

## 19 Warranty Analysis

*Money back guaranties or the Good Housekeeping seal of approval is of damn little comfort to the rifleman on the battlefield when his rifle jams. It is of little comfort to the commander who has a five-year or fifty-thousand-mile warranty on a truck if it breaks down on the way to the front with ammunition that may be critical to winning the battle.*

—Former Commandant General Paul X. Kelly, USMC,  
June 1994 (Brennan 1994)

A warranty is a guarantee by the manufacturer or seller, usually in the form of a contract with a buyer, that defines a responsibility with respect to the product or service provided. Manufacturers provide warranties to attract customers and assure them of the high quality of their products. The primary role of warranties is protective, for both buyer and seller. Protection for the buyer is provided by warranty terms that require compensation in some form for defective items, namely repair of the item or replacement by a new item at reduced cost to the consumer or at no cost at all. The manufacturer is protected by specifications in the warranty terms of conditions under which the warranty is invalidated (e.g., use of the product for a purpose other than that intended, failure to perform proper maintenance, and use of a consumer product in a commercial application), and by limiting the amount of compensation, specifying the length of coverage, excluding specified components, limiting or precluding claims for ancillary damages, and so forth.

Customers value a good warranty as an economic protection, but a product is generally not considered good if it fails during the customer's perception of the product's useful life, regardless of the warranty. In other words, customers want the product that they purchase to function properly for some specified time in its application environment. Consumers would prefer that the product they purchase never needs to be returned for repair or replacement during its useful life. This is especially true if the malfunction involves safety, such as the loss of drivability of a vehicle.

## 19.1 Product Warranties

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To protect consumers from defective products and misleading product specifications, Congress empowered the Federal Trade Commission (FTC), along with state and local governments, to enforce consumer protection mandates. The Magnusson–Moss Warranty Act of July 1975 defined the terms for a “full warranty” (Federal Consumer Product Warranties Law 1975). The act states that a defective product manufactured with a warranty labeled “full” must be fixed within a reasonable amount of time and without charge to the customer for parts and labor. In addition, the warrantor cannot impose any unreasonable repair conditions on a customer, unless approved by the FTC. All warranties that offer anything less than the “full warranty” requirements must be designated as “limited.” In such instances, should the product ever need servicing, the consumer may be responsible for labor costs.

A warranty can also be categorized based on whether it is express or implied. An express warranty is any specific statement in the manufacturer’s written warranty, including those in brochures, advertisements, and other documents provided by a sales representative about a specific product. Claiming that a car “purrs like a kitten” is not considered an express warranty due to the general, as opposed to specific, nature of the specification, and is not recoverable under warranty. Specifying that a car has a six-cylinder engine constitutes an express warranty because it is a measurable specification subject to evaluation for accuracy. Thus, express warranties are based upon specific factual aspects of the product, such as “four vs. six-cylinder” engines, which are subject to evaluation for compliance to the specifications of the product.

An implied warranty is an unwritten warranty that is implied by state law. Accepted by all 50 states, an implied warranty can be based either on merchantability or fitness for a particular purpose. Under the Uniform Commercial Code (UCC), merchantability is defined as being fit for the regular purposes for which the products are intended. An example of merchantability is the ability of an automobile to stop when the brakes are applied. On the other hand, the implied warranty of fitness for a particular purpose is based on the seller’s skill or judgment. An example of an implied warranty for fitness for a particular purpose might be selling a truck and verbally stating it is capable of towing a large boat of a particular weight. The consumer is relying upon the professional judgment of the salesperson to evaluate the capability of the product to perform a specific task.

Three remedy options are typically offered in warranties: a free replacement, a pro rata refund, or a combination of free replacement and pro rata refund. Free replacement warranties require the seller to pay the entire cost of the remedy if the product fails before the end of the warranty period. Under a pro rata warranty, if a product fails before the end of the warranty period, the seller is responsible for repair or replacement, and the cost extent of the seller’s obligation is determined based upon the age or wear of the product at the time of failure. For example, it is common to purchase tires with the condition that the replacement cost will be determined by measurement of tread wear and determination of the proportional amount of use of the tire prior to the appearance of the defect or malfunction. Warranty policies can be a combination of a free replacement and pro rata refund when an initial free replacement period is followed by a pro rata period. Full warranties, under the Magnusson–Moss Act, place an obligation on the manufacturer to provide remedies for claims within a reasonable period of time at no charge, or refund the purchase

price less a reasonable depreciation if the defect cannot be remedied within a reasonable number of attempts.

A warranty pertains to the obligations of the manufacturer in relation to materials, workmanship, and fitness of a product for ordinary use or reasonably intended purposes throughout the duration of the warranty period defined in the express warranty. “Nonconformity” means a defect or other condition that substantially impairs the use, value, or safety of the product, but does not include a defect or condition that is the result of abuse, neglect, or unauthorized modification or alteration of the product. Thus, if a vehicle had been driven off-road, when it was expressly intended by the manufacturer for on-road use, there may be no liability under a consumer or commercial warranty.

The purchase price of a product generally incorporates the projected warranty cost, which includes repair cost, administration cost, and field service cost. The repair cost consists of all labor and material costs to accomplish repair of the warranted product to restore it to a serviceable condition, including the costs associated with the return of the product to the manufacturer, failure verification, repair, testing, and possible shipping to return the product to the consumer. Repair cost is typically the most significant component of the total warranty cost. Administration cost includes all cost for repair documentation, processing of warranty data, and the preparation of required status reports. Field service cost includes the cost incurred by the manufacturer during the repair, such as electricity, transportation, loaner items, diagnosis by an independent contractor of the problem, and other general administrative and overhead expenses.

## 19.2 Warranty Return Information

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Projections from early warranty data may not capture pending problems if there is an increasing failure rate with time. Wearout failure mechanisms may initially exhibit an extremely low rate of occurrence, but have an increasing failure rate. The root cause of failure must therefore be assessed to determine if failures are caused by wearout failure mechanisms. Constant failure rate statistics or prediction (e.g., Mil-Hdbk-217, Telcordia, and PRISM) should not be used. Furthermore, even if early returns are not a result of wearout mechanisms, it should not be assumed that wearout will not occur.

Warranty data will underestimate problems if owners go to sources that do not document repairs for warranty claims. Thus, even small warranty return rates must be assessed, especially if the product is driveability, emissions, or safety related.

A manufacturer should assume that all field returns are field failures and treat them as such. For example, a company that produces a drivability-, safety-, or emission-regulated product should assume that every return of that product is a failure, and take on full responsibility of ascertaining the root cause. A thorough root-cause analysis must be conducted so that potential and nonobvious failure mechanisms are not overlooked.

Root-cause analysis should include diagnostic monitoring, troubleshooting, and “ABA” testing.<sup>1</sup> The manufacturer has the responsibility to conduct both tests and

<sup>1</sup>In ABA testing, a good module is used to replace the “apparently failed” module, and then the failed module is reinstalled to ensure that a system integration problem (i.e., connector) was not the cause of the problem.

teardowns of every product (regardless of warranty stages), or determine why an apparently good product has been returned.

Testing of a product will often require combining multiple and time operating and environmental loading (stress) conditions for extended periods of time.<sup>2</sup> (Mitra and Patankar 1997) It must be recognized that a unique sequence of environmental and operating conditions can trigger a fault; for example, computer users are familiar with such situations where a unique and often complex sequence of keystrokes can initiate an unpredictable fault, which appears intermittent, and cannot be duplicated.

Both nondestructive and destructive evaluations must be conducted after testing and must involve investigating every potential failure mode and failure site to assess degradation. If no defect can be determined and there is no other verifiable and auditable explanation, then the module should still be considered a field failure, because electrical failures can often be quite elusive and intermittent in nature. Terms such as trouble not identified (TNI) or no-fault-found (NFF) should not be used to ignore problems. It is misleading to state that an unidentified problem means there was no problem, just because limited investigations, tests, and failure analyses have not yet enabled observation of the defect.

Incentives should be provided to the supply-chain participants to track all failure statistics of products that are out of warranty. Just because a product is out of warranty does not mean that the hazard rates and the root causes of any failures should not be determined. Out of warranty wear-out mechanisms that cause problems cannot be ignored. The additional data that these products provide can also help engineers to design more reliable products in the future.

The warranty and maintenance plan of a product should directly correspond to the “customer’s expectations” of useful life. The customer must be told that the product must be replaced or maintained in some manner at the end of the warranty, especially if a failure of the product can affect safety.

### 19.3 Warranty Policies

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Two types of warranty policies have been widely applied in practice: replacement warranty policy and pro rata warranty policy. For a replacement warranty policy, the seller is required to either repair the product or provide a new product at no cost to the buyer from the time of initial purchase. Such a policy is usually offered with repairable products. Pro rata warranty policy requires the seller to provide replacement at some cost, which is called pro rata cost, to the buyer. Pro rata cost can be either a linear or nonlinear function of the remaining time in the warranty length. Pro rata warranty policy is usually offered with nonrepairable products.

A combination of the replacement and pro rata warranty policies is also common because it has a significant promotional value to the seller while at the same time providing adequate control over costs for both buyer and seller in most applications

<sup>2</sup>The engineering specification of a module should specify the worst-case operating conditions, and backed by tests conducted to assess all combinations of worst-case applications and environments. If any module fails to meet the engineering specifications, no further modules should be placed into a vehicle. It is strongly recommended that IEC Standard 60134 be followed and applied to all electronic products that are drivability, emissions, or safety related.

(Chukova et al. 2004). One type of the combinations is the fully renewing combination free replacement and pro rata warranty (FRW/PRW) policy. Denote  $W$  as the warranty length of the product. Under the FRW/PRW policy, the seller agrees to replace the product that fails prior to the time point  $W'$ , where  $W' < W$ , from the time of purchase with a new product at no cost to the buyer; meanwhile, any failure in the time interval from  $W'$  to  $W$  results in a pro rata replacement, that is, any product is replaced with a new item at pro rata cost to the buyer.

## 19.4 Warranty and Reliability

Topics that will be dealt with in the sections that follow are warranty cost analysis, the relationship between warranty and reliability, and management of warranty.

Consumers are seldom in a position to evaluate all products adequately and have little information concerning product performance and reliability before making a purchase decision. Warranty terms provide signals of these characteristics, generous warranty terms conveying the message that the risk is low. As a result, warranty is used as an advertising tool in competition with other manufacturers. Note that the manufacturer must use caution in setting warranty terms. Terms that are too generous may lead to excessive future warranty costs.

There are several technical business and legal decisions that need to be made by the seller and buyer in determining warranty policies and warranty contracts. The seller has to consider issues related to warranty policies (terms, length, and so forth), costs, a function of the warranty parameters (e.g., length, amount of rebate, and repair vs. replace options), servicing products and warranty, data (historic, test, field, claims, etc.) needed for warranty management and how to obtain them, and impact on product and process design.

The buyer has to take into account the cost of warranty, including the additional cost options (e.g., length of warranty), comparison with other sellers, needed lifetime, and options of extended warranty. Addressing these issues is a difficult problem for both manufacturer and buyer. In addition, there are many other aspects of warranty, and many disciplines are involved in the analysis of these issues. Product reliability has an impact, directly or indirectly, in nearly all of these areas (Blischke and Murthy 1996).

Some other variations and definitions regarding warranty should also be mentioned. A warranty is renewing if on failure of an item, a replacement is made and this replacement item carries a warranty that is identical with that of the original item. In effect, the warranty period begins anew.

There are three main categories of warranties, including consumer, commercial, and defense acquisition warranties. For consumer goods, the most common warranties are various versions of the free replacement and pro rata warranties (which involve repair or replacement at no cost or prorated cost); cash rebates on failure of the item; and a combination free replacement/pro rata warranty.

Commercial and industrial warranties are those offered in sales by a manufacturer to another company. Sales of aircraft engines, seats, windshields, radar systems, and so forth, to an aircraft manufacturer are examples. These warranties often are of the same basic type as those offered on consumer products, but additional features may be involved. For example, groups or lots of items may be warrantied rather than

individual items. Warranties of this type are called cumulative or fleet warranties (Berke and Zaino 1991; Guin 1984; Zaino and Berke 1994).

Warranties on items procured by the government include all of the above plus some special warranties, particularly in acquisition of defense products. The best known of these special warranties is the reliability improvement warranty, which includes provisions for product development and improvement.

There are of course many different combinations of warranties when we take into account all the possibilities.

- *Nonrenewing Free Replacement Warranty (FRW)*. Under this warranty, the manufacturer will repair or replace a failed item free of charge up to time  $W$  from the time of initial purchase. The repaired or replaced item is warranted only for the time remaining in the original warranty period. The nonrenewing FRW is most often offered on repairable items (with repair almost always involving replacement of a faulty part or component). Examples of consumer products are household appliances, electronic items such as PCs and television sets, and automobile parts. Commercial products include tools, motors, and heavy equipment. This warranty is one of the more costly to the manufacturer.
- *Renewing Pro Rata Warranty (PRW)*. Under the renewing PRW, on failure at time  $X$  of an item, the manufacturer will provide a replacement item at cost to the buyer of  $[1 - X/W]C$ . The replacement item is covered under warranty identical to that of the item originally purchased. This warranty features linear proration. Although not used in practice, the proration function could be nonlinear as well, for example, quadratic, pro rata warranties are most commonly used for nonrepairable items. Nearly all automobile tires and batteries are covered by warranties of this type. This warranty favors the seller, that is, is less costly than the FRW.
- *Renewing Combination FRW/PRW*. The manufacturer will provide a free replacement for a failed item up to time  $W_1 < W$  from the time of initial purchase, and at pro-rated cost from  $W_1$  to  $W$ . The replacement item is warranted under the same terms as the original purchased item.

This is the most common combination warranty. It is typically offered on items ranging from automobile tires to appliances. It is a compromise between the FRW and PRW in that the cost of this warranty is less than that of the FRW and more than the PRW.

Other combination warranties are those that cover different components for different periods of time (common in TV sets where the picture tube carries a separate warranty, refrigerators, where the compressor is warranted for a longer period of time than other components, and automobiles, where there are many separate warranties.)

A classification of warranties in accordance with the various features offered is given in Blischke and Murthy (1993). This taxonomy of warranties is given in Figure 19.1 and is based on the following characteristics:

Whether or not the warranty includes product development (redesign, design or process improvements, and so forth). This is typically only found in reliability improvement warranties (RIW).

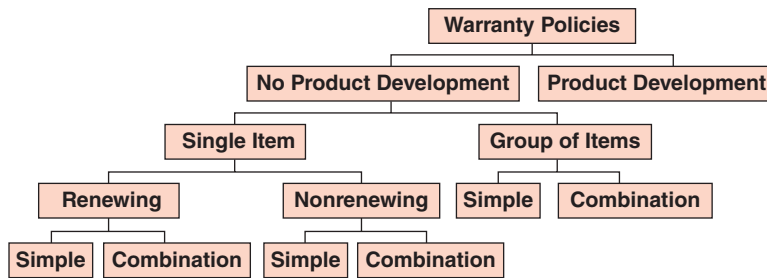


Figure 19.1 Taxonomy of warranties.

- Whether the warranty covers a single item purchased or a lot of items.
- Whether or not the warranty is renewing.
- Whether or not the warranty is a combination warranty.
- Whether the warranty is one-dimensional or more.
- Whether the warranty is single-attribute or covers multiple attributes.
- Whether or not the warranty has the option of extended coverage.

Burn-in can be considered as a part of the production process in which the manufactured products are operated under accelerated stresses for a short time period, which is called burn-in time, before their release. The principal motivation of burn-in is to detect the situation that latent defects exist in the early stage of introducing products. According to Nguyen and Murthy (1982), for the products with an initially high failure rate, burn-in can be used to reduce the warranty cost. Kar and Nachlas (1997) presented a model to study the warranty policy and burn-in together in order to examine the benefits for product management. Determination of optimal burn-in time to minimize the associated cost is always an interesting topic in the literature. Nguyen and Murthy (1982) derived the optimal burn-in time for repairable and non-repairable products sold under the failure-free and rebate policy. Yun et al. (2002) determined the optimal burn-in time to minimize the total mean cost, which is the sum of the manufacturing cost, the burn-in cost, and the cumulative warranty cost, under free replacement warranty policy. Sheu and Chien (2005) developed a generalized burn-in and field-operation model for the repairable products sold under warranty.

## 19.5 Warranty Cost Analysis

There are many aspects to the analysis of the cost of a warranty. First, it is necessary to develop adequate cost models. These depend on the perspective (buyer or seller), the basis on which the costs are to be assessed, and the probabilistic structure of the random elements involved. In this section, we discuss these factors and present cost models for the FRW, PRW, and a few additional warranties. The various types of information needed to estimate the models will also be discussed.

### 19.5.1 Elements of Warranty Cost Models

There are many types of models used in analysis of warranties. These include sales and demand models in marketing, cost and other models in economics, engineering models for analysis of various aspects of design, product reliability, production and quality control, operational models for servicing of warranties, and so forth. As is apparent, cost models must be developed separately for seller and buyer.

The cost of warranty depends on a number of factors, including at least the following:

- *Type of Warranty.* All other aspects being equal, generally manufacturer's costs for the FRW are higher than for a combination FRW/PRW, and these are higher than those for the PRW. Length of the warranty, renewability, and other features of the warranty can also have a significant impact on cost. Buyer's costs, at least in the long run, are affected in an inverse fashion to those of the seller.
- *Failure Pattern of New Items.* The mean time to failure (MTTF) is an important measure of product performance. Different statistical distributions of time to failure, however, can significantly affect costs, even when the MTTF is the same.
- *Repairability of Failed Items.* If an item is repairable and can be repaired at less cost than providing a replacement item, warranty costs may be reduced.
- *Failure Pattern of Repaired or Replaced Items.* The distribution of failure times of replacement items (which may be different from that of the original item) or repaired items (which may depend on the type of repair and the number of times an item has been repaired) may also significantly impact costs.
- *Incidental Costs.* These include warranty administration, shipping, the cost of service centers, cost of spare's storage, and many related items.

### 19.5.2 Failure Distributions

Item failure is a random process. As such, it is modeled by a probability distribution. Which distribution is appropriate depends on a number of factors, including product characteristics (determined by engineering design, raw materials, process design, decisions regarding outsourcing and selection of suppliers, and perhaps other factors, such as time of production and production rate), type and rate of usage, determined primarily by the buyer/user, age of the item, maintenance, again determined primarily by the buyer, but influenced by the warranty requirements, environmental factors, some under control or partial control of the user (e.g., protection from weather, extremes of heat and cold, and moisture), and some not (the weather itself).

### 19.5.3 Cost Modeling Calculation

There are a number of approaches to the costing of warranty. Costs clearly are different for buyer and seller. In addition, the following are some of the methods for calculating costs that might be considered:



- Cost, to the seller, per item sold. This per unit cost may be calculated as the total cost of warranty, as determined by general principles of accounting, divided by number of items sold.
- Cost per item to the buyer, averaging over all items purchased plus those obtained free or at reduced price under warranty.
- Life-cycle cost of ownership of an item with or without warranty, including purchase price and operating and maintenance cost, and finally including cost of disposal.
- Life-cycle cost of an item and its replacements, whether purchased at full price or replaced under warranty, over a fixed time horizon.
- Cost per unit of time.

The selection of an appropriate cost basis depends on the product, the context, and perspective. The type of customer—individual, corporation, government—is important, as are many other factors.

The cost of offering a warranty clearly depends on the reliability of the item. The precise role of product reliability will become more apparent in this section and in the remainder of the presentation. Warranty expenses also depend on a number of other factors. These include at least the following:

- *The Proportion of Legitimate Claims That Are Made.* This is called warranty execution. For various reasons (too much trouble, not worth the effort near the end of a pro rata warranty, desire to switch brands), claims are not made for all items that fail within the warranty period.
- *The Proportion of Claims That Are Not Legitimate.* Some typical bogus claims are those made after expiration of the warranty and not verified as such, failures due to misuse, and intentionally failed items.
- *Servicing Policy.* Factors include whether buyer or seller pays shipping costs, number and location of service centers, coverage of parts and labor or parts only, repair versus replace decisions, company versus contracted warranty service, and so forth. The list is a long one and many management decisions must be made.
- *Administrative Costs.* Processing of claims, cost of setting up and maintaining a warranty department, and so forth are included.
- *Incidental Costs.* Setting up a warranty information system; general overhead items.

#### 19.5.4 Modeling Assumptions and Notation

As is always the case in constructing mathematical models, many assumptions are made. Models that are useful are those for which the assumptions provide a reasonable approximation of the true nature of the phenomenon being studied and are not unduly sensitive to minor violations of these assumptions. In modeling seller's warranty costs, we make the following assumptions, some of which can be relaxed but at the expense of, for our purposes, unnecessarily increased complexity:

- Warranty claims are made on all items that fail within the warranty period (warranty execution is 100%).
- No illegitimate claims are made.
- Claims are made immediately on failure of the item. In practice, this means that the time from item failure to warranty claim is small relative to the warranty period and the lifetime of the product.
- Compensation under warranty (repair or replacement) is instantaneous. Again, this is interpreted as in Item 3.
- Constant, fixed cost per claim. In practice, this cost is a random variable. The analysis in effect uses an average repair cost.
- Identical items. All repaired or replaced items have the same life distribution as that of the original item. Thus no design changes that would affect the lifetime of the item have been made and any repairs bring the item back to “good as new.” (Some of the models can be modified in a relatively straight-forward manner to accommodate other repair or replace regimes.)
- Statistical independence. This in effect says that there is no relationship between lifetimes of items.
- No brand switching. The buyer is assumed to purchase the same make and model item on failure of a product, whether or not it is covered by warranty.
- All parameters are known. Costs of supplying an item to the buyer, repair or replacement costs, life distribution of the items and their parameters, and so forth, are all known to the analyst or can be reliably estimated.

**19.5.5 Cost Models Examples**

In expressing the cost models, we require the following additional notation:

$$\mu_T = \int_0^T x dF(x), \tag{19.1}$$

$\mu_T$  is called the partial expectation of  $X$ ; it is the average time to failure of all items that fail with lifetimes less than some given time value  $T$ , for the time to failure random variable with cumulative distribution function  $F(x)$ . In addition, we have

$$M(T) = \text{Expected number of failures in the interval } (0, T),$$

where  $M(T)$  is the renewal function. It is defined as the solution to an integral equation, and can be obtained in closed form only for a few distributions. For the exponential distribution, for example,  $M(t) = \lambda t$ . In other cases, it can be evaluated by means of computer algorithms and has been extensively tabulated (Baxter et al. 1982; Blischke and Murthy 1994).

Finally, we use  $C_s$  to denote the seller’s average cost per item (including development, production, distribution, marketing, etc.),  $C_b$  to denote the buyer’s cost, and  $C_r$  to denote the average cost of repair.

**19.5.5.1 Nonrenewing FRW** We look first at the seller’s cost per unit sold for non-repairable items sold under nonrenewing FRW with warranty period  $W$ . The analysis

of this warranty by Menke (1969) and Lowerre (1968) was one of the first theoretical analyses of warranty costs. The analyses presented were first-failure models, ignoring the possibility of multiple failures during the warranty period. Nonetheless, some useful first approximations were obtained. These results were extended by Blischke and Scheuer (1975, 1981), providing the model for seller's expected cost, say  $E[C_m(W)]$ , in this case as

$$E[C_m(W)] = C_s [1 + M(W)]. \quad (19.2)$$

**Example 19.1**

Suppose that a TV picture tube has a lifetime that is exponentially distributed with  $MTTF = 6.5$  years. Then the failure rate is  $\lambda = 1 / 6.5 = 0.1538$  per year. Suppose that the cost of supplying a new tube (original or under warranty) is \$67.20. Compare the costs of 6-month and 1-year nonrenewing free replacement warranties.

For this distribution, the renewal function is  $M(t) = \lambda t$ . For a 6-month warranty, the average cost to the seller per unit sold is

$$\$67.20[1 + 0.1538(0.5)] = \$72.37.$$

For 1-year warranty:

$$\text{Avg. cost} = \$67.20[1 + 0.1538(1)] = \$77.54.$$

For a 1-year warranty, this cost would be \$77.54. Note that the warranty cost to the seller has doubled, from about 7.7% of the production price to about 15%.

For repairable items, the situation is somewhat more complicated. The average cost to the seller depends on the repair policy, the average cost of repairing an item, and the life distribution of repaired items. Some useful results for repairable items are given by Nguyen and Murthy (1984).

If repairable items are repaired good-as-new, that is, repaired items have the same failure distribution as new items (which may occur if failure of the item is due to failure of a component that has a much higher failure rate than any other component), then the expected cost is given by:

$$E[C_m(W)] = C_s + C_r M(W). \quad (19.3)$$

Many additional models for FRW, from buyer's and seller's points of view, and under various assumptions regarding costs and reparability and a number of other factors, are given in Newman and Nesbitt (1978).

**19.5.5.2 Nonrenewing PRW** A nonrenewing warranty is equivalent to a warranty that provides a rebate on failure of an item. The buyer may or may not use the rebate to buy an identical replacement item of the same brand. Whether or not this occurs, from the seller's point of view the cost is based on the cost of supplying an item plus the selling price, because that is what the rebate is based on. The expected cost to the seller, per item, for this type of pro rata warranty is

$$E[C_m(W)] = C_s + C_b [F(W) - \mu_w / W]. \quad (19.4)$$

Formulas for evaluation of the partial expectation are available for a number of distributions, but again lead to computational difficulties in most cases. Computation of the CDF is also sometimes difficult. Computer analysis of these cost equations, however, is reasonably straightforward.

For the exponential distribution, the partial expectation is given by

$$\mu_W = \lambda^{-1}[1 - (1 + \lambda W)e^{-\lambda W}]. \quad (19.5)$$

### Example 19.2

There is a TV picture tube with an exponential lifetime and MTTF of 6.5 years, and cost to seller of \$67.20. Suppose that the selling price is  $C_b = \$105$  and that the tube has a one-year pro rata warranty, with a pro-rated rebate rather than a replacement at pro-rated price. From Equation (19.4) and Equation (19.5), we find that the expected cost to the seller, including warranty, is

$$E(C_m) = \$67.20 + 105[1 - e^{-0.1538} - 0.0674] = \$74.88.$$

Note that this is less than the cost to the seller of the nonrenewing FRW. (The warranty cost is 11% rather than 15%.) The cost for the renewing PRW would be even less, because it involves a new item (with cost based on  $C_s$ , rather than  $C_b$ ) rather than a cash rebate.

## 19.5.6 Information Needs

Information regarding both the technical and commercial aspects is necessary for effective warranty management. Technical aspects include product design, manufacturing, test results, quality control, and many related issues. Commercial aspects include marketing strategy, pricing, warranty, service policy, and so forth. Here we discuss a few key issues.

**19.5.6.1 Requirements for Successful Application of the Models** In order to apply the models discussed earlier and the many other cost models in Newman and Nesbitt (1978), and elsewhere, it is necessary to know:

- *The Form of the Failure Distribution.* This may be obtained from theoretical considerations based on an understanding of the physical mechanism of failure, or empirically, through fitting of various distributions graphically or by use of other statistical methods.
- *Type of Warranty.* The examples given in the previous section are two of the more simple warranties. There are many other possibilities (Blichke and Murthy 1993, 1996; Guin 1984) and many forms of each, involving renewability, length of the warranty period, and so forth. The decision as to warranty policy is an important managerial responsibility, and there are many potential cost ramifications.
- *Parameter Values.* As is apparent, the cost models require many inputs, some of which may be poorly known or unknown. These include cost parameters as well as the parameters of the life distribution.

- *Rectification Policy.* Here we are concerned with servicing policy, for example, repair versus replacement, and under what conditions. The type of warranty influences this and parameter values as listed in Item 3 must be determined for each scenario under consideration.
- *Other Cost Information.* Many direct and indirect costs are included in the models. A number of these were discussed previously. The impact of these and how they are assessed depend on the company and its policies and methods of accounting, the product itself, and many other factors.
- *The Broader Picture.* For a proper interpretation of the model output (and input), the analyst, engineer, and manager must be aware of the overall company perspective. (Here we have looked at the process from the seller's point of view. Similar considerations are relevant in the case of the buyer.) Important factors include the overall organizational strategy, management objectives for this product in that context, the marketplace, including demand and competition, marketing strategy, and many more.

**19.5.6.2 Information Sources** In any organization, a great deal of information is available. The problem is to determine exactly what is, indeed, accessible and the relevance of available data to the problem at hand. For assessment of product reliability and warranty costs, at least the following should be sought:

- *Test Data.* This may include data on testing of prototypes, data on various designs under consideration, and so forth. Data on tests of this specific product are, of course, of most relevance. It is important that the data result from a well-designed, comprehensive experiment.
- *Part and Component Data.* Much useful information can be obtained from tests at the part and component level. Some tests of this type will be done on parts and components obtained from suppliers, as well, for example, acceptance testing data. In reliability studies, the use of this information requires detailed models of the relationship between the reliability of these items and that of the system (product). See any text on reliability, for example, Blischke and Murthy (1998), Kapur and Lamberson (1977), Barlow and Proschan (1965). Data of this type are invaluable in Bayesian reliability analysis (Martz and Waller 1982, 1990), as they form the basis of the prior distribution that is used in this analysis.
- *Data on Similar Products.* In many cases, much information will have been collected on prior versions of the product being analyzed. In fact, it is often the case that some of the same parts and components used previously will be included in the current product design. Data of this type may be useful in predicting the reliability of the new product. Again, Bayesian methods may be an effective approach.
- *Vendor Data.* When parts or components are produced by a supplier, extensive testing may not be done by the manufacturer of the product but may have been done by the producer of the part. These data may be very useful for the purposes described earlier, and should be requested of the supplier or made a part of the purchase contract.

- *Subjective Information.* “Engineering judgment” often plays an important role in reliability analysis, particularly when little information of other types is available. It is also frequently used in Bayesian analysis.
- *Claims Data.* After introduction of a product into the marketplace, other types of data become available. The most important of these is data on warranty claims. Although there are often problems with this type of data (e.g., validity of claims, claims execution, actual time of failure, identification of failure cause, and censoring at time  $W$ ), it can provide valuable information concerning the validity of models and assessment of model parameters.
- *Operational Data.* Field data on operation of an item, when available, is another important source of information that can be obtained after product introduction. Operational data provide information on failure causes and rates in real operational conditions rather than simulated laboratory environments.

### 19.5.7 Other Cost Models

A schematic representation of warranty cost is shown in Figure 19.2. In the previous sections, we have looked at only a few specific warranties and even fewer cost models. Among the many extension of these warranties, models and related results are the following (Blischke and Murthy 1993, 1994, 1996; Murthy and Blischke 1991a, 1991b):

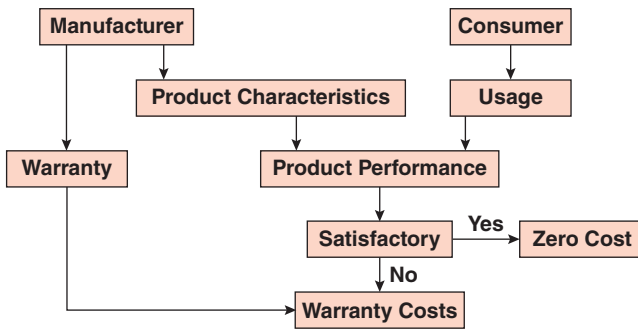
*Other Types of Warranties.* There are many versions of the basic FRW and PRW, many combinations of these, and a nearly unlimited number of exclusions and limitations on coverage. Cumulative and extended warranties offer still additional possibilities. In competitive markets, careful management and analysis of the possibilities is essential.

*More General Failure Distributions.* The exponential distribution, which features a constant failure rate, is realistic in many situations. (It has the added advantage of being mathematically tractable where most other distributions are not.) When the assumption of constant failure rate is not realistic, however, other distributions should be used in modeling failures. Distributions that may be considered include the Weibull, gamma, lognormal, inverse Gaussian, truncated normal, and extreme value, to name a few.

*Higher Dimensional Warranties.* Two-dimensional warranties (e.g., based on calendar time and usage) as well as some three-dimensional warranties have been discussed previously. In theory, there are an endless number of possible versions of these, and the modeling and analysis problems become even more difficult.

*Life-Cycle Cost Models.* Models that represent life-cycle costs may be defined in various ways and have been discussed briefly previously. These typically present additional analytical difficulties, for example, involving even more complex renewal-type equations for solution, but are essential in any analysis of long term costs.

*Various Cost Bases.* The two models given in the previous section look at costs from the seller’s point of view. Cost models from the buyer’s point of view, including life-cycle costs, have also been developed. Other cost bases—unit,



**Figure 19.2** Schematic representation of warranty cost.

time, and so forth—have been discussed previously and models of these types have been developed as well.

*Discounted Costs.* Discounting of future costs to the present value of money is an important aspect of the analysis of long-term warranties. A number of models include this feature. The selection of an appropriate discount rate is always an additional uncertainty, but comparing the results of analyses assuming various possibilities can be enlightening.

*Indifference Price Structures.* A different approach to determining to cost or value of a warranty to either buyer or seller is to look at the indifference price. The idea is as follows: Suppose that a product is sold without warranty at price  $C_b$ ; determine a price  $c^*$  ( $> C_b$ ) for selling an item with warranty such that the buyer (or seller, as the case may be) would incur the same cost if the item is sold with or without warranty. Again, this may be calculated in various ways, depending on whether it is the indifference price for the buyer or seller, the cost basis, whether or not discounting to present value is involved, and so forth.

In analysis of the process, important considerations are:

- *Cause and Effect Relationships.* Here, as noted, the most important is the relationship between product reliability and warranty cost.
- *Sequence of Model Elements.* The sequence in the warranty context is apparent in the chart describing the system characterization of the warranty process. Both buyer and seller have an impact on ultimate warranty costs.
- *Static versus Dynamic Elements.* Some engineering design changes that may affect reliability can be made after release of the product. Some cannot, at least not without excessive cost.
- *Level of Complexity.* Realistic models are nearly always complex. The result is that realistic models are difficult to analyze and compromises must be made.
- *Level of Realism.* The tradeoff between analytical tractability and realism is well known. The analyst must be aware of the reality of the models used. In warranty analysis, the problems are quite difficult, but the models may be a fair representation of the process. As has been said by many analysts, “No model is correct, but some are useful!”
- *Deterministic versus Stochastic Models.* In warranty analysis, many elements that are stochastic are modeled as though they are deterministic (expected

values are used). If the distribution has small variance, this approach is reasonable, but the analyst must be aware of the possible implications.

- *Generalizability.* Many models developed for a specific application can, in fact, be applied much more generally. To determine whether or not this is the case, studies of the sensitivity of the results to the assumptions made must be undertaken. One such study in the context of warranty is given in Blischke and Vij (1997). It was found that, indeed, distributional assumptions can have a significant impact on predicted costs.

## 19.6 Warranty and Reliability Management

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There is an inverse relationship between the reliability of a product and the future cost of warranty to the manufacturer of the product. Analysis of this cost trade-off is essential for effective management of both issues.

A systems analysis provides valuable insight in that, as usual, the quantification of the problem that it imposes on the analyst, engineer, and manager tends to focus on the longer term and the overall objectives as well as on the methodology. We briefly summarize the systems approach, emphasizing some of the details that are particularly important in the warranty context. The important aspects are:

- *System Characterization.* The first step in an analysis is determining a basis for description of the system, that is, a system characterization. This involves definition of the important variables that can be used to describe the system and attempting to list the possible relationships between them. How much detail is included will depend on the level of understanding of the system at this point and the point of view—buyer, seller, and so on.
- *Mathematical Modeling.* There is inherent uncertainty in the warranty process, as is evident in our previous discussion of warranty analysis. This must be taken into consideration in model formulation, and introduces probabilistic (stochastic) elements into the analysis. The life distribution of the items models an essential part of this uncertainty and plays an important role in the analysis.

Many of these were discussed in Chapter 3 (e.g., exponential and Weibull).

- *Analysis.* The exponential distribution can be dealt with analytically in many cases. Most other important life distributions cannot, particularly with regard to evaluation of renewal functions and other complex functions encountered in warranty analysis, and numerical methods are required, but this is not a major problem—either numerical evaluation of some of the complex integrals or simulation can be used.
- *Model Validation.* Having formulated a mathematical model representing the warranty process, including its stochastic aspects, validation by use of data is required. Claims data are important, as are the other data sets mentioned previously.



- *Interpretation.* Interpretation of the results of all of the above in the context of the real problem in the real world is essential. Even more important is:
- *Implementation.* In order to achieve actual gains in reducing warranty costs, it is necessary to understand the tradeoffs, in the long term, between the cost of increasing reliability and the savings realized from reducing the cost of warranty. This is a managerial decision that must be made early on, and implemented beginning at the product concept and next at the product design stage.

A simplified system characterization of the important elements involved in the analysis of warranties is given in the following chart. The roles of both manufacturer and buyer are indicated and both are clearly important. More detailed characterizations of the warranty process, with extensive flow charts, are given by Murthy and Blischke (1991a, 1991b) and Blischke and Murthy (1993, 1994).

## 19.7 Summary

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In this chapter, we have looked at warranty from several points of view, buyer/seller, engineer/manager (or both), warranty versus reliability and the cost trade-offs of the two, and so forth. Some of the important points we have made with regard to the management of warranty (for both buyer and seller) are the following:

- Determine precisely what the warranty terms are, or are to be.
- Are these negotiable?
- What is the failure distribution of the item?
- How are costs related to this?
- What is (or should be) the warranty servicing policy?
- How shall warranty be managed (for both seller and buyer)?
- There are many other important considerations, for example, what data are available.

Attention must be paid to warranty costs and their relation to product reliability and to methods for predicting and managing both. This has always been important but is crucial in today's marketplace, where products are often nearly indistinguishable and warranty is used as a competitive tool to increase market share.

## Problems

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19.1 Discuss the role of a warranty from different points of view. Consider both the seller's and the buyer's viewpoints.

19.2 Draw a tree diagram of warranty policies for consumer products.

19.3 How can a burn-in test reduce warranty cost?

*19.4* What are the factors that should be considered when developing a warranty cost model?

*19.5* Suppose that the cost of producing an electronic instrument is \$2000. The instrument is repaired good-as-new after failures under an ordinary 1-year free-replacement warranty. The average cost of servicing a warranty claim is \$100, and the MTTF of the instruments is 5 years. Assuming that the failures are exponentially distributed, what is the average cost to the manufacturer per unit sold?

*19.6* A manufacturer plans to provide a 12-month ordinary free-replacement warranty for a new laptop. Assume that all failures result in replacements instead of repair. The lifetimes of the laptops are exponentially distributed with MTBF of 7.5 years. The manufacturer's average cost of servicing a warranty claim is \$120. The fixed cost of providing warranty coverage for this laptop is also considered. (The fixed cost can include administrative costs to run the warranty department for this laptop.) Assume that the fixed cost is \$10,000 for 500,000 laptops sold. What warranty reserve should be put in place (discount rate is ignored)? That is, how much money should the manufacturer of the laptop budget to satisfy the promised warranty?

*19.7* Suppose that the electronic instrument in Problem 19.5 has a 1-year pro rata warranty with a pro-rated rebate rather than replacement at a pro-rated price. What sales price will the manufacturer set if the same cost is expected per unit sold?

*19.8* Suppose that the electronic instrument in Problem 19.5 has a 1-year pro rata warranty with a replacement at a prorated price and that multiple failures are considered within the warranty period. What sales price will the manufacturer set if the same cost is expected per unit sold?

*19.9* List as many sources as possible where information can be obtained for warranty cost modeling and effective warranty management.

*19.10* Examples of prognostics-based warranty models have been provided in this chapter. Think of one more warranty model that could be enabled by PHM, and describe how this warranty model could be implemented.