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# 4 Design for Six Sigma

The concept of Six Sigma originated at Motorola in the 1980s by Dr. Mikel Harry, an engineer. He realized that systems can be improved by measuring and reducing variation. General Electric in the 1990s started implementing these concepts in all their divisions. Impressive quality improvements were experienced. Estimates are that cost savings due to the application of Six Sigma exceeded $300 million within the first 2 years and more than $1 billion by 1999.

Sigma ($\sigma$) is a Greek letter used to denote standard deviation, which is used to compare expected outcomes versus failures in a population. Six Sigma is the definition of outcomes as close as possible to perfection. With Six Sigma, the goal is to arrive at 3.4 defects per million opportunities, or 99.9997% perfection. As an example, with Six Sigma, an airline would lose only three to four pieces of luggage for every 1 million pieces that it handles. This is what the Six Sigma process strives to achieve. Over the last 25 years, Six Sigma has been successfully implemented in many industries, from large manufacturers to small businesses, from financial services and the insurance industry to healthcare systems (Barry et al. 2002; Harry and Schroeder 2000; Hoerl 1998; Pande et al. 2000; Pyzdek and Keller 2009).[1]

## 4.1 What Is Six Sigma?

In many organizations, Six Sigma is a business management process that provides tangible business results to the bottom line by continuous process improvement and variation reduction. As a data-driven, statistically based approach, Six Sigma aims to deliver near-zero defects for every product, process, and transaction within an organization.

The concept of Six Sigma was developed based on the assumption that the process characteristic follows a normal distribution. The objective of Six Sigma is to achieve a target of at most 3.4 defectives per million items, even if the process mean shifts by 1.5 times its standard deviation over a period of time. This is possible only if the

[1]http://www.ge.com/sixsigma/.

*Reliability Engineering*, First Edition. Kailash C. Kapur and Michael Pecht.

**Table 4.1** Defects per million for normal distribution

| Quality level | Defects per million without any process shift | Defects per million with 1.5 Sigma shift |
|---|---|---|
| 2 Sigma | 45,500 | 308,771 |
| 3 Sigma | 2,700 | 66,803 |
| 4 Sigma | 63 | 6,200 |
| 5 Sigma | 0.57 | 233 |
| 6 Sigma | 0.002 | 3.4 |
| 7 Sigma | 0.0000026 | 0.019 |

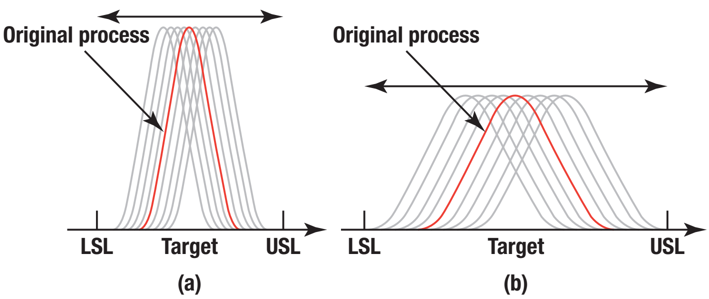


**Figure 4.1** Effect of shifting process mean for (a) a Six Sigma process and (b) an ordinary process.

process variation is considerably reduced such that six times the standard deviation on either side of the process mean is within the specifications. Six Sigma actually aims at producing less than 0.002 defectives per million. However, considering the possible shifting of the process mean by 1.5 Sigma over time, it aims at less than 3.4 defectives per million. Table 4.1 and Figure 4.1 clarify these concepts. If the mean of a process which is not Six Sigma qualified shifts, it will produce a large number of defectives compared to a Six Sigma process.

A Six Sigma process can also be interpreted in terms of process capability. The typical definition for the process capability index, $C_{pk}$, is

$$C_{pk} = \min\left\{\frac{\mathrm{USL} - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - \mathrm{LSL}}{3\hat{\sigma}}\right\}, \tag{4.1}$$

where USL is the upper specification limit, LSL is the lower specification limit, $\hat{\mu}$, is the point estimator of the mean, and $\hat{\sigma}$ is the point estimator of the standard deviation for the underlying quality characteristic. If the process is centered at the middle of the specifications, which is also interpreted as the target value (for this model), then the Six Sigma process means that $C_{pk} = 2$. If the process shifts by 1.5 Sigma, the $C_{pk}$ will be 1.5, leading to less than or equal to 3.4 defectives per million. Six Sigma is a continuous process and is a strategy to improve the present process capability (say $C_{pk} = 1$) to the Six Sigma capability ($C_{pk} = 2$).

## 4.2 Why Six Sigma?

Six Sigma is a methodology for structured, process-oriented, and systematic quality improvement. The primary reason for the success of Six Sigma is that it provides a

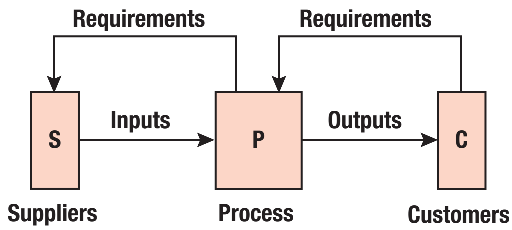


**Figure 4.2** Process mapping.

systematic approach for quality and process improvement, rather than being just a collection of tools. The Six Sigma strategy is a good way to integrate such methods as design of experiments (DoE or DoX), statistical process control (SPC), failure mode, effects and criticality analysis (FMECA), fault tree analysis (FTA), and quality function deployment (QFD). Implemented project by project, Six Sigma provides an overall process that clearly shows how to link and sequence individual tools.

Many companies, such as Motorola,[2] GE,[3] and Honeywell,[4] began continuous process improvement with Six Sigma methodology. The method has a customer focus and is data-driven and analytically sound. Six Sigma is a rigorous, data-driven, decision-making approach to analyzing the root causes of problems and improving process capability.

## 4.3 How Is Six Sigma Implemented?

Improving processes is very important for businesses to stay competitive in today's marketplace, where customers are demanding better and better products and services.

Understanding the meaning of a process before trying to improve it is very important. A typical process is shown in Figure 4.2. There are many ways to visualize and model a process; one good model uses the Suppliers–Inputs–Process–Outputs–Customers (SIPOC) diagram. Other good models are Taguchi's P-diagram and the Ishikawa/fishbone diagram. In use, the model requires identification of the supplier(s), process inputs, process, associated outputs, and customer(s). The model also shows the feedback loop based on the requirements of the customer and the process.

A process consists of many input variables and one or more output variables. The input variables include both controllable factors and uncontrollable, or noise, factors. For instance, for an electric circuit designed to obtain a target output voltage, the designer can specify the nominal values of resistors and capacitors, but cannot control the influence of temperature or moisture, degradation over time, and measurement error. A typical process with one output variable is given in Figure 4.3, where $X_1$, $X_2, \ldots, X_n$ are the controllable variables and $y$ is the realization of the random output variable $Y$.

[2]http://www.motorola.com/Business/US-EN/Motorola+University.
[3]http://www.ge.com/en/company/companyinfo/quality/whatis.htm.
[4]http://www51.honeywell.com/honeywell/our-culture-n3n4/continually-improving.html?c=11.

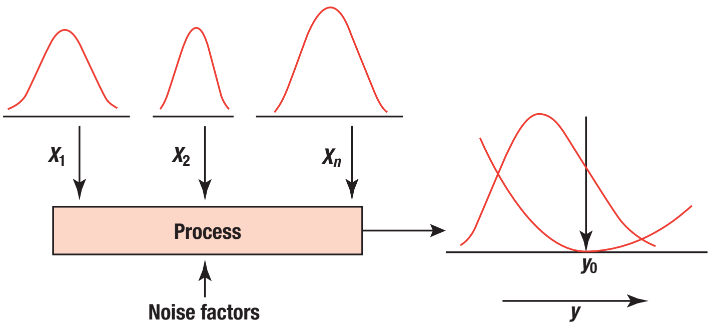


**Figure 4.3** A general process with one output variable.


**Figure 4.4** Six phases for the Six Sigma process.

### 4.3.1 Steps in the Six Sigma Process

At the strategic level, the goal of Six Sigma is to align an organization to its marketplace and deliver real improvements to the bottom line. At the operational level, Six Sigma strives to move product or process characteristics into the specifications required by customers, shrink process variation to a Six Sigma level, and reduce the causes of defects that negatively affect quality (Bertels 2003). A typical procedure for Six Sigma quality improvement has six well-known and highly utilized phases: define, measure, analyze, improve, control, and technology transfer (DMAICT), as shown in Figure 4.4. The process of DMAICT stays on track by establishing deliverables for each phase, by creating engineering models over time to reduce the process variation, and by continuously upgrading the predictability of system performance. Each of the six phases in the DMAICT process is critical to achieving success.

***4.3.1.1 Step 1: Define—What Problem Needs to Be Solved?*** In this phase, there are three critical factors: the scope of the project, the customer, and issues that are critical to quality (CTQ) are identified and the core processes are defined. It is important to define the scope, expectations, resources, and timelines for the selected project.

Once an organization decides to launch a Six Sigma project, it needs to first define the improvement activities involved. Usually, the following two factors are considered in the define phase.

**Identifying and Prioritizing Customer Requirements** Methods such as benchmarking surveys, spider charts, and customer needs mapping are used to ensure that the customer's requirements are properly identified. The critical-to-quality (CTQ) characteristics (also called external CTQs) are specified. The external CTQs need to be translated into internal CTQs that are key process requirements. This translation is the key step in the measure phase.

**Selecting the Project** Six Sigma process improvement is a reactive approach that is initiated when a process does not deliver a satisfactory result (according to the customer affected by the process). Based on customer requirements, a target project is selected by analyzing the gap between the current process performance and the requirements of customers.

For a selected project, a charter must be developed that specifies project scope, expectations, resources, milestones, and core processes. The charter identifies and documents necessary information before the measurement (M) step is applied. Charter development companies follow or use the following steps.

**STEP 1.1:** DRAFT A PROJECT CHARTER Drafting a project charter is the first step in the Six Sigma methodology. The project charter should include the following: business case; goals and objectives of the project; milestones; project scope, constraints, and assumptions; team membership; roles and responsibilities; and a preliminary project plan.

**STEP 1.2:** IDENTIFY AND DOCUMENT THE PROCESS The Six Sigma approach focuses on one process at a time, either a core process (product development and customer service) or a support process (human resources and information system). One process is chosen for the project. After the process for improvement is identified, a process model (P-diagram (Phadke 1989) or SIPOC) is selected and used to model and analyze the process. Once the project is understood and the baseline performance is documented, it is time to do an analysis of the process. In this phase, Six Sigma applies statistical tools to validate the root causes of problems. The objective is to understand the process in sufficient detail so that options for improvement can be formulated.

**STEP 1.3:** IDENTIFY, ANALYZE, AND PRIORITIZE CUSTOMER REQUIREMENTS In Six Sigma, the customer is defined not so much as the traditional buyer, but as the environment, producer, seller, or buyer that is affected by the process. Because quality is measured from the customer's perspective, there has to be a link between the output that a process delivers and the quality that the customer expects. There are two types of customer requirements: product output requirements, which must be translated from voice of the customer (VOC) into design parameters; and service-level requirements, which involve establishing the service needs of the customer, often with some level of abstraction and subjectivity.

Elements of this step include selecting critical-to-quality (CTQ) characteristics using tools such as quality function development (QFD) (Akao 1989) and failure modes, effects, and criticality analysis (FMECA) to translate the external CTQs into internal requirements denoted by *Y*s. Some of the objectives for this step include defining performance standards by: defining, constructing, and interpreting the QFDs; participating in a customer needs mapping exercise; applying (FMECA) to the process of selecting CTQ characteristics; identifying CTQs and internal *Y*s; and analyzing and determining the priority of the customer requirements. Since the customer's expectations often include multiple requirements, these need to be ranked by importance.

**STEP 1.4:** DEVELOP APPROPRIATE MEASUREMENT SYSTEMS Once the SIPOC elements and the customer's functional requirements are identified, measurement tools are used to evaluate the current performance. (Note that Six Sigma strategy focuses only on an existing process that needs rework or improvement.) After the product requirements, *Y*s, and measurement standards for the *Y*s are defined, QFD, FMECA, and process mapping can be used to establish internal measurement standards.

***4.3.1.2 Step 2: Measure—What Is the Current Capability of the Process?*** Design for Six Sigma is a data-driven approach that requires quantifying and benchmarking the process using actual data. In this phase, the performance or process capability for the identified CTQ characteristics is evaluated.

Measurement is a very important element in the Six Sigma strategy. This step involves data collection and data processing before proceeding to the analysis step. Notice that if this step is wrongly executed, a statistical error could result in a measurement error, leading to an incorrect analysis and wrongly executed procedures. The first step in the measurement stage is to select which of the process elements needs to be measured. Generally, the relevant measures include both input and output measures.

Input measures may involve data stratification. One input variable may be the output of another input. Cause-and-effect relationships can lead to the lowest independent input variables that may influence the output values.

Output measures include CTQ data, such as the lower and upper specification limits and defect counts. It is necessary to develop a data collection strategy that defines sampling frequency, the method of measurement, the format of data collection forms, and the measurement instruments. The team also must consider the possibility of Type II statistical error (and also measurement error) and use a well-planned strategy to tackle it.

In addition, the team should consider the type of data (discrete vs. continuous) and the sampling method. Thus, the steps for the measurement phase may be summarized as follows:

**Step 2.1: Establish Product Capability** The current product capability, associated confidence levels, and sample size are established by statistical analysis. The typical definition for the process capability index, $C_{pk}$, is

$$C_{pk} = \min\left\{\frac{\mathrm{USL} - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - \mathrm{LSL}}{3\hat{\sigma}}\right\}, \tag{4.2}$$

where USL is the upper specification limit, LSL is the lower specification limit, $\hat{\mu}$ is the point estimator of the mean, and $\hat{\sigma}$ is the point estimator of the standard deviation. If the process is centered at the middle of the specifications, which is also interpreted as the target value—that is,

$$\hat{\mu} = \frac{\mathrm{USL} + \mathrm{LSL}}{2} = y_0, \tag{4.3}$$

then the Six Sigma process means that $C_{pk} = 2$. If the process mean shifts by 1.50σ, which is typically assumed in the literature for Six Sigma methodology, then $C_{pk} = 1.50$. It is this 1.50σ shift that results in 3.4 defects per million opportunities (DPMO).

**Step 2.2: Define Performance Objectives** The performance objectives are defined to establish a balance between improving process capability (and thus, customer satisfaction) and the available technology capability.

**Step 2.3: Identify Sources of Variation** This step begins to identify the causal variables that affect the product requirements or the responses of the process. Some of these causal variables might be used to control the response *Y*s.

***4.3.1.3 Step 3: Analyze—What Are the Root Causes of Process Variability?*** Once the project is understood and the baseline performance is documented, it is time to do an analysis of the process. In this phase, the Six Sigma approach applies statistical tools to validate the root causes of problems. The objective is to understand the process at a level sufficient to be able to formulate options for improvement. We should be able to compare the various options with each other to determine the most promising alternatives. In general, during the process of analysis, the collected data are analyzed and process maps are used to determine the root causes of defects and prioritize opportunities for improvement.

The collected data can be used to find patterns, trends, outliers, and other differences that could support or reject theories (hypothesis testing) about cause and effect. The methods frequently used include design of experiments (Hicks and Turner 1999; Montgomery 2001), the Shanin method,[5] root cause analysis, cause–effect diagrams, failure modes and effects analysis (FMEA), Pareto charts, and validation of root cause.

Process analysis uses tools such as value stream mapping, process management, and the process mapping technique to analyze nonvalue-adding steps that result in nonconformity. Thus, the steps for the analysis phase may be summarized in the following sections.

**Step 3.1: Discover Variable Relationships** In the previous stage, causal variables, *X*s, are identified with a possible prioritization as to their importance in controlling *Y*s. In this step, the impact of each vital *X* on the response *Y*s is explored. A system transfer function (STF) is developed as an empirical model relating *Y*s and the vital *X*s.

**Step 3.2: Establish Operating Tolerances** After understanding the functional relationship between the vital *X*s and the response *Y*s, we need to establish the operating tolerances of *X*s that optimize the performance of *Y*s. Mathematically, we develop a variance transmission equation (VTE) that transfers the variances of the vital *X*s to variances of *Y*s.

**Step 3.3: Optimize Variable Settings** The STF and VTE are used to determine the key operating parameters and tolerances to achieve the desired performance of the *Y*s. Optimization models are developed to determine the optimum values for both the means and variances for these vital *X*s.

[5]http://www.shainin.com/.

***4.3.1.4 Step 4: Improve—How Can the Process Capability Be Improved?*** During the improvement phase, ideas and solutions are established to initialize the needed changes. Based on the root causes discovered and validated for the existing opportunity, the target process is improved by designing creative solutions to fix and prevent problems. Some experiments and trials may be necessary in order to find the best solution. If a mathematical model is developed, then optimization methods are utilized to determine the best solution.

After completing the analysis step, the team should be able to identify the root causes of nonconformity. If the root cause is identified by data analysis tools, finding the solutions to fix the process could either be easy or hard, because analysis tools point directly to the nonconformity culprit. Sometimes, the solutions applied can fix the problem indicated by the analysis tools, but may also result in another problem caused by other variables. This is due to the interdependence of the variables. The team can use brainstorming or the theory of inventive problem-solving called TRIZ to tackle the problem. If the root cause is identified by the process analysis, the Six Sigma team can use process management techniques, such as process simplification, parallel processing, and bottleneck elimination.

***4.3.1.5 Step 5: Control—What Controls Can Be Put in Place to Sustain the Improvement?*** The key to the overall success of Six Sigma methodology is its sustainability, which seeks to make the process incrementally better on a continuous basis. The sum of all these incremental improvements can be substantial. Without continuous sustenance, over time, the process will worsen until finally it is time for another effort toward improvement. As part of the Six Sigma approach, performance tracking mechanisms and measurements are put in place to assure that the gains made in the project are not lost over time and that the process remains on the new course. The steps for the control phase may be summarized in the following sections.

**Step 5.1: Validate the Measurement System** The measurement system tools first applied in Step 1.4 are now used for the *X*s.

**Step 5.2: Implement Process Controls** Statistical process control is a critical element in maintaining a Six Sigma level. Control charting is the major tool used to control the few vital *X*s. Special causes of process variations are identified through the use of control charts, and corrective actions are implemented to reduce variations.

**Step 5.3: Document the Improvement** The project is not complete until the changes are documented in the appropriate quality management system, such as QS9000/ISO9000. A translation package and plan should be developed for possible technology transfer.

Once improvements have been made, proper documentation and standards should be established to monitor the process. If the process is improved by process management, new process standards should be established. If the process is improved by eliminating the root causes of bad performance, the new performance should be measured consistently by controlling the critical variable related to the chart.

***4.3.1.6 Step 6: Technology Transfer*** Ideas and knowledge developed in one part of the organization can be transferred to other parts of the organization. In addition, the methods and solutions developed for one product or process can be applied to other similar products or processes. With technology transfer, the Six Sigma approach can create exponentially increasing returns.

### 4.3.2 Summary of the Six Sigma Steps

The DMAICT process stays on track by reducing process variations and establishing deliverables at each phase. In each phase, several quality improvement methods, tools, and techniques can be used. Each organization has different ways of summarizing these steps, based on the nature of its products, processes, and customers. Table 4.2 is one such summary. Another is given in Table 4.3 for comparison.

***4.3.2.1 Future Trends of Six Sigma*** Although Six Sigma originated in the manufacturing industry, it has been successfully adopted throughout the public and private sectors in applications from financial services and health care to information technology and knowledge management. Its successful implementation over 20 years supports the hypothesis that the basic theory and methods of Six Sigma have lasting value, regardless of what names they are marketed under. These ideas can be integrated into other productivity improvement methods—for example, the recent

**Table 4.2** Summary of key steps in Six Sigma process improvement and Six Sigma tools at each step

| Key steps in Six Sigma process improvement | Six Sigma steps | Six Sigma tools |
|---|---|---|
| Define (D) | ■ Draft project charter<br>■ Identify and document process<br>■ Identify VOC | ■ SIPOC modeling<br>■ P-diagram<br>■ Ishikawa diagram<br>■ Kano analysis<br>■ Quality function deployment (QFD) |
| Measure (M) | ■ Select measurement variables<br>■ Develop data collection plan<br>■ Calculate process Sigma level | ■ Statistical process capability<br>■ DOE |
| Analyze (A) | ■ Data analysis<br>■ Process analysis | ■ Root cause analysis<br>■ Cause–effect diagram<br>■ FMEA<br>■ Pareto chart<br>■ DOE<br>■ Shainin method |
| Improve (I) | ■ Statistical improvement: eliminate the root cause of inconsistency<br>■ Process improvement: increase the value-adding processes and decrease nonvalue-adding processes | ■ Brainstorming<br>■ TRIZ |
| Control (C) | ■ Statistical control: develop a data collection strategy to ensure consistency of performance<br>■ Process control: establish new standards | ■ SPC<br>■ EPC<br>■ Documentation |

**Table 4.3** DMAICT framework

| | Phase | Tools |
|---|---|---|
| D | Define the scope and objective of the project, the critical-to-quality (CTQ) issues, and the potential opportunities. | ■ Project charter<br>■ Benchmarking surveys<br>■ Spider charts<br>■ Flowchart |
| M | Measure the process performance, especially the CTQ issues, to analyze the operations of the current system. | ■ Quality function deployment (QFD)<br>■ Failure modes, effects, and criticality analysis (FMECA)<br>■ Gage R&R |
| A | Analyze data collected and use process maps to determine root causes of defects and prioritize opportunities for improvement. Apply statistical tools to guide the analysis. | ■ Histogram/Pareto chart/run chart<br>■ Scatter plot/cause and effect diagram<br>■ Product capability analysis |
| I | Improve the process by designing creative solutions to fix and prevent problems. Some experiments may be performed in order to find the best solution. Optimization methods are utilized to determine the optimum solution. | ■ Quality function deployment (QFD)<br>■ FMECA<br>■ Statistical experimental design and analysis<br>■ Simulation |
| C | Control the process on the new course. Performance tracking mechanisms and measurements are put in place to ensure that the gains are not lost over time. The key to overall success is sustainability. | ■ Gage R&R<br>■ Statistical process control/ control charts<br>■ QS9000/ISO9000 |
| T | Transfer ideas and knowledge developed in one project to other sections of the organization. Transfer the methods and solutions developed for one product or process to other similar products or processes. | ■ Project management<br>■ Collaborative team effort and cross-functional teams |

emergence of Lean Six Sigma, based on Toyota's production system (Liker 2003; Ohno 1988; Shingo 1989).

## 4.4 Optimization Problems in the Six Sigma Process

Many optimization problems occur in the six phases of this methodology. In this section, optimization models to improve the quality of the system to the Six Sigma level are reviewed. Various methods and tools of probabilistic design, robust design, design of experiments, multivariable optimization, and simulation techniques can be used for this purpose. The methodology can be improved and extended for the analysis and improvement phases of the Six Sigma process (Kapur and Feng 2005). In the analysis phase, the system transfer function and variance transmission equations need to be developed to enable formulating options for improvement by understanding the system. Based on the system transfer function or variance transmission equation, optimization models are formulated and solved to obtain the best decisions. These topics are briefly discussed in the following section.

### 4.4.1 System Transfer Function

A typical system consists of many input variables and one or more output variables. The input variables include both controllable factors and uncontrollable, or noise, factors. For a system with one output variable, as given in Figure 4.3, $X_1$, $X_2, \ldots, X_n$ are the controllable variables, and $y$ is the realization of random output variable $Y$.

As we discussed, in the measurement phase of the DMAICT process, the critical-to-quality (CTQ) characteristics are developed. In order to understand the system, we need to analyze the functional relationship between the output variable and the input variables, which can be described as a system transfer function (STF):

$$y = g(x_1, x_2, \ldots, x_n) + \varepsilon, \tag{4.4}$$

where $\varepsilon$ is the system error caused by the noise factors. Let $y$, $x_1$, $x_2, \ldots, x_n$ be the realization of random variables $Y$, $X_1$, $X_2, \ldots, X_n$, respectively.

The CTQ characteristics in the system are linked together through the system transfer functions. The CTQ flow-down tree (Kapur and Feng 2006) in Figure 4.5 illustrates how the system transfer functions establish the relationships among the CTQs at different levels.

The process can be improved during the design phase by reducing the bias or variance of the system output—that is, by changing the mean and variance of the quality characteristics of the output. Statistical methods for process optimization, such as experimental design, response surface methods (Myers and Montgomery 2002), and Chebyshev's orthogonal polynomials, can be used.

Integrated optimization models are developed to minimize the total cost to both producers and customers by determining the distribution of the controllable factors. For many complex systems, the analytical forms of the STF are explicitly known; even

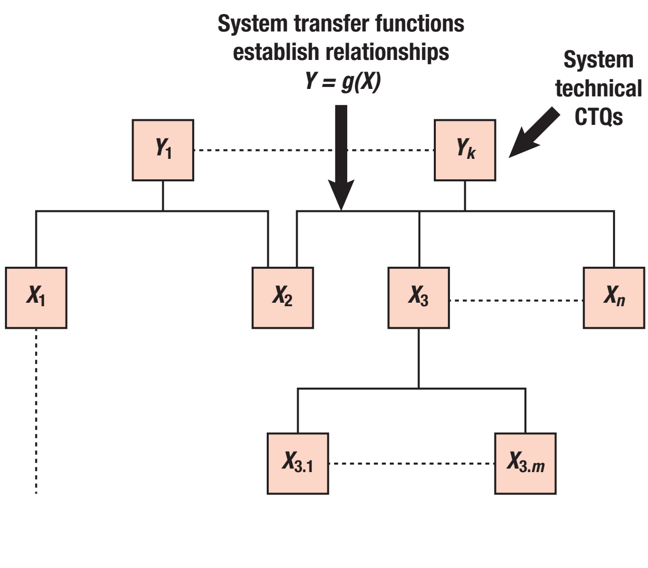


**Figure 4.5** The CTQ flow-down tree diagram.

so, it is usually very complicated to work with them. Given a set of values for the input variables of the system, the corresponding values of the response variables can be obtained through computer simulations or actual experiments. Based on the simulated or experimental data, an empirical model of the system transfer function can be developed using the regression method. The mean and variance models can be obtained by applying conditional expectation and variance operators to the regression model. Myers and Montgomery discuss this approach to obtain the mean and variance response models.

### 4.4.2 Variance Transmission Equation

Six Sigma methodology strives to improve quality by reducing the variation of a process. Given a particular requirement of the system, one of the problems in the Six Sigma process is to determine the optimal variances of the input variables. Instead of finding the system transfer function, what must be found is the relationship of the variances between the input and output variables. Letting $\sigma_Y^2$ denote the variance of the output variable $Y$ and $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$ denote the variances of the input variables $X_1$, $X_2, \ldots, X_n$, Six Sigma methodology strives to improve quality by reducing the variation of a process. Given a particular requirement of the system, one of the problems in the Six Sigma process is to determine the optimal variances of the input variables. Instead of finding the system transfer function, what must be found is the relationship of the variances between the input and output variables. Letting denote the variance of the output variable Y and denote the variances of the input variables $X_1$, $X_2$, $\ldots$, $X_n$, the functional relationship of the variances can be expressed as a variance transmission equation (VTE) as given below:

$$\sigma_Y^2 = h\left(\sigma_1^2, \ldots, \sigma_n^2\right) + \varepsilon, \tag{4.5}$$

where ε is the error. The VTE transfers the variances of the input variables to the variance of the response variable.

Different approaches can be used to develop the variance transmission equation based on the information we have. If the STF is known and differentiable, the VTE can be approximated using Taylor's expansion:

$$\sigma_Y^2 \approx \sum_{i=1}^{n} \left[ \frac{\partial g(\mu_1, \mu_2, \ldots, \mu_n)}{\partial x_i} \right]^2 \sigma_i^2, \tag{4.6}$$

where $\mu_i$ is the expected value of $x_i$, $i = 1, 2, \ldots, n$, and we assume that $X_1, X_2, \ldots, X_n$ are independent variables. Kapur and Lamberson (1977) give an example commonly used in reliability design to analyze the error in this approximation method.

If the STF is not known in an analytical form, the VTE can be developed using statistical tools such as linear regression, design of experiments, and response surface methodology. Computer simulations or actual experiments are used to obtain data for the analysis of variance. The emphasis on fractional factorial design may limit the number of real experiments.

***4.4.2.1 Taguchi's VTE for Fixed Effect Model*** Taguchi (1987) constructed a variance transmission equation with the assumption of no interaction between the components. The equation is intuitively appealing but has no solid theoretical basis, and the interactions between the components are overlooked. The VTE developed by Taguchi's methods has the advantage that it can be developed by experimentation or simulation even if the analytical form for STF is not known. Using Taguchi's "three-level factorial experiments," the total evaluations of the function are significantly fewer than that required by a Monte Carlo simulation.

In practice, it may be necessary to choose the levels of the design factors at random. For the random effects models, the VTE should be based on the expected mean square (EMS) values. An analysis of variance for a random effect model is used to develop the variance of treatment effects.

### 4.4.3 Economic Optimization and Quality Improvement

The ultimate objective of the Six Sigma strategy is to minimize the total cost to both the producer and the consumer, or the cost of the whole system. The cost to the consumer is related to the expected quality loss of the output variable, and it is caused by the deviation from the target value. The cost to the producer is associated with the changing probability distributions of input variables. If the system transfer function and the variance transmission equation are available, and if the cost functions for different grades of input factors are given, then the general optimization models can be developed and are briefly discussed.

We usually consider the first two moments of the probability distributions of input variables, and then the optimization models focus on the mean and variance values. Therefore, the expected quality loss to the consumer consists of two parts: the bias of the process and the variance of the process. The strategy to reduce bias is to find adjustment factors that do not affect variance and thus are used to bring the mean closer to the target value. Design of experiments can be used to find these adjustment factors, although it will incur some cost to the producer. In order to reduce the variance of $Y$, the designer should reduce the variances of the input variables, which will also increase cost. The problem is to balance the reduced expected quality loss with the increased cost for the reduction of the bias and variances of the input variables. Typically, the variance control cost for the $i$th input variable, $X_i$, is denoted by $\sum_{i=1}^{n} C_i\left(\sigma_i^2\right)$, and the mean control cost for the $i$th input variable, $X_i$, is denoted by $D_i(\mu_i)$. Focusing on the first two moments of the probability distributions of $X_1$, $X_2$, . . . , $X_n$, the general optimization model is formulated as follows:

Minimize

$$TC = \sum_{i=1}^{n} C_i\left(\sigma_i^2\right) + \sum_{i=1}^{n} D_i(\mu_i) + k\left[\sigma_Y^2 + (\mu_Y - y_0)^2\right], \tag{4.7}$$

subject to

$$\mu_Y \approx m(\mu_1, \mu_2, \ldots, \mu_n) \tag{4.8}$$

$$\sigma_Y^2 \approx h\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2\right). \tag{4.9}$$

In this objective function, the first two terms,

$$\sum_{i=1}^{n} C_i\left(\sigma_i^2\right) \quad \text{and} \quad \sum_{i=1}^{n} D_i(\mu_i),$$

are the control costs on the variances and means of input variables, or the cost to the producer; the term $k\left[\sigma_Y^2 + (\mu_Y - y_0)^2\right]$ is the expected quality loss to the customer, where $k$ is a constant in the quality loss function and $y_0$ is the target value of $y$. The first constraint, $\mu_Y \approx m(\mu_1, \mu_2, \ldots, \mu_n)$, is the mean model of the system, which can be obtained through the system transfer function. The second constraint, $\sigma_Y^2 \approx h\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2\right)$, is the variance transmission equation.

### 4.4.4 Tolerance Design Problem

Assuming that the bias reduction has been accomplished, the general optimization problem given by Equation 4.7 can be simplified as a tolerance design problem, which is given below:

Minimize

$$TC = \sum_{i=1}^{n} C_i\left(\sigma_i^2\right) + k\sigma_Y^2, \tag{4.10}$$

subject to

$$\sigma_Y^2 \approx h\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2\right). \tag{4.11}$$

The objective of the tolerance design is to determine the tolerances (which are related to variances) of the input variables to minimize the total cost, which consists of the expected quality loss due to variation, $k\sigma_Y^2$, and the control cost on the tolerances of the input variables, $\sum_{i=1}^{n} C_i\left(\sigma_i^2\right)$. Typically, $C_i\left(\sigma_i^2\right)$ is a nonincreasing function of each $\sigma_i^2$.

***4.4.4.1 The Dual Problem of Tolerance Design*** In addition, given the constraint on the cost of the control of tolerances, the dual problem of the tolerance design problem can be developed to minimize the variance of response as given below:

Minimize

$$\sigma_Y^2 \approx h\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2\right), \tag{4.12}$$

subject to

$$\sum_{i=1}^{n} C_i\left(\sigma_i^2\right) \le C^*, \tag{4.13}$$

where $C^*$ is the maximum allowable cost to the producer, or the control cost on the tolerances of input variables.

## 4.5 Design for Six Sigma

While the Six Sigma process improvement approach leaves the fundamental structure of a process unchanged, Design for Six Sigma (DFSS) involves changing or redesigning the process at the early stages of the product and/or process life cycle. DFSS becomes necessary when the current process has to be replaced, rather than repaired or just improved; the required quality level cannot be achieved by just improving an existing process; when an opportunity is identified to offer a new process; and/or when there is a breakthrough and new disruptive technologies are available.

DFSS (Yang and El-Haik 2003) is the other strategy used to achieve Six Sigma process capability. However, the main difference between the Six Sigma process improvement strategy and DFSS is the approach taken to reach Six Sigma process capability. Six Sigma process improvement focuses on the improvement of the process after it has been developed and is in operation. Therefore, data are already available for measurement and analysis. DFSS focuses on the design steps that ensure the problem will not happen in the first place. DFSS is usually applied before the production routine operation is started or the product is in the field. Because of this, Six Sigma process improvement is easier to analyze in terms of cost–benefit analysis, because it works from the existing operation, from which data can be collected before and after the improvement strategy implementation and compared in terms of Six Sigma process levels. Consequently, DFSS is considered more of philosophical tool than a practical tool, because it can only be used at a preoperations level and its focus is on the research, development, and design phases. Table 4.4 shows an example of a product/process life cycle and Six Sigma tasks and tools.

**Table 4.4** Product/process life cycle and Six Sigma tasks and tools

| Product life cycle stages | Six Sigma tasks | Six Sigma strategy | Six Sigma tools |
|---|---|---|---|
| 1. Impetus/ ideation | ■ Identify project scope, customers, suppliers, customer needs | DFSS | ■ Customer research, process analysis, Kano analysis, QFD |
| 2. Concept development | ■ Develop new process concept to come up with right functional requirements<br>■ Ensure that new concept can lead to sound system design, free of design vulnerabilities<br>■ Ensure the new concept is robust for downstream development | DFSS | ■ QFD<br>■ Taguchi methods/robust design<br>■ TRIZ<br>■ Axiomatic design<br>■ DOE<br>■ Simulation/optimization<br>■ Reliability-based design |

(*Continued*)

**Table 4.4** (*Continued*)

| Product life cycle stages | Six Sigma tasks | Six Sigma strategy | Six Sigma tools |
|---|---|---|---|
| 3. Process design/tryout | ■ Ensure process can deliver desired functions<br>■ Ensure process will perform consistently and robustly<br>■ Validate process for performance and consistency | DFSS | ■ Taguchi methods/robust design<br>■ DOE<br>■ Simulation/optimization<br>■ Reliability-based design/testing and estimation<br>■ Statistical validation |
| 4. Process and routine operations | ■ Ensure process will perform consistently | Six Sigma process improvement | ■ SPC<br>■ Troubleshooting and diagnosis<br>■ Error-proofing |
| 5. Process Improvement | ■ Improve to satisfy new requirements | Six Sigma process improvement | ■ DMAICT strategy<br>■ Customer analysis, Kano analysis<br>■ QFD<br>■ Statistical measurement system<br>■ DOE, Shanin methods, multivariate analysis, regression analysis<br>■ Process analysis, value stream mapping<br>■ SPC |

The major objective of the Six Sigma improvement process is "to do it right and do it right all the time." The major objective of DFSS is "to design it right the first time" to avoid complications during the product life cycle. Most managers who are unable to improve Six Sigma process performances retreat to the design phase to reach Six Sigma process capability. Generally, a bad design results in a bad performance. The sources for bad design are either conceptual vulnerabilities that exist due to violations of design axioms (Yang and andEl-Haik 2003)[6] and principles, or operational vulnerabilities that exist due to lack of robustness in the usage environment.

DFSS aims to tackle both operational vulnerabilities and conceptual vulnerabilities. Conceptual vulnerabilities are generally anticipated by using quality engineering (Taguchi 1986; Taguchi 1987), TRIZ (Altshuller 1984), axiomatic design (Suh 1990, 2001), and theory of probability and statistical modeling. Operational vulnerabilities are generally anticipated by using robust design, DMAIC Six Sigma process improvement, and tolerance design/tolerance analysis. Conceptual vulnerabilities are usually overlooked or underestimated because of the lack of a compatible systematic approach to finding an ideal solution; overcoming the errors of the designer; the pressure of schedule deadlines; and budget limitations.

Traditional quality methods focus on improvement, since the process is already ongoing, and it is therefore easy to measure the costs and benefits. This triggers an endless cycle of design-test-fix-retest, because the quality improvement process is based on the necessity to tackle operational vulnerabilities that have been overlooked at the conceptual phase. Corrective action to improve conceptual vulnerabilities by

[6]An axiom is a truth that cannot be derived but for which no counterexamples or exceptions exist.

repairing operational vulnerabilities is not an efficient approach. That is why many of current problem-solving techniques are hard to implement and very costly.

Traditional design is based on both empirical data (experience) and subjectivity (creativity). Nonscientific design produces less-than-optimal solutions for achieving Six Sigma process capability. Axiomatic design is introduced to provide a scientifically based design principle.

Human nature in the business world is typically more reactive than proactive and focuses attention on the later phases of the design cycle. Although DFSS takes more effort at the beginning, it will benefit an organization in the long run by designing Six Sigma quality into products and processes. There are several methodologies for DFSS, such as define, measure, analyze, design, and verify (DMADV) and identify, design, characterize the design, optimize, and validate (ICOV [or IDOV]). DMADV is a popular methodology, and basically follows the DMAICT model (omitting the transfer step). ICOV is a well-known design methodology, especially in the manufacturing world. Thus, DFSS integrates many of the well-known methods, tools, and philosophies for quality and reliability improvement; research, development, and design strategies; and management strategies to build teamwork and collaboration from cradle to grave for products and processes in any organization.

The suggested ICOV DFSS strategy has four phases:

1. Identify (I) the requirements.
2. Characterize (C) the design.
3. Optimize (O) the design.
4. Verify (V) the design.

### 4.5.1 Identify (I)

The design project can be categorized as design and redesign.

*Step 1: Draft Project Charter.* This is the same as the DMAIC strategy. However, the draft project charter in the DFSS project is longer because, as argued earlier, the design phase takes longer than the improvement phase. The latter is like patching a hole, while design involves creating something from nothing.

*Step 2: Identify Customer Requirements.* Since all processes are defined in terms of customer satisfaction, the customer requirements need to be identified before they can be translated and mapped into engineered functional requirements.

QFD and Kano analysis are examples of early-stage tools that can be used to help identify critical-to-quality customer requirements. An algorithmic approach is used to ensure that all the elements of the customer requirements are identified and included. The approach involves the following steps:

1. Identify methods of ascertaining customer needs and wants.
2. Ascertain customer needs and wants.
3. Translate the voice of the customer (VOC) into functional and measurable requirements.
4. Finalize requirements: establish minimum requirements definition; identify and fill gaps in customer-provided requirements; validate application and usage environments.

5. Identify points that are critical to quality (CTQ), critical to delivery (CTD), and critical to cost (CTC). (CTQ, CTD, and CTC can be referred to as CTXs.)
6. Quantify CTXs: establish metrics for CTXs; establish performance levels and operating windows; and perform flow-down of CTXs.

### 4.5.2 Characterize (C)

The customer's requirements may be too abstract to be meaningful to the product/process engineer. Therefore, the CTQ, CTD, and CTC elements must be translated into product/process functional requirements—that is, those things necessary so that the product can function at the level of customer satisfaction. After these functional requirements (FRs) have been identified, the design parameters and process variables can be determined. Thus, it is a very important step to tackle conceptual vulnerabilities. Some tools used in this phase include: TRIZ, QFD, axiomatic design, robust design, Design for X (X = manufacture and assembly, reliability, maintainability, serviceability, environmentality, life-cycle cost), DFMEA and PFMEA, design review, CAD, simulation, and process management. The characterization phase comprises a few strategic and algorithmic steps to aid designers in good design:

*Step 1: Translate CTS into Process-Functional Requirements*.

Step 2: *Generate Design Alternatives*. Sometimes, the existing technology is unable to deliver the CTS. It is therefore very important to design alternatives to deliver the CTS. The new design can be creatively begun from scratch or incrementally evolved from the baseline design.

*Step 3: Evaluate Design Alternatives to Select the Best Process-Functional Requirements*.

### 4.5.3 Optimize (O)

There are many combinations of parametric designs that engineers can use to satisfy functional requirement goals. Optimization aims to identify the best way to tailor the functional requirements and minimize operational vulnerabilities. The objective is to provide a logical and objective basis for setting manufacturing tolerances. Once the optimal parameter design is established, engineers can determine the level of system robustness best suited to the environment. Some tools used in this phase include: design/simulation tools, DOE (design of experiments), the Taguchi method, parameter design, and tolerance design, reliability-based design, and robustness assessment.

### 4.5.4 Verify (V)

Once the parameters of the design are optimized, a validation or design inspection is performed before the design is launched into mass production and process implementation.

*Step 1: Conduct Pilot Testing and Refining.* Pilot testing can be used to test and evaluate real-life performance.

*Step 2: Validate the Results of the Pilot Testing.* Confirm that process variables accounted for in the parameter design are mapped to functional requirements and can produce the identified customer attributes (quality functional validation). Confirm that the final process can produce Six Sigma process capability (statistical validation).

*Step 3: Roll Out the Product Commercially and Hand It Over to the New Process Owner.* The following tools are used in this phase: process capability modeling, DOE (design of experiment), reliability testing, poka-yoke, error-proofing, confidence analysis, process control plan, and training.

A summary of the key steps for DFSS and the applicable Six Sigma tools is given in Table 4.5. Table 4.6 shows the key differences between Design for Six Sigma (DFSS) and the traditional Six Sigma process improvement strategy based on DMAICT.

Six Sigma and other continuous improvement strategies are extremely valuable tools in today's global competition. The ideas presented in this chapter are important in terms of both the research and its applications for the analysis and improvement phases of the DMAICT process. Six Sigma will contribute to the design of many products and processes and also improve the quality and productivity in any organization.

**Table 4.5** Summary of key steps in DFSS and the Six Sigma tools available at each step

| Key DFSS steps | | Six Sigma tools |
|---|---|---|
| (I) Identify | ■ Draft project charter<br>■ Identify and document process<br>■ Identify VOC | ■ SIPOC modeling<br>■ P-diagram<br>■ Ishikawa diagram<br>■ Kano Analysis<br>■ QFD |
| (C) Characterize | ■ Translate CTS to process functional requirements<br>■ Define design alternatives<br>■ Map FR to DP (design parameter) | ■ TRIZ<br>■ QFD<br>■ Axiomatic design<br>■ Robust design<br>■ DFMEA<br>■ PFMEA<br>■ Design for "X"<br>■ CAD<br>■ Simulation |
| (O) Optimize | ■ Parametric design optimization to determine optimal process variable (PV) | ■ DOE<br>■ Taguchi method<br>■ Tolerance design<br>■ Reliability-based design<br>■ Robustness assessment |
| (V) Verify | ■ Validation of experimental data to customer satisfaction attributes | ■ Process capability modeling<br>■ DOE<br>■ Reliability testing<br>■ Poka-yoke<br>■ Confidence analysis<br>■ Process control plan<br>■ Training |

**Table 4.6** The key differences between Six Sigma process improvement and design for Six Sigma

| Key aspects | Six Sigma process improvement | Design for Six Sigma |
|---|---|---|
| Strategy and approaches | ■ DMAIC: define, measure, analyze, improve, control | ■ DMADV: define, measure, analyze, design, verify<br>■ DMADOV: define, measure, analyze, design, optimize, verify |
| Operating mode | ■ Reactive | ■ Proactive |
| Focus | ■ Fixing problems in existing process | ■ Up-front design of the process to prevent problems from happening |
| Benefits | ■ Easier to quantify in dollars | ■ Hard to quantify but tend to be greater long-term |

## 4.6 Summary

Successful industrial, manufacturing, and service organizations are interested in reducing variance in their products and processes. Customers as well judge the quality of a process or product based on the variance in quality that they encounter in their transactions with processes or repeated uses of a product. Six Sigma is a business management process that companies implement to achieve a reduction in variance and continuously improve their products and processes. The ideal goal of Six Sigma is to deliver near-zero defects for every product, process, and transaction within an organization.

There are six steps in the Six Sigma process; these steps can be remembered by the acronym DMAICT. The first step is to define the problem that needs to be solved. The second step is to measure the current capability of the process. The third step is to analyze the root causes of process variability. The fourth step is to improve the process capability. The fifth step is to determine which controls can be put in place to sustain the improvement. The sixth step is technology transfer. The Six Sigma process can be optimized in various ways to fit the specific needs of an organization. In addition to process optimization, which more or less leaves a company's current processes intact, there is also design for Six Sigma (DFSS), which involves changing or redesigning a process at the early stages of the product or process life cycle. Each company must weigh the various costs and benefits of implementing either Six Sigma process optimization or Design for Six Sigma.

## Problems

*4.1* What is a Six Sigma process? Why was this process developed and who developed it?

*4.2* Explain briefly why it is necessary and good strategy to consider variability around the target value for the underlying quality characteristic from the viewpoint of the customer.

*4.3* Consider a Three Sigma process where the mean is at the target value. As discussed in this chapter, this gives a probability of meeting the specifications as 0.9973 which corresponds to 2,700 defective parts per million.

(a) If a product consists of an assembly of 100 independent parts and all of these parts must be nondefective for the product to function successfully, what is the probability that any specific unit of the product is nondefective?

(b) If a complex product has 10,000 parts that function independently, what is the probability that any specific unit of the product will be nondefective?

(c) Now suppose that we have a Six Sigma process without mean shift for each part of the complex product that has 10,000 parts. What is the probability that any specific unit of the product will be nondefective?

(d) Now suppose that we have a Six Sigma process with mean shifted for each part of the complex product which has 10,000 parts. What is the probability that any specific unit of the product will be nondefective?

*4.4* The upper specification limit for a product is 10.00 and the lower specification limit is 8.00. It is assumed that the target value is 9.00. Based on statistical process control, it was found the process is in control and the mean of the process is 9.26 and the standard deviation is 0.21.

(a) What is the $C_{pk}$ index for this process?

(b) What is the DPMO for this process?

*4.5* This chapter covered the DMAIC process and explained the steps for such a process. Some companies have tollgates between each of the major steps in the DMAIC process. Briefly explain the need and importance of these tollgates.

*4.6* Suppose an organization for a particular product is operating at 4 level (where the mean is shifted from the target). This will result in 6210 DPMO. The objective is to achieve 6 performance (3.4 DMPO). Suppose the organization quality improvement effort is 25% annual improvement in quality level. How many years will it take to achieve 6 performance?

*4.7* Suppose your business is operating at Three Sigma quality level, and the project has an average annual improvement rate of 50%. How many years will it take to achieve Six Sigma quality?

*4.8* During the analysis phase, an organizations finds that it has discovered the solution for the underlying problem. Please discuss whether the solution should be immediately implemented and remaining steps of DMAIC process are abandoned.

*4.9* Consider any type of service system that you use and are very familiar with it. What are the CTQs for such a system and how will you apply the Six Sigma DMAIC process to this service system?

*4.10* What is the difference between Six Sigma DMAIC process and the DFSS?

*4.11* Explain various steps for DMADV process for any organization whose products you are familiar with.

*4.12* Explain briefly various steps of ICOV, especially for manufacturing, which is used for DFSS for any organization that you are familiar with. Briefly explain some of the tools, methods, and philosophies that you will use during various steps of ICOV.