11

11 Probabilistic Design for Reliability and the Factor of Safety

To ensure the production of a reliable product, reliability activities must start early in the product development cycle. In order to analyze the reliability of a product, we have to first understand how to analyze the reliability of its components. In order to achieve the desirable reliability level, various reliability methodologies and tools can be used throughout the life cycle of the product—from the early planning stages through design, development, production, field testing, and customer use.

This chapter covers basic models and principles to quantify and evaluate reliability during the design stage. It presents the probabilistic design approach and relationships between reliability and safety factor. The relationship between tolerances on the characteristics of the parts and reliability is also discussed. Probabilistic design requires analysis of the functions of random variables.

11.1 Design for Reliability

Reliability is a design parameter and must be incorporated into a product at the design stage. One way to quantify reliability during design and to design for reliability is the probabilistic approach to design (Haugen 1968; Kececioglu 1991; Kececioglu and Cormier 1968). The design variables and parameters are random variables, and hence the design methodology must consider them as random variables.

The basic premise in reliability analysis from the viewpoint of probabilistic design methodology is that a given component has a certain strength which, if exceeded, will result in the failure of the component. The factors that determine the strength of the component are random variables, as are the factors that determine the stresses or load acting on the component. Stress is used to indicate any agency that tends to induce failure, whereas strength indicates any agency resisting failure. Failure is taken to mean failure to function as intended; it occurs when the actual stress exceeds the actual strength for the first time.

Reliability Engineering, First Edition. Kailash C. Kapur and Michael Pecht.

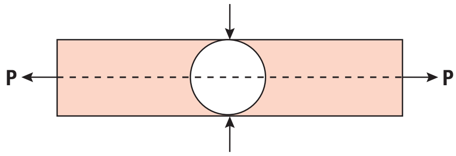


Figure 11.1 Design of a tension element.

11.2 Design of a Tension Element

Let us consider the design of a tension element for a tensile load of $P = 4{,}000$ units of load, as shown in Figure 11.1. The design engineer is considering the failure mode due to tensile fracture. Based on the material and its manufacturing processes, the designer finds the value of the ultimate tensile strength to be 10,000 units of load per square inch. This value typically is some average or mean value for the strength.

The classical approach to design uses the equation (where A is the cross-sectional area of the element)

$$\text{Mean Strength} \geq \text{Mean Stress} \times \text{Factor of Safety}$$

$$\text{or } 100{,}000 \geq (4{,}000 / A) \times 2$$

$$\text{or } A \geq 0.08 \text{ in}^2.$$

If we consider that the element has a circular cross-section with diameter D, then we can calculate that $D = 0.3192$ in.

Thus, it is clear that this approach does not consider the concept of reliability. We cannot answer the following questions:

- How reliable is this design? The answer is not provided by the above design approach and analysis.
- If a certain level of reliability is specified for a given mode of failure, what should be the value of the design variable (the diameter) of the tension element?

We do know that

- The load is a random variable due to varying conditions of customer usage and environmental factors.
- The ultimate tensile strength is a random variable due to material variation and manufacturing processes.
- The diameter of the element is a random variable due to manufacturing variability and is typically dealt with by introducing tolerances.

Thus we want to know what effect all types of variability have on the reliability.

The concept of design by probability, or probabilistic design, recognizes the reality that loads or stresses, and the strength of products subjected to these stresses, cannot be identified as specific values but have ranges of values with a probability of occurrence associated with each value in the range. Figure 11.2 shows $f(x)$ as the probability density function (pdf) for the stress random variable X, and $g(y)$ as the pdf for the strength random variable Y.

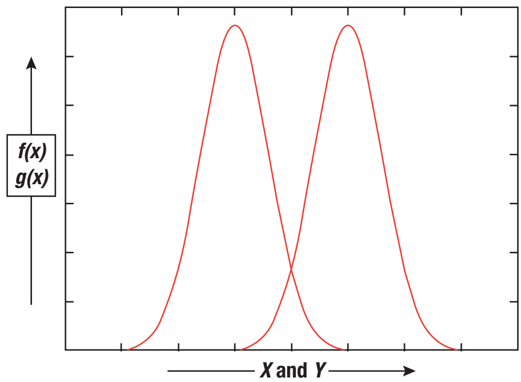


Figure 11.2 Stress and strength distributions.

The words *stress* and *strength* are used here in a broad sense applicable in a variety of situations well beyond traditional mechanical or structural systems. As mentioned, stress is used to indicate any agency that tends to induce failure, while strength indicates any agency resisting failure. Formulaically, we can say that

$$\text{Reliability} = R = P[\text{Strength} > \text{Stress}].$$

The reliability of the component is the probability that the strength of the component will be greater than the stress to which it will be subjected. The factor of safety, represented by number n, is the ratio of strength (Y) and the stress (X). Since both Y and X are random variable, one definition of the factor of safety is

$$n = \frac{\mu_Y}{\mu_X}. \tag{11.1}$$

There are four basic ways in which the designer can increase reliability:

1. *Increase Mean Strength.* This is achieved by increasing size or weight of materials, using stronger materials, and so on.
2. *Decrease Average Stress.* This can be done by controlling loads or using higher dimensions.
3. *Decrease Stress Variations.* This variation is harder to control, but can be effectively truncated by putting limitations on use conditions.
4. *Decrease Strength Variation.* The inherent part-to-part variation can be reduced by improving the basic process, controlling the process, and utilizing tests to eliminate less desirable parts.

11.3 Reliability Models for Probabilistic Design

For a certain mode of failure, let $f(x)$ and $g(y)$ be the probability density functions for the stress random variable X and the strength random variable Y, respectively.

Also, let $F(x)$ and $G(y)$ be the cumulative distribution functions for the random variables X and Y, respectively. Then the reliability, R, of the product for a failure mode under consideration, with the assumption that the stress and the strength are independent random variables, is given by

$$\begin{aligned} R &= P\{Y > X\} \\ &= \int_{-\infty}^{\infty} g(y)\left\{\int_{-\infty}^{y} f(x)dx\right\}dy \\ &= \int_{-\infty}^{\infty} g(y)F(y)dy \\ &= \int_{-\infty}^{\infty} f(x)\left\{\int_{x}^{\infty} g(y)dy\right\}dx \\ &= \int_{-\infty}^{\infty} f(x)\{1-G(x)\}dx. \end{aligned} \tag{11.2}$$

Consider a product where the stress and strength are normally distributed. Specifically, stress random variable X is normally distributed, with mean μ_X and with the standard deviation as σ_X. Similarly, the strength random variable Y is normally distributed, with mean μ_Y and standard deviation σ_Y. The reliability, R, for this mode of failure can be derived by:

$$R = P[Y > X] = P[(Y - X) > 0]. \tag{11.3}$$

It is known that $U = Y - X$ is also normally distributed with

$$\begin{aligned} \mu_U &= \mu_Y - \mu_X \\ \sigma_U^2 &= \sigma_Y^2 + \sigma_X^2. \end{aligned} \tag{11.4}$$

Hence,

$$R = P[U > 0] = \Phi\left[Z > \left(\frac{-\mu_U}{\sigma_U}\right)\right] = \Phi\left[Z < \left(\frac{\mu_U}{\sigma_U}\right)\right], \tag{11.5}$$

or

$$R = \Phi\left[\frac{\mu_Y - \mu_X}{\sqrt{\sigma_Y^2 + \sigma_X^2}}\right], \tag{11.6}$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal variable.

The reliability computations for other distributions, such as exponential, lognormal, gamma, Weibull, and several extreme value distributions, have been developed (Kapur and Lamberson 1977). In addition, the reliability analysis has been generalized when the stress and strength variables follow a known stochastic process.

Example 11.1

Suppose we have the following data for strength Y and stress X for a particular failure mode:

$$\mu_Y = 40{,}000 \quad \sigma_Y = 4000$$

$$\mu_X = 30{,}000 \quad \sigma_X = 3000.$$

One definition for the factor of safety as mentioned before is that it is the ratio of mean value of strength divided by the mean value of stress. Hence, for this problem,

$$\text{Factor of safety} = 40{,}000/30{,}000 = 1.33.$$

Using Equation 11.6, the reliability for this failure mode is

$$R = \Phi\left[\frac{40{,}000 - 30{,}000}{\sqrt{(4000)^2 + (3000)^2}}\right] = \Phi(2.0) = 0.97725.$$

From the failure perspective, there would be 2275 failures per 100,000 use conditions. The above reliability calculation is for a factor of safety of 1.33. We can increase the factor of safety by changing the design (such as increasing dimensions). This change makes μ_X equal to 20,000, increasing the factor of safety to 2. Thus, higher reliability can be expected, given as

$$R = \Phi\left[\frac{40{,}000 - 20{,}000}{\sqrt{(4{,}000)^2 + (3{,}000)^2}}\right] = \Phi(4.0) = 0.99997.$$

Now there are only three failures per 100,000 use conditions, which is a tremendous decrease from the previous situation. Reliability could also have been increased by decreasing the stress and strength variation.

11.4 Example of Probabilistic Design and Design for a Reliability Target

Let us now illustrate how to solve the design of the tension element given in Section 11.2 by using the probabilistic approach, which considers variability in the load conditions, ultimate tensile strength, and the diameter of the element. Suppose the variability of the load is quantified by its standard deviation as 100 units, and the ultimate tensile strength has a standard deviation of 5×10^3 units of strength. Thus we have

$$\text{Load:} \quad \mu_P = 4\times10^3, \quad \sigma_P = 100.$$

$$\text{Ultimate tensile strength:} \quad \mu_Y = 100\times10^3, \quad \sigma_Y = 5\times10^3.$$

Now we want to design the element for a specified reliability = R = 0.99990, with tensile fracture as the failure mode. Suppose the standard deviation of the diameter

Table 11.1 Relationship between dimensional tolerances and reliability

% Tolerances on *D*	Reliability
0	0.999915
1.0	0.999908
1.5	0.999900
3.0	0.999847
7.0	0.999032

Table 11.2 Relationship between strength variability and reliability

Standard deviation for strength	Reliability
2×10^3	0.99999
4×10^3	0.99996
5×10^3	0.99990
8×10^3	0.99157
10×10^3	0.97381

based on manufacturing processes is 0.5% of the diameter. The standard deviation can be converted to tolerances based on the idea of using $\pm k\sigma$ tolerances, where k is typically chosen as 3. If k is equal to 3 and the underlying random variable has normal distribution, then 99.73% of the values of the variable will fall within $\pm 3\sigma$ (it must be emphasized that it can be any other number depending on the requirements). Thus, for our example, $\pm 3\sigma$ tolerances will be $\pm 1.5\%$ of the diameter of the element. Then we can design (Kapur and Lamberson 1977) the tension element using a probabilistic approach, and the mean value of the diameter will be $\bar{D} = 0.2527$, compared with 0.3192 (calculated in Section 11.2).

We can also do a sensitivity analysis of reliability with respect to all the design variables. For example, consider the effect of tolerances or the standard deviation of the diameter of the design element. Table 11.1 shows the effect on the diameter of tolerances, based on the nature of the manufacturing processes, as a percent of the dimension.

Similarly, we can study the sensitivity of R with respect to the standard deviation of the strength, which also may be a reflection of the design and manufacturing processes. This is given in Table 11.2.

11.5 Relationship between Reliability, Factor of Safety, and Variability

When stress and strength are normally distributed,

$$R = \Phi\left[\frac{\mu_Y - \mu_X}{\sqrt{\sigma_Y^2 + \sigma_X^2}}\right]. \tag{11.7}$$

The factor of safety is some ratio of the strength and stress variables. Since both are random variables, the question arises of which measure of the strength or the stress should be used in the computation of the factor of safety. One definition is based on using the mean values of the strength and the stress variables; then, the factor of safety, n, is defined as

$$n = \frac{\mu_Y}{\mu_X} = \text{factor of safety}. \tag{11.8}$$

The variability of any random variable can be quantified by its coefficient of variation, which is the ratio of the standard deviation and the mean value. Thus, the coefficient of variation is a dimensionless quantity. If it is 0.05, we can say that the standard deviation is 5% of the mean value, and if we use $\pm 3\sigma$ tolerances, we can say that the tolerances are $\pm 15\%$ of the mean value of the underlying variable. Thus,

$$\begin{aligned} CV_Y &= \text{coefficient of variation for strength random variable} \\ &= \frac{\sigma_Y}{\mu_Y} \end{aligned} \tag{11.9}$$

$$\begin{aligned} CV_X &= \text{coefficient of variation for stress random variable} \\ &= \frac{\sigma_X}{\mu_X}. \end{aligned} \tag{11.10}$$

Then, Equation 11.7 can be rewritten as (by dividing both the numerator and the denominator by μ_Y):

$$R = \Phi\left[\frac{n-1}{\sqrt{CV_Y^2 n^2 + CV_X^2}}\right]. \tag{11.11}$$

Thus, the above relation can be used to relate reliability, factor of safety, coefficient of variation for the stress random variable, and coefficient of variation for the strength random variable.

Example 11.2

The stress (X) and the strength (Y) random variables for a given failure mode of a component follow the normal distributions with the following parameters:

$$\mu_X = 10{,}000 \quad \mu_X = 2400$$

$$\mu_Y = 15{,}000 \quad \mu_Y = 2000.$$

(a) Find the reliability for the component for this failure mode.

Solution:

$$R = \Phi\left(\frac{\mu_Y - \mu_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right) = \Phi\left(\frac{5000}{3124.1}\right) = \Phi(1.60) = 0.9452.$$

(b) The customer wants a reliability of 0.9990. The only thing that the designer can change for this failure mode is the mean value for the strength random variable Y (thus increasing the factor of safety). Find the new target for μ_Y to achieve the reliability goal.

Solution:

$$R = \Phi\left(\frac{\mu_Y - \mu_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right) = 0.9990$$

$$3.10 = \frac{\mu_Y - \mu_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}} = \frac{\mu_Y - 10{,}000}{3124.1}$$

$$\mu_Y = 3.1 \times 3124.2 + 10{,}000 = 19{,}684.7.$$

Thus, the new target for the mean value of the strength should be 19,684.7 units.

Example 11.3

The stress (X) and the strength (Y) for a given failure mode of a component follow a normal distribution with the following information about their coefficient of variation, CV:

$$CV_X = 0.25 \quad CV_Y = 0.17.$$

The customer wants a reliability of 0.99990 for this failure mode. What is the safety factor that the designer must use to meet the requirements of the customer?

Solution:

$$R = 0.99990 = \Phi\left[\frac{n-1}{\sqrt{n^2 CV_Y^2 + CV_X^2}}\right] = \Phi(3.715)$$

$$3.715 = \frac{n-1}{\sqrt{n^2 0.17^2 + 0.25^2}}$$

$$(n-1)^2 = \left(0.0289n^2 + 0.0625\right) \times 13.8012$$

$$0.601145n^2 - 2n + 0.137423 = 0$$

$$\begin{aligned} n &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{2 \pm \sqrt{4 - 4 \times 0.601145 \times 0.137423}}{2 \times 0.601145} \\ &= \frac{2 \pm 1.91561}{1.20229} = 3.26 \quad \text{or} \quad 0.0702. \end{aligned}$$

Choose $n = 3.26$. Note that the other root of the quadratic equation gives us the value of the unreliability.

11.6 Functions of Random Variables

For any design problem, there is one design variable that is a function of several other design variables. For example, for the tension element considered in Section 11.2, the stress random variable, X, for the circular cross-section tension element is given by

$$X = \frac{4P}{\pi D^2}, \tag{11.12}$$

where P and D are both random variables. Generally, it is very difficult to find the probability density function for one variable that is a nonlinear function of several other variables. In such cases, for engineering analysis, knowledge of the first and the second moments of the transformed variable is quite useful. This knowledge can be used for probabilistic design. Consider the following general model:

$$Y = f(X_1, X_2, \ldots, X_n), \tag{11.13}$$

where Y is a function of n other variables, represented by vector $X = (X_1, X_2, \ldots, X_n)$.

This represents a general equation where a design random variable, Y, is a function of other design random variables. Information about the mean and variance of these n random variables is given as

$$\begin{aligned} E[X_i] &= \mu_i, \quad i = 1, 2, \ldots, n \\ V[X_i] &= \sigma_i^2, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{11.14}$$

Then, we can find the approximate values for μ_Y and σ_Y^2 using Taylor's series approximations as follows:

$$\begin{aligned} \mu_Y &\cong f(\mu_1, \mu_2, \ldots, \mu_n) + \frac{1}{2}\sum_{i=1}^{n} \frac{\partial^2 f(X)}{\partial X_i^2}\bigg| X = \mu V(X_i) \\ V_Y = \sigma_Y^2 &\cong \sum_{i=1}^{n} \left\{ \frac{\partial f(X)}{\partial X_i}\bigg| X = \mu \right\}^2 V(X_i), \end{aligned} \tag{11.15}$$

where

$$X = (X_1, X_2, \ldots, X_n) \quad \text{and} \quad \mu = (\mu_1, \mu_2, \ldots, \mu_n). \tag{11.16}$$

For approximate analysis, the second derivative terms for the mean are typically ignored.

For engineering analysis, designers think in terms of tolerances. Let the tolerances about the mean value be denoted by

$$X = \mu_X \pm t_X. \tag{11.17}$$

If X is normally distributed with a mean of μ_X and variance of σ_X^2, and we use 3σ limits—that is, $t_X = 3\sigma_X$, then for symmetrical tolerances about the mean, 0.27% of the items will be outside the 3σ limits.

If we use 4σ limits, then $t_X = 4\sigma_X$ and 99.993666% of the values of X are within 4σ limits, or 0.006334% will be outside 4σ limits.

Example 11.4

Consider the probabilistic analysis of part of an electrical circuit that has two resistances in parallel. The terminal resistance, R_T, as a function of the two other resistances, R_1 and R_2, is given by

$$R_T = f(R_1, R_2) = \frac{R_1 R_2}{R_1 + R_2}.$$

Suppose 3σ tolerances on R_1 are 100 ± 30, and 3σ tolerances on R_2 are 200 ± 45. Thus,

$$\begin{aligned} \mu_{R_1} &= 100\,\Omega \quad \sigma_{R_1} = 10\,\Omega \\ \mu_{R_2} &= 200\,\Omega \quad \sigma_{R_2} = 15\,\Omega \end{aligned}.$$

Then, using Equation 11.15,

$$E[R_T] = f(100, 200) = \frac{100 \times 200}{100 + 200} = 66.67\,\Omega$$

$$\frac{\partial f}{\partial R_1} = \frac{R_2^2}{(R_1 + R_2)^2} \quad \frac{\partial f}{\partial R_2} = \frac{R_1^2}{(R_1 + R_2)^2}$$

$$\left.\frac{\partial f}{\partial R_1}\right| R = \mu = 0.444 \quad \left.\frac{\partial f}{\partial R_2}\right| R = \mu = 0.111$$

$$\sigma_{R_T}^2 = 0.444^2 \times 10^2 + 0.111^2 \times 15^2 = 22.4858 \quad \text{or} \quad \sigma_{R_T} = 4.74\,\Omega.$$

Thus, the three tolerances on R_T are 66.67 ± 14.22. These statistical tolerances are much tighter than the tolerances based on worst-case analysis, which may consider minimum and maximum values of the resistances based on 3σ tolerances for the two resistances.

Example 11.5

Determine the tolerance for the volume of a cylinder having the following 3σ tolerances for the diameter, D, and its length, L

$$D = 2.5 \pm 0.002\text{ m}, \quad L = 4.0 \pm 0.005\text{ m}.$$

Also assume that D and L are probabilistically independent.

Solution:
The volume of the cylinder is given by

$$V = \frac{\pi D^2 L}{4}.$$

so the mean volume is approximately given by

$$\mu_V \approx f(\mu_D, \mu_L) = \pi \frac{\mu_D^2 \mu_L}{4} = \pi \frac{2.5^2 \times 4.0}{4} = 19.635.$$

The partial derivatives are:

$$\frac{\partial V}{\partial D} = \frac{\pi DL}{2} \quad \frac{\partial V}{\partial L} = \frac{\pi D^2}{4},$$

so,

$$\sigma_V^2 = \left\{ \frac{\partial V}{\partial D}\bigg|_{\mu_D,\mu_L} \right\}^2 \sigma_D^2 + \left\{ \frac{\partial V}{\partial L}\bigg|_{\mu_D,\mu_L} \right\}^2 \sigma_L^2$$
$$= \left(\frac{\pi \mu_D \mu_L}{2} \right)^2 \sigma_D^2 + \left(\frac{\pi \mu_D^2}{4} \right)^2 \sigma_L^2.$$

If we let the tolerance on V as $T_V = 3\sigma_V$, then the above equation can be written in terms of tolerances as,

$$T_V^2 == \left\{ \frac{\partial V}{\partial D}\bigg|_{\mu_D,\mu_L} \right\}^2 T_D^2 + \left\{ \frac{\partial V}{\partial L}\bigg|_{\mu_D,\mu_L} \right\}^2 T_L^2$$
$$= \left(\frac{\pi \mu_D \mu_L}{2} \right)^2 T_D^2 + \left(\frac{\pi \mu_D^2}{4} \right)^2 T_L^2 = 0.001589.$$

Hence the tolerances on $V = \mu_V \pm T_V = 19.635 \pm 0.0399$ m^3.

Example 11.6

A random variable, Y, for a product is a function of three other random variables, X_1, X_2, X_3, and is given by

$$Y = \frac{2\sqrt{X_1}}{X_2 X_3} \quad \text{or} \quad Y = \frac{2X_1^{1/2}}{X_2 X_3}.$$

(a) Find the expected value and the standard deviation of the random variable Y, given the following information on the three-sigma tolerances for the variables using Taylor's series approximation:

$$X_1 = 4.00 \pm 1.20$$

$$X_2 = 2.00 \pm 0.60$$

$$X_3 = 1.00 \pm 0.30$$

$$\mu_Y \cong \frac{2\sqrt{\mu_{X_1}}}{\mu_{X_2}\mu_{X_3}} = \frac{2\times\sqrt{4}}{2\times 1} = 2.$$

Now, the derivatives are:

$$\frac{\partial Y}{\partial X_1} = \frac{1}{\sqrt{X_1}X_2X_3}; \frac{\partial Y}{\partial X_2} = -\frac{2\sqrt{X_1}}{X_2^2X_3};$$

$$\frac{\partial Y}{\partial X_3} = -\frac{2\sqrt{X_1}}{X_2X_3^2}.$$

Evaluate the derivatives at μ:

$$\left.\frac{\partial Y}{\partial X_1}\right|_\mu = \frac{1}{\sqrt{4}\times 2\times 1} = \frac{1}{4}; \left.\frac{\partial Y}{\partial X_2}\right|_\mu = -\frac{2\sqrt{4}}{2^2\times 1} = -1$$

$$\left.\frac{\partial Y}{\partial X_3}\right|_\mu = -\frac{2\sqrt{4}}{2\times 1^2} = -2.$$

To find the variance, we have:

$$\begin{aligned}\sigma_Y^2 &\cong \left(\left.\frac{\partial Y}{\partial X_1}\right|_\mu\right)\sigma_{X_1}^2 + \left(\left.\frac{\partial Y}{\partial X_1}\right|_\mu\right)\sigma_{X_2}^2 + \left(\left.\frac{\partial Y}{\partial X_1}\right|_\mu\right)\sigma_{X_3}^2 \\ &= \left(\frac{1}{4}\right)^2(0.4)^2 + (-1)^2(0.2)^2 + (-2)^2(0.1)^2 \\ &= 0.01 + 0.04 + 0.04 = 0.09.\end{aligned}$$

Hence, $\sigma_Y = \sqrt{0.09} = 0.3$.

(b) The upper specification limit on Y is 3.00, and the lower specification limit is 1.50. What percent of the products produced by this process will be out of specifications?

$$\begin{aligned}&= \left[1-\Phi\left(\frac{3-2}{0.3}\right)\right] + \left[1-\Phi\left(\frac{2-1.5}{0.3}\right)\right] \\ &= [1-\Phi(3.33)] + [1-\Phi(1.67)] \\ &= [1-0.9995658] + [1-0.95254] \\ &= 0.04789.\end{aligned}$$

11.7 Steps for Probabilistic Design

Considering the total design-for-reliability program, the steps related to the probabilistic approach may be summarized as follows:

- Define the design problem. Develop system functional and reliability block diagrams to the lowest level of detail.
- Identify the design variables and parameters needed to meet customer's requirements for each component. Focus on understanding the physics/chemistry/biology of failure.
- Conduct a failure modes, mechanisms, and effects analysis (FMMEA). Focus on understanding failure mechanisms.
- Select and verify the significant design parameters.
- Formulate the relationship between the critical parameters and the failure-governing criteria related to the underlying components.
- Determine the stress function governing failure.
- Determine the failure governing stress distribution.
- Determine the failure governing strength function.
- Determine the failure governing strength distribution.
- Calculate the reliability associated with these failure-governing distributions for each critical failure mode.
- Iterate the design to obtain the design reliability goal.
- Optimize the design in terms of other qualities, such as performance, cost, and weight.
- Repeat optimization for each critical component.
- Calculate system reliability.
- Iterate to optimize system reliability.

11.8 Summary

Producing a reliable product requires planning for reliability from the earliest stages of product design. There are models and principles that can be used to quantify and evaluate reliability in the design stage. One approach is known as probabilistic design for reliability. The basic premise of probabilistic design for reliability is that a given component has a certain strength which, if exceeded, will result in failure. The factors that determine the strength of the component are random variables, as are the factors that determine the stresses or load acting on the component. Stress is used to indicate any agency that tends to induce failure, whereas strength indicates any agency resisting failure. The factor of safety is some ratio of the strength and stress variables. Since both are random variables, the engineers designing a product must determine which measures of strength or the stress should be used in the computation of the factor of safety based on probability. Following the steps for probabilistic design provided in this chapter can result in the production of a product that will achieve the desired level of reliability in its application environment.

Problems

11.1 The stress (X) and the strength (Y) random variables for a given failure mode of a product follow the normal distributions with the following parameters:

$$\begin{aligned} \mu_X &= 11{,}000 \quad \sigma_X = 2400 \\ \mu_Y &= 15{,}000 \quad \sigma_Y = 1500 \end{aligned}.$$

(a) Find the system reliability for this failure mode.

(b) The customer wants a reliability of 0.99990. The only thing that the designer can change for this failure mode is the mean value for the strength random variable Y (thus increasing the factor of safety). Find the new target for μ_Y to achieve the reliability goal.

11.2 The stress (X) and the strength (Y) are random variables for a given failure mode and follow the normal distribution with the following information (CV is the coefficient of variation):

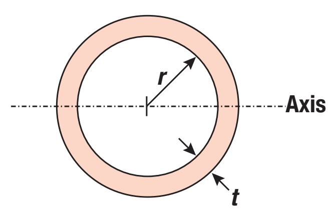
$$CV_X = 0.20 \quad CV_Y = 0.15.$$

(a) The customer wants a reliability of 0.9990. What is the safety factor that the designer must use to meet the requirements of the customer?

(b) The customer wants a reliability of 0.990. What is the safety factor that the designer must use to meet the requirements of the customer?

(c) Another customer wants a reliability of 0.9990. The design team does not want to increase the safety factor any more. The only thing the team can easily change is the variation of the strength variable. What should be the value of CV_Y to meet the requirements of the customer?

11.3 A beam with a tubular cross-section, shown in the figure below, is to be used in an automobile assembly.



To compute the stresses, the moment of inertia (I) of the beam about the neural axis is calculated as

$$I = \pi r^3 t.$$

The mean radius and thickness of the tubular cross-section have the following dimensions with 3σ tolerances:

$$r = 2.00 \pm 0.06$$
$$t = 0.11 \pm 0.015.$$

Find the mean value of the moment of inertia and its standard deviation.

11.4 A random variable, Y, for a product is a function of three random variables, X_1, X_2, X_3, and is given by

$$Y = \frac{4\sqrt{X_1}}{X_2 X_3^2} \quad \text{or} \quad Y = \frac{4X_1^{1/2}}{X_2 X_3^2}.$$

(a) Find the expected value and the standard deviation of the random variable Y, given the following information on the 3σ tolerances for the variables using Taylor's series approximation:

$$X_1 = 4.00 \pm 0.60$$
$$X_2 = 2.00 \pm 0.40$$
$$X_3 = 1.00 \pm 0.15.$$

(b) The upper specification limit on Y is 5.00 and the lower specification limit is 3.00. What percent of the products produced by this process will be out of specifications?

(c) The design team thinks that the percent of nonconforming products, as calculated in part (b), is relatively high. If the tolerances can be reduced on only one random variable, which variable would you pick? Let us say that we can decrease the tolerances on the chosen random variable by half. What percent of the products will be out of specification with the new process?

11.5 A random variable, Y, which determines the function of a system, is a function of three other random variables, X_1, X_2, X_3, and is given by

$$Y = \frac{3X_1^{\frac{1}{2}}}{X_2^2 X_3}.$$

Find the expected value and the standard deviation of the random variable Y, given the following information using the first-order Taylor's series approximation:

$$\mu_{X_1} = 9.00 \quad \sigma_{X_1} = 0.60$$
$$\mu_{X_2} = 2.00 \quad \sigma_{X_2} = 0.20.$$
$$\mu_{X_3} = 1.50 \quad \sigma_{X_3} = 0.15$$

Also develop the 3σ tolerance limits for Y.

11.6 Suppose a mechanism is made of three components with dimensions X_1, X_2, X_3. The response of this mechanism, Y, is related to X_is by: $Y = 2X_1 + X_2 - 3X_3$.

The 3σ tolerances on the dimensions are as follows:

$$X_1 = 5.00 \pm 0.18$$
$$X_2 = 4.00 \pm 0.12$$
$$X_3 = 2.00 \pm 0.15.$$

All the dimensions, X_i, $i = 1, 2, 3$, follow the normal distribution.

(a) Find the mean and variance for the random variable Y, and specify its 3σ tolerance limits.

(b) If the specification limits on the response, Y, of this mechanism are 8.00 ± 0.50, what percentage of the mechanism will not meet these specifications?

(c) Another response, Z, of the mechanism is given by:

$$Z = \frac{X_1^2 X_2}{X_3}.$$

Find the mean and variance of the random variable Z and specify its 3σ tolerance limits.