CHAPTER 21

Retaining Walls

21.1 DIFFERENT TYPES (TOP-DOWN, BOTTOM-UP)

There are many different types of retaining walls, but they are generally classified into two main categories: bottom-up walls and top-down walls. *Bottom-up walls* are walls that are built before the soil is placed behind the wall. In this case the backfill is compacted in lifts from the bottom of the wall to the top of the wall, often with inclusions (e.g., metal strips, geosynthetics) being installed on the way up. *Top-down walls* are walls that are built in the ground; then the excavation in front of the wall takes place in stages, most often with inclusions (e.g., anchors, tiebacks, nails) being installed through the wall as excavation proceeds. Examples of bottom-up walls are gravity walls and mechanically stabilized earth (MSE) walls (Figure 21.1). Examples of top-down walls are cantilever walls, soil-nailed walls, and anchored walls (also known as tieback walls).

The design of retaining walls requires calculations regarding:

1. Earth pressure distribution behind the wall
2. Deflection of the wall
3. Drainage issues

The body of knowledge regarding the issue of earth pressure is much more developed than that on the issue of deflection.

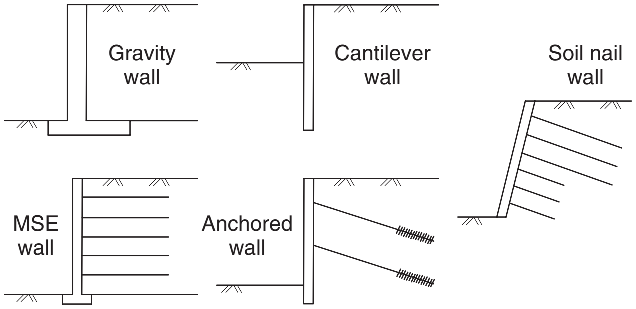


Figure 21.1 Types of retaining walls.

Geotechnical Engineering: Unsaturated and Saturated Soils Jean-Louis Briaud

One of the reasons is that historically, earth pressure theories came first.

21.2 ACTIVE, AT REST, PASSIVE EARTH PRESSURE, AND ASSOCIATED DISPLACEMENT

Consider an imaginary wall in a lake. The water pressure u_w on both sides of the wall would be hydrostatic and equal to $\gamma_w z$ where u_w is the water pressure against the wall at depth z below the water surface, and γ_w is the unit weight of water. As a result, the pressure diagram is triangular and the resultant is located at two-thirds of the wall height from the top of the wall. Note that the water pressure is the same in all directions, including horizontal and vertical, because water has a negligible resistance to shear (the Mohr circle for water is a point). Now consider an imaginary wall in the ground (Figure 21.2). The at-rest earth pressure σ_{oh} exists on both sides of the wall. If you push the wall horizontally, the pressure will increase on the side that penetrates into the soil up to soil failure (passive pressure σ_{ph}) and decrease on the other side where the wall is moving away from the soil down to soil failure (active pressure σ_{ah}). Note that if you push the wall far enough and if the soil is strong enough because of true or apparent cohesion, the pressure may become zero on the side where the wall is moving away from the soil and a gap opens up.

On the passive side, the soil is pushed away and upward as a wedge of failing soil forms in front of the wall (Figure 21.3); as a result, the soil imposes an upward friction force on the

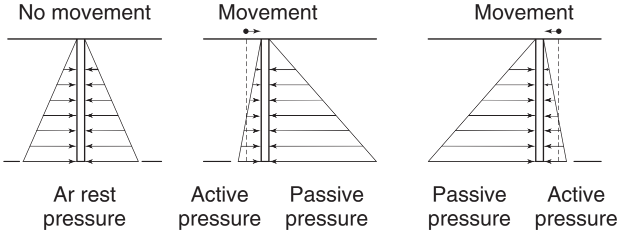


Figure 21.2 Imaginary wall and earth pressures.

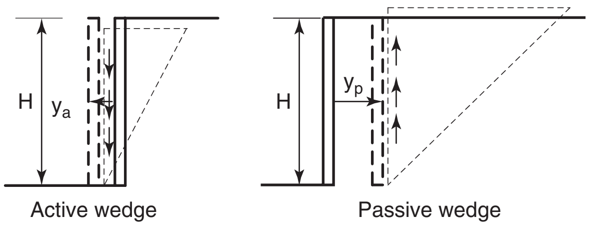


Figure 21.3 Earth pressures wedges.

wall. On the active side, the soil falls against the wall and downward as a wedge of failing soil forms behind the wall; as a result, the soil imposes a downward friction force on the wall. The passive wedge is much larger than the active wedge and requires more displacement to be mobilized. This is why the displacement required to mobilize the passive earth pressure is larger than the displacement required to mobilize the active earth pressure. The relationship between the soil pressure against the wall and the horizontal displacement of the wall is shown in Figure 21.4.

Now let's zoom in at the interface between the soil and the wall as shown in Figure 21.5. The soil particles contact the wall at several points where forces are transmitted between the soil and the wall. Between the particle contacts are the voids in the soil. These voids can be either completely filled with water (saturated soil) or filled with air and water (unsaturated soil). In the case of the saturated soil, the water will exert a pressure u_w against the wall. This water stress can be compression below the groundwater level (GWL) or tension within the capillary zone above the GWL. The water stress times the area of wall over which the water acts is the force transmitted by the water on the wall. The horizontal force on the wall is the sum of the forces at the particle contacts and the force contributed by the water stress u_w. Then we divide by the total area and, as in the case of vertical stress (see section 10.13), the total horizontal stress σ_h is

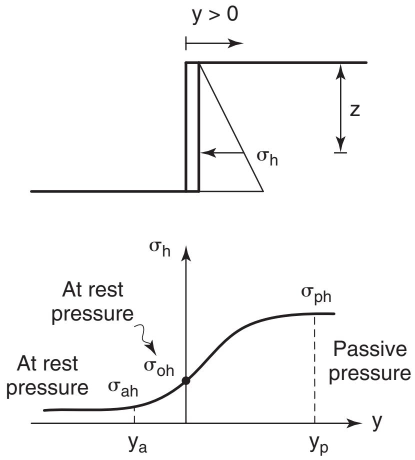


Figure 21.4 Earth pressure versus wall displacement.

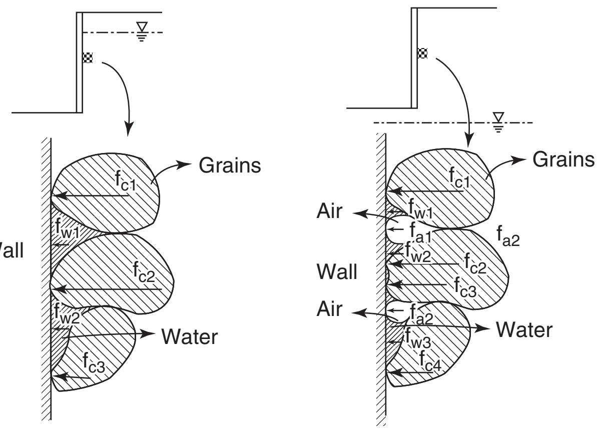


Figure 21.5 View of the soil-wall contact.

$\sigma'_h + u_w$ where σ'_h is the effective horizontal stress and u_w is the water stress (compression or tension).

If the soil is unsaturated, the horizontal force on the wall is the sum of the forces at the particle contacts, the forces transmitted through the water, and the forces transmitted through the air. If the air is occluded in the water phase and does not contact the wall, then σ_h is still equal to $\sigma'_h + u_w$, but the water phase is more compressible. Air tends to be occluded when the degree of saturation S is above 85%. If the air is not occluded ($S < 85\%$), there is a continuous air path to the ground surface and the air stress is atmospheric or zero gage pressure. In this case (see section 10.13), the horizontal stress σ_h is $\sigma'_h + \alpha u_w$ where σ'_h is the effective horizontal stress, α is the ratio of the water area in contact with the wall over the total area, and u_w is the water stress (which is in tension in this case). As pointed out in section 10.13, α can be estimated as the degree of saturation with a $\pm 30\%$ precision or by Khalili rule. The effect of the water tension in unsaturated soil will be to decrease the active horizontal pressure and increase the passive horizontal pressure compared to the case of the saturated soil with water in compression. Note that the active earth pressure and the passive earth pressure correspond to soil failure. Therefore, they should be thought of as strength rather than stress.

21.3 EARTH PRESSURE THEORIES

21.3.1 Coulomb Earth Pressure Theory

The earth pressure theories make the general assumption that the soil is at failure. In that sense, the earth pressures obtained by using these theories are similar to the concept of ultimate bearing capacity in foundation engineering; they represent strengths at failure rather than stresses at working loads. Coulomb, in 1776, was the first person to work on earth pressures. Charles Augustin de Coulomb was a French physicist who worked on this topic just before the French Revolution in the late 1700s, although he is better known for his work

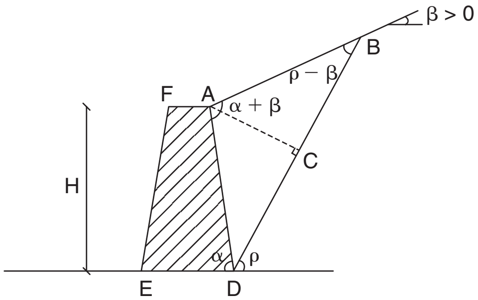


Figure 21.6 General geometry of the Coulomb soil wedge.

on electromagnetism. To develop his earth pressure theory, Coulomb made the following assumptions (Figure 21.6):

1. The problem is a plane strain problem
2. The soil has friction (φ') and cohesion (c')
3. The soil has no water
4. The failure wedge is a rigid body
5. The failure surface and the ground surface are planes
6. The friction coefficient between the wall and the soil wedge is $\tan\delta$

Let's first calculate the weight of the wedge W per unit length of wall. The area A of the triangle ABD is:

$$A = \frac{1}{2}BD \times AC = \frac{1}{2}AD\left(\frac{\sin(\alpha+\beta)}{\sin(\rho-\beta)}\right) \times AD\sin(180-\alpha-\rho) \quad (21.1)$$

Because

$$AD = \frac{H}{\sin\alpha} \quad (21.2)$$

then

$$A = \frac{H^2}{2\sin^2\alpha}\sin(\alpha+\rho)\left(\frac{\sin(\alpha+\beta)}{\sin(\rho-\beta)}\right) \quad (21.3)$$

and

$$W = \frac{\gamma H^2}{2\sin^2\alpha}\sin(\alpha+\rho)\left(\frac{\sin(\alpha+\beta)}{\sin(\rho-\beta)}\right) \quad (21.4)$$

In the case of the active earth pressure (Figure 21.7), the external forces acting on the wedge are the weight W, the active force P_a on the wall side AD, and the resultant force R on the soil side BD.

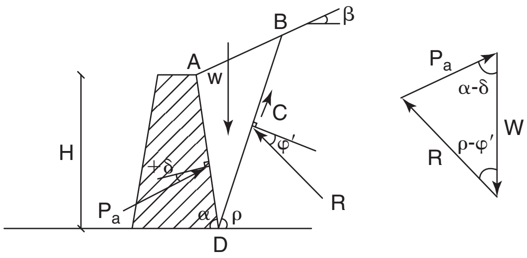


Figure 21.7 Free body of the active soil wedge.

The force P_a is inclined at an angle δ with the normal to the wall-soil interface. If the wall does not settle excessively, the wedge goes down with respect to the wall and the wall friction acts upward on the wedge (positive δ value for the active case). The resultant R is inclined at an angle φ' with the normal to the soil-soil failure plane at the back of the wedge. Because the wedge goes down with respect to the soil mass beyond the wedge, the friction force acts upward on the wedge. We will neglect the cohesion force at this time. Then the polygon of forces can be drawn (Figure 21.7) and the law of sines gives:

$$\frac{P_a}{\sin(\rho-\varphi')} = \frac{W}{\sin(180-\rho+\varphi'-\alpha+\delta)} \quad (21.5)$$

and

$$P_a = \frac{\gamma H^2}{2\sin^2\alpha}\sin(\alpha+\rho)\left(\frac{\sin(\alpha+\beta)}{\sin(\rho-\beta)}\right) \times \frac{\sin(\rho-\varphi')}{\sin(180-\rho+\varphi'-\alpha+\delta)} \quad (21.6)$$

Equation 21.6 shows that P_a is a function of a number of factors, including the angle ρ which is an unknown variable. The active earth pressure force will correspond to the value of ρ that leads to the lowest value of P_a, because that will be the first value reached as the wall is pulled away from the soil. Therefore, the ρ value corresponding to the active force is the one that minimizes P_a. For this we set:

$$\frac{\partial P_a}{\partial\rho} = 0 \quad (21.7)$$

and solve for ρ as was done in section 11.4.2. The final result for P_a is:

$$P_a = \frac{\gamma H^2}{2}\frac{\sin^2(\alpha+\varphi')}{\sin^2\alpha\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\varphi'+\delta)\sin(\varphi'-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2} = \frac{1}{2}K_a\gamma H^2 \quad (21.8)$$

and the coefficient of active earth pressure K_a giving the magnitude of the vector P_a is:

$$K_a = \frac{\sin^2(\alpha+\varphi')}{\sin^2\alpha\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\varphi'+\delta)\sin(\varphi'-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2} \quad (21.9)$$

Note that the direction of the force P_a is not horizontal, but rather acts at an angle 90—$\alpha + \delta$ with the horizontal. The horizontal component P_{ah} is:

$$P_{ah} = \frac{1}{2}K_a\gamma H^2 \cos(90 - \alpha + \delta) = \frac{1}{2}K_a\gamma H^2 \sin(\alpha - \delta)$$
$$= \frac{1}{2}K_{ah}\gamma H^2 \tag{21.10}$$

Therefore, the coefficient of active earth pressure K_{ah} giving the horizontal component P_{ah} of the active push P_a is:

$$K_{ah} = \frac{\sin^2(\alpha + \varphi')}{\sin^2\alpha\left[1 + \sqrt{\dfrac{\sin(\varphi' + \delta)\sin(\varphi' - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \tag{21.11}$$

In the simpler case where the backfill is horizontal, the wall is vertical, and there is no soil-wall friction (conservative), then $\beta = \delta = 0$, $\alpha = 90^\circ$, and K_a becomes:

$$K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'} \tag{21.12}$$

In the case of the passive earth pressure (Figure 21.8), the external forces acting on the wedge are the weight W, the passive force P_p on the wall side AD, and the resultant force R on the soil side BD.

The force P_p is inclined at an angle δ with the normal to the wall-soil interface. As the wall pushes against the wedge, the wedge goes up with respect to the wall and the wall friction acts downward on the wedge (positive δ value for the passive case). The resultant R is inclined at an angle φ' with the normal to the soil-soil failure plane at the back of the wedge. Because the wedge goes up with respect to the soil mass beyond the wedge, the friction force acts downward on the wedge. We will neglect the cohesion force at this time. Then the polygon of forces can be drawn (Figure 21.8) and

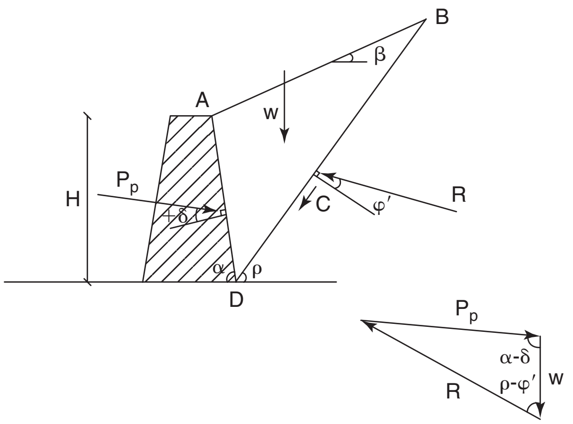


Figure 21.8 Free body of the passive soil wedge.

the derivation proceeds as for the active case. In the end, the equation for P_p is:

$$P_p = \frac{1}{2}K_p\gamma H^2 \tag{21.13}$$

The passive earth pressure coefficient giving the magnitude of the vector P_p is:

$$K_p = \frac{\sin^2(\alpha - \varphi')}{\sin^2\alpha\sin(\alpha + \delta)\left[1 - \sqrt{\dfrac{\sin(\varphi' + \delta)\sin(\varphi' + \beta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^2} \tag{21.14}$$

Note that the direction of the force P_p is not horizontal, but rather acts at an angle $\alpha + \delta - 90$ with the horizontal. The horizontal component P_{ph} is:

$$P_{ph} = \frac{1}{2}K_p\gamma H^2 \cos(\alpha + \delta - 90) = \frac{1}{2}K_p\gamma H^2 \sin(\alpha + \delta)$$
$$= \frac{1}{2}K_{ph}\gamma H^2 \tag{21.15}$$

Therefore, the coefficient of passive earth pressure K_{ph} giving the horizontal component P_{ph} of the passive push P_p is:

$$K_{ph} = \frac{\sin^2(\alpha - \varphi')}{\sin^2\alpha\left[1 - \sqrt{\dfrac{\sin(\varphi' + \delta)\sin(\varphi' + \beta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^2} \tag{21.16}$$

In the simpler case where the backfill is horizontal, the wall is vertical, and there is no soil-wall friction (conservative), then $\beta = \delta = 0$, $\alpha = 90^\circ$, and K_p becomes:

$$K_p = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} \tag{21.17}$$

and the product $K_a \times K_p$ is equal to 1.

21.3.2 Rankine Earth Pressure Theory

In 1857, Rankine took a different approach to the same problem. William J. Rankine was a Scottish civil engineer, physicist, and mathematician. He made the following assumptions:

1. The problem is a plane strain problem
2. The soil has friction (φ') but no cohesion ($c' = 0$)
3. The soil has no water
4. The soil mass is in a state of plastic failure
5. The failure surface and the ground surface are planes
6. There is no friction between the soil and the wall

Coulomb considered the equilibrium of a rigid body wedge and reasoned in terms of equilibrium of forces, whereas Rankine considered the equilibrium of stresses at the element level in a failing mass. Rankine theory predates the work of Otto Mohr and the Mohr circle around 1882, but it is easiest to explain Rankine theory through the use of the Mohr circle, which will be done in section 21.3.3. The active and passive earth pressures are as follows (Figures 21.9 and 21.10):

$$\sigma_a = K_a \sigma_v = K_a \gamma z \tag{21.18}$$

$$\sigma_p = K_p \sigma_v = K_p \gamma z \tag{21.19}$$

where σ_a and σ_p are the active and passive earth stresses on the wall, K_a and K_p are the active and passive coefficients, γ is the soil unit weight, and z is the depth below the ground surface. Note that the stress vectors σ_a and σ_p are parallel to the ground surface and therefore inclined at an angle β with the horizontal (Figures 21.9 and 21.10). Rankine obtained the following expressions for K_a and K_p:

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi'}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi'}} \tag{21.20}$$

$$K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi'}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi'}} \tag{21.21}$$

As can be seen from Eqs. 21.18 and 21.19, the stresses on the wall increase linearly with z. By integration of these two equations between 0 and H, the height of the wall, the active force P_a and the passive force P_p can be obtained and are given by Eqs. 21.8 and 21.13, but with different expressions

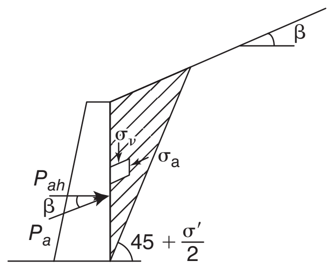


Figure 21.9 Active pressure mass (Rankine).

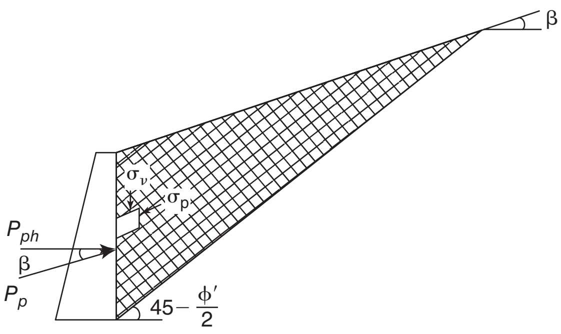


Figure 21.10 Passive pressure mass (Rankine).

for K_a and K_p given in Eqs. 21.20 and 21.21. Note also that the forces P_a and P_p are not horizontal, but rather parallel to the ground surface, which is at an angle β with the horizontal (Figures 21.9 and 21.10). The horizontal components P_{ah} and P_{ph} are:

$$P_{ah} = \frac{1}{2} K_a \gamma H^2 \cos\beta = \frac{1}{2} K_{ah} \gamma H^2 \tag{21.22}$$

$$P_{ph} = \frac{1}{2} K_p \gamma H^2 \cos\beta = \frac{1}{2} K_{ph} \gamma H^2 \tag{21.23}$$

Therefore, the coefficient of active earth pressure K_{ah} giving the horizontal component P_{ah} of the active force P_a and the coefficient of passive earth pressure K_{ph} giving the horizontal component P_{ph} of the passive force P_p are:

$$K_{ah} = \cos^2\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi'}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi'}} \tag{21.24}$$

$$K_{ph} = \cos^2\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi'}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi'}} \tag{21.25}$$

In the simple case where the backfill is horizontal, then $\beta = 0$, and K_a and K_p become:

$$K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'} \tag{21.26}$$

$$K_p = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} \tag{21.27}$$

So, should we use Coulomb or Rankine earth pressure coefficients? The Coulomb solution is a limit equilibrium solution giving upper-bound values because the chosen failure surface and mechanism is not necessarily the weakest one. In this context, Coulomb passive earth pressure coefficients tend to be very optimistic (too large). In contrast, the Rankine solution is an equilibrium of stresses solution that gives lower-bound values. Therefore, if a lower bound is conservative, one could choose Rankine; if an upper bound is conservative, one could choose Coulomb. Note that for extreme values of the geometry parameters, it is advisable to use engineering judgment, as the K_a and K_p values can become unreasonable. Note also that for the simple case of a vertical wall, no wall friction, and horizontal backfill, both theories give the same answers (Eqs. 21.12, 21.17, 21.26, and 21.27). The most common values vary from 0.25 to 0.40 for K_a and from 2.5 to 4 for K_p.

21.3.3 Earth Pressure Theory by Mohr Circle

Consider an element of soil behind a retaining wall (Figure 21.11). This element is in an at-rest state of stress to start with. The vertical effective stress is σ'_{ov}, the horizontal effective stress is σ'_{ov}, and the corresponding Mohr circle is shown in Figure 21.11. If the wall is pulled very slightly away from the soil, the horizontal effective stress will decrease until the

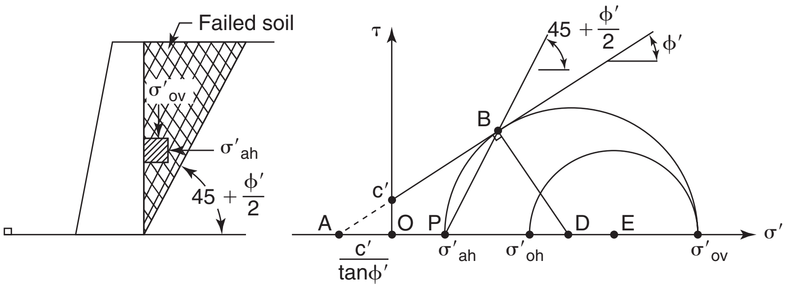


Figure 21.11 Element of soil and Mohr circle (active case).

Mohr circle touches the failure envelope. At that point the soil element will be in a state of failure: It will have mobilized all the shear strength it can offer to support itself, but will still need σ'_{ah} from the wall to avoid collapse. This value σ'_{ah} is the active earth pressure.

From triangle ABD in Figure 21.11, we can write that:

$$\sin\varphi' = \frac{BD}{AO+OD} = \frac{0.5(\sigma'_{ov} - \sigma'_{ah})}{\frac{c'}{\tan\varphi'} + 0.5(\sigma'_{ov} + \sigma'_{ah})} \tag{21.28}$$

which reduces to:

$$\sigma'_{ah} = \sigma'_{ov}\left(\frac{1-\sin\varphi'}{1+\sin\varphi'}\right) - 2c'\sqrt{\frac{1-\sin\varphi'}{1+\sin\varphi'}} \tag{21.29}$$

or:

$$\sigma'_{ah} = \sigma'_{ov}K_a - 2c'\sqrt{K_a} \quad \text{with} \quad K_a = \frac{1-\sin\varphi'}{1+\sin\varphi'} \tag{21.30}$$

The direction of the failure lines can be found by using the Pole method (see section 10.5). The stress point on the Mohr circle at σ'_{ah} corresponds to a stress acting on a vertical plane, so a vertical line will intersect the circle at two points: the stress point and the Pole. Because the vertical line is tangent to the circle, the two points are the same and the Pole is at point P on Figure 21.11. A line from the Pole to the failure point B gives the direction of the failure plane on the diagram. From geometry considerations, the angle of this plane with the horizontal is equal to $45 + \varphi'/2$. Because the entire mass is at failure, a set of parallel failure lines exists.

Now if the wall is pushed into the soil instead of pulled away (Figure 21.12), the horizontal effective stress will increase, pass the value of the vertical effective stress σ'_{ov}, and continue to increase until the Mohr circle touches the failure envelope. At that point the soil element will be in a state of failure: It will have mobilized all the shear strength it can offer to resist the wall push and σ'_{ph} will be generated. This value σ'_{ph} is the passive earth pressure. From triangle ABD in Figure 21.12, we can write that:

$$\sin\varphi' = \frac{BD}{AO+OD} = \frac{0.5(\sigma'_{ph} - \sigma'_{ov})}{\frac{c'}{\tan\varphi'} + 0.5(\sigma'_{ph} + \sigma'_{ov})} \tag{21.31}$$

which reduces to:

$$\sigma'_{ph} = \sigma'_{ov}\left(\frac{1+\sin\varphi'}{1-\sin\varphi'}\right) + 2c'\sqrt{\frac{1+\sin\varphi'}{1-\sin\varphi'}} \tag{21.32}$$

or

$$\sigma'_{ph} = \sigma'_{ov}K_p + 2c'\sqrt{K_p} \quad \text{with} \quad K_p = \frac{1+\sin\varphi'}{1-\sin\varphi'} \tag{21.33}$$

The direction of the failure lines can be found by using the Pole method (see section 10.5). The stress point on the Mohr circle at σ'_{ph} corresponds to a stress acting on a vertical plane, so a vertical line will intersect the circle at two points: the stress point and the Pole. Because the vertical line is tangent to the circle, the two points are the same and the Pole is at point P on Figure 21.12. A line from the Pole to the failure point B gives the direction of the failure plane on the diagram. From geometry considerations, the angle of this plane with the horizontal is equal to $45 - \varphi'/2$. Because the entire mass is at failure, a set of parallel failure lines exists. The conjugate failure lines on Figure 21.12 come from the failure point on the bottom part of the Mohr circle at failure that is not shown on the figure.

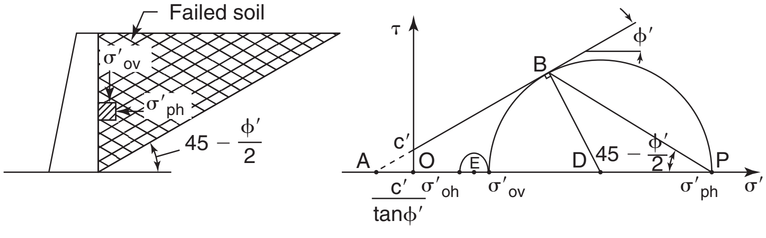


Figure 21.12 Element of soil and Mohr circle (passive case).

21.3.4 Water in the Case of Compression Stress (Saturated)

Up to this point we have calculated the effective horizontal stress for the active case and the passive case. The wall is subjected to the total horizontal stress. When the soil next to the wall is saturated and the water is in compression, the total active and passive earth pressures become:

$$\sigma_{ah} = \sigma'_{ov} K_a - 2c'\sqrt{K_a} + u_w \tag{21.34}$$

$$\sigma_{ph} = \sigma'_{ov} K_p + 2c'\sqrt{K_p} + u_w \tag{21.35}$$

The water stress u_w is obtained as follows:

$$u_w = \gamma_w h_p \tag{21.36}$$

where γ_w is the unit weight of water, h_p is the distance from the groundwater level to the point considered if there is no flow, and h_p is the pressure head obtained from a flow net if there is flow.

Note that there is a big difference between the pressure against a wall that has to retain a soil without water and the pressure against a wall that has to retain a soil with a water level at the ground surface. For example, if a wall is 3 m high and retains a dry sand with a unit weight of 18 kN/m^3 and a friction angle of 30°, the active earth pressure behind the bottom of the wall will be:

$$\sigma_{ah} = 3 \times 18 \times 0.33 = 18\ \text{kN/m}^2 \tag{21.37}$$

However, if the water rises to the top of the wall, increasing the unit weight of the soil to 20 kN/m^3, and if the water stress is hydrostatic, the active pressure behind the bottom of the wall becomes:

$$\sigma_{ah} = (3 \times 20 - 3 \times 10) \times 0.33 + 3 \times 10 = 40\ \text{kN/m}^2 \tag{21.38}$$

As can be seen, the pressure doubles due to the presence of the water. If we had assumed that no water could be present and designed the wall for a factor of safety of 2, the wall would have been close to failure when the water accumulated behind it. It is extremely important to pay great attention to water when designing retaining walls.

21.3.5 Water in the Case of Tension Stress (Unsaturated or Saturated)

If the soil behind the wall is above the groundwater level, the water is in tension and the soil is either saturated or unsaturated. In both cases, the water stress u_w is negative. This increases the shear strength of the soil because it increases the effective stress. Thus, one would expect the active earth pressure to decrease and the passive earth pressure to increase. Equations 21.34 and 21.35 become:

$$\sigma_{ah} = \sigma'_{ov} K_a - 2c'\sqrt{K_a} + \alpha u_w \tag{21.39}$$

$$\sigma_{ph} = \sigma'_{ov} K_p + 2c'\sqrt{K_p} + \alpha u_w \tag{21.40}$$

where α is the water area ratio, which can be estimated as the degree of saturation S or by using the Khalili rule (see section 10.13). Note that the term αu_w is also embedded in σ'_{ov}. Regrouping gives:

$$\sigma_{ah} = \sigma_{ov} K_a - 2c'\sqrt{K_a} + (1 - K_a)\alpha u_w \tag{21.41}$$

$$\sigma_{ph} = \sigma_{ov} K_p + 2c'\sqrt{K_p} + (1 - K_p)\alpha u_w \tag{21.42}$$

Equations 21.41 and 21.42 show that water tension decreases the active earth pressure and increases the passive earth pressure. However, it is very important to consider if the water tension used in these equations will always be present or if it is a seasonal occurrence. Furthermore, it would be uncommon for the water tension to pull on the wall. In the case of unsaturated soils and for earth pressure calculations, it is therefore prudent to consider that the water stress is equal to zero.

21.3.6 Influence of Surface Loading (Line Load, Pressure)

Load is often applied at the top of a retaining wall (Figure 21.13) either during construction (e.g., compaction rollers) or after construction (e.g., bridge abutment, additional fill). In the case of a pressure δp that covers the entire surface area at the top of the retaining wall, the active and passive earth pressures have an added term $K_a \Delta p$ and $K_p \Delta p$ respectively. The reason is that the pressure Δp simply adds to the total stress σ_{ov}.

In the case of a line load Q (kN/m) parallel to the wall crest and located at a perpendicular distance x from the wall, the increase in horizontal stress against the wall at a depth z below the top of the wall can be calculated by:

$$\Delta\sigma_h = \frac{4Q}{\pi}\frac{x^2 z}{(z^2 + x^2)^2} \tag{21.43}$$

If the load is a point load P (kN) applied at a perpendicular distance x from the wall, the maximum increase in horizontal pressure against the wall at a depth z below the top of the wall can be calculated by:

$$\Delta\sigma_h = \frac{P}{\pi(z^2 + x^2)}\left(\frac{3x^2 z}{\left(z^2 + x^2\right)^{3/2}} - \frac{(z^2 + x^2)^{1/2}(1 - 2\nu)}{(z^2 + x^2)^{1/2} + z}\right) \tag{21.44}$$

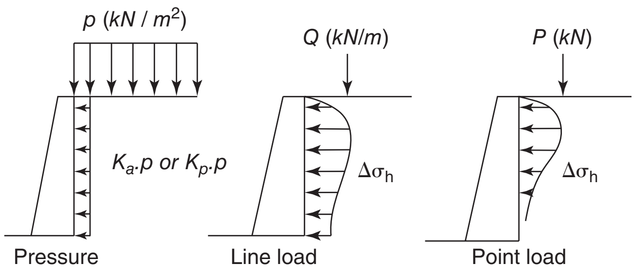


Figure 21.13 Horizontal pressures due to surface loading.

The values obtained from Eqs. 21.43 and 21.44 are added to both the active earth pressure and the passive earth pressure. Solutions for other surface loading can be found in the Canadian Foundation Engineering Manual (2007) and the AASHTO Bridge Specifications (2007).

21.3.7 General Case and Earth Pressure Profiles

In the general case, the total active and passive earth pressures are given by:

$$\sigma_{ah} = \sigma'_{ov} K_a - 2c'\sqrt{K_a} + \Delta\sigma_h + \alpha u_w \tag{21.45}$$

$$\sigma_{ph} = \sigma'_{ov} K_p + 2c'\sqrt{K_p} + \Delta\sigma_h + \alpha u_w \tag{21.46}$$

where σ_{ah} is the total active earth pressure on the wall at a depth z below the top of the wall, σ'_{ov} is the vertical effective stress at depth z, K_a is the coefficient of active earth pressure, $\Delta\sigma_h$ is the earth pressure due to surface loading, c' is the effective stress cohesion of the retained soil, α is the water area ratio, u_w is the water stress (tension or compression), σ_{ph} is the total passive earth pressure on the wall at a depth z below the top of the wall, and K_p is the coefficient of passive earth pressure.

These are the equations to use when calculating the active or passive earth pressure against the wall at a chosen depth z where σ'_{ov} and $\Delta\sigma_h$ exist. Keep in mind that these pressures or stresses may not be horizontal if the ground surface is not horizontal, the back of the wall is not vertical, or the wall friction is not assumed to be zero. A distinction is made in this respect between K_a and K_{ah} on the one hand and K_p and K_{ph} on the other (sections 21.3.1 and 21.3.2).

The next problem is to generate the complete profile of pressure against the wall versus depth. This is done by preparing a series of profiles using the following steps:

1. Profile of total vertical stress σ_{ov} versus depth
2. Profile of water stress u_w versus depth
3. Profile of water area ratio α versus depth
4. Profile of effective vertical stress ($\sigma'_{ov} = \sigma_{ov} - \alpha\ u_w$) versus depth
5. Profile of effective horizontal stress ($\sigma'_{ah} = \sigma'_{ov} K_a - 2c' K_a{}^{0.5}$) versus depth
6. Profile of horizontal stress due to surface loads ($\Delta\sigma_{ah}$) versus depth
7. Profile of total horizontal stress ($\sigma_{ah} = \sigma'_{ov} K_a - 2c' K_a{}^{0.5} + \Delta\sigma_{ah} + \alpha\ u_w$) versus depth

Figure 21.14 shows an example of the series of profile steps. The same sequence is followed for the passive earth pressure profiles.

If the soil is layered, the earth pressure has to be calculated twice at the depth of the layer boundary: once with the upper-layer soil parameters and once with the lower-layer soil parameters. As a result, there is typically a discontinuity in the earth pressure profile at the boundary between two soil layers (Figure 21.15).

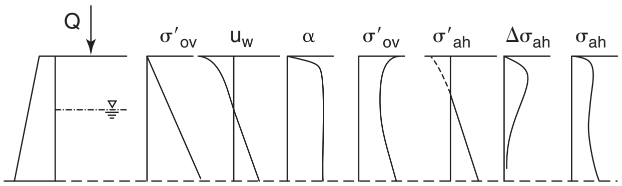


Figure 21.14 Series of profiles to generate earth pressure profile versus depth.

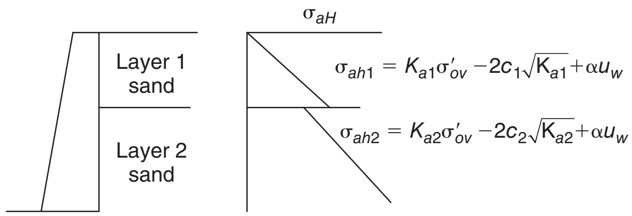


Figure 21.15 Active pressures at a soil layer boundary.

21.4 SPECIAL CASE: UNDRAINED BEHAVIOR OF FINE-GRAINED SOILS

As discussed in section 15.16, the equations for the undrained behavior of a fine-grained soil can be obtained from the effective stress equations by a simple transformation or correspondence principle:

1. Effective unit weight becomes total unit weight

$$\gamma_{eff} \rightarrow \gamma_t \tag{21.47}$$

2. Effective stress becomes total stress

$$\sigma' \rightarrow \sigma \tag{21.48}$$

3. Effective stress cohesion becomes undrained shear strength

$$c' \rightarrow s_u \tag{21.49}$$

4. Effective stress friction angle becomes zero

$$\varphi' \rightarrow 0 \tag{21.50}$$

Using this transformation on Eqs. 21.45 and 21.46, the following equations are obtained for the undrained behavior active and passive earth pressures:

$$\sigma_{ah} = \sigma_{ov} - 2s_u + \Delta\sigma_h \tag{21.51}$$

$$\sigma_{ph} = \sigma_{ov} + 2s_u + \Delta\sigma_h \tag{21.52}$$

where s_u is the undrained shear strength of the soil. These equations tend to give active earth pressures that are too low and passive pressures that are too high. One reason is that they assume that the soil is uniform with no fissures.

These equations should be used with great caution and proper judgment. For example, imagine that you have to design a wall for a clay that has an undrained shear strength of 100 kPa and a unit weight of 20 kN/m^3. Equation 21.51 says that no wall is needed until a depth of 10 m, as the active earth pressure is negative down to that depth. Now imagine that this clay has many fissures that are about 0.3 meters apart. The sample you tested was taken from one of the blocks between fissures and gave 100 kPa for s_u, but the soil mass is actually much weaker because of the fissures; the sample strength is not representative of the mass strength. If you dug a trench in such a material, it would be very surprising if you could dig down to 10 meters without a major collapse before that point. In contrast, if the material is truly uniform with no fissures (very rare), the theory says that you could dig to 10 m without support.

21.5 AT-REST EARTH PRESSURE

The *at-rest earth pressure* is the horizontal stress that exists in the soil under geostatic stresses and without displacement. The coefficient of at-rest earth pressure K_o is defined as:

$$K_o = \frac{\sigma'_{oh}}{\sigma'_{ov}} \tag{21.53}$$

where σ'_{ov} and σ'_{ov} are the horizontal and vertical effective stresses respectively. Note that K_o is the ratio of the effective stresses, not the total stresses; also, K_o does not involve the cohesion c', whereas the ratios K_a and K_p incorporate c' in their definition:

$$K_a = \frac{\sigma'_{ah}}{\sigma'_{ov}} + \frac{2c'\sqrt{K_a}}{\sigma'_{ov}} \tag{21.54}$$

$$K_p = \frac{\sigma'_{ph}}{\sigma'_{ov}} - \frac{2c'\sqrt{K_p}}{\sigma'_{ov}} \tag{21.55}$$

Thus, it is theoretically possible for K_o to have values higher than K_p and lower than K_a. For example, if $\sigma'_{ph} = 300$ kPa, $\sigma'_{ov} = 100$ kPa, and c' = 20 kPa, and if a high horizontal stress at rest is locked up tectonically at the value of σ'_{ph}, then K_p is 2.4 and K_o is 3.

In elasticity, the ratio of the horizontal stress to the vertical stress for a condition with no lateral movement (at-rest condition) is obtained in cylindrical coordinates from:

$$\varepsilon_h = \frac{1}{E}(\sigma'_{oh} - \nu(\sigma'_{ov} + \sigma'_{oh})) = 0 \tag{21.56}$$

where ε_h is the horizontal strain, E is a modulus of deformation of the soil, and ν is Poisson's ratio. Therefore

$$K_o = \frac{\nu}{1-\nu} \tag{21.57}$$

A commonly used value of Poisson's ratio for a drained case is 0.33; then K_o is equal to 0.5. However, measured K_o values have been reported in the range of 0.4 to more than 2. A value of 2 would require a Poisson's ratio equal to 0.67, which is possible for soils that dilate during compression, a well-known phenomenon. Such high K_o values are found in cases where high horizontal stresses have developed during geological events that densify or overconsolidate the soil. They may also be generated during compaction of shallow layers.

The coefficient of at-rest earth pressure is very difficult to measure, essentially because any instrument placed in the ground to measure K_o will create disturbance and change the at-rest state of stress. The best measurements are thought to be possible with a self-boring pressuremeter. However, even the self-boring pressuremeter creates significant disturbance due to shearing and side friction upon descent of the probe. Furthermore, the choice of zero volume of the probe can significantly affect the value of K_o obtained.

The early part of the preboring pressuremeter test offers another way to obtain an estimate of the horizontal stress. As the horizontal pressure applied by the pressuremeter probe on the borehole wall is increased, it goes through the threshold of pressure corresponding to the at-rest horizontal pressure. The curved line that describes the horizontal pressure versus increase in radius until the elastic portion of the curve is reached could be used. A construction much like the Casagrande construction for the preconsolidation pressure in the consolidation test would be needed, but calibration of such an idea has not been performed.

The step blade test consists of pushing a series of flat blades of increasing thickness into the soil while recording the horizontal stress on each blade. The idea was to extrapolate the horizontal stresses obtained on each blade back to a blade with zero thickness so as to find the at-rest horizontal stress. Although this idea was very clever, unfortunately the superposition of a penetration event and a lateral expansion event made the extrapolation unreliable.

One method consists of measuring the water tension developing in fine-grained soils upon extrusion of saturated samples. When the saturated sample comes out of the sampling tube, it decompresses and the total stress suddenly becomes zero—but the sample cannot readily expand because of the low hydraulic conductivity, and the water goes into tension to prevent any increase in volume. This results in a transfer from the mean effective stress to the water tension.

Sample at depth z:

$$\sigma_{mean} = \frac{1}{3}(\sigma'_{ov} + 2\sigma'_{oh}) + u_w \tag{21.58}$$

Sample extruded:

$$0 = \frac{1}{3}(\sigma'_{ov} + 2\sigma'_{oh}) + u_w \quad \text{or} \quad -u_w = \frac{1}{3}\sigma'_{ov}(1 + 2K_o) \tag{21.59}$$

Equation 21.59 shows that the water tension in the sample is a function of the horizontal effective stress. K_o can then be calculated knowing the vertical effective stress.

A K_o triaxial test can be used to obtain a value of K_o. This test consists of loading the sample vertically while increasing the horizontal stress (cell pressure) independently and in such a way that no lateral deformation will take place. During the test, the water stress is measured and the ratio between the horizontal effective stress (cell pressure minus water stress) and the vertical effective stress gives the K_o value. Alternatively, consolidometer tests with an instrumented ring can be used to obtain a value of K_o. The metal ring in which the sample is placed is instrumented with strain gages to measure the hoop strain in the metal, thereby giving the hoop stress that prevents lateral expansion. The radial stress is then obtained as:

$$\sigma_{oh} = \frac{\sigma_\theta t}{r} \tag{21.60}$$

where σ_{oh} is the radial or horizontal stress exerted by the soil on the metal ring that prevents expansion, σ_θ is the hoop stress in the metal obtained from the hoop strain measurements, t is the thickness of the metal ring, and r is the radius of the consolidometer. Knowing the vertical stress σ_{ov} imposed on the sample, and assuming that zero water stress is in the sample at the end of consolidation, gives data to calculate K_o. One of the difficulties with this approach is to ensure that the strain gages are sensitive enough to detect the strain in the metal ring under the relatively small radial stresses imposed by the soil.

Many correlations have also been proposed. The first one may be attributed to Jacky (1944), expressed as:

$$K_o = 1 - \sin\varphi' \tag{21.61}$$

This equation was later revised to include the effect of the overconsolidation ratio (OCR) for uncemented sands and clays of low to medium sensitivity:

$$K_o = (1 - \sin\varphi')OCR^{\sin\varphi'} \tag{21.62}$$

where φ' is the effective stress friction angle of the soil, and OCR is the overconsolidation ratio, defined as the ratio of the effective preconsolidation stress σ'_p over the current effective vertical stress. For clean quartz sand in chamber tests, Mayne (2007a, b) proposed:

$$K_o = 0.192\left(\frac{q_c}{\sigma_a}\right)^{0.22}\left(\frac{\sigma_a}{\sigma'_{ov}}\right)^{0.31}(OCR)^{0.27} \tag{21.63}$$

where q_c is the CPT point resistance, σ_a is the atmospheric pressure, σ'_{ov} is the vertical effective stress, and OCR is the overconsolidation ratio.

21.6 EARTH PRESSURE DUE TO COMPACTION

When soil is compacted behind bottom-up walls, the compaction process induces horizontal stresses that are higher than active earth pressures. This has been clearly documented

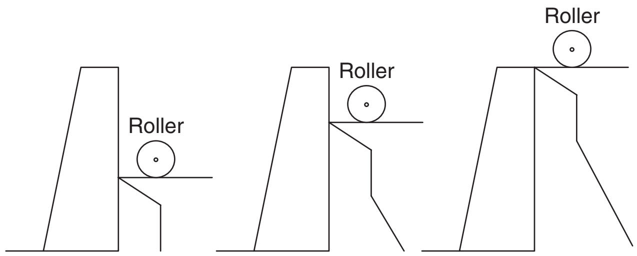


Figure 21.16 Compaction earth pressure during backfilling.

(Duncan and Seed 1986; Chen and Fang 2008). The compaction roller creates high vertical stresses, which in turn create high horizontal stresses during compaction. Because the soil does not return to an undeformed state after unloading (not elastic), and because the soil locks in plastic strains after unloading, high horizontal stresses remain after the roller moves on. This horizontal prestressing is actually very beneficial for improving the behavior of pavement base courses. For retaining walls, this means that designing for the active earth pressure case may not be prudent. At the same time, the depth of influence of the roller is limited and after several lifts of compaction have been completed the high stresses at depth (Figure 21.16) become smaller than the at-rest stresses at that depth.

The U.S. Navy (1982) made some recommendations for earth pressures due to compaction, which, considering more recent data, lead to the profile shown in Figure 21.17. The pressure diagram starts at a slope equal to the passive earth pressure coefficient. From the surface to a depth where the horizontal pressure reaches the value σ_h, the passive earth pressure profile, $K_p\gamma z$, is used. Then the pressure remains constant at a value of σ_h equal to:

$$\sigma_h = \frac{L}{a+L}\sqrt{\frac{2P\gamma}{\pi}} \tag{21.64}$$

where L is the length of the roller, a is the distance between the edge of the wall and the closest roller position, P is the

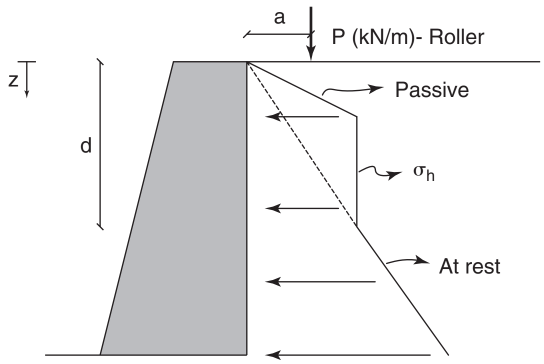


Figure 21.17 Wall pressure diagram including compaction stresses. (After U.S. Navy 1982.)

line load imposed by the roller (weight of the roller plus the centrifugal force for vibratory rollers divided by the length of the roller), and γ is the unit weight of the soil being compacted. At a depth d, the pressure diagram joins the at-rest earth pressure profile, $K_o \gamma z$, which is used beyond that point. That depth d is therefore equal to (Figure 21.17):

$$d = \frac{L}{K_o(a+L)}\sqrt{\frac{2P}{\pi\gamma}} \tag{21.65}$$

where K_o is the at-rest earth pressure coefficient.

21.7 EARTH PRESSURES IN SHRINK-SWELL SOILS

When the backfill of a bottom-up wall or the soil behind a top-down wall has a high plasticity index (Ip) or swell index (Is), it is necessary to consider the soil shrink-swell behavior in calculating the pressure diagram. Hong et al. (2010) studied this issue and made the following recommendation (Figure 21.18).

Three diagrams come into play in the resultant pressure diagram (Figure 21.18): the passive earth pressure diagram, the swell pressure diagram, and the at-rest earth pressure diagram. Both the passive and at-rest diagrams increase with depth according to K_p and K_o respectively. The swell pressure diagram, however, typically decreases with depth because the overburden pressure increases with depth and limits the swell pressure.

The pressure diagram starts at a slope equal to the passive earth pressure coefficient. Although the swell pressure is higher than the passive pressure within that zone, the soil fails in shear before it can reach the swell pressure. When the passive pressure profile reaches the swell pressure profile, the swell pressure limits the earth pressure against the wall and the pressure diagram follows the swell pressure profile. When the swell pressure profile reaches the at-rest pressure profile, the at-rest pressure is maintained against the wall because the swell pressure is smaller than that. As a result, the pressure diagram switches to the at-rest pressure profile. The coefficients K_p and K_o have been discussed in previous sections. The swell pressure profile can be obtained by performing swell tests on samples from the retained soil.

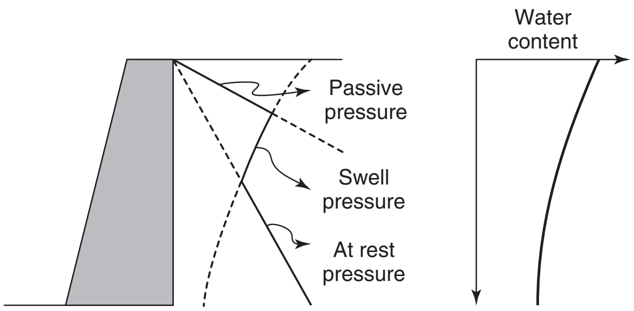


Figure 21.18 Wall pressure diagram including swelling pressure. (After Hong et al. 2010.)

21.8 DISPLACEMENTS

Figure 21.4 showed the general form of the earth pressure σ_h or p vs. displacement y curve. This curve, sometimes called a P-y curve, represents the plane strain behavior of the wall at a depth z. Figure 21.19 shows some values coming from measurement and numerical simulations (Briaud and Kim 1998). The vertical axis is a generalized earth pressure coefficient K, which is discussed further in section 21.12, and the horizontal axis is the horizontal displacement normalized by the height of the wall.

The amount of movement necessary to generate the active earth pressure σ_{ah} is y_a and the amount of movement necessary to generate the passive earth pressure σ_{ph} is y_p. Table 21.1 shows some possible values of y_a/H and y_p/H (H is the height of the wall) for different soil types. This means that if the wall is high, it will take more movement to mobilize the earth pressure than if the wall is low. The argument in favor

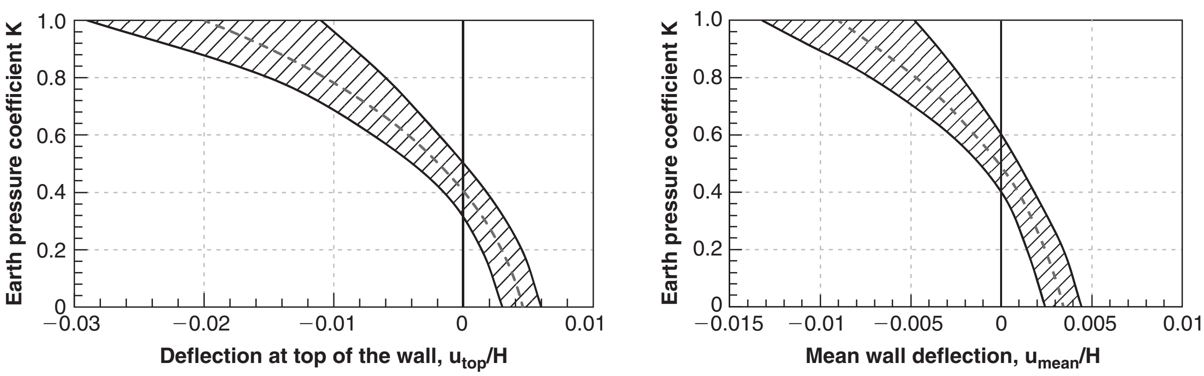


Figure 21.19 Measured earth pressure coefficient versus normalized displacement of a wall (Briaud and Kim 1998).

Table 21.1 Possible Range of Displacement to Generate Active and Passive Earth Pressures

Soil Type	Active, y_a/H	Passive, y_p/H
Loose sand	0.003 to 0.005	0.03 to 0.05
Dense sand	0.001 to 0.002	0.01 to 0.03
Soft clay	0.01 to 0.02	0.03 to 0.05
Stiff clay	0.005 to 0.01	0.01 to 0.03

of this concept is that if the wall is high, the earth pressure wedge will be large and it will take more movement to completely fail the wedge of soil behind a high wall compared to a low wall.

While it is clear that earth pressures depend on movement, and while it is also clear that predicting movements is important, our ability to make such predictions is not as good as our ability to calculate foundation settlement. Often the design of walls takes place solely on the basis of earth pressure distributions (ultimate limit state) rather than a combination of earth pressures and movements. Nevertheless, the trend in practice is toward increased inclusion of movement calculations in retaining wall design. Because the intact mass is the one deforming during such earth pressure problems, and because the overall strain level is quite small for well-designed systems, small strain moduli are most useful and can be obtained from cross hole sonic tests.

21.9 GRAVITY WALLS

Gravity walls are bottom-up walls usually made of reinforced concrete (Figure 21.20). In the early days they were heavy, massive blocks (concrete gravity wall), but such systems were soon replaced by walls that use less concrete weight and more backfill weight as dead weight to resist the soil push (cantilever gravity walls). In cantilever gravity walls, the slab under the retained portion of the backfill is subjected to the backfill weight, which increases the sliding resistance and the resistance to overturning. Cantilever gravity walls have to be heavily reinforced, as a high bending moment develops at the connection between the slab and the stem. The word *cantilever* is also used for a type of top-down wall; this is why the word *gravity* is added to *cantilever* to designate the wall shown in Figure 21.20.

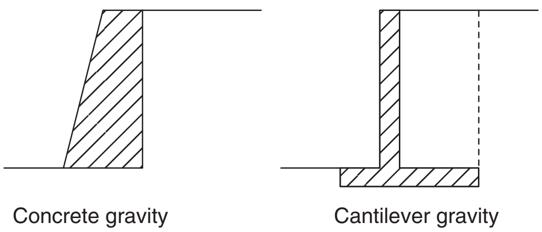


Figure 21.20 Types of gravity walls.

The geotechnical design of gravity walls consists of a number of steps aimed at ensuring the safety (low probability of failure) and functionality (low probability of intolerable movements) of the wall. The purpose of the design is to satisfy the ultimate limit state and the serviceability limit state of the wall as it is subjected to the earth pressures behind the wall and in front of the wall. For gravity walls, however, the serviceability limit state is rarely addressed, as movements are difficult to estimate and often small. The design steps include estimating the pressure distribution behind the wall (active pressure), the pressure distribution in front of the wall (passive pressure), the resultant active force and its location, the resultant passive force and its location, the sliding ultimate limit state, the overturning ultimate limit state, the bearing capacity ultimate limit state, the slope stability ultimate limit state, and the settlement serviceability limit state (rare). Figure 21.21 identifies the possible failure modes for a gravity wall.

1. Active pressure behind the wall σ_{ah}. For this, the steps in section 21.3.7 are followed and the profile of total active earth pressure is prepared. Special earth pressure conditions, such as compaction stresses, stresses due to shrink-swell soils, and stresses due to surface loading, are considered in arriving at the design active pressure diagram.
2. Passive pressure in front of the wall σ_{ph}. This refers to any embedded portion of the wall that could generate a passive resistance. Here again, the steps of section 21.3.7 are followed and the profile of passive earth pressure is prepared.
3. The resultant active push P_a (kN/m) is calculated as the area under the active earth pressure diagram (Figure 21.22):

$$P_a = \int_{z=0}^{z=H+D} \sigma_{ah}\, dz = \sum_{i=1}^{n} A_{ai} \tag{21.66}$$

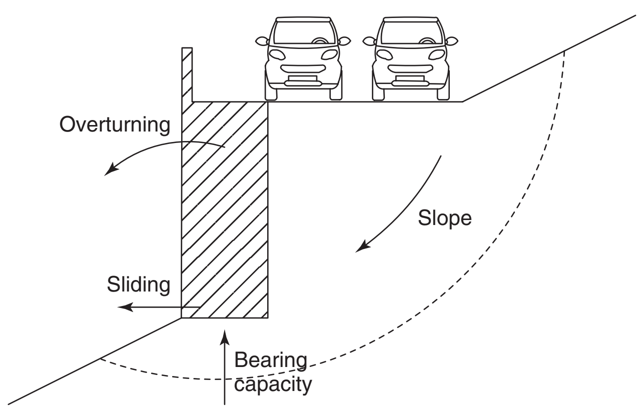


Figure 21.21 Failure modes.

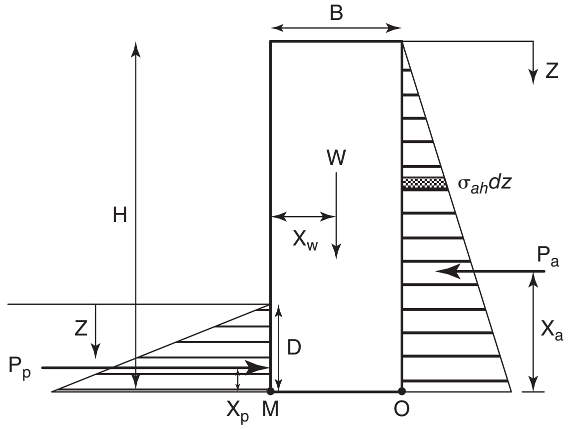


Figure 21.22 Forces acting on a gravity wall.

where σ_{ah} is the horizontal active earth pressure at depth z below the top of the wall, H is the height of the wall, D is the embedded depth, and A_a is the area under the active earth pressure diagram. If there is more than one soil layer, P_a is given by the sum of the areas A_{ai} (corresponding to layer i) under the active pressure diagram.

4. The resultant passive push P_p (kN/m) is calculated as the area under the passive earth pressure diagram (Figure 21.22):

$$P_p = \int_{z=0}^{z=D} \sigma_{ph}\, dz = \sum_{i=1}^{m} A_{pi} \tag{21.67}$$

where σ_{ph} is the horizontal passive earth pressure at depth z below the bottom of the wall, H is the height of the wall, D is the embedded depth, and A_p is the area under the passive earth pressure diagram. If there is more than one soil layer, P_p is given by the sum of the areas A_{pi} (corresponding to layer i) under the passive earth pressure diagram.

5. The point of application of P_a is found by writing that the moment around a chosen point (often the bottom of the wall, O in Figure 21.22) created by the active earth pressure diagram is the same as the moment created by the resultant P_a:

$$P_a x_a = \int_{z=0}^{z=H+D} \sigma_{ah}(H + D - z)dz = \sum_{i=1}^{n} A_{ai}a_{ai} \tag{21.68}$$

where x_a is the moment arm of P_a, and a_{ai} is the moment arm of the individual areas under the pressure diagram corresponding to A_{ai}. Of course, if the active earth pressure diagram is a simple triangle, then x_a is $0.33(H + D)$.

6. The point of application of P_p is found by writing that the moment around a chosen point (often the bottom of the wall, O in Figure 21.22) created by the passive earth pressure diagram is the same as the moment created by the resultant P_p:

$$P_p x_p = \int_{z=0}^{z=D} \sigma_{ph}(D - z)dz = \sum_{i=1}^{n} A_{pi}a_{pi} \tag{21.69}$$

where x_p is the moment arm of P_p, and a_{pi} is the moment arm of the individual areas under the pressure diagram corresponding to A_{pi}. Of course, if the passive earth pressure diagram is a simple triangle, then x_p is 0.33D.

7. Sliding ultimate limit state is checked by evaluating the following equation:

$$\gamma_1 P_{a1} + \gamma_2 P_{a2} \leq \varphi_1 W \tan\delta + \varphi_2 P_p \tag{21.70}$$

where γ_1 is the load factor associated with the active push P_{a1} due to soil weight, γ_2 is the load factor associated with the active push P_{a2} due to surcharge, φ_1 is the resistance factor for the resistance to sliding due to soil weight, φ_2 is the resistance factor for the resistance to sliding due to the passive earth pressure in front of the wall, and δ is the friction angle for the interface between the bottom of the wall and the soil on which it rests. The angle δ is usually taken as equal to the friction angle φ' of the soil for rough interfaces. The load factor γ_1 is typically taken as 1.5, and γ_2 as 1.75. The resistance factor for the sliding resistance due to the weight of the wall is in the range of 0.8 to 0.9, whereas the resistance factor for the sliding resistance due to the passive earth pressure is usually around 0.5.

8. Overturning ultimate limit state is checked by evaluating the following equation related to the moment around the front of the wall (point M in Figure 21.22):

$$\gamma_1 P_{a1} x_{a1} + \gamma_2 P_{a2} x_{a2} \leq \varphi_1 W x_w + \varphi_2 P_p x_p \tag{21.71}$$

where γ_{a1}, γ_{a2}, P_{a1}, and P_{a2} are as defined in step 7; x_{a1}, x_{a2}, and x_p are the moment arms of the forces P_{a1}, P_{a2} and P_p respectively; φ_1 and φ_2 are the same resistance factors as in step 7; W is the weight of the wall, and x_w is the corresponding moment arm. The values of the load and resistance factors for this ultimate limit state are the same as the values for step 7.

9. Bearing capacity ultimate limit state is checked as a shallow foundation subjected to the combination of W, P_a, and P_p (see section 17.4). This combination leads to the case of an inclined, eccentric load.
10. Slope ultimate limit state is checked in the same way as a slope with a wall loading the soil surface (see Chapter 19). The load and resistance factors were presented in section 19.2.
11. Serviceability limit state is usually not addressed in current practice.

The following comments may be made on the movement of gravity walls. For gravity walls founded on competent soil, the movement usually takes place by rotation around the bottom of the wall (point O in Figure 21.22). Most of the horizontal movement tends to occur during construction and corresponds to the order of magnitude given in Table 21.1 for the active case. Note that the main source of horizontal movement comes from rotation of the base under the overturning moment. Indeed, if the sliding ultimate limit state is satisfied, sliding movement should be very small. Vertical settlement of the wall will occur if the downdrag from the backfill and the high stresses under the front edge of the wall due to the applied moment compress the soil under the wall. Because this compression is uneven, with more settlement under the front edge, the wall will rotate with more horizontal movement at the top. To this extent, the settlement factors giving the settlement at the center and at the edge of the foundation (see section 17.7 on the load settlement curve approach) can be used to infer the rotation and movement of the wall. In that respect it is useful to study the case of a foundation subjected to a line load Q (kN/m) and an overturning moment M (kN.m/m) (Figure 21.23). The eccentricity e of the load Q is given by:

$$e = \frac{M}{Q} \tag{21.72}$$

The pressure diagram under the foundation is shown in Figure 21.23. The maximum pressure $p_{\max}$ and minimum pressure $p_{\min}$ under the foundation with a width B are given by:

$$p_{\max} = \frac{Q}{B}\left(1 + \frac{6e}{B}\right) \tag{21.73}$$

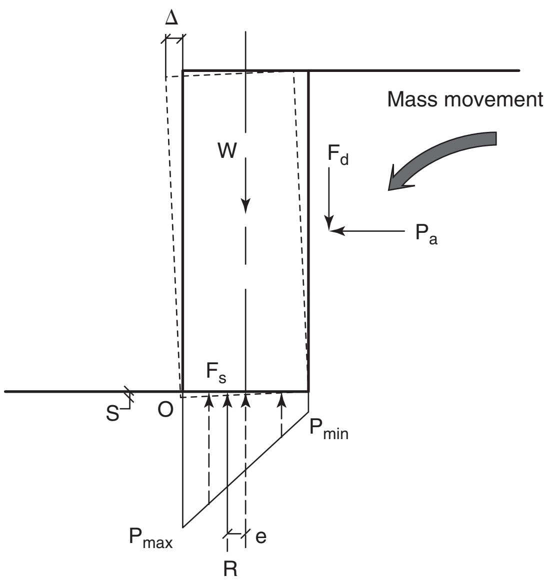


Figure 21.23 Pressure under a gravity wall.

$$p_{\min} = \frac{Q}{B}\left(1 - \frac{6e}{B}\right) \tag{21.74}$$

Equation 21.74 indicates that $p_{\min}$ becomes zero when the eccentricity becomes equal to B/6. If $p_{\min}$ becomes zero, the instability of the wall is more likely, as the foundation cannot develop tensile resistance to overcome further increase in eccentricity. As long as the point of application of the resultant stays within a distance of $B/6$ from the axis of symmetry, the wall is more likely to be stable and experience limited movement. This is called the *rule of the middle third* as e can be $\pm$ $B/6$. Note that the wall can also move in the other direction (more horizontal movement at the bottom of the wall) if a slope stability problem exists.

21.10 MECHANICALLY STABILIZED EARTH WALLS

Mechanically stabilized earth (MSE) walls are bottom-up walls made mostly of soil with some reinforcement. Henri Vidal, a French engineer, is credited with inventing reinforced earth in 1957. This technology is to geotechnical engineering what reinforced concrete is to structural engineering. It consists of placing inclusions in the soil to give it significant tensile strength. These walls were called reinforced earth walls in the beginning and are now called mechanically stabilized earth walls (MSE walls). An MSE wall is built by placing a layer of soil (say, 0.7 m thick), then a layer of reinforcement, then a layer of soil, then a layer of reinforcement, and so on until the desired wall height is reached (Figure 21.24). Panels are placed in front of and attached to the reinforcement for esthetic purposes and to retain any soil that might fall between reinforcement layers close to the front. The pressure on the panels is very small, as most of the earth pressure is taken up by the reinforcement. The reinforcement can be galvanized steel strips, steel grids, or geosynthetics. The success of MSE walls is due to their lower cost compared to cantilever gravity walls, particularly for very high walls (Figure 21.25). Indeed, MSE walls built to 50 meters in height have performed very well.

The design of MSE walls includes an external stability design and an internal stability design.

21.10.1 External Stability

For this case, the MSE wall is considered to be a gravity wall consisting of the front panels, the reinforcement, and the soil between the reinforcement. This reinforced soil mass (ABCD in Figure 21.24) is the gravity wall and has to satisfy the design criterion of a gravity wall outlined in section 21.9. These include the sliding ultimate limit state, the overturning ultimate limit state, the bearing capacity ultimate limit state, the slope stability ultimate limit state, and the settlement serviceability limit state (rare). The design steps are identical to the steps detailed in section 21.9.

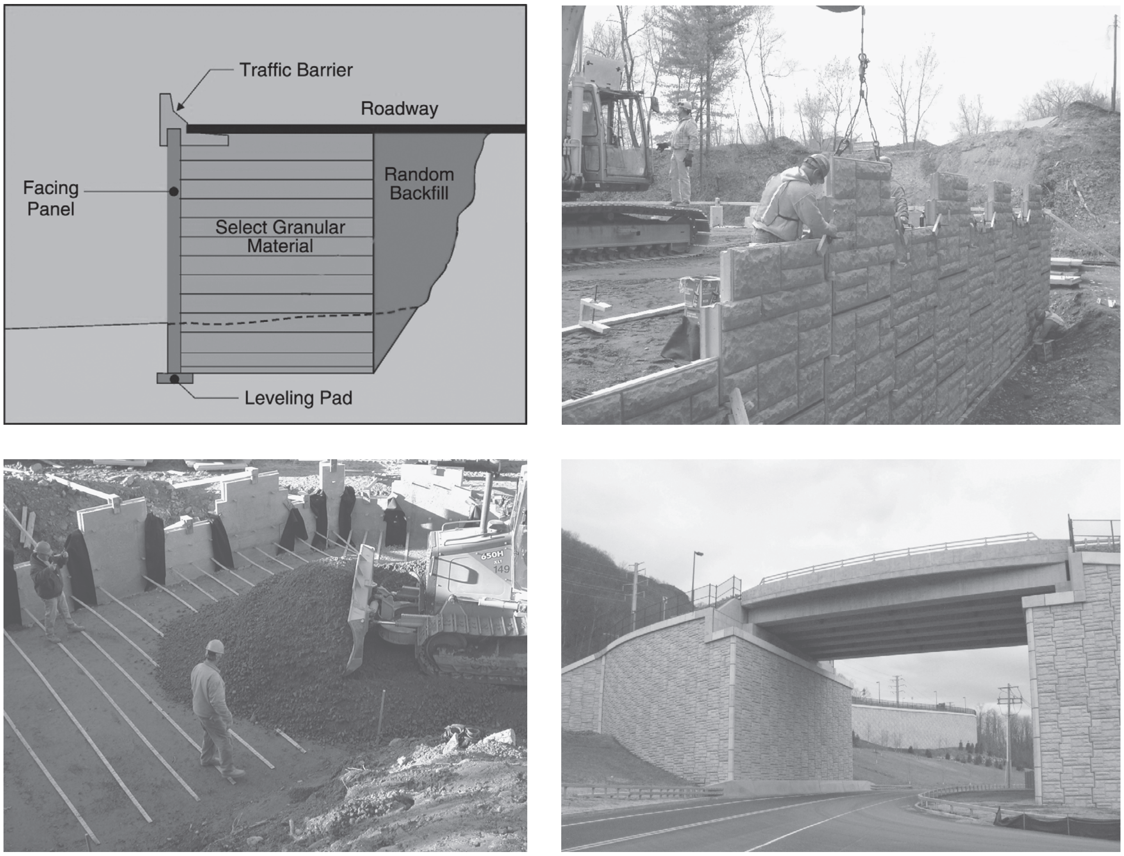


Figure 21.24 MSE wall. (Courtesy of The Reinforced Earth Company.)

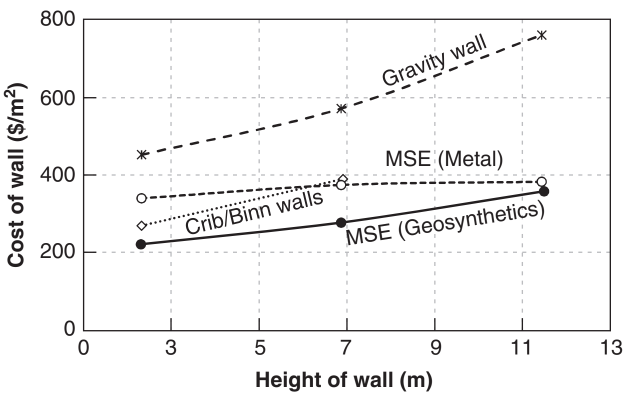


Figure 21.25 Cost of bottom-up walls (After Koerner and Soong 2001.)

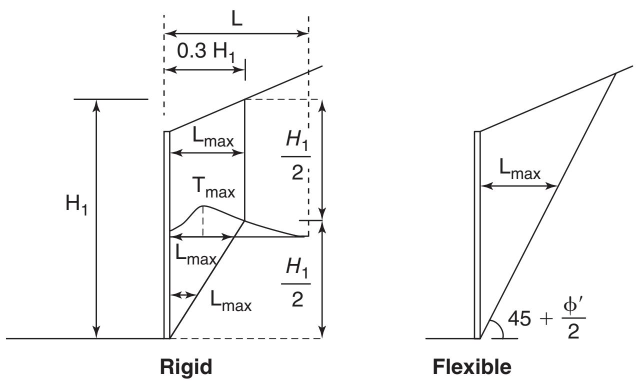


Figure 21.26 Load in the reinforcement.

21.10.2 Internal Stability

Pull-out capacity and yield of the reinforcement are the two aspects of internal stability of an MSE wall. Let's address pull-out capacity first.

Pull-Out Design

This design consideration ensures that the load in the reinforcement will not be high enough to pull the reinforcement out of the soil. An understanding of the load distribution in the reinforcement is necessary. Figure 21.26 shows the variation of the tension load T (kN) in the reinforcement as a function of the distance from the front of the wall.

At the wall facing, the load T in the reinforcement is very small, and then it increases as the instability of the wedge of soil near the wall is transferred to the tension T (kN) in the reinforcement. At a distance L_{max} from the front, the tension T reaches a maximum T_{max}. Beyond T_{max}, the tension decreases as the load is transferred to the stable soil mass and reaches zero at a certain distance from the front. The true embedment or anchoring length L_a available to resist the active pressure force against the wall is $L - L_{max}$ where L is the total length of the reinforcement. The design requires knowledge of L_{max}, which is to be ignored in the length required to resist T_{max}. L_{max} is given in Figure 21.26; as can be seen for rigid inclusions, it is equal to 0.3 H in the top half of the wall and decreases to zero at the bottom of the wall. For flexible inclusions (geosynthetics), it is taken as the width of the active wedge. These recommendations are partially based on measurement and simulation data.

The force T_{max} is calculated as follows:

$$T_{max} = s_v s_h \sigma_h \tag{21.75}$$

where T_{max} is the maximum line load (kN) to be resisted by the layer of reinforcement at depth z, s_v is the vertical spacing between reinforcement layers at depth z, s_h is the horizontal spacing between reinforcement inclusions at depth z, and σ_h is the total horizontal stress at depth z. The stress σ_h is calculated as:

$$\sigma_h = k_r \sigma_{ov} + \Delta\sigma_h \tag{21.76}$$

where k_r is a coefficient of earth pressure defined in Figure 21.27 as a function of K_a. The reason that k_r is higher than K_a for rigid inclusions is that during compaction of the backfill, the rigid inclusions (e.g., steel strips) can lock in higher horizontal stresses. Flexible inclusions (geosynthetics) do not lock in additional compaction stresses. As a result, k_r is equal to K_a for flexible inclusions.

Now that we have calculated the load T_{max}, we need to find the length of reinforcement that will safely carry this load without pulling out of the soil. The pull-out capacity $T_{pullout}$ (kN) of the reinforcement inclusion is given by:

$$T_{pull\ out} = 2 f_{max} b L_a \tag{21.77}$$

where f_{max} is the maximum shear stress that can be developed on both sides of the interface between the reinforcement and the soil, b is the width of the inclusion, and L_a is the anchoring length beyond L_{max}, the width of the active failure zone. The shear stress f_{max} is evaluated as follows:

$$f_{max} = F^* \sigma'_v \alpha \tag{21.78}$$

where F^* is the friction factor given in Figure 21.28; σ'_v is the vertical effective stress on the reinforcement; and α is a scale

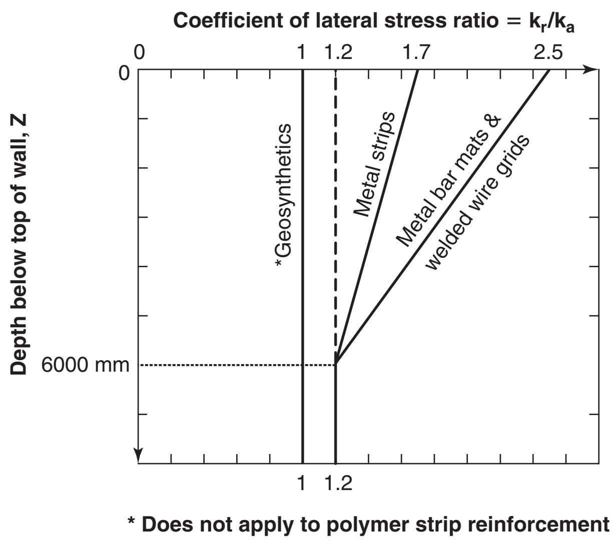


Figure 21.27 Earth pressure coefficient for load in the reinforcement.

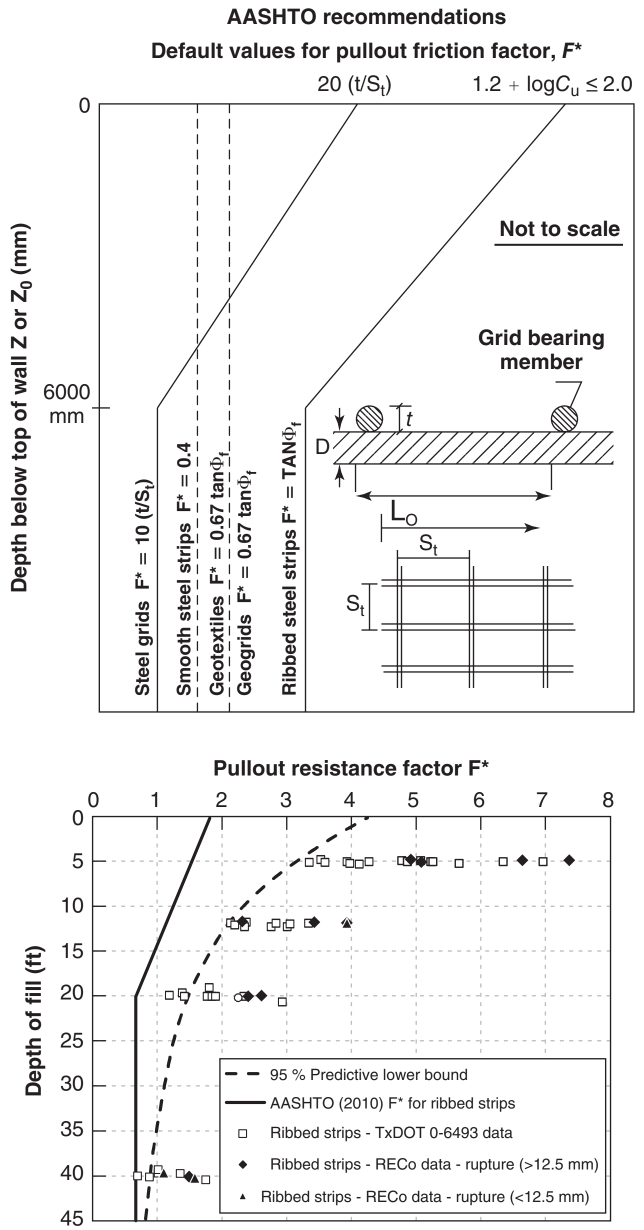


Figure 21.28 Friction coefficient F* for MSE wall reinforcement.

factor taken as 1 for steel reinforcement, 0.8 for geogrids, and 0.6 for geotextiles. Note that although the recommended F^* values can be as high as 2 at the ground surface, values of F^* much higher than 2 have been measured. The reason the coefficient of friction may be higher than 1 is that a combination of friction and bearing capacity is involved in the sliding-out of the reinforcement. The bearing capacity component comes from the protruding ribs for strips and from the transverse bars for grids.

Then the ultimate limit state for pull-out must be satisfied:

$$\gamma_1 T_{\max 1} + \gamma_2 T_{\max 2} = \phi T_{pull\ out} \tag{21.79}$$

where γ_1 is the load factor for the active earth pressure due to soil weight ($\gamma_1 = 1.35$), γ_2 is the load factor for the active earth pressure due to any surcharge on top of the wall ($\gamma_2 = 1.50$), φ is the resistance factor ($\varphi = 0.9$), $T_{\max1}$ is the part of the load in the reinforcement due to the soil weight, $T_{\max2}$ is the part of the load in the reinforcement due to any surcharge on top of the wall, and $T_{pullout}$ is the pull-out resistance calculated in Eq. 21.77. The required safe length L_a of the reinforcement is given by:

$$L_a = \frac{(\gamma_1 k_r \sigma'_{ov} + \gamma_2 \Delta\sigma_h) s_v s_h}{2\varphi F * \sigma'_{ov} \alpha b} \tag{21.80}$$

The total length L of the reinforcement is largest at the top of the wall, but because it is common practice to keep the reinforcement length L constant, L is given by:

$$L = L_a + L_{\max} = \frac{(\gamma_1 k_r \sigma'_{ov} + \gamma_2 \Delta\sigma_h) s_v s_h}{2\varphi F * \sigma'_{ov} \alpha b} + 0.3H \tag{21.81}$$

In the simple case, where $k_r = K_a = 0.33$, there is no surcharge, s_v and s_h are 0.75 m, F^* is 1, α is 1, and b is 0.05 m, then the length L_a is 2.8 m and independent of the wall height H. The reason is that the vertical stress contributes equally to the load and the resistance. The total length of reinforcement is $L = 2.8 + 0.3H$, where H is the height of the wall. For a 7 m high wall (common case of an overpass), L is approximately $0.7H$, which is a common recommendation. For higher walls, the 0.7 H rule is conservative, and for smaller walls a minimum of about 3 m reinforcement length is imposed.

Yield of the Reinforcement Design

We need to make sure that the reinforcement can safely carry the load $T_{\max}$ without yielding or rupturing. For this, we write the ultimate limit state as:

$$\gamma_1 T_{\max 1} + \gamma_2 T_{\max 2} = \phi T_{yield} \tag{21.82}$$

where γ_1 is the load factor for the active earth pressure due to soil weight ($\gamma_1 = 1.35$), γ_2 is the load factor for the active earth pressure due to any surcharge on top of the wall ($\gamma_2 = 1.50$), φ is the resistance factor ($\varphi = 0.75$ for strips, 0.65 for grids, and 0.9 for geosynthetics), $T_{\max1}$ is the part of the load in the reinforcement due to the soil weight, $T_{\max2}$ is the part of the load in the reinforcement due to any surcharge on top of the wall, and T_{yield} is the load corresponding to the yield strength of the reinforcement. T_{yield} for steel reinforcement is given by:

$$T_{yield} = \sigma_{yield} A \tag{21.83}$$

where σ_{yield} is the yield strength of the reinforcement and A is the cross-sectional area.

For geosynthetics, see section 27.6.2. For steel reinforcement, one issue is corrosion. This is addressed by using a thickness larger than required by the ultimate limit state for yield. Corrosion rates for nonaggressive soils are in the range of 0.005 to 0.015 mm/yr (AASHTO 2007). This means that a 1 mm excess thickness corresponds to a typical 75-year design life. Nonaggressive soils are recommended for backfill and are defined in Table 21.2.

Table 21.2 Characteristics of Nonaggressive Soils for Corrosion

pH	5 to 10
Resistivity	>3000 Ohm.cm
Chlorides	<100 ppm
Sulfates	<200 ppm
Organic content	<1%

(After AASHTO 2007.)

Movement

The movement of MSE walls is not typically calculated. If necessary, the settlement should be checked according to the procedures outlined in sections 17.7 and 17.8 and discussed in section 21.9, design step 11. The maximum horizontal movement $\Delta_{\max}$ of MSE walls during construction can be estimated for normal conditions and little or no surcharge as follows:

$$\text{For rigid inclusions} \quad \Delta_{\max} = 0.004 H \delta_r \tag{21.84}$$

$$\text{For flexible inclusions} \quad \Delta_{\max} = 0.013 H \delta_r \tag{21.85}$$

where H is the height of the wall and δ_r is given in Figure 21.29.

21.11 CANTILEVER TOP-DOWN WALLS

Cantilever walls are top-down walls, though they are sometimes confused with cantilever gravity walls. They are made, for example, of bored piles drilled side by side or sheet pile Z sections driven side by side (Figures 21.30 and 21.31). They can be used to retain soil up to a height of about 7 m; beyond that height anchored walls are more economical. The design

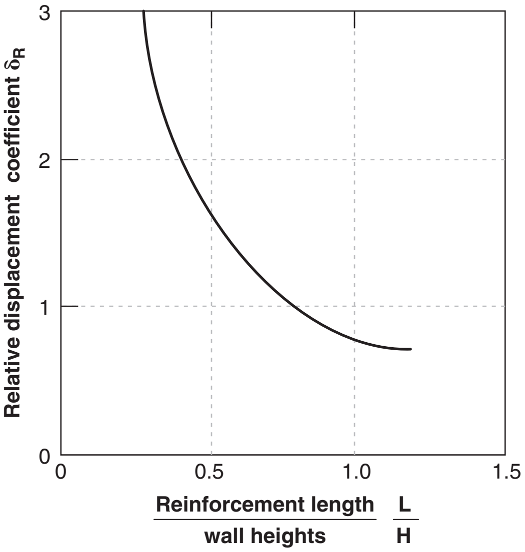


Figure 21.29 Movement parameter for estimating MSE wall horizontal displacement during construction (AASHTO 2010).

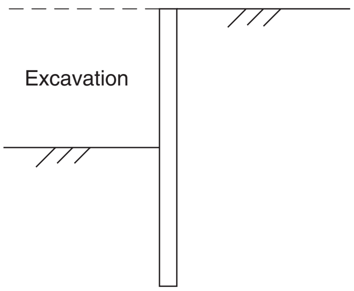


Figure 21.30 Cantilever top-down wall.

of such walls consists of satisfying the ultimate limit state (safety) and the serviceability limit state (limited movement). The parameters to be selected in the design are the depth of embedment D and the section of the wall to resist the maximum bending moment.

21.11.1 Depth of Embedment and Pressure Diagram

The pressure diagram is the first step. It is assumed that the wall will move into the excavation by an amount sufficient to generate the active earth pressure behind the wall, and that part of the passive earth pressure will be generated in front of the wall to resist the push (Figure 21.32). Of course, both the active push and the passive resistance depend on the depth of embedment D.

In the simplest case (no water, no surcharge, one uniform soil with no cohesion), the active push P_a (kN/m) is:

$$P_a = \frac{1}{2} K_a \gamma (H + D)^2 \tag{21.86}$$

where K_a is the active earth pressure coefficient, γ is the unit weight of the retained soil, H is the excavation height, and D is the depth of embedment. On the passive side, the passive pressure diagram is truncated at a depth where the passive pressure reaches half of the passive pressure at the embedment depth. This is done to acknowledge that the movement decreases with depth and may not be sufficient to generate the complete passive pressure at depth. This assumption brings into play the concept of both safety and serviceability, although it does not address that concept directly. As a result, the mobilized passive resistance P_{pm} is given by:

$$P_{pm} = \frac{3}{8} K_p \gamma D^2 \tag{21.87}$$

The point of application of P_a and P_{pm} are at a distance X_a and X_{pm} from the bottom of the wall respectively:

$$X_a = \frac{1}{3}(H + D) \tag{21.88}$$

$$X_{pm} = \frac{7}{18} D \tag{21.89}$$

Now we might be tempted to write horizontal equilibrium and we would find a depth D. The problem is that, even if we satisfied $P_a = P_{pm}$, the wall still could not be in moment equilibrium. For moment equilibrium to be satisfied, a force R is necessary at the bottom of the wall, and comes from the deflection pattern (Figure 21.32). By writing moment equilibrium around the bottom of the wall, we get the equation that leads to the value of D:

$$P_a X_a - P_{pm} X_{pm} = 0 \tag{21.90}$$

or

$$D = \frac{H}{\left(\frac{7}{8}\frac{K_p}{K_a}\right)^{0.33} - 1} \tag{21.91}$$

For a common ratio of K_p/K_a equal to 10, then:

$$D \approx H \tag{21.92}$$

This result shows that cantilever walls need an embedment at least equal to the excavation height. More detailed analysis shows that $D = 1.2\text{H}$ is more appropriate as a minimum for a uniform soil. Of course, more complex soil layering, surcharge, and water conditions will lead to a different result.

21.11.2 Displacement of the Wall, Bending Moment, and P-y Curves

The calculations shown in section 21.11.1 can give an estimate of the embedment depth D. Then the horizontal displacement of the wall and bending moment profile in the wall can be calculated by using a P-y curve analysis (see sections 18.6.8 and 11.4.4). The parameter P represents the load on the wall at depth z and the parameter y represents the horizontal deflection of the wall from the unloaded position. In the P-y curve analysis, a repeatable width of wall, usually one meter width,

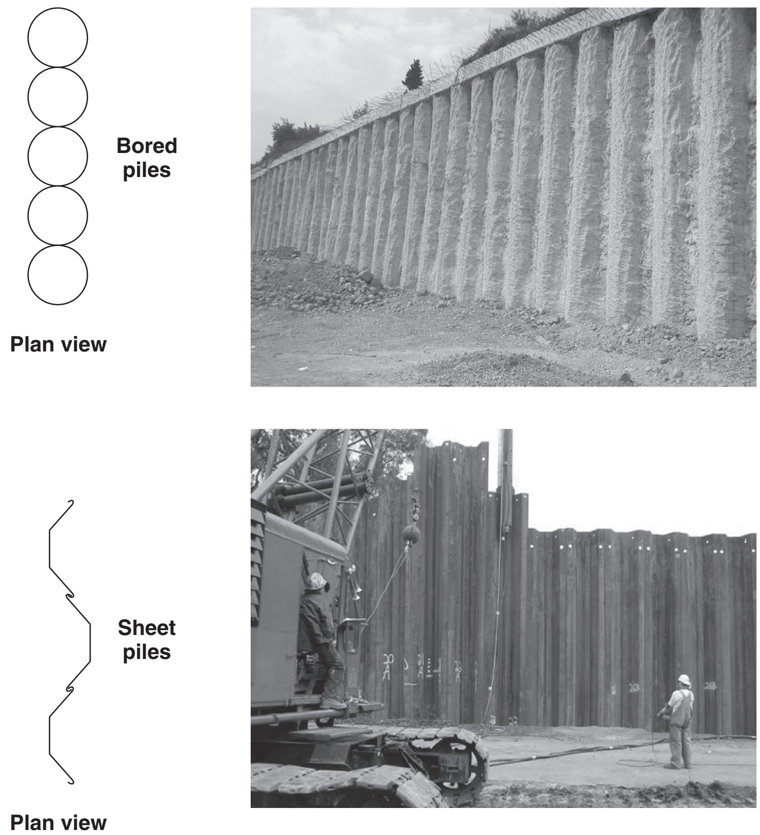


Figure 21.31 Bored piles and sheet pile cantilever walls. (d: Courtesy of Associated Pacific Constructors, Inc.)

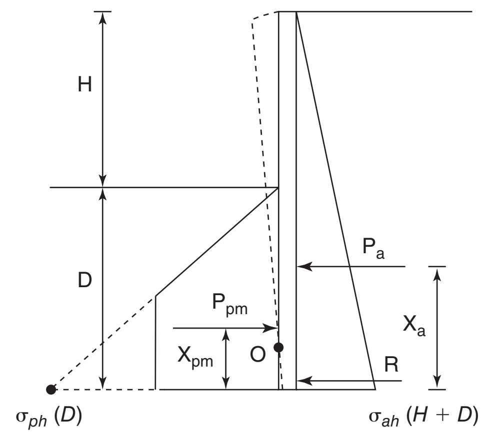


Figure 21.32 Simple approach for cantilever walls.

is simulated as a structural member and the soil mass is simulated by a series of nonlinear springs (*P*-*y* curves) tied to the wall and describing the response of the soil to the wall deflection. The first step in this analysis problem is to discretize the wall into elements (Figure 21.33); a minimum of 10 elements is recommended. The input to the problem includes.

1. Length of the wall (excavation height *H* plus depth of embedment *D*).
2. Length of the elements ($\Delta H < (H + D)/10$).
3. Bending stiffness EI of the wall (E modulus of elasticity, I moment of inertia) for the cross section corresponding to the repeatable wall section. This is usually a one meter width for continuous walls or the section tributary to one pile if a line of pile is involved.
4. P-y curves as a function of depth (one curve at each node).

The governing differential equation (GDE) and its finite difference method (FDM) solution are described in section

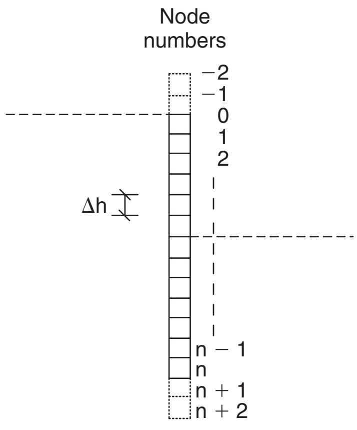


Figure 21.33 Cantilever wall discretized into elements.

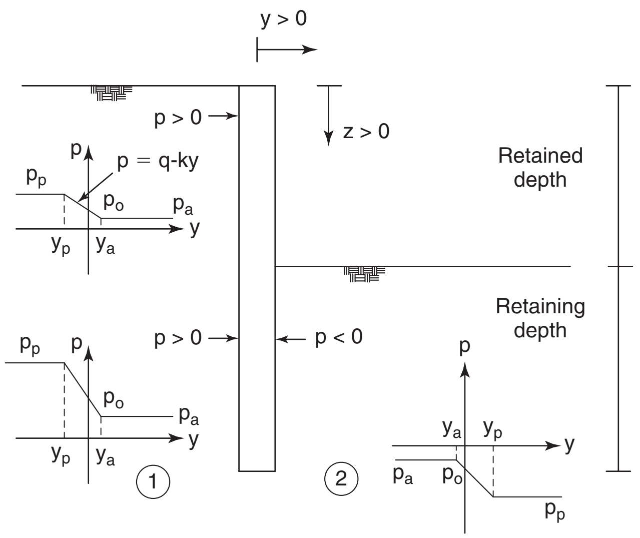


Figure 21.34 P-y curves for a cantilever top-down wall.

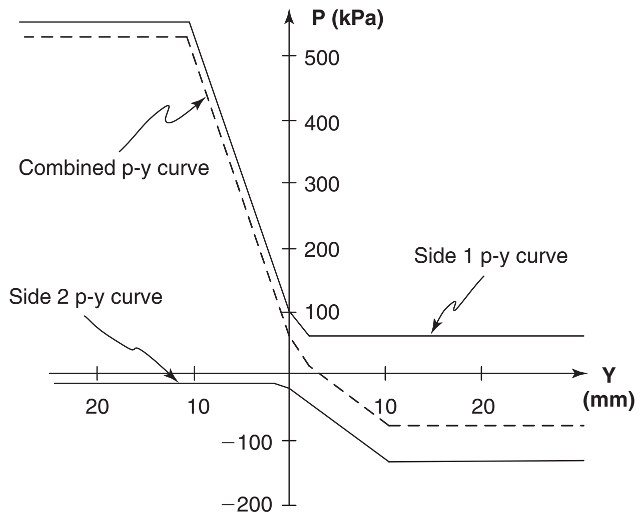


Figure 21.35 Combining P-y curves below the excavation level.

11.5.2 with a solved example. The *P-y* curves are constructed as follows (Figure 21.34). Above the excavation level in the retained soil zone, the soil is on only one side of the wall and the *P-y* curve is as shown in Figure 21.34. In that zone, if the wall moves toward the soil ($y < 0$), the P value increases from the P_o value corresponding to the at-rest earth pressure to the P_p value corresponding to the passive earth pressure when a displacement y_p is reached. In all cases the P values are given by:

$$P = \sigma_h \times b \times \Delta h \tag{21.93}$$

where σ_h is the horizontal stress (obtained as discussed in the previous sections in this chapter), Δh is the wall element length (vertical), and b is the width of the repeatable section (horizontal). The displacement y_p can be estimated by using Table 21.1. In the retained soil zone, the soil pushes in the chosen positive direction; therefore P is positive. In that zone also, if the wall moves away from the soil, the P value decreases from the P_o value corresponding to the at-rest earth pressure to the P_a value corresponding to the active earth pressure. The soil still pushes in the positive direction.

Below the excavation level, the soil is on both sides of the wall and there are two P-y curves: one for the retained soil side (Side 1 in Figure 21.34) and one for the retaining soil side (Side 2 in Figure 21.34). The *P-y* curve on Side 1 is similar to the one above the excavation except that the values of P are higher, because the depth is larger. The *P-y* curve on Side 2 is prepared as follows. If the wall moves toward Side 1 ($y < 0$), the magnitude of the P value decreases from the P_o value corresponding to the at-rest earth pressure to the P_a value corresponding to the active earth pressure. Because the soil pushes in a direction opposite to the chosen positive direction, P is negative. If the wall moves toward Side 2 ($y > 0$), the magnitude of the P value increases from the P_o value corresponding to the at-rest earth pressure to the P_p value corresponding to the passive earth pressure. Because the soil pushes in a direction opposite to the chosen positive direction, P is negative. The net *P-y* curve for the zone below the excavation level is constructed by combining the two curves for Side 1 and Side 2 (Figure 21.35).

Note that this *P-y* curve preparation is done for each node along the discretized wall. Then a finite difference program or spreadsheet is used and the solution gives the following parameters as a function of depth: wall deflection $y(z)$, slope of the wall $y'(z)$, bending moment in the wall $M(z)$, shear force in the wall $V(z)$, and pressure on the wall $p(z)$. Sample outputs are shown in Figures 21.36 and 21.37. The deflection profile predicted by this method tends to underpredict the deflections observed in practice. The reason is that the mass movement of the retained soil is not included in the *P-y* curve in a theoretically sound manner. However, the bending moment profile predicted by this method and the maximum bending moment for design are much more consistently reliable than any hand calculation based on an assumed pressure distribution, such as shown in Figure 21.32 (Briaud and Kim 1998). For improved prediction of deflections including mass movement, the finite element method should be used; nevertheless, a problem remains concerning the quality of the input parameters and the selection of the soil model.

21.12 ANCHORED WALLS AND STRUTTED WALLS

Anchored walls (or tieback walls) and strutted walls are top-down walls (Figure 21.38). The wall portion may be a solid concrete wall built by the slurry wall method, a sheet pile wall, a bored pile wall, a deep soil mixing wall, or a soldier pile and lagging wall, to name a few. Concrete slurry walls are built by excavating the soil one rectangular panel at a time with a clamshell rig and under slurry if necessary, lowering the reinforcing cage into the slurry-filled hole, and placing the concrete in liquid paste through a tremie (tube that goes to the bottom of the hole) from the bottom of the panel to the top while displacing the slurry out of the rectangular hole

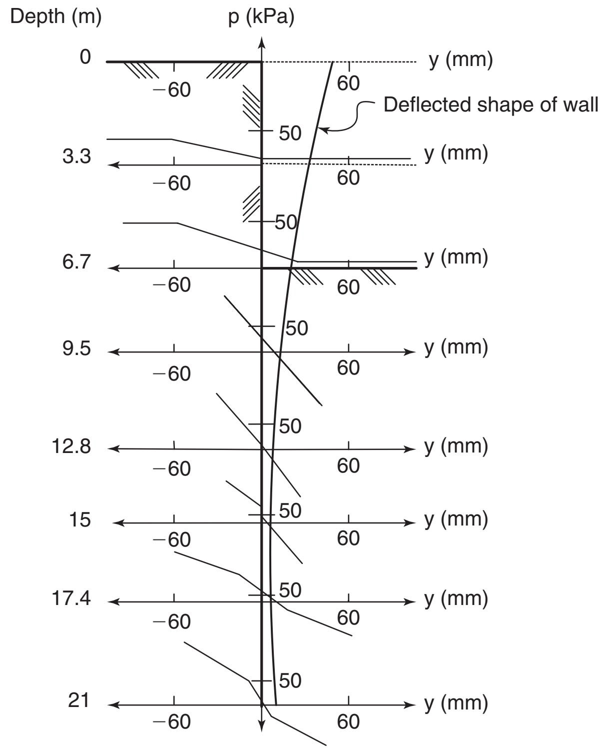


Figure 21.36 P-y curves and deflected shape of cantilever wall (Briaud et al. 1983).

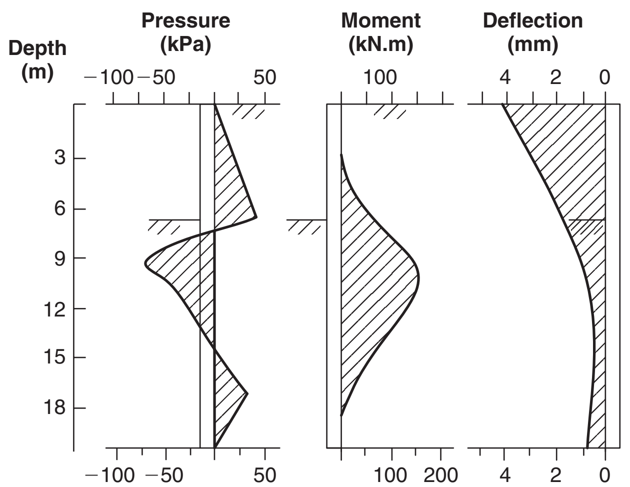


Figure 21.37 Pressure, bending moment, and deflection (Briaud et al. 1983).

(Figure 21.38). Bored pile walls and sheet pile walls were discussed in section 21.11. In this section they are anchored to be able to retain larger depth of soil. Deep soil mixing walls are like bored pile walls except that the bored piles are drilled by mixing the soil with about 20% cement; the resulting piles are not as strong, but they are less expensive. Soldier pile and lagging walls are constructed by driving or drilling piles in line on a 2 to 3 meter spacing and excavating in front of this line of piles while placing wood lagging to retain the soil between piles. Anchored and strutted walls are very convenient in tight settings like urban areas because they do not require much space for construction. On the one hand, struts clutter the excavation; on the other hand, anchors may hit underground utility lines.

The design of anchored walls and strutted walls includes many parts, with the main ones being estimating the pressure distribution behind the wall, calculating the anchor or strut loads, calculating the maximum bending moment in the wall, estimating the horizontal and vertical movements, and calculating the necessary length of anchors.

21.12.1 Pressure Distribution

Consider the pressure distribution behind a cantilever top-down retaining wall with the active pressure on the retained side (Figure 21.32). If you install an anchor within the excavated depth to hold the wall back, and if you stress that anchor in tension, the anchor head (plate) is going to press against the wall while you pull on the tendon, thereby increasing the local pressure (Figure 21.39). As a result, the pressure behind the anchor will be higher than the active pressure and will correspond to the prestressing load of the anchor or the strut.

It is very common to stress all the anchors to the same load, so that the pressure behind the wall in the retained soil depth (above excavation level) is nearly constant and equal to the sum of the anchor loads divided by the retained soil area. This is what led Terzaghi et al. (1996) to recommend a constant pressure diagram for strutted walls. Based on full-scale measurements, they recommended pressure diagrams for sand, for soft to medium clays, and for stiff fissured clays (Figure 21.40). The maximum total pressure σ_h is as follows:

For sands $$\sigma_h = 0.65K_0\sigma'_{ov} + u_w \quad (21.94)$$

For soft to medium clays $$\sigma_h = \gamma H - 4\,\mathrm{m}s_u \quad (21.95)$$

For stiff fissured clay $$\sigma_h = 0.2\gamma H \text{ to } 0.4\gamma H + u_w(?) \quad (21.96)$$

where K_a is the coefficient of earth pressure at rest, σ'_{ov} is the effective vertical stress on the retained soil (sand) side at the bottom of the excavation, γ is the total unit weight of the clay, H is the height of the excavation, s_u is the clay undrained shear strength, and m is a parameter that depends on the depth of the soft to medium clay layer below the excavation. It is taken as equal to 1 if the soft to medium clay layer stops at the bottom of the excavation, and as equal to 0.4 if the clay layer goes much deeper than the bottom of the excavation. Note that for sand, the analysis is an effective stress analysis and the water pressure must be added if water is present. For soft to medium clay, the analysis is an undrained analysis and the water pressure is included in γH. For stiff fissured clays, the coefficient 0.2 would correspond to less fissured clays and 0.4 to more fissured clays. Also, if the fissures are large enough that water will exert pressure on the wall, the water pressure must be added.

(a) Slurry wall digging the wall
(b) Slurry wall lowering cage
(c) Slurry wall final with anchors
(d) Deep soil mixing wall
(e) Soldier pile and lagging wall
(f) Strutted wall
(g)

Figure 21.38 Various anchored and strutted wall techniques. (a, b, c, d: Courtesy of Nicholson Construction; e, f, g: Courtesy of Schnabel Foundation Company.)

21.12.2 Pressure vs. Movement

Briaud and Kim (1998) collected a number of full-scale case histories on anchored walls and performed numerical simulations. For the case histories, the anchor loads were known, as were the horizontal deflections of the wall. The mean pressure σ_h behind the wall was calculated as the ratio of the sum of the individual anchor loads F_i divided by the total wall area A of soil retained by the anchors:

$$\sigma_h = \frac{\sum_{i=1}^{n} F_i}{A} \tag{21.97}$$

The mean pressure σ_h behind the wall was associated with the horizontal movement at the top of the wall u_{top} and the mean horizontal deflection u_{mean}. Note that one case history led to more than one combination of pressure and displacement, as the construction sequence included several excavation levels and several anchor installations. The earth pressure coefficient K was calculated as the ratio of the mean pressure σ_h over the vertical effective stress behind the wall at the bottom of the excavation:

$$K = \frac{\sigma_h}{\sigma'_{ov}(at\ \mathrm{z} = \mathrm{H})} \tag{21.98}$$

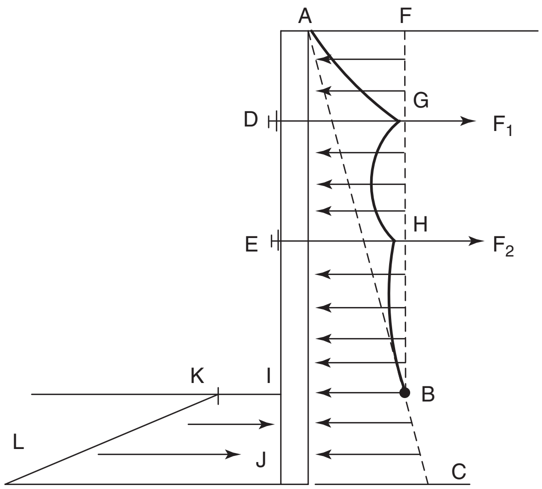


Figure 21.39 Influence of anchor stressing on pressure diagram.

Figure 21.41 shows the range of values of K versus u_{top}/H and K versus u_{mean}/H.

Terzaghi and Peck's earth pressure value of $0.65K_a\gamma$H for strutted excavations in sand leads to a K value of 0.21 if the friction angle is 30° ($K_a = 0.33$). For a K value of 0.21, Figure 21.41 gives a range of u_{top}/H from 0.002 to 0.0045 and u_{mean}/H from 0.0015 to 0.0035. However, in the case of anchored walls, the engineer can choose the wall deflection by choosing the anchor loads. Indeed, if the anchor loads are very high, the wall could actually move back and go into passive resistance. In contrast, if the anchor loads are very low, there will be a lot of wall deflection toward the excavation. Figure 21.41 helps the engineer to select a K factor that will generate a targeted amount of wall movement. It appears that a K value of about 0.4 will lead to minimal displacements. For a given wall height H and for a chosen horizontal displacement u_{top} or u_{mean}, the total earth pressure σ_h at depth z is calculated according to:

$$\sigma_h = K\sigma'_{ov}(\text{at z} = \text{H}) + u_w \tag{21.99}$$

where K is read on Figure 21.41 at the corresponding relative displacement, σ'_{ov} is the vertical effective stress at the bottom of the wall, and u_w is the water pressure at depth z. Note that the term $K\sigma'_{ov}$ is a constant independent of depth, whereas u_w increases with depth (Figure 21.42).

21.12.3 Base Instability

In the case of clays, one concern is an inverted bearing capacity failure. In the case of sands, the concern is a loss of effective stress and the development of a quick condition at the bottom of the excavation. In clays, the bottom of the excavation may be unstable if the soil is not strong enough to sustain the lack of overburden on the excavated side. The factor of safety F against base instability is (Figure 21.43):

$$F = \frac{N_c s_u}{\sigma_{ov}(\text{at z} = \text{H})} \tag{21.100}$$

where N_c is a bearing capacity factor for a strip footing (Figure 17.7), s_u is the undrained shear strength, and $\sigma_{ov(\text{at } z = H)}$ is the vertical total stress behind the bottom of the wall.

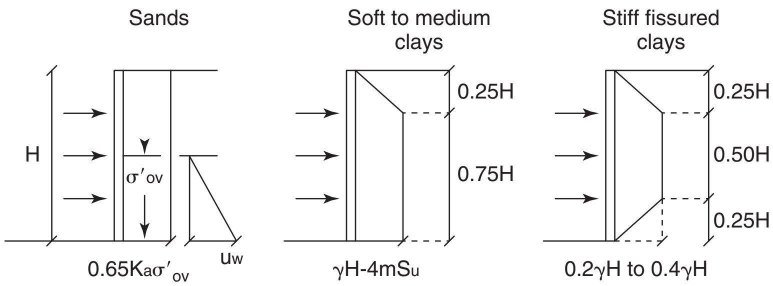


Figure 21.40 Pressure distribution for strutted walls. (After Terzaghi et al. 1996.)

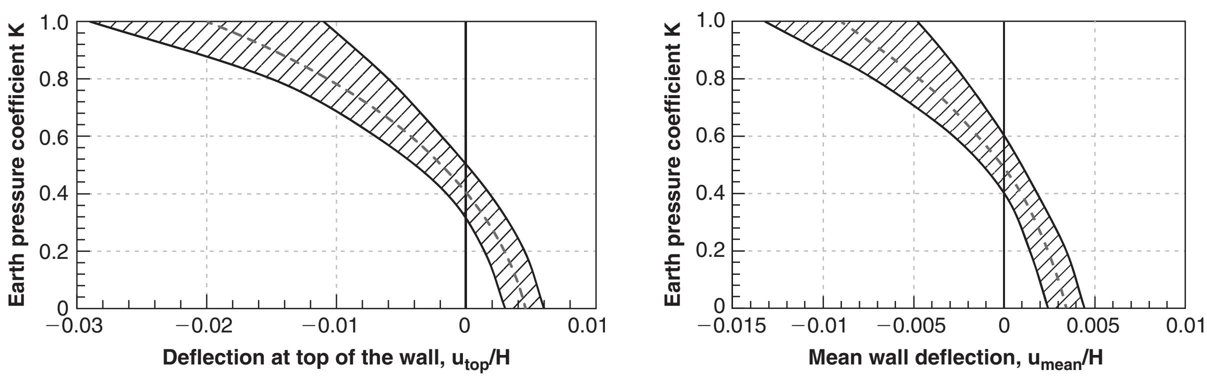


Figure 21.41 Measured earth pressure coefficient versus normalized displacement of the wall (Briaud and Kim 1998).

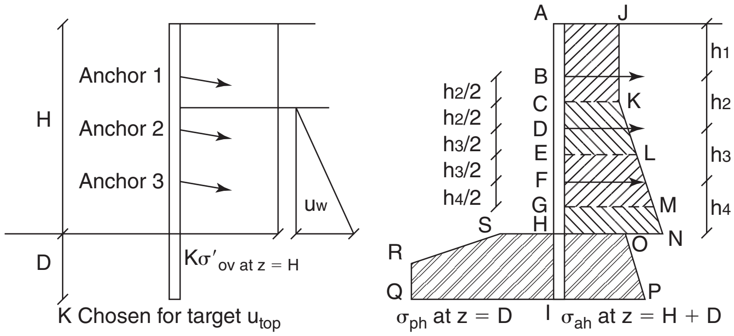


Figure 21.42 Pressure diagram for anchored walls.

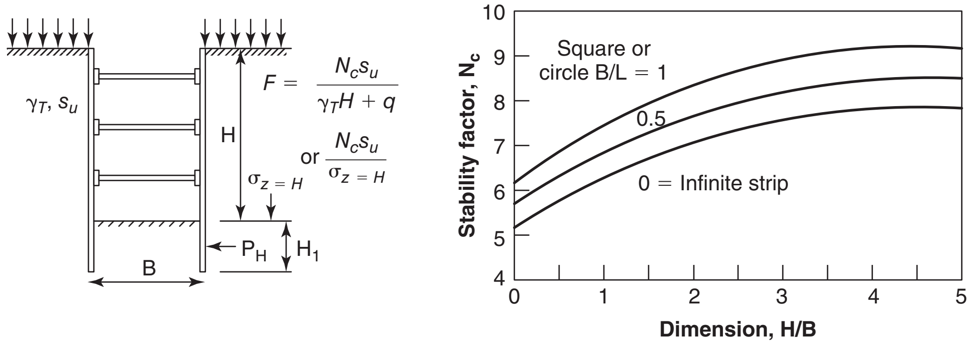


Figure 21.43 Base instability.

21.12.4 Movement of Wall and Ground Surface

The general shape of the deformed wall and adjacent ground surface has two components (Figure 21.44): a cantilever movement and a movement associated with deep deformations. The first one is associated with the lack of lateral support leading to the soil mass leaning into the excavation, and the second with the slope stability/bearing capacity type of deformation deeper in the soil mass. Predicting these displacements is not simple. Peck (1969) collected data on the movement of the ground surface near excavations and presented it in a very useful fashion (Figure 21.45). Peck divided the behavior according to soil type and to the value of the factor of safety F against base instability. He showed that the maximum settlement of the top of the wall can reach 0.01 H for excavations in sand or in soft to hard clay, it can reach 0.02 H for soft clays when F is larger than 1.3, and it can be larger than 0.02 H for soft clays when F is less than 1.3. Regarding the lateral extent over which the ground surface would be depressed, Peck found that it could be up to 2 H for excavations in sand or in soft to hard clay, it could be up to 4 H for soft clays when F is larger than 1.3, and larger than 4 H for soft clays when F is less than 1.3.

Clough and O'Rourke (1990) collected additional data and revised Peck's plots accordingly (Figure 21.46). In their work, Clough and O'Rourke also proposed a method to predict the maximum lateral movement of the wall depending on the relative stiffness L of the wall and the factor of safety F

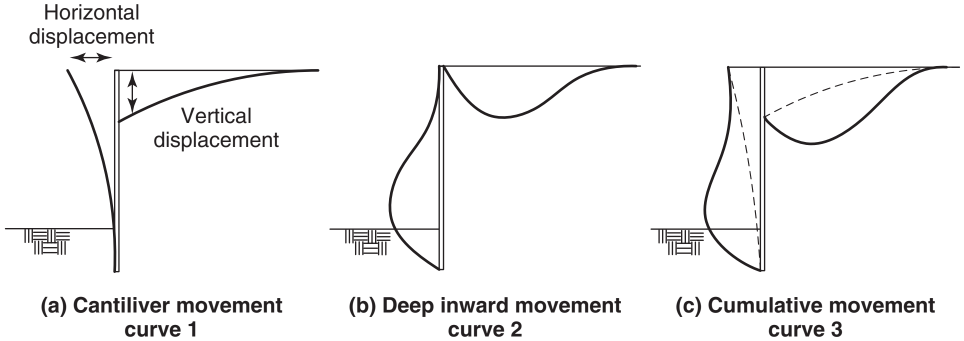


Figure 21.44 Components of excavation movements.

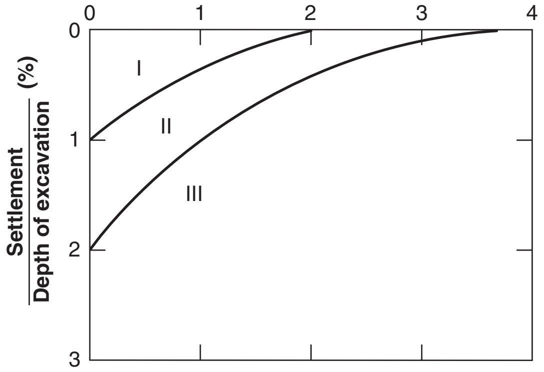


Figure 21.45 Peck diagram for ground surface settlement (Peck 1969).

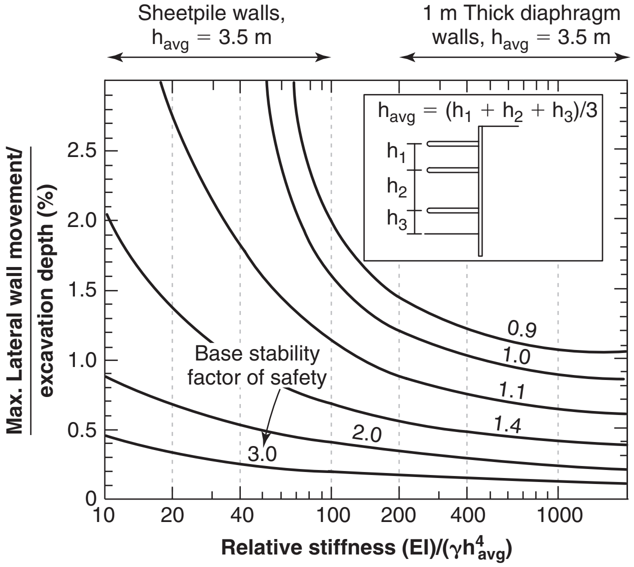


Figure 21.47 Clough and O'Rourke chart for maximum lateral movement. (After Clough and O'Rourke 1990)

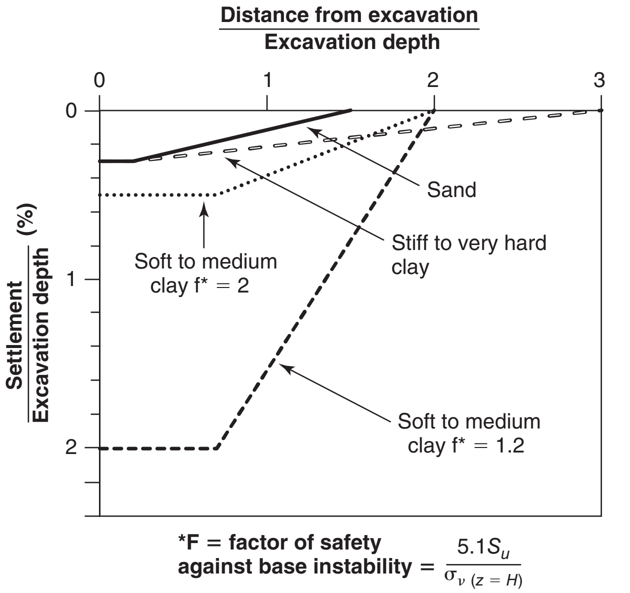


Figure 21.46 Clough and O'Rourke diagram for ground surface settlement. (After Clough and O'Rourke 1990.)

against base instability (Figure 21.47). The relative stiffness L(m) is defined as:

$$L = \frac{EI}{\gamma h^4} \tag{21.101}$$

where E is the modulus of elasticity of the wall material, I is its moment of inertia around the bending moment axis, γ is the unit weight of the soil, and h is the average vertical distance between anchors. For a given case, the relative stiffness L and the factor of safety for base instability are defined, the correct curve on Figure 21.47 is selected, and the corresponding ratio between the maximum horizontal deflection and the excavation height is read on the vertical axis. In general, the vertical and horizontal displacements of excavations are on the same order of magnitude unless the soil is very dilatant or collapsible. Some of the ways to decrease movements are to place the first anchor as shallow and as early as reasonably possible and to use high anchor loads ($K = 0.4$ in Figure 21.41).

21.12.5 Anchors

Anchors can be constructed in different ways, but the most common way (Figure 21.48) is to drill a hole through the wall when the anchor depth is reached, insert a rod or multiple-strand cable in the open hole with centralizers, fill the hole with grout, wait for the grout to set, then tension the anchor, subject it to a proof test, and then lock the anchor at the design load. Sometimes a second injection of grout is performed through tubes left in place during the first injection to increase the anchor capacity. The rod or strand is in a bond-breaking sheath from the anchor head to a certain distance called the *tendon unbonded length* L_u. The sheath stops at L_u; the rest of the rod or strand is barren and is called the *tendon bond anchor length* L_b. The length of the anchor in the active wedge is called the *discounted anchor length* L_d. The rest of the anchor is called the *anchor bond length* L_a. The total length of the anchor is L_t:

$$L_t = L_u + L_b = L_d + L_a \tag{21.102}$$

The length L_d is taken as the length of the anchor within the active wedge behind the wall (Figure 21.48). An example

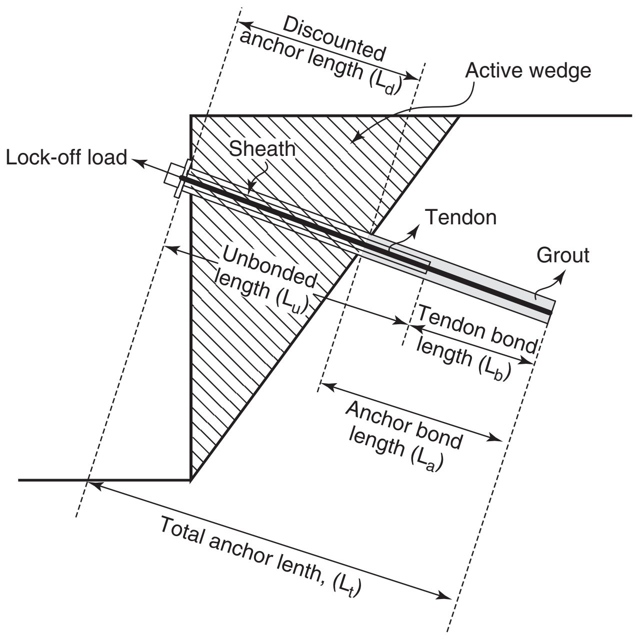


Figure 21.48 Anchor or tieback.

of load distribution in the anchor under tension is shown in Figure 21.49. A long unbonded length is best for anchors in tension because it maximizes the length of grout in compression (Briaud et al. 1998).

The design of anchors or tiebacks has two parts: calculating the anchor loads and calculating the required anchor capacity and associated length. The anchor load is determined by using the tributary method. Once the pressure diagram is obtained (Figure 21.42), the horizontal component A_{hi} of the load in anchor i is obtained by using the part of the pressure diagram tributary to anchor i. For example, the tributary area of the pressure diagram in Figure 21.42 for anchor 2 is CKLE. The expression is:

$$A_{hi} = p_i \left(\frac{h_i}{2} + \frac{h_{i+1}}{2} \right) s_h \tag{21.103}$$

where A_{hi} is the horizontal component of the anchor load A_i, p_i is the mean pressure behind the wall within the tributary depth, h_i is the anchor spacing above anchor i, h_{i+1} is the anchor spacing below anchor i, and s_h is the anchor spacing

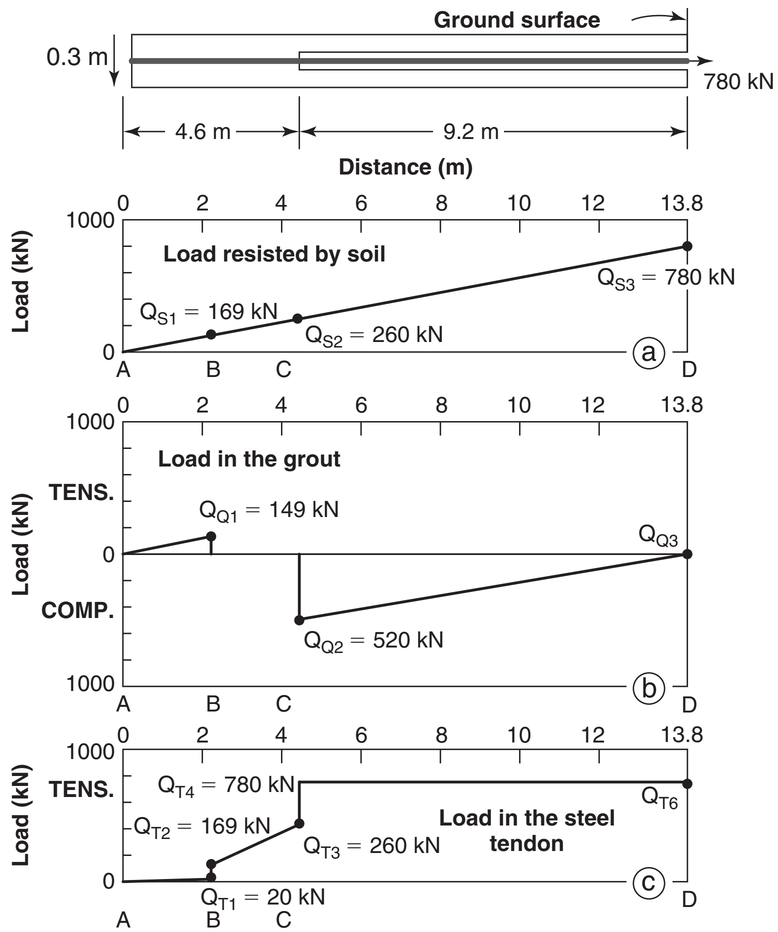


Figure 21.49 Example of load distribution in an anchor (Briaud et al. 1998).

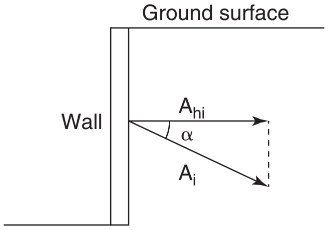


Figure 21.50 Anchor load components.

in the horizontal direction. Equation 21.103 applies to all anchor loads except the top anchor, where it becomes:

$$A_{h1} = p_1 \left(h_1 + \frac{h_2}{2} \right) s_h \tag{21.104}$$

Often the anchor is not horizontal, but rather inclined at an angle α to the horizontal (15° to 30°). Thus, the anchor load A_i is (Figure 21.50):

$$A_i = \frac{A_{hi}}{\cos \alpha} \tag{21.105}$$

Once the anchor load is determined, the anchor resistance and length can be calculated. The LRFD equation gives:

$$\gamma A_1 = \varphi R_1 \tag{21.106}$$

where γ is the load factor ($\gamma = 1.35$), A_i is the anchor load, φ is the resistance factor, and R_i is the ultimate resistance of the anchor. If anchors are not proof tested, then φ is between 0.35 and 0.45. However, all anchors are usually proof tested and therefore there is little uncertainty as to the anchor capacity; in that case a resistance factor close to 1 can be used.

Once R_i is obtained, the length of anchor necessary to obtain R_i is calculated. The design is very similar to the case of a pile in tension, and R_i is given by:

$$R_i = \pi D L_a f_{\max} = F_{\max} L_a \tag{21.107}$$

where D is the diameter of the anchor, L_a is the anchor bond length, $f_{\max}$ is the shear strength of the soil-grout interface, and $F_{\max}$ is the maximum load that can be resisted per unit length of anchor. The parameter $f_{\max}$ is estimated as follows for various soils:

For sand and gravel $f_{\max} = \alpha_s \sigma'_{ov}$ with α_s from Table 21.3 (21.108)

For silts and clays $f_{\max} = \alpha_c s_u$ with α_c from Figure 21.51 (21.109)

Tables 21.4, 21.5, and 21.6 present some presumptive values of $f_{\max}$ as recommended by AASHTO (2007). Furthermore, the values of $F_{\max}$ in Table 21.7 can be used for anchors satisfying the following criteria:

- Diameter between 150 to 200 mm
- Grout pressure of about 1000 kPa

Table 21.3 Values of α_s Anchorage Factor for Sand and Gravel

Soil Type	Relative Density: Loose	Medium	Dense
Silt	0.1	0.4	1.0
Fine sand	0.2	0.6	1.5
Medium sand	0.5	1.2	2.0
Coarse sand, gravel	1.0	2.0	3.0

(Canadian Foundation Manual 2007.)

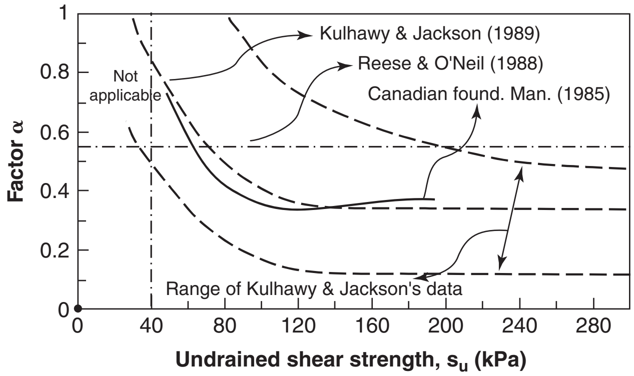


Figure 21.51 α factor for grouted anchors in clay (Briaud et al. 1998).

- Center-to-center spacing vertically and horizontally larger than 4 anchor diameter D

Design rules based on pressuremeter data for calculating the ultimate resistance of anchors also exist (Briaud 1992). These rules, established by the LCPC in France, make a distinction between several construction techniques for the anchors.

Typically, all anchors are tested after installation and curing time. These tests include proof tests, performance tests, creep tests, and 70-day load-hold tests (Briaud et al. 1998). The proof test is the most common and consists of increasing the load in steps up to 1.33 times the design load. In the United States, anchors are accepted if the creep movement at that load is less than 2 mm per log cycle of time. This creep movement is due to the creep in the steel tendon, the progressive cracking of the grout in tension, and the creep of the soil in shear. The loading history for the proof test and the result of a test are shown in Figure 21.52.

21.12.6 Embedment Depth and Downdrag

Another issue to be addressed is the embedment depth below the excavation level. You might think that the anchored wall would not need much embedment, since the anchors hold the soil back. The following reasoning shows the danger of having very little embedment (Briaud and Lim 1999).

Table 21.4 Presumptive f_{max} Values for Fine-Grained Soils

Anchor Type (Grout Pressure)	Soil Type	Shear Strength of Soil s_u (kPa)	Shear Strength of Soil-Grout Interface f_{max} (kPa)
Gravity grouted anchors (< 350 kPa)	Silt-clay mixtures	Stiff to very stiff 50 to 200 kPa	30 to 70 kPa
Pressure grouted anchors (350 to 2800 kPa)	High-plasticity clay	Stiff (50 to 120 kPa)	30 to 100 kPa
		Very stiff (120 to 200 kPa)	70 to 170 kPa
	Medium-plasticity clay	Stiff (50 to 120 kPa)	100 to 250 kPa
		Very stiff (120 to 200 kPa)	140 to 350 kPa
	Medium-plasticity sandy silt	Very stiff (120 to 200 kPa	280 to 380 kPa

(After AASHTO 2007)

Table 21.5 Presumptive f_{max} Values for Coarse-Grained Soils

Anchor Type (Grout Pressure)	Soil Type	Relative Density and SPT N Value N (blows/0.3 m)	Shear Strength of Soil-Grout Interface f_{max} (kPa)
Gravity grouted anchors (< 350 kPa)	Sand or sand/gravel mixtures	Medium dense to dense (N = 10 to 50)	70 to 140 kPa
Pressure grouted anchors (350 to 2800 kPa)	Fine to medium sand	Medium dense to dense (N = 10 to 50)	80 to 380 kPa
	Medium to coarse sand with gravel	Medium to dense (N = 10 to 30)	110 to 670 kPa
		Dense to very dense (N = 30—50+)	250 to 950 kPa
	Silty sand		170 to 400 kPa
	Sandy gravel	Medium dense to dense (N = 10 to 40)	210 to 1400 kPa
		Dense to very dense (N = 40 to 50+)	280 to 1400 kPa
	Glacial till	Dense (N = 30 to 50)	300 to 520 kPa

(After AASHTO 2007)

Table 21.6 Presumptive f_{max} Values for Rock

Rock Type	Shear Strength of Soil-Grout Interface
Soft shale	200 to 800 kPa
Weathered sandstone	700 to 800 kPa
Sandstone	800 to 1700 kPa
Slate and hard shale	800 to 1400 kPa
Soft limestone	1000 to 1400 kPa
Dolomite limestone	1400 to 2100 kPa
Granite or basalt	1700 to 3100 kPa

(After AASHTO 2007)

When the excavation takes place, the soil mass behind the wall tends to move toward the excavation and downward (Figure 21.53). The downward movement drags the wall down, and if the embedment is insufficient, the downward movement can be significant. Even if the anchors are performing well, the wall can rotate; indeed, the anchors keep the soil from moving horizontally but not vertically. This rotation will generate horizontal movement as well. Therefore, to minimize horizontal movement, it is necessary to have well-designed anchors and a well-designed embedment depth to resist downdrag and the vertical component of the anchor loads.

The embedment depth must also resist the unbalanced lateral load from the bottom of the pressure diagram. This is

Table 21.7 Values of Anchor Load Transfer Capacity

Soil or Rock Type	Strength (SPT N Values)	Estimated Load Transfer (kN/m)
Sand and gravel	Loose (N = 4 to 10)	145
	Medium (N = 10 to 30)	210
	Dense (N = 30 to 50)	290
Sand	Loose (N = 4 to 10)	100
	Medium (N = 10 to 30)	145
	Dense (N = 30 to 50)	190
Sand and silt	Loose (N = 4 to 10)	75
	Medium (N = 10 to 30)	100
	Dense (N = 30 to 50)	130
Low-plasticity silt and clay	Stiff (N = 10 to 20)	30
	Hard (N = 20 to 40)	60
Soft shale		145
Slate and hard shale		360
Soft limestone		430
Sandstone		430
Dolomite limestone		580
Granite or basalt		730

(Canadian Foundation Engineering Manual 2007; FHWA 1984)

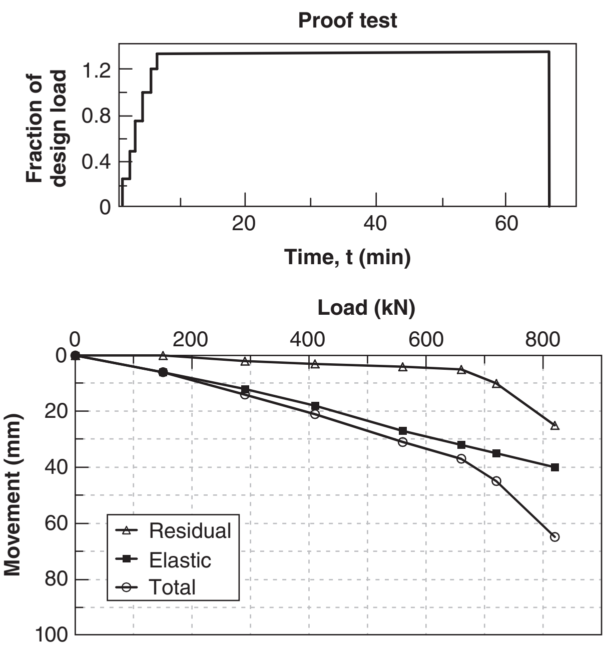


Figure 21.52 Loading history for an anchor proof test (Briaud et al. 1998).

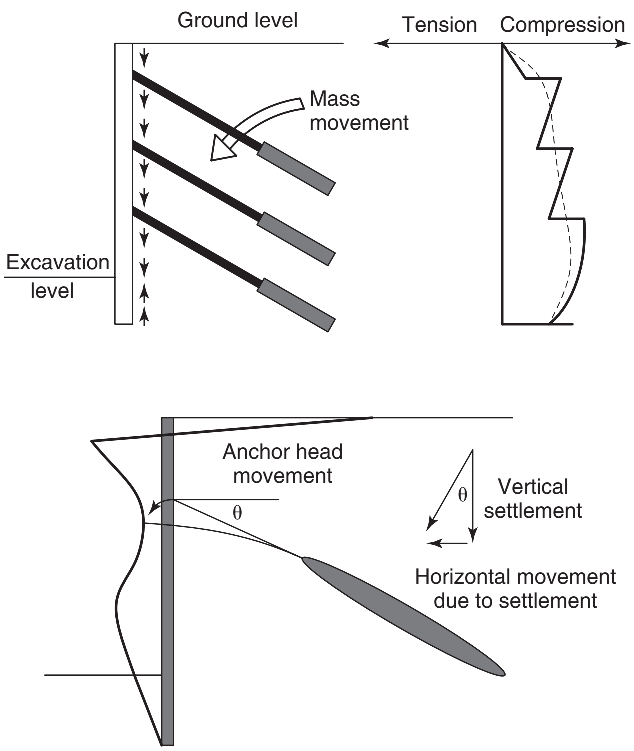


Figure 21.53 Downdrag creates horizontal movement (Briaud and Kim 1998).

area GMNH in Figure 21.42. The depth of embedment for lateral resistance is obtained by designing the bottom part of the wall (GHI in Figure 21.42) to resist the pressure from areas GMNH and HOPI with the factored passive resistance SHIQR. This design follows the approach described for the cantilever top-down wall. As a guide, the depth of embedment required for lateral resistance is on the order of 1.5 times the distance GH. The downdrag design requirement may be

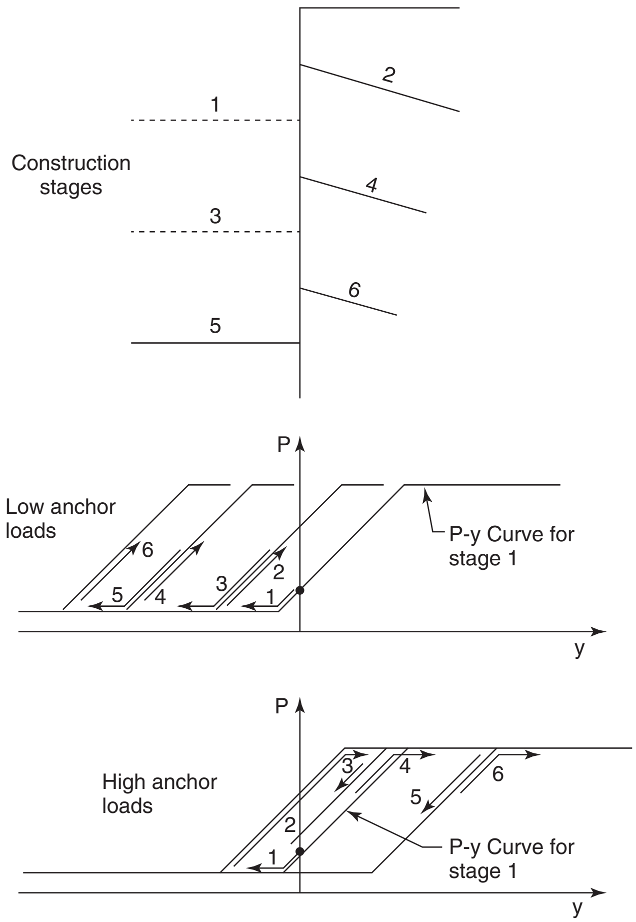


Figure 21.54 P-y curve path during construction sequence (Briaud and Kim 1998).

larger (see section 18.6). Note that two cases occur for the embedment depth: the case where the wall is continuous below the excavation level (e.g., slurry wall, sheet pile wall), and the case where only a row of piles exists below the excavation level (e.g., soldier pile and lagging wall). The depth of embedment of a system using a row of piles will have to be larger than that of a continuous wall system.

21.12.7 P-y Curve Approach and FEM Approach

The P–y curve approach described in section 21.11 is also applicable to anchored walls, and represents the best way to obtain the bending moment versus depth profile for the wall (Briaud and Kim 1998). In the process of preparing the P-y curves, it is possible to follow the construction sequence as shown in Figure 21.54. The anchors must have their separate P-y curves, as shown in Figure 21.55. A sample result for the P-y curve approach is shown in Figure 21.56. This is a comparison between the P-y curve predictions and the actual measurement for a full-scale instrumented wall at Texas A&M University (Figure 21.57). The P-y curve approach is not as reliable for predicting movements as it is for predicting bending moments. For better movement predictions, the FEM is preferred, provided quality soil parameters are obtained and a realistic soil model is selected (Briaud and Lim 1999).

Figure 21.58 shows a sample result for the FEM approach. This is a comparison of the FEM predictions with the same full-scale wall at Texas A&M University (Figure 21.57).

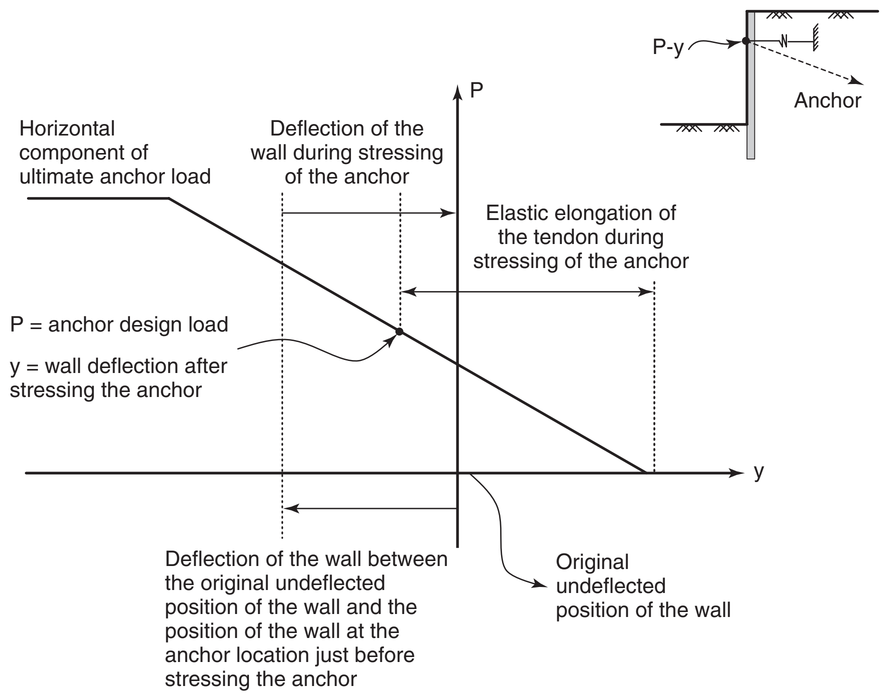


Figure 21.55 Anchor P-y curve (Briaud and Kim 1998).

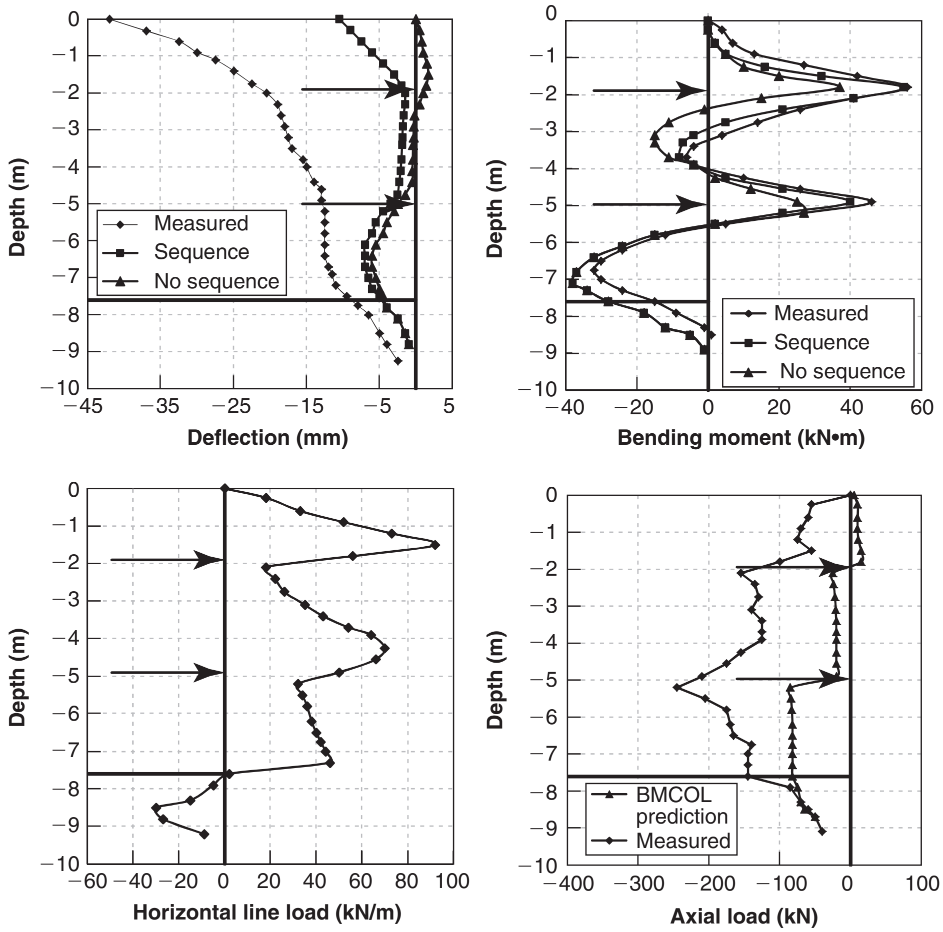


Figure 21.56 Predicted and measured result for the P-y curve method (Briaud and Kim 1998).

The predictions from the P-y curve approach and the FEM approach can be compared (Figure 21.56 and 21.58).

21.13 SOIL NAIL WALLS

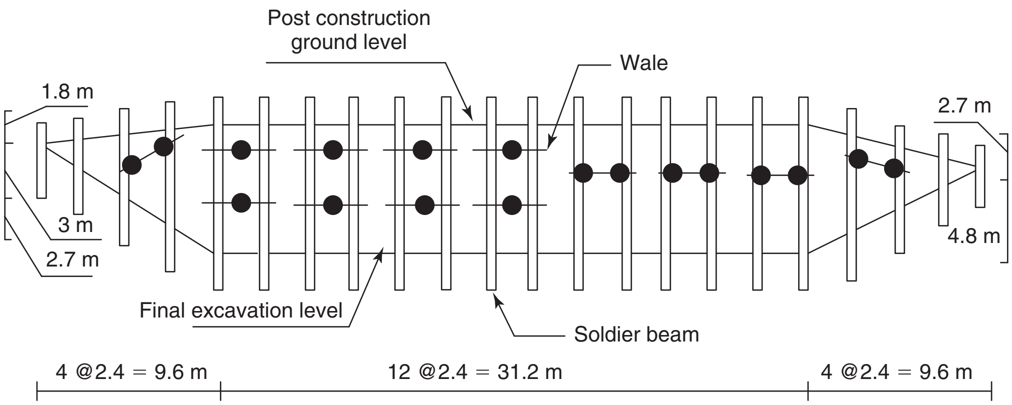
Soil nail walls are top-down walls reinforced with rigid inclusions. They are to the top-down walls what MSE walls are to the bottom-up walls. *Soil nails* are rigid inclusions that are built much like anchors, by drilling a 100 to 300 mm diameter hole, inserting a steel bar with centralizers in the open hole, and backfilling the hole with grout. Unlike anchors, however, they are not posttensioned. The load in the nail develops as the soil mass deforms. The spacing between soil nails is typically quite a bit smaller than the spacing between anchors. Whereas all anchors and tiebacks are load tested, only a small percentage of soil nails are load tested. Soil nail walls are particularly suited for cases where the soil can stand unsupported for a height of 1 to 2 m long enough to place a row of nails (a few hours) and where the drill hole can stand open long enough for nail insertion and grouting. The front of the wall is typically covered with shotcrete to a thickness of 100 to 200 mm projected over a reinforcement mesh. Figures 21.59 and 21.60 show the construction sequence.

Much like the case of MSE walls, the design must consider external stability and internal stability as well as deformations.

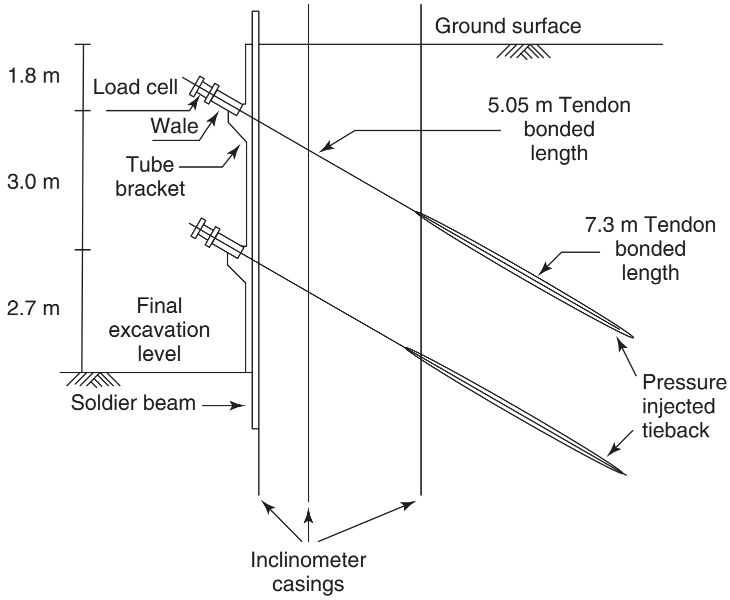
21.13.1 External Stability

External stability includes global stability, sliding, and bearing capacity. Sliding and bearing capacity are handled in a fashion similar to the MSE wall design (section 21.12). The global stability, however, is different from the MSE wall approach. It is a slope stability type of analysis that considers failure along a surface through the nails. This surface can be a circle, two straight lines, or one line (Figure 21.61). The one-line solution is the simplest and is discussed here. Computer programs such as SNAIL (CALTRANS 1991) and GOLDNAIL (Golder, 1993) can be used to solve the problem for more complex failure surfaces.

Consider the soil nail wall of Figure 21.62. At equilibrium, the force resisted by the nails is T, the weight of the wedge is W, the surcharge force is Q, the shear force on the failure plane is S, and the normal force on the failure plane is N. The dimensions and angles involved are defined in Figure 21.62. The problem is to find the value of T to obtain a target factor



(a) Elevation view



(b) Two row anchored wall – section view

Figure 21.57 Full-scale instrumented wall at Texas A&M University (Briaud and Lim 1999).

of safety F (chosen value) on the ultimate shear resistance S. The factor of safety F is defined as:

$$F = \frac{\sum \text{maximum resisting shear forces on failure plane}}{\sum \text{driving shear forces on failure plane}} = \frac{\sum \mathrm{R}}{\sum L} \tag{21.110}$$

Alternatively, the LRFD expression would be:

$$\gamma \sum L = \varphi \sum \mathrm{R} \tag{21.111}$$

Writing equilibrium equations normal and along the plane of failure gives:

$$\sum \text{normal forces} = (W + Q)\cos\psi + T\sin(\psi + i) - N = 0 \tag{21.112}$$

$$\sum \text{tangent forces} = (W + Q)\sin\psi - T\cos(\psi + i) - S = 0 \tag{21.113}$$

The maximum value of the force S is S_{max} corresponding to the shear strength of the soil:

$$S = \frac{S_{max}}{F} = \frac{c'L + N\tan\varphi'}{F} \tag{21.114}$$

where c' and φ' are the effective stress cohesion and friction angle of the soil and F is the chosen factor of safety by design. The unknowns are N, S, and T and the three equations (21.112, 21.113, and 21.114) give the three quantities. Actually, the angle ψ corresponding to the lowest factor of safety is not known either, and must be found by trial and error. Once this is done, the load that must be safely carried by the nails is T.

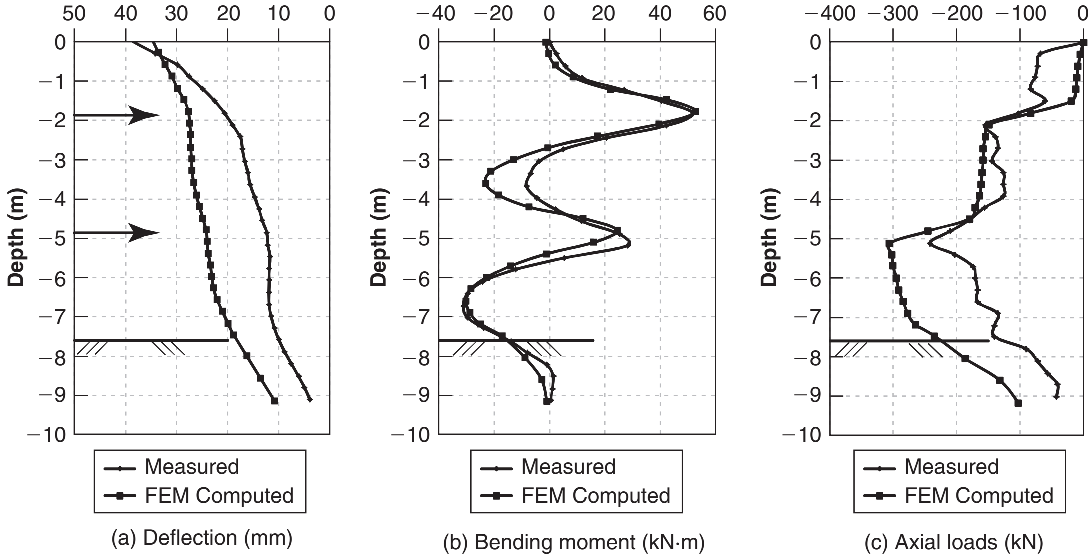


Figure 21.58 Predicted and measured result for the FEM method (Briaud and Lim 1999).

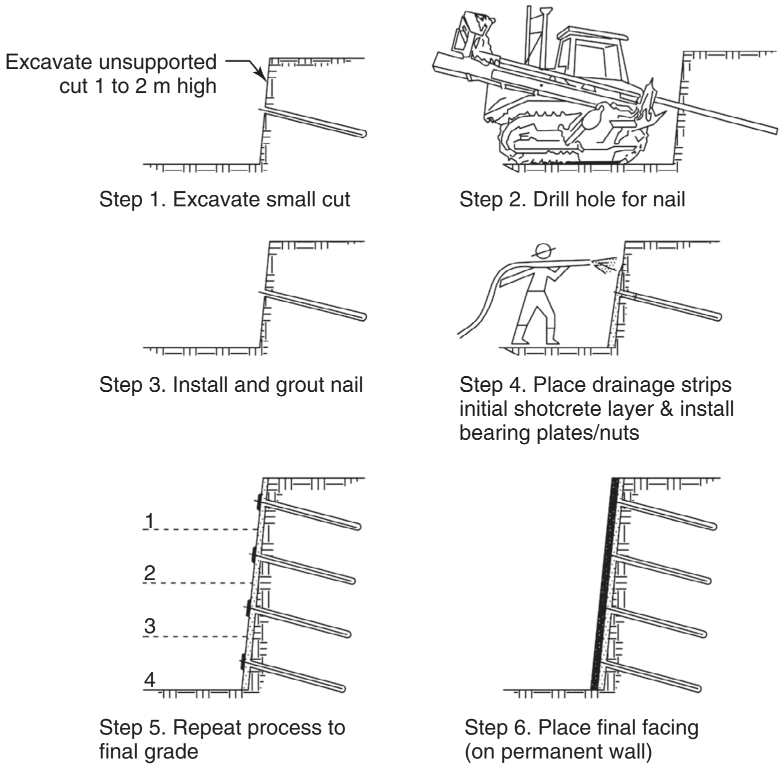


Figure 21.59 Soil nail wall construction sequence (FHWA 1998).

21.13.2 Internal Stability

Internal stability includes pull-out of the nails at the grout-soil interface, pull-out of the steel bar at the grout-bar interface, tensile yielding, and bending and shearing of the nails. The pull-out of the steel bar at the grout-bar interface is usually not controlling if threaded bars are used. Bending and shearing also do not appear to have a major influence on the behavior of the mass (Lazarte et al. 2003). Let's look first at pull-out at the grout-soil interface.

Pullout at Grout-Soil Interface

The equation for the ultimate axial resistance R of a nail in tension is:

$$R = \pi D L_p f_{\max} \tag{21.115}$$

Figure 21.60 Soil nail wall construction (Courtesy of FHWA www.fhwa.dot.gov/publications/publicroads/11septoct/alongroad.cfm).

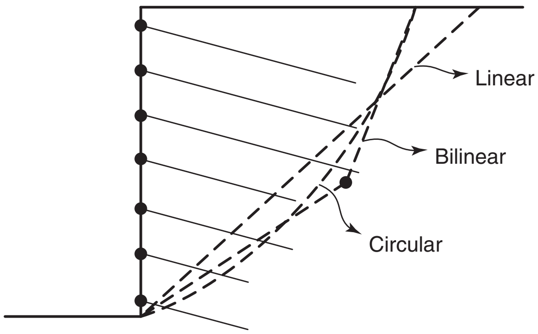


Figure 21.61 Soil nail wall failure surfaces.

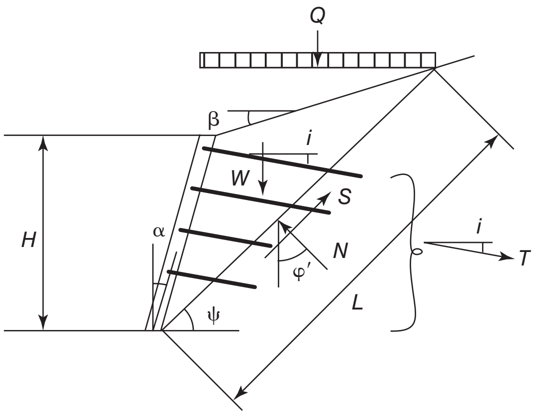


Figure 21.62 Soil nail wall global stability analysis.

where D is the diameter of the nail (drill hole), L_p is the useful length of the nail beyond the failure zone, and f_{max} is the shear strength at the grout-soil interface. Table 21.8 gives some estimated values of f_{max}.

Tensile Force Distribution in the Nail

Figure 21.63 shows a simplified distribution of the tension in a nail within the reinforced soil mass. As in the case of the MSE wall, the tension load is lower at the wall face (T_o), then increases as the soil transfers the load to the nail until a maximum value is reached (T_{max}), and then decreases to zero as the load is transferred from the nail to the surrounding soil. At the wall face, the nail is usually connected to a plate pressed against the soil by a nut threaded on the nail steel bar. The load T_o varies from 0.6 to 1 times the maximum load T_{max}. The load T_{max} starts to decrease at a distance L_p from the end of the nail. The measured locus of T_{max}, the failure plane, and the distance L_p are shown in Figure 21.64. The load that must be globally carried by the nails is T, as calculated in the external stability analysis. The ultimate resistance that can be developed by individual nails over the length L_p is R_i, which must satisfy:

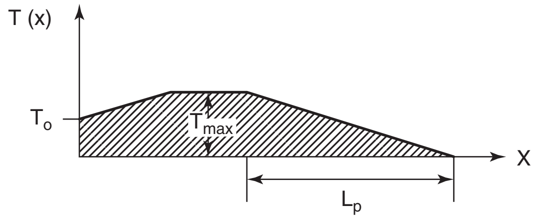


Figure 21.63 Tension load distribution in a soil nail.

$$\gamma T = \varphi \sum_{i=1}^{n} R_i \qquad (21.116)$$

The distribution of R_i among the nails is not precisely defined and experience plays a role in that determination. In general, shorter nails are placed at the bottom of the wall and longer ones at the top. A pattern such as the one shown in Figure 21.65 is not uncommon.

Length of Nails

The required length of each nail L_{pi} to resist R_i is calculated by using Eq. 21.115. The total length for nail i is L_{ti}; it is obtained by adding the length L_{pi} required to safely carry the

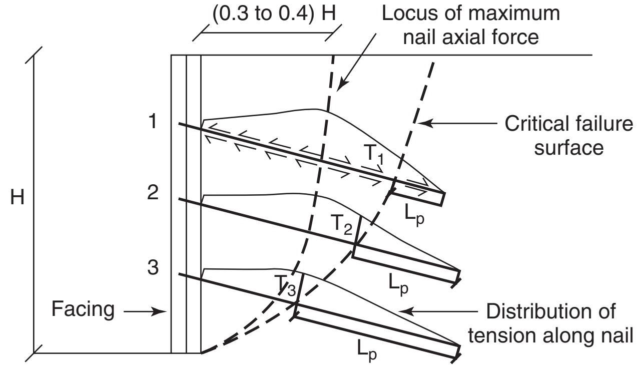


Figure 21.64 Load in the nails and available resisting length.

Table 21.8 Estimated Ultimate Grout-Soil Shear Strength, f_{max}

Material	Construction Method	Soil/Rock Type	Ultimate Grout-Soil Shear Strength, f_{max} (kPa)
Coarse-grained soils	Rotary drilling	Sand/gravel	100-180
		Silty sand	100-150
		Silt	60-75
		Piedmont residual	40-120
		Fine colluvium	75-150
	Driven casing	Sand/gravel	
		low overburden	190-240
		high overburden	280-430
		Dense moraine	380-480
		Colluvium	100-180
	Augered	Silty sand fill	20-40
		Silty fine sand	55-90
		Silty clayey sand	60-140
	Jet grouted	Sand	380
		Sand/gravel	700
Fine-grained soils	Rotary drilling	Silty clay	35-50
	Driven casing	Clayey silt	90-140
	Augered	Loess	25-75
		Soft clay	20-30
		Stiff clay	40-60
		Stiff clayey silt	40-100
		Calcareous sandy clay	90-140
Rock	Rotary drilling	Marl/limestone	300-400
		Phyllite	100-300
		Chalk	500-600
		Soft dolomite	400-600
		Fissured dolomite	600-1000
		Weathered sandstone	200-300
		Weathered shale	100-150
		Weathered schist	100-175
		Basalt	500-600
		Slate/Hard shale	300-400

(Elias and Juran 1991)

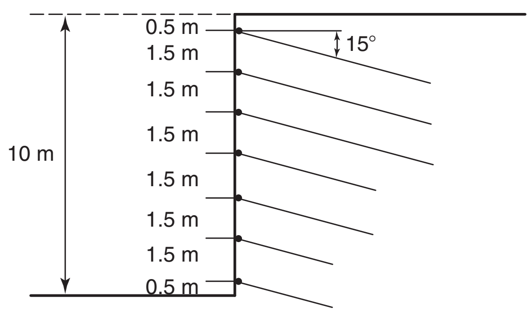


Figure 21.65 Typical pattern of nail length distribution.

required load R_i plus the discounted length L_{di} within the failure zone (Figure 21.64):

$$L_{ti} = L_{di} + L_{pi} \tag{21.117}$$

Tensile Yielding of Nails

The nails must be designed in such a way that the load applied does not break the nails. In calculating the tensile strength of the nail, the resistance of the grout is ignored and only the steel is considered. The area of the steel bar must satisfy:

$$\gamma T_{max} = \varphi A_t \sigma_y \tag{21.118}$$

where γ and φ are the load and resistance factors respectively, T_{max} is the highest load in the nail, A_t is the steel bar cross section, and σ_y is the yield strength of the steel. According to Briaud and Lim (1997) and Lazarte et al. (2003), the value of T_{max} is given by:

$$T_{max} = 0.65 \ to \ 0.75 \ K_a \gamma H s_v s_h \tag{21.119}$$

where K_a is the active earth pressure coefficient, γ is the total unit weight of the soil, H is the height of soil retained, and s_v and s_h are the vertical and horizontal nail spacing respectively. Note that Eq. 21.119 assumes that there is no water within the retained depth of soil or rock.

21.13.3 Wall Movement

The movement of soil nail walls is similar to the movement of anchored and strutted walls. According to Lazarte et al. (2003), for soil nail walls with ratios of length of nails to height of wall between 0.7 and 1.0, negligible surcharge loading, and typical load and resistance factors (safety factors), empirical data show that the maximum long-term horizontal and vertical wall displacements at the top of the wall, δ_h and δ_v, can be estimated by the values in Table 21.9 where H is the wall height. The parameter C helps to estimate the extent of the movement behind the wall (Figure 21.66):

$$D = CH(1 - \tan\alpha) \tag{21.120}$$

where D is the horizontal distance of influence of the excavation measured from the front of the wall where settlement of the ground surface takes place, C is the coefficient in Table 21.9, H is the height of the wall, and α is the batter of the wall (Figure 21.66).

21.13.4 Other Issues

Other issues include the details of the connection plates at the nail head, punching and bending of the wall cover at the front of the nail, corrosion resistance, and seismic loading. For more details on these matters, see Lazarte et al. (2003).

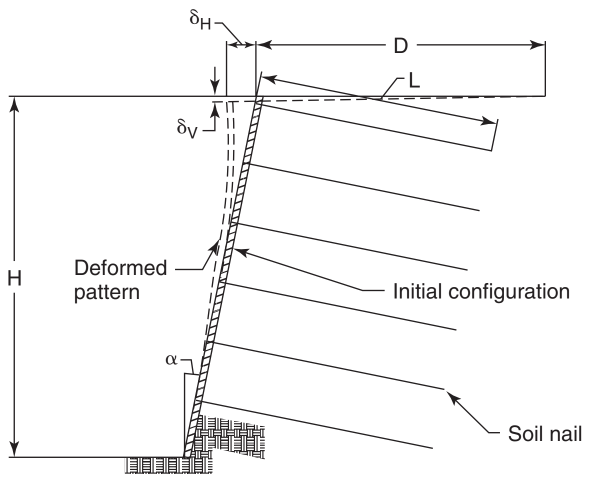


Figure 21.66 Deformation of soil nail walls.

Table 21.9 Estimates of Soil Nail Wall Movements

Variable	Weathered Rock or Stiff Soil	Sandy Soil	Fine-Grained Soil
δ_h/H and δ_v/H	0.001	0.002	0.003
C	1.25	0.8	0.7

(Lazarte et al. 2003.)

21.14 SPECIAL CASE: TRENCH

Trenches are narrow and fairly shallow excavations often used for placing utilities in congested areas. In the case of the undrained behavior of fine-grained soils, the relationship between the vertical total stress σ_{ov} and horizontal total stress σ_{oh} at failure of the trench is given by:

$$\sigma_{oh} = \sigma_{ov} - 2s_u \tag{21.121}$$

where s_u is the undrained shear strength of the soil.

The most stressed element in the trench is the soil element at the bottom of the trench, as shown in Figure 21.67. For that element, the vertical total stress σ_{ov} is:

$$\sigma_{ov} = \gamma h \tag{21.122}$$

where γ is the total unit weight of the soil and h is the depth of the trench. Initiation of failure of the trench corresponds to failure of the element shown in Figure 21.67. For this element, σ_{oh} is zero and the depth h_f at which initiation of failure starts is:

$$h_f = \frac{2s_u}{\gamma} \tag{21.123}$$

So, for example, if s_u is 100 kPa and γ is 20 kN/m^3, then h_f is 10 m and a safe depth might be 5 m. Would you go and work at the bottom of an open, unprotected trench 1 m wide and 5 m deep? You should not, and you should not allow anyone else to work in such a situation. The risk of collapse is too great, as evidenced by the number of related deaths every year. There is an average of 50 deaths per year due to trench accidents in the United States. Even going into a 1.2 m deep trench is not safe. You might think that as long as your head is above ground, you will be safe: Not true! If your head is above ground, you can open your mouth to

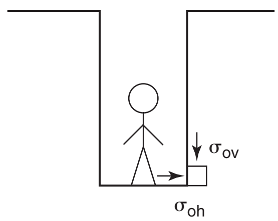


Figure 21.67 Initiation of failure in a trench.

take air in, but if your chest is below ground, you cannot expand your lungs, no air goes in, and you die. Do not go into an open, unsupported trench (Figure 21.68). You may go into a trench that is supported by what is called a trench box (Figure 21.69).

The theory leading to Eq. 21.101 is correct, but the assumptions may not match the reality. It is assumed that the soil is uniform and that every part of it has a minimum undrained shear strength s_u. This may not be the case in the field, for many reasons: the soil may be fissured and you tested the soil between fissures rather than at the fissures, which may control the mass strength; you may have tested the soil in the summer when it is harder (e.g., water tension higher) and the trench is opened in the winter; the undrained shear strength may not be the appropriate strength if the soil drains during and after the trench is open.

Figure 21.68 Do not go in there! (Courtesy of CDC's Public Health Image Library.)

Figure 21.69 Much safer when protected by a trench box. (Courtesy of www.cobletrenchsafety.com/jobprofile.php?id=95)

Additionally, if someone becomes partially buried in soil, do not try to pull that person out by rope and mechanical means. The tensile strength of the body is typically less than the pull-out capacity or force generated and you can imagine the result! Excavate around the body to free the person.

PROBLEMS

21.1 Show the pressure diagram, calculate the resultant push, and give its location for a 10 m high wall due to
   a. Water only
   b. Dry soil with unit weight of 20 kN/m^3 and a friction angle of 30° (active and passive)
   c. The same soil but with water to the top of the wall (active and passive)

21.2 Solve Coulomb's wedge analysis for the passive case of a soil with friction and cohesion. Write vertical and horizontal equilibrium and demonstrate equation 21.17.

21.3 Plot Coulomb and Rankine active and passive earth pressure coefficients for a vertical wall, no wall friction, as a function of the ground surface inclination β. Which one would you use?

21.4 Evaluate the influence of wall friction on the active and passive earth pressure coefficients by comparing Rankine value (no friction) and Coulomb values (varying friction angle from 0 to φ') for a vertical wall and horizontal backfill. Which one would you use?

21.5 Demonstrate that the direction of the plane of failure for the active pressure case (PB in Figure 21.11) is equal to 45 + $\varphi'/2$.

21.6 A 6 m high retaining wall has a backfill made of unsaturated sandy silt with a water tension equal to -1000 kPa and an area ratio (α) equal to 0.3. The total unit weight is 20 kN/m^3. The wall has no effective stress cohesion ($c' = 0$), and an effective stress friction angle equal to 30° ($\varphi' = 30$). The backfill is horizontal and the wall friction is neglected. Calculate the active and passive earth pressure diagram for this wall.

21.7 A wall is to be placed in a soil as described in Figure 21.1s. Prepare the active pressure diagram and the passive pressure diagram for that soil profile.

21.8 A 10 m high retaining wall has a horizontal backfill made of soil without water. The soil properties are $\gamma = 20$ kN/m^3, $c' = 0$, $\varphi' = 30^\circ$. Draw the active pressure diagram against the wall due to the following surcharges at the top of the wall:
   a. Uniform surcharge equal to 20 kPa
   b. Line load of 20 kN/m at a distance of 1 m from the edge of the wall
   c. A point load of 20 kN at a distance of 1 m from the edge of the wall

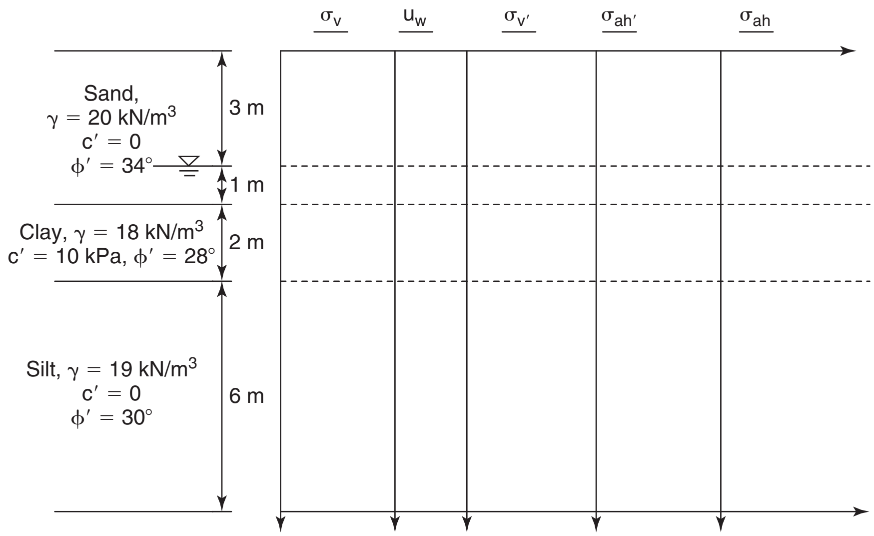


Figure 21.1s Soil profile.

21.9 How deep would you dig an unsupported trench in a stiff clay with an undrained shear strength of 75 kPa and a unit weight of 18 kN/m^3? The contract requires that you do the digging yourself while working at the bottom of the trench.

21.10 Plot the coefficient of earth pressure at rest K_o as a function of OCR for an overconsolidated clay with a friction angle φ' equal to 28°. On the same graph, plot K_a and K_p.

21.11 Draw the earth pressure diagram for a 7 m high gravity retaining wall with a backfill compacted with a vibratory roller. The roller weighs 150 kN, has a centrifugal force amplitude of 50 kN, is 2 m wide, and gets as close as 1 m to the top edge of the wall. The soil has a unit weight of 19 kN/m^3, a passive earth pressure coefficient equal to 3, and an at-rest earth pressure coefficient equal to 0.6.

21.12 An 8 m high top-down wall is retaining a shrink-swell soil with a swell pressure profile decreasing with depth from 500 kPa at the ground surface down to 50 kPa at the bottom of the wall. The soil has a friction angle φ' equal to 28° and no cohesion c'. K_o is 0.6. Draw the pressure diagram for the wall.

21.13 Draw the displacement y_a and y_p necessary to mobilize the active and passive earth pressure as a function of the wall height H for a dense sand.

21.14 Derive equations 21.73 and 21.74.

21.15 For the retaining wall shown in Figure 21.2s, calculate the pressure distribution against the wall, the resultant push, the factor of safety against sliding, and the factor of safety against overturning.

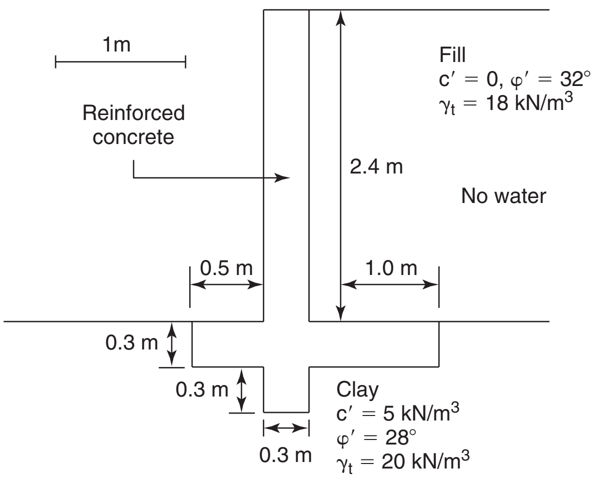


Figure 21.2s Retaining wall.

21.16 Design the soil reinforcing strips required for a 20 m high MSE wall. The precast concrete panels are 1.5 m by 1.5 m. The vertical and horizontal spacing between strips are 750 mm and 450 mm respectively. The unit weight of the backfill material is 19 kN/m^3 with an angle of internal friction of 34° and a coefficient of uniformity of 4.4. The location of the first layer of strips, measured from the finished grade, is 375 mm. Neglect the traffic surcharge.

21.17 A cantilever retaining wall is embedded 6 m below excavation level and retains 5 m of soil. An impervious layer exists 4 m below the bottom of the wall. The water level is at the ground surface on both sides of the wall and the soil deposit is uniform and deep. Draw the water pressure diagram against the wall on both sides of the wall, assuming that the water pressure is hydrostatic. Then draw a flow net and develop the water pressure diagram on both sides of the wall. Compare and comment.

21.18 Demonstrate Equation 21.91.

21.19 What is the depth of embedment d required for a cantilever wall retaining a height of sand H? Express the results as a function of H, K_p/K_a, and a factor of safety F applied to σ_p, the passive pressure. (Note: There is no water.)

21.20 For the anchored slurry wall shown in Figure 21.3s, calculate the pressure distribution on both sides of the wall for a deflection of 25 mm at the top of the wall. Calculate the anchor forces. How important is the vertical capacity of the wall? Explain your answer. What would happen if the water level rose on both sides of the excavation to the top of the wall? What would happen if the water level rose to the top of the wall on the retained-soil side of the excavation and to 2 m below that on the excavated side?

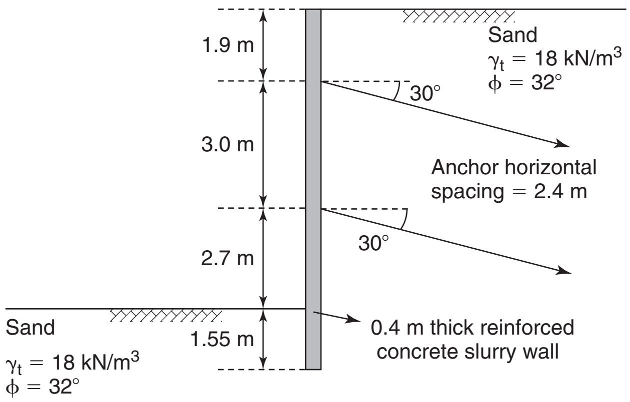


Figure 21.3s Anchored slurry wall.

21.21 Explain Figure 21.49.

21.22 Use Tables 21.4 and 21.5 and add a column giving the back-calculated alpha values.

21.23 For Figure 21.62, the height H is 9 m, α is 17°, β is 18°, ψ is 44°, and i is 10°. The stiff clay weighs 20 kN/m^3 with some cohesion c' (to be ignored), and a friction angle φ' of 32°. A uniformly applied surcharge of 10 kPa is to be considered on top of the wall. Calculate the required nail force T for a factor of safety against shear failure along the chosen plane to be 1.5. Distribute that force among the four nails and find the required length for each nail.

21.24 A 3 m wide strutted excavation is planned in a clay with an undrained shear strength equal to 40 kPa and a total unit weight of 19 kN/m^3. What depth of excavation corresponds to a factor of safety against base failure equal to 1.5?

Problems and Solutions

Problem 21.1

Show the pressure diagram, calculate the resultant push, and give its location for a 10 m high wall due to:

a. Water only
b. Dry soil with unit weight of 20 kN/m^3 and a friction angle of 30° (active and passive)
c. The same soil but with water to the top of the wall (active and passive)

Solution 21.1 (Figure 21.4s)

a. Water only

$$u_w = \gamma_w \times H$$
$$u_w = 9.81 \times 10 = 98.1 \text{ kPa}$$

The resultant push per unit length of wall is:

$$P_w = \frac{u_w \times H}{2} = \frac{\gamma_w \times H^2}{2}$$
$$P_w = \frac{9.81 \times (10)^2}{2} = 490.5 \text{ kN/m}$$

Location from the bottom of the wall:

$$z = \frac{h}{3} = 3.33 \text{ m}$$

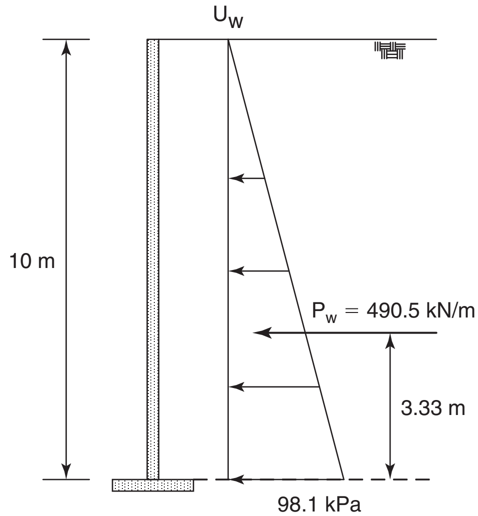


Figure 21.4s Pressure diagram for water only behind the wall.

b. Dry soil with unit weight of 20 kN/m^3 and a friction angle of 30° (active and passive)
The active force is:

$$\sigma'_{ov} = \gamma_d \times H = 20 \times 10 = 200 \text{ kPa}$$
$$P_a = \frac{K_a \times \gamma_d \times H^2}{2}$$
$$K_a = \frac{1 - \sin\varphi}{1 + \sin\varphi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 0.33$$
$$P_a = \frac{0.33 \times 20 \times (10)^2}{2} = 333.3 \text{ kN/m}$$

The passive force is:

$$P_p = \frac{K_p \times \gamma_d \times H^2}{2}$$
$$K_p = \frac{1 + \sin\varphi}{1 - \sin\varphi} = \frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} = 3.0$$
$$P_p = \frac{3 \times 20 \times (10)^2}{2} = 3000 \text{ kN/m}$$

The location of the active and passive force measured from the bottom of the wall (shown in Figure 21.5s) is:

$$z = \frac{h}{3} = 3.33 \text{ m}$$

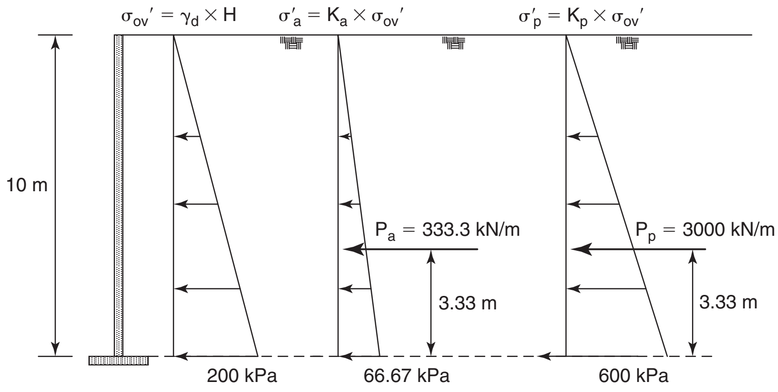


Figure 21.5s Pressure diagram for dry soil: Active and passive pressure profile.

c. Soil with water to the top of the wall (active and passive)

The vertical effective stress at the bottom of the wall is:

$$\sigma'_{ov} = \gamma_t \times H - \gamma_w \times H = 20 \times 10 - 9.81 \times 10 = 101.9 \text{ kPa}$$

The active force is

$$P_a = \frac{1}{2} H K_a \sigma'_{ov} + P_w$$

$$K_a = \frac{1 - \sin\varphi}{1 + \sin\varphi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 0.33$$

$$P_a = \frac{10 \times 0.33 \times 101.9}{2} + 490.5 = 660.2 \text{ kN/m}$$

The passive force is:

$$P_a = \frac{1}{2} H K_p \sigma'_{ov} + P_w$$

$$K_p = \frac{1 + \sin\varphi}{1 - \sin\varphi} = \frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} = 3.0$$

$$P_p = \frac{10 \times 3 \times 101.9}{2} + 490.5 = 2019 \text{ kN/m}$$

The location of the active and passive force measured from the bottom of the wall (shown in Figure 21.6s) is:

$$z = \frac{h}{3} = 3.33 \text{ m}$$

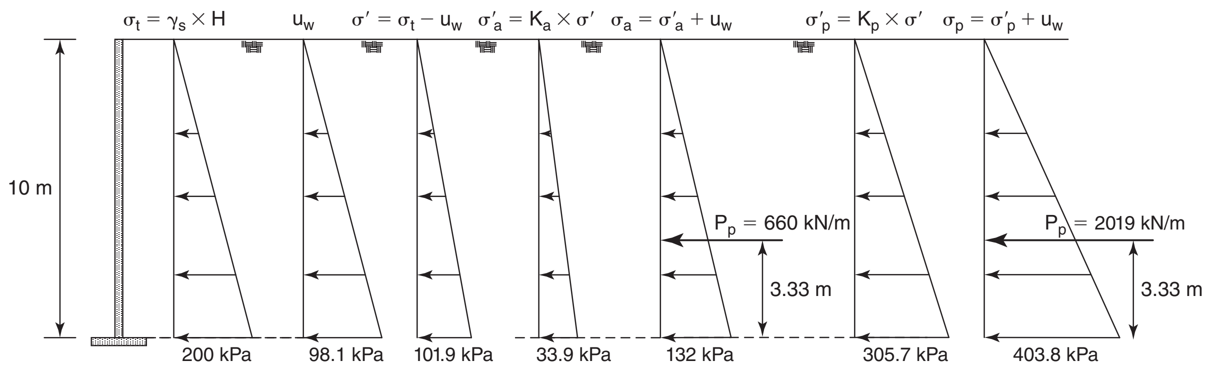


Figure 21.6s Pressure diagram for wall with water at ground surface.

Problem 21.2

Solve Coulomb's wedge analysis for the passive case of a soil with friction and cohesion. Write vertical and horizontal equilibrium and demonstrate equation 21.17.

Solution 21.2

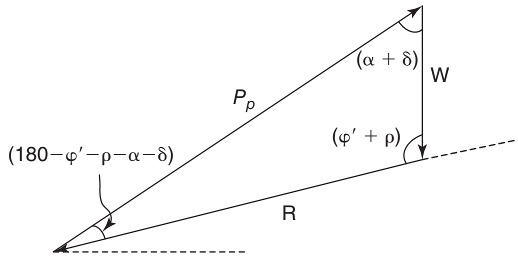


Figure 21.7s Coulomb wedge analysis for the passive case.

$$\frac{P_p}{\sin(\rho+\varphi')} = \frac{W}{\sin(180-\varphi'-\rho-\alpha-\delta)}$$

$$P_p = \frac{W\sin(\rho+\varphi')}{\sin(\varphi'+\rho+\alpha+\delta)}$$

$$W = \frac{\gamma H^2}{2\sin^2(\alpha)}\sin(\alpha+\rho)\left(\frac{\sin(\alpha+\beta)}{\sin(\rho-\beta)}\right)$$

$$P_p = \frac{\gamma H^2}{2\sin^2(\alpha)}\sin(\alpha+\rho)\left(\frac{\sin(\alpha+\beta)}{\sin(\rho-\beta)}\right)\frac{\sin(\rho+\varphi')}{\sin(\varphi'+\rho+\alpha+\delta)}$$

$$\frac{\partial P_p}{\partial\rho} = 0 \rightarrow \qquad P_p = \frac{\gamma H^2}{2}\frac{\sin^2(\alpha-\varphi')}{\sin^2(\alpha)\sin(\alpha+\delta)\left[1-\sqrt{\dfrac{\sin(\varphi'+\delta)\sin(\varphi'+\beta)}{\sin(\alpha+\delta)\sin(\alpha+\beta)}}\right]^2}$$

Problem 21.3

Plot Coulomb and Rankine active and passive earth pressure coefficients for a vertical wall, no wall friction, as a function of the ground surface inclination β. Which one would you use?

Solution 21.3

Coulomb theory

a. Figure 21.8s shows the wedge analysis in Coulomb theory for this case. As stated, the wall is vertical, and there is no wall friction. Therefore, the active earth force is acting horizontally. Note that β is the ground surface inclination, φ is the soil friction angle, and α is the failure plane inclination. H is the height of the wall, P_a is the maximum active force acting on the wall, W is the weight of the wedge, and R is the resultant force.

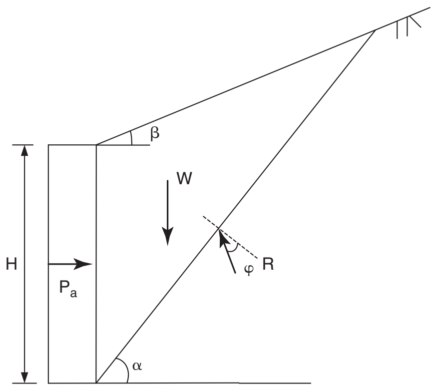


Figure 21.8s Illustration of active wedge analysis in Coulomb theory.

In this case, the expression of Coulomb active earth pressure coefficient simplifies to:

$$K_{ah} = \frac{\cos^2\varphi}{\left[1 + \sqrt{\frac{\sin\varphi \sin(\varphi - \beta)}{\cos\beta}}\right]^2}$$

To find the relationship between K_{ah} and β, a soil friction angle equal to 30° is assumed. The plot between K_a and β is shown in Figure 21.9s.

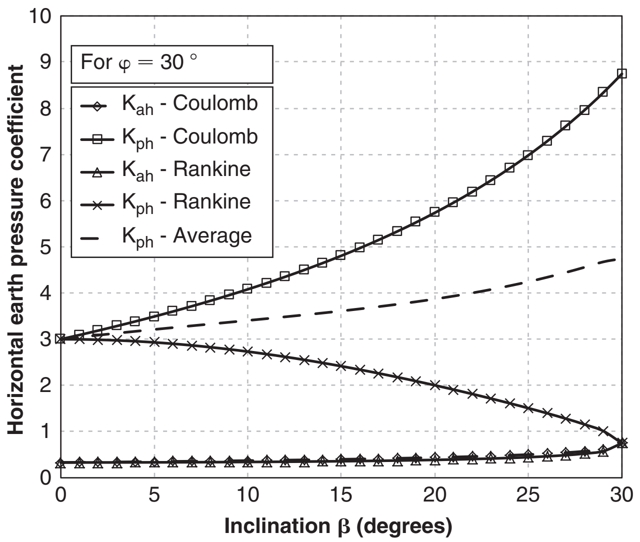


Figure 21.9s Plot of K_a and K_p vs. β using both Coulomb theory and Rankine theory.

b. Figure 21.10s shows the wedge analysis in Coulomb theory for this case. As stated, the wall is vertical, and there is no wall friction. Therefore, the passive earth force is acting horizontally. Note that β is the ground surface inclination, φ is the soil friction angle, and α is the failure plane inclination. H is the height of the wall, P_p is the maximum passive force acting on the wall, W is the weight of the wedge, and R is the resultant force.

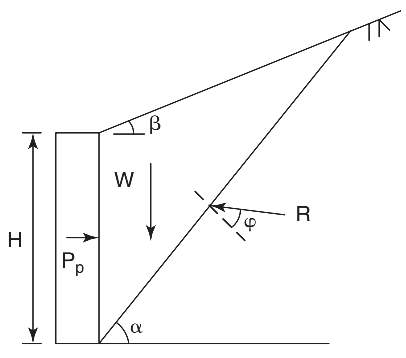


Figure 21.10s Illustration of passive wedge analysis in Coulomb theory.

In this case, the expression of Coulomb passive earth pressure coefficient simplifies to:

$$K_{ph} = \frac{\cos^2\varphi}{\left[1 - \sqrt{\frac{\sin\varphi \sin(\varphi+\beta)}{\cos\beta}}\right]^2}$$

To find the relationship between K_{ph} and β, a soil friction angle equal to 30° is assumed. The plot between K_{ph} and β is shown in Figure 21.9s.

Rankine theory

Fig 21.11s shows an illustration of the retaining wall analyzed using Rankine theory.

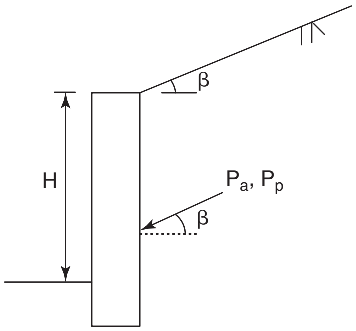


Figure 21.11s Retaining wall analyzed using Rankine theory.

Note that β is the ground surface inclination, φ is the soil friction angle, H is the height of the wall, P_p is the passive earth force, and P_a is the active earth force.

a. Based on Rankine theory, the active earth pressure coefficient that gives the horizontal component of P_a is:

$$K_{ah} = \cos^2\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}$$

To find the relationship between K_{ah} and β, a soil friction angle equal to 30° is assumed. The plot between K_{ah} and β is shown in Figure 21.9s.

b. Based on Rankine soil theory, the passive earth pressure coefficient that gives the horizontal component of P_a is:

$$K_{ph} = \cos^2\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}$$

To find the relationship between K_{ph} and β, a soil friction angle equal to 30° is assumed. The plot between K_{ph} and β is shown in Figure 21.9s.

Discussion

Figure 21.9s shows that for K_{ph} the Coulomb solution is a limit equilibrium solution giving upper-bound values, whereas the Rankine solution is an equilibrium-of-stresses solution that gives lower-bound values. Therefore, if a lower bound is conservative, one should choose Rankine theory; if an upper bound is conservative, one should choose Coulomb theory. To that end one would be tempted to use an average of the two values as a more reasonable estimate; however such an average is not based on any theoretical reasoning. Note that in the case of extreme values of the geometry parameters, it is advisable to use engineering judgment, as the K_a and K_p values can become unreasonable.

Problem 21.4

Evaluate the influence of wall friction on the active and passive earth pressure coefficients by comparing Rankine value (no friction) and Coulomb values (varying friction angle from 0 to φ') for a vertical wall and horizontal backfill. Which one would you use?

Solution 21.4

Active Pressure

For a vertical wall and horizontal backfill, the Coulomb value K_{ah} that gives the horizontal component P_{ah} of the active push P_a is:

$$K_{ah} = \frac{\sin^2(90+\varphi')}{\left[1+\sqrt{\frac{\sin(\varphi'+\delta)\sin\varphi'}{\sin(90-\delta)}}\right]^2}$$

and the Rankine value is:

$$K_{ah} = \frac{1 - \sin\varphi'}{1 + \sin\varphi}$$

Figure 21.12s shows the K_{ah} values versus friction angle for both the Coulomb and Rankine solutions. For Coulomb, different curves are presented for different values of the wall friction. As can be seen from the figure, the Coulomb value of K_{ah} decreases as the wall friction increases; the Rankine value does not change. The maximum value of Coulomb K_{ah} is reached for zero wall friction, which is equal to the Rankine value.

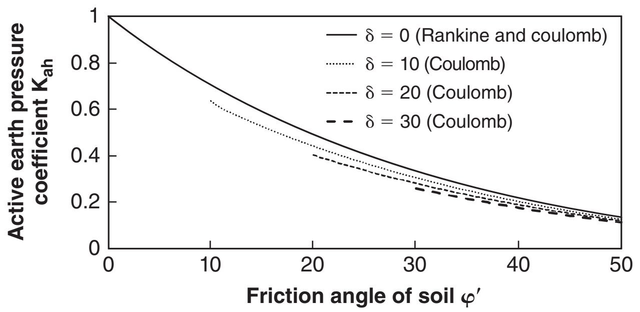


Figure 21.12s K_{ah} vs. soil friction angle for different wall friction angle.

Passive Pressure

Coulomb:

$$K_{ph} = \frac{\sin^2(90 - \varphi')}{\left[1 - \sqrt{\frac{\sin\left(\varphi' + \delta\right)\sin\varphi'}{\sin(90 + \delta)}}\right]^2}$$

Rankine:

$$K_{ph} = \frac{1 + \sin\varphi'}{1 - \sin\varphi}$$

Figure 21.13s shows the K_{ph} values versus friction angle for both the Coulomb and Rankine solutions. For Coulomb, different curves are presented for different values of the wall friction.

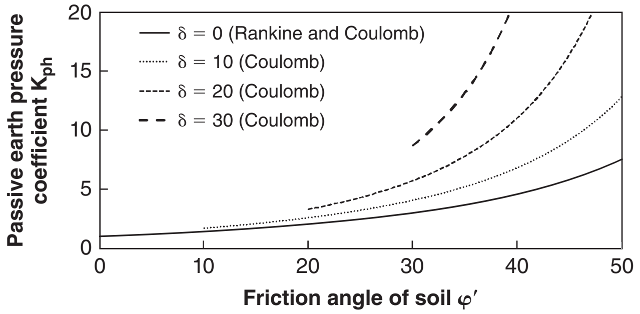


Figure 21.13s K_{ph} vs. soil friction angle for different wall friction angle.

Which one would you use? Rankine solution generally gives reasonable values. Coulomb theory is also reasonable, in that it takes into account wall friction in the case of the active earth pressure, but Coulomb's passive earth pressure values are quite optimistic and should not be used. The problem is that the failure surface is optimistically chosen as a straight line instead of a curved surface which would offer less resistance.

Problem 21.5

Demonstrate that the direction of the plane of failure for the active pressure case (PB in Figure 21.11) is equal to $45 + \varphi'/2$.

Solution 21.5

First we find the Pole on the failure circle in the active case (Figure 21.14s). The failure point is shown by point T on the failure circle. If we draw a line from the left of the failure circle M to T, the angle $\widehat{MTO}$ will be 90°. In the triangle TOM:

$$\widehat{TMO} = 180 - (90 + \varphi') = 90 - \varphi'$$

In the triangle TPM:

$$\widehat{MPT} = \widehat{MTP}$$

$$2 \times \widehat{MPT} + (90 - \varphi') = 180 \rightarrow \widehat{MPT} = 45 + \frac{\varphi'}{2}$$

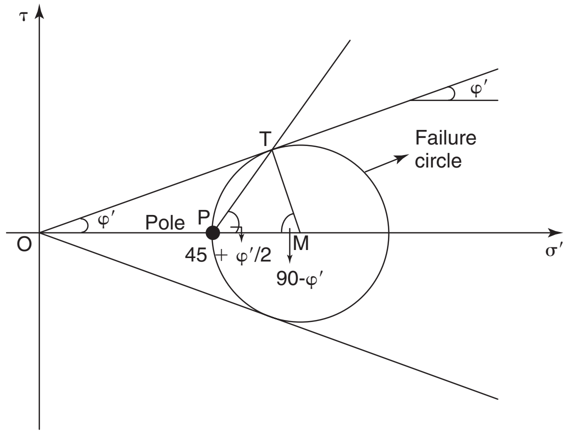


Figure 21.14s Pole method.

Problem 21.6

A 6 m high retaining wall has a backfill made of unsaturated sandy silt with a water tension equal to -1000 kPa and an area ratio (α) equal to 0.3. The total unit weight is 20 kN/m^3. The wall has no effective stress cohesion ($c' = 0$), and an effective stress friction angle equal to 30° ($\varphi' = 30$). The backfill is horizontal and the wall friction is neglected. Calculate the active and passive earth pressure diagram for this wall.

Solution 21.6

Use Rankine theory to solve this problem.

Active Earth Pressure

The active earth pressure coefficient for this problem is:

$$K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'} = \frac{1}{3}$$

Because the soil behind the wall is uniform, we only need to choose two calculation points: point a and b, shown in Figure 21.15s.

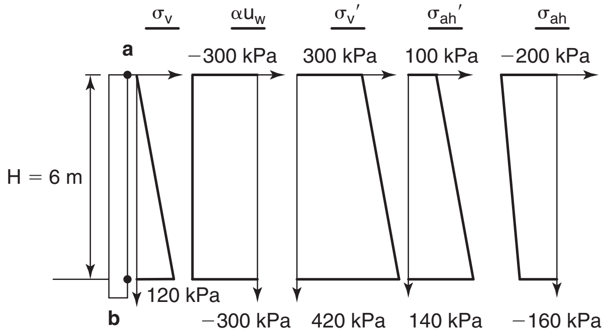


Figure 21.15s Active earth pressure diagram.

Point a: total vertical stress $\sigma_v = 0$

Therefore, effective vertical stress $\sigma'_v = \sigma_v - \alpha u_w = 0 + 0.3 \times (-1000) = 300$ kPa

Effective active horizontal stress is $\sigma'_{ah} = K_a \sigma'_v = \frac{1}{3} \times 300 = 100$ kPa

Total active horizontal stress is $\sigma_{ah} = \sigma'_{ah} + \alpha u_w = 100 + (-300) = -200$ kPa

Point b: total vertical stress $\sigma_v = \gamma_t H = 20 \times 6 = 120$ kPa

Therefore, effective vertical stress $\sigma'_v = \sigma_v - \alpha u_w = 120 - 0.3 \times (-1000) = 420$ kPa

Effective active horizontal stress is $\sigma'_{ah} = K_a \sigma'_v = \frac{1}{3} \times 420 = 140$ kPa

Total active horizontal stress is $\sigma_{ah} = \sigma'_{ah} + \alpha u_w = 140 + (-300) = -160$ kPa

The active earth pressure diagram is shown in Figure 21.15s. Practically, the suction should be ignored as it could disappear in the rainy season or cracks could develop in the backfill and the active earth pressure diagram would be the same as if the soil had no water.

Passive Earth Pressure:

The passive earth pressure coefficient for this problem is

$$K_p = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} = 3$$

Point a: Effective passive horizontal stress is $\sigma'_{ph} = K_p \sigma'_v = 3 \times 300 = 900$ kPa

Total passive horizontal stress is $\sigma_{ph} = \sigma'_{ph} + \alpha u_w = 900 + (-300) = 600$ kPa

Point b: Effective passive horizontal stress is $\sigma'_{ph} = K_p \sigma'_v = 3 \times 420 = 1260$ kPa

Total passive horizontal stress is $\sigma_{ph} = \sigma'_{ph} + \alpha u_w = 1260 + (-300) = 960$ kPa

The passive earth pressure diagram is shown in Figure 21.16s. Practically, the suction would be ignored and the passive earth pressure diagram would be the same as if the soil had no water.

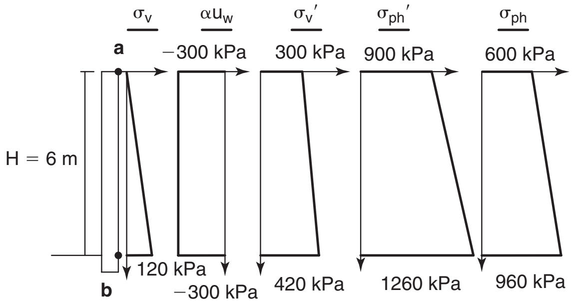


Figure 21.16s Passive earth pressure diagram.

Problem 21.7

A wall is to be placed in a soil as described in Figure 21.1s. Prepare the active pressure diagram and the passive pressure diagram for that soil profile.

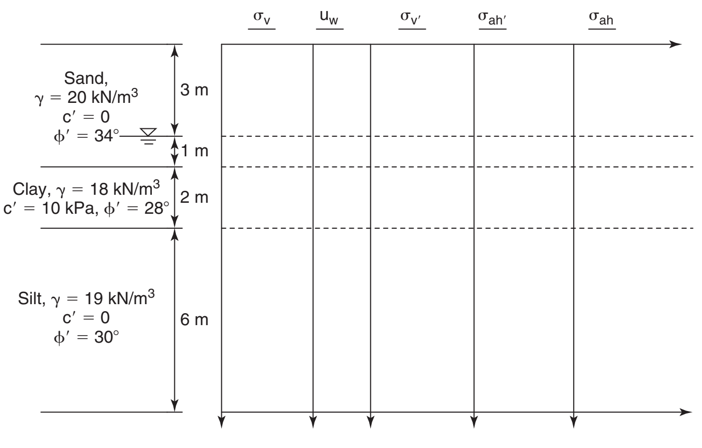


Figure 21.1s Soil profile.

Solution 21.7

Use Rankine theory to solve this problem.

For each layer, the active and passive earth pressure coefficients are calculated as follows:

$$\text{Sand}: K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'} = 0.283,\ K_p = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} = 3.54$$

$$\text{Clay}: K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'} = 0.361,\ K_p = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} = 2.77$$

$$\text{Silt}: K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'} = 0.333,\ K_p = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} = 3$$

Active Earth Pressure

Because several soil layers and a groundwater level are involved in this problem, 5 calculation points are chosen (Figure 21.17s).

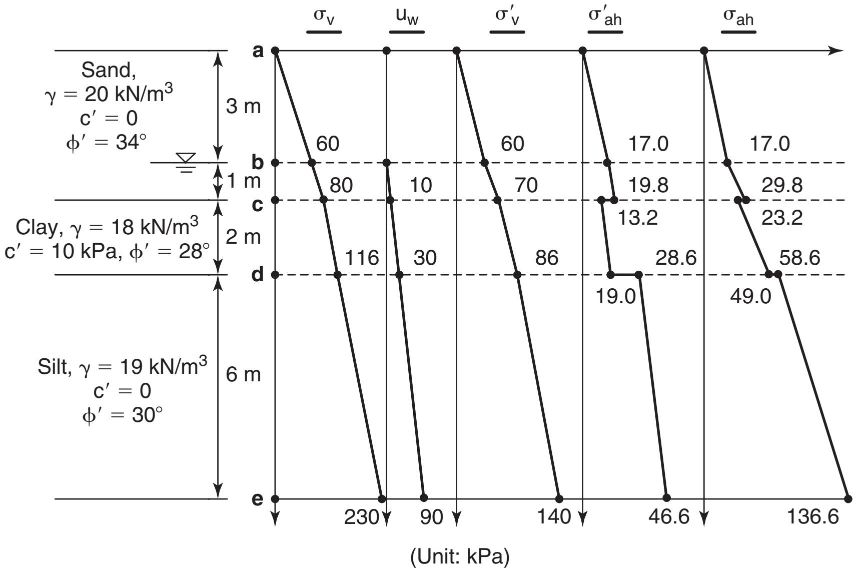


Figure 21.17s Active earth pressure diagram.

Point a: Total vertical stress: $\sigma_v = 0$, $u_w = 0$

Therefore, effective vertical stress:

$$\sigma_v' = \sigma_v - u_w = 0$$

Effective active horizontal stress is $\sigma_{ah}' = K_a\sigma_v' - 2c'\sqrt{K_a} = 0$

Total active horizontal stress is $\sigma_{ah} = \sigma_{ah}' + u_w = 0$

Point b: Total vertical stress:

$\sigma_v = \gamma h = 20 \times 3 = 60$ kPa, $u_w = 0$

Therefore, effective vertical stress:

$$\sigma_v' = \sigma_v - u_w = 60 \text{ kPa}$$

Effective active horizontal stress is $\sigma_{ah}' = K_a\sigma_v' - 2c'\sqrt{K_a} = 0.283 \times 60 - 2 \times 0 = 17.0$ kPa

Total active horizontal stress is $\sigma_{ah} = \sigma_{ah}' + u_w = 17.0$ kPa

Point c: Total vertical stress $\sigma_v = \gamma h = 60 + 20 = 80$ kPa, $u_w = 10 \times 1 = 10$ kPa

Therefore, effective vertical stress:

$$\sigma_v' = \sigma_v - u_w = 70 \text{ kPa}$$

Note that point c is on the interface between two different layers, so the effective active horizontal stress at that point should be calculated individually in each layer.

Point c, sand: Effective active horizontal stress is $\sigma'_{ah} = K_a\sigma'_v - 2c'\sqrt{K_a} = 0.283 \times 70 - 2 \times 0 = 19.8$ kPa
Total active horizontal stress is

$$\sigma_{ah} = \sigma'_{ah} + u_w = 19.81 + 10 = 29.8 \text{ kPa}$$

Point c, clay: Effective active horizontal stress is $\sigma'_{ah} = K_a\sigma'_v - 2c'\sqrt{K_a} = 0.361 \times 70 - 2 \times 10 \times \sqrt{0.361} =$ 13.2 kPa
Total active horizontal stress is

$$\sigma_{ah} = \sigma'_{ah} + u_w = 13.2 + 10 = 23.2 \text{ kPa}$$

Point d: Total vertical stress: $\sigma_v = \sum \gamma h = 80 + 18 \times 2 = 116$ kPa, $u_w = 10 \times 3 = 30$ kPa
Therefore, effective vertical stress: $\sigma'_v = \sigma_v - u_w = 116 - 30 = 86$ kPa
Note that point d is on the interface between two different layers, so the effective active horizontal stress at that point should be calculated individually in each layer.
Point d, clay: Effective active horizontal stress is $\sigma'_{ah} = K_a\sigma'_v - 2c'\sqrt{K_a} = 0.361 \times 86 - 2 \times 10 \times \sqrt{0.361} =$ 19.0 kPa
Total active horizontal stress is $\sigma_{ah} = \sigma'_{ah} + u_w = 19.0 + 30 = 49.0$ kPa
Point d, silt: Effective active horizontal stress is $\sigma'_{ah} = K_a\sigma'_v - 2c'\sqrt{K_a} = 0.333 \times 86 - 2 \times 0 = 28.6$ kPa
Total active horizontal stress is $\sigma_{ah} = \sigma'_{ah} + u_w = 28.6 + 30 = 58.6$ kPa
Point e: Total vertical stress: $\sigma_v = \sum \gamma h = 116 + 19 \times 6 = 230$ kPa, $u_w = 10 \times 9 = 90$ kPa
Therefore, effective vertical stress:

$$\sigma'_v = \sigma_v - u_w = 230 - 90 = 140 \text{ kPa}$$

Effective active horizontal stress is $\sigma'_{ah} = K_a\sigma'_v - 2c'\sqrt{K_a} = 0.333 \times 140 - 2 \times 0 = 46.6$ kPa
Total active horizontal stress is $\sigma_{ah} = \sigma'_{ah} + u_w = 46.6 + 90 = 136.6$ kPa
The active earth pressure diagram is shown in Figure 22.17s.

Passive Earth Pressure

Because several soil layers and a groundwater level are involved in this problem, 5 calculation points are chosen (Figure 21.18s.).

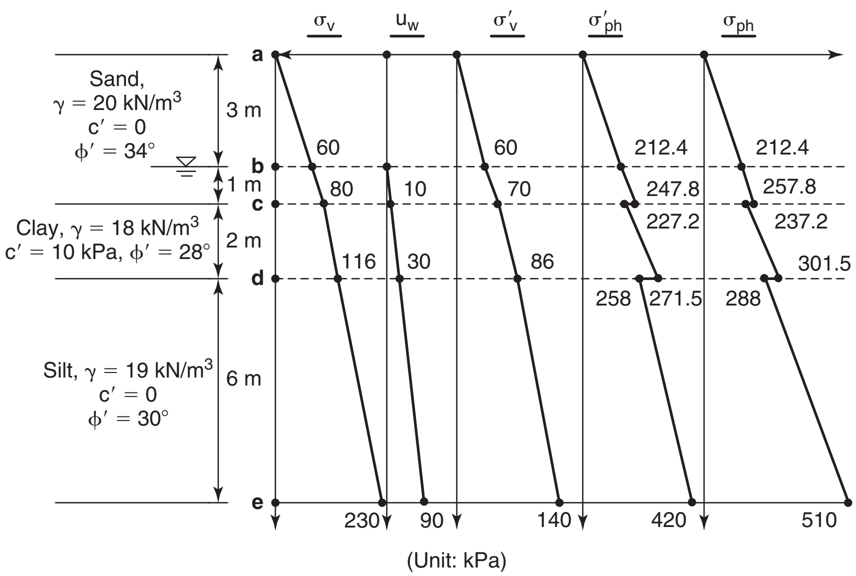


Figure 21.18s Passive earth pressure diagram.

For those calculation points, the vertical stresses, water stress, and effective vertical stresses are the same as for the active earth pressure. Here we only provide the calculation of effective passive horizontal stress and total passive horizontal stress at those five points.

Point a: Effective passive horizontal stress is

$$\sigma'_{ph} = K_p \sigma'_v + 2c' \sqrt{K_p} = 0$$

Total passive horizontal stress is

$$\sigma_{ph} = \sigma'_{ph} + u_w = 0$$

Point b: Effective passive horizontal stress is

$$\sigma'_{ph} = K_p \sigma'_v + 2c' \sqrt{K_p} = 3.54 \times 60 + 2 \times 0 = 212.4 \text{ kPa}$$

Total passive horizontal stress is

$$\sigma_{ph} = \sigma'_{ph} + u_w = 212.4 \text{ kPa}$$

Point c: Note that point c is on the interface between two different layers, so the effective passive horizontal stress at that point should be calculated individually in each layer.Point c, sand: Effective passive horizontal stress is

$$\sigma'_{ph} = K_p \sigma'_v + 2c' \sqrt{K_p} = 3.54 \times 70 + 2 \times 0 = 247.8 \text{ kPa}$$

Total passive horizontal stress is

$$\sigma_{ph} = \sigma'_{ph} + u_w = 247.8 + 10 = 257.8 \text{ kPa}$$

Point c, clay: Effective passive horizontal stress is

$$\sigma'_{ph} = K_p \sigma'_v + 2c' \sqrt{K_p} = 2.77 \times 70 + 2 \times 10 \times \sqrt{2.77} = 227.2 \text{ kPa}$$

Total passive horizontal stress is

$$\sigma_{ph} = \sigma'_{ph} + u_w = 227.2 + 10 = 237.2 \text{ kPa}$$

Point d: Note that point d is on the interface between two different layers, so the effective passive horizontal stress at that point should be calculated individually in each layer.

Point d, clay: Effective passive horizontal stress is

$$\sigma'_{ph} = K_p \sigma'_v + 2c' \sqrt{K_p} = 2.77 \times 86 + 2 \times 10 \times \sqrt{2.77} = 271.5 \text{ kPa}$$

Total passive horizontal stress is

$$\sigma_{ph} = \sigma'_{ph} + u_w = 271.5 + 30 = 301.5 \text{ kPa}$$

Point d, silt: Effective passive horizontal stress is

$$\sigma'_{ph} = K_p \sigma'_v + 2c' \sqrt{K_p} = 3 \times 86 + 2 \times 0 = 258 \text{ kPa}$$

Total passive horizontal stress is

$$\sigma_{ph} = \sigma'_{ph} + u_w = 258 + 30 = 288 \text{ kPa}$$

Point e: Effective passive horizontal stress is

$$\sigma'_{ph} = K_p\sigma'_v + 2c'\sqrt{K_p} = 3 \times 140 + 2 \times 0 = 420 \text{ kPa}$$

Total passive horizontal stress is

$$\sigma_{ph} = \sigma'_{ph} + u_w = 420 + 90 = 510 \text{ kPa}$$

The passive earth pressure diagram is shown in Figure 21.18s.

Problem 21.8

A 10 m high retaining wall has a horizontal backfill made of soil without water. The soil properties are $\gamma = 20$ kN/m^3, c$' = 0$, $\varphi' = 30°$. Draw the active pressure diagram against the wall due to the following surcharges at the top of the wall:

a. Uniform surcharge equal to 20 kPa
b. Line load of 20 kN/m at a distance of 1 m from the edge of the wall
c. A point load of 20 kN at a distance of 1 m from the edge of the wall

Solution 21.8 (Figure 21.19s)

$$K_a = \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3}$$

$$\sigma_{ah} = \sigma'_{ov}K_a + \Delta\sigma_h$$

a.

$$\sigma_{ah} = \frac{1}{3} \times (20z + 20) = 6.67(z + 1)$$

b.

$$\sigma_{ah} = \frac{1}{3} \times 20z + \frac{4 \times 20}{\pi}\frac{1^2 z}{(z^2 + 1)^2} = 6.67z + \frac{25.46z}{(1 + z^2)^2}$$

c. Assume that $v = 0.35$

$$\sigma_{ah} = \frac{1}{3}z + \frac{20}{\pi(z^2 + 1)}\left(\frac{3z}{\left(z^2 + 1\right)^{\frac{3}{2}}} - \frac{(z^2 + 1)^{\frac{1}{2}}(1 - 2v)}{(z^2 + 1)^{\frac{1}{2}} + z}\right)$$

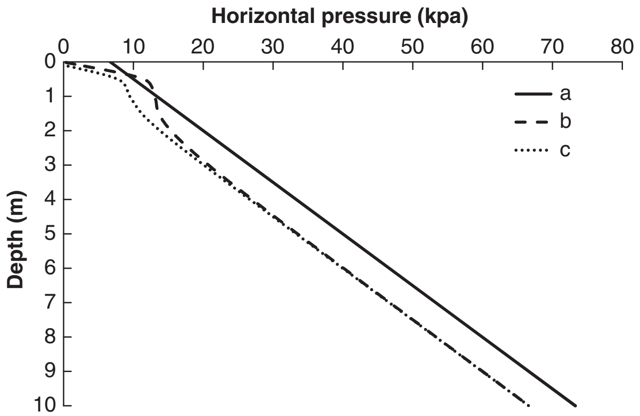


Figure 21.19s Horizontal pressure diagram.

Problem 21.9

How deep would you dig an unsupported trench in a stiff clay with an undrained shear strength of 75 kPa and a unit weight of 18 kN/m^3? The contract requires that you do the digging yourself while working at the bottom of the trench.

Solution 21.9

Assuming that the soil is truly uniform, with no fissures:

$$\sigma_{ah} = \sigma_{0v} - 2S_u$$
$$\sigma_{ah} = \gamma H - 2S_u = 18H - 2 \times 75 = 0 \to H = 4.16 \text{ m}$$

I would dig the trench to a depth of 1 m and stop there. Before going deeper, I would place a trench box to protect myself against trench collapse. Then I would dig further.

Problem 21.10

Plot the coefficient of earth pressure at rest K_o as a function of OCR for an overconsolidated clay with a friction angle φ' equal to 28°. On the same graph, plot K_a and K_p.

Solution 21.10 (Figure 21.20s)

$$K_0 = (1 - \sin\varphi')OCR^{\sin\varphi'}$$
$$K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 28^\circ}{1+\sin 28^\circ} = 0.36$$
$$K_p = \frac{1+\sin\varphi}{1-\sin\varphi} = 2.77$$

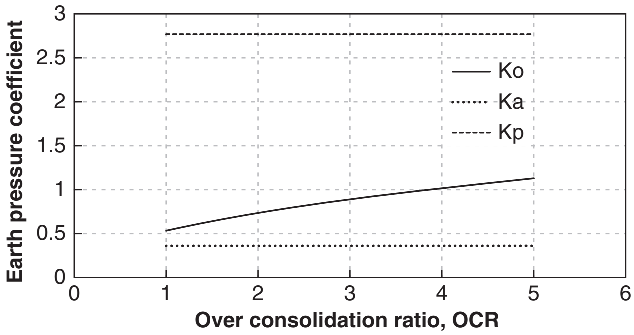


Figure 21.20s Earth pressure coefficients vs. OCR.

Problem 21.11

Draw the earth pressure diagram for a 7 m high gravity retaining wall with a backfill compacted with a vibratory roller. The roller weighs 150 kN, has a centrifugal force amplitude of 50 kN, is 2 m wide, and gets as close as 1 m to the top edge of the wall. The soil has a unit weight of 19 kN/m^3, a passive earth pressure coefficient equal to 3, and an at-rest earth pressure coefficient equal to 0.6.

Solution 21.11 (Figure 21.21s)

$$\sigma_h = \frac{L}{a+L}\sqrt{\frac{2P\gamma}{\pi}} = \frac{2}{1+2}\sqrt{\frac{2\left(\frac{150+50}{2}\right) \times 19}{\pi}} = 23.2 \text{ kN/m}^2$$
$$d = \frac{L}{K_o(a+L)}\sqrt{\frac{2P}{\pi\gamma}} = \frac{2}{0.6(1+2)}\sqrt{\frac{2\left(\frac{150+50}{2}\right)}{\pi \times 19}} = 2.0 \text{ m}$$
$$\mathrm{K_o}\gamma \mathrm{z} = 0.6 \times 19 \times 7 = 79.8 \text{ kN/m}^2$$

L: the length of the roller
a: the distance between the edge of the wall and the closest roller position
P: the line load imposed by the roller weight of the roller plus the centrifugal force for vibratory rollers
γ: the unit weight of the soil

K_0: the at-rest earth pressure coefficient
d: depth to which the pressure diagram is modified due to the roller

The depth z to reach the horizontal pressure equal to 23.2 kPa is such that $K_p\gamma z = 23.2$ kPa, therefore, z = 0.41 m.

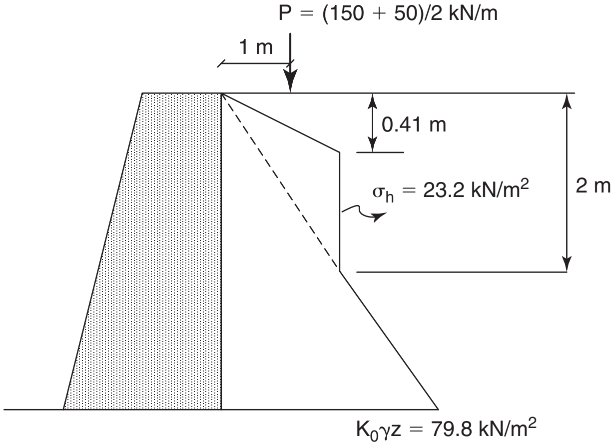


Figure 21.21s Earth pressure diagram.

Problem 21.12

An 8 m high top-down wall is retaining a shrink-swell soil with a swell pressure profile decreasing with depth from 500 kPa at the ground surface down to 50 kPa at the bottom of the wall. The soil has a friction angle φ' equal to 28° and no cohesion c'. K_o is 0.6. Draw the pressure diagram for the wall.

Solution 21.12 (Figure 21.22s)

$$K_0 = 0.6$$

$$K_p = \frac{1+\sin\varphi'}{1-\sin\varphi'} = \frac{1+\sin 28^\circ}{1-\sin 28^\circ} = 2.77$$

Assuming that the soil unit weight is γ =18 kN/m^3:

$$K_o\gamma h = 0.6 \times 18 \times 8 = 86.4 \text{ kPa}$$

$$K_p\gamma h = 2.77 \times 18 \times 8 = 398.88 \text{ kPa}$$

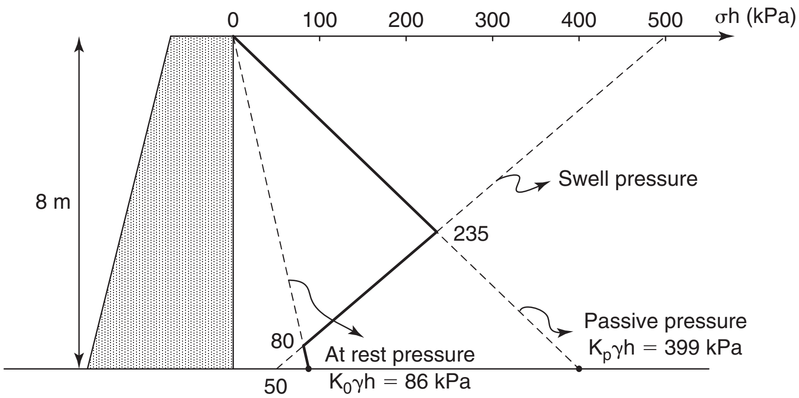


Figure 21.22s Earth pressure diagram.

Problem 21.13

Draw the displacement y_a and y_p necessary to mobilize the active and passive earth pressure as a function of the wall height H for a dense sand.

Solution 21.13 (Figures 21.23s, 21.24s)

From Table 21.1, the average displacements needed to generate active and passive earth pressures for different soil types are:

Loose sand	Soft clay
$\frac{y_a}{H} = 0.004,\ \frac{y_p}{H} = 0.04$	$\frac{y_a}{H} = 0.015,\ \frac{y_p}{H} = 0.04$
Dense sand	Stiff clay
$\frac{y_a}{H} = 0.0015,\ \frac{y_p}{H} = 0.02$	$\frac{y_a}{H} = 0.0075,\ \frac{y_p}{H} = 0.02$

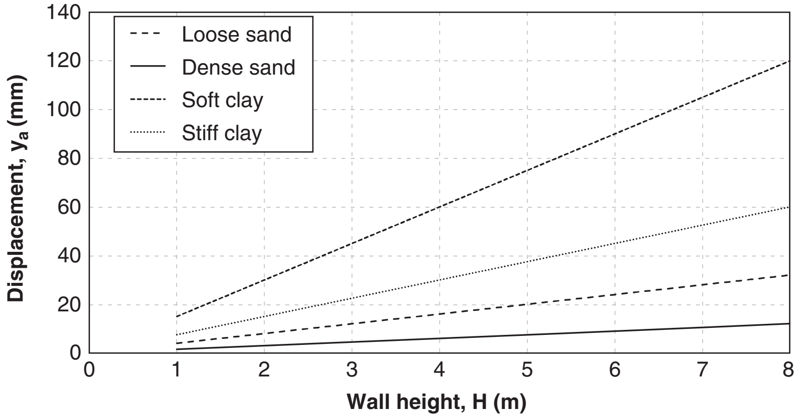


Figure 21.23s Active displacement vs. wall height.

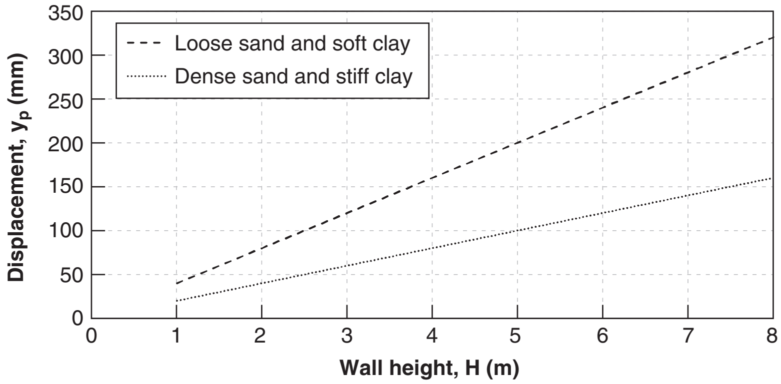


Figure 21.24s Passive displacement vs. wall height.

Problem 21.14

Derive equations 21.73 and 21.74.

Solution 21.14

The uniform soil pressure p_1 due to the line load is:

$$p_1 = \frac{Q}{B}$$

The soil pressure p_2 due to overturning moment is the maximum pressure at the edge of the triangular distribution under the foundation. The pressure distribution under the foundation must resist the moment. Writing the moment equilibrium gives:

$$\frac{1}{2}p_2\frac{B}{2} \times \frac{2}{3}\frac{B}{2} \times 2 = M \Rightarrow p_2 = \pm\frac{6M}{B^2}$$

$$p = \frac{Q}{B} \pm \frac{6M}{B^2} \& e = \frac{M}{Q} \Rightarrow \begin{cases} p_{\max} = \frac{Q}{B}\left(1 + \frac{6e}{B}\right) \\ p_{\min} = \frac{Q}{B}\left(1 - \frac{6e}{B}\right) \end{cases}$$

Problem 21.15

For the retaining wall shown in Figure 21.2s, calculate the pressure distribution against the wall, the resultant push, the factor of safety against sliding, and the factor of safety against overturning.

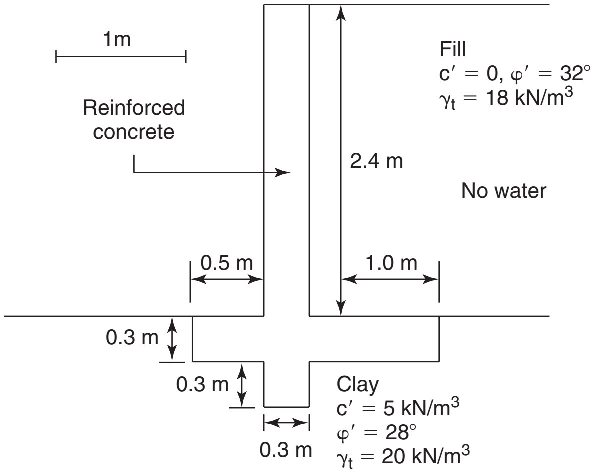


Figure 21.2s Retaining wall.

Solution 21.15 (Figures 21.25s, 21.26s)

Passive earth pressure, active earth pressure:

$$\sigma_{ph} = \mathrm{K_p}\sigma'_{ov} + 2c'\sqrt{\mathrm{K_p}} + \alpha u \qquad \sigma_{ah} = \mathrm{K_a}\sigma'_{ov} - 2c'\sqrt{\mathrm{K_a}} + \alpha u$$

$$\mathrm{K_p} = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} \qquad K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}$$

a. Calculate active earth pressure (active side):

$$\mathrm{K_{a1}} = \frac{1 - \sin 32}{1 + \sin 32} = 0.307$$

$$\mathrm{K_{a2}} = \frac{1 - \sin 28}{1 + \sin 28} = 0.361$$

$\sigma'_{ah} = 0.307 \text{ x } 43.2 = 13.26$ kPa at a depth of 2.4 m (in the fill)

$\sigma'_{ah} = 0.361 \text{ x } 43.2–2 \text{ x } 5\sqrt{0.361} = 9.59$ kPa at a depth of 2.4 m (in the clay)

$\sigma'_{ah} = 0.361 \text{ x } 55.2–2 \text{ x } 5\sqrt{0.361} = 13.92$ kPa at a depth of 3.0 m (in the clay)

Since there is no water $\sigma'_{ah} = \sigma_{ah}$

b. Calculate passive earth pressure (passive side):

$$K_p = \frac{1+\sin\varphi'}{1-\sin\varphi'} = \frac{1+\sin 28}{1-\sin 28} = 2.77$$

$$\sigma'_{ph} = K_P\sigma'_{ov} + 2c'\sqrt{K_p} = 16.64 \text{ kPa at the ground level}$$

$$\sigma'_{ph} = K_P\sigma'_{ov} + 2c'\sqrt{K_p} = 2.77 \times 12 + 10\sqrt{2.77} = 49.88\,\text{kPa at a depth of } 0.6\,\text{m}$$

Since there is no water $\sigma'_{ph} = \sigma_{ph}$

c. Draw diagram (Figure 21.2s).

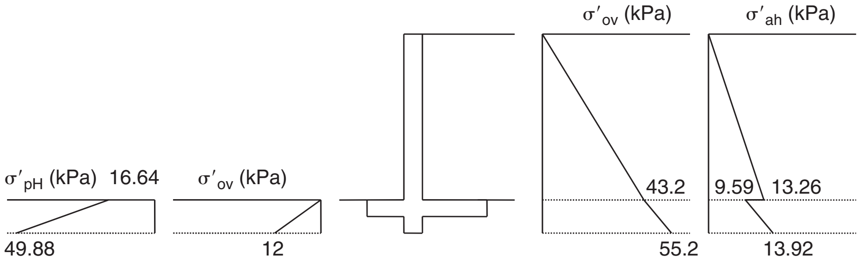


Figure 21.25s Earth pressure diagram.

$$P_{ah} = \frac{1}{2} \times 13.26 \times 2.4 + 9.59 \times 0.6 + \frac{1}{2} \times (13.92 - 9.59) \times 0.6 = 22.96\,\text{kN}$$

$$X_a = \frac{\frac{1}{2} \times 13.26 \times 2.4 \times 1.4 + 9.59 \times 0.6 \times 0.3 + \frac{1}{2} \times (13.92 - 9.59) \times 0.6 \times 0.2}{22.965} = \frac{24.26}{22.96} = 1.06\,\text{m}$$

$$P_{ph} = 16.64 \times 0.6 + \frac{1}{2} \times (49.88 - 16.64) \times 0.6 = 19.96\,\text{kN}$$

$$X_p = \frac{16.64 \times 0.6 \times 0.3 + \frac{1}{2} \times (49.88 - 16.64) \times 0.6 \times 0.2}{19.956} = \frac{4.99}{19.96} = 0.25\,\text{m}$$

$$W_{soil1} = 2.4\text{ m} \times 1\text{ m} \times 18\text{ kN/m}^2 = 43.2\text{ kN/m}$$

$$W_{soil2} = 0.5\text{ m} \times 0.3\text{ m} \times 20\text{ kN/m}^2 = 3\text{ kN/m}$$

$$W_{soil3} = 1\text{ m} \times 0.3\text{ m} \times 20\text{ kN/m}^2 = 6\text{ kN/m}$$

$$W_{stem} = (2.4\text{ m} + 0.3\text{ m}) \times 0.3\text{ m} \times 25\text{ kN/m}^2 = 20.25\text{ kN/m}$$

$$W_{base} = 1.8\text{ m} \times 0.3\text{ m} \times 25\text{ kN/m}^2 = 13.5\text{ kN/m}$$

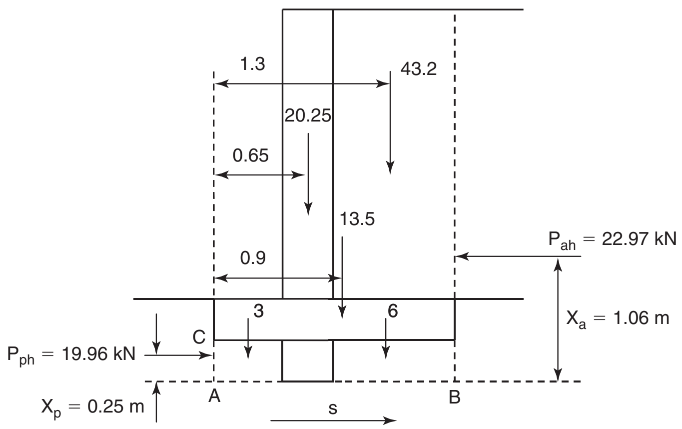


Figure 21.26s Forces diagram.

Considering a sliding failure along AB:

$$F_{\text{sliding}} = \frac{\sum W \tan\varphi' + P_{pH}}{P_{aH}} = \frac{(43.2 + 20.25 + 13.5 + 6 + 3) \times \tan 28^\circ + 19.96}{22.97}$$

$$= \frac{45.70 + 19.96}{22.97} = 2.84 > 2 \rightarrow OK$$

Considering a rotation failure around point A:

$$F_{\text{overturning}} = \frac{M_{\text{max,resist}}}{M_{\text{driving}}} = \frac{43.2 \times 1.3 + 20.25 \times 0.65 + 13.5 \times 0.9 + 19.96 \times 0.25}{22.97 \times 1.06}$$

$$= \frac{86.46}{24.35} = 3.55 > 2 \rightarrow OK$$

It is also reasonable to consider rotation failure around point C:

$$F_{\text{overturning}} = \frac{M_{\text{max,resist}}}{M_{\text{driving}}} = \frac{43.2 \times 1.3 + 20.25 \times 0.65 + 13.5 \times 0.9}{22.97 \times 0.76} = \frac{81.47}{17.46} = 4.67 > 2 \rightarrow OK$$

Problem 21.16

Design the soil reinforcing strips required for a 20 m high MSE wall. The precast concrete panels are 1.5 m by 1.5 m. The vertical and horizontal spacing between strips are 750 mm and 450 mm respectively. The unit weight of the backfill material is 19 kN/m^3 with an angle of internal friction of 34° and a coefficient of uniformity of 4.4. The location of the first layer of strips, measured from the finished grade, is 375 mm. Neglect the traffic surcharge.

Solution 21.16

$$\text{Panel section} = 1.5 \text{ m} \times 1.5 \text{ m}$$
$$s_v = 0.75 \text{ mm}$$
$$s_h = 450 \text{ mm}$$
$$\gamma_{\text{soil}} = 19 \text{ kN/m}^3$$
$$C_u = 4.4$$

a. Design for Pullout

The maximum line load ($T_{\max}$) to be resisted by the reinforcement inclusions at depth z can be computed as:

$$T_{\max} = s_v s_h \sigma_h$$

The horizontal stress σ_h can be calculated as:

$$\sigma_h = k_r \sigma_{ov} + \Delta\sigma_h$$
$$\sigma_h = k_r \sigma_{ov}$$

The coefficient of earth pressure k_r is computed using Figure 21.27s (AASHTO). The k_a value is computed as:

$$k_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'} = \frac{1 - \sin 34}{1 + \sin 34} = 0.283$$

Then the k_r value is computed as:
a-1. If z_i is less than 6 m, then:

$$\frac{k_r}{k_a} = 1.7 - \frac{z_i}{12}$$

a-2. If z_i is larger than 6 m, then:

$$\frac{k_r}{k_a} = 1.2$$

The calculation of T_{max} for the different strips is summarized in Table 21.1s.

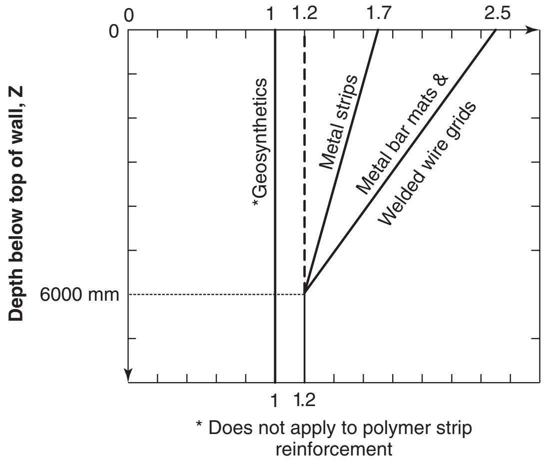


Figure 21.27s Coefficient of lateral stress ratio = k_r/k_a.

Table 21.1s Summary of Calculation of T_{max}

Strip No.	Depth (m)	k_a	k_r/k_a	k_r	σ_v (kPa)	σ_h (kPa)	T_{max} (kN)
1	0.375	0.283	1.67	0.472	7.1	3.365	1.14
2	1.125	0.283	1.61	0.455	21.4	9.716	3.28
3	1.875	0.283	1.54	0.437	35.6	15.564	5.25
4	2.625	0.283	1.48	0.419	49.9	20.907	7.06
5	3.375	0.283	1.42	0.402	64.1	25.747	8.69
6	4.125	0.283	1.36	0.384	78.4	30.082	10.15
7	4.875	0.283	1.29	0.366	92.6	33.913	11.45
8	5.625	0.283	1.23	0.348	106.9	37.240	12.57
9	6.375	0.283	1.20	0.340	121.1	41.134	13.88
10	7.125	0.283	1.20	0.340	135.4	45.973	15.52
11	7.875	0.283	1.20	0.340	149.6	50.813	17.15
12	8.625	0.283	1.20	0.340	163.9	55.652	18.78
13	9.375	0.283	1.20	0.340	178.1	60.491	20.42
14	10.125	0.283	1.20	0.340	192.4	65.331	22.05
15	10.875	0.283	1.20	0.340	206.6	70.170	23.68
16	11.625	0.283	1.20	0.340	220.9	75.009	25.32
17	12.375	0.283	1.20	0.340	235.1	79.848	26.95
18	13.125	0.283	1.20	0.340	249.4	84.688	28.58
19	13.875	0.283	1.20	0.340	263.6	89.527	30.22
20	14.625	0.283	1.20	0.340	277.9	94.366	31.85
21	15.375	0.283	1.20	0.340	292.1	99.206	33.48
22	16.125	0.283	1.20	0.340	306.4	104.045	35.12
23	16.875	0.283	1.20	0.340	320.6	108.884	36.75
24	17.625	0.283	1.20	0.340	334.9	113.724	38.38
25	18.375	0.283	1.20	0.340	349.1	118.563	40.01
26	19.125	0.283	1.20	0.340	363.4	123.402	41.65
27	19.875	0.283	1.20	0.340	377.6	128.241	43.28

Now that we have calculated the load $T_{\max}$, we need to find the length of reinforcement that will safely carry that load without pulling out of the soil. The pull-out capacity $T_{pullout}$ (kN) of the reinforcement inclusion is given by:

$$T_{pullout} = 2 \times f_{\max} \times b \times L_a$$
$$f_{\max} = F^* \times \sigma'_{ov} \times \alpha$$

Using the ultimate limit state procedure, we have:

$$\gamma T_{\max} = \phi T_{pullout}$$

The active length of the reinforcement strip required to resist the pullout load is:

$$T_{pullout} = \frac{\gamma T_{\max}}{\varphi}$$
$$L_a = \frac{T_{pullout}}{2 \times f_{\max} \times b}$$
$$L_a = \frac{(\gamma_1 k_r \sigma'_{ov}) \times s_v \times s_h}{2 \times \varphi \times F^* \times \sigma'_{ov} \times \alpha \times b}$$
$$L_a = \frac{(\gamma_1 k_r) \times s_v \times s_h}{2 \times \varphi \times F^* \times \alpha \times b}$$
$$L = L_a + L_{\max} = \frac{(\gamma_1 k_r) \times s_v \times s_h}{2 \times \varphi \times F^* \times \alpha \times b} + 0.3H$$

The value of α is taken as 1.0 for strip reinforcements (Section 21.10.2). The resistance and load factors are taken as 0.9 and 1.35 respectively. The coefficient of friction (F^*) is computed according to AASHTO LRFD using Figure 21.28s.

If z_i is less than 6 m, then:

$$F^* = 1.2 + Log\ C_u = 1.8435\ at\ z = 0 \text{ m}$$
$$F^* = 0.6745\ at\ z = 6 \text{ m}$$
$$F^* = 1.8435 - 0.1948 \times z_i$$

If z_i is larger than 6 m, then:

$$F^* = \tan\phi = 0.6745$$

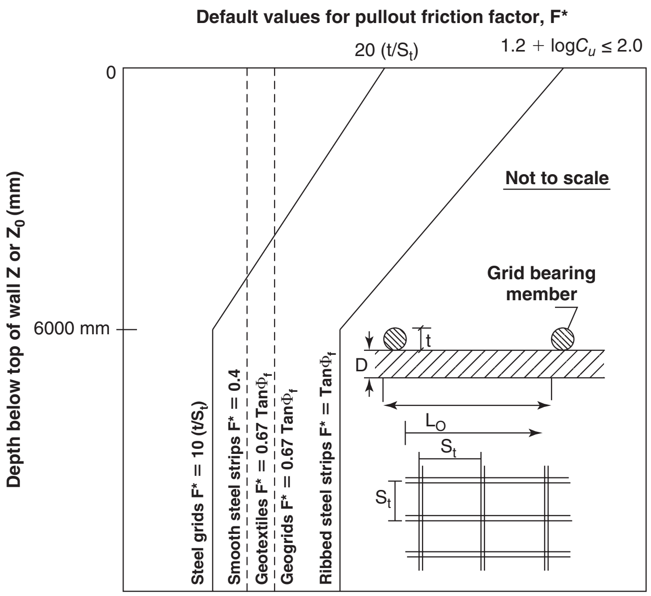


Figure 21.28s Friction coefficient F* for MSE wall reinforcement.

Table 21.2s Summary of Calculation of Total Strip Length (L)

Strip No.	Depth (m)	T_{max} (kN)	F*	f_{max} (kPa)	L_a (m)	L (m)
1	0.375	1.14	1.770	12.61	1.350	7.350
2	1.125	3.28	1.624	34.72	0.944	6.944
3	1.875	5.25	1.478	52.66	0.997	6.997
4	2.625	7.06	1.332	66.44	1.062	7.062
5	3.375	8.69	1.186	76.06	1.143	7.143
6	4.125	10.15	1.040	81.51	1.246	7.246
7	4.875	11.45	0.894	82.79	1.382	7.382
8	5.625	12.57	0.748	79.92	1.573	7.573
9	6.375	13.88	0.675	81.70	1.699	7.699
10	7.125	15.52	0.675	91.31	1.699	7.699
11	7.875	17.15	0.675	100.92	1.699	7.699
12	8.625	18.78	0.675	110.53	1.699	7.699
13	9.375	20.42	0.675	120.15	1.699	7.699
14	10.125	22.05	0.675	129.76	1.699	7.699
15	10.875	23.68	0.675	139.37	1.699	7.699
16	11.625	25.32	0.675	148.98	1.699	7.699
17	12.375	26.95	0.675	158.59	1.699	7.699
18	13.125	28.58	0.675	168.20	1.699	7.699
19	13.875	30.22	0.675	177.82	1.699	7.699
20	14.625	31.85	0.675	187.43	1.699	7.699
21	15.375	33.48	0.675	197.04	1.699	7.699
22	16.125	35.12	0.675	206.65	1.699	7.699
23	16.875	36.75	0.675	216.26	1.699	7.699
24	17.625	38.38	0.675	225.87	1.699	7.699
25	18.375	40.01	0.675	235.48	1.699	7.699
26	19.125	41.65	0.675	245.10	1.699	7.699
27	19.875	43.28	0.675	254.71	1.699	7.699

b. Design for Yielding

Using the ultimate limit state procedure, we have:

$$\gamma T_{\max} = \phi T_{yield}$$

The resistance and load factors are taken as 0.75 and 1.35 respectively. The T_{yield} for steel reinforcement is given by:

$$T_{yield} = \sigma_{yield} \times A$$
$$T_{yield} = \sigma_{yield} \times b \times E_c$$

The value of A is the cross-sectional area of the strip after accounting for corrosion (AASHTO 2010). The structural thickness of the strip at the end of the service life is computed according to AASHTO LRFD as:

$$\textit{Service Life of Zinc Coating}\ (0.086\ \text{mm/year}) = 2\ \text{years} + \frac{0.086 - 2 \times 0.015}{0.004}\text{years}$$
$$\textit{Service Life of Zinc Coating}\ (0.086\ \text{mm/year}) = 16\ \text{years}$$

Use a strip thickness 50 mm wide and 5 mm thick:

$$E_c = 5\text{ mm} - 2E_s$$
$$E_c = 5\text{ mm} - 2 \times (75\text{ years} - 16\text{ years}) \times 0.12\text{ mm/year}$$
$$E_c = 3.58\text{ mm}$$

Then:

$$T_{yield} = 448159.2\text{ kPa} \times (0.00358\text{ mm} \times 0.05\text{ mm})$$
$$T_{yield} = 80.2\text{ kN}$$

Then, using the result of $T_{\max}$ at the bottom layer of strips where the maximum tension load is expected, we have:

$$0.75 \times 80.2\text{ kN} > 1.35 \times 43.3\text{ kN}$$
$$60.2\text{ kN} > 58.5\text{ kN} \quad \therefore\ OK$$

Detailed calculations for all the strips are shown in Table 21.3s.

Table 21.3s Summary of Calculations for Strip Resistance to Yielding

Strip No.	Depth (m)	T_{max} (kN)	R	φR	γT_{max}	Check
1	0.375	1.14	80.2	60.2	1.5	OK
2	1.125	3.28	80.2	60.2	4.4	OK
3	1.875	5.25	80.2	60.2	7.1	OK
4	2.625	7.06	80.2	60.2	9.5	OK
5	3.375	8.69	80.2	60.2	11.7	OK
6	4.125	10.15	80.2	60.2	13.7	OK
7	4.875	11.45	80.2	60.2	15.5	OK
8	5.625	12.57	80.2	60.2	17.0	OK
9	6.375	13.88	80.2	60.2	18.7	OK
10	7.125	15.52	80.2	60.2	20.9	OK
11	7.875	17.15	80.2	60.2	23.2	OK
12	8.625	18.78	80.2	60.2	25.4	OK
13	9.375	20.42	80.2	60.2	27.6	OK
14	10.125	22.05	80.2	60.2	29.8	OK
15	10.875	23.68	80.2	60.2	32.0	OK
16	11.625	25.32	80.2	60.2	34.2	OK
17	12.375	26.95	80.2	60.2	36.4	OK
18	13.125	28.58	80.2	60.2	38.6	OK
19	13.875	30.22	80.2	60.2	40.8	OK
20	14.625	31.85	80.2	60.2	43.0	OK
21	15.375	33.48	80.2	60.2	45.2	OK
22	16.125	35.12	80.2	60.2	47.4	OK
23	16.875	36.75	80.2	60.2	49.6	OK
24	17.625	38.38	80.2	60.2	51.8	OK
25	18.375	40.01	80.2	60.2	54.0	OK
26	19.125	41.65	80.2	60.2	56.2	OK
27	19.875	43.28	80.2	60.2	58.4	OK

Note: Bearing capacity and slope stability failure were not checked as part of this problem. However, they must be checked to ensure that the system is safe against these failure modes.

Problem 21.17

A cantilever retaining wall is embedded 6 m below excavation level and retains 5 m of soil. An impervious layer exists 4 m below the bottom of the wall. The water level is at the ground surface on both sides of the wall and the soil deposit is uniform and deep. Draw the water pressure diagram against the wall on both sides of the wall, assuming that the water pressure is hydrostatic. Then draw a flow net and develop the water pressure diagram on both sides of the wall. Compare and comment.

Solution 21.17

The hydrostatic water pressure diagram is shown in Figure 21.29s and the flow net in Figure 21.30s.

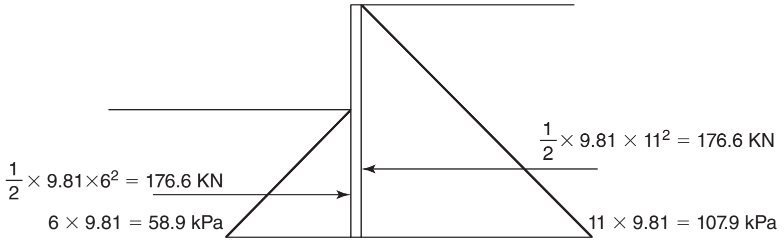


Figure 21.29s Water pressure in hydrostatic conditions.

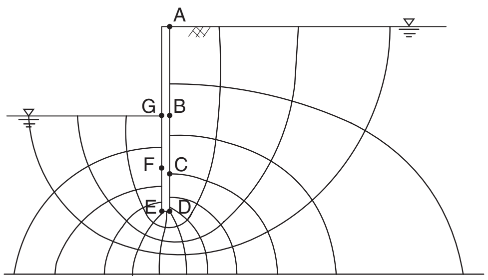


Figure 21.30s Flow net.

Water pressures are calculated at points A, B, C, D, E, F, and G on each side of the wall to generate the water stress profile. The loss of total head through the flow net is 5 m. The loss of total head through each flow field is $5/12 = 0.417$ m. The total head $h_{t(M)}$ at any point M is calculated by:

$$h_{t(M)} = h_{t(beg)} - n_d \Delta h_t$$

where $h_{t(beg)}$ is the total head at the beginning of the flow net (11 m), n_d is the number of equipotential drops to go from the beginning of the flow net to point M, and Δh_t is the drop of total head across any flow field. Then the elevation head $h_{e(M)}$ is measured on the scaled drawing and the pressure head $h_{p(M)}$ is obtained as the difference between the total head and the elevation head (Table 21.4s.).

Table 21.4s

Point	Total Head (m)	Elevation Head (m)	Pressure Head (m)	WaterStress (kPa)
A	15	15	0	0
B	14.33	9.6	4.73	46.40
C	13.75	6.0	7.75	76.03
D	12.5	4	8.5	83.38
E	11.66	4	7.66	75.14
F	10.62	6.3	4.32	42.38
G	10	10	0	0

The water pressure diagram from the flow net is shown in Figure 21.31s together with the hydrostatic diagram. As can be seen, the hydrostatic diagram is more conservative.

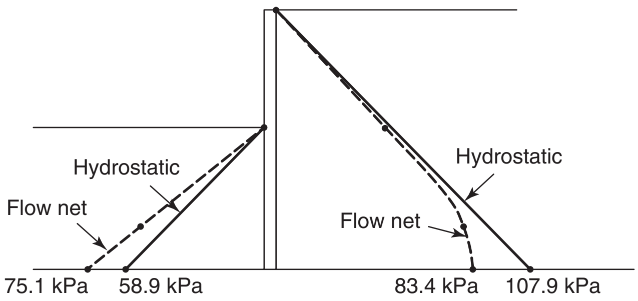


Figure 21.31s Water pressure under flow conditions.

Problem 21.18

Demonstrate Equation 21.91.

Solution 21.18

Equation 21.90 expresses moment equilibrium at the bottom of a wall:

$$P_a X_a - P_{pm} X_{pm} = 0$$

Using Equations 21.86–21.88, and 21.89 in Eq. 21.90, we get:

$$\left(\frac{1}{2}K_a\gamma(H+D)^2\right)\left(\frac{1}{3}(H+D)\right) - \left(\frac{3}{8}K_p\gamma D^2\right)\left(\frac{7}{18}D\right) = 0$$

$$\left(\frac{3}{8}K_p D^2\right)\left(\frac{7}{18}D\right) = \left(\frac{1}{2}K_a(H+D)^2\right)\left(\frac{1}{3}(H+D)\right)$$

$$\frac{21}{144}K_p D^3 = \frac{1}{6}K_a(H+D)^3$$

$$\frac{7}{8}K_p D^3 = K_a(H+D)^3$$

$$\sqrt[3]{\frac{7}{8}\frac{K_p}{K_a}D^3} = H + D$$

$$\left(\left(\frac{7}{8}\frac{K_p}{K_a}\right)^{0.33} D\right) - D = H$$

$$D\left(\left(\frac{7}{8}\frac{K_p}{K_a}\right)^{0.33} - 1\right) = H$$

$$D = \frac{H}{\left(\frac{7}{8}\frac{K_p}{K_a}\right)^{0.33} - 1}$$

Problem 21.19

What is the depth of embedment d required for a cantilever wall retaining a height of sand H? Express the results as a function of H, K_p/K_a, and a factor of safety F applied to σ_p, the passive pressure. (Note: There is no water.)

Solution 21.19

$$\frac{1}{2}K_a\gamma(H+D)^2 \times \frac{1}{3}(H+D) - \frac{\frac{1}{2}K_p\gamma D^2 \times \frac{1}{3}D}{F.S} = 0$$

$$K_a(H+D)^3 = \frac{1}{F.S}K_pD^3$$

$$\frac{H+D}{D} = \left(\frac{1}{F.S}\frac{K_p}{K_a}\right)^{0.33}$$

$$D = \frac{H}{\left(\frac{1}{F.S}\frac{K_p}{K_a}\right)^{0.33} - 1}$$

Problem 21.20

For the anchored slurry wall shown in Figure 21.3s, calculate the pressure distribution on both sides of the wall for a deflection of 25 mm at the top of the wall. Calculate the anchor forces. How important is the vertical capacity of the wall? Explain your answer. What would happen if the water level rose on both sides of the excavation to the top of the wall? What would happen if the water level rose to the top of the wall on the retained-soil side of the excavation and to 2 m below that on the excavated side?

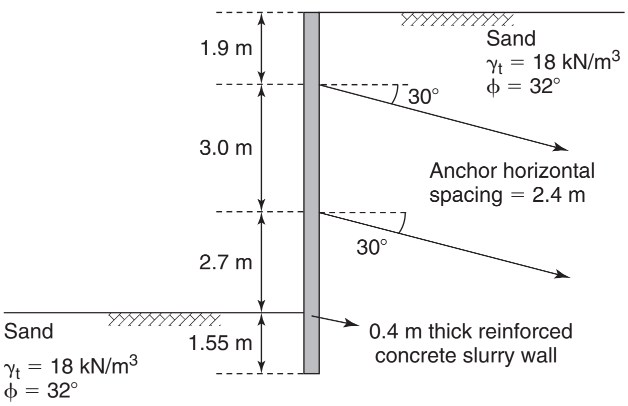


Figure 21.3s Anchored slurry wall.

Solution 21.20

From Figure 21.19 and $u_{top}/H = 0.025/7.6 = 0.003$, K behind the wall is 0.2 (average). Using Eq. 21.99, the constant pressure from $z = 0$ to $z = H$ is:

$$\begin{aligned}\sigma_h &= K\sigma'_{ov}(at\,z = H) + u_w \\ &= 0.2(18)(7.6) \\ &= 27.4\ \text{kN/m}^2\end{aligned}$$

For $z = H$, just below the constant pressure, the K_a active earth pressure is used:

$$\begin{aligned}\sigma_{ah} &= K_a\sigma_v \\ &= \frac{1-\sin(30)}{1+\sin(30)}(18)(7.6) \\ &= 45.5\ \text{kN/m}^2\end{aligned}$$

For $z = H + D$, the active pressure is:

$$\sigma_{ah} = K_a \sigma_v = \frac{1}{3}(18)(9.15) = 54.8\ \text{kN/m}^2$$

For $z = H$ on the excavation side, the passive pressure is 0 and at $z = H + D$, the passive pressure is:

$$\sigma_{ph} = K_p \sigma_v = 3(18)(1.55) = 83.7\ \text{kN/m}^2$$

Using the tributary area for the top anchor, the horizontal component for that anchor is:

$$F_{1h} = \sigma_h A_1 = 27.4 \times (1.9 + 1.5) \times 2.4 = 223.6\ \text{kN}$$

Using the tributary area for the bottom anchor, the horizontal component for that anchor is:

$$F_{2h} = \sigma_h A_2 = 27.4 \times (1.5 + 1.35) \times 2.4 = 187.4\ \text{kN}$$

Because the anchors are inclined at 30°, the actual loads in the anchors are:

$$F_1 = \frac{F_{1h}}{\cos\alpha} = \frac{223.6}{\cos 30} = 258.2\ \text{kN}$$

$$F_2 = \frac{F_{2h}}{\cos\alpha} = \frac{187.4}{\cos 30} = 216.4\ \text{kN}$$

The vertical capacity is important because the soil mass tends to move toward the excavation and downward. The downward movement imposes downdrag on the wall. If the vertical capacity is insufficient, the wall will move downward and rotate around the anchor. This will cause horizontal movement as well.

If water rises on both sides to the top of the wall, the water pressure on both sides will cancel out and the soil horizontal stress will decrease from the total stress ($K\gamma H$) to the effective stress ($K\gamma' H$). This would lead to a pressure on the wall of about one-half of the pressure with no water on either side. If there was a difference in level of 2 m, there would be a net water pressure equal to 2 m of water on the wall in addition to the $K\gamma' H$.

Problem 21.21

Explain Figure 21.49.

Solution 21.21

Figure 21.49 shows an example of load distribution in an anchor in tension. The load resisted by the soil increases steadily from the back of the anchor to the front of the anchor. The load in the tendon is constant and equal to the anchor load along the tendon unbonded length because the greased sheath that covers the anchor does not permit any load transfer. Then the load in the tendon drops off as the grout contributes to the load being resisted. Within the zone where the grout is in tension, the tendon is the only one carrying load, because the grout cracks and contributes no load to the resistance. Within the tensile strains where the grout can resist tension, some of the load is carried by the tendon and some by the grout. The grout has zero load at the ground surface and the load increases in compression over the unbonded tendon length because the grout moves with respect to the soil and is loaded in compression. Beyond the tendon unbonded length, the grout is in tension to such a level that it cracks and cannot contribute to the resistance. Then, in the back of the anchor, the tension load decreases to the point where the strains are low enough and the grout can resist some tension.

Problem 21.22

Use Tables 21.4 and 21.5 and add a column giving the back-calculated alpha values.

Solution 21.22 (Table 21.5s)

$$f_{\max} = \alpha_c s_u$$
$$\alpha_c = \frac{f_{\max}}{s_u}$$

Sample calculation: Stiff silt-clay mixture $s_u = 50$ kPa, $f_{\max} = 30$ kPa, $\alpha_c = \frac{30}{50} = 0.6$

Table 21.5s

Anchor Type (Grout Pressure)	Soil Type	Shear Strength of Soil s_u (kPa)	Shear Strength of Soil-Grout Interface f_{max} (kPa)	α_s
Gravity grouted anchors (< 350 kPa)	Silt-clay mixtures	Stiff to very stiff 50 to 200	30 to 70	0.35-0.6
Pressure grouted anchors (350 to 2800 kPa)	High-plasticity clay	Stiff (50 to 120) Very stiff (120 to 200)	30 to 100 70 to 170	0.6-0.83 0.58-0.85
	Medium-plasticity clay	Stiff (50 to 120) Very stiff (120 to 200)	100 to 250 140 to 350	2-2.1 1.2-1.75
	Medium-plasticity sandy silt	Very stiff (120 to 200)	280 to 380	2.3-1.9

Problem 21.23

For Figure 21.62, the height H is 9 m, α is 17°, β is 18°, ψ is 44°, and i is 10°. The stiff clay weighs 20 kN/m^3 with some cohesion c' (to be ignored), and a friction angle φ' of 32°. A uniformly applied surcharge of 10 kPa is to be considered on top of the wall. Calculate the required nail force T for a factor of safety against shear failure along the chosen plane to be 1.5. Distribute that force among the four nails and find the required length for each nail.

Solution 21.23 (Figure 21.32s)

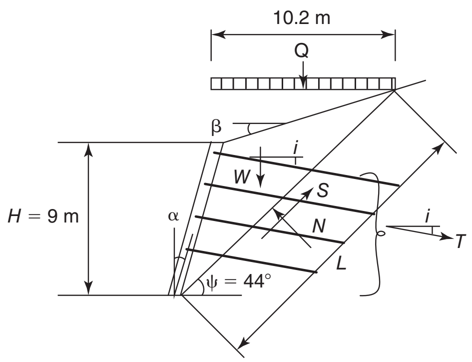


Figure 21.32s Nailed wall.

Equations 22.112, 22.113, and 22.114 are used to find the three unknowns N, S, and T:

$$(W + Q)\cos\psi + T\sin(\psi + i) - N = 0$$
$$(W + Q)\sin\psi - T\cos(\psi + i) - S = 0$$
$$S = \frac{S_{\max}}{F} = \frac{c'L + N\tan\varphi'}{F}$$

The weight of the soil is obtained by multiplying the unit weight of the soil by the area of the triangle. The area of the triangle is 41.4 m^2 per meter perpendicular to the page and W is 828 kN/m. Substituting these values in the equations:

$$(828 + 10.2 \times 10) \cos 44 + T \sin(44 + 15) - N = 0$$
$$(828 + 10.2 \times 10) \sin 44 - T \cos(44 + 15) - S = 0$$
$$\frac{N \tan 30}{1.5} - S = 0$$

Then:

$$669 + 0.857T - N = 0$$
$$646 - 0.515T - S = 0$$
$$0.385N - S = 0$$

After solving the system of equations, $N = 1065$ kN/m, $S = 410$ kN/m, and $T = 462$ kN/m. For simplification, if all the nails in the wall carry the same force, then the force T is divided by the four nails and the force at each nail is approximately 115.5 kN/m. The required length of the nails can be found using Eq. 22.115 plus a factor of safety F:

$$R_a = \frac{T}{n} = \frac{\pi D L_p f_{\max}}{F}$$

where R_a is the allowable load on each nail, T is the total nail load, n is the number of nails, D is the diameter of the nail (drill hole), L_p is the useful length of the nail, and $f_{\max}$ is the shear strength at the grout-soil interface. The shear strength $f_{\max}$ depends on the soil and the construction method and is estimated using Table 22.8. For a stiff clay, an $f_{\max}$ of 50 kPa can be used and a diameter of 200 mm.

$$L_p = \frac{FT}{n\pi D f_{\max}} = \frac{1.5 \times 462}{4\pi \times 0.2 \times 50} = 5.5 \text{ m}$$

The required length L_p of each nail is 4.4 m. The length of the nail inside the failure zone is the discounted length L_d. The total length of each nail is the sum of the required length and the discounted length. The total length of each nail is:

$$L_{ti} = L_{di} + L_{pi}$$
$$L_{t1} = L_{d1} + L_{p1} = 5.6 + 5.5 = 11.1 \text{ m}$$
$$L_{t2} = L_{d2} + L_{p2} = 4.3 + 5.5 = 9.8 \text{ m}$$
$$L_{t3} = L_{d3} + L_{p3} = 3 + 5.5 = 8.5 \text{ m}$$
$$L_{t4} = L_{d4} + L_{p4} = 1.8 + 5.5 = 7.3 \text{ m}$$

Problem 21.24

A 3 m wide strutted excavation is planned in a clay with an undrained shear strength equal to 40 kPa and a total unit weight of 19 kN/m^3. What depth of excavation corresponds to a factor of safety against base failure equal to 1.5?

Solution 21.24

The safety factor for the base failure can be calculated using the following equation:

$$F = \frac{N_c s_u}{\sigma_{ov(z=H)}}$$

Assume that $H = 3$m and $H/B = 1$. Then, using the Skempton chart, the $N_c = 6.4$

$$1.5 = \frac{6.4 \times 40}{19 \times H} \text{ and H} = 8.9 \text{ m}$$

Assume that $H = 9$ and $H/B = 3$. Then, using the Skempton chart, the $N_c = 7.3$

$$1.5 = \frac{7.3 \times 40}{19 \times H} \text{ and H} = 10.24 \text{ m}$$

Assume that $H = 10$ and $H/B = 3.33$. Then, using the Skempton chart, the $N_c = 7.3$

$$1.5 = \frac{7.3 \times 40}{19 \times H} \text{ and } H = 10.24 \text{ m}$$

$$H = 10 \text{ m}$$