained from an orthogonal array experiment without providing noise factors.

#### Split Analysis for Two-Level Orthogonal Array

To simplify explanation, an  $L_8$  orthogonal array is used. (In practice, however, it is recommended that one use  $L_{12}$ ,  $L_{18}$ , or  $L_{36}$  arrays.) Two-level control factors  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ , and  $G$  are assigned to the inner array. A three-level signal factor  $M$  is assigned to the outer array. Table 12.27 shows the layout.

Assume that zero-point proportional equations are used for  $M$  and  $y$ . The SN ratio of  $A_1$  is calculated from the data for runs 1, 2, 3, and 4. The data for  $A_1$  are shown in Table 12.28.

The 12 data points in Table 12.28 are decomposed as follows:

$$(f = 12) \begin{cases} S_T & (f = 3) \\ S_{\text{row}} & (f = 1) \\ S_\beta & (f = 1) \\ S_e & (f = 8) \end{cases}$$

$S_{\text{row}}$  is the variation between rows 1, 2, 3, and 4. It is the variation caused by control factors  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ , and  $G$ . This variation belongs neither to the signal nor to the noise, and therefore is separated out.

Total variation:

$$S_T = y_{11}^2 + y_{12}^2 + \cdots + y_{43} \quad (f = 1) \quad (12.222)$$

**Table 12.27**

Results without noise factors

No.	A	B	C	D	E	F	G	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	Total
1	1	1	1	1	1	1	1	y <sub>11</sub>	y <sub>12</sub>	y <sub>13</sub>	y <sub>1</sub>
2	1	1	1	2	2	2	2	y <sub>21</sub>	y <sub>22</sub>	y <sub>23</sub>	y <sub>2</sub>
3	1	2	2	1	1	2	2	y <sub>31</sub>	y <sub>32</sub>	y <sub>33</sub>	y <sub>3</sub>
4	1	2	2	2	2	1	1	y <sub>41</sub>	y <sub>42</sub>	y <sub>43</sub>	y <sub>4</sub>
Total								y <sub>-1</sub>	y <sub>-2</sub>	y <sub>-3</sub>	
5	2	1	2	1	2	1	2	y <sub>51</sub>	y <sub>52</sub>	y <sub>53</sub>	y <sub>5</sub>
6	2	1	2	2	1	2	1	y <sub>61</sub>	y <sub>62</sub>	y <sub>63</sub>	y <sub>6</sub>
7	2	2	1	1	2	2	1	y <sub>71</sub>	y <sub>72</sub>	y <sub>73</sub>	y <sub>7</sub>
8	2	2	1	2	1	1	2	y <sub>81</sub>	y <sub>82</sub>	y <sub>83</sub>	y <sub>8</sub>

Totals of M:

$$\begin{aligned}
 y_{.1} &= y_{11} + y_{21} + y_{31} + y_{41} \\
 y_{.2} &= y_{12} + y_{22} + y_{32} + y_{42} \\
 y_{.3} &= y_{13} + y_{23} + y_{33} + y_{43} \\
 y_{.4} &= y_{14} + y_{24} + y_{34} + y_{44}
 \end{aligned}
 \tag{12.223}$$

Variation of linear term:

$$S_B = \frac{1}{\gamma_0 \gamma} (M_1 y_{.1} + M_2 y_{.2} + M_3 y_{.3})^2 \quad (f = 1)
 \tag{12.224}$$

where  $\gamma_0$  is the number of data points under M:

$$\gamma_0 = 4
 \tag{12.225}$$

**Table 12.28**

Results of A<sub>1</sub>

No.	1	2	3	4	5	6	7	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	Total
1	1	1	1	1	1	1	1	y <sub>11</sub>	y <sub>12</sub>	y <sub>13</sub>	y <sub>1</sub>
2	1	1	1	2	2	2	2	y <sub>21</sub>	y <sub>22</sub>	y <sub>23</sub>	y <sub>2</sub>
3	1	2	2	1	1	2	2	y <sub>31</sub>	y <sub>32</sub>	y <sub>33</sub>	y <sub>3</sub>
4	1	2	2	2	2	1	1	y <sub>41</sub>	y <sub>42</sub>	y <sub>43</sub>	y <sub>4</sub>
Total								y <sub>-1</sub>	y <sub>-2</sub>	y <sub>-3</sub>	

Effective divider:

$$\gamma = M_1^2 + M_2^2 + M_3^2 \quad (12.226)$$

Variation between rows:

$$S_{\text{row}} = \frac{1}{k} (y_1^2 + y_2^2 + y_3^2 + y_4^2) - \frac{(y_1 + y_2 + y_3 + y_4)^2}{\gamma_0 k} \quad (f = \gamma_0 - 1) \quad (12.227)$$

where  $k$  is the number of data points in each run:

$$k = 3 \quad (12.228)$$

Error variation:

$$S_e = S_T - S_B - S_{\text{row}} \quad (f = 8) \quad (12.229)$$

Error variance:

$$V_e = \frac{S_e}{8} \quad (12.230)$$

SN ratio:

$$\eta = 10 \log \frac{(1/\gamma_0 \gamma)(S_B - V_e)}{V_e} \quad (12.231)$$

The SN ratio of  $A_2$  is calculated similarly from the results for  $A_2$ . The SN ratio of  $B_1$  is calculated from the results for  $B_1$ , as shown in Table 12.29.

### Split Analysis for Mixed Orthogonal Arrays

Calculation of the SN ratios for a three-level orthogonal array is exactly the same as that for a two-level orthogonal array. An  $L_{18}$  orthogonal array is used as an example to explain the calculations in Table 12.30.

Table 12.31 shows the layout of an experiment. A two-level control factor,  $A$ , and seven three-level control factors,  $B, C, D, E, F, G$ , and  $H$ , are assigned to an  $L_{18}$  orthogonal array. The signal factor  $M$  is assigned to outside the  $L_{18}$  array. There are no noise factors.

First, the calculation of the SN ratio of  $A_1$  is explained. The results under  $A_1$  are shown in Table 12.31.

Total variation:

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{9k}^2 \quad (f = 9k) \quad (12.232)$$

**Table 12.29**

Results of  $B_1$

No.	$M_1$	$M_2$	$M_3$	Total
1	$y_{11}$	$y_{12}$	$y_{13}$	$y_1$
2	$y_{21}$	$y_{22}$	$y_{23}$	$y_2$
5	$y_{51}$	$y_{52}$	$y_{53}$	$y_5$
6	$y_{61}$	$y_{62}$	$y_{63}$	$y_6$
Total	$y_{\cdot 1}$	$y_{\cdot 2}$	$y_{\cdot 3}$	



**Table 12.30**

Results without noise factors

No.	A	B	C	D	E	F	G	H	$M_1$	$M_2$	...	$M_k$	Total
1	1	1	1	1	1	1	1	1	$y_{11}$	$y_{12}$	...	$y_{1k}$	$y_1$
2	1	1	2	2	2	2	2	2	$y_{21}$	$y_{22}$	...	$y_{2k}$	$y_2$
3	1	1	.	.	.	.	.	.	.	.	...	.	.
4	1	2	.	.	.	.	.	.	.	.	...	.	.
5	1	2	.	.	.	.	.	.	.	.	...	.	.
6	1	2											
7	1	3											
8	1	3											
9	1	3											
10	2	1											
11	2	1											
12	2	1											
13	2	2											
14	2	2											
15	2	2											
16	2	3											
17	2	3											
18	2	3	3	2	1	2	3	1	$y_{18-1}$	$y_{18-2}$	...	$y_{18k}$	$y_{18}$

**Table 12.31**

Results of  $A_1$

No.	$M_1$	$M_2$	...	$M_k$	Total
1	$y_{11}$	$y_{12}$	...	$y_{1k}$	$y_1$
2	$y_{21}$	$y_{22}$	...	$y_{2k}$	$y_2$
3	.				
4	.				
5	.				
6	.				
7					
8					
9	$\frac{y_{91}}{y_{\cdot 1}}$	$\frac{y_{92}}{y_{\cdot 2}}$	...	$\frac{y_{9k}}{y_{\cdot k}}$	$y_9$
Total	$y_{\cdot 1}$	$y_{\cdot 2}$	...	$y_{\cdot k}$	

Column totals:

$$\begin{aligned} y_{.1} &= y_{11} + y_{21} + \dots + y_{91} \\ y_{.2} &= y_{12} + y_{22} + \dots + y_{92} \\ &\vdots \\ y_{.k} &= y_{1k} + y_{2k} + \dots + y_{9k} \end{aligned} \quad (12.233)$$

Variation of linear term:

$$S_B = \frac{1}{\gamma_0 \gamma} (M_1 y_{.1} + M_2 y_{.2} + \dots + M_k y_{.k})^2 \quad (f = 1) \quad (12.234)$$

where  $\gamma_0$  is the number of runs under  $M_1$ :

$$\gamma_0 = 9 \quad (12.235)$$

Effective divider:

$$\gamma_0 \gamma = 9(M_1^2 + M_2^2 + \dots + M_k^2) \quad (12.236)$$

Variation between rows:

$$\begin{aligned} S_{\text{row}} &= \frac{1}{k} (y_1^2 + y_2^2 + \dots + y_9^2) - \frac{1}{\gamma_0 k} (y_1 + y_2 + \dots + y_9)^2 \\ &\quad (f = \gamma_0 - 1 = 8) \end{aligned} \quad (12.237)$$

where  $k$  is the number of data points in each run.

Error variation:

$$S_e = S_T - S_B - S_{\text{row}} \quad (f = 9k - 9) \quad (12.238)$$

Error variance:

$$V_e = \frac{S_e}{9k - 9} \quad (12.239)$$

SN ratio:

$$\eta = 10 \log \frac{(1/\gamma_0 \gamma)(S_B - V_e)}{V_e} \quad (12.240)$$

The SN ratio of  $B_1$  is calculated from the results under  $B_1$ , as shown in Table 12.32.

$$S_T = y_{11}^2 + y_{12}^2 + \dots + y_{12-k}^2 \quad (f = 6k) \quad (12.241)$$

$$y_{.1} = y_{11} + y_{21} + y_{31} + y_{10-1} + y_{12-1}$$

$$y_{.2} = y_{12} + y_{22} + y_{32} + y_{10-2} + y_{11-2} + y_{12-2}$$

$\vdots$

$$y_{.k} = y_{1k} + y_{2k} + y_{3k} + y_{10-k} + y_{11-k} + y_{12-k} \quad (12.242)$$

$$S_B = \frac{1}{\gamma_0 \gamma} (M_1 y_{.1} + M_2 y_{.2} + \dots + M_k y_{.k})^2 \quad (12.243)$$

**Table 12.32**  
Results of  $B_1$

No.	$M_1$	$M_2$	...	$M_k$	Total
1	$y_{11}$	$y_{12}$	...	$y_{1k}$	$y_1$
2	$y_{21}$	$y_{22}$	...	$y_{2k}$	$y_2$
3	$y_{31}$	...			.
10	$y_{10-1}$	...			.
11	$y_{11-1}$	...	...		.
12	$y_{12-1}$	$y_{12-2}$	...	$y_{12k}$	$y_{12}$
Total	$y_{\cdot 1}$	$y_{\cdot 2}$	...	$y_{\cdot k}$	

where

$$\gamma_0 = 6 \quad (12.244)$$

$$\gamma = M_1^2 + M_2^2 + \dots + M_k^2 \quad (12.245)$$

$$S_{\text{row}} = \frac{1}{k} (y_1^2 + y_2^2 + \dots + y_{12}^2) - \frac{(y_1 + y_2 + \dots + y_{12})^2}{\gamma_0 k} \quad (12.246)$$

$(f = \gamma_0 - 1 = 5)$

$$S_e = S_T - S_B - S_{\text{row}} \quad (f = 9k - 9) \quad (12.247)$$

$$V_e = \frac{S_e}{9k - 9} \quad (12.248)$$

$$\eta = 10 \log \frac{(1/\gamma_0 \gamma)(S_B - V_e)}{V_e} \quad (12.249)$$

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