9 Specification Tolerancing

9.1. Introduction

In traditional quality control, the fraction defective of a manufacturing process is considered to be the quality level of the product. When a defective product is discovered, it is not shipped. So if all products are within specifications, are these products perfect? Obviously they are not, because many of the products shipped, considered as being good or nondefective, have problems in the marketplace. Problems occur due to lack of robustness against a user's conditions. The only responsibility that a manufacturer has is to ship nondefective products, that is, products within specifications. Defective products cause a loss to the company. In other words, it is not a quality problem but a cost problem. It is important to realize that the tolerances, the upper and lower limits, stated in the drawings are for inspection purposes, *not* for quality control.

Again, as in school, any mark above 60 is a passing grade, with a full mark being 100. A student passing with a grade of 60 is not perfect and there is little difference between a student with a grade of 60 and one with a grade of 59. It is clear that if a product is assembled with the component parts marginally within tolerance, the assembled product cannot be a good one.

When troubles occur from products that are within tolerance, the product design engineer is to be blamed. If robust designs were not well performed and specifications were not well studied, incorrect tolerances would come from the design department.

Should the tolerance be tightened? If so, the loss claims from the market could be reduced a little. But then manufacturing cost increases, giving the manufacturer a loss and ending up with an increased loss to society.

 Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

Although the determination of tolerance is important, there have been no methods with a sound basis. A safety factor such as 4 has been widely used but not for sound reasons. From Taguchi's definition, *quality is the loss that a product imparts to society after the product is shipped*. The quality problem for a manufacturing process is in taking countermeasures to minimize the loss to society.

However, it is difficult to actually measure the societal loss associated with a product. We might produce 1000 units of product, distribute them to various type of customers, and let the customers use the product for the designed life, measure the loss due to problems, and take the average. Such an approach is realistically impossible. Even if it were possible, after the designed year is passed, the product would become obsolete. It is therefore necessary to forecast the quality level. The quality loss function provides a basis for the determination of tolerance.

In quality engineering, tolerance is not the deviation between products. Tolerance is defined as *a deviation from target*. Tolerance is determined so that the loss caused by the manufacturer and the one caused by the customer are balanced.

❐ Example

To determine a specification, it is necessary to determine two values: functional tolerance and customer loss. For every product characteristic we can find a value at which 50% of customers view the product as not functioning. This value represents an *average customer viewpoint* and is referred to as the *functional tolerance* or LD_{50} (denoted as Δ_0). The average loss occurring at LD_{50} is referred to as the *customer loss, A_0. The functional tolerance and consumer loss are required to establish a loss function*.

Let us set up the loss function for a color TV power supply circuit where the target value of y (output voltage) is $m = 115$ V. Suppose that the average cost for

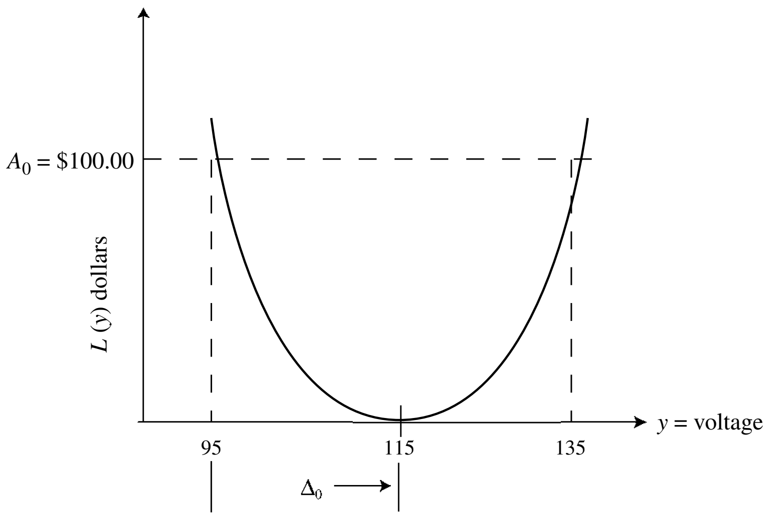

Figure 9.1
Loss function for TV power supply

repairing the color TV is \$100. This occurs when y goes out of the range 115 ± 20 V in the hands of the consumer (Figure 9.1). We see that L = \$100 when y = 95 or 135 V, consumer tolerance $\Delta_0 = \pm 20$ V, and consumer loss A_0 = \$100. Then we have

$$L(y) = k(y - m)^2 \tag{7.4}$$

$$A_0 = k\Delta_0^2 \tag{9.1}$$

$$k = \frac{A_0}{\Delta_0^2} = \frac{\$100}{(20\text{ V})^2} = \$0.25 \text{ per volt}^2 \tag{9.2}$$

The loss function can now be rewritten as

$$L = 0.25(y - 115)^2 \qquad \text{dollars/piece} \tag{9.3}$$

When the output voltage becomes 95 or 135 V, somebody is paying \$100. As long as the output is 115 V, society's financial loss is minimized.

With the loss function established, let's look at its various uses. Suppose that a circuit was shipped with an output of 110 V. It is imparting a loss of

$$L = \$0.25(110 - 115)^2 = \$6.25 \tag{9.4}$$

This means that on the average, someone is paying \$6.25. This figure is a rough approximation of loss imparted to society due to inferior quality (Figure 9.2).

With information about the after-shipping or consumer tolerance, we can calculate the prior-to-shipping or manufacturing tolerance. *The manufacturing tolerance is the economical break-even point for rework for scrap*.

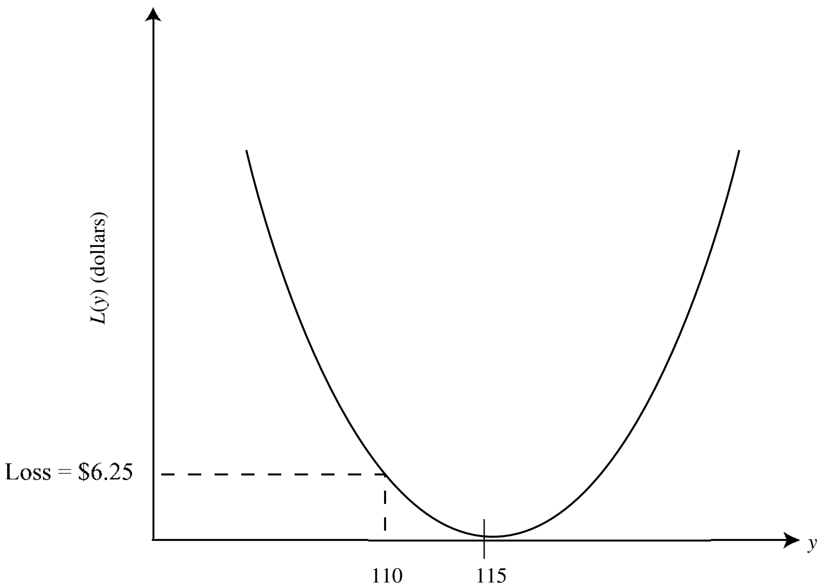

Figure 9.2
Loss function for 110-V circuit

Suppose that the output voltage can be recalibrated to target at the end of the production line at a cost of \$2. What is the manufacturing tolerance? Stated differently, at what output voltage should the manufacturer spend \$2 to fix each set?

The manufacturing tolerance, Δ, is obtained by setting L at \$2.

$$L(y) = 0.25(y - m)^2 \tag{9.5}$$

$$\$2 = \$0.25(y - 115)^2 \tag{9.6}$$

$$\begin{aligned} y &= 115 \pm \sqrt{\frac{2.00}{0.25}} \\ &= 115 \pm \sqrt{8} \\ &= 115 \pm 2.83 \\ &\approx 115 \pm 3 \text{ V} \end{aligned} \tag{9.7}$$

$$\begin{aligned} A &= k\Delta^2 \\ &= \frac{A_0}{\Delta_0^2}\Delta^2 \end{aligned} \tag{9.8}$$

or

$$\frac{A}{A_0}\Delta_0^2 = \Delta^2 \tag{9.9}$$

$$\Delta = \sqrt{\frac{A}{A_0}}\,\Delta_0 \tag{9.10}$$

As long as y is within 115 ± 3, the factory should not spend \$2 for rework because the loss without the rework will be less than \$2. The manufacturing tolerance sets the limits for shipping a product. It represents a break-even point between the manufacturer and the consumer. Either the customer or the manufacturer can spend \$2 for quality (Figure 9.3).

Suppose that a circuit is shipped with an output voltage of 110 V without rework. The loss is

$$L(y) = \$0.25(110 - 115)^2 = \$6.25 \tag{9.11}$$

The factory saves \$2 by not reworking, but it is imparting a loss of \$6.25 to society. This loss becomes apparent to the manufacturer through customer dissatisfaction, added warranty costs, consumer's expenditure of time and money for repair, damaged reputation, and long-term loss of market share.

9.2. Safety Factor

As described before, a safety factor of 4 or 5 has often been used in industry. In most cases it is determined by considering the technological possibilities or the frequencies of actual problem occurrence. More specifically, special research was conducted when a high safety factor was needed in an especially demanding area

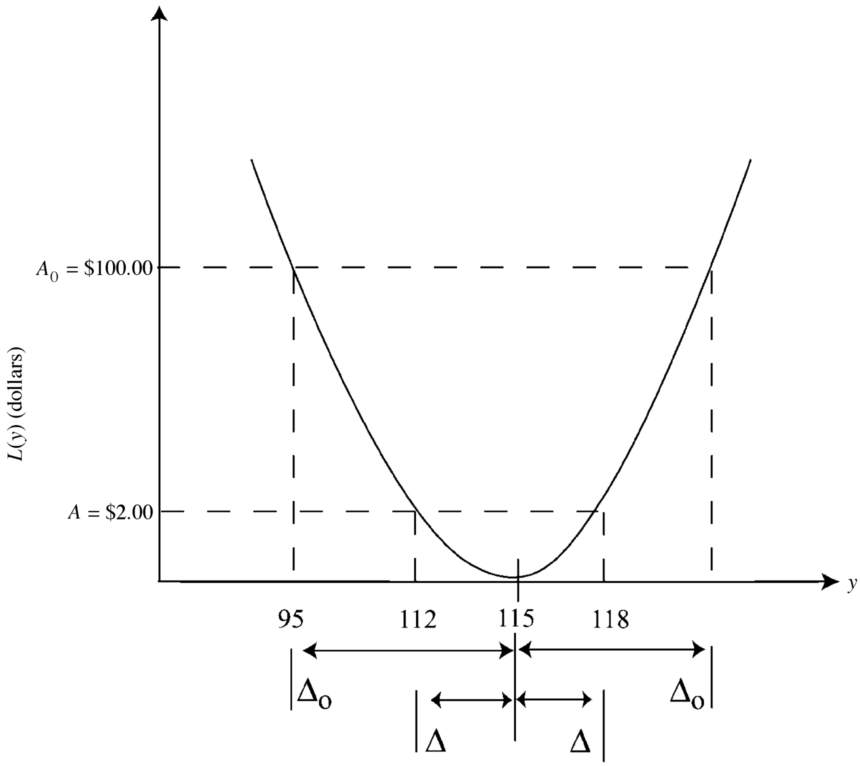

Figure 9.3
Tolerancing using the loss function

(military, communication), and a higher level of safety was attained through technological advancement.

In advanced countries where such high-precision products can be produced, those new technologies were applied for general products, and tolerances for general products have become more and more stringent. In Japan, industrial standards (JIS) are reviewed every five years, and of course, tolerances are included in the list of reviews.

Again, let us denote the function limit as Δ_0. Δ_0 represents the point at which the function actually fails. It is determined as the point where the function fails due to the deviation of y by Δ_0 from m, assuming that other characteristics and operating conditions are in the standard (nominal value) state.

Determination of the function limit, assuming that the other conditions are in the standard state, means that the other conditions are in the normal state. The normal state means the median or mode condition, or where half of the conditions are more severe and the other half are less severe. It is abbreviated as LD_{50} (lethal dosage). In most cases, a test for determining the function limit may be conducted appropriately only under standard conditions.

The value of Δ_0 can easily be determined. For example, the LD_{50} value is determined by finding the voltage at which ignition fails by varying the voltage from the mean value of 20 kV of the ignition voltage of an engine. Suppose that the ignition fails at 8 kV when the voltage is lowered under the standard operating conditions; the lower function limit is 8 kV − 20 kV = −12 kV. On the other hand, the upper function limit is determined where problems begin to occur by increasing the output voltage. When the voltage is high, failures occur due to corona discharge, and so on. Let's assume that the function limit is 20 kV, for example. When function limits are different above and below the standard conditions, different tolerance values may sometimes be given. The tolerances in this case are as follows:

$$m \begin{matrix} -\Delta_1 \\ \\ +\ \Delta_2 \end{matrix} \qquad (9.12)$$

Tolerance values Δ_1 and Δ_2 are determined separately for above and below the upper and lower function limits, Δ_{01} and Δ_{02}, with separate safety factors, ϕ_1 and ϕ_2, respectively.

$$\Delta_i = \frac{\Delta_{0i}}{\phi_i} \qquad (i = 1,2) \qquad (9.13)$$

The function limit Δ_0 is often determined by experiments or calculations. However, the determination of a safety factor is based on experience and is unclear. Therefore, recently some people have begun to determine the tolerance by introducing the concept of probability. The author believes that the probability method is not a good approach. One of the reasons is that a safety factor of about 1000 with respect to LD_{50} is adopted in cases of side effects of drugs and harmful contents in food. How can the people who deal with probability theory determine the probability? It is likely that probability theory is useless in such cases. The form of the distribution is unknown, not to mention probability calculations.

The following equation was proposed for the safety factor ϕ, which is now beginning to be accepted:

$$\phi = \sqrt{\frac{\text{average loss when the function limit is exceeded}}{\text{loss at the factory when the factory standard is not met}}} \qquad (9.14)$$

A_0 and A are used for the numerator and denominator under the square root in equation (9.14), where A_0 is the average loss when the function limit is exceeded (after shipping) and A is the loss at the factory when the factory standard is not met (sale price or adjustment cost).

Therefore, equation (9.14) is written as

$$\phi = \sqrt{\frac{A_0}{A}} \qquad (9.15)$$

Let us explain this reason first. Let the characteristic value be y and the target value be m. The loss due to deviation of y from the target value m is considered here. The actual economic loss is $L(y)$ when a product with a characteristic value, y, is shipped and used under various conditions. N is the size of the total market where the product can potentially be used. $L_i(t,y)$ is the actual loss occurring at location i after t years.

$L_i(t,y)$ is equal to zero in most cases. If the function fails suddenly, an actual loss is incurred. The average loss $L(y)$ caused by using the product at all locations during the entire period of the design life is

$$L(y) = \frac{1}{N}\sum_{i=1}^{n}\int_0^T L_i(t,y)\ dt \tag{9.16}$$

Although each $L_i(t,y)$ is a discontinuous function, $L(y)$ becomes approximately a continuous function when it is averaged for the losses of many individuals. As mentioned before, the loss function above is approximated by a quadratic equation and expressed by the following equations using the function limit Δ_0 and the loss A_0 caused by failure of the function.

$$L(y) = \frac{A_0}{\Delta_0^2}(y - m)^2 \tag{9.17}$$

$(y - m)$ in equation (9.17) is the deviation from the target value. Assuming that the tolerance level of the deviation is the tolerance Δ and A is the value of the point where quality and cost balance in the tolerance design, equation (9.17) yields

$$A = \frac{A_0}{\Delta_0^2}\Delta^2 \tag{9.18}$$

$$\Delta = \sqrt{\frac{A}{A_0}}\,\Delta_0 = \frac{\Delta_0}{\phi} \tag{9.19}$$

$$\phi = \sqrt{\frac{A_0}{A}} \tag{9.20}$$

$\phi = \sqrt{A_0/A}$ is called the *safety factor*.

❒ Example

On January 5, 1988, an illumination device weighing 1.6 tons fell in a disco-dancing hall in Tokyo. Three youths died and several were injured. The device was hung with six wires, but these wires could stretch easily, so when the chain used for driving the device broke, the device fell.

The design was such that the wires stretched and the device fell when the chain broke. The main function of the chain was to move the device up and down. Because of its design, when the chain broke and stopped functioning, human lives were threatened. The tensile strength of the chain had a safety factor of 4 with respect to the weight of the illuminating device, 1.6 tons. Two chains with a tensile strength of 3.2 tons had been used. A safety factor of 4 to 5 is adopted in many corporations in the United States and quite a few Japanese companies have adopted the same value.

Because the weight of the illuminating device was 1.6 tons, the function limit, Δ_0, of the chain was apparently 1.6 tons_f. On average, several people are located in the area underneath the illuminating device at any time. Assuming that the loss caused by the death of one person is \$1,550,000 and that four people died, the loss caused by losing the suspension function is

$$A_0 = (1{,}550{,}000)(4) = \$6{,}200{,}000 \tag{9.21}$$

By applying a larger-the-better characteristic, the loss function for strength y of the chain is as follows:

$$L = \frac{A_0 \Delta_0^2}{y^2} = \frac{(\$6{,}200{,}000)(1.6^2)}{y^2} \tag{9.22}$$

When the safety factor is 4, $y = (1.6)(4) = 6.4$ tons$_f$. This leads to a poor quality level, as follows:

$$\begin{aligned} L &= \frac{(\$6{,}200{,}000)(1.6^2)}{6.4^2} \\ &= \$387{,}500 \end{aligned} \tag{9.23}$$

If the price of the chain were \$387,500, the price and quality would have been balanced. Actually, a chain costs about \$1000, or \$2000 for two. Therefore, the loss of quality that existed in each chain was about 190 times the price. If deterioration of the chain were considered, the loss of quality would be even larger. Therefore, the problem existed in selection of the safety factor.

In this case, the graver error was the lack of safety design. Safety design is not for improvement of the functioning rate or reliability. It is for reducing the loss to merely the repair cost in the case of malfunction. Some examples of safety design: At JR (Japan Railroad), design is done so that a train will stop whenever there is any malfunction in the operation of the train. At NTT, there is a backup circuit for each communication circuit, and when any type of trouble occurs, the main circuit is switched to the backup circuit; customers feel no inconvenience. Instead of allowing a large safety factor for prevention of a breakdown of the chain, the device in the disco-dancing hall should have had shorter wires so that it would have stopped above the heads of the people if the chain did break. In this case, the safety design could have been done at zero cost. If no human lives were lost, the loss caused by the breakdown of the chain would probably be only about \$10,000 for repair. The loss in this case would be

$$\begin{aligned} L &= \frac{(\$10{,}000)(1.6^2)}{6.4^2} \\ &= \$625 \end{aligned} \tag{9.24}$$

This is less than the price of the chains, \$2000. Therefore, the quality of chains having 6.4 tons$_f$ strength is excessive, and even cheaper chains may be used.

Table 9.1 compares the loss with and without safety design. For example, when there is no safety design and one chain is used, the safety factor is equal to 2. Since this is the case of a larger-the-better characteristic, the equality is calculated as

$$L = \frac{A_0 \Delta_0^2}{y^2} = \frac{(\$6{,}200{,}000)(1.6^2)}{(1.6 \times 2)^2} = \$1{,}550{,}000 \tag{9.25}$$

Table 9.1
Loss function calculation of chain (×$10)[a]

Number of chains	Price	No Safety Design		With Safety Design	
		Quality	Total	Quality	Total
1	100	155,000	155,100	250	350
2	200	38,750	38,950	60	*260
3	300	17,220	17,520	30	330
4	400	9,690	10,090	20	420
6	500	4,310	4,910	7	607
8	800	2,420	3,220	4	804
10	1,000	1,550	2,550	3	1,003
12	1,200	1,080	2,280	2	1,202
14	1,400	790	*2,190		
15	1,500	610	2,200		

[a] Asterisks denote an optimum solution.

Since the cost of a chain is $1000, the total loss is

$$1{,}550{,}000 + 1000 = \$1{,}551{,}000 \qquad (9.26)$$

When there is safety design using one chain, the loss, A_0, is equal to a repair cost of $10,000. The quality is calculated as

$$A_0\Delta_0^2 = (10{,}000)(1.6^2)$$

$$L = \frac{A_0\Delta_0^2}{[(1.6)(2)]^2} = \frac{(10{,}000)(1.6^2)}{[(1.6)(2)]^2} = \$2500 \qquad (9.27)$$

Adding the cost of the chain, the total loss is

$$2500 + 1000 = \$3500 \qquad (9.28)$$

This calculates the sum of price and quality for cases with and without safety design corresponding to the number of chains (1,2,...). In actual practice, a factor 2 or 3 is often given to the purchase price, considering the ratio of purchase price to the sales price and the interest, because the price is the first expenditure for a consumer. Regarding the deterioration of quality, it is only necessary to think about the mode condition, or the most possible condition. Although it is usually better to consider the deterioration rate in the mode condition, it is ignored here because the influence of deterioration is small.

As we can see, the optimum solution for the case without a safety design is 14 chains and $14,000 for the price of the chains. For the case with a safety design, the solution is two chains and $2000 as the cost of the chains. The total loss that indicates productivity is $21,900 and $2600 for the optimum solutions, respectively. Therefore, the productivity of the design with a safety design included is 8.4 times higher.

9.3. Determination of Low-Rank Characteristics

Usually, the purchaser gives the specifications for parts and materials to the parts manufacturers or materials manufacturers. For finalproduct manufacturers, the planning section usually gives the standards of a product. Regarding these objective characteristics, parameter design and tolerance design consider how to determine the standard for the characteristics of the cause. Parameter design shows how to obtain the level of the parameter for the cause of low-rank characteristics. That is explained following Chapter 10, where the method of determining the central value is discussed. Now we discuss how to determine tolerance assuming that the central value of the low-rank characteristics has been determined.

When the shipping standards of characteristics of a product are given, the shipping standards are the high-rank characteristics with respect to the characteristics of subsystems and parts used in the product. When the characteristic values of parts and materials influence the characteristic values of a subsystem, the level of the characteristic values of the subsystem is high with respect to the characteristic values of the parts and materials. If the characteristic values of the product shipped from a company influence the characteristic values of the worked products of a purchaser, the characteristic values of the purchaser are high-rank characteristic values.

❒ Example

A stamped product is made from a steel sheet. If the shape after press forming is defective, an adjustment is needed at a cost, A_0, of $12. The specification for a pressed product dimension is $m \pm 300$ μm. The dimension of the product is affected by the hardness and thickness of the steel sheet. When the hardness of steel sheet changes by a unit quantity (in Rockwell hardness H_R), the dimension changes by 60 μm. When the thickness of the steel sheet changes by 1 μm, the dimension changes by 6 μm.

Let's determine the tolerance for the hardness and thickness of the steel sheet. When either the hardness or thickness fails to meet the specifications, the product is scrapped. The loss is $3 for each stamped product.

The method of determining the tolerance of the cause or the low-rank characteristics is to establish an equation for a loss function for the high-rank characteristics and to convert it to a formula for the characteristics of the cause. When the standard of the high-rank characteristic is $m_0 + \Delta_0$ and the loss for failing to meet the standard is A_0, the loss function is given by the following equation, where the high-rank characteristic is y.

$$L = \frac{A_0}{\Delta_0^2}(y - m_0)^2 \tag{9.29}$$

For a low-rank characteristic x, the right-hand side of equation (9.29) becomes the following, due to the linear relationship between y and x, where b is the influence of a unit change of x to the high-rank characteristic y:

$$\frac{A_0}{\Delta_0^2}[b(x - m)]^2 \tag{9.30}$$

where m is the central value of the low-rank characteristic x. Substituting this expression into the right-hand side of equation (9.29), and substituting the loss, A dollars, due to rejection by the low-rank characteristic for L on the left-hand side, we get

$$A = \frac{A_0}{\Delta_0^2}[b(x - m)]^2 \tag{9.31}$$

By solving this equation for $\Delta = x - m$, the tolerance, Δ, for the low-rank characteristic, x, is given by

$$\Delta = \sqrt{\frac{A}{A_0}}\,\frac{\Delta_0}{b} \tag{9.32}$$

The parameters are as follows:

A_0: loss due to high-rank characteristics (objective characteristics) failing to meet the standard

Δ_0: tolerance for the high-rank characteristics

A: loss due to the low-rank characteristics failing to meet the standard

b: the influence of a unit change in the low-rank characteristics to the high-rank characteristics

Both A_0 and A are losses for a product failing to meet the standard found by the purchaser or in the factory of the company. The cost for inspection is not included here. A_0 is the loss for adjustment in the case when the shipping standard is not satisfied in the company. In the case of high-rank characteristics of the purchaser, the loss occurs when the standard is not satisfactory to the purchaser.

In the case of hardness, A_0 is the loss when the purchaser finds a defective shape.

A_0: \$12

A: \$3

Δ_0: 300 μm

b: 60 μm/H_R

Therefore, the tolerance for hardness is

$$\Delta = \sqrt{\frac{3}{12}}\left(\frac{300}{60}\right) = 2.5H_R \tag{9.33}$$

Accordingly, the tolerance of hardness for shipping is $m \pm 2.5H_R$. Similarly, the tolerance, Δ, of the thickness is derived as follows, where A_0, A, and Δ_0 are the same as before and $b = 6$ μm/μm:

$$\Delta = \sqrt{\frac{3}{12}}\left(\frac{300}{6}\right) = 25.0\ \mu m \tag{9.34}$$

Therefore, the tolerance of the thickness of the product shipped is ±25.0 μm. In this case, the average of the thickness is assumed to agree with the target value of the dimension after press forming. In this example, the purchaser or the press former is the one who determines the tolerance of the dimension.

In many cases, the relationship between the low- and high-rank characteristics can be approximated by a linear function. When this is not possible, a graphic approach is employed. For example, the influence of the change of a low-rank characteristic to the objective characteristic is found to be as shown in Figure 9.4. To study this relationship, it is important to keep other low-rank characteristics and environmental conditions constant.

The function limits, Δ_{10} and Δ_{20}, of the low-rank (cause) characteristic, x, are determined from the point where the high-rank (objective) characteristic, y, crosses the upper and lower limits. These are the points where the objective characteristic, y, exceeds the tolerance under the condition that all the other low-rank characteristics are in the standard state. When the upper and lower tolerance limits of x are to be determined separately, the lower tolerance limit, Δ_1, and the upper tolerance limit, Δ_2, are

$$\Delta_1 = \frac{\Delta_{10}}{\phi} \qquad \phi_1 = \sqrt{\frac{A_0}{A_1}} \tag{9.35}$$

$$\Delta = \frac{\Delta_{20}}{\phi} \qquad \phi_1 = \sqrt{\frac{A_0}{A_2}} \tag{9.36}$$

A_0 is the loss when the high-rank characteristic, y, exceeds the tolerance, A_1 is the loss when the low-rank characteristic, x, does not satisfy the lower limit for x

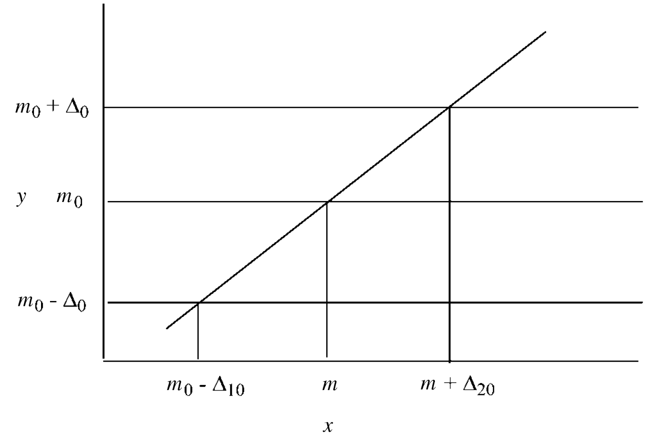

Figure 9.4
Relationship between low- and high-rank characteristics

(usually, parts and materials cannot be repaired; the loss is their price) A_2 is the loss when x exceeds the upper limit for x. ϕ_1 and ϕ_2 are safety factors. Because it is cumbersome to give different tolerances for above and below the standard, generally

$$\Delta_x = \min(\Delta_1, \Delta_2) \tag{9.37}$$

is used and the specification for x is given by $m \pm \Delta_x$, where Δ_x is the tolerance for x:

9.4. Distribution of Tolerance

Let's assume that there are k low-rank characteristics of parts and materials that influence the objective characteristics of assembled products. When the price of the ith part (more exactly, the loss when the i-th characteristic fails to meet the specification) is A_i (i = 1,2, ..., k) and the tolerance for each part is Δ_i, the variability range, Δ, of the objective characteristic, y, is given by equation (9.38). This is so because Δ_i can be calculated from equation (9.32) by assuming the additivity of variability.

$$\Delta^2 \frac{A_1 + A_2 + \cdots + A_k}{A_0} \Delta_0^2 \tag{9.38}$$

There are different relationships between A_0, which is the loss caused by the characteristics of an assembled product being unable to meet the tolerance, and the total sum of the price of component parts ($A_1 + \cdots + A_k$).

$$\frac{A_1 + A_2 + \cdots + A_k}{A_0} << 1 \tag{9.39}$$

In the case of equation (9.39), if an assembled product fails to meet the standard, it must be discarded. Usually, the loss caused by rejection of an assembled product is several times the total of the price of the component parts. If it is four times, the process capability index of the characteristics of the assembled product will be twice that of the component parts. Of course, there is no rejection in such a case.

$$\frac{A_1 + A_2 + \cdots + A_k}{A_0} >> 1 \tag{9.40}$$

With equation (9.40), the assembled product can be fixed or adjusted at a low cost. There may be many rejections, but it is all right because the cost for adjustment is low.

$$\frac{A_1 + A_2 + \cdots + A_k}{A_0} \approx 1 \tag{9.41}$$

The case of equation (9.41) seldom occurs. The cost for adjustment is just equal to the total sum of the price of parts. Distribution of the tolerance is justified only in this case.

Generally speaking, it is wrong to distribute the tolerance to each characteristic of a part so that the characteristic of the assembled product falls within the tolerance limit around the target value. The best way is to determine tolerances separately for each part, as shown in this chapter. The most common situations are equations (9.39) and (9.40).

9.5. Deteriorating Characteristics

By definition, when the standard value of the high-rank characteristic fails to meet the standard at $m_0 \pm \Delta_0$, the loss is A_0 dollars.

The tolerance, Δ, of the initial value of the low-rank characteristic, x, which affects the high-rank characteristic, y, at a coefficient b for a change of unit value is given by the following equation, as was also shown in equation (9.32):

$$\Delta = \sqrt{\frac{A}{A_0}}\,\frac{\Delta_0}{|b|} \tag{9.42}$$

where A is the loss due to the low-rank (part) characteristic failing to meet the specification, A_0 the loss due to the high-rank characteristic failing to meet the specification, Δ_0 the tolerance for the high-rank characteristic, and $|b|$ is the change in the high-rank characteristic due to a unit change in the low-rank characteristic.

The loss function for deterioration per year is obtained by calculating the variance due to deterioration. The variance, δ^2, of the objective characteristic is obtained by considering the deterioration of the low-rank characteristic in T years:

$$\begin{aligned}\sigma^2 &= \frac{1}{T}\int_0^T (b\beta t)^2\,dt\\ &= \frac{1}{T}\left(\frac{b^2\beta^2 t^3}{3}\right)_0^t\\ &= \frac{b^2\beta^2 T^2}{3}\end{aligned} \tag{9.43}$$

where β is the coefficient of deterioration. Therefore,

$$L = \frac{A_0}{\Delta_0^2}\frac{b^2\beta^2T^2}{3} \tag{9.44}$$

If the tolerance for the deterioration in one year is Δ*,

$$\Delta^* = \sqrt{\frac{3A^*}{A_0}}\,\frac{\Delta_0}{bT} \tag{9.45}$$

by substituting Δ* in β. A_0 and Δ_0 are the same as above, and A^* and T are defined as follows:

A^*: loss due to the low-rank characteristic failing to meet the standard

T: design life (years)

❒ Example

A quality problem occurs when a luminance changes by 50 lx, and the social loss for repair is $150:

Δ_0: 50 lx

A_0: $150

When the luminous intensity of a lamp changes by a unit amount of 1 cd in the manufacturing process, the illuminance changes by 0.8 lx. When the initial luminous intensity of the lamp fails to meet the specification, it can be adjusted by a cost of A − $3. When the rejection is due to deterioration, the lamp is discarded, and the loss, A^*, is $32.

If the design life is 20,000 hours,

b: 0.8 lx/cd

A: $3

A^*: $32

T: 20,000 hours

Using equations (9.42) and (9.45), the tolerance Δ for the initial luminous intensity and the tolerance Δ* for deterioration are given as follows:

$$\Delta = \sqrt{\frac{A}{A_0}}\,\frac{\Delta_0}{|b|} = \sqrt{\frac{3}{150}}\left(\frac{50}{0.8}\right) = 8.8 \text{ cd} \tag{9.46}$$

$$\begin{aligned}\Delta^* &= \sqrt{\frac{3A^*}{A_0}}\,\frac{\Delta_0}{|\beta|}\,\frac{1}{T} \\ &= \sqrt{\frac{(3)(32)}{150}}\left(\frac{50}{0.8}\right)\left(\frac{1}{20{,}000}\right) = 0.00225 \text{ cd}\end{aligned} \tag{9.47}$$

Therefore, the tolerance for the initial luminous intensity of the lamp is +8.8 cd, and the tolerance for the coefficient of deterioration is less than 0.00225 cd/h. Even if the lamp has sufficient luminous intensity, the loss due to cleaning must be considered if the luminance is lost rapidly because of stains on the lamp.

In Chapter 10, we discuss tolerance design in more detail.