## CHAPTER 22

## Earthquake Geoengineering

This chapter serves as an introduction to the large and complex field of geotechnical earthquake engineering. The book by Kramer (1996), the book by Towhata (2008), and the FHWA manual by Kavazanjian et al. (2011) are excellent references for further study.

### 22.1 BACKGROUND

Plate tectonics is the main reason for earthquakes on our planet. The Earth's crust is made of six continental plates (Figure 22.1) that have travelled large distances over geologic times. The plates move because of the thermal difference between the earth surface and the deeper layers. This thermal difference creates convection currents in the rock mass, which move the plates. The boundaries between plates are called faults. The problem is that the movement of the plates with respect to one another is not smooth. Indeed, the interface between plates or faults is rough and stresses accumulate along the fault over time. When the stress becomes equal to the strength of the fault surface, the fault shears in a dramatic motion known as an earthquake (Figure 22.2). A bit of lubricant would solve that problem!

An earthquake originates at some depth below the ground surface; this point is called the hypocenter. The point on the ground surface directly above the hypocenter is the epicenter. The distance between a site on the ground surface and the epicenter is the epicentral distance. An earthquake starts at one location, propagates along a fault, then propagates up into the rock mass and then into the soil mass (earthquake), and then sometimes into ocean (tsunami). This propagation is in the form of compression waves and shear waves (see section 8.2.1). Seismographs record the passage of the waves. A compression wave moves faster than the corresponding shear wave and will arrive first. The difference in time between the arrival of the compression wave and the shear wave can be used to determine the distance $d$ between the location of the seismograph and the epicenter:

$$
\begin{equation*}
d=\frac{\Delta t_{p-s}}{\frac{1}{v_{s}}-\frac{1}{v_{p}}} \tag{22.1}
\end{equation*}
$$

where $\Delta t_{p-s}$ is the difference in time between the arrival of the $p$ wave and the $s$ wave, $v_{s}$ is the shear wave velocity, and $v_{p}$ is the compression wave velocity. One seismograph can give the distance $d$ but not the direction of the wave generating the signal; three seismographs are necessary to locate the epicenter (Foster 1988) (Figure 22.3).

### 22.2 EARTHQUAKE MAGNITUDE

The size of an earthquake can be quantified in several ways. The first and oldest way is the earthquake intensity, which is a qualitative description of the effect of the earthquake. The Mercalli scale (1883) is the best known and goes from I (not felt) to XII (total destruction). Giuseppe Mercalli was an Italian seismologist and volcanologist who proposed this scale in the late 1800s. It was revised a few times after that. The problem with the Mercalli scale is that it relies on human reactions and structural damage observations, both of which depend on more than just the size of the earthquake.

The Richter scale is the most well-known of the magnitude scales (Richter 1935). The Richter magnitude ( $M_{L}$, with the subscript $L$ used to designate local magnitude) is defined as the logarithm base 10 of the magnitude trace amplitude in micrometers recorded on a Wood-Anderson seismometer located 100 km from the epicenter of the earthquake. Seismic instruments were developed and installed around 1930 and are used extensively today to quantify earthquake magnitude. The Richter scale has been modified over the years and led to the use of the body wave magnitude and surface wave magnitude scales.

The body wave magnitude $\left(m_{b}\right)$ is calculated from the amplitude of compression waves with periods of about 1 sec toward the beginning of the record. The surface wave magnitude $\left(M_{s}\right)$ is calculated from the amplitude of Rayleigh waves with periods of about 20 sec . One limitation with these scales is that they are unable to recognize large earthquakes; this is called saturation. Saturation occurs at a magnitude of about 6.2 for $m_{b}$ and 8 for $M_{s}$. Saturation is due to the fact that very large earthquakes release more of their energy at longer


Figure 22.1 Tectonic plates on Earth. (Photo by United States Geologic Survey [USGS])


Figure 22.2 Movement of tectonic plates.
periods; because the periods associated with the $m_{b}$ and $M_{s}$ calculations are fixed, they cannot acknowledge higher periods and therefore larger earthquakes. Another limitation of these magnitude scales is that they do not address the amount of time associated with the shaking.

The moment magnitude ( $M_{w}$ ) takes that aspect into account and is broadly used today. It is rooted in the seismic moment
$M_{o}$ associated with the work done by the earthquake along the fault:

$$
\begin{equation*}
M_{o}=G A D \tag{22.2}
\end{equation*}
$$

where $G$ is the shear modulus of the rock, $A$ is the area over which the slip occurs, and D is the amount of slip movement. Because $M_{o}$ is a very large number, and because


Figure 22.3 Locating the epicenter with three seismographs.
the public is used to the Richter scale, Kanamori (1977) proposed a transformation that makes $M_{w}$ consistent with the other scales, including the Richter scale. The moment magnitude $M_{w}$ is then obtained by:

$$
\begin{equation*}
M_{w}=0.66 \log M_{0}(\mathrm{~N} . \mathrm{m})-6.05 \tag{22.3}
\end{equation*}
$$

As an example, let's calculate the moment magnitude of the December 26, 2004, Sumatra-Andaman earthquake. As reported by Lay et al. (2005), the fault surface was 1300 km long and 220 km deep, and the slip distance was 5 m . For a typical value for $G$ of $3 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}$, the seismic moment $M_{o}$ was $4.7 \times 10^{22} \mathrm{~N} . \mathrm{m}$. Then the moment magnitude is very close to 9 or a huge earthquake. Though this is the moment magnitude, the news media would report that the earthquake registered as 9 on the Richter scale. Note that all these scales are $\log$ scales, so that, for example, a magnitude 9 earthquake is 10 times larger than a magnitude 8 earthquake.
Yet another way to classify an earthquake is to calculate the energy released during the slip. Bath (1966) proposed to obtain the energy $E$ from:

$$
\begin{equation*}
\log E=5.24+1.44 M \tag{22.4}
\end{equation*}
$$

where $E$ is the energy in N.m or joules and $M$ is the magnitude. So if $M$ is 9 , then $E$ is $1.6 \times 10^{18}$ joules.

To give you an idea of how much energy this represents, it is enough to cover the electrical consumption of the entire United States for one month. So if we could harness that energy and turn it to good use, it would be very valuable, and unfortunately it seems to be renewable energy!

### 22.3 WAVE PROPAGATION

For specifics on wave propagation, see section 8.2.1.

### 22.4 DYNAMIC SOIL PROPERTIES

Dynamic soil properties have been discussed in previous chapters:

- See section 7.2 for the seismic CPT
- See section 7.11.5 for the lightweight deflectometer test
- See sections 8.2.2, 8.2.3, 8.2.4, and 8.2.5 for dynamic in situ tests based on wave propagation
- See section 9.13 for the resonant column test, shear modulus, and damping ratio
- See section 14.10 for the initial tangent shear modulus $G_{\text {max }}$
- See section 14.11 for the normalized shear modulus $G / G_{\max }$ and damping ratio vs. shear strain $\gamma$ curves
- See sections 14.15 and 14.16 for the resilient modulus
- See sections 18.8.7 and 18.8.8 for the rate of loading and cyclic loading effects


### 22.5 GROUND MOTION

During an earthquake, the rock fault shears and sends shear waves and compression waves through to the ground surface. This shaking of the rock and soil mass can be recorded using instruments sensitive to motion. These are generally accelerometers that use the piezoelectric effect. They contain microscopic crystal structures (crystal quartz) that get stressed by inertia forces and react by creating a change in voltage. This voltage is measured and correlated by calibration to accelerations. While the soil motion created by an earthquake is in three directions, the horizontal motion is usually the one of most interest because it tends to cause the most damage. Figure 22.4 shows an acceleration record for an earthquake along with the velocity and the displacement. The velocity and the displacement are obtained by integrating once and then twice the acceleration versus time.

These time domain signals are quite complex, and there is a need to report simpler parameters to describe an earthquake. These parameters include information on the amplitude $A$, the frequency $f$, and the duration $t$ of the acceleration $a$; velocity $v$; and displacement $u$. The amplitudes of $a, v$, and $u$ can be characterized by the peak values, which are the highest values in the signal. The PGA is the peak ground acceleration, the PGV is the peak ground velocity, and the PGD is the peak ground displacement. The PGA, PGV, and PGD are indicated in Figure 22.4. A huge earthquake can generate $10 \mathrm{~m} / \mathrm{s}^{2}$ or 1 g acceleration, whereas acceleration of $0.1 \mathrm{~m} / \mathrm{s}^{2}$ or 0.01 g is associated with small earthquakes. Figure 22.5 shows a PGA map of the United States prepared by the United States Geologic Service (USGS) for two distinct return periods: 2275 years and 475 years. Also useful are the effective acceleration (acceleration closest to the structural response and damage of the structure), the sustained maximum acceleration (acceleration sustained for 3 or 5 cycles), and the effective design acceleration (peak acceleration after filtering accelerations above 8 Hz ).

A more detailed inspection of Figure 22.4 shows that the frequencies associated with the acceleration signal are higher than the frequencies associated with the velocity signal, which are themselves higher than the frequencies associated with the displacement signal. The frequency content is different and is best obtained by performing a Fourier transform


Figure 22.4 Ground motion for an earthquake (FHWA 1998).
analysis (Kramer 1996). This transformation is a mathematical transformation named after the work of Jean Baptiste Joseph Fourier, a French mathematician and physicist, who developed it around 1800. It transforms the signal from a plot of amplitude vs. time into a plot of amplitude vs. frequency (Figure 22.6) or from the time domain to the frequency domain. This amplitude vs. frequency plot is called a Fourier spectrum, so one will have a Fourier acceleration spectrum, a Fourier velocity spectrum, and a Fourier displacement spectrum. The Fourier spectra describe the ground motion.

Another spectrum is the response spectrum to a particular earthquake input motion. A response spectrum is a plot of the maximum response ( $a, v$, or $u$ ) of a linear single degree of freedom (SDOF) system to an earthquake input motion versus the natural period T of the system for a given damping ratio $\beta$. The natural period $T$ of an undamped SDOF system is given by:

$$
\begin{equation*}
T=2 \pi \sqrt{\frac{m}{k}} \tag{22.5}
\end{equation*}
$$

where $m$ is the mass of the system $(\mathrm{kg})$ and $k$ is the spring stiffness ( $\mathrm{N} / \mathrm{m}$ ).

The damping ratio $\beta$ in percent is given by:

$$
\begin{equation*}
\beta=\frac{c}{c_{c r i t}} \times 100=\frac{c}{2 \sqrt{m k}} \times 100 \tag{22.6}
\end{equation*}
$$

where $c$ is the damping coefficient of the dashpot ( $\mathrm{N} . \mathrm{s} / \mathrm{m}$ ) and $c_{c r i t}$ is the critical damping.

The critical damping is what brings the system back to zero without oscillations. Nowadays many doors are equipped with critically damped pistons so they close back without oscillations. (Old saloon bar doors, for example, did not have critical dampers.) In earthquake engineering, a damping ratio equal to $5 \%$ is common. Three response spectra are typically created: one each for the acceleration $\left(S_{a}\right)$, the velocity $\left(S_{v}\right)$, and the displacement $\left(S_{d}\right)$.
The process followed to obtain an acceleration response spectrum is illustrated in Figure 22.7 and an example is shown in Figure 22.8. The step-by-step procedure is as follows:

1. Choose the input motion signal for the earthquake.
2. Choose a value of the damping ratio $\beta$ and the mass m for the SDOF.
3. Select a value of the stiffness $k$ of the SDOF and excite the system with the selected earthquake motion.
4. Record the highest value (acceleration, velocity, displacement) of the output motion of the SDOF.
5. Repeat steps 3 and 4 for different values of $k$.
6. Plot the maximum values of step 4 (spectral value) versus the fundamental period of the SDOF. This is the response spectrum.
Note that structures have fundamental periods that increase with their height (Figure 22.9); these are generally in the


Figure 22.5 USGS maps of peak ground acceleration for $2 \%$ and $10 \%$ probability of exceedance over 50 years corresponding to 2275-year and 475-year return periods respectively. (Courtesy of USGS.)
range of 0.1 seconds for very small buildings to 10 seconds for extremely tall and flexible buildings. The spectral acceleration on the ordinate of the spectrum depends on the rock motion, the soil properties, the damping ratio, and the ratio of the SDOF stiffness over mass. It does not depend on $k$ and $m$ separately, because of the mathematics behind the problem.

The spectrum itself-meaning the curve of $a$ vs. $T$-is a function of the rock motion, the soil properties, and the damping ratio, but not of the stiffness and the mass of the SDOF. This unique property makes it possible to recommend a single design spectrum that can be used for any structure, given a damping ratio.


Figure 22.6 Fourier transforms of signals.


Figure 22.7 Development of an acceleration response spectrum. (After Matasovic 1993.)

The response spectrum is very useful because, for a building with a given fundamental period, this spectrum defines the highest spectral acceleration to which the structure is likely to be subjected. This acceleration times the mass of the building
gives the inertia force to be resisted by the structure and the foundation. Table 22.1 shows orders of magnitudes of accelerations and velocities for the soil and the structures placed on it.

The duration of the earthquake's strong motion has a major influence on the amount of damage inflicted. The most common way to measure duration is to use the bracketed duration, which is defined as the time between the first and last exceedance of a chosen threshold of acceleration. This threshold is often taken as 0.05 g . Figure 22.10 shows an example in which the bracketed duration is 15 seconds.

### 22.6 SEISMIC HAZARD ANALYSIS

Now that we know how to characterize ground motion, we need to establish what parameters to consider for the site where the construction will take place, or where the stability must be evaluated, or where liquefaction is an issue. A distinction is made here between a deterministic analysis and a probabilistic analysis. In a seismic hazard deterministic analysis, the steps are as follows:

1. Identify all earthquake sources capable of creating significant ground motion at the site.
2. Determine the distance between the source and the site.
3. Select the controlling earthquake, that is, the earthquake most likely to produce the highest level of shaking at the site.


Figure 22.8 Examples of response spectrum.


Figure 22.9 Relationship between fundamental period and height of a building.
4. Determine the ground motion parameters at the site associated with the controlling earthquake: peak acceleration, peak velocity, response spectrum.

Step 3 requires the use of an attenuation relationship, a relationship that gives the decrease in acceleration, for example, as a function of the distance from the source. Such attenuation relationships have been developed based mostly on experimental data. These empirical equations (e.g., Cornell et al. 1979) are typically of the form:

$$
\begin{equation*}
\log (P G A)=A+B M-C \log (R+D) \tag{22.7}
\end{equation*}
$$

Table 22.1 Order of Magnitude of Soil and Structure Horizontal Acceleration and Velocity for Different Earthquake Magnitudes

|  | Accelerations <br> (Gravities) |  |  | Velocity <br> (mm/second) |  |
| :--- | :--- | :---: | :--- | :--- | :--- |
|  | Ground <br> Motion | Structure |  | Ground <br> Motion | Structure |
| 8 | 0.60 | 0.33 |  | 740 | 410 |
| 7.5 | 0.45 | 0.22 |  | 560 | 280 |
| 7 | 0.30 | 0.15 |  | 360 | 180 |
| 5.5 | 0.12 | 0.1 |  | 150 | 130 |

(After Hall and Newmark 1977)
where PGA is the peak ground acceleration, $M$ is the earthquake magnitude, $R$ is the distance from the source to the site, and $A, B, C$, and $D$ are calibration constants. Figure 22.11 shows a correlation by Boore et al. (1997), including the data points used.

In a seismic hazard probabilistic analysis, the steps are somewhat different and consist of the following:

1. Identify all earthquake sources capable of creating significant ground motion at the site. This is the same step as in a deterministic analysis, except that a probability distribution is associated with the location of the source to quantify that uncertainty.


Figure 22.10 Example of bracketed duration.



Figure 22.11 Attenuation of peak horizontal ground acceleration (Boore et al. 1997).
2. Determine the magnitude and recurrence of earthquakes from each source. Small earthquakes occur more often than large earthquakes. The magnitude-recurrence relationship gives the number $N$ of earthquakes of a given magnitude or higher that may occur every year. Gutenberg and Richter (1944) proposed the following model:

$$
\begin{equation*}
\log N=a-b M \tag{22.8}
\end{equation*}
$$

where $N$ is the number of earthquakes per year of magnitude $M$ or greater, and a and b are regional parameters.
This model has been revised (Figure 22.12) based on further measurements over the years and also by including geologic and geodetic data. Note that the
reciprocal of N is called the recurrence interval or return period. In the building industry, the recurrence interval of the earthquake to use for insuring collapse prevention is the 2500-year earthquake; in the bridge industry, it is the 1000-year earthquake.
3. Determine the ground motion at the site by using an appropriate attenuation relationship. This is the same step as in a deterministic seismic hazard analysis, except that the uncertainty regarding the attenuation is now included as shown in Figure 22.11.
4. The uncertainties in steps 1 through 3 are combined to obtain the probability that the ground motion parameters will be exceeded over a chosen period of time and ensure that this probability meets a target value chosen as acceptable by design.


Figure 22.12 Magnitude-recurrence relationship. (Kavazanjian et al. 2011, based on Gutenberg and Richter 1944, and Schwartz and Coppersmith 1984.)

### 22.7 GROUND RESPONSE ANALYSIS

When a rock fault slides, it shakes the adjacent rock into motion. This motion is propagated in all directions, including upward toward the ground surface. Although most of the travel is through rock, the last few 100 meters may be through soil. The propagation through soil may have a significant impact on the motion of the ground surface, and this ground response is addressed in this section. Propagation analysis can be done as a one-dimensional (1D), two-dimensional (2D), or three-dimensional (3D) analysis. The theory for such analyses can get quite complex. Two simple cases of one-dimensional analysis are presented here. More advanced coverage is given in Kramer (1996).

### 22.7.1 One-Dimensional Solution for Undamped Linear Soil on Rigid Rock

In the case of undamped linear soil on rigid rock (Figure 22.13), the shaking of the rock generates, among other waves, a shear wave travelling in the soil at a shear wave velocity $v_{s}$ and generating a horizontal movement equal to $u(t, z)$ where $t$ is time and $z$ is depth. The equation of motion for an element of soil (see section 18.3.4) is:

$$
\begin{equation*}
\frac{\partial^{2} u}{\partial t^{2}}=v_{s}{ }^{2} \frac{\partial^{2} u}{\partial z^{2}} \tag{22.9}
\end{equation*}
$$

If the rock imposes a harmonic motion at the base of the soil layer which is $H$ thick, two waves will be generated in


Figure 22.13 Linear soil on rigid rock.
the soil layer: one going up and one going down. The solution to this differential equation in complex notation reflects this decomposition and is of the form:

$$
\begin{equation*}
u(z, t)=a e^{i(\omega t+k z)}+b e^{i(\omega t-k z)} \tag{22.10}
\end{equation*}
$$

where $a$ and $b$ are the amplitude of the wave going up and the wave going down respectively, $\omega$ is the circular frequency of the harmonic motion, and $k$ is a wave number given by:

$$
\begin{equation*}
k=\frac{\omega}{v_{s}} \tag{22.11}
\end{equation*}
$$

Recall that:

$$
\begin{equation*}
T=\frac{1}{f}=\frac{2 \pi}{\omega} \tag{22.12}
\end{equation*}
$$

where $T$ is the period $(s), f$ is the frequency $(\mathrm{Hz})$, and $\omega$ is the circular frequency ( $\mathrm{rd} / \mathrm{s}$ ) of the harmonic motion. Because the ground surface is considered to be a free boundary, the shear stress $\tau(z, t)$ and shear strain $\gamma(z, t)$ must be zero on that boundary $(z=0)$. The shear strain $\gamma(0, t)$ must therefore satisfy:

$$
\begin{equation*}
\gamma(0, t)=\frac{\partial u(0, t)}{\partial z}=0 \tag{22.13}
\end{equation*}
$$

This leads to the condition that $\mathrm{a}=\mathrm{b}$ and the final expression for u is:

$$
\begin{equation*}
u(z, t)=2 a \cos (k z) e^{i \omega t} \tag{22.14}
\end{equation*}
$$

This represents a stationary wave (a wave that remains in a constant position) due to the superposition of the upward wave and the downward wave. Remember that we are interested in transforming the motion of the rock at the base of the soil layer into a motion at the ground surface. The transfer function $F(\omega)$ is therefore:

$$
\begin{equation*}
F(\omega)=\frac{u_{\max }(0, t)}{u_{\max }(H, t)}=\frac{1}{\cos (k H)}=\frac{1}{\cos \left(\frac{\omega H}{v_{s}}\right)} \tag{22.15}
\end{equation*}
$$

While in the general case the transfer function will be a complex number, in this simple case it is a scalar. To obtain the horizontal displacement vs. time signal at the soil surface, the horizontal displacement vs. time at the rock boundary is simply multiplied by the transfer function for each time in the
record. Note that if $\omega H / v_{s}$ is equal to $\pi / 2$, Eq. 22.15 indicates that the transfer function becomes infinite and the soil is in resonance with the rock motion. Therefore, the natural period $T$ of a soil layer with a height $H$ and a shear wave velocity $v_{s}$, also called the characteristic site period, is:

$$
\begin{equation*}
\frac{\omega H}{v_{s}}=\frac{\pi}{2} \quad \text { or } \quad \frac{2 \pi H}{T v_{s}}=\frac{\pi}{2} \quad \text { or } \quad T=\frac{4 H}{v_{s}} \tag{22.16}
\end{equation*}
$$

Equation 22.15 shows that the important factors in the response of a soil layer to an earthquake are the frequency of the rock motion, the thickness of the layer, and its shear wave velocity or small strain shear modulus, which are closely related (see section 8.2.1). In the United States, the natural period of soil deposits is on the order of 0.4 to 2 seconds.

### 22.7.2 One-Dimensional Solution for Damped Linear Soil on Rigid Rock

Let's assume that the damping in a soil layer can be represented by a Kelvin-Voigt model (see section 12.2.1). In the case of a damped linear soil on rigid rock, the governing differential equation becomes a bit more complicated (Kramer 1996):

$$
\begin{equation*}
\frac{\partial^{2} u}{\partial t^{2}}=v_{s}{ }^{2} \frac{\partial^{2} u}{\partial z^{2}}+\frac{\eta}{\rho} \frac{\partial^{3} u}{\partial z^{2} \partial t} \tag{22.17}
\end{equation*}
$$

where $\eta$ is the soil viscosity ( $\mathrm{N} . \mathrm{s} / \mathrm{m}^{2}$ ) and $\rho$ is the mass density of the soil $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$. The solution is similar to the preceding undamped case except that the wave number $k$ is now a complex number $k^{*}$ with a real part $k_{1}$ and a complex part $k_{2}$ :

$$
\begin{equation*}
u(z, t)=a e^{i\left(\omega t+k^{*} z\right)}+b e^{i\left(\omega t-k^{*} z\right)} \tag{22.18}
\end{equation*}
$$

with:

$$
\begin{equation*}
k^{*}=\frac{\omega}{v_{s}}(1-i \beta) \tag{22.19}
\end{equation*}
$$

where $\beta$ is the damping ratio, which is usually a small number between 0.05 and 0.1 . The transfer function is:

$$
\begin{equation*}
F(\omega)=\frac{1}{\cos (k * H)} \tag{22.20}
\end{equation*}
$$

and the modulus of that function or amplification function is the ratio of the movement at the ground surface over the movement at the rock level:

$$
\begin{aligned}
|F(\omega)| & =\frac{u_{\max }(0, t)}{u_{\max }(H, t)}=\frac{1}{\sqrt{\cos ^{2}\left(\frac{\omega H}{v_{s}}\right)+\sinh ^{2} \beta\left(\frac{\omega H}{v_{s}}\right)}} \\
& \approx \frac{1}{\sqrt{\cos ^{2}\left(\frac{\omega H}{v_{s}}\right)+\left(\beta \frac{\omega H}{v_{s}}\right)^{2}}}
\end{aligned}
$$



Figure 22.14 Amplification function.
This amplification function $|F(\omega)|$ is shown in Figure 22.14 for several values of the damping ratio $\beta$. As can be seen, the amplification is maximum for the lower frequencies.
Solutions can also be found for more complex situations, such as damped linear soil on elastic rock, layered damped soil on elastic rock, and nonlinear soil behavior (Kramer 1996).

### 22.7.3 Layered Soils

One very useful solution is that for a layered system. This solution was coded by Schnabel et al. (1972) with the program SHAKE and modified by Idriss and Sun (1992) with the program SHAKE91. It solves the problem of a soil deposit made of n layers $(i=1$ to $n), \Delta z$ thick, with a shear modulus $\mathrm{G}_{\mathrm{i}}$ for each layer $i$. A column of soil is considered and each layer is represented by an element being deformed in simple shear (Figure 22.15).
The horizontal cross section of the column is $1 \mathrm{~m} \times 1 \mathrm{~m}$. Horizontal equilibrium of an element leads to:

$$
\begin{align*}
& \left(\tau+\frac{\partial \tau}{\partial z} d z\right) 1 \times 1-\tau \times 1 \times 1=m a=1 \times 1 \times d z \times \rho \\
& \quad \times \frac{\partial^{2} u}{\partial t^{2}} \tag{22.22}
\end{align*}
$$



Figure 22.15 Column of soil elements deformed in simple shear during shear wave propagation.
or:

$$
\begin{equation*}
\frac{\partial \tau}{\partial z}=\rho \frac{\partial^{2} u}{\partial t^{2}} \tag{22.23}
\end{equation*}
$$

where $\tau$ is the shear stress on the horizontal plane, $z$ is depth, $\rho$ is the soil mass density, $u$ is the horizontal displacement, and $t$ is time. This equation is solved by the finite difference method (see section 11.5.1) and ends up for a centered expression as:

$$
\begin{equation*}
\frac{\tau_{i+1, t}-\tau_{i-1, t}}{\Delta z}=\rho \frac{u_{i, t+\Delta t}-2 u_{i, t}+u_{i, t-\Delta t}}{\Delta t^{2}} \tag{22.24}
\end{equation*}
$$

where $\tau_{i, t}$ is the shear stresses at time $t$ on element $i, \Delta z$ is the increment of depth, $\rho$ is the mass density of the soil, $u_{i, t}$ is the displacement of element $i$ at time $t$, and $\Delta t$ is the increment of time.

Boundary conditions exist at the bottom and at the top of the soil column. At the bottom, the displacement is equal to the input rock displacement at any time $t$. At the top, the shear stress is zero. The shear strain $\gamma_{i, t}$ is linked to the horizontal displacements of the nodes of the soil column as follows:

$$
\begin{equation*}
\gamma_{i, t}=\frac{u_{i+1, t}-u_{i, t}}{\Delta z} \tag{22.25}
\end{equation*}
$$

The shear stress $\tau_{i, t}$ can then be calculated as:

$$
\begin{equation*}
\tau_{i, t}=G_{i} \gamma_{i, t}+\eta_{i} \frac{\partial \gamma_{i, t}}{\partial t} \tag{22.26}
\end{equation*}
$$

where $G$ is the shear modulus of the soil and $\eta$ is the viscosity of the soil. It can be shown (Kramer 1996) that the viscosity $\eta$ of the soil is linked to the damping ratio $\beta$ by:

$$
\begin{equation*}
\eta=\frac{2 G}{\omega} \beta \tag{22.27}
\end{equation*}
$$

where $G$ is the shear modulus and $\omega$ is the circular frequency of the motion.

The input of the problem consists of a shear modulus, a damping ratio, and a mass density for each element in the soil column. Then, the boundary conditions together with Eqs. 22.24, 22.25, 22.26, and 22.27 are written for all nodes in the soil column to solve for the unknown displacements in all elements. The solution consists of starting at $t=0$ and stepping into time an amount $\Delta t$ per step. The boundary conditions provide the first values of the displacements and shear stress, which are usually zero for most nodes except for the boundary condition nodes. At the ground surface, the shear stress is always zero, whereas the displacement at the rock level is set equal to the displacement of the bottom element. The displacement at the rock level at the beginning of the earthquake provides the first value.

More realistic analyses include the strain level dependency of the shear modulus and damping ratio and the influence of the confinement on the shear modulus. The process of deconvolution is the reverse process, where the ground surface motion is observed during an earthquake and the rock motion is back-calculated at the base of the soil column. The use of programs like SHAKE and other techniques has helped produce graphs like the one in Figure 22.16, which shows the acceleration at the ground surface for a given acceleration at the rock level.

### 22.8 DESIGN PARAMETERS

The design approach often considers two levels of earthquakes: a rare earthquake and an expected earthquake. A rare earthquake may be defined as an earthquake with a $2 \%$ probability of exceedance in 50 years, whereas an expected earthquake would correspond to a $10 \%$ probability of exceedance in 50 years (Figure 22.5). These definitions correspond approximately to a return period of 2500 years and 500 years respectively. The design parameters, including ground


Figure 22.16 Amplification of rock motion at soil sites (After Idriss. 1990).
motion, are selected in one of two ways. The first way is to perform a site-specific analysis (as discussed in section 22.7) or by using recommendations outlined in building codes. This next section addresses the code approach.

### 22.8.1 Site Classes A-E for Different Soil Stiffness

The design spectrum depends on the soil at the site and in particular its stiffness. This is why site classes have been defined, ranging from site class A for hard rock to site class E for soft soil, as shown in Table 22.2. Site class F is a special category for which a site-specific dynamic site response analysis is recommended (section 22.7) instead of a code approach. The site classes use the average soil parameters within the top 30 m from the surface as a classification basis, because this depth is most influential in determining the dynamic response. The shear wave velocity is the parameter of choice, but the SPT blow count and the undrained shear strength are also helpful. Once the soil is classified according to the definitions listed in Table 22.2, the amplification factors can be selected to modify the acceleration spectrum.

### 22.8.2 Code-Based Spectrum

In the code approach, the acceleration response spectrum is constructed from the analysis of existing data, past experience, and engineering judgment. Such a spectrum is shown in Figure 22.17. First the reference spectrum is developed assuming that the soil at the site is rock (site class B) and then the values obtained for the reference spectrum are modified for the proper site class. The reference spectrum parameters are:

1. The spectral acceleration at a period equal to 0 seconds taken as the peak ground acceleration PGA. The PGA is used here because for a fundamental period of 0 seconds, the structure is infinitely stiff and the maximum acceleration of the structure is the same as the maximum acceleration from the ground.
2. The spectral acceleration at a short period equal to 0.2 seconds, called $S_{s}$.
3. The spectral acceleration at a long period equal to 1 second, called $S_{1}$.

These values come from the selection of the design earthquake (e.g., 1000-year or 2500-year return period) and the use

Table 22.2 Site Class Definitions

|  |  | Average Properties in Top 30 m |  |
| :--- | :--- | :--- | :--- |
|  |  | Standard <br> penetration <br> resistance, N <br> (blow/0.3m) | Undrained shear <br> strength, $\mathrm{s}_{\mathrm{u},}(\mathrm{kPa})$ |
| A Class | Soil Profile Name | velocity, $\bar{v}_{s},(\mathrm{~m} / \mathrm{s})$ | $\mathrm{N} / \mathrm{A}$ |
| B | Hard rock | $\bar{v}_{s}>1500$ | $\mathrm{~N} / \mathrm{A}$ |

Any profile with more than 3 m of soil having the following characteristics:

1. Plasticity index $\mathrm{PI}>20_{\mathrm{s}}$
2. Moisture content $\omega \geq 40 \%$, and
3. Undrained shear strength $\mathrm{s}_{\mathrm{u}}<25 \mathrm{kPa}$

Any profile containing soils having one or more of the following characteristics:

1. Soils vulnerable to potential failure or collapse under seismic loading, such as liquefiable soils, quick and highly sensitive clays, collapsible weakly cemented soils.
2. Peats and/or highly organic clays ( $H>3 \mathrm{~m}$ of peat and/or highly organic clay where $H=$ thickness of soil)
3. Very high-plasticity clay $(H>7.5 \mathrm{~m}$ with plasticity index $\mathrm{PI}>75)$
4. Very thick soft/medium stiff clays $(H>36 \mathrm{~m})$


Figure 22.17 Design code acceleration response spectrum. (After Kavazanjian et al. 2011)
of the dedicated USGS web site, for example (Kavazanjian et al. 2011).
The site-specific spectrum is obtained from the values of the reference spectrum. The site-specific spectrum parameters are:

1. The spectral acceleration at a period equal to 0 seconds, called As:

$$
\begin{equation*}
A_{S}=F_{P G A} \times P G A \tag{22.28}
\end{equation*}
$$

where $F_{P G A}$ is the site factor for the $P G A$ found in Table 22.3.
2. The spectral acceleration at a short period equal to 0.2 seconds, called $S_{D S}$ :

$$
\begin{equation*}
S_{D S}=F_{A} \times S_{S} \tag{22.29}
\end{equation*}
$$

Table 22.3 Site Factor $\mathbf{F}_{\text {PGA }}$

|  | Mapped <br>  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Spectral <br> at Short Periods |  |  |  |  |  |
| Site | PGA | PGA | PGA | PGA | PGA |
| Class | $\leq 0.1 \mathrm{~g}$ | $=0.2 \mathrm{~g}$ | $=0.3 \mathrm{~g}$ | $=0.4 \mathrm{~g}$ | $\geq 0.5 \mathrm{~g}$ |
|  |  |  |  |  |  |
| A | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| B | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| C | 1.2 | 1.2 | 1.1 | 1.0 | 1.0 |
| D | 1.6 | 1.4 | 1.2 | 1.1 | 1.0 |
| E | 2.5 | 1.7 | 1.2 | 0.9 | 0.9 |
| F | a | a | a | a | a |

a: Site-specific geotechnical investigation and dynamic site response analysis are required in this case.
(After Kavazanjian et al. 2011)

Table 22.4 Site Factor $\mathbf{F}_{\mathrm{A}}$

\left.|  | Mapped Spectral Response Accelerations |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| at Short Periods |  |  |  |  |  |$\right]$

a: Site-specific geotechnical investigation and dynamic site response analysis are required in this case.
(After Kavazanjian et al. 2011)
where $F_{A}$ is the site factor for $\mathrm{S}_{\mathrm{S}}$, found in Table 22.4.
3. The spectral acceleration at a long period equal to 1 second, called $S_{D 1}$ :

$$
\begin{equation*}
S_{D 1}=F_{V} \times S_{1} \tag{22.30}
\end{equation*}
$$

where $F_{V}$ is the site factor for $S_{1}$, found in Table 22.5.
4. The period $T_{S}$ corresponding to the end of the spectrum plateau is given by:

$$
\begin{equation*}
T_{S}=\frac{S_{D 1}}{S_{D S}} \tag{22.31}
\end{equation*}
$$

5. The period $T_{o}$ corresponding to the beginning of the spectrum plateau:

$$
\begin{equation*}
T_{o}=0.2 T_{S} \tag{22.32}
\end{equation*}
$$

Table 22.5 Site Factor $\mathbf{F}_{\mathbf{V}}$

| Site Class | Mapped Spectral Response Accelerations at 1-Second Periods |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & S_{1} \\ \leq & 0.1 \mathrm{~g} \end{aligned}$ | $\begin{aligned} & S_{1} \\ = & 0.2 \mathrm{~g} \end{aligned}$ | $\begin{aligned} & S_{1} \\ = & 0.3 \mathrm{~g} \end{aligned}$ | $\begin{gathered} \\ S_{1} \\ = \\ 0.4 \mathrm{~g} \end{gathered}$ | $\begin{aligned} & S_{1} \\ \geq & 0.5 \mathrm{~g} \end{aligned}$ |
| A | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| B | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| C | 1.7 | 1.6 | 1.5 | 1.4 | 1.3 |
| D | 2.4 | 2.0 | 1.8 | 1.6 | 1.5 |
| E | 3.5 | 3.2 | 2.8 | 2.4 | 2.4 |
| F | a | a | a | a | a |

a: Site-specific geotechnical investigation and dynamic site response analysis are required in this case.
(After Kavazanjian et al. 2011)

In that fashion the design spectrum is completely defined (Figure 22.17) and can be used for structural or geotechnical analysis. In Figure 22.17, the elastic seismic coefficient $\mathrm{C}_{\mathrm{sm}}$ is the ratio between the design horizontal shear force due to inertia and the effective weight of the structure.

### 22.8.3 Hazard Levels

The severity of an earthquake is described by the hazard level, which ranges from I to IV. Each level is tied to the $S_{D 1}$ or $S_{D S}$ value. Recall that $S_{D 1}$ is the long-period (1 second) spectral acceleration, adjusted for the site factor, and $S_{D S}$ is the short-period ( 0.2 second) spectral acceleration, also adjusted for the site factor. The hazard levels are defined in Table 22.6.

### 22.9 LIQUEFACTION

### 22.9.1 Phenomenon

When a loose coarse-grained soil under the groundwater level is shaken rapidly enough, it tends to decrease in volume. The decrease in volume causes the water to be pushed out of the pores. If the water cannot escape fast enough, the water stress $u_{w}$ increases and can reach a value equal to the vertical total stress $\sigma_{o v}$. At that point the effective stress ( $\sigma_{o v}^{\prime}=\sigma_{o v}-u_{w}=$ 0 ) becomes zero: The soil loses its strength and behaves like a thick liquid. This is the phenomenon of liquefaction. Loose sands under the groundwater level are particularly sensitive to this condition. Dense coarse-grained soils and fine-grained soils are much less sensitive. Liquefaction of the soil leads to flow slides, lateral spreading, loss of bearing capacity, increased earth pressures against retaining walls as the soil becomes a heavy liquid, and postearthquake settlement as the water stress dissipates.

### 22.9.2 When to Do a Liquefaction Study?

The need for liquefaction studies is tied first to the severity of the earthquake. This severity is described by hazard levels ranging from I to IV, as described in Table 22.6. For hazard

Table 22.6 Seismic Hazard Levels

| Hazard Level | $\mathrm{S}_{\mathrm{D} 1}=\mathrm{F}_{\mathrm{V}} \mathrm{S}_{1}$ | $\mathrm{~S}_{\mathrm{DS}}=\mathrm{F}_{\mathrm{A}} \mathrm{S}_{\mathrm{S}}$ |
| :--- | :--- | :--- |
| I | $\mathrm{S}_{\mathrm{D} 1}<0.15$ | $\mathrm{~S}_{\mathrm{D} 1}<0.15$ |
| II | $0.15<\mathrm{S}_{\mathrm{D} 1}<0.25$ | $0.15<\mathrm{S}_{\mathrm{D} 1}<0.35$ |
| III | $0.25<\mathrm{S}_{\mathrm{D} 1}<0.40$ | $0.35<\mathrm{S}_{\mathrm{D} 1}<0.60$ |
| IV | $0.40<\mathrm{S}_{\mathrm{D} 1}$ | $0.60<\mathrm{S}_{\mathrm{D} 1}$ |

Note: These hazard levels apply for site classifications A, B, C, and D. Further description and conditions apply for site classifications E and F (see Kavazanjian et al. 2011).
levels I and II, a liquefaction study is not required. For hazard level IV, a liquefaction study is always required. For hazard level III, a liquefaction study is required unless:

1. The mean earthquake magnitude for the design event is less than 6 , or
2. The mean magnitude is between 6 and 6.4, and $N_{1-60}>$ 20 (mean normalized SPT blow count; see section 7.1)
3. The mean magnitude is between 6 and $6.4, N_{1-60}>15$, and $\mathrm{S}_{\mathrm{DS}}<0.35 \mathrm{~g}$
If the soil is resistant to liquefaction, a liquefaction study is not necessary even for hazard levels III and IV. Liquefactionresistant soils include:

## 1. Bedrock

2. Fine-grained soils with more than $15 \%$ clay, liquid limit $\mathrm{w}_{\mathrm{L}}$ higher than $35 \%$, and water content lower than $0.9 \mathrm{w}_{\mathrm{L}}$
3. Sands with $N_{1-60}>30 \mathrm{bpf}$ or $q_{t 1}>160$ (mean corrected and normalized cone penetrometer resistance; see section 7.2)
4. Soils where the water table is deeper than 15 m below the ground surface

Note that quick clays should be considered as potentially liquefiable; however, the liquefaction is not due to the same process as the liquefaction of fine sands discussed here.

### 22.9.3 When Can a Soil Liquefy?

To predict whether a soil can liquefy, cyclic tests can be performed in the laboratory by cyclic triaxial testing, or (better) by cyclic simple shear testing, or (even better) by cyclic torsional shear testing. During such tests, the sample is subjected to an initial effective stress and then a chosen value of shear stress or deviator stress is applied cyclically in twoway symmetrical shearing. This means that the shear stress varies between $+\tau_{c}$ and $-\tau_{c}$. The frequency of the cycles is selected to be representative of earthquake frequencies (say, 1 to 10 Hz ). Typical results for cyclic simple shear tests on saturated sand are shown in Figures 22.18 and 22.19. Liquefaction may or may not occur after a number of cycles or an amount of time consistent with typical earthquakes (less than 30 seconds for most cases and up to 2 minutes for a huge earthquake).
The cyclic stress ratio CSR is the ratio of the horizontal shear stress $\tau_{c}$ applied cyclically to a soil at a depth $z$ over the vertical effective stress $\sigma_{v o}^{\prime}$ on the soil at the same depth. The lowest value of the CSR that triggers liquefaction is called the cyclic resistance ratio or CRR. Figure 22.20 shows the results of shaking table tests performed by De Alba et al. (1976); that figure indicates how the CRR decreases as the number of cycles increases. The goal of liquefaction studies is to calculate the CSR and the CRR within the depth of interest. Liquefaction is predicted if:

$$
\begin{equation*}
C R R<C S R \tag{22.33}
\end{equation*}
$$




Figure 22.18 Undrained constant stress cyclic simple shear test results (stress-strain curves).


Figure 22.19 Undrained constant stress cyclic simple shear test results (stress path and water stress).


Figure 22.20 Cyclic stress ratio to reach liquefaction. (From De Alba et al. 1976. With permission from ASCE)

The drawbacks of using laboratory tests to predict liquefaction include sample disturbance, difficulty in reproducing in situ stresses, and difficulties in reproducing a true earthquake cyclic shear loading. As a result, the preferred approach in design has been to use earthquake case histories at sites where liquefaction did or did not occur.

The combination of the average shear stress $\tau_{a v}$ due to the earthquake shaking, a measure of the soil strength, and the knowledge of whether the soil liquefied are used to produce design charts. The results of in situ tests are preferred in this approach to quantify the soil strength. The first charts, such as the one shown in Figure 22.21, were based on the SPT blow count. Additional charts were then proposed based on cone penetrometer data (Figure 22.22) and then shear wave velocity data (Figure 22.23). The chart based on the dilatometer is preliminary in nature (Figure 22.24).

In these charts, the vertical axis is the cyclic stress ratio CSR, defined as $\tau_{a v} / \sigma_{o v}^{\prime}$ where $\tau_{a v}$ is the average shear stress generated during the design earthquake and $\sigma_{o v}^{\prime}$ is the vertical effective stress at the depth investigated and at the time of the in situ soil test. The shear stress $\tau_{a v}$ is related to the maximum shear stress $\tau_{\text {max }}$, which is obtained from a site response analysis (e.g., using the program SHAKE) for the design earthquake, or, more simply, by using the peak ground acceleration PGA obtained from maps such as the one show in Figure 22.5. If the PGA is used to obtain $\tau_{a v}$ for a 7.5 magnitude, the expression is (Seed and Idriss 1971):

$$
\begin{equation*}
C S R=\frac{\tau_{a v}}{\sigma_{v o}^{\prime}}=0.65\left(\frac{a_{\max }}{g}\right)\left(\frac{\sigma_{v o}}{\sigma_{v o}^{\prime}}\right) r_{d} \tag{22.34}
\end{equation*}
$$

where $a_{\max }$ is the PGA for the design earthquake, $g$ is the acceleration due to gravity, $\sigma_{v o}$ is the total vertical stress


Figure 22.21 SPT-based liquefaction chart for magnitude 7.5. (After Youd and Idriss 1997)


Figure 22.22 CPT-based liquefaction chart for magnitude 7.5. (After Robertson and Wride 1998)


Figure 22.23 Shear wave velocity-based liquefaction chart for magnitude 7.5. (After Andrus and Stokoe 2000)
at the depth being investigated, $\sigma_{v o}^{\prime}$ is the effective vertical stress at the depth being investigated, and $r_{r}$ is a flexibility factor. The flexibility factor depends on the depth at which the liquefaction is being evaluated. Such a factor is necessary because the PGA is acting at the ground surface while the possibility of liquefaction is evaluated at a depth $z$. Figure 22.25, after Seed and Idriss (1971), gives a range of values for $r_{d}$.


Figure 22.24 DMT-based liquefaction chart for magnitude 7.5. (After Monaco et al. 2005)


Figure 22.25 Soil flexibility factor to modify the PGA for a depth z (Seed and Idriss 1971)

On the horizontal axis of the charts in Figures 22.21 to 22.24 is the in situ test parameter normalized and corrected for the effective stress level in the soil at the time of the test and for fine content. The SPT blow count is $N_{1-60}$ and the procedure to correct it for effective stress level is described
in section 7.1. The correction for fine content is embedded in the chart of Figure 22.21. The CPT point resistance is $q_{t 1}$ and the procedure to correct for effective stress level and fine content is described in section 7.2. The shear wave velocity is $v_{s 1}$ and the procedure to correct for effective stress is:

$$
\begin{equation*}
v_{s 1}=v_{s}\left(\frac{\sigma_{a}}{\sigma_{v o}^{\prime}}\right)^{0.25} \tag{22.35}
\end{equation*}
$$

where $v_{s}$ is the shear wave velocity measured in the field, $\sigma_{a}$ is the atmospheric pressure, and $\sigma_{v o}^{\prime}$ is the vertical effective stress at the depth investigated. Once the soil parameter is corrected, it is entered on the horizontal axis of the chart and the CRR is read on the liquefaction design curve of the corresponding chart. Note that the charts in Figures 22.21 to 22.24 give the cyclic resistance ratio (CRR) for an earthquake of magnitude 7.5 . For earthquakes of different magnitude, a magnitude scaling factor (MSF) is applied as follows:

$$
\begin{equation*}
C R R_{M}=M S F \times C R R_{M=7.5} \tag{22.36}
\end{equation*}
$$

Kavazanjian et al. (2011), building on the work of Youd and Idriss (1997), suggested that the hatched area in Figure 22.26 be used for MSF.

In summary, the way to use the charts is (Figure 22.27):

1. Obtain the soil parameter profile.
2. Correct the profile for stress level due to depth effects and fine content if necessary. Prepare a corrected soil parameter profile.
3. Enter the chart corresponding to the soil parameter and read the cyclic resistance ratio. Prepare a CRR profile.
4. Modify the CRR profile for a magnitude different from 7.5.


Figure 22.26 Magnitude scaling factor (MSF). (After Kavazanjian et al. 2011; Youd and Idriss 1997)


Figure 22.27 Profiles of liquefaction analysis (Idriss and Boulanger 2008).
5. Calculate the CSR generated by the design earthquake. Prepare the CSR profile.
6. Compare the CSR profile and the CRR profile.
7. The zone of potential liquefaction is the zone where CSR > CRR.

### 22.10 SEISMIC SLOPE STABILITY

Seismic slope stability was covered in section 19.18. The following summarizes the procedure to select the appropriate value of the horizontal seismic coefficient $k$ for a pseudostatic analysis.

The step-by-step procedure that follows is as recommended by Kavazanjian et al. (2011):

1. Perform a static slope stability analysis without any earthquake loading to ensure that the slope is stable and that the factor of safety $F$ is sufficient in the case of no earthquake (say, 1.5).
2. Using maps (USGS map, such as the ones shown in Figure 22.5, for example), obtain the peak ground acceleration PGA and the spectral acceleration at one second $S_{1}$ for site class B at the base of the slope for the design earthquake.
3. Select the site adjustment factor $F_{P G A}$ from Table 22.3 and the factor $F_{V}$ from Table 22.5 for the correct site class and the correct acceleration value.
4. Calculate the value of the maximum horizontal seismic inertia coefficient $k_{\text {max }}$ as:

$$
\begin{equation*}
k_{\max }=F_{P G A} \times P G A \tag{22.37}
\end{equation*}
$$

5. Calculate the value of the average horizontal seismic inertia coefficient $k_{a v}$ as follows. The coefficient $k_{a v}$ is lower than $k_{\text {max }}$ because the average horizontal acceleration over the slope mass is less than the PGA due to wave scattering:

$$
\begin{equation*}
k_{a v}=\gamma\left(1+0.01 H\left(0.5 \frac{F_{V} S_{1}}{k_{\max }}-1\right)\right) k_{\max } \tag{22.38}
\end{equation*}
$$

where $\gamma$ is equal to 1 for all site classes except for site classes A and B , where it is taken as $1.2 ; H$ is the height of the slope; and $F_{v}$ is the site factor from Table 22.5.
6. If the slope can tolerate a movement of 25 to 50 mm , the value of $k_{a v}$ can be further reduced by a factor of 2 . In the end, the factor $k_{h}$ is given by:

$$
\begin{equation*}
k_{h}=0.5 \gamma\left(1+0.01 H\left(0.5 \frac{F_{V} S_{1}}{k_{\max }}-1\right)\right) k_{\max } \tag{22.39}
\end{equation*}
$$

7. Under the combined static and earthquake inertia loading, the target factor of safety should be at least 1.1.

### 22.11 SEISMIC DESIGN OF RETAINING WALLS

The design of retaining walls under static conditions is described in Chapter 21. This section addresses what happens under earthquake conditions. Gravity walls are considered first, followed by MSE walls, cantilever walls, and tieback walls.

### 22.11.1 Seismic Design of Gravity Walls

The horizontal force $P_{a}$ per unit length of wall due to the active earth pressure behind a retaining wall when there is no earthquake and the water table is below the bottom of the wall (see sections 21.3.1 and 21.9) is given by:

$$
\begin{equation*}
P_{a}=\frac{1}{2} K_{a} \gamma H^{2} \tag{22.40}
\end{equation*}
$$

where $\gamma$ is the soil unit weight, $H$ is the wall height, and $K_{a}$ is the active earth pressure coefficient expressed as (Figure 22.28):

$$
\begin{equation*}
K_{a}=\frac{\sin ^{2}\left(\alpha+\varphi^{\prime}\right)}{\sin ^{2} \alpha \sin (\alpha-\delta)\left[1+\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}-\beta\right)}{\sin (\alpha-\delta) \sin (\alpha+\beta)}}\right]^{2}} \tag{22.41}
\end{equation*}
$$

where $\alpha$ is the angle of the back of the wall with the horizontal, $\varphi^{\prime}$ is the effective stress friction angle of the soil behind the wall, $\delta$ is the angle of friction between the soil and the back of the wall, and $\beta$ is the angle of the ground surface behind the wall with the horizontal.
In the case of earthquake loading on a gravity wall, the earth pressure is increased by the horizontal shaking of the soil and the associated horizontal inertia force. The vertical acceleration can also modify the weight of soil acting on the wall, but this vertical inertia force is usually ignored, mainly because it does not occur at the same time as the horizontal force; indeed, the horizontal and vertical accelerations are rarely in phase, so the peak horizontal and peak vertical accelerations do not occur simultaneously. The horizontal


Figure 22.28 Gravity retaining wall with earthquake loading: active case.
inertia force generated by the earthquake is written as $k_{h} W$ where $k_{h}$ is the seismic coefficient and $W$ is the weight of the soil wedge. The coefficient $k_{h}$ is similar to the coefficient used for slope stability; it is taken as $k_{a v}$ (Eq. 22.38) if the wall cannot tolerate any movement and as $k_{h}$ (Eq. 22.39) if a movement of 25 to 50 mm is tolerable.
In the case of an earthquake, the force $P_{a}$ becomes $P_{a e}$, which is written as:

$$
\begin{equation*}
P_{a e}=\frac{1}{2} K_{a e} \gamma H^{2} \tag{22.42}
\end{equation*}
$$

where $P_{a e}$ is the active force per unit length of wall due to the active earth pressure during an earthquake, $K_{a e}$ is the active earth pressure coefficient in the earthquake case, $\gamma$ is the soil unit weight, and $H$ is the wall height. The coefficient $K_{a e}$ is obtained in the same fashion as $K_{a}$ (see section 21.3) except that the earthquake force $k_{h} W$ is added to the equilibrium equations.
The final expression of $K_{a e}$ after finding the most critical wedge angle is credited to Mononobe and Okabe (Okabe 1926; Mononobe and Matsuo 1929):

$$
\begin{equation*}
K_{a e}=\frac{\sin ^{2}\left(\alpha+\varphi^{\prime}-\psi\right)}{\cos \psi \sin ^{2} \alpha \sin (\alpha-\delta-\psi)\left[1+\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}-\beta-\psi\right)}{\sin (\alpha-\delta-\psi) \sin (\alpha+\beta)}}\right]^{2}} \tag{22.43}
\end{equation*}
$$

where $\alpha$ is the angle of the back of the wall with the horizontal, $\beta$ is the angle of the ground surface behind the wall with the horizontal, $\delta$ is the angle of friction between the back of the wall and the soil, $\varphi^{\prime}$ is the friction angle of the soil, and $\psi$ is the angle representing the earthquake inertia force through:

$$
\begin{equation*}
\psi=\tan ^{-1}\left(\frac{k_{h}}{1-k_{v}}\right) \tag{22.44}
\end{equation*}
$$

where $k_{h}$ and $k_{v}$ are the horizontal and vertical seismic coefficients respectively. Note that $k_{v}$ is often ignored (taken as equal to zero). The angle of the critical surface with the horizontal is flatter in the active earthquake case than in the static case (Kramer 1996). Note also that $P_{a e}$ includes the static component $P_{a}$ and a dynamic component $\Delta P_{a e}$ of the active push (Figure 22.28) and can be rewritten as:

$$
\begin{equation*}
P_{a e}=P_{a}+\Delta P_{a e} \tag{22.45}
\end{equation*}
$$

For the passive earth pressure, the equations become:

$$
\begin{equation*}
P_{p e}=\frac{1}{2} K_{p e} \gamma H^{2} \tag{22.46}
\end{equation*}
$$

where $P_{p e}$ is the passive force per unit length of wall due to the passive earth pressure during an earthquake, $K_{p e}$ is the passive earth pressure coefficient in the earthquake case, $\gamma$ is


Figure 22.29 Gravity retaining wall with earthquake loading: passive case.
the soil unit weight, and $H$ is the wall height. The expression of $K_{p e}$ (Figure 22.29) is:

$$
\begin{equation*}
K_{p e}=\frac{\sin ^{2}\left(\alpha-\varphi^{\prime}+\psi\right)}{\cos \psi \sin ^{2} \alpha \sin (\alpha+\delta+\psi)\left[1-\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}+\beta+\psi\right)}{\sin (\alpha+\delta+\psi) \sin (\alpha+\beta)}}\right]^{2}} \tag{22.47}
\end{equation*}
$$

Note that $P_{p e}$ includes the static component $P_{p}$ and dynamic component $\Delta P_{p e}$ of the passive push (Figure 22.29) and can be rewritten as:

$$
\begin{equation*}
P_{p e}=P_{p}+\Delta P_{p e} \tag{22.48}
\end{equation*}
$$

The point of application of the static component of the active and passive forces, $P_{a}$ and $P_{p}$, is located at $0.33 H(H$ is wall height) from the bottom of the wall in the simplest case of a uniform soil. Note that the static pressure distribution is triangular, but the pressure distribution associated with
the earthquake inertia force is not triangular. Recall that $K_{a e}$ was obtained from a Coulomb wedge analysis, which gives a global force solution, and not a Rankine stress analysis, which gives a pressure distribution. In fact, the point of application of the seismic component is different from the point of application of the static component. The point of application of the dynamic components of the active and passive forces, $\Delta P_{a e}$ and $\Delta P_{p e}$, is higher than the one for the static components, because the amplitude of the soil movement due to the earthquake generally increases as the shear wave propagates upward. As a result, the point of application of $\Delta P_{a e}$ and $\Delta P_{p e}$ is located at $0.6 H$ from the bottom of the wall.

The Monobe-Okabe expressions of $K_{a e}$ and $K_{p e}$ in Eqs. 22.43 and 22.47 have the advantage of being simple to use. They also have shortcomings. One of them is that the failure surface is assumed to be the same for the static case and the dynamic case, which is not true. In the active case, the slope of the failure surface is flatter for the dynamic case than for the static case. Second, the effect of cohesion is not included, although it can reduce the effect of the dynamic part of the active pressure. Figure 22.30 shows the influence of the cohesion $c^{\prime}$ on $K_{a e}$ for a friction angle of $35^{\circ}$ and for different values of the horizontal seismic coefficient $k_{h}$. The cohesion $c^{\prime}$ is normalized in the figure by $\gamma H$ where $\gamma$ is the soil unit weight and $H$ is the wall height. Another shortcoming is that the wedge approach assumes a straight-line failure surface, which is not necessarily the weakest surface. This difference is particularly severe for the passive resistance $P_{p e}$, which can be seriously overestimated and should be used with caution if at all. A log spiral failure surface gives more conservative values for $K_{p e}$ and should


Figure 22.30 Influence of cohesion $c^{\prime}$ for $\varphi^{\prime}=35^{\circ}$ on the earthquake active earth pressure coefficient $K_{a e}$ for different values of the horizontal seismic coefficient $k_{h}$ (NCHRP 2008).


Figure 22.31 Coefficient of passive earth pressure in the case of earthquake loading for a log spiral failure surface and a wall friction angle equal to $2 / 3$ of the soil friction angle (NCHRP 2008, Kavazanjian et al. 2011).
be preferred. Such values are shown in Figure 22.31 (after NCHRP 2008).

### 22.11.2 Water Pressures on Walls during Earthquake

It is generally desirable to ensure that the groundwater table is below the bottom of the retaining wall, as the water pressure significantly increases the active force. This is also true for walls in earthquake-prone areas. However, this may not be possible; a high water level is often encountered for walls in harbors or near shore. In such instances it is necessary to account for the water behavior during an earthquake in addition to the hydrostatic pressure associated with the static case.

## Water on the Side That Has No Soil

If there is water on the side of the wall that has no soil (e.g., berthing wall in a harbor, earth dam), the pressure in the static case $p_{w h}$ is hydrostatic and given by:

$$
\begin{equation*}
p_{w h}=\gamma_{w} z \tag{22.49}
\end{equation*}
$$

where $\gamma_{w}$ is the unit weight of water and $z$ is the depth below the water level. The dynamic pressure during an earthquake $\Delta p_{w e}$ is given by Westergaard (1931):

$$
\begin{equation*}
\Delta p_{w e}=\frac{7}{8} k_{h} \gamma_{w} \sqrt{z H_{w}} \tag{22.50}
\end{equation*}
$$

where $k_{h}$ is the horizontal seismic coefficient, $z$ is the depth below the water level, and $H_{w}$ is the total height of water against the wall (Figure 22.32). The assumptions made by Westergaard to develop this solution limit the application of this formula to the case where the earthquake frequency is below the fundamental frequency $f_{w}$ of the water body. This frequency is given by:

$$
\begin{equation*}
f_{w}=\frac{v_{p}}{4 H_{w}} \tag{22.51}
\end{equation*}
$$

where $v_{p}$ is the compression wave velocity. Note that the dynamic pressure works alternatively in both directions. The most detrimental condition for the retaining wall is likely to be when the dynamic pressure decreases the hydrostatic


Figure 22.32 Water pressure on wall due to earthquake.
pressure, thereby decreasing the stabilization effect of the water. By integrating the expression in Eq. 22.50, we can obtain the resultant force $\Delta P_{w e}$ :

$$
\begin{equation*}
\Delta P_{w e}=\frac{7}{12} k_{h} \gamma_{w} H_{w}^{2} \tag{22.52}
\end{equation*}
$$

The point of application of $\Delta P_{w e}$ can be calculated by moment equilibrium and is found to be $0.6 H_{w}$ below the water surface (Figure 22.32).

## Water on the Retained-Soil Side

If there is water in the backfill, the problem becomes a bit more complicated, as the inertia force is proportional to the total unit weight $\gamma$ and the shear resistance is proportional to the effective unit weight $\gamma^{\prime}$. Therefore, Eq. 22.42 must be altered to reflect this dual effect. In the case where the water level in the backfill is at the ground surface and no excess water stress is generated, Towhata (2008) recommends the following approach:

1. Use $\gamma^{\prime}$ in the active earth pressure equation
2. Increase the horizontal seismic coefficient $k_{h}$ to reflect the increase in inertia force
3. Add the hydrostatic pressure

The equation then becomes (Figure 22.32):

$$
\begin{equation*}
P_{a e}^{\prime}=\frac{1}{2} K_{a e} \gamma^{\prime} H^{2} \tag{22.53}
\end{equation*}
$$

However, the horizontal seismic coefficient $k_{h}$ is increased to $k_{h}^{\prime}$ :

$$
\begin{equation*}
k_{h}^{\prime}=\frac{\gamma}{\gamma^{\prime}} k_{h} \tag{22.54}
\end{equation*}
$$

This nearly doubles the value of $k_{h}$. Then the angle $\psi$ used in the expression of $K_{a e}$ is:

$$
\begin{equation*}
\psi=\tan ^{-1}\left(\frac{k_{h}^{\prime}}{1-k_{v}}\right)=\tan ^{-1}\left(\frac{k_{h}}{1-k_{v}} \times \frac{\gamma}{\gamma^{\prime}}\right) \tag{22.55}
\end{equation*}
$$

The hydrostatic thrust must then be added:

$$
\begin{align*}
P_{w} & =\frac{1}{2} \gamma_{w} H^{2}  \tag{22.56}\\
P_{a e} & =P_{w}+P_{a e}^{\prime}=\frac{1}{2} \gamma_{w} H^{2}+\frac{1}{2} K_{a e} \gamma^{\prime} H^{2} \tag{22.57}
\end{align*}
$$

Because of the triangular distribution of pressures, both $P_{a e}^{\prime}$ and $P_{w}$ act at 0.67 H from the top of the wall in the simplest case of a uniform soil. Kramer (1996) gives recommendations for the more complex case where the water level behind the wall is not at the ground surface.

### 22.11.3 Seismic Design of MSE Walls

MSE walls retain the soil through a reinforced soil mass. The earthquake design of these types of walls follows the same approach as the static design (see section 21.10), except that the coefficient $K_{a}$ is replaced by the coefficient $K_{a e}$ in the calculation.

### 22.11.4 Seismic Design of Cantilever Walls

Cantilever walls retain the soil without anchors or strut simply by the resistance of their embedment into the foundation soil. The earthquake design of these types of walls follows the same approach as the static design (see section 21.11), except that the coefficients used for the earth pressure are $K_{a e}$ and $K_{p e}$ instead of $K_{a}$ and $K_{p}$.

### 22.11.5 Seismic Design of Anchored Walls

Anchored walls retain the soil through the use of anchors or struts and through their depth of embedment. The earthquake design of these types of walls follows the same approach as the static design (see section 21.12), except that the coefficient $K$ used for the earth pressure above the excavation level is increased by the ratio $K_{a e} / K_{a}$. Below the excavation level, the earth pressure coefficients are $K_{a e}$ and $K_{p e}$.

### 22.12 SEISMIC DESIGN OF FOUNDATIONS

During an earthquake, a foundation and the soil around it will interact. Two kinds of interactions are identified: kinematic and inertial. Kinematic interaction refers to the interaction between the soil and the foundation as the foundation modifies the free field movement of the soil because of its presence. Inertial interaction refers to the interaction between the soil and the foundation as the foundation movement due to soil shaking generates accelerations throughout the building and associated inertia forces at the foundation level. In many instances, kinematic interaction can be neglected, and the foundation need only be designed to resist the inertia forces due to the inertial interaction.

The approaches used for the design of foundations to resist earthquake loading are the same for shallow foundations and deep foundations. There are two main categories: the design code approach and the dynamic analysis approach. Both approaches aim at obtaining the inertia forces and moments on the foundation and then designing the foundation to handle these forces on a pseudostatic basis.

In the design code approach, a response spectrum is specified, and then the fundamental period of the building is calculated. This fundamental period is entered on the horizontal axis of the spectrum and the corresponding spectral acceleration is obtained. The horizontal force $H$ to be resisted by the foundation is the product of the spectral acceleration and the associated mass of the building. In this approach, the ductility of the structure and foundation are not considered. This ductility tends to reduce the inertia force and is included through the use of a reduction factor $R_{f}$. Table 22.7 shows such reduction factors for bridge substructures. The reduced force used for design purposes is $H / R_{f}$. This force is applied to the foundation and the ultimate limit state is checked to ensure safety against failure. Because earthquake is considered to be

Table 22.7 Force Reduction Factor $\mathbf{R}_{\mathbf{f}}$ for Bridges (Kavazanjian et al. 2011)

|  | Importance Category |  |  |
| :--- | :---: | :---: | :---: |
| Substructure | Critical | Essential | Other |
| Wall-type piers, larger | 1.5 | 1.5 | 2.0 |
| $\quad$ dimension |  |  |  |
| Reinforced concrete pile bents |  |  |  |
| $\quad$ - Vertical piles only | 1.5 | 2.5 | 3.0 |
| - With batter piles | 1.5 | 1.5 | 2.0 |
| Single columns | 1.5 | 2.9 | 3.0 |
| Steel or composite steel and <br> concrete pile bents |  |  |  |
| - Vertical pile only | 1.5 | 3.5 | 5.0 |
| - With batter piles | 1.5 | 2.0 | 3.0 |
| Multiple-column bents | 1.5 | 3.5 | 5.0 |

an extreme event, the load and resistance factors are close to 1 if not equal to 1 .

In the dynamic analysis approach, the structure and foundation are simulated numerically. The foundation is usually simplified and represented by a system of translational and rotational springs and dashpots. The simulation gives the inertia forces and moments applied to the foundation. This approach has the advantage of including the ductility of the structure more directly. Again, this force is applied to the foundation and the ultimate limit state is checked to ensure safety against failure. Because earthquake is considered to be an extreme event, the load and resistance factors are close to 1 if not equal to 1 .

There is a trend toward displacement base design (service limit state) rather than load-based design. In this approach the displacement due to the earthquake loads are calculated and allowance is made for what leads to no damage, medium damage, heavy damage but still standing, and total collapse.

In the case of deep foundations, it is possible for the liquefied soil to load the piles by flowing past them. The load generated by liquefied soil must be added to the inertia load. This brings in the importance of the shear strength of liquefied soils. Seed and Harder (1990) proposed a correlation of the liquefied soil shear strength to the corrected standard penetration test (SPT) blow count $\left(N_{1}\right)_{60}$ (see section 7.1). Further correction was added to the $\left(N_{1}\right)_{60}$ value for the presence of fines, which can be approximated as follows:

$$
\begin{equation*}
\left(N_{1}\right)_{60-c s}=\left(N_{1}\right)_{60}+\frac{P}{10} \tag{22.58}
\end{equation*}
$$

where $\left(N_{1}\right)_{60}$ is the SPT blow count corrected for stress and energy level, $P$ is the percent finer than 0.075 mm (expressed in percent), and $\left(N_{1}\right)_{60-\mathrm{cs}}$ is the SPT blow count further corrected for the fine content. The correction increases the value of N to bring it back to the value that would have been obtained had the sand not contained fines (cs means clean sand). Olson and Stark (2002) further developed the original work of Seed and Harder, added data, and proposed the following equation on the basis of the corrected SPT blow count and the corrected CPT point resistance as follows (Figures 22.33 and 22.34):

$$
\begin{align*}
& \frac{s_{u-l i q}}{\sigma_{v o}^{\prime}}=0.03+0.0075\left(N_{1}\right)_{60} \quad \text { for } \quad N_{1} \leq 12 \mathrm{bpf}  \tag{22.59}\\
& \frac{s_{u-l i q}}{\sigma_{v o}^{\prime}}=0.03+0.0143 \times q_{c 1} \quad \text { for } \quad q_{c 1} \leq 6.5 \mathrm{MPa} \tag{22.60}
\end{align*}
$$

where $s_{u-l i q}$ is the shear strength of the liquefied soil, $\sigma_{v o}^{\prime}$ is the prefailure vertical effective stress in the soil, and $\left(N_{1}\right)_{60}$ and $q_{c 1}$ are the prefailure corrected SPT blow count and CPT point resistance respectively (see sections 7.1 7.2).


Figure 22.33 Shear strength of liquefied coarse-grained soils based on SPT blow count. (After Olson and Stark 2002)


Figure 22.34 Shear strength of liquefied coarse-grained soils based on CPT point resistance. (After Olson and Stark 2002)

The pressure generated by the liquefied soil flowing past the pile can be estimated as 7 times the shear strength of the liquefied soil:

$$
\begin{equation*}
p_{u}=7 s_{u-l i q} \tag{22.61}
\end{equation*}
$$

where $p_{u}$ is the pressure generated on the pile by the flowing soil and $s_{u-l i q}$ is the shear strength of the liquefied soil.

## PROBLEMS

22.1 After an earthquake, a seismograph installed in the bedrock records the arrival of a compression wave and 10 seconds later the arrival of a shear wave. The rock has a compression wave velocity equal to $3000 \mathrm{~m} / \mathrm{s}$ and a shear wave velocity equal to $1500 \mathrm{~m} / \mathrm{s}$. How far is the earthquake epicenter from the seismograph? How would you find the exact location of the epicenter?
22.2 An earthquake takes place along a fault and creates 2 m of relative displacement between two tectonic plates. The area over which the slip takes place is 500 km by 100 km and the shear modulus of the rock is 20 GPa . Calculate the seismic moment $M_{o}$, the moment magnitude $M_{w}$, and the energy E of the earthquake.
22.3 Search the Pacific Earthquake Engineering Research (PEER) Center web site (http://peer.berkeley.edu/nga/) and select an earthquake ground acceleration vs. time record. From this record, determine the peak ground acceleration. Then integrate the acceleration record to generate the velocity vs. time record and find the peak ground velocity. Then integrate the velocity record to generate the displacement vs. time record and find the peak ground displacement.
22.4 From the acceleration record of problem 22.3, find the bracketed duration for a threshold acceleration of 0.05 g and the sustained maximum acceleration for 3 cycles and then for 5 cycles.
22.5 An event with a return period $T$ has a yearly probability of exceedance equal to $1 / T$. The equation linking the return period $T$ of an event to the probability of exceedance $P$ over a period of time $L$ is:

$$
P=1-(1-1 / T)^{L}
$$

Calculate (a) the return period for an earthquake that has a $2 \%$ probability of exceedance in 50 years and (b) the return period for an earthquake that has a $10 \%$ probability of exceedance in 50 years.
22.6 An 828 m tall tower weighs 6000 MN and has an equivalent stiffness of $200 \mathrm{MN} / \mathrm{m}$.
a. Calculate the natural period of the tower. A one-story house weighs 1.4 MN and has a natural period of 0.15 seconds.
b. What is the equivalent stiffness of the house?
22.7 Search the PEER Center web site (http://peer.berkeley.edu/nga/) and select an earthquake ground acceleration vs. time record. For the acceleration record,
a. Develop the Fourier acceleration spectrum
b. Develop the response spectrum, for a damping ratio of $5 \%$, by choosing $m$ and varying $k$.
c. Choose a first set of values for $k$ and $m$ and find the spectral acceleration $a_{1}$, then find the spectral acceleration $a_{2}$ for a second set of values equal to 2 k and 2 m . Compare $a_{1}$ and $a_{2}$.
22.8 The PGA for a magnitude 6 earthquake is 0.5 g . What is the most likely PGA 50 km away?
22.9 What is the likely return period or recurrence interval for a magnitude 6 earthquake?
22.10 Calculate the natural period of a 20 m thick stiff soil layer if the soil shear wave velocity is $200 \mathrm{~m} / \mathrm{s}$. Then calculate the natural period of a 50 m thick soft soil layer if the shear wave velocity is $100 \mathrm{~m} / \mathrm{s}$.
22.11 What is the transfer function (amplification factor) for the displacement at the ground surface during an earthquake if the natural period of the deposit is 1 second and the depth of soil layer above rock level is 100 m ? Assume an undamped linear soil on rigid rock. Redo the calculation for a damped linear soil on rigid rock if the damping ratio is 5\%. The shear wave velocity of the soil is $250 \mathrm{~m} / \mathrm{s}$.
22.12 A soil has a shear wave velocity equal to $250 \mathrm{~m} / \mathrm{s}$ and an SPT blow count equal to 30 bpf . The design earthquake corresponds to a PGA equal to 0.3 g . Develop the response spectrum according to Figure 22.17 if the reference spectrum has the following characteristics: spectral acceleration at 0.2 seconds $=0.5 \mathrm{~g}$, spectral acceleration at 1 second $=0.2 \mathrm{~g}$.
22.13 At a depth of 5 m below the ground surface, a saturated sand deposit has a corrected SPT blow count equal to 10 bpf, a CPT corrected and normalized point resistance of 90 , and a corrected shear wave velocity of $170 \mathrm{~m} / \mathrm{s}$. The groundwater level is at the ground surface and the soil has a total unit weight of $18 \mathrm{kN} / \mathrm{m}^{3}$. Will the soil liquefy in a magnitude 7.5 earthquake if the PGA is 0.6 g ? What would be the highest magnitude for which the soil would not liquefy?
22.14 A slope is cut in a medium-stiff clay with an undrained shear strength $s_{u}$ equal to 50 kPa . The site has a site class B, a PGA of 0.45 g , and a spectral acceleration at 1 second equal to 0.3 g . Calculate the horizontal seismic coefficient $k_{h}$ to be used in the slope earthquake stability analysis.
22.15 Write the expression of the earthquake active earth pressure coefficient and the corresponding static active earth pressure coefficient. Plot the ratio versus $k_{h}$ for $k_{v}=0$, vertical back wall, horizontal backfill, frictionless wall, and a $30^{\circ}$ friction angle for the backfill.
22.16 A 3 m high vertical gravity retaining wall has a dry horizontal backfill with a friction angle equal to $30^{\circ}$ and a unit weight of $20 \mathrm{kN} / \mathrm{m}^{3}$. It must be designed for a horizontal seismic coefficient equal to 0.2 . Calculate:
a. Static coefficient of active and passive earth pressure, $K_{a}$ and $K_{p}$
b. Seismic coefficient of active and passive earth pressure, $K_{a e}$ and $K_{p e}$
c. The static component and dynamic component of the active push against the wall and their point of application, $P_{a}, \Delta P_{a e}, X_{a}$, and $X_{a e}$
d. The static and dynamic components of the passive push against the wall if the wall was pushed into the soil backfill and their point of application, $P_{p}, \Delta P_{p e}, X_{p}$, and $X_{p e}$
22.17 The wall of problem 22.16 has water on the no-soil side and water in the backfill up to the ground surface. The water depth on the no-soil side is 2 m . Calculate:
a. The hydrostatic pressure and the resultant water push on both sides of the wall, $p_{w 1}, p_{w 2}, P_{w 1}$, and $P_{w 2}$
b. The earthquake pressure and the resultant push on both sides of the wall if the horizontal seismic coefficient is 0.2
22.18 Demonstrate that the point of application of the dynamic water pressure in Eq. 22.50 is $0.6 H_{w}$ from the top of the water level.
22.19 An anchored wall retains 10 m of sand with a blow count of 18 bpf and a unit weight of $20 \mathrm{kN} / \mathrm{m}^{3}$. The water level is deeper than the excavation level. The design earthquake will generate a horizontal seismic coefficient equal to 0.25 . Calculate:
a. The pressure $p$ against the wall above the excavation in the case of no earthquake
b. The pressure $p_{e}$ against the wall above the excavation in the case of an earthquake
c. The average load per anchor in both cases if the anchors are inclined at $15^{\circ}$ and the vertical and horizontal spacing between anchors is 2.5 m .
22.20 A building is 60 m tall, weighs 500 MN , and has a horizontal stiffness of $400 \mathrm{MN} / \mathrm{m}$. The design earthquake gives the response spectrum shown in Figure 22.1s. Calculate the horizontal force that must be resisted by the foundation.


Figure 22.1s Response spectrum for problem 22.20.

## Problems and Solutions

## Problem 22.1

After an earthquake, a seismograph installed in the bedrock records the arrival of a compression wave and 10 seconds later the arrival of a shear wave. The rock has a compression wave velocity equal to $3000 \mathrm{~m} / \mathrm{s}$ and a shear wave velocity equal to $1500 \mathrm{~m} / \mathrm{s}$. How far is the earthquake epicenter from the seismograph? How would you find the exact location of the epicenter?

## Solution 22.1

The distance between the epicenter and seismograph is:

$$
d=\frac{\Delta t_{\mathrm{p}-\mathrm{s}}}{\frac{1}{v_{s}}-\frac{1}{v_{p}}}=\frac{10}{\frac{1}{1500}-\frac{1}{3000}}=30,000 \mathrm{~m}
$$

where $\Delta t_{p-s}$ is the arrival time difference of a shear wave and compression wave, $v_{s}$ is the shear wave velocity, and $v_{p}$ is the compression wave velocity. The earthquake epicenter is $30,000 \mathrm{~m}$ away from the seismograph. Three seismographs are needed to find the exact location of the epicenter: The intersection of the three circles gives the location.

## Problem 22.2

An earthquake takes place along a fault and creates 2 m of relative displacement between two tectonic plates. The area over which the slip takes place is 500 km by 100 km and the shear modulus of the rock is 20 GPa . Calculate the seismic moment $M_{o}$, the moment magnitude $M_{w}$, and the energy $E$ of the earthquake.

## Solution 22.2

Seismic moment $M_{o}$ :

$$
M_{o}=G A D=\left(20 \times 10^{9}\right) \times\left(5 \times 10^{10}\right) \times(2)=2 \times 10^{21} \mathrm{~N} \cdot \mathrm{~m}
$$

where $G$ is the shear modulus of the rock, $A$ is the area over which the slip occurs, and $D$ is the amount of slip movement.
Moment magnitude $M_{w}$ :

$$
M_{w}=0.66 \log M_{o}(\mathrm{~N} \cdot \mathrm{~m})-6.05=0.66 \log \left(2 \times 10^{21}\right)-6.05=8
$$

Energy $E$ :

$$
\log E=5.24+1.44 M=5.24+1.44 \times 8=16.76
$$

Therefore, the energy $E$ is $E=10^{16.76}=5.8 \times 10^{16} \mathrm{~N} . \mathrm{m}=5.8 \times 10^{16}$ joules.

## Problem 22.3

Search the Pacific Earthquake Engineering Research (PEER) Center web site (http://peer.berkeley.edu/nga/) and select an earthquake ground acceleration vs. time record. From this record, determine the peak ground acceleration. Then integrate the acceleration record to generate the velocity vs. time record and find the peak ground velocity. Then integrate the velocity record to generate the displacement vs. time record and find the peak ground displacement.

Solution 22.3


Figure 22.2s Acceleration, velocity, and displacement of an earthquake record.
A sample record chosen from the PEER web site is the Loma Prieta Station Gilroy \#2 record. From Figure 22.2s:
Peak ground acceleration in gs $(\mathrm{PGAg})=0.322 \mathrm{~g}$
Peak ground acceleration $(\mathrm{PGA})=3.159\left(\mathrm{~m} / \mathrm{s}^{2}\right)$

Peak ground velocity $(\mathrm{PGV})=0.391 \mathrm{~m} / \mathrm{s}$
Peak ground displacement $(\mathrm{PGD})=0.121(\mathrm{~m})$

## Problem 22.4

From the acceleration record of problem 22.3, find the bracketed duration for a threshold acceleration of 0.05 g and the sustained maximum acceleration for 3 cycles and then for 5 cycles.

## Solution 22.4



Figure 22.3s Bracketed duration of the ground acceleration.

The horizontal lines on Figure 22.3s show the threshold accelerations of $\pm 0.05 \mathrm{~g}$. The bracketed duration for this earthquake (the time between the first and last exceedance) is 15.26 seconds.

The maximum acceleration for three cycles is 0.145 g .
The maximum acceleration for five cycles is 0.13 g .

## Problem 22.5

An event with a return period T has a yearly probability of exceedance equal to $1 / \mathrm{T}$. The equation linking the return period T of an event to the probability of exceedance $P$ over a period of time $L$ is:

$$
\mathrm{P}=1-(1-1 / \mathrm{T})^{\mathrm{L}}
$$

Calculate (a) the return period for an earthquake that has a $2 \%$ probability of exceedance in 50 years and (b) the return period for an earthquake that has a $10 \%$ probability of exceedance in 50 years.

## Solution 22.5

a. For an earthquake with a $2 \%$ probability of exceedance in 50 years, the return period is:

$$
\begin{aligned}
P & =1-(1-1 / T)^{L} \\
1-P & =(1-1 / T)^{L} \\
(1-P)^{1 / L} & =(1-1 / T) \\
1 / T & =1-(1-P)^{1 / L} \\
T & =\frac{1}{1-(1-P)^{1 / L}} \\
T & =\frac{1}{1-(1-0.02)^{1 / 50}}=2476 \text { years }
\end{aligned}
$$

b. For an earthquake with a $10 \%$ probability of exceedance in 50 years, the return period is:

$$
\begin{aligned}
T & =\frac{1}{1-(1-P)^{1 / L}} \\
T & =\frac{1}{1-(1-0.10)^{1 / 50}}=475 \text { years }
\end{aligned}
$$

## Problem 22.6

An 828 m tall tower weighs 6000 MN and has an equivalent stiffness of $200 \mathrm{MN} / \mathrm{m}$. Calculate the natural period of the tower. A one-story house weighs 1.4 MN and has a natural period of 0.15 seconds. What is the equivalent stiffness of the house?

Solution 22.6
a. The natural period T of the tower is:

$$
\begin{aligned}
T & =2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{W}{g k}} \\
T & =2 \pi \sqrt{\frac{(6000 / 9.81)}{200}}=10.98 \mathrm{sec}
\end{aligned}
$$

b. Rearranging the natural period equation, the stiffness k of the house is:

$$
k=4 \pi^{2} \frac{W}{g T^{2}}=4 \times 3.14^{2} \times \frac{1.4}{9.81 \times 0.15^{2}}=250.1 \mathrm{MN} / \mathrm{m}
$$

## Problem 22.7

Search the PEER Center web site (http://peer.berkeley.edu/nga/) and select an earthquake ground acceleration vs. time record. For the acceleration record:
a. Develop the Fourier acceleration spectrum
b. Develop the response spectrum, for a damping ratio of $5 \%$, by choosing m and varying $k$.
c. Choose a first set of values for $k$ and $m$ and find the spectral acceleration $a_{1}$, then find the spectral acceleration $a_{2}$ for a second set of values equal to 2 k and 2 m . Compare $a_{1}$ and $a_{2}$.

Solution 22.7
The record selected for this example is the station Gilroy $\# 2$ on soil. The Fourier acceleration spectrum and the three response spectra (acceleration, velocity, and displacement) are given in Figures 22.4 s and 22.5 s.


Figure 22.4s Fourier acceleration spectrum.




Figure 22.5s Fourier response spectrum.

Note: The response spectra are defined as the response of the SDOF with a natural period $T$. It is obtained by solving the equation of motion:

$$
m \ddot{x}+c \dot{x}+k x=-m a(t)
$$

By setting $\omega=\sqrt{k / m}$ and $c=2 m \omega \beta$, and then dividing by m , the equation becomes:

$$
\ddot{x}+2 \beta \omega \dot{x}+\omega^{2} x=-a(t)
$$

Thus, by multiplying $k$ and $m$ by the same value, $\omega$ will not change and the response spectrum will not change, including the spectral acceleration. However, a change in $\beta$ (damping) will change the response spectrum.

## Problem 22.8

The PGA for a magnitude 6 earthquake is 0.5 g . What is the most likely PGA 50 km away?

## Solution 22.8

The peak ground acceleration at a distance $R(\mathrm{~km})$ for a magnitude M earthquake can be estimated by using Figure 22.6s. A line is drawn parallel to the trend line starting at the PGA value of 0.5 g . Then the PGA value is read on that line at a distance of 50 km .


Figure 22.6s Attenuation of peak horizontal ground acceleration.

## Problem 22.9

What is the likely return period or recurrence interval for a magnitude 6 earthquake?
Solution 22.9
According to Figure 22.12, the recurrence interval of a magnitude 6 earthquake is about 200 years.

## Problem 22.10

Calculate the natural period of a 20 m thick stiff soil layer if the soil shear wave velocity is $200 \mathrm{~m} / \mathrm{s}$. Then calculate the natural period of a 50 m thick soft soil layer if the shear wave velocity is $100 \mathrm{~m} / \mathrm{s}$.

## Solution 22.10

Using Eq. 22.6, the natural period is:

$$
T=\frac{4 H}{v_{s}}
$$

Stiff soil:

$$
T=\frac{4 \times 20}{200}=0.4 \mathrm{sec}
$$

Soft soil:

$$
T=\frac{4 \times 50}{100}=2 \mathrm{sec}
$$

## Problem 22.11

What is the transfer function (amplification factor) for the displacement at the ground surface during an earthquake if the natural period of the deposit is 1 second and the depth of soil layer above rock level is 100 m ? Assume an undamped linear soil on rigid rock. Redo the calculation for a damped linear soil on rigid rock if the damping ratio is $5 \%$. The shear wave velocity of the soil is $250 \mathrm{~m} / \mathrm{s}$.

## Solution 22.11

The 1 second period is used to find $\omega$ :

$$
T=\frac{2 \pi}{\omega}=1
$$

Therefore, $\omega=2 \pi$. The first calculation is when the soil is undamped, $\beta=0$. The transfer function is:

$$
\begin{aligned}
& F(\omega)=\frac{1}{\sqrt{\cos ^{2}\left(\frac{\omega H}{v_{s}}\right)+\left(\beta \frac{\omega H}{v_{s}}\right)^{2}}} \\
& F(\omega)=\frac{1}{\sqrt{\cos ^{2}\left(\frac{2 \pi \times 100}{250}\right)+0}}=1.24
\end{aligned}
$$

With damping, $\beta=0.05$, the transfer function is:

$$
F(\omega)=\frac{1}{\sqrt{\cos ^{2}\left(\frac{2 \pi \times 100}{250}\right)+\left(.05 \frac{2 \pi \times 100}{250}\right)^{2}}}=1.22
$$

## Problem 22.12

A soil has a shear wave velocity equal to $250 \mathrm{~m} / \mathrm{s}$ and an SPT blow count equal to 30 bpf . The design earthquake corresponds to a PGA equal to 0.3 g . Develop the response spectrum according to Figure 22.17 if the reference spectrum has the following characteristics: spectral acceleration at 0.2 seconds $=0.5 \mathrm{~g}$, spectral acceleration at 1 second $=0.2 \mathrm{~g}$.

## Solution 22.12

The site-specific spectral parameters are found in Table 22.2. With the given soil parameters:

$$
v_{s}=250 \mathrm{~m} / \mathrm{s} \text { and } \mathrm{N}_{S P T}=30 \Rightarrow \text { Soil clasification is " } \mathrm{D} " .
$$

From Table 22.3, with a PGA $=0.3 \mathrm{~g}$ and a site classification of $\mathrm{D}, F_{P G A}=1.2$.

$$
\begin{aligned}
A_{S} & =F_{P G A} \times P G A \\
A_{s} & =1.2 \times 0.3 \mathrm{~g}=0.36 \mathrm{~g}
\end{aligned}
$$

$S_{D S}=0.5 \mathrm{~g}$ and $\mathrm{S}_{\mathrm{Dl}}=0.2 \mathrm{~g}$ (from the problem), therefore:

$$
\begin{aligned}
T_{S} & =\frac{S_{D 1}}{S_{D S}} \\
T & =\frac{0.2 g}{0.5 g}=0.4 \\
T_{o} & =0.2 T_{S} \\
T_{o} & =0.2 \times 0.4=0.08
\end{aligned}
$$

The constants found using the site classification are used to develop the site-specific acceleration response spectrum, shown in Figure 22.7s.


Figure 22.7s Design code acceleration response spectrum.

## Problem 22.13

At a depth of 5 m below the ground surface, a saturated sand deposit has a corrected SPT blow count equal to 10 bpf, a CPT corrected and normalized point resistance of 90 , and a corrected shear wave velocity of $170 \mathrm{~m} / \mathrm{s}$. The fine percentage is less than $5 \%$. The groundwater level is at the ground surface and the soil has a total unit weight of $18 \mathrm{kN} / \mathrm{m}^{3}$. Will the soil liquefy in a magnitude 7.5 earthquake if the PGA is 0.6 g ? What would be the highest magnitude for which the soil would not liquefy?

## Solution 22.13

a. The cyclic stress ratio $C S R=\frac{\tau_{a v}}{\sigma_{v o}^{\prime}}=0.65\left(\frac{a_{\max }}{g}\right)\left(\frac{\sigma_{v o}}{\sigma_{v o}^{\prime}}\right) r_{d}$

$$
a_{\max }=0.6 g, \sigma_{v o}=\gamma H=18 \times 5=90\left(\mathrm{kN} / \mathrm{m}^{2}\right), \sigma_{v o}^{\prime}=\gamma^{\prime} H=(18-9.8) \times 5=41\left(\mathrm{kN} / \mathrm{m}^{2}\right)
$$

Fig. $22.25 \rightarrow r_{d}=0.95$.
$C S R=\frac{\tau_{a v}}{\sigma_{v o}^{\prime}}=0.65(0.6)(2.195) 0.95=0.81$
Fig. 22.21 $\xrightarrow{N_{S P T}=10}$ Liquefy,
Fig. 22.22 $\xrightarrow{q=90}$ Liquefy,
Fig. $22.23 \xrightarrow{v_{s}=170(\mathrm{~m} / \mathrm{s})}$ Liquefy
b. $\operatorname{CSR}=\frac{\tau_{a v}}{\sigma_{v o}^{\prime}}=0.65\left(\frac{a_{\max }}{g}\right)\left(\frac{\sigma_{v o}}{\sigma_{v o}^{\prime}}\right) r_{d}$

Fig. $22.21 \xrightarrow{C S R=0.1} a_{\text {max }}=0.07 \mathrm{~g}$
Fig. $22.22 \xrightarrow{\text { CSR }=0.13} a_{\text {max }}=0.095 \mathrm{~g}$
Fig. $22.23 \xrightarrow{C S R=0.13} a_{\text {max }}=0.095 \mathrm{~g}$

## Problem 22.14

A slope is cut in a medium-stiff clay with an undrained shear strength $s_{u}$ equal to 50 kPa . The height of the slope is 10 m . The site has a site class B, a PGA of 0.45 g , and a spectral acceleration at 1 second equal to 0.3 g . Calculate the horizontal seismic coefficient $k_{h}$ to be used in the slope earthquake stability analysis.

## Solution 22.14

$$
\begin{aligned}
\text { Class } B, S_{1} & =0.3 \mathrm{~g} \xrightarrow{\text { Table } 22.3} F_{P G A}=1.0 \\
\text { Class } B, P G A & =0.45 \mathrm{~g} \xrightarrow{\text { Table } 22.5} F_{V}=1.0 \\
k_{\max } & =F_{P G A} \times P G A=0.45 \mathrm{~g} \\
k_{a v} & =\gamma\left(1+0.01 H\left(0.5 \frac{F_{V} S_{1}}{k_{\max }}-1\right)\right) k_{\max } \\
k_{h} & =k_{a v}=1.2\left(1+0.01 \times 10\left(0.5 \frac{1 \times 0.3 \mathrm{~g}}{0.45 \mathrm{~g}}-1\right)\right) 0.45 \mathrm{~g}=0.504 \mathrm{~g}
\end{aligned}
$$

Problem 22.15
Write the expression of the earthquake active earth pressure coefficient and the corresponding static active earth pressure coefficient. Plot the ratio versus $k_{h}$ for $k_{v}=0$, vertical back wall, horizontal backfill, frictionless wall, and a $30^{\circ}$ friction angle for the backfill.

## Solution 22.15

The expression for the active earth pressure coefficient in the earthquake case, $K_{a e}$, is found after finding the most critical wedge angle. $K_{a e}$ is:

$$
K_{a e}=\frac{\sin ^{2}\left(\alpha+\varphi^{\prime}-\psi\right)}{\cos \psi \sin ^{2} \alpha \sin (\alpha-\delta-\psi)\left[1+\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}-\beta-\psi\right)}{\sin (\alpha-\delta-\psi) \sin (\alpha+\beta)}}\right]^{2}}
$$

where $\alpha$ is the angle of the back of the wall with the horizontal, $\beta$ is the angle of the ground surface behind the wall with the horizontal, $\delta$ is the angle of friction between the back of the wall and the soil, $\varphi^{\prime}$ is the friction angle of the soil, and $\psi$ is the angle representing the earthquake inertia force as:

$$
\psi=\tan ^{-1}\left(\frac{k_{h}}{1-k_{v}}\right)
$$

where $k_{h}$ and $k_{v}$ are the horizontal and vertical seismic coefficients respectively. The expression for the static active earth pressure coefficient, $K_{a}$, is:

$$
K_{a}=\frac{\sin ^{2}\left(\alpha+\varphi^{\prime}\right)}{\sin ^{2} \alpha \sin (\alpha-\delta)\left[1+\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}-\beta\right)}{\sin (\alpha-\delta) \sin (\alpha+\beta)}}\right]^{2}}
$$

The ratio of $K_{a e} / K_{a}$ for a vertical wall $(\alpha=90)$, no wall friction $(\delta=0)$, horizontal backfill $(\beta=0)$, and a $30^{\circ}$ angle of friction for the backfill can be plotted as in Figure 22.8s.


Figure 22.8s Ratio of the earthquake active earth pressure coefficient and the corresponding active earth pressure coefficient versus $k_{h}$.

## Problem 22.16

A 3 m high vertical gravity retaining wall has a dry horizontal backfill with a friction angle equal to $30^{\circ}$ and a unit weight of $20 \mathrm{kN} / \mathrm{m}^{3}$. It must be designed for a horizontal seismic coefficient equal to 0.2 . Calculate:
a. Static coefficient of active and passive earth pressure, $K_{a}$ and $K_{p}$
b. Seismic coefficient of active and passive earth pressure, $K_{a e}$ and $K_{p e}$
c. The static component and dynamic component of the active push against the wall and their point of application, $P_{a}$, $\Delta P_{a e}, X_{a}$, and $X_{a e}$
d. The static and dynamic components of the passive push against the wall if the wall was pushed into the soil backfill and their point of application, $P_{p}, \Delta P_{p e}, X_{p}$, and $X_{p e}$.

## Solution 22.16

a. Static coefficient of active and passive earth pressure, $K_{a}$ and $K_{p}$ :

$$
\begin{aligned}
& K_{a}=\frac{1-\sin \varphi^{\prime}}{1+\sin \varphi^{\prime}}=\frac{1-\sin (30)}{1+\sin (30)}=0.333 \\
& K_{p}=\frac{1+\sin \varphi^{\prime}}{1-\sin \varphi^{\prime}}=\frac{1+\sin (30)}{1-\sin (30)}=3
\end{aligned}
$$

b. Seismic coefficient of active and passive earth pressure, $K_{a e}$ and $K_{p e}$ :

$$
\psi=\tan ^{-1}\left(\frac{k_{h}}{1-k_{v}}\right)=\tan ^{-1}\left(\frac{0.2}{1-0}\right)=11.3^{\circ}
$$

The seismic coefficient of active earth pressure is:

$$
\begin{aligned}
K_{a e}= & \frac{\sin ^{2}\left(\alpha+\varphi^{\prime}-\psi\right)}{\cos \psi \sin ^{2} \alpha \sin (\alpha-\delta-\psi)\left[1+\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}-\beta-\psi\right)}{\sin (\alpha-\delta-\psi) \sin (\alpha+\beta)}}\right]^{2}} \\
K_{a e}= & \frac{\sin ^{2}(90+30-11.3)}{\cos (11.3) \sin ^{2}(90) \sin (90-0-11.3)\left[1+\sqrt{\frac{\sin (30+0) \sin (30-0-11.3)}{\sin (90-0-11.3) \sin (90+0)}}\right]^{2}}=0.473
\end{aligned}
$$

The seismic coefficient of passive earth pressure is:

$$
\begin{aligned}
K_{p e}= & \frac{\sin ^{2}\left(\alpha-\varphi^{\prime}+\psi\right)}{\cos \psi \sin ^{2} \alpha \sin (\alpha+\delta+\psi)\left[1-\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}+\beta+\psi\right)}{\sin (\alpha+\delta+\psi) \sin (\alpha+\beta)}}\right]^{2}} \\
K_{p e}= & \frac{\sin ^{2}(90-30+11.3)}{\cos (11.3) \sin ^{2}(90) \sin (90+0+11.3)\left[1-\sqrt{\frac{\sin (30+0) \sin (30+0+11.3)}{\sin (90+0+11.3) \sin (90+0)}}\right]^{2}}=5.29
\end{aligned}
$$

c. The static component of active push is:

$$
\begin{aligned}
P_{a} & =\frac{1}{2} K_{a} \gamma H^{2} \\
P_{a} & =\frac{1}{2} \times 0.33 \times 20 \times(3)^{2}=30 \frac{\mathrm{kN}}{\mathrm{~m}}
\end{aligned}
$$

The dynamic component of active push is:

$$
\begin{aligned}
\Delta P_{a e} & =\frac{1}{2}\left(K_{a e}-K_{a}\right) \gamma H^{2} \\
\Delta P_{a e} & =\frac{1}{2}(0.473-0.333)(20)(3)^{2}=12.6 \frac{\mathrm{kN}}{\mathrm{~m}^{3}}
\end{aligned}
$$

The point of application of $P_{a}$ from the bottom of the wall is:

$$
X_{a}=\frac{1}{3} H=\frac{1}{3}(3)=1 \mathrm{~m}
$$

The point of application of $P_{a e}$ from the bottom of the wall is:

$$
X_{a e}=0.6 H=0.6(3)=1.8 \mathrm{~m}
$$

d. The static component of passive push is:

$$
\begin{aligned}
P_{p} & =\frac{1}{2} K_{p} \gamma H^{2} \\
P_{p} & =\frac{1}{2}(3)(20)(3)^{2}=270 \frac{\mathrm{kN}}{\mathrm{~m}^{3}}
\end{aligned}
$$

The dynamic component of passive push is:

$$
\begin{aligned}
\Delta P_{p e} & =\frac{1}{2}\left(K_{p e}-K_{p}\right) \gamma H^{2} \\
\Delta P_{p e} & =\frac{1}{2}(5.29-3)(20)(3)^{2}=206.1 \frac{\mathrm{kN}}{\mathrm{~m}^{3}}
\end{aligned}
$$

The point of application of $P_{a}$ from the bottom of the wall is:

$$
X_{p}=\frac{1}{3} H=\frac{1}{3}(3)=1 \mathrm{~m}
$$

The point of application of $P_{a e}$ from the bottom of the wall is:

$$
X_{p e}=0.6 \mathrm{H}=0.6 \times 3=1.8 \mathrm{~m}
$$

## Problem 22.17

The wall of problem 22.16 has water on the no-soil side and water in the backfill up to the ground surface. The water depth on the no-soil side is 2 m . Calculate:
a. The hydrostatic pressure and the resultant water push on both sides of the wall, $p_{w 1}, p_{w 2}, P_{w 1}$, and $P_{w 2}$
b. The earthquake pressure and the resultant push on both sides of the wall if the horizontal seismic coefficient is 0.2

## Solution 22.17

a. The hydrostatic pressure and the resultant water push on both sides of the wall, $p_{w 1}, p_{w 2}, P_{w 1}$, and $P_{w 2}$ :

The hydrostatic pressure on the no-soil side is:

$$
\begin{aligned}
& p_{w 1}=\gamma_{w} z_{1} \\
& p_{w 1}=9.81(2)=19.62 \frac{\mathrm{kN}}{\mathrm{~m}^{2}}
\end{aligned}
$$

The resultant push for the hydrostatic pressure on the no-soil side is:

$$
P_{w 1}=\frac{1}{2} p_{w 1} z_{1}=\frac{1}{2}(19.62)(2)=19.62 \frac{\mathrm{kN}}{\mathrm{~m}}
$$

The hydrostatic pressure on the backfill side is:

$$
\begin{aligned}
& p_{w 2}=\gamma_{w} z_{2} \\
& p_{w 2}=9.81(3)=29.43 \frac{\mathrm{kN}}{\mathrm{~m}^{2}}
\end{aligned}
$$

The resultant push for the hydrostatic pressure on the backfill side is:

$$
P_{w 2}=\frac{1}{2} p_{w 2} z_{2}=\frac{1}{2}(29.43)(3)=44.15 \frac{\mathrm{kN}}{\mathrm{~m}}
$$

b. The earthquake pressure and the resultant push on both sides of the wall if the horizontal seismic coefficient is 0.2 :

The earthquake water pressure on the no-soil side is:

$$
\begin{aligned}
\Delta p_{w e 1} & =\frac{7}{8} k_{h} \gamma_{w} \sqrt{z H_{w}} \\
\Delta p_{w e 1} & =\frac{7}{8}(0.2)(9.81) \sqrt{(2)(2)}=3.43 \frac{\mathrm{kN}}{\mathrm{~m}^{2}}
\end{aligned}
$$

The resultant push for the hydrostatic pressure on the no-soil side is:

$$
\begin{aligned}
& \Delta P_{w e 1}=\frac{7}{12} k_{h} \gamma_{w} H_{w}{ }^{2} \\
& \Delta P_{w e 1}=\frac{7}{12}(0.2)(9.81)(2)^{2}=4.58 \frac{\mathrm{kN}}{\mathrm{~m}}
\end{aligned}
$$

The resultant water push on the no-soil side is:

$$
P_{w 1}+\Delta P_{w e 1}=19.62+4.58=24.2 \frac{\mathrm{kN}}{\mathrm{~m}}
$$

The earthquake push on the backfill side is obtained as follows:

$$
\begin{aligned}
\gamma^{\prime}= & \gamma-\gamma_{h}=20-9.81=10.19 \frac{\mathrm{kN}}{\mathrm{~m}^{3}} \\
\psi= & \tan ^{-1}\left(\frac{k_{h}}{1-k_{v}} \times \frac{\gamma}{\gamma^{\prime}}\right) \\
\psi= & \tan ^{-1}\left(\frac{0.2}{1-0} \times \frac{20}{10.19}\right)=21.4^{\circ} \\
K_{a e}= & \frac{\sin ^{2}\left(\alpha+\varphi^{\prime}-\psi\right)}{} \begin{aligned}
& \cos \psi \sin ^{2} \alpha \sin (\alpha-\delta-\psi)\left[1+\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}-\beta-\psi\right)}{\sin (\alpha-\delta-\psi) \sin (\alpha+\beta)}}\right]^{2} \\
& K_{a e}= \frac{\sin ^{2}(90+30-21.4)}{\cos (21.4) \sin ^{2}(90) \sin (90-0-21.4)\left[1+\sqrt{\frac{\sin (30+0) \sin (30-0-21.4)}{\sin (90-0-21.4) \sin (90+0)}}\right]^{2}}=0.685 \\
& P_{a e}^{\prime}= \frac{1}{2} K_{a e} \gamma^{\prime} H^{2} \\
& P_{a e}^{\prime}= \frac{1}{2}(0.685)(10.19)(3)^{2}=31.41 \frac{\mathrm{kN}}{\mathrm{~m}^{3}}
\end{aligned}
\end{aligned}
$$

The resultant push on the backfill side is:

$$
\begin{aligned}
& P_{a e}=P_{w 2}+P_{a e}^{\prime} \\
& P_{a e}=44.15+31.41=75.6 \frac{\mathrm{kN}}{\mathrm{~m}}
\end{aligned}
$$

## Problem 22.18

Demonstrate that the point of application of the dynamic water pressure in Eq. 22.50 is $0.6 H_{w}$ from the top of the water level.

## Solution 22.18

By writing the moment equation:

$$
\bar{Z} . \Delta P_{w e}=\int_{z=0}^{z=H_{W}}\left(\Delta p_{w e} \times z\right) d z
$$

From Eqs. 22.50 and 22.52, we have:

$$
\begin{aligned}
\Delta p_{w e} & =\frac{7}{8} k_{h} \gamma_{w} \sqrt{z H_{w}} \\
\Delta P_{w e} & =\frac{7}{12} k_{h} \gamma_{w} H_{w}{ }^{2}
\end{aligned}
$$

By plugging Eqs. 22.50 and 22.52 into:

$$
K_{a e}=\frac{\sin ^{2}(90+30-11.3)}{\cos (11.3) \sin ^{2}(90) \sin (90-0-11.3)\left[1+\sqrt{\frac{\sin (30+0) \sin (30-0-11.3)}{\sin (90-0-11.3) \sin (90+0)}}\right]^{2}}=0.473
$$

we get:

$$
\begin{aligned}
& \bar{Z}=\frac{\int_{0}^{H_{w}}\left(\frac{7}{8} k_{h} \gamma_{w} \sqrt{z H_{w}} z\right) \mathrm{d} z}{\frac{7}{12} k_{h} \gamma_{w} H_{w}^{2}} \\
& \bar{Z}=\frac{12 \int_{0}^{H_{w}} z^{\frac{3}{2}} \mathrm{~d} z}{8 H_{w}^{\frac{3}{2}}} \\
& \bar{Z}=\frac{3}{2} \times \frac{1}{{ }^{\frac{3}{2}}} \times \frac{2}{5} H_{w}^{\frac{5}{2}}=\frac{6}{10} H_{w}
\end{aligned}
$$

## Problem 22.19

An anchored wall retains 10 m of sand with a blow count of 18 bpf and a unit weight of $20 \mathrm{kN} / \mathrm{m}^{3}$. The wall is vertical, the backfill is horizontal, and the wall friction is zero. The water level is deeper than the excavation level. The allowable movement at the top of the wall is 30 mm . The design earthquake will generate a horizontal seismic coefficient equal to 0.25 . Calculate:
a. The pressure $p$ against the wall above the excavation in the case of no earthquake
b. The pressure $p_{e}$ against the wall above the excavation in the case of an earthquake

## Solution 22.19

1. The constant pressure $p$ against the wall above the excavation in the case of no earthquake

Fig. $21.19 u_{\text {top }}=30 \mathrm{~mm}, H=10000 \mathrm{~mm} \rightarrow \frac{u_{\text {top }}}{H}=0.003 \xrightarrow{\text { Fig. } 21.19} K=0.2$

$$
p=K \times \gamma H=0.2 \times 20 \times 10=40 \mathrm{kN} / \mathrm{m}^{2}
$$

2. The constant pressure $p_{e}$ against the wall above the excavation in the case of the earthquake

Calculate the coefficient of active earth pressure in the case of no earthquake
Based on Fig. $15.12 \rightarrow \varphi^{\prime}=33^{\circ} K_{a}=\frac{1-\sin 33^{\circ}}{1+\sin 33^{\circ}}=0.3$
Calculate the coefficient of active earth pressure in the case of earthquake

$$
K_{a e}=\frac{\sin ^{2}\left(\alpha+\varphi^{\prime}-\psi\right)}{\cos \psi \sin ^{2} \alpha \sin (\alpha-\delta-\psi)\left[1+\sqrt{\frac{\sin \left(\varphi^{\prime}+\delta\right) \sin \left(\varphi^{\prime}-\beta-\psi\right)}{\sin (\alpha-\delta-\psi) \sin (\alpha+\beta)}}\right]^{2}}
$$

Where $\psi=\tan ^{-1}\left(\frac{k_{h}}{1-k_{v}}\right)=\tan ^{-1}\left(\frac{0.25}{1-0}\right)=14^{\circ}$

$$
K_{a e}=\frac{\sin ^{2}(33-14)}{\cos (14) \sin ^{2}(90) \sin (90-0-14)\left[1+\sqrt{\frac{\sin (33+0) \sin (33-0-14)}{\sin (90-0-14) \sin (90+0)}}\right]^{2}}=0.466
$$

Calculate the pressure against the wall in the case of the earthquake.

$$
p_{e}=\frac{K_{a e}}{K_{a}} \times K \gamma H=\frac{0.466}{0.3} \times 0.2 \times 20 \times 10=62.1 \mathrm{kN} / \mathrm{m}^{2}
$$

Problem 22.20
A building is 60 m tall, weighs 500 MN , and has a horizontal stiffness of $400 \mathrm{MN} / \mathrm{m}$. The design earthquake gives the response spectrum shown in Figure 22.1s. Calculate the horizontal force that must be resisted by the foundation.


Figure 22.1s Response spectrum for problem 22.20.

Solution 22.20
Fundamental period: $T=2 \pi \sqrt{\frac{M}{K}}=2 \pi \sqrt{\frac{500 / g}{400}}=2.24 \mathrm{sec}$

$$
\begin{aligned}
T \xrightarrow{\text { Spectrum }} a & =0.245 \mathrm{~g} \\
F & =M a=(500 / 9.81) \times 0.245 * 9.81=122.5 \mathrm{MN}
\end{aligned}
$$

