14

14 Process Control and Process Capability

One traditional approach to manufacturing and addressing quality is to depend on production to make the product and on quality control to inspect the final product, screening out the items that do not meet the requirements of the customer. This detection strategy using after-the-fact inspection is highly uneconomical, since the rejected products have already been produced. A better strategy is to avoid waste by not producing unacceptable output in the first place, focusing on prevention rather than screening. Statistical process control (SPC) is an effective prevention strategy to manufacture products that will meet the requirements of the customer (Duncan 1986; Montgomery 2005; Shewhart 1931).

This chapter covers process control systems, the different types of variation and how they affect the process output, and control charts and their use. It also covers how control charts and statistical methods identify whether a problem is due to special or common causes and the benefits that can be expected from using the control charts. It also covers what is meant by a process being in statistical control and process capability and its various indices and their applications.

14.1 Process Control System

A process control system (see Figure 14.1) is a kind of feedback system. Four elements of that system are important to the discussions that will follow:

1. *The Process.* The process means the whole combination of people, equipment, input materials, methods, and environment that work together to produce output. The total performance of the process—the quality of its output and its productive efficiency—depends on the way the process has been designed and built and on the way it is operated. The rest of the process control system is useful only if it contributes to improved performance of the process.

Reliability Engineering, First Edition. Kailash C. Kapur and Michael Pecht.

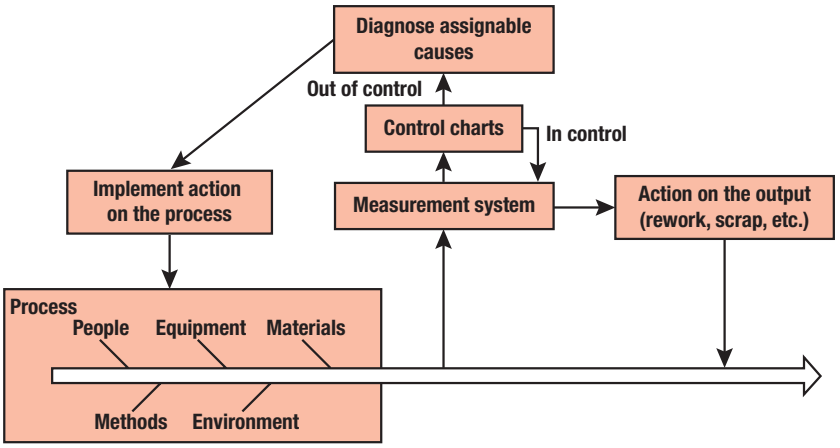


Figure 14.1 Process control system.

2. *Information about Performance.* Much information about the actual performance of the process can be learned by studying the process output. In a broad sense, process output includes not only the products that are produced, but also any intermediate outputs that describe the operating state of the process, such as temperatures, cycle times, and so on. If this information is gathered and interpreted correctly, it can indicate whether action is necessary to correct the process or the product. If timely and appropriate actions are not taken, however, any information-gathering effort is wasted.
3. *Action on the Process.* Action on the process is future oriented, because it is taken when necessary to prevent the production of nonconforming products. This action might consist of changes in the operation (e.g., operator training and changes to the incoming materials) or in the more basic elements of the process itself (e.g., the equipment, which may need rehabilitation, or the design of the process as a whole, which may be vulnerable to changes in shop temperature or humidity).
4. *Action on the Output.* Action on the output is past oriented, because it involves detecting out-of-specification output already produced. Unfortunately, if current output does not consistently meet customer requirements, it may be necessary to sort all products and to scrap or rework any nonconforming items. This must continue until the necessary corrective action on the process has been taken and verified, or until the product specifications have been changed.

It is obvious that inspection followed by action only on the output is a poor substitute for using an effective process performance from the start. Therefore, the discussions that follow focus on gathering process information and analyzing it so that action can be taken to correct the process itself.

Process control plays a very important role in the effort for improvement. When a process is well controlled, analysis and improvement naturally result; and when we try to make an improvement, we naturally come to understand the importance of control. Breakthroughs occur only after achieving control. Without process control, we cannot set appropriate standards or identify needed improvements. Improvement can only be achieved through process analysis.

14.1.1 Control Charts: Recognizing Sources of Variation

A control chart is a type of trend chart (displaying data over time) with statistically determined upper and lower control limits; it is used to determine if the process is under control. A process is said to be under control when the variation within the process is consistently only random and within predictable (control) limits. Random variation results from the interaction of the steps within a process. When the performance falls outside the control limits, assignable variation may be the cause. Assignable variation can be attributed to a number of special causes. A control chart will help determine what type of variation is present within the process. Control charts are also used to assess process variations and their sources and to monitor, control, and improve process performance over time. A control chart focuses attention on detecting and monitoring process variation over time. Using one can allow us to distinguish special causes of variation from common causes of variation. Control charts can serve as an ongoing control tool and help improve a process to perform consistently and predictably. They also provide a common language for discussing process performance.

14.1.2 Sources of Variation

As discussed earlier, the sources of variability in a process are classified into two types: chance or random causes and assignable causes. Chance causes, or common causes, are sources of inherent variability, which cannot be removed easily from the process without fundamental changes in the process itself. Assignable causes, or special causes, arise in somewhat unpredictable fashion, such as operator error, material defects, or machine failure. The variability due to assignable causes is comparably larger than that for chance causes, and can send the process out of control. Table 14.1 compares the two sources of variation, including some examples.

14.1.3 Use of Control Charts for Problem Identification

Control charts by themselves do not correct problems. They indicate that something is wrong and requires corrective action. Assignable causes due to a change in

Table 14.1 Sources of variation

Common or chance causes	Special or assignable causes
Include many individual causes.	Include one or just a few individual causes.
Any one chance cause results in only a minute amount of variation. (However, many chance causes together may result in a substantial amount of variation.)	Any one assignable cause can result in a large amount of variation.
As a practical matter, chance variation cannot be economically eliminated—the process may have to be changed to reduce variability.	The presence of assignable variation can be detected (by control charts), and action to eliminate the causes is usually economically justified.
Examples: ■ Slight variations in raw materials ■ Slight vibrations of a machine ■ Lack of human perfection in reading instruments or setting controls	Examples: ■ Batch of defective raw materials ■ Faulty setup ■ Untrained operator

manpower, materials, machines, or methods, or a combination of these, can cause the process to go out of control.

Assignable causes relating to manpower:

- New or wrong person on the job
- Careless workmanship and attitudes
- Incorrect instructions
- Domestic, personal problems.

Assignable causes relating to materials:

- Improper work handling
- Stock too hard or too soft
- Wrong dimensions
- Contamination, dirt, and so on
- Improper flow of materials.

Assignable causes relating to machines or methods:

- Dull tools
- Poor housekeeping
- Inaccurate machine adjustment
- Improper machine tools, jigs, fixtures
- Improper speeds, feeds, and so on
- Inadequate maintenance
- Worn or improperly placed locators.

When assignable causes are present, as shown in Figure 14.2, the probability of nonconformance may increase, and the process quality deteriorates significantly. The

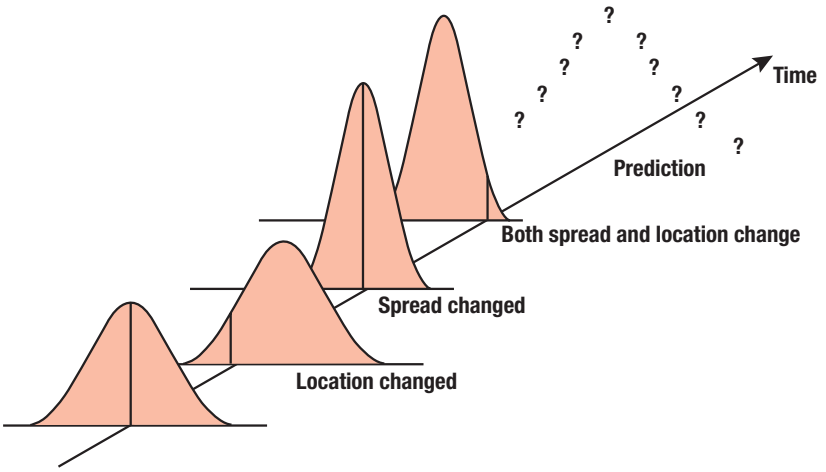


Figure 14.2 Unstable and unpredictable process resulting from assignable causes.

eventual goal of SPC is to improve quality by reducing variability in the process. As one of the primary SPC techniques, the control chart can effectively detect the variation due to assignable causes and reduce process variability if the identified causes can be eliminated from the process.

SPC techniques aim to detect changes over time in the parameters (e.g., mean and standard deviation) of the underlying distribution for the process. In general, the statistical process control problem can be described as follows (Stoumbos et al. 2000). Let X denote a random variable for a quality characteristic with the probability density function $f(x; \theta)$, where θ is a set of parameters. If the process is operating with $\theta = \theta_0$, it is said to be in statistical control; otherwise, it is out of control. The value of θ_0 is not necessarily equal to the target (or ideal) value of the process. Due to experimental design and process adjustment techniques, a process is assumed to start with the in-control state (Box and Luceno 1997; Hicks and Turner 1999; Montgomery 2001). After a random length of time, variability in the process will possibly cause deterioration of or a shift in the process. This shift can be reflected by a change in θ from the value of θ_0; then the process is said to be out of control. Therefore, the basic goal of control charts is to detect changes in θ that can occur over time.

A process is said to be operating in statistical control when the only source of variation is common causes. The status of statistical control is obtained by eliminating special causes of excessive variation one by one.

Process capability is determined by the total variation that comes from common causes. A process must first be brought into statistical control before its capability to meet specifications can be assessed. We will discuss the details of process capability analysis in later sections.

14.2 Control Charts

The basic concept of control charts was proposed by Walter A. Shewhart of the Bell Telephone Laboratories in the 1920s; this was the formal beginning of statistical quality control. The effective use of the control chart involves a series of process improvement activities. For a process variable of interest, someone must observe data from the process over time, monitor the process, and apply a control chart to detect process changes. When the control chart signals the possible presence of an assignable cause, effort should be made to diagnose the assignable cause(s), implement corrective actions to remove them so as to reduce variability, and improve the process quality. The long history of control charting application in many industries has proven the technique's effectiveness in improving productivity, preventing defects, and providing information about diagnostic and process capability.

Control charts must be investigated in order to identify in-control and out-of-control processes and detect common causes and special causes of the out-of-control state. In interpreting control charts, it is important to note that attribute data control charts measure variation among samples. Variations among subgroups over time can be measured by the first variable data control chart, while variations within subgroups over time can be measured by a second chart.

Also, the chart analyst should determine if the process mean (center line) is where it should be relative to production specifications or objectives. If not, then either the process or the objectives have changed. To distinguish between common causes and

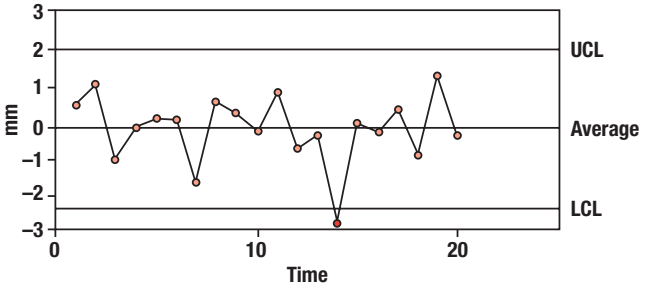


Figure 14.3 A typical control chart (X-bar chart).

special causes, data relative to control limits must be analyzed. Upper and lower control limits (UCL/LCL) are not specification limits and do not imply a value judgment (good, bad, and marginal) about a process. The judgment is derived with other tools, such as benchmarking "stretch" goals. UCL/LCL is only a statistical tool. If a process is consistently performing above the command UCL, the reason must be discovered to enable process improvements. A typical control chart is given in Figure 14.3.

The basic model for Shewhart control charts consist of a center line, an upper control limit (UCL), and a lower control limit (LCL) (ASTM Publication STP-15D 1976).

$$\begin{aligned} \text{UCL} &= \mu_s + L\sigma_s \\ \text{Center line} &= \mu_s \\ \text{LCL} &= \mu_s - L\sigma_s, \end{aligned} \tag{14.1}$$

where μ_s and σ_s are the mean and standard deviation of the sample statistic, such as the sample mean (X-bar chart), sample range (R chart), and sample proportion defective (p chart). $L\sigma_s$ is the distance of the control limits from the center line, and it is most often set at three times the standard deviation of the sample statistic. Constructing a control chart requires specifying the sample size and sampling frequency. The common wisdom is to take smaller samples at short intervals or larger samples at longer intervals, so that the sampling effort can be allocated economically. An important concept related to sampling scheme is the rational subgroup approach, recommended by Shewhart. In order to maximize the detection of assignable causes between samples, the rational subgroup approach takes samples in a way that the within-sample variability is only due to common causes, while the between-sample variability should indicate assignable causes in the process. Further discussion of the rational subgroup can be found in Montgomery (2005).

An out-of-control signal is given when a sample statistic falls beyond the control limits, or when a nonrandom pattern presents. Western Electric rules are used to identify the nonrandom pattern in the process. According to Western Electric rules (Western Electric 1956), a process is considered out of control if any of the rules given in Table 14.2 are met. More decision rules or sensitizing rules can be found in Montgomery's textbook.

Figure 14.5 gives some examples of an out-of-control condition based on the guidelines given in Table 14.2 and Figure 14.4. The measurements of quality characteristics are typically classified as attributes or variables. Continuous measurements, such as length, thickness, or voltage, are variable data. Discrete measurements, such as the

Table 14.2 Rules to detect out-of-control processes

1. One or more points fall outside control limits
2. Two out of three consecutive points are in zone A
3. Four out of five consecutive points are in zone A or B
4. Nine consecutive points are on one side of the average
5. Six consecutive points are increasing or decreasing
6. Fourteen consecutive points alternate up and down
7. Fifteen consecutive points within zone C

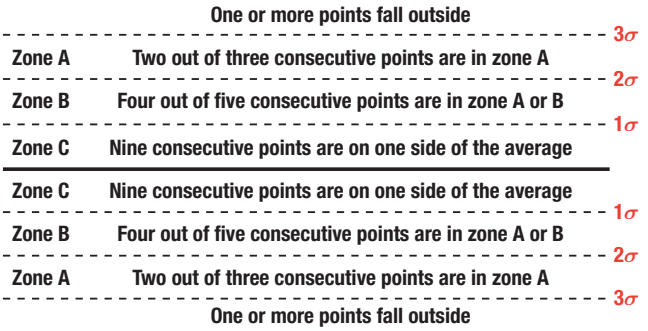


Figure 14.4 Guidelines to distinguish out-of-control process.

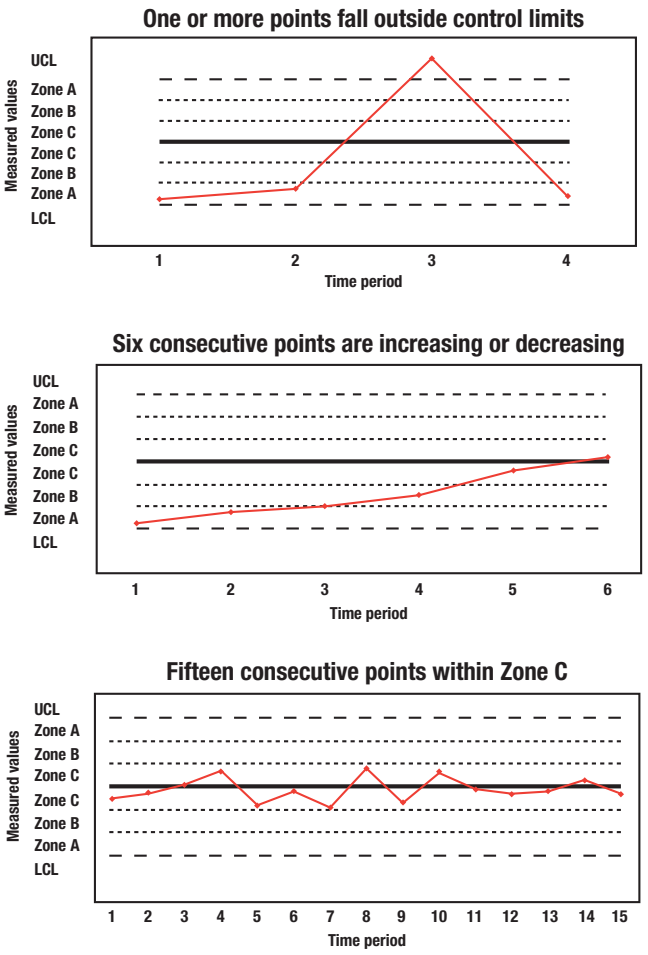


Figure 14.5 Examples of out-of-control situations.

Table 14.3 The most commonly used Shewhart control charts

Symbol	Description	Sample size
Variable charts		
X-bar and *R*	The average (mean) and range of measurements in a sample	Must be constant
X-bar and *S*	The average (mean) and standard deviation of measurements in a sample	May be variable
Attributes charts		
p	The percent of defective (nonconforming) units in a sample	May be variable
np	The number of defective (nonconforming) units in a sample	Must be constant
c	The number of defects in a sample	Must be constant
u	The number of defects per unit	May be variable

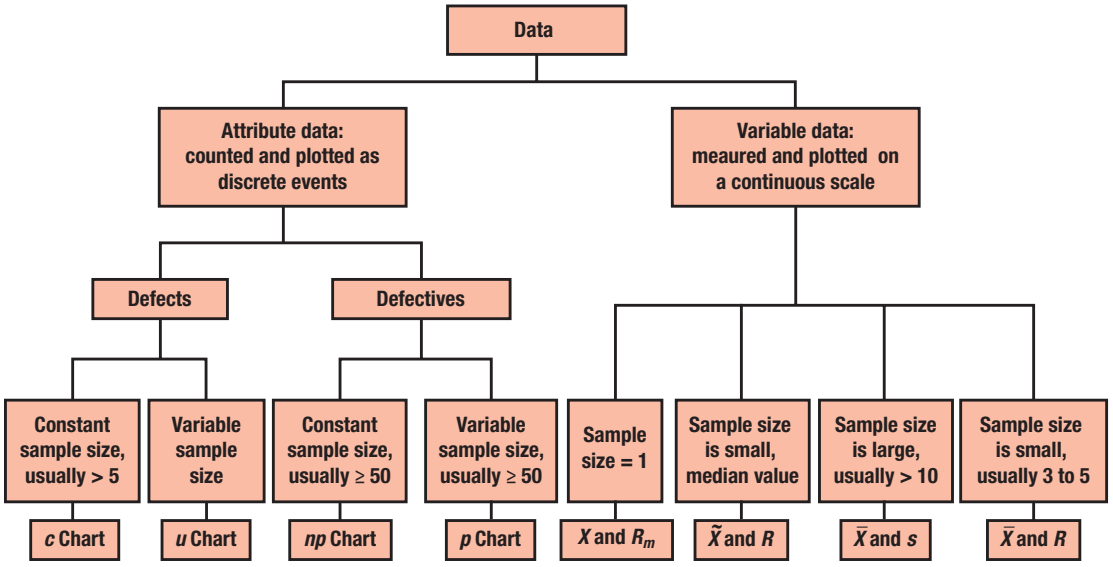


Figure 14.6 A process to select the appropriate control chart.

number of defective units or number of nonconformities per unit, are attributes. The most commonly used Shewhart control charts for both attributes and variables are summarized in Table 14.3.

To draw a control chart a series of guidelines must be considered. Different kinds of control charts can be selected, considering different kinds of data. Figure 14.6 and Figure 14.7 show a guideline to select the control.

To construct a control chart, follow the steps shown in Figure 14.8. To calculate appropriate statistics, it is necessary to know the method being used and the constants for that method. Constants and different formulae used in construction control charts are shown in Table 14.4 and Table 14.5 for variable and attribute data, respectively. Table 14.6 and Table 14.7 give the values of the constants needed for the variable control charts.

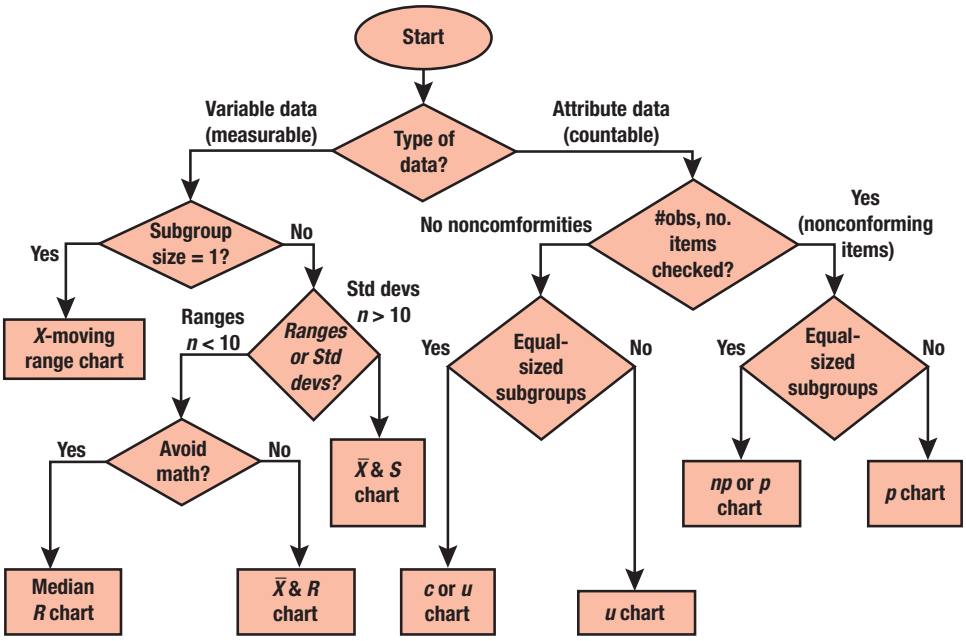


Figure 14.7 Guidelines for selecting control charts.

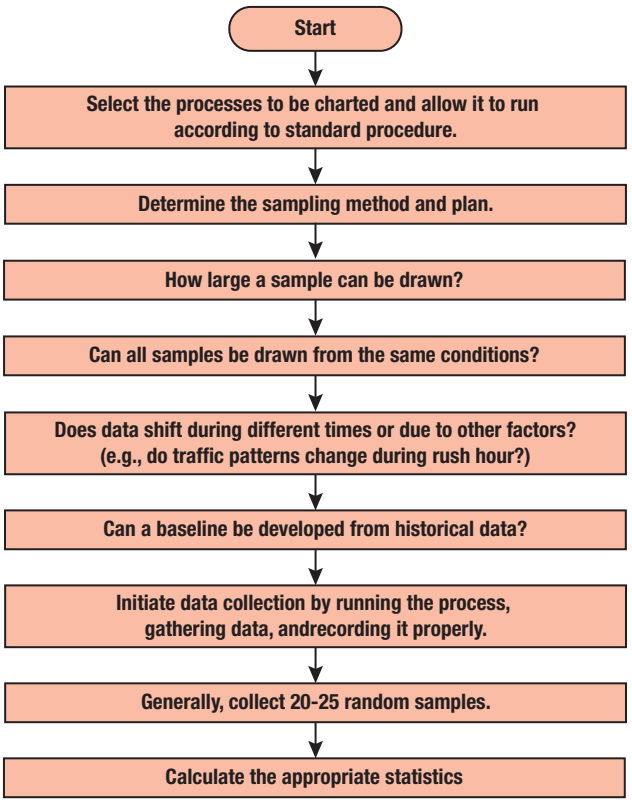


Figure 14.8 Ten steps in control chart construction.

Table 14.4 Variable data table

Type control chart	Sample size	Central line	Control limits
Average and range *X*-bar and *R*	<10, but usually 3–5	$\bar{\bar{X}} = \frac{(\bar{X}_1 + \bar{X}_2 + \cdots + \bar{X}_k)}{k}$ $\bar{R} = \frac{(R_1 + R_2 + \cdots + R_k)}{k}$	$\text{UCL}_{\bar{x}} = \bar{\bar{X}} + A_2\bar{R}$ $\text{LCL}_{\bar{x}} = \bar{\bar{X}} - A_2\bar{R}$ $\text{UCL}_R = D_4\bar{R}$ $\text{LCL}_R = D_3\bar{R}$
Average and standard deviation *X*-bar and *s*	Usually > or = 10	$\bar{\bar{X}} = \frac{(\bar{X}_1 + \bar{X}_2 + \cdots + \bar{X}_k)}{k}$ $\bar{S} = \frac{(S_1 + S_2 + \cdots + S_k)}{k}$	$\text{UCL}_{\bar{x}} = \bar{\bar{X}} + A_3\bar{S}$ $\text{LCL}_{\bar{x}} = \bar{\bar{X}} - A_3\bar{S}$ $\text{UCL}_S = B_4\bar{S}$ $\text{LCL}_S = B_3\bar{S}$
Median and range $\tilde{X}$ and *R*	<10, but usually 3–5	$\bar{\tilde{X}} = \frac{(\tilde{X}_1 + \tilde{X}_2 + \cdots + \tilde{X}_k)}{k}$ $\bar{R} = \frac{(R_1 + R_2 + \cdots + R_k)}{k}$	$\text{UCL}_{\tilde{x}} = \bar{\tilde{X}} + \tilde{A}_2\bar{R}$ $\text{LCL}_{\tilde{x}} = \bar{\tilde{X}} - \tilde{A}_2\bar{R}$ $\text{UCL}_R = D_4\bar{R}$ $\text{LCL}_R = D_3\bar{R}$
Individuals and moving range	1	$\bar{X} = \frac{(X_1 + X_2 + \cdots + X_k)}{k}$ $R_m = \lvert X_{i=1} - X_i \rvert$ $\bar{R}_m = \frac{(R_1 + R_2 + \cdots + R_{k-1})}{k-1}$	$\text{UCL}_x = \bar{X} + E_2\bar{R}_m$ $\text{LCL}_x = \bar{X} - E_2\bar{R}_m$ $\text{UCL}_{R_m} = D_4\bar{R}_m$ $\text{LCL}_{R_m} = D_3\bar{R}_m$

Table 14.5 Attribute data table

Type/control chart	Sample size	Central line	Control limits
Fraction defective *p* Chart	Variable, usually > or = 50	For each subgroup: $p = np/n$ For all subgroups: $\bar{p} = \frac{\sum np}{\sum n}$	$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ $\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
Number defective *np* Chart	Constant, usually > or = 50	For each subgroup: np = no. of defective units For all subgroups: $n\bar{p} = \frac{\sum np}{k}$	$\text{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ $\text{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$
Number of defects *c* Chart	Constant	For each subgroup: c = no. of defects For all subgroups: $\bar{c} = \frac{\sum c}{k}$	$\text{UCL}_c = \bar{c} + 3\sqrt{\bar{c}}$ $\text{LCL}_c = \bar{c} - 3\sqrt{\bar{c}}$
Number of defects per unit *u* Chart	Variable	For each subgroup: $u = c/n$ For all subgroups: $\bar{u} = \frac{\sum c}{\sum n}$	$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$ $LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$

Table 14.6 Table of constants for control charts

Sample size n	*X*-bar and *R*-bar chart			*X*-bar and *s*-bar chart			
	A_2	D_3	D_4	A_3	B_3	B_4	C_4
2	1.880	0	3.267	2.659	0	3.267	0.7979
3	1.023	0	2.574	1.954	0	2.568	0.8862
4	0.729	0	2.282	1.628	0	2.266	0.9213
5	0.577	0	2.114	1.427	0	2.089	0.9400
6	0.483	0	2.004	1.287	0.030	1.970	0.9000
7	0.419	0.076	1.924	1.182	0.118	1.882	0.9594
8	0.373	0.136	1.864	1.099	0.184	1.815	0.9650
9	0.337	0.184	1.816	1.032	0.239	1.761	0.9693
10	0.308	0.223	1.777	0.975	0.284	1.716	0.9727

Table 14.7 Table of constants for charts

Sample size n	*X*-median and *R*-bar chart			*X* and R_m chart			
	$\tilde{A}_2$	D_3	D_4	E_2	D_3	D_4	d_2
2	–	0	3.267	2.659	0	3.267	1.128
3	1.187	0	2.574	1.772	0	2.574	1.693
4	–	0	2.282	1.457	0	2.282	2.059
5	0.691	0	2.114	1.290	0	2.114	2.326
6	–	0	2.004	1.184	0	2.004	2.534
7	0.509	0.076	1.924	1.109	0.076	1.924	2.704
8	–	0.136	1.864	1.054	0.136	1.864	2.847
9	0.412	0.184	1.816	1.010	0.184	1.816	2.970
10	–	0.223	1.777	0.975	0.223	1.777	3.078

After identifying an out-of-control process, a series of actions must be taken in order to bring the process back to under control status. The following are common questions for investigating an out-of-control process. A team should consider any "yes" answer to the question as a potential source of a special cause:

- Are there differences in the measurement accuracy of instruments/methods used?
- Are there differences in the methods used by different personnel?
- Is the process affected by the environment—for example, temperature, and humidity?
- Has there been a significant change in the environment?
- Is the process affected by predictable conditions, such as tool wear?
- Were any untrained personnel involved in the process at the time?
- Has the source of input for the process changed (e.g., raw materials)
- Is the process affected by employee fatigue?
- Has there been a change in policies or procedures (e.g., maintenance procedures)?
- Is the process adjusted frequently?

- Did the samples come from different parts of the process/shifts/individuals?
- Are employees afraid to report "bad news"?

14.2.1 Control Charts for Variables

When a quality characteristic is measured as a variable, both the process mean and standard deviation must be monitored. For grouped data, use the X-bar chart to detect the process mean shift (between-group variability), and the R or S chart to monitor the process variation (within-group variability). The control limits of each chart are constructed based on the Shewhart model in Equation 14.1. When using X-bar, R, and S charts, assume that the underlying distribution of the quality characteristic is normal, and that the observations exhibit no correlation over time. If the quality characteristic is extremely nonnormal or the observations are autocorrelated, other control charts, such as the exponentially weighted moving average chart (EWMA) or the time series model (ARIMA), may be used instead.

In practice, the parameters of the underlying distribution of a quality characteristic are not known. The process mean and standard deviation are estimated based on the preliminary data. It can be shown that an unbiased estimate of the standard deviation is $\hat{\sigma} = \bar{s}/c_4$, where s-bar is the average sample standard deviation. A more convenient approach in quality control applications is the range method, where the range of the sample, R, is used to estimate the standard deviation, and is obtained as $\hat{\sigma} = \bar{R}/d_2$ where R-bar is the average value of the sample ranges. The resulting control charts using different estimators of standard deviation are the R chart and the S chart, respectively.

14.2.2 *X*-Bar and *R* Charts

When the sample size is not very large ($n < 10$), the X-bar and R charts, due to their simplicity of application, are widely used to monitor variable quality characteristics. In order to use the basic Shewhart model for X-bar and R charts, we need to estimate $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$, μ_R and σ_R first.

It is obvious that we can use the grand average to estimate μ_X and μ_R, that is, $\hat{\mu}_{\bar{X}} = \bar{\bar{X}}$ and $\hat{\mu}_R = \bar{R}$. Using the range method, $\hat{\sigma}_{\bar{x}} = \hat{\sigma}/\sqrt{n} = \bar{R}/\left(d_2\sqrt{n}\right)$ and $\hat{\sigma}_R = d_3\hat{\sigma} = d_3\bar{R}/d_2$. The control limits for X-bar and R charts are

$$\begin{aligned} LCL &= \bar{\bar{x}} - A_2\bar{R} \qquad & LCL &= D_3\bar{R} \\ CL &= \bar{\bar{x}} \qquad \text{and} & CL &= \bar{R} \\ ULC &= \bar{\bar{x}} + A_2\bar{R} \qquad & UCL &= D_4\bar{R}, \end{aligned}$$

respectively, where

$$A_2 = \frac{3}{d_2\sqrt{n}}, \quad D_3 = 1 - \frac{3d_3}{d_2}, \quad \text{and} \quad D_4 = 1 + \frac{3d_3}{d_2}.$$

The values of d_2, d_3, A_2, D_3, and D_4 can be obtained from most books on control charts for n up to 25 (Montgomery 2005). For a sample size up to 10, these values are given in Table 14.6 and Table 14.7. Normally, the preliminary data used to establish the control limits is about 20–25 samples, with a sample size of 3–5. The

Table 14.8 Organize data in a chart

Group no.	*A*	*B*	*C*	*D*	*E*	*X*-bar	*R*
1	1.4	1.2	1.3	1.4	1.2		
2	1.3	1.2	1.3	1.5	1.3		
3	1.7	1.3	1.4	1.2	1.2		
4	1.4	1.2	1.3	1.3	1.4		
5	1.5	1.1	1.7	1.3	1.3		
6	1.8	1.2	1.5	1.5	1.4		
7	1.5	1.2	1.3	1.3	1.2		
8	1.7	1.7	1.2	1.2	1.1		
9	1.8	1.8	1.7	1.8	1.5		
10	1.1	1.2	1.8	1.6	1.3		
11	1.2	1.3	1.4	1.4	1.4		
12	1.3	1.9	1.9	1.5	1.5		
13	1.4	1.8	1.7	1.1	1.3		
14	1.8	1.9	1.5	1.4	1.4		
15	1.1	1.3	1.1	1.8	1.5		
16	1.8	1.9	1.7	1.6	1.3		
17	1.2	1.4	1.3	1.2	1.4		
18	1.1	1.1	1.7	1.2	1.3		
19	1.8	1.6	1.5	1.7	1.8		
20	1.1	1.3	1.3	1.4	1.3		
Total							

established control limits are then used to check if the preliminary samples are in control. The *R* chart (or *S* chart) should be checked first to ensure that the process variability is in statistical control, and then the X-bar chart is checked for the process mean shift. Once a set of reliable control limits is constructed, they can be used for process monitoring.

14.2.2.1 **X-*Bar and* R *Chart Example*** In this example, information was needed to analyze the weight of a specific part made in a machine shop. The machine shop sampled the parts at twenty different times (groups), and each group had five measurements (samples), as given in Table 14.8. Since there are variable data with a constant sample size = 5, choose the X-bar and R charts.

Compute the Mean and Range for Each Group The mean (X-bar) = the sum of the samples within the group divided by the group size. For example, group 1 has an = (1.4 + 1.2 + 1.3 + 1.4 + 1.2) / 5 = 1.3. The range (R) = the difference between the largest observation within a group and the smallest observation within that group. The R-value for group 1 is $R_1 = (1.4 - 1.2) = 0.2$. The computed values of X-bar and R are given in Table 14.9.

Compute the Average Mean and Average Range The overall average (X-bar) = the total/total number of groups = 28.54/20 = 1.427. This is also called the grand average. This is used as the centerline for the chart. The average of all group ranges (R-bar) = the total R/total number of groups = 9.0/20 = 0.45 is used as the centerline (average) for the range chart.

Table 14.9 Add Calculated Data to the Chart

Group no.	A	B	C	D	E	X-bar	R
1	1.4	1.2	1.3	1.4	1.2	1.30	0.2
2	1.3	1.2	1.3	1.5	1.3	1.32	0.3
3	1.7	1.3	1.4	1.2	1.2	1.36	0.5
4	1.4	1.2	1.3	1.3	1.4	1.32	0.2
5	1.5	1.1	1.7	1.3	1.3	1.38	0.6
6	1.8	1.2	1.5	1.5	1.4	1.48	0.6
7	1.5	1.2	1.3	1.3	1.2	1.30	0.3
8	1.7	1.7	1.2	1.2	1.1	1.38	0.3
9	1.8	1.8	1.7	1.8	1.5	1.72	0.3
10	1.1	1.2	1.8	1.6	1.3	1.40	0.7
11	1.2	1.3	1.4	1.4	1.4	1.34	0.2
12	1.3	1.9	1.9	1.5	1.5	1.62	0.6
13	1.4	1.8	1.7	1.1	1.3	1.46	0.7
14	1.8	1.9	1.5	1.4	1.4	1.60	0.5
15	1.1	1.3	1.1	1.8	1.5	1.36	0.7
16	1.8	1.9	1.7	1.6	1.3	1.66	0.6
17	1.2	1.4	1.3	1.2	1.4	1.30	0.2
18	1.1	1.1	1.7	1.2	1.3	1.28	0.6
19	1.8	1.6	1.5	1.7	1.8	1.68	0.3
20	1.1	1.3	1.3	1.4	1.3	1.28	0.3
						28.54	9.0

Determine Control Limits

$$\mathrm{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 1.427 + (0.577 \times 0.45) = 1.687$$
$$\mathrm{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 1.427 - (0.577 \times 0.45) = 1.168.$$

About 99.73% (3 sigma limits) of the average values should fall between 1.168 and 1.687.

$$\mathrm{UCL}_R = D_4\bar{R} = 2.114 \times 0.45 = 0.951$$
$$\mathrm{LCL}_R = D_3\bar{R} = 0 \times 0.45 = 0.$$

About 99.73% (3 sigma limits) of the sample ranges should fall between 0 and 0.951. The *X*-bar chart is shown in Figure 14.9, and the *R* chart is shown in Figure 14.10. This shows that the average based on subgroup 9 is outside the upper control limit, and hence the process is out of control. We have to investigate the reasons for this situation and find the assignable causes and eliminate or remove them from the system.

14.2.3 Moving Range Chart Example

Now, we present an example of the moving range chart. In this example, information was needed to analyze the weights of a specific part made in the machine shop. Only one sample existed per observation. Since there are variable data and only one unit

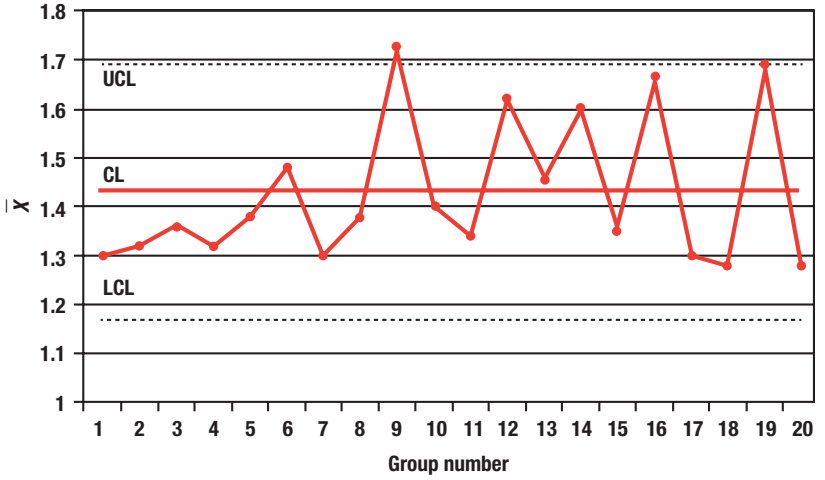


Figure 14.9 *X*-bar chart.

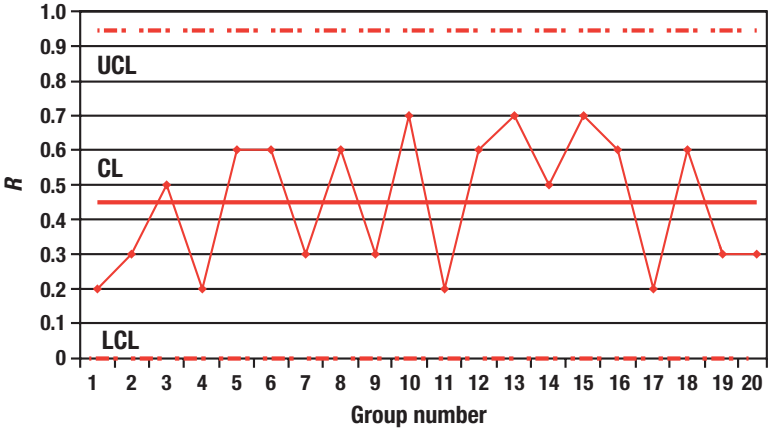


Figure 14.10 Range chart.

in each sample, the moving range chart is most appropriate. The data are given in Table 14.10. We also use the symbol R_m for moving range (MR) and they are used interchangeably in this chapter.

14.2.3.1 Compute the Moving Range (MR) MR = $|R_n - R_{n-1}|$ = Absolute value of the difference between consecutive range values. It is also known as the two-sample moving range (the most common form of moving range.) There is no range for the first observation. The first MR value works out to $MR_1 = |1.4 - 1.3| = 0.1$. The computed values of the MR are given in Table 14.11.

14.2.3.2 Compute the Average Mean and Group Range The overall average (*X*-bar) = sum of the measurements/number of observations = 28.90/20 = 1.45. Here, *X*-bar is also called the grand average, and *X*-bar is used as the centerline for the X chart. The average of all group ranges MR-bar = Total MR/number of ranges = 6.9/19 = 0.36. MR-bar is used as the centerline (average) for the MR chart.

Table 14.10 Organize data in a chart

Observation no.	Sample (*X*)	MR
1	1.4	
2	1.3	
3	1.7	
4	1.4	
5	1.5	
6	1.8	
7	1.5	
8	1.7	
9	1.8	
10	1.1	
11	1.2	
12	1.3	
13	1.4	
14	1.8	
15	1.1	
16	1.8	
17	1.2	
18	1.0	
19	1.8	
20	1.1	
Total	28.0	

Table 14.11 Add calculated data to the chart

Observation no.	Sample (X)	MR
1	1.4	N/A
2	1.3	0.1
3	1.7	0.4
4	1.4	0.3
5	1.5	0.1
6	1.8	0.3
7	1.5	0.3
8	1.7	0.2
9	1.8	0.1
10	1.1	0.7
11	1.2	0.1
12	1.3	0.1
13	1.4	0.1
14	1.8	0.4
15	1.1	0.7
16	1.8	0.7
17	1.2	0.6
18	1.0	0.2
19	1.8	0.8
20	1.1	0.7
Total	28.9	6.9

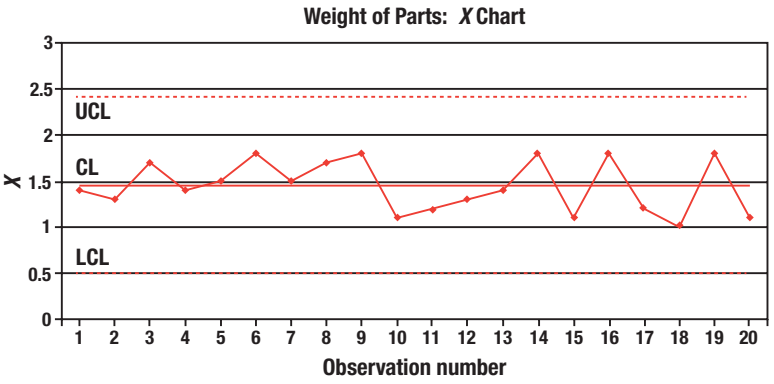


Figure 14.11 *X* chart.

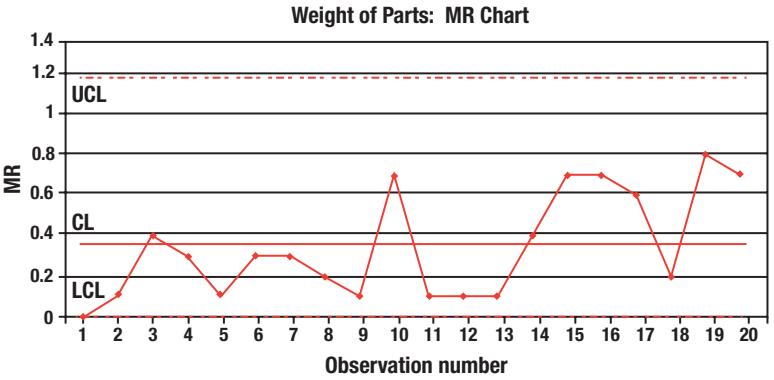


Figure 14.12 MR chart.

14.2.3.3 Determine Control Limits

$$\text{UCL}_X = \bar{X} + \left(E_2 \times \bar{MR}\right) = 1.45 + (2.659 \times 0.36) = 2.41$$
$$\text{LCL}_X = \bar{X} - \left(E_2 \times \bar{MR}\right) = 1.45 - (2.659 \times 0.36) = 0.49$$

$$\text{UCL}_{MR} = D_4 \times \bar{MR} = 3.267 \times 0.36 = 1.18$$
$$\text{LCL}_{MR} = D_3 \times \bar{MR} = 0 \times 0.36 = 0.$$

The sample size used to obtain the values for E_2, D_3, and D_4 is 2 in this case, since we are using a two-sample moving range. If a three-sample moving range is used, the number of ranges will reduce to 18, and the values of the constants used will change accordingly. The *X* chart is given in Figure 14.11, and the MR chart is given in Figure 14.12.

14.2.4 *X*-Bar and *S* Charts

When the sample size is relatively large ($n > 10$), or the sample size is variable, the *X*-bar and *S* charts are preferred to *X*-bar and *R* charts. To construct the control limits, first estimate the mean and standard deviation of *X*-bar and *S*—that is, $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$, μ_S and σ_S. We have $\hat{\mu}_{\bar{x}} = \bar{\bar{x}}$ and $\hat{\mu}_S = \bar{S}$. Using $\hat{\sigma} = s/c_4$, we have $\hat{\sigma}_{\bar{x}} = \hat{\sigma}/\sqrt{n} = \bar{s}/\left(c_4\sqrt{n}\right)$, and $\hat{\sigma}_s = \bar{s}\sqrt{1-c_4^2}/c_4$. Therefore, the control limits for *X*-bar and *S* charts are

$$\begin{aligned} \text{LCL} &= \bar{\bar{x}} - A_3\bar{S} & \qquad \text{LCL} &= B_3\bar{S} \\ \text{CL} &= \bar{\bar{x}} \quad \text{and} & \text{CL} &= \bar{S} \\ \text{ULC} &= \bar{\bar{x}} + A_3\bar{S} & \text{UCL} &= B_4\bar{S}, \end{aligned}$$

respectively, where

$$A_3 = \frac{3}{c_3\sqrt{n}}, \quad B_3 = 1 - \frac{3}{c_4}\sqrt{1-c_4^2}, \quad \text{and} \quad B_4 = 1 + \frac{3}{c_4}\sqrt{1-c_4^2}.$$

The values of c_4, A_3, B_3, and B_4 can be obtained from most books on control charts for n up to 25 (see Table 14.6).

14.2.5 Control Charts for Attributes

When quality characteristics are expressed as attribute data, such as defective or conforming items, control charts for attributes are established. Attribute charts can handle multiple quality characteristics jointly because the unit is classified as defective if it fails to meet the specifications on one or more characteristics. The inspection of samples for attribute charts is usually cheaper because it requires less precision. Attribute charts are particularly useful in quality improvement efforts where numerical data are not easily obtained, such as service industrial and health care systems. In the context of quality control, the attribute data include the proportion of defective items and the number of defects per item. A defective unit may have one or more defects due to nonconformance to standards with regard to one or more quality characteristics. Nevertheless, a unit with several defects may not necessarily be classified as a defective unit. This requires two different types of attribute charts: control charts for the proportion defective (*p* chart and *np* chart), and control charts for the number of defects (*c* chart and *u* chart).

14.2.6 *p* Chart and *np* Chart

The proportion defective is defined as the ratio of the number of defective units to the total number of units in a population. We usually assume that the number of defective units in a sample is a binomial variable—that is, each unit in the sample is produced independently, and the probability that a unit is defective is a constant, p. Using preliminary samples, we can estimate the defective rate—that is, $\bar{p} = \sum_{i=1}^{m} D_i/mn$—where D_i is the number of defective units in sample i, n is the sample size, and m is the number of samples taken. The formula used to calculate control limits is then

$$\begin{aligned} \text{UCL}_p &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\ \text{Centerline} &= \bar{p} \\ \text{LCL}_p &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}. \end{aligned}$$

Sometimes, it may be easier to interpret the number defective instead of the proportion defective. That is why the *np* chart came into use:

$$\begin{aligned} \text{UCL} &= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \\ \text{Centerline} &= n\bar{p} \\ \text{LCL} &= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}. \end{aligned}$$

The developed trial control limits are then used to check if the preliminary data are in statistical control, and assignable causes may be identified and removed if a point is out of control. As the process improves, we expect a downward trend in the *p* or *np* control chart.

14.2.7 *np* Chart Example

In this example, 10 weeks of defect data have been collected with a sample size of 50. Since we have attribute data with a constant sample size, we use the *np* chart. The data are given in Table 14.12.

14.2.7.1 Determine the Averages The average percent defective = *p*-bar = total defectives/totaled sampled.

$$\bar{p} = \frac{46}{(n)(\text{weeks})} = \frac{46}{(50)(10)} = 0.092.$$

The grand average = $n \times p$-bar (centerline) also = total defectives/total number of samples.

$$\begin{aligned} n\bar{p} &= (50)(0.092) = 4.6 \\ n\bar{p} &= 46/10 = 4.6. \end{aligned}$$

Table 14.12 Organize data in a chart

Week no.	Number of defectives
1	9
2	7
3	4
4	2
5	4
6	5
7	2
8	3
9	5
10	5
Total	46

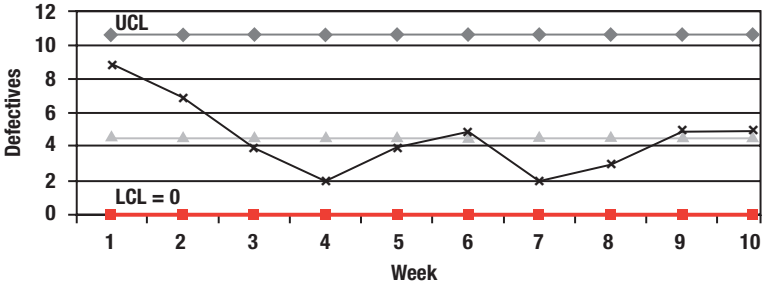


Figure 14.13 *np* chart.

14.2.7.2 Determine Control Limits

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 4.6 + 3\sqrt{4.6(1-0.092)} = 10.731$$
$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 4.6 - 3\sqrt{4.6(1-0.092)} = 0.$$

Note: Since LCL is less than zero, use zero. The *np* control chart is shown in Figure 14.13.

14.2.8 *c* Chart and *u* Chart

Control charts for monitoring the number of defects per sample are constructed based on the Poisson distribution. With this assumption of a reference distribution, the probability of occurrence of a defect at any area is small and constant, the potential area for defects is infinitely large, and defects occur randomly and independently. If the average occurrence rate per sample is a constant, c, both the mean and the variance of the Poisson distribution are the constant c. Therefore, the parameters in the c chart for the number of defects are

$$\text{LCL} = c - 3\sqrt{c}$$
$$\text{CL} = c$$
$$\text{UCL} = c + 3\sqrt{c},$$

where c can be estimated by the average number of defects in a preliminary sample. To satisfy the assumption of a constant rate of occurrence, the sample size must be constant.

For variable sample sizes, a u chart should be used instead of a c chart. Compared with the c chart, which is used to monitor the number of defects per sample, the u chart is designed to check the average number of defects per inspection unit. Usually, a sample may contain one or more inspection units. For example, in a textile finishing plant, dyed cloth is inspected for defects per 50 m^2, which is one inspection unit. A roll of cloth of 500 m^2 is thus one sample with 10 inspection units. Different rolls of cloth may vary in area; hence there is a variable sample size. As a result, it is not appropriate to use a c chart, because the occurrence rate of defects in each sample is not a constant. The alternative is to monitor the average number of defects per

Table 14.13 Organize data in a chart

Week no.	Number of specification
1	9
2	7
3	4
4	2
5	4
6	15
7	2
8	3
9	5
10	5
Total	56

inspection unit in a sample, $u_i = c_i/n_i$. In this way, the parameters in the u chart are given as

$$\begin{aligned} \text{LCL} &= \overline{u} - 3\sqrt{\frac{\overline{u}}{n}} \\ \text{CL} &= \overline{u} \\ \text{UCL} &= \overline{u} + 3\sqrt{\frac{\overline{u}}{n}}, \end{aligned}$$

where $\overline{u} = \sum_{i=1}^{m} u_i / m$ is an estimation of the average number of defects in an inspection unit. For variable sample sizes, the upper and lower control limits vary for different n values.

14.2.9 c Chart Example

In this example, a company tracks the number of times a specification was changed by either an engineering change proposal (ECP) or by a letter from the contracting officer. The attribute data summarize changes to 50 contracts over a 10-week period (as shown in Table 14.13). Since we have attribute data with a constant sample size, and the number of changes is represented by the number of defects, we use a c chart.

14.2.9.1 Determine Centerline (C bar) and Control Limits

C bar = Total defects found/total number of groups = 56/10 = 5.6 (changes per week). Determine control limits. If LCL is less than zero, set LCL = 0.

$$\begin{aligned} \text{UCL} &= \overline{c} + 3\sqrt{\overline{c}} = 5.6 + 3\sqrt{5.6} = 12.699 \\ \text{LCL} &= \overline{c} - 3\sqrt{\overline{c}} = 5.6 - 3\sqrt{5.6} = 0. \end{aligned}$$

14.2.9.2 Draw the c Chart

The c chart is shown in Figure 14.14.

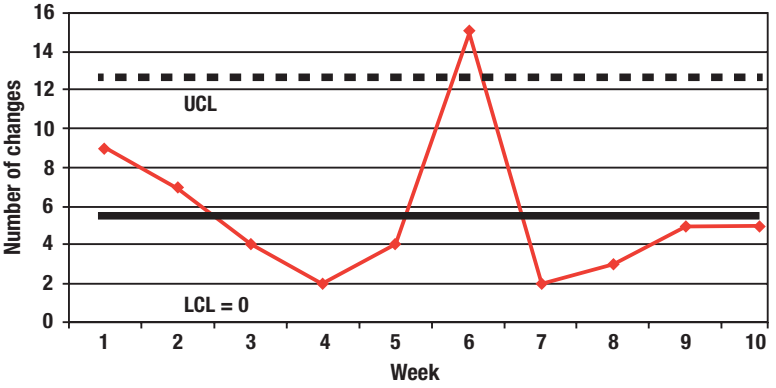


Figure 14.14 *c* chart.

14.3 Benefits of Control Charts

In this section, we summarize some of the important benefits that can come from using control charts.

- Control charts are simple and effective tools to achieve statistical control. They lend themselves to being maintained at the job station by the operator. They give the people closest to the operation reliable information on when action should and should not be taken.
- When a process is in statistical control, its performance to specification will be predictable. Thus, both producer and customer can rely on consistent quality levels, and both can rely on stable costs for achieving that quality level.
- After a process is in statistical control, its performance can be further improved to reduce variation. The expected effects of proposed improvements in the system can be anticipated, and the actual effects of even relatively subtle changes can be identified through the control chart data. Such process improvements will:
  - Increase the percentage of output that meets customer expectations (improve quality).
  - Decrease the output requiring scrap or rework (improve cost per good unit produced).
  - Increase the total yield of acceptable output through the process (improve effective capacity).
- Control charts provide a common language for communication about the performance of a process between the two or three shifts that operate a process; between line production (operator and supervisor) and support activities (maintenance, material control, process engineering, and quality control); between different stations in the process; between supplier and user; and between the manufacturing/assembly plant and the design engineering activity.
- Control charts, by distinguishing special from common causes of variation, give a good indication of whether any problems are likely to be correctable

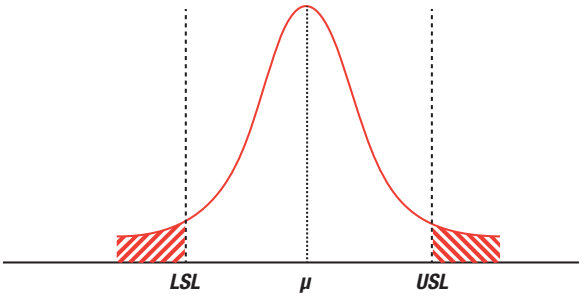


Figure 14.15 Average outgoing quality.

locally or will require management action. This minimizes the confusion, frustration, and excessive cost of misdirected problem-solving efforts.

14.4 Average Outgoing Quality

A measure of part quality is average outgoing quality (AOQ). It is typically defined as the total number of parts per million (ppm) that are outside manufacturer specification limits during the final quality control inspection. A high AOQ indicates a high defective count, and therefore a poor quality level.

For example, manufacturers conduct visual, mechanical, and electrical tests to measure the AOQ of electronic parts. Visual and mechanical tests review marking permanency, dimensions, planarity, solderability, bent leads, and hermeticity (if applicable). Electrical tests include functional and parametric tests at room temperature, high temperature, and low temperature. AOQ is defined in Equation 14.2, referring to Figure 14.15.

$$\text{AOQ} = \frac{\text{Shaded area under the process curve}}{\text{Total area under the process curve}} \times 10^6, \tag{14.2}$$

where *USL* is the upper specification limit, *LSL* is the lower specification limit, and μ is the process mean.

The formulae for AOQ calculations may differ among manufacturers. Xilinx provides AOQ based on JEDEC Standard JESD 16–A [2], which is

$$\begin{aligned} \text{AOQ} &= P \times \text{LAR} \times 10^6 \\ P &= \frac{D}{N} \\ \text{LAR} &= \frac{\text{AL}}{\text{TL}}. \end{aligned} \tag{14.3}$$

where D is the total number of defective parts, N is the total number of parts tested, LAR is the lot acceptance rate, AL is the total number of accepted lots, and TL is the total number of lots tested. IDT provided AOQ based on the following formula:

$$\begin{aligned} \text{AOQ} &= P \times 10^6 \\ P &= \frac{D}{N}. \end{aligned} \tag{14.4}$$

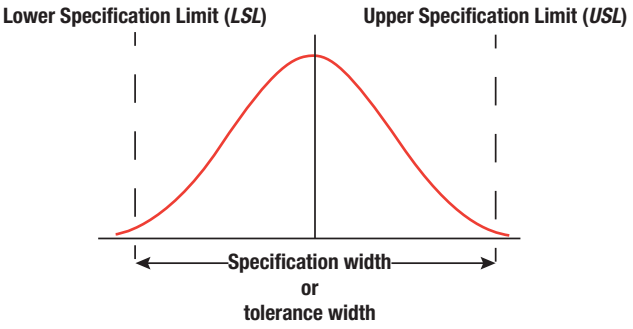


Figure 14.16 Measuring conformance to the customer requirements.

14.4.1 Process Capability Studies

AOQ is a measure of the quality of parts as they leave the production facility. Process capability is a measure of conformance to customer requirements and is typically measured at key process steps. A process capability assessment is conducted to determine whether a process, given its natural variation, is capable of meeting established customer requirements or specifications. It can help to identify changes that have been done in the process, and determine the percent of product or service that does not meet the customer requirements. If the process is capable of making products that conform to the specifications, the specifications can remain the same.

Figure 14.16 shows the specification limits of a product. Specification limits are used to determine if the products will meet the expectations of the customer. Recognize that these specification limits are based solely on the customer requirements and are not meant to reflect on the capability of the process. Figure 14.16 overlays a normal distribution curve on top of the specification limits. In all mathematics related to process capability, an underlying normal distribution of the parameters being examined is assumed.

1. To determine the process capability, the following steps are followed. Determine the process grand average, $\bar{\bar{X}}$, and the average range, *R*-bar.
2. Determine the *USL* and the *LSL*.
3. Calculate the process standard deviation, σ, from the information on the control chart by

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \quad \text{or} \quad \hat{\sigma} = \frac{\bar{s}}{c_4}, \tag{14.5}$$

where *R*-bar and *s*-bar are the averages of the subgroup ranges and standard deviation for a period when the process was known to be in control, and d_2 and c_4 are the associated constant values based on the subgroup sample sizes. The process average can be estimated by $\bar{\bar{X}}$ or $\tilde{\bar{X}}$.

A stable process can be represented by a measure of its variation—six standard deviations. Comparing six standard deviations of the process variation to the customer specifications provides a measure of capability. Some measures of capability include C_p, C_r (inverse of C_p), C_{pl}, C_{pu}, and C_{pk}. C_p is calculated using the following formula:

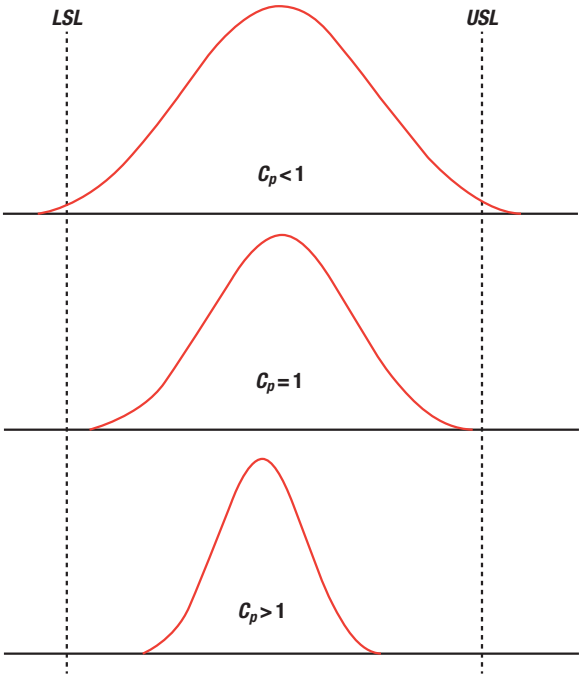


Figure 14.17 *Cp*, simple process capability.

$$C_P = \frac{USL - LSL}{6\hat{\sigma}}. \tag{14.6}$$

When $C_p < 1$, the process variation exceeds specification and defectives are being made, as shown in Figure 14.17. When $C_p = 1$, the process is just meeting specification. A minimum of 0.27% defectives will be made, more if the process is not centered. When $C_p > 1$, the process variation is less than the specification; however, defectives might be made if the process is not centered on the target value.

The indices C_{pl} and C_{pu} (for single-sided specification limits) and C_{pk} (for two-sided specification limits) measure not only the process variation with respect to the allowable specification, but they also take into account the location of the process average. Capability describes how well centered the curve is in the specification spread and how tight the variation is. C_{pk} is considered a measure of the process capability and is the smaller of either C_{pl} or C_{pu}. If the process is near normal and in statistical control, C_{pk} can be used to estimate the expected percentage of the defective products.

$$C_{pl} = \frac{\bar{\bar{X}} - LSL}{3\hat{\sigma}}, \quad C_{pu} = \frac{USL - \bar{\bar{X}}}{3\hat{\sigma}} \tag{14.7}$$

$$C_{pk} = \min\{C_{pu}, C_{pl}\}. \tag{14.8}$$

Figure 14.18 shows an example of a process not capable of meeting targets. For the process in this figure, $C_p > 1$, but the incapability of the process arises because the process is not centered between *LSL* and *USL*.

If the process is capable of consistently making parts to specification, common causes of the variation in the process must be identified and corrected. Examples of

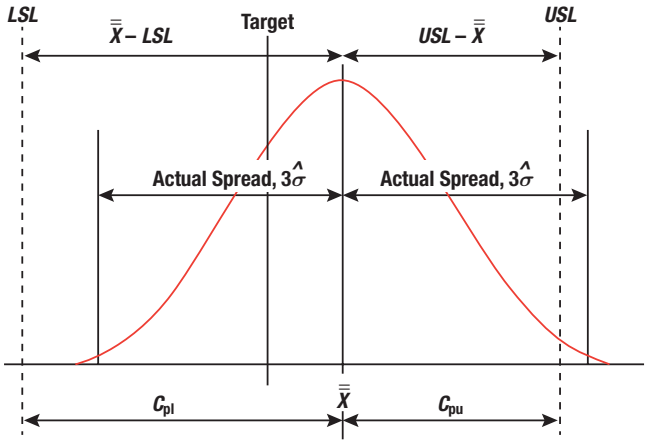


Figure 14.18 Process not capable of meeting specifications.

common remedies include assigning another machine to the process, procuring a new piece of equipment, providing additional training to reduce operator variations, or requiring vendors to implement statistical process controls. In some cases the process may have to be changed, or the specification may have to be relaxed or broadened.

Example 14.1

In a die-cutting process, a control chart was maintained, producing the following statistics: $\bar{\bar{X}} = 212.5$, $\bar{R} = 1.2$, and $n = 5$. The specification limit for this process is 210 ± 3; that means that $USL = 213$, and $LSL = 207$. Calculate C_p and C_{pk} for this process. Also find the number of defects.

Solution:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{1.2}{2.326} = 0.516$$

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{213 - 207}{6(0.516)} = \frac{6}{3.096} = 1.938$$

$$C_{pl} = \frac{\bar{\bar{X}} - LSL}{3\hat{\sigma}} = \frac{212.5 - 207}{3(0.516)} = \frac{5.5}{1.548} = 3.553$$

$$C_{pu} = \frac{USL - \bar{\bar{X}}}{3\hat{\sigma}} = \frac{213 - 212.5}{3(0.516)} = \frac{0.5}{1.548} = 0.323$$

$$C_{pk} = \min\{C_{pl}, C_{pu}\} = 0.323.$$

Since $C_{pk} < 1$, defective material is being made. Figure 14.19 shows the schematic of the problem.

Defects Calculation:
If the process is near normal and in statistical control, the process of calculating C_{pk} can also be used to estimate the expected percent of defective material. The area under

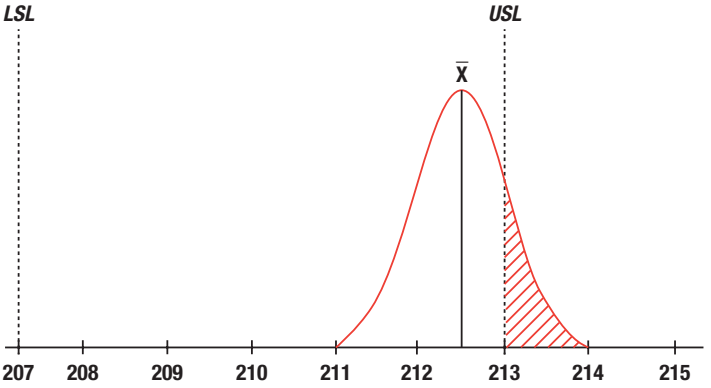
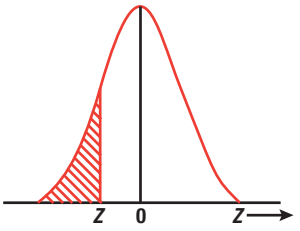


Figure 14.19 Process not capable.

z	0	0.02	0.04	0.06	0.08
-4.00	0.0000	0.0000	0.0000	0.0000	0.0000
-3.80	0.0001	0.0001	0.0001	0.0001	0.0001
:	:	:	:	:	:
-3.00	0.0013	0.0014	0.0015	0.0016	0.0018
-2.90	0.0019	0.0020	0.0021	0.0023	0.0024
-2.80	0.0026	0.0027	0.0029	0.0031	0.0033
-2.70	0.0035	0.0037	0.0039	0.0041	0.0044
:	:	:	:	:	:
0.00	0.5000	0.5080	0.5160	0.5239	0.5319
0.10	0.5398	0.5478	0.5557	0.5636	0.5714
:	:	:	:	:	:
0.70	0.7580	0.7642	0.7704	0.7764	0.7823
0.80	0.7881	0.7939	0.7995	0.8051	0.8106
0.90	0.8159	0.8212	0.8264	0.8315	0.8365
1.00	0.8413	0.8461	0.8508	0.8554	0.8599
:	:	:	:	:	:
2.80	0.9974	0.9976	0.9977	0.9979	0.9980
2.90	0.9981	0.9982	0.9984	0.9985	0.9986
3.00	0.9987	0.9987	0.9988	0.9989	0.9990



F(z) = P(Z < z) is the area under the curve to the left of the z-value (shaded area)

F(z) = P(Z < z)

P(Z < 0.969) ≈ 0.832

Figure 14.20 Sample cumulative normal distribution table.

the curve outside the specification limits is used to determine the number of defects. To determine the area under the curve, the following factors must be calculated:

$$z_1 = \frac{LSL - \bar{\bar{X}}}{\hat{\sigma}} = \frac{207 - 212.5}{0.516} = -10.68$$

$$z_2 = \frac{USL - \bar{\bar{X}}}{\hat{\sigma}} = \frac{213 - 212.5}{0.516} = 0.969.$$

Defects for the value of $z < LSL = \Phi(z_1) = 0$ (approximately). Defects for the value of $z > USL = [1 - \Phi(z_2)]$. Here $[1 - \Phi(z_2)] = [1 - 0.832] = 0.168$. Here, $\Phi(z) = P(Z < z)$

is the cumulative distribution value for any value of z obtained from the standard normal distribution table.

$$\text{Total defects} = \Phi(z_1) + [1 - \Phi(z_2)] = 16.8\%.$$

Example 14.2

We have the following information for a process

$$\hat{\mu} = 0.738, \quad \hat{\sigma} = 0.0725, \quad USL = 0.9, \quad \text{and} \quad LSL = 0.5.$$

Since the process has two-sided specification limits,

$$\begin{aligned} Z_{\min} &= \min\left(\frac{USL - \hat{\mu}}{\hat{\sigma}}, \frac{\hat{\mu} - LSL}{\hat{\sigma}}\right) \\ &= \min\left(\frac{0.9 - 0.738}{0.0725}, \frac{0.738 - 0.5}{0.0725}\right) \\ &= \min(2.23, 3.28) = 2.23, \end{aligned}$$

and the proportion of process fallout would be:

$$p = 1 - \Phi(2.23) + \Phi(-3.28) = 0.0129 + 0.0005 = 0.0134.$$

The process capability index would be:

$$C_{pk} = \frac{Z_{\min}}{3} = 0.74.$$

If the process could be adjusted toward the center of the specification, the proportion of process fallout might be reduced, even with no change in σ:

$$\begin{aligned} Z_{\min} &= \min\left(\frac{USL - \hat{\mu}}{\hat{\sigma}}, \frac{\hat{\mu} - LSL}{\hat{\sigma}}\right) \\ &= \min\left(\frac{0.9 - 0.7}{0.0725}, \frac{0.7 - 0.5}{0.0725}\right) = 2.76, \end{aligned}$$

and the proportion of process fallout would be:

$$p = 2\Phi(-2.76) = 0.0058.$$

The process capability index would be:

$$C_{pk} = \frac{Z_{\min}}{3} = 0.92.$$

To improve the actual process performance in the long run, the variation from common causes must be reduced. To consider variability in terms of mean, standard deviation, and the target value, another index is defined as:

$$\hat{C}_{pm} = \frac{USL - LSL}{6\hat{\tau}},$$

where $\hat{\tau}$ is an estimator of the expected square deviation from the target, T, and is given by

$$\tau^2 = E\left[(X-T)^2\right] = \sigma^2 + (\mu - T)^2.$$

Therefore, if we know the estimate of C_p, we can estimate C_{pm} as:

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+\left(\frac{\hat{\mu}-T}{\hat{\sigma}}\right)^2}}.$$

At this point, the process has been brought into statistical control and its capability has been described in terms of the process capability index, or $Z_{\min}$. The next step is to evaluate the process capability in terms of meeting customer requirements. The fundamental goal is never-ending improvement in process performance. In the near term, however, priorities must be set as to which processes should receive attention first. This is essentially an economic decision. The circumstances vary from case to case, depending on the nature of the particular process in question. While each such decision could be resolved individually, it is often helpful to use broader guidelines to set priorities and promote consistency of improvement efforts. For instance, certain procedures require $C_{pk} > 1.33$, and further specify $C_{pk} = 1.50$ for new processes. These requirements are intended to assure a minimum performance level that is consistent among characteristics, products, and manufacturing sources.

Whether in response to a capability criterion that has not been met, or in response to the continuing need for improvement in cost and quality performance even beyond the minimum capability requirement, the action required is the same: Improve the process performance by reducing the variation that comes from common causes. This means taking management action to improve the system.

14.5 Advanced Control Charts

In order to effectively detect small process shifts (on the order of 1.5σ or less), a cumulative sum (CUSUM) control chart and the exponentially weighted moving average (EWMA) control chart may be used instead of Shewhart control charts. In addition, there are many situations where we need to simultaneously monitor two or more correlated quality characteristics. The control charts for multivariate quality characteristics will also be discussed in the next section.

The major disadvantage of the Shewhart control chart is that it uses the information in the last plotted point and ignores information given by the sequence of points. This makes it insensitive to small shifts. Thus, either the CUSUM or EWMA charts may be more useful.

14.5.1 Cumulative Sum Control Charts

CUSUM control charts incorporate all the information in the sequence of sample values by plotting the CUSUM of deviations of the sample values from a target value, defined as

$$C_i = \sum_{j=1}^{i} (\bar{x}_j - T). \tag{14.9}$$

A significant trend developed in C_i is an indication of the process mean shift. Therefore, CUSUM control charts would be more effective than Shewhart charts in detecting small process shifts. Two statistics are used to accumulate deviations from the target, T:

$$\begin{aligned} C_i^+ &= \max\left[0, \quad x_i - (T + K) + C_{i-1}^+\right] \\ C_i^- &= \max\left[0, \quad (T - K) - x_i + C_{i-1}^-\right], \end{aligned} \tag{14.10}$$

where $C_0^+ = C_0^- = 0$, and K is the slack value; it is often chosen about halfway between the target value and the process mean after the shift. If either C^+ or C^- exceeds the decision interval H (a common choice is $H = 5\sigma$), the process is considered to be out of control.

14.5.2 Exponentially Weighted Moving Average Control Charts

As discussed earlier, using Western electric rules increases the sensitivity of Shewhart control charts to detect nonrandom patterns or small shifts in a process. A different approach to highlight small shifts is to use a time average over past and present data values as an indicator of recent performance. The exponentially weighted moving average (EWMA) indicator considers the past data values and remembers them with geometrically decreasing weight. For example, we denote the present and past values of a quality characteristic, x, by x_t, x_{t-1}, x_{t-2}, . . . , then the EWMA y_t with discount factor q is

$$y_t = a\left(x_t + qx_{t-1} + q^2 x_{t-2} + \cdots\right), \tag{14.11}$$

where a is a constant that makes the weights add up to 1 and is equal to $1 - q$. In the practice of process monitoring, the constant $1 - q$ is given the distinguishing symbol λ. Using λ, the EWMA can be expressed as $y_t = \lambda x_t + (1 - \lambda)\, y_{t-1}$, which is a more convenient formula for updating the value of EWMA at each new observation. It is observed from the formula that a larger value of λ results in weights that die out more quickly and places more emphasis on recent observations. Therefore, a smaller value of λ is recommended to detect small process shifts, usually $\lambda = 0.05$, 0.10, or 0.20. An EWMA control chart with appropriate limits is used to monitor the value of the EWMA. If the process is in statistical control with a process mean of μ and a standard deviation of σ, the mean of the EWMA would be μ, and the standard deviation of the EWMA would be

$$\sigma\left(\frac{\lambda}{2-\lambda}\right)^{1/2}.$$

Thus, given a value of λ, three-sigma or other appropriate limits can be constructed to monitor the value of EWMA.

14.5.3 Other Advanced Control Charts

The successful use of Shewhart control charts and the CUSUM and EWMA control charts has led to the development of many new techniques over the last 30 years. A brief summary of these techniques and references to more complete descriptions are provided here.

The competitive global market expects lower defect rates and higher quality levels, which requires 100% inspection of output products. The recent advancement of sensing techniques and computer capacity makes one hundred percent inspection more feasible. Due to the reduced intervals between sampling in a 100% inspection, the complete observations will be correlated over time. However, one of the assumptions for Shewhart control charts is the independence between observations over time. When the observations are autocorrelated, Shewhart control charts will give misleading results in the form of many false alarms. Time series models (ARIMA) are used to remove autocorrelation from the data, and then control charts are applied to the residuals. Further discussion of SPC with auto-correlated process data can be found in Box and Luceno (1997) and Montgomery (2005). It is often necessary to simultaneously monitor or control two or more related quality characteristics. Using individual control charts to monitor the independent variables separately can be very misleading. Multivariate SPC control charts were developed based on multivariate normal distribution by Hotelling (1947). The use of control charts requires the selection of sample size, sampling frequency, or interval between samples, and the control limits for the charts. The selection of these parameters has economic consequences in that the cost of sampling, the cost of false alarms, and the cost of removing assignable causes will affect the choice of the parameters. Therefore, the economic design of control charts has also been discussed in the literature.

14.6 Summary

Process control is an effective prevention strategy to manufacture products that will meet the requirements of the customer. There are four elements of process control systems: the process, information about performance, action on the process, and action on the output. The process refers to the combination of people, equipment, input materials, methods, and environment that work together to produce output. The total performance of the process depends on the way the process has been designed and built and on the way it is operated. Information about performance can be learned by studying the process output. Action on the process is future oriented, because it is taken when necessary to prevent the production of nonconforming products. Action on the output is past oriented, because it involves detecting out-of-specification output already produced.

A control chart is a type of trend chart that displays data over time with statistically determined upper and lower control limits; it is used to determine if a process is under control. Control charts by themselves do not correct problems. They indicate that something is wrong and requires corrective action. Assignable causes due to a change in manpower, materials, machines, or methods, or a combination of these, can cause the process to go out of control.

Control charts must be investigated in order to identify in-control and out-of-control processes and detect common causes and special causes of the out-of-control

state. A process is said to be operating in statistical control when the only source of variation are common causes. An out-of-control signal is given when a sample statistic falls beyond the control limits, or when a nonrandom pattern is detected.

Process capability is determined by the total variation that comes from common causes. A process must first be brought into statistical control before its capability to meet specifications can be assessed. Process capability is a measure of conformance to customer requirements and is typically measured at key process steps. A process capability assessment is conducted to determine whether a process, given its natural variation, is capable of meeting established customer requirements or specifications. It can help to identify changes that have been done in the process and determine the percent of product or service that does not meet the customer requirements.

The process control and process capability techniques described in this chapter can help to ensure the production of quality products. These techniques can help manufacturers to avoid waste by not producing unacceptable output in the first place, focusing on prevention rather than screening. Statistical process control is an effective prevention strategy to manufacture products that will meet the requirements of customers.

Problems

14.1 For each of the datasets given, identify which of the following control charts should be used to plot the data for process control: *c* chart, *u* chart, *p* chart, *np* chart, *X*-bar–*R* chart, or X–R_m chart. For each case, state why you selected the particular chart type.

a	An equal number of samples of process output have been monitored each week for the last 5 weeks. Ten defective parts were found the first week, eight the second week, six the third week, nine the fourth week, and seven the fifth week.
b	Different numbers of samples (between 40 and 60) of process output have been monitored each week for the last 4 weeks. In the first week, 1.2 defects per sample were observed. In the second week, 1.5 defects per sample were observed. In the third week, 1 defect per sample was observed. In the fourth week, 0.8 defects per sample were observed.
c	The thicknesses of 10 samples were measured each day for a week.
d	An equal number of samples of process output have been monitored each week for the last four weeks. In the first week, 8 defects were observed. In the second week, 12 defects were observed. In the third week, 10 defects were observed. In the fourth week, 9 defects were observed.
e	The thickness of a single sample was measured each day for a week.
f	A process has been observed each week for the last 3 weeks. Ten percent of the parts were found to be defective the first week, 20% were found to be defective the second week, and 15% were found to be defective the third week

14.2 The copper content of a plating bath is measured three times per day and the results are reported in ppm. The *X*-bar and *R*-values for 10 days are shown in the following tables.

Day	X-bar	R
1	5.45	1.21
2	5.39	0.95
3	6.85	1.43
4	6.74	1.29
5	5.83	1.35
6	7.22	0.88
7	6.39	0.92
8	6.50	1.13
9	7.15	1.25
10	5.92	1.05

(a) Determine the upper and lower control limits.

(b) Is the process in statistical control?

(c) Estimate the C_p and C_{pk} given that the specification is 6.0 ± 1.0. Is the process capable?

14.3 Printed circuit boards are assembled by a combination of manual assembly and automation. The reflow soldering process is used to make the mechanical and electrical connections of the leaded components to the board. The boards are run through the solder process continuously, and every hour five boards are selected and inspected for process-control purposes. The number of defects in each sample of five boards is noted. The results for 20 samples are shown in the table. What type of control chart is appropriate for this case and why? Construct the control chart limits and draw the chart. Is the process in control? Does it need improvement?

Sample no.	No. of defects	Sample no.	No. of defects
1	6	11	9
2	4	12	15
3	8	13	8
4	10	14	10
5	9	15	8
6	12	16	2
7	16	17	7
8	2	18	1
9	3	19	7
10	10	20	13

14.4 The number of nonconforming switches in samples of size 150 is shown here. Construct a fraction nonconforming control chart for these data. Does the process appear to be in control? If not, assume that assignable causes can be found for all points outside the control limits and calculate the revised control limits.

Sample no.	No. of noncomformings	Sample no.	No. of noncomformings
1	8	11	6
2	1	12	0
3	3	13	4
4	0	14	0
5	2	15	3
6	4	16	1
7	0	17	15
8	1	18	2
9	10	19	3
10	6	20	0

14.5 The diameter of a shaft with nominal specifications of 60 ± 3 mm is measured six times each hour and the results are recorded. The X-bar and R values for 8 hours are shown in the table below:

Hour	X-bar	R
1	62.54	1.95
2	60.23	2.03
3	58.46	1.43
4	59.95	1.29
5	61.58	0.78
6	57.93	1.48
7	61.56	0.86
8	57.34	1.35

(a) Determine the upper and lower control limits.

(b) Determine if the process is in statistical control.

(c) Estimate the C_p and C_{pk} for the process. Is the process capable?

14.6 The specification for a shaft diameter is 212 ± 2 mm. Provided below are 30 recorded observations for the diameter of a shaft (in mm) taken at 30 different points in time.

First observation: 212.1	214.2	213.7	212.7	212.5	Sixth observation: 212.7
212.8	213.0	212.9	212.3	212.5	212.1
211.8	213.5	212.0	213.0	214.5	212.3
212.2	211.9	213.2	212.7	211.9	212.3
212.0	212.8	213.9	212.6	214.0	Thirtieth observation: 212.4

(a) Develop X and three sample MR charts and determine control limits from the data.

(b) Determine from the control charts whether the process is under control or not.

(c) Determine the capability indices (C_p and C_{pk}) for the process.

(d) Determine the percent defective shafts produced by the process.

14.7 A high-voltage power supply should have a normal output voltage of 350 V. A sample of four units is selected each day and tested for process-control purposes. The data shown give the difference between the observed reading on each unit and the nominal voltage times ten; that is X_i = (observed voltage on unit i − 350) × 10.

Sample no.	X_1	X_2	X_3	X_4
1	6	9	10	15
2	10	4	6	11
3	7	8	10	5
4	8	9	6	13
5	9	10	7	13
6	12	11	10	10
7	16	10	8	9
8	7	5	10	4
9	9	7	8	12
10	15	16	10	13
11	8	12	14	16
12	6	13	9	11
13	16	9	13	15
14	7	13	10	12
15	11	7	10	16
16	15	10	11	14
17	9	8	12	10
18	15	7	10	11
19	8	6	9	12
20	13	14	11	15

(a) Set up X-bar and R charts on this process. Does this process seem to be in statistical control? If necessary, revise the trial control limits.

(b) If specifications are at 350 ± 5 V, what can you say about process capability?

14.8 Vane-opening measurements are as follows. Set up X-bar and s charts on this process. Does this process seem to be in statistical control? If necessary, revise the trial control limits.

Sample no.	X_1	X_2	X_3	X_4	X_5	X-bar	R	s
1	33	29	31	32	33	31.6	4	1.67
2	33	31	35	37	31	33.4	6	2.61
3	35	37	33	34	36	35.0	4	1.58
4	30	31	33	34	33	32.2	4	1.64
5	33	34	35	33	34	33.8	2	0.84
6	38	37	39	40	38	38.4	3	1.14
7	30	31	32	34	31	31.6	4	1.52
8	29	39	38	39	39	36.8	10	4.38
9	28	33	35	36	43	35.0	15	5.43
10	38	33	32	35	32	34.0	6	2.55
11	28	30	28	32	31	29.8	4	1.79
12	31	35	35	35	34	34.0	4	1.73

(*Continued*)

Sample no.	X_1	X_2	X_3	X_4	X_5	X-bar	R	s
13	27	32	34	35	37	33.0	10	3.81
14	33	33	35	37	36	34.8	4	1.79
15	35	37	32	35	39	35.6	7	2.61
16	33	33	27	31	30	30.8	6	2.49
17	35	34	34	30	32	33.0	5	2.00
18	32	33	30	30	33	31.6	3	1.52
19	25	27	34	27	28	28.2	9	3.42
20	35	35	36	33	30	33.8	6	2.39

14.9 A supply chain engineering group monitors shipments of materials through the company distribution network. Errors on either the delivered material or the accompanying documentation are tracked on a weekly basis. Fifty randomly selected shipments are examined and the errors are recorded. Data for 20 weeks are shown in the table below.

(a) Establish a *u* chart to monitor this process.

(b) Does this process seem to be in statistical control? If necessary, revise the trial control limits.

(c) Do we need to take any action based on our data? Why? If yes, what action?

Sample no.	Sample size	No. of errors X_i (nonconformities)	Average no. of units per unit, $u_i = x_i/n$
1	50	2	0.04
2	50	3	0.06
3	50	8	0.16
4	50	1	0.02
5	50	1	0.02
6	50	4	0.08
7	50	1	0.02
8	50	4	0.08
9	50	5	0.10
10	50	1	0.02
11	50	8	0.16
12	50	2	0.04
13	50	4	0.08
14	50	3	0.06
15	50	4	0.08
16	50	1	0.02
17	50	8	0.16
18	50	3	0.06
19	50	7	0.14
20	50	4	0.08
		74	1.48