8 Quality Loss Function for Various Quality Characteristics

8.1. Classification of Quality Characteristics

Quality loss function is used for the nominal-the-best, smaller-the-better, larger-the-better characteristics. The *nominal-the-best* characteristic is the type where there is a finite target point to achieve. There are typically upper and lower specification limits on both sides of the target. For example, the plating thickness of a component, the length of a part, and the output current of a resistor at a given input voltage are nominal-the-best characteristics.

A *smaller-the-better* output response is the type where it is desired to minimize the result, with the ideal target being zero. For example, the wear on a component, the amount of engine audible noise, the amount of air pollution, and the amount of heat loss are smaller-the-better output responses. Notice that all these examples represent things that we do not want, not the intended system functions. In the smaller-the-better characteristic, no negative data are included.

The *larger-the-better* output response is the type where it is desired to maximize the result, the ideal target being infinity. For example, strength of material, and fuel efficiency are larger-the-better output responses. Percentage yield seems to be the larger the better, but it does not belong to the larger-the-better category in quality engineering, since the ideal value is 100%, not infinity. In the larger-the-better characteristic, negative data are not included.

8.2. Nominal-the-Best Characteristic

To demonstrate the criteria for quality evaluation, consider Figure 8.1, which contains the frequency distribution curves for TV set color density. One curve

 Taguchi's Quality Engineering Handbook. Genichi Taguchi, Subir Chowdhury and Yuin Wu

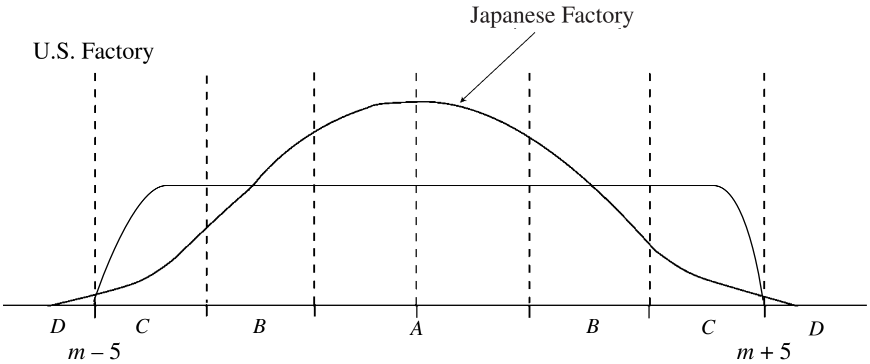

Figure 8.1
Distribution of color density in TV sets

represents the color density frequency distribution associated with sets built in Japan, and the other curve represents the same distribution for sets built in the United States. The two factories belong to the same manufacturing company.

In 1979, an article appeared in *The Asahi* (a newspaper) relative to the preference of American consumers for TV sets built by a company in Japan. Apparently, identical subsidiary plants had been built in the United States and in Japan. Both facilities were designed to manufacture sets for the U.S. market and did so using identical designs and tolerances. However, despite the similarities, the American consumer displayed a preference for sets that had been made in Japan.

Referring again to Figure 8.1, the U.S.-built sets were defect-free; that is, no sets with color density out of tolerance were shipped from the San Diego plant. However, according to the article and as can be seen in Figure 8.1, the capability index, C_p, of the Japan-built sets was $C_p = 1.0$ and represented a defect rate of 3 per 1000 units. Why, then, were the Japanese sets preferred?

Before answering that question, it is important to note that:

- ❑ Conformance to specification limits is an inadequate measure of quality or of loss due to poor quality.
- ❑ Quality loss is caused by customer dissatisfaction.
- ❑ Quality loss can be related to product characteristics.
- ❑ Quality loss is a financial loss.

Traditional methods of satisfying customer requirements by controlling component and/or subsystem characteristics have failed. Consider Figure 8.2, which represents the inspection-production-oriented traditional concept, whereby all products or processes that exist or function within some preestablished limits are considered to be equally good, and all products or processes that are outside these limits are considered to be equally bad. The fallacy in this type of judgment criteria was illustrated in the example where although all of the sets manufactured in the United States were within specifications, the customer requirements were apparently better satisfied by the sets built in Japan.

Taguchi compared specification limits to pass/fail criteria often used for examinations. In school, for example, 60% is generally considered to be a passing grade, but the difference between a 60% student who passes the course and a 59%

Figure 8.2
Conformance to requirements

student who fails is nil. If 100 points represent perfection, the student with 60 points is more like the student with 59 than he or she is like the perfect student. Similarly, the manufactured product that barely conforms to the specification limits is more similar to a defective part than to a perfect part.

Specification limits only provide the criteria for acceptance and/or rejection. A product that just barely conforms to some preestablished limits functions relative to optimum just as the 60% student functions relative to perfection. For some customers, such deviation from optimum is not acceptable. In Figure 8.1, the quality characteristic has been graded depending on how close its value is to the target or best value denoted by m. The figure shows that the majority of the sets built in Japan were grade A or B, whereas the sets built in the United States were distributed uniformly in categories A, B, and C.

The grade-point average of the TV sets built in Japan was higher than that of the U.S.-built sets. This example rejects the quality evaluation criteria that characterize product acceptance by inspection with respect to specification limits. How, then, can quality be evaluated? Since perfection for any student is 100%, loss occurs whenever a student performs at a lower level. Similarly, associated with every quality characteristic is a best or target value: that is, a best length, a best concentricity, a best torque, a best surface finish, and so on. Quality loss occurs whenever the quality characteristic deviates from this best value. Quality should, therefore, be evaluated as a function of deviation of a characteristic from target.

If a TV color density of m is best, quality loss must be evaluated as a function of the deviation of the color density from this m value. This new definition of product quality is the uniformity of the product or component characteristics around the target, not the conformity of these characteristics to specification limits. Specification limits have nothing to do with quality control. As quality levels increase and the need for total inspection as a tool for screening diminishes, specifications become the means of enunciating target values. Associated with every product characteristic, however, there exists some limit outside of which the product, as viewed by 50% of the customers, does not function. These limits are referred to as the *customers' tolerance*.

The rejection of quality assurance by means of inspection is not new. The Shewhart concept also rejects quality assurance by inspection and instead tries to assure quality by control methods. From an economic standpoint, however, this concept has, a weakness. If the cost for control is larger than the profit realized from the resulting reduced variation, we should, of course, do nothing. In other words, it is the duty of a production department to reduce variation while maintaining the profit margin required.

More Than One Piece

Suppose that four factories are producing the same product under the same engineering specifications. The target values desired are denoted by m. The outputs are as shown in Figure 8.3. Suppose further that the four factories carry out 100% inspection and ship out only pieces within specification limits.

Although the four factories are delivering products that meet specifications, factory 4 offers more products at or near the desired target value and exhibits less piece-to-piece variability than the other factories. Factory 4 is likely to be selected as the preferred vendor. If a person selects factory 4, it is probably because he or she believes in the loss function but being within specifications is not the entire story.

The loss function, L, as described previously, is used for evaluating 1 unit of product:

$$L = k\,(y - m)^2 \tag{7.4}$$

where k is a proportionality constant and y is the output. More often, the qualities of all pieces of products are evaluated. To do this, the average of $(y - m)^2$, which is called the *mean-squared deviation* (MSD), is used. When there is more than one piece of product, the loss function is given by

$$L = k(\text{MSD}) \tag{8.1}$$

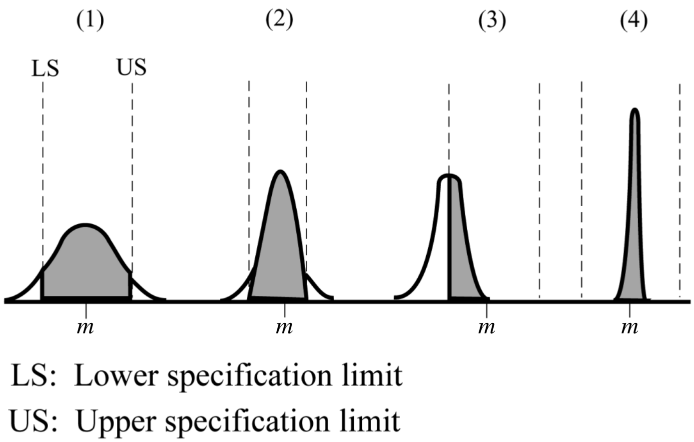

Figure 8.3
Outputs for four factories

For n pieces of products with output $y_1, y_2, \ldots, y_n$, the average loss is

$$L = \frac{k(y_1 - m)^2 + k(y_2 - m)^2 + \cdots + k(y_n - m)^2}{n}$$

$$= k\,\frac{(y_1 - m)^2 + (y_2 - m)^2 + \cdots + (y_n - m)^2}{n} \tag{8.2}$$

$$\text{MSD} = \frac{1}{n}\sum_{i=1}^{n}(y_i - m)^2$$

$$= \frac{1}{n}\sum_{i-1}^{n}(y_i - \bar{y})^2 + (\bar{y} - m)^2$$

$$= \frac{(y_1 - \bar{y})^2 + \cdots + (y_n - \bar{y})^2}{n} + (\bar{y} - m)^2$$

$$= \sigma^2 + (\bar{y} - m)^2 \tag{8.3}$$

where $\bar{y}$ is the average of y. The loss function for more than one piece then becomes

$$L = k[\sigma^2 + (\bar{y} - m)^2] \tag{8.4}$$

where σ^2 is the variability around the average and $\bar{y}$ is the deviation of the average from the target.

We can now evaluate the quality of all our outputs. To reduce the loss, we must reduce the MSD. This can be accomplished with:

δ^2: reducing the variability around the average

$(y - m)^2$: adjusting the average to the target

The quality loss function gives us a quantitative means of evaluating quality. Let us reexamine our comparison of the four factories with different output distributions (Table 8.1).

❒ Example

The following analysis involves a sample size of 13 pieces each. Suppose that $k = 0.25$. Then, using the nominal-the-best format:

$$L = k[\sigma^2 + (\bar{y} - m)^2] = k(\text{MSD}) \tag{8.5}$$

For factory 3:

$$\bar{y} = \frac{112 + 113 + \cdots + 114}{13}$$

$$= 113$$

$$(\bar{y} - m)^2 = (113 - 115)^2$$

$$= 4 \tag{8.6}$$

Table 8.1
Data for the four factories

Factory	Data	MSD	Loss per Piece
1	115 113 113 114 114 115 115 116 116 117 117 115 118	2.92	$0.73
2	113 114 114 114 115 115 115 115 115 116 116 116 113	1.08	$0.27
3	112 113 112 113 112 113 114 115 112 113 114 112 114	4.92	$1.23
4	114 115 116 114 115 116 114 115 116 114 115 116 115	0.62	$0.15

$$\sigma^2 = \frac{(y_1 - \bar{y})^2 + (y_2 - \bar{y})^2 + \cdots + (y_n - \bar{y})^2}{n}$$

$$= \frac{(112 - 113)^2 + (113 - 113)^2 + \cdots + (114 - 113)^2}{n} = 0.92 \quad (8.7)$$

$$\text{MSD} = \sigma^2 + (\bar{y} - m)^2$$
$$= 0.92 + 4$$
$$= 4.92 \quad (8.8)$$

$$L = k(\text{MSD})$$
$$= 0.25(4.92)$$
$$= \$1.23 \text{ per piece} \quad (8.9)$$

Losses for factories 1, 2, and 4 are calculated in the same way. The results are summarized in Figure 8.4. The 73-cent loss for factory 1, for example, is interpreted as follows: As a rough approximation, one randomly selected product shipped from factory 1 is, on average, imparting a loss of 73 cents. Somebody spends the 73 cents: a customer, the company itself, an indirect consumer, and so on. Does factory 4 still appear to be the best choice?

Notice that in all cases, the smaller the MSD, the less the average loss to society. *Our job, to obtain high quality at low cost, is to reduce the MSD.* This can be accomplished through use of parameter design and tolerance design.

The loss function offers a way to quantify the benefits achieved by reducing variability around the target. It can help to justify a decision to invest $20,000 to improve a process that is already capable of meeting specifications.

Suppose, for example, that you are an engineer at factory 2 and you tell your boss that you would like to spend $20,000 to raise the quality level of your process to that of factory 4. What would your boss say? How would you justify such an investment?

Let's assume that monthly production is 100,000 pieces. Using the loss function, the improvement would account for a savings of $(0.27 − 0.15)(100,000) = $12,000 per month. The savings would represent customer satisfaction, reduced warranty costs, future market share, and so on.

8.3. Smaller-the-Better Characteristic

The loss function can also be determined for cases when the output response is a smaller-the-better response. The formula is a little different, but the procedure is much the same as for the case of nominal-the-best (Figure 8.5). For the case of smaller-the-better, where the target is zero, the loss function becomes

$$L = ky^2 \qquad k = \frac{A_0}{y_0^2} \tag{8.10}$$

$$L = k(\text{MSD}) \qquad \text{MSD} = \sum_{i=1}^{n} \frac{y_i^2}{n} = \sigma^2 + \bar{y}^2 \tag{8.11}$$

where A_0 is the consumer loss and y_0 is the consumer tolerance.

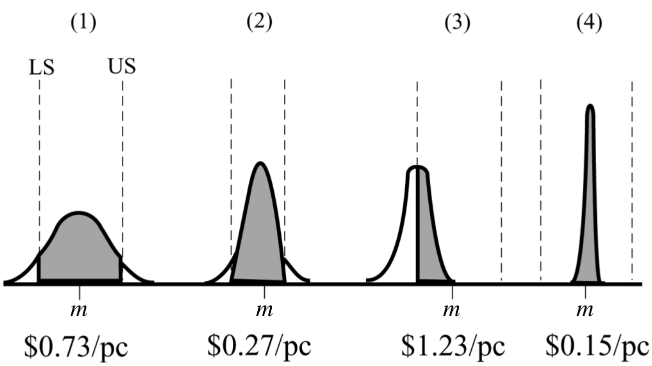

Figure 8.4
Evaluating four factories

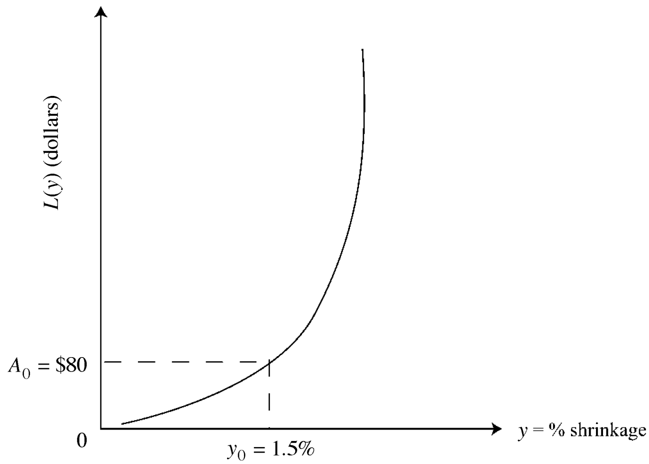

Figure 8.5
Smaller-the-better curve (% shrinkage)

❒ Example

The smaller-the-better case is illustrated for the manufacturing of speedometer cable casing, where the output response is

$$y = \%\text{ shrinkage of speedometer casing}, \qquad y \geq 0$$

When y is 1.5%, the customer complains about 50% of the time and brings the product back for replacement. The replacement cost is $80.

$$A_0 = \$80$$

$$y_0 = 1.5$$

Thus,

$$k = \frac{80}{(1.5)^2} = 35.56 \tag{8.12}$$

The loss function can then be written as

$$L(y) = 35.56y^2 \tag{8.13}$$

The data in Table 8.2 are percent shrinkage values of the casings made from two different materials. The losses were computed using the formula

$$L = 35.56\,(\bar{\delta}^2 + \bar{y}^2) \tag{8.14}$$

While the shrinkage measurements from both materials meet specifications, the shrinkage from material type B is much less than that of type A, resulting in a much smaller loss. If both materials cost the same, material type B would be the better choice.

Table 8.2
Smaller-the-better data (% shrinkage)

Type of Material	Data		$\bar{\delta}^2$	$\bar{y}$	MSD	*L*
A	0.28	0.24	0.00227	0.0729	0.0751	$2.67
	0.33	0.30				
	0.18	0.26				
	0.24	0.33				
B	0.08	0.12	0.00082	0.00439	0.0052	$0.19
	0.07	0.03				
	0.09	0.06				
	0.05	0.03				

8.4. Larger-the-Better Characteristic

For a larger-the-better output response where the target is infinity, the loss function is (Figure 8.6)

$$L = k\frac{1}{y^2} \qquad k = A_0 y_0^2 \tag{8.15}$$

$$L = k(\text{MSD}) \qquad \text{MSD} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_1^2}$$
$$= \frac{1}{n}\left(\frac{1}{y_1^2} + \frac{1}{y_2^2} + \cdots + \frac{1}{y_n^2}\right) \tag{8.16}$$

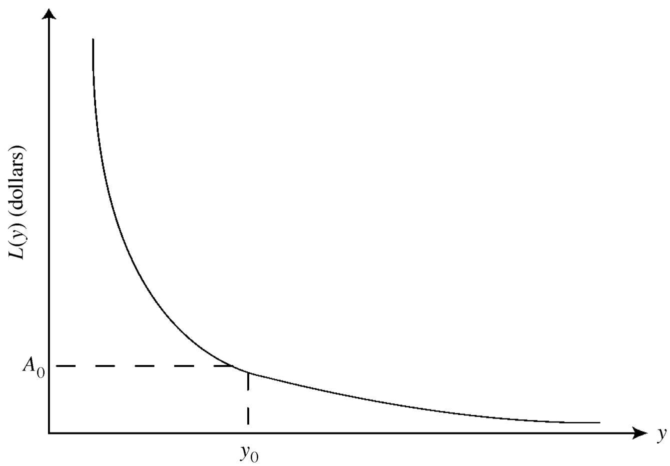

Figure 8.6
Larger-the-better curve

❐ Example

A company wants to maximize the weld strength of its motor protector terminals. When the weld strength is 0.2 lb/in², some welds have been known to break and result in an average replacement cost of A_0 = \$200.

1. Find k and set up the loss function.
2. If the scrap cost at production is \$2 per unit, find the manufacturing tolerance.
3. In an experiment that is conducted to optimize the existing semimanual welding process (Table 8.3), compare the "before" and "after."

8.5. Summary

The equations to calculate the loss function for different quality characteristics are summarized below.

1. Nominal-the-best (Figure 8.7)

$$L = k(y - m)^2 \qquad k = \frac{A_0}{\Delta_0^2}$$

$$L = k(\text{MSD}) \qquad \text{MSD} = \frac{1}{n}\sum_{i=1}^{n}(y_1 - m)^2$$

$$= \sigma^2 + (\bar{y} - m)^2$$

2. Smaller-the-better (Figure 8.8)

$$L = ky^2 \qquad k = \frac{A_0}{y_0^2}$$

Table 8.3
Larger-the-better data (weld strength)

	Data	MSD	*L*
Before experiment	2.3 2.0 1.9 1.7 2.1 2.2 1.4 2.2 2.0 1.6	0.28529	\$2.28
After experiment	2.1 2.9 2.4 2.5 2.4 2.8 2.1 2.6 2.7 2.3	0.16813	\$1.35

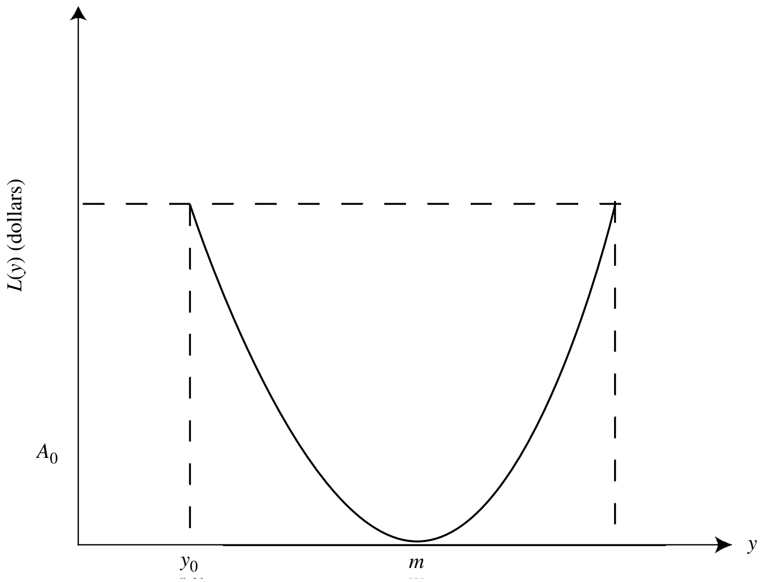

Figure 8.7
Nominal-the-best curve

$$L = k(\text{MSD}) \qquad \text{MSD} = \sum_{i=1}^{n} \frac{1}{y_i^2}$$

$$= \sigma^2 + \bar{y}^2$$

3. Larger-the-better (Figure 8.9)

$$L = k\frac{1}{y^2} \qquad k = A_0 y_0^2$$

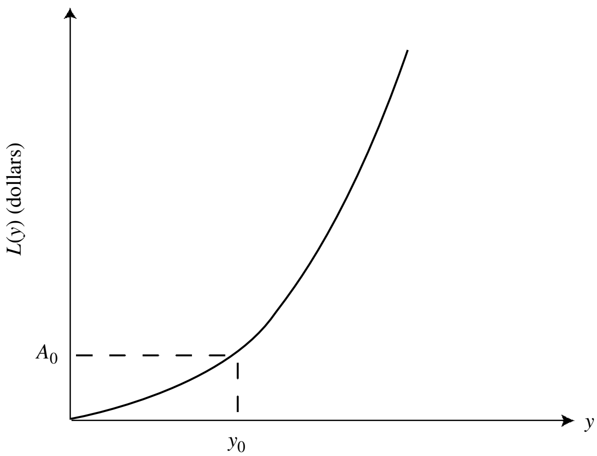

Figure 8.8
Smaller-the-better curve

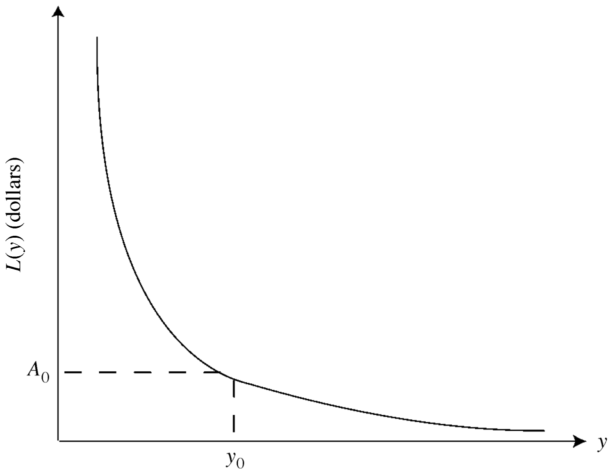

Figure 8.9
Larger-the-better curve

$$L = k(\text{MSD}) \qquad \text{MSD} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}$$

$$= \frac{1}{n}\left(\frac{1}{y_i^2} + \frac{1}{y_2^2} \cdots + \frac{1}{y_n^2}\right)$$