CHAPTER 16

Thermodynamics for Soil Problems

16.1 GENERAL

Heat flow in soils involves several different phenomena: convection, radiation, and conduction. *Convection* takes place when a fluid flows over a solid that is at a different temperature than the fluid. When you set up a fan to blow air toward your body and cool yourself down in the summer, you use convection heat transfer. *Radiation* refers to the fact that all bodies continuously emit energy because of their temperature. This energy propagates to other nearby fluids or bodies through electromagnetic waves. Hot radiators that you may use in the winter to warm yourself up operate by radiation heat transfer. *Conduction* is a heat transfer mechanism whereby energy moves from a region of high temperature to a region of lower temperature. The phenomenon is due to the motion and impact of molecules, which increase as the temperature rises. Conduction of heat in soil is very similar to the flow of water through soil and is the most important mechanism of heat transfer through soils.

16.2 DEFINITIONS

Because of the analogy between temperature propagation and water flow, it is useful to draw a parallel between the parameters used in both fields of geotechnical engineering. Heat, Q, is a quantity of energy measured in joules (N × m). It is named after James Prescott Joule (1818–1889), an English physicist. The heat Q is equivalent to the volume of water V (m^3) in flow problems.

Temperature, T, is a measure of how hot a material is; it is sometimes measured in degrees Kelvin (K), but more commonly in degrees Celsius (C). The Kelvin is named after the British engineer and physicist William Thomson, First Baron Kelvin (1824–1907). The Celsius is named after the Swedish astronomer Anders Celsius (1701–1744). The Kelvin scale starts at absolute zero temperature, which is -273°C. There is in fact a lower bound to the temperature scale: It corresponds to the point where none of the molecules are moving. There is no known upper bound to the temperature scale. The temperature T is equivalent to the total head h_t(m) in flow problems.

The *temperature gradient* i_t is defined between two points in the soil mass; it is the ratio between the change in temperature dT over the distance dx separating the two points and is expressed in K/m. It corresponds to the hydraulic gradient i for the flow problem:

$$i_t = \frac{dT}{dx} \quad in \quad \text{K/m} \tag{16.1}$$

The *heat transfer rate* H is the amount of heat transferred per amount of time and is expressed in joules per second or watts, named after the Scottish engineer James Watt (1736–1819) (J/s or W). The heat transfer rate is equivalent to the flow rate Q (m^3/s) in flow problems:

$$H = \frac{dQ}{dt} \quad in \quad \text{J/s} \tag{16.2}$$

The *heat flow* q is the amount of heat dQ per unit time dt and per unit area A or the heat transfer rate H per unit area A. It is expressed in watts per meter square (W/m^2) or in joules per second and per meter square (J/s.m^2). It is equivalent to the velocity v (m/s) in the flow problem:

$$q = \frac{dQ}{dt} \times \frac{1}{A} = \frac{H}{A} \quad in \quad \text{J/s.m}^2 \tag{16.3}$$

The *thermal conductivity* k_t is a property of the soil. It takes units of J/s.K.m and is defined through Fourier's law (section 16.3) as the ratio between the heat flow and the thermal gradient:

$$k_t = \frac{q}{\frac{dT}{dx}} \quad in \quad \text{J/s.K.m} \tag{16.4}$$

The thermal conductivity is an indication of the speed with which the heat flows through the soil under a given temperature gradient. It is equivalent to the hydraulic conductivity for the flow problem.

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The *specific heat* c is a property of the soil and takes units of J/kg.K. It is defined as:

$$c = \frac{1}{m}\frac{dQ}{dT} \quad in \quad \text{J/kg.K} \tag{16.5}$$

where m is the mass of the soil element considered, and dQ is the increase in heat stored in the element when the temperature is raised by dT. In the flow problem, the compressibility of the soil skeleton plays the role of the inverse of the specific heat. The inverse of the specific heat tells you how much heat you can squeeze out of the soil for a given change in temperature, much like the compressibility tells you how much water you can squeeze out the soil if you apply a change in effective stress.

The *diffusivity* α appears in the governing differential equation. It is in m^2/s and is defined as:

$$\alpha = \frac{k}{\rho c} \quad in \quad \text{m}^2/\text{s} \tag{16.6}$$

The diffusivity gives the speed with which the temperature will decay in a soil. It is closely linked to the thermal conductivity k_t, but is also influenced by the specific heat, which indicates how much heat can be squeezed out of the soil for a given change in temperature. In other words, you could have two soils with the same thermal conductivity but different specific heats. In this instance the heat would flow at the same speed in both soils for the same thermal gradient, but if the heat source stopped, the one with the highest specific heat would cool down the slowest because it would be harder to squeeze the heat out of the soil.

Table 16.1 Equivalency between Thermal Conductivity and Hydraulic Conductivity

Parameter	Flow of water	Flow of heat
Quantity	Volume V (m^3)	Heat Q (J)
Potential	Head h_t(m)	Temperature T (K)
Gradient	Hydraulic gradient i_h(unitless)	Temperature gradient i_t(K/m)
Flux	Flow rate Q (m^3/s)	Heat transfer rate H (J/s)
Flux density	Velocity v (m/s)	Heat flow q (J/s.m^2)
Conductivity	Hydraulic conductivity k_h(m/s)	Thermal conductivity k_t(J/s.K.m)
Law	Darcy	Fourier
Storage	Compressibility	Specific heat c (J/kg.K)
Decay coefficient	Coefficient of consolidation $c_v(m^2/s)$	Thermal diffusivity $\alpha(m^2/s)$

Table 16.1 summarizes the equivalency between soil thermal flow problems and soil hydraulic flow problems.

16.3 CONSTITUTIVE AND FUNDAMENTAL LAWS

Fourier's law is the constitutive law for heat flow. It is named after Joseph Fourier (1768–1830), a French mathematician and physicist. Fourier's law (Fourier 1822) states that the heat flow q is linearly related to the temperature gradient through the thermal conductivity k_t:

$$q = -k_t i_t = -k_t \frac{dT}{dx} \tag{16.7}$$

where q is the heat flow, k_t is the thermal conductivity, i_t is the temperature gradient, T is the temperature, and x is the length in the direction of the heat flow. Therefore, the units of thermal conductivity are J/s.K.m. The minus sign indicates that heat flows in the direction of decreasing temperatures. Fourier's law is equivalent to Darcy's law in the flow problem. By the way, the R rating of house insulation comes from Eq. 16.7 and is based on very much the same concept as the resistance of an electrical conductor:

$$R = \frac{dT}{q} = \frac{dx}{k_t} \quad or \quad dT = R\,q \tag{16.8}$$

The fundamental law is the conservation of energy. For the purpose of this chapter, this law states that during an amount of time dt, the amount of heat dQ_{in} flowing into an element of soil is equal to the amount of heat dQ_{out} flowing out of the element plus the heat stored or extracted dQ_{stored} from the element.

$$\frac{dQ}{dt}_{in} = \frac{dQ}{dt}_{out} + \frac{dQ}{dt}_{stored} \tag{16.9}$$

16.4 HEAT CONDUCTION THEORY

Let's first address the problem of one-dimensional heat conduction. An example is the penetration of frost into a surface layer of soil due to low air temperature in the winter months. To solve this problem, we follow the normal steps (see section 11.4):

1. Consider an element of soil dx wide, dy long, and with a unit length perpendicular to the page (Figure 16.1).

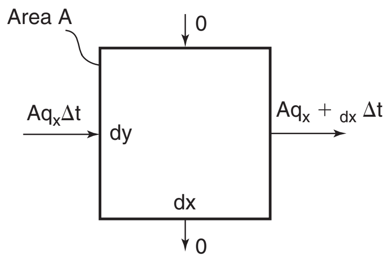

Figure 16.1 Element of soil.

2. The heat flows through the element volume, which has a cross-sectional area $A(dy \times 1)$ and a length dx. During a time dt, the quantity of heat entering the element is $Aq_x dt$, whereas the quantity of heat leaving the element is $Aq_{x+dx}dt$.
3. Conservation of energy allows us to state that the difference $(Aq_x dt - Aq_{x+dx}dt)$ is equal to the stored heat in the element.

$$dQ = A\ q_x dt - A\ q_{x+dx} dt \tag{16.10}$$

4. The constitutive law is Fourier's law:

$$q(x,t) = -k\frac{dT(x,t)}{dx} \tag{16.11}$$

where q is the heat flow (J/s.m^2), k is the thermal conductivity (J/s.K.m), T is the temperature (K), and x is the length (m) in the direction of the heat flow.
5. The second constitutive law is associated with the definition of specific heat. The amount of heat dQ will generate an increase in temperature dT in the element of mass m such that:

$$dQ = m\ c\ dT = A\ dx\ \rho\ c\ dT \tag{16.12}$$

where ρ is the mass density of the material (kg/m^3) and c is the specific heat of the material (J/kg.K).
6. Regrouping Eqs. 16.10 and 16.12 gives:

$$Aq_x dt - Aq_{x+dx} dt = A\ dx\ \rho\ c\ dT \tag{16.13}$$

Or, in partial derivative form:

$$-\frac{\partial q}{\partial x} = \rho\ c\ \frac{\partial T}{\partial t} \tag{16.14}$$

Combining Eqs. 16.11 and 16.14, we get:

$$k\frac{\partial^2 T}{\partial x^2} = \rho\ c\frac{\partial T}{\partial t} \tag{16.15}$$

If we define the thermal diffusivity α as:

$$\alpha = \frac{k}{\rho c} \tag{16.16}$$

where α is the diffusivity in m^2/s, then the governing differential equation for one-dimensional conduction heat is:

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2} \tag{16.17}$$

In three dimensions, it becomes:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\ \frac{\partial T}{\partial t} \tag{16.18}$$

7. Now the boundary and initial conditions have to be expressed. This depends on the problem at hand. The complexity of the solution depends on the complexity of the boundary conditions, but numerical methods can always be used to solve such problems. Note that Eq. 16.17 is identical to Eq. 11.56 for the consolidation theory, where the temperature T is replaced by the excess water stress u_e. Therefore, the solutions are identical for identical boundary conditions. Jumikis (1977) presents the solution for a sinusoidal temperature fluctuation input at the ground surface to replicate seasonal variations.

16.5 AXISYMMETRIC HEAT PROPAGATION

In the case of an axisymmetric geometry, Eq. 16.18 becomes:

$$\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{16.19}$$

where r is the radial distance from the axis, t is time, T is temperature, and α is the thermal diffusivity of the soil. Carslaw and Jaeger (1947) solved this problem in the case of an infinitely long cylindrical heat source of radius R_o maintained at a temperature T_o at the center of a full space, which was initially at a temperature equal to zero. In this case the time t required for a given temperature T_m to reach a certain distance R into the soil is given by:

$$t = T_F\frac{R_o^2}{\alpha} \tag{16.20}$$

where T_F is the time factor (Figure 16.2), and is a function of the ratio R/R_o and T_m/T_o. This equation is very similar to the consolidation equation, which yields the time for excess water stress dissipation around a pile. At first glance, Eq. 16.20 seems to indicate that t increases with ${R_o}^2$. But in fact, t decreases as R_o increases, because T_F decreases with R_o faster than R_o^2 increases.

The following reasoning illustrates this point. In Eq. 16.20, if R_o is multiplied by $\sqrt{10}$, the time t is not multiplied by 10 because the time factor T_F is not the same in both cases. If t was multiplied by 10, it would mean that it would take 10 times longer for the temperature to reach a value T_m at a distance $R-R_o$ from the boundary in the case of the large-radius heat source ($\sqrt{10}R_o$) than for the same temperature T_m to be reached at the same distance $R - R_o$ in the case of the smaller-radius heat source (R_o). This does not make sense: Because the heat source is larger, it should take less time—and indeed it does, because the time factor T_F decreases more than by a ratio of 10 in this case (Figure 16.2). Therefore, as R_o increases, t in fact decreases nonlinearly.

For example, consider a hot cone penetrometer with a radius R_o of 20 mm that is kept at a temperature T_o of 100°C in a soil with an initial temperature of 20°C and a diffusivity of 1 mm^2/s. Let's calculate the time it will take for the temperature to reach 40°C at a distance of R equal to $R_o + 100$ mm $= 120$ mm. Considering that the temperature of the soil is at 20°C initially, the ratio of net

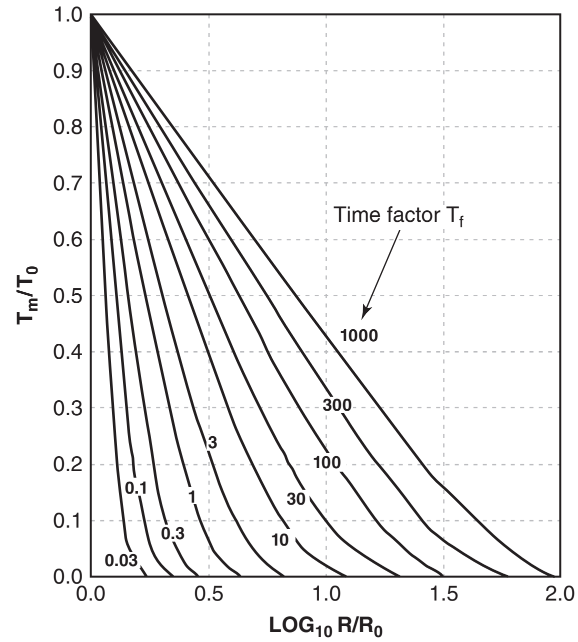

Figure 16.2 Time factor. (After Carslaw and Jaeger 1947)

temperature increase is $(40 - 20)/(100 - 20) = 0.25$. For this temperature ratio and a radius ratio of $120/20 = 6$, Figure 16.2 yields a time factor T_F equal to 32, and the time for the temperature to reach 40°C at $R = 120$ mm is:

$$t = 32\frac{20^2}{1} = 12800 \text{ s} = 3.55 \text{ hours} \tag{16.21}$$

Now consider a hot oil conductor in the bottom of the Gulf of Mexico with a radius R_o of 500 mm, that is kept at a temperature T_o of 100°C. Let's calculate the time it will take for the temperature to reach a temperature of 40°C at a distance of R equal to $R_o + 100$ mm $= 600$ mm. Considering that the temperature of the soil is at 20°C initially, the ratio of net temperature increase is $(40 - 20)/(100 - 20) = 0.25$. For this temperature ratio and a radius ratio of $600/500 = 1.2$, Figure 16.2 yields a time factor T equal to 0.02, and the time for the temperature to reach 40°C at $R = 600$ mm is:

$$t = 0.02\frac{500^2}{1} = 5000 \text{ s} = 1.39 \text{ hours} \tag{16.22}$$

16.6 THERMAL PROPERTIES OF SOILS

Any material can be found in solid, liquid, or gas form. For water, the transition from solid to liquid is at 0°C and the transition from liquid to gas is at 100°C. These temperatures correspond to 1 atmosphere of pressure, but would be different at different pressure levels. Figure 16.3 shows the pressure-temperature phase diagram for water and its triple point. By the way, the *latent heat* of a material is the heat necessary to change the phase of the material (solid to gas, for example).

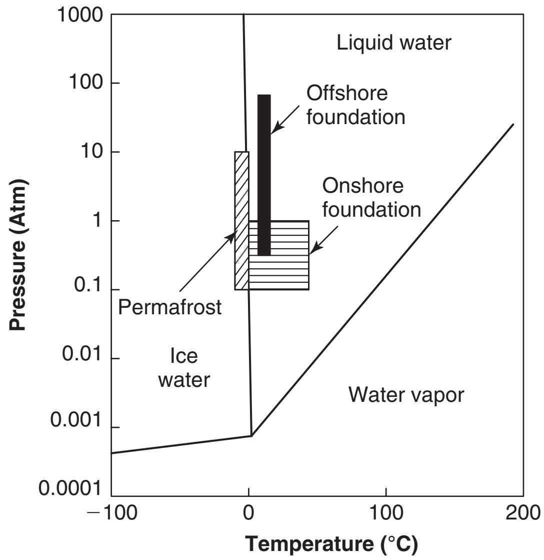

Figure 16.3 Temperature phases for water.

The temperature on the Earth varies from about −50°C to about +50°C. The temperature in the Earth varies from −50°C on the surface to 5500°C at the center of the Earth. Rocks and soil particles melt at a temperature varying between 600°C and 1200°C. The temperature gradient in the Earth varies and may be taken as 15°C per km over the first 100 km of depth. The deepest types of projects involving the geotechnical engineer may be offshore platforms and the associated retrieval of oil. The water depth in which the largest platforms are constructed reaches several kilometers. At the bottom of such oceans, the temperature is only a few degrees Celsius. The oil reservoir may be at a depth of 15 km; thus, the temperature of the oil can easily be 100°C when it comes back up to the surface (Figure 16.4). So, for

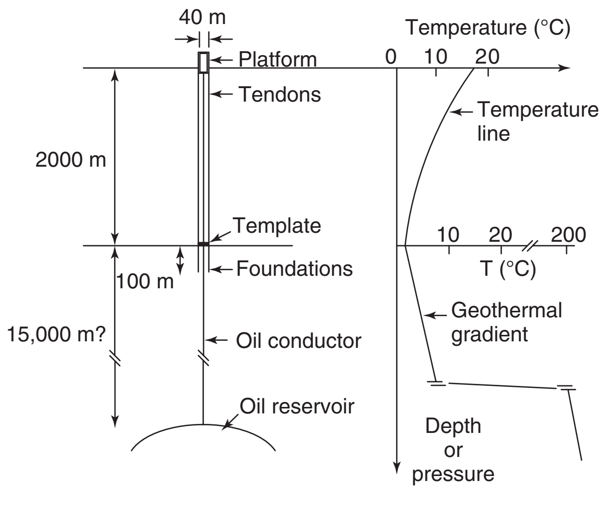

Figure 16.4 Temperature gradient for an offshore platform. (After Briaud and Chaouch 1997)

the geotechnical engineer, soil particles and rocks remain in solid form. However, within the range of Earth surface temperatures, water can be in liquid or in solid form (frozen), and the thermal properties of the soil may differ depending on whether the soil is frozen or not.

The thermal properties of interest are the thermal conductivity k(J/s.m.°C), the specific heat C(J/kg.°C), and the diffusivity $\alpha(\text{m}^2/\text{s})$. A high value of thermal conductivity means that heat travels easily through the material; a high value of specific heat means that it takes a lot of heat to raise the temperature of the material; and a high value of diffusivity means that it will take little time for the temperature to rise in the material. These thermal properties depend on a number of factors, among which are the temperature level T, the pressure level p, the moisture content w, and the density ρ. Table 16.2 shows an estimate of the range of values one can expect for those thermal properties at ordinary temperature and pressure levels. The range of values in this table helps one to understand the factors affecting the thermal properties. For example, a dry soil will have a thermal conductivity lower than the same soil once saturated, because air has a lower thermal conductivity than water. Also, sand in a very dense state will have a higher thermal conductivity than the same sand in a very loose state, because soil particles have a higher thermal conductivity than air or water.

16.7 MULTILAYER SYSTEMS

Heat can flow through a layered system, such as an asphalt-concrete pavement over soil in the heat of the summer or a layer of snow covering the soil surface in the winter. Consider the case in which heat flows parallel to the interface of the two layers (Figure 16.5), where the starting temperature and the ending temperature are maintained at T_A and T_B.

These temperatures exist at two points separated by a horizontal distance L. Layer 1 is h_1 thick and layer 2 is h_2 thick. The thermal gradient is the same in both layers:

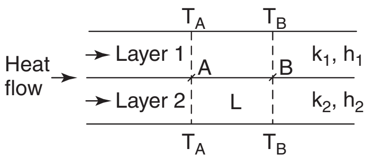

Figure 16.5 Horizontal heat flow through two layers.

$$i_1 = i_2 = \frac{T_B - T_A}{L} = i_e \tag{16.23}$$

However, the total heat transfer rate H is the sum of the heat transfer rate H_1 in layer 1 plus the heat transfer rate H_2 in layer 2:

$$H = q_e(h_1 + h_2) \times 1 = H_1 + H_2 = q_1 h_1 \times 1 + q_2 h_2 \times 1 \tag{16.24}$$

where q_e is the total heat flow, q_1 is the heat flow in layer 1, and q_2 is the heat flow in layer 2. Using Fourier's law gives:

$$k_e i_e (h_1 + h_2) \times 1 = k_1 i_1 h_1 \times 1 + k_2 i_2 h_2 \times 1 \tag{16.25}$$

where k_e is the equivalent thermal conductivity, k_1 is the thermal conductivity of layer 1, k_2 is the thermal conductivity of layer 2, i_e is the equivalent gradient, i_1 is the gradient in layer 1, and i_2 is the gradient in layer 2. Therefore:

$$k_e = \frac{k_1 h_1 + k_2 h_2}{h_1 + h_2} \tag{16.26}$$

This result can be generalized for n layers:

$$k_e = \frac{\sum_{i=1}^{n} k_i h_i}{\sum_{i=1}^{n} h_i} \tag{16.27}$$

Table 16.2 Thermal Properties for Various Earth Materials at Standard Conditions of Temperature and Pressure

Material	Density $\rho(\text{kg/m}^3)$	Specific Heat c(J/kg.°C)	Thermal Conductivity k(J/s.m.°C)	Thermal Diffusivity $\alpha(\text{mm}^2/\text{s})$
Air	1 to 1.4	1000 to 1050	0.02 to 0.03	13 to 30
Water	960 to 1000	4190 to 4220	0.5 to 0.8	0.13 to 0.17
Ice	917 to 920	1960 to 2110	2.0 to 2.6	1.24 to 1.52
Clay (unfrozen)	1400 to 1800	750 to 920	0.8 to 2.8	0.1 to 1.66
Clay (frozen)	1400 to 1800	650 to 800	1.0 to 3.6	0.15 to 2.3
Sand (unfrozen)	1500 to 2200	630 to 1460	2.3 to 3.8	0.87 to 3.0
Sand (frozen)	1500 to 2200	500 to 1200	2.9 to 4.7	1.2 to 4.2
Rock	2200 to 3000	710 to 920	2 to 6	1.1 to 3.0

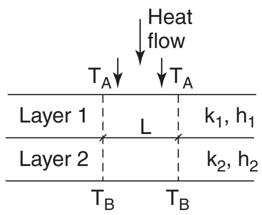

Figure 16.6 Vertical heat flow through two horizontal layers.

Now consider the case in which heat flows perpendicular to two layers (Figure 16.6), where the starting temperature and the ending temperature are maintained at T_A and T_B.

Layer 1 is h_1 thick and layer 2 is h_2 thick. The temperatures T_A and T_B exist at two points separated by a distance $(h_1 + h_2)$. The heat transfer rate is the same in both layers:

$$H = H_1 = H_2 \quad and$$
$$H = k_e i_e L \times 1 = k_1 i_1 L \times 1 = k_2 i_2 L \times 1 \qquad (16.28)$$

The change in temperature, however, is additive:

$$\Delta T = \Delta T_1 + \Delta T_2 = T_A - T_B \qquad (16.29)$$

But

$$i_e = \frac{\Delta T}{h_1 + h_2} \quad and \quad i_1 = \frac{\Delta T_1}{h_1} \quad and \quad i_2 = \frac{\Delta T_2}{h_2} \qquad (16.30)$$

Therefore

$$\frac{h_1 + h_2}{k_e L} H = \frac{h_1}{k_1 L} H_1 + \frac{h_2}{k_2 L} H_2 \qquad (16.31)$$

and

$$k_e = \frac{h_1 + h_2}{\frac{h_1}{k_1} + \frac{h_2}{k_2}} \qquad (16.32)$$

This result can be generalized for n layers:

$$k_e = \frac{\sum_{i=1}^{n} h_i}{\sum_{i=1}^{n} \frac{h_i}{k_i}} \qquad (16.33)$$

16.8 APPLICATIONS

Let's consider a soil deposit in a cold country (Figure 16.7). The question is: How deep will the frost penetrate during a very cold period? At depth, where the soil is not frozen, the temperature is T_d. The air is at a temperature T_a, much lower than 0° Celsius. It is assumed that the temperature of the soil surface T_s is the same as the air temperature T_a. The temperature at the bottom of the frozen soil is assumed to be 0° Celsius (C).

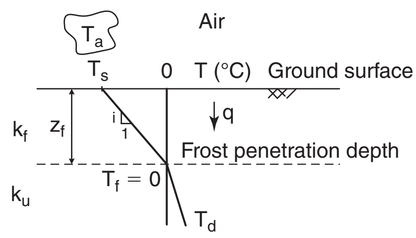

Figure 16.7 Frost penetration depth for a uniform soil.

The gradient of temperature in the frozen layer is i and is associated with a heat flow q and a thermal conductivity k_f. Therefore, the depth of the frozen soil is:

$$z_f = \frac{0 - T_s}{i} = -\frac{T\,k_f}{q} \qquad (16.34)$$

Now let's consider that a layer of snow covers the ground surface (Figure 16.8). The question here is: Would the depth of the frozen soil z_f be the same? The thickness of the snow cover is h_s and the thermal conductivity of the snow is k_s. The air temperature is T_a, and the temperature of the snow surface is T_s and is assumed equal to T_a. The thermal conductivity of the frozen soil is k_f.

The difference in temperature between the bottom of the frozen soil layer at 0°C and the surface of the snow layer at T_s can be written as:

$$0 - T_s = 0 - T_1 + T_1 - T_s \qquad (16.35)$$

where T_1 is the temperature at the interface between the bottom of the snow layer and the soil surface (Figure 16.8). By using the definition of the temperature gradient and then Fourier's law, Eq. 16.35 can be rewritten as:

$$0 - T_s = i_{sn} h_{sn} + i_f z_f = \frac{q_{sn}}{k_{sn}} h_{sn} + \frac{q_f}{k_f} z_f \qquad (16.36)$$

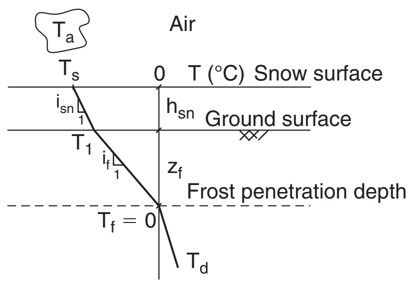

Figure 16.8 Frost penetration depth for a two-layer system.

For continuity purposes, though, the heat flow has to be the same in the snow and the frozen part of the soil layer. Then:

$$q_{sn} = q_f = q \tag{16.37}$$

and the frost penetration depth z_f is:

$$z_f = \frac{-T_s k_f}{q} - h_{sn}\frac{k_f}{k_{sn}} \tag{16.38}$$

As can be seen, the snow cover reduces the frost penetration depth by $h_{sn}\frac{k_f}{k_{sn}}$.

16.9 FROZEN SOILS

The general term *frozen soils* regroups problems of freezing soils, frozen soils, and thawing soils. Frozen soils are usually classified in three categories: soils with nonvisible ice (N), soils with visible ice and ice lenses less than 25 mm thick (V), and soils with visible ice with ice lenses larger than 25 mm thick (ICE). *Permafrost* is a term indicating that the ground, including soil and rock, is at or below 0° Celsius for more than two consecutive years. The temperature at which the water in the voids will freeze depends on many factors, including the salt content. The more salt there is, the lower the temperature has to be before the water will freeze. Generally, freezing starts at around −1°C, and at −20°C most soils are completely frozen. Figure 16.9 shows typical temperature profiles in frozen soils. It indicates that close to the surface there is usually a zone that freezes and thaws each year, called the *active zone*.

The water very close to the mineral surface of a particle can be tightly bound to the particle, especially for very small particles. This adsorbed water layer practically never freezes. Therefore, clays tend to resist freezing more than sands. The water film the furthest away from that boundary is the first one to freeze. Figure 16.10 shows conceptually the evolution of the water content of a soil as the temperature plunges below zero. As can be seen, the equilibrium frozen water content is higher for clayey soils than for sandy soils.

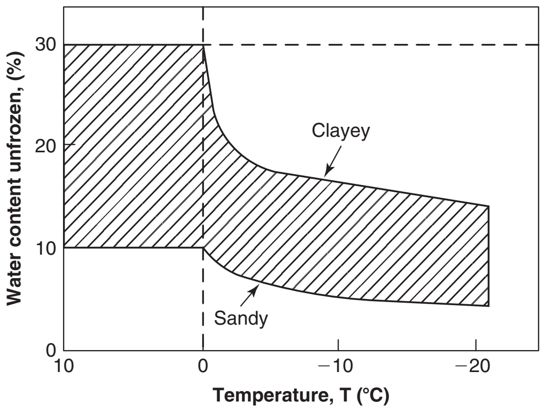

Figure 16.10 Evolution of water content with temperature.

Table 16.3 Frost Susceptibility and Soils

Soil Type	Frost Susceptibility
High-plasticity clays	Negligible
Low-plasticity clays, clays with sand and gravel	Moderate
Silty clays	Moderate to severe
Silts, silty sands, very fine sands	Severe
Gravels and sands with fines	Moderate
Clean sands and gravels	Negligible

Frost susceptibility is smallest for clean gravels and clean sands, on the one hand, and for high-plasticity clays on the other. The most frost-susceptible soils are silts, as shown in Table 16.3. The reason is that frost heave requires the soil to have the ability to lift water by capillary action and let the water flow through its voids. Clean gravels and clean sands have high hydraulic conductivity but little ability for capillary action; in other words, it is easy for the water to move, but the water has no energy to go anywhere. High-plasticity clays, in contrast, have a very high ability for capillary action but a very low hydraulic conductivity; in other words, the water has plenty of energy, but it is very hard to move through

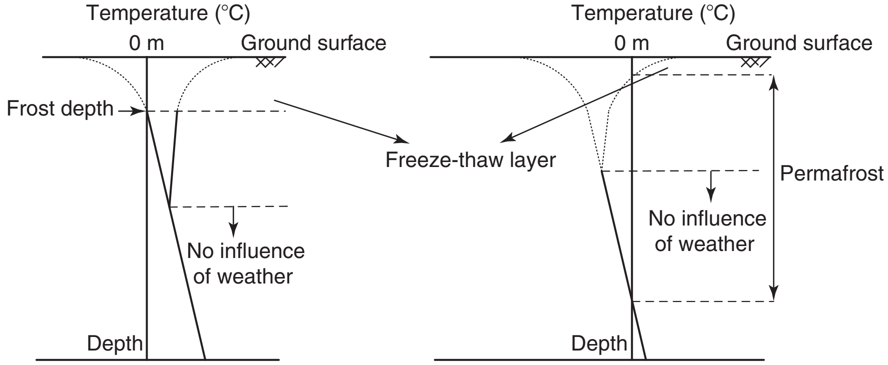

Figure 16.9 Typical temperature profiles in frozen soils.

the clay. Silts optimize the two requirements of capillary potential and water flow and are therefore some of the most frost-susceptible soils.

Note that the unit weight of ice is about 10% less than the unit weight of water. Therefore, if a certain weight of water becomes ice, it will occupy about 10% more volume. This is why icebergs float with only one-tenth of the iceberg mass showing up above the water level and 90% below it (hence the expression "this is only the tip of the iceberg"). If a soil becomes frozen, it will expand according to the increase in volume of the water becoming ice. These *ice lenses*, once started, continue to attract water and become thicker by something called the *cryosuction process*. Such ice lenses have significant lifting potential; the uplift pressures can be several hundreds of kPa and can reach 2000 kPa if the heave is confined. Heave magnitudes of 50 to 75 mm are common. The frozen soil can also develop an "adfreeze" bond with neighboring objects such as foundation piles. This bond can generate shear stresses from 50 to 150 kPa.

Frozen soils have four phases instead of three. Note that nearly all frozen soils contain liquid water. The phase diagram is shown in Figure 16.11. For the water content, a distinction must be made between the unfrozen water content and the frozen water or ice content. They are defined as:

Total water content $$w = \frac{W_w + W_i}{W_s} \quad (16.39)$$

Unfrozen water content $$w_u = \frac{W_w}{W_s} \quad (16.40)$$

Frozen water (ice) content $$w_i = \frac{W_i}{W_s} \quad (16.41)$$

$$w = w_u + w_i \quad (16.42)$$

where W_w is the weight of water, W_i is the weight of ice, and W_s is the weight of solids (Figure 16.11). In all other index parameters it is necessary to state what is included and what is not. For example, the degree of saturation, the void ratio, and the porosity can be defined by including or not including the ice.

The thermal properties of a frozen soil are the combination of the properties of the water, the ice, the air, and the soil skeleton. Table 16.2 shows these properties for each material individually and the impact they have on the soil. As can be seen, the frozen soil will have a higher thermal conductivity, a lower specific heat, and a higher thermal diffusivity. In other words, the heat will flow faster in the frozen soil, and it will be easier to squeeze the heat out of the frozen soil.

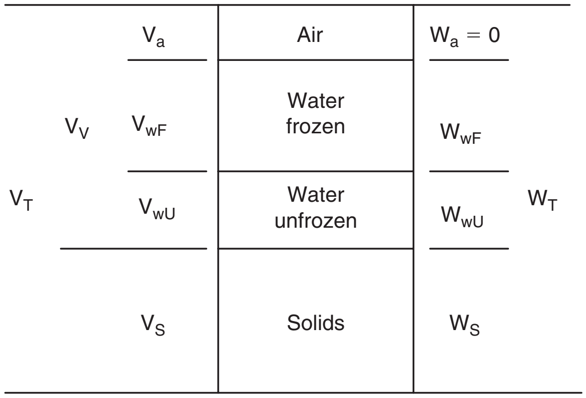

Figure 16.11 Phase diagram for a frozen soil.

The mechanical properties will also be affected. The shear strength will increase significantly, as the ice will contribute to increasing the cohesion intercept. The stiffness will also increase, as the ice essentially increases the amount of solids in the soil. However, the creep component of the settlement will be increased as the ice content increases. Indeed, ice exhibits creep properties that depend on the ice temperature; the lower the ice temperature, the less it will creep. The viscous exponent mentioned in Eq. 15.56 varies in the range of 0.1 to 0.5 for ice. Recall that the same exponent for unfrozen clays was 0.02 to 0.05 (see Figure 15.18). As a result, a frozen soil will creep more than the unfrozen soil under constant load, but the initial movement will be less. The hydraulic conductivity will decrease, as there is less area for the water to flow through. In that sense, frozen soils act according to the same principles as unsaturated soils. The best way to obtain the mechanical properties of frozen soils is to perform a laboratory or in situ test that duplicates the conditions under which the soil will be stressed in the project at hand.

There is a close analogy between frozen soils and unsaturated soils, and more interaction between these two fields is likely to be very rewarding.

PROBLEMS

16.1 A house is built on a frozen soil layer. The house generates heat such that it maintains a temperature of 20°C in the house. If the thermal conductivity of the frozen soil is $k_{frozen} = 1.3$ W/m.K, if the thermal conductivity of thawed-out soil is $k_{unfrozen} = 1.1$ W/m.K, and the temperature gradient in the frozen soil $i_{frozen} = -15$°C/m, what thickness of soil will thaw out?

16.2 A building is to be built with a geothermal foundation in a soil with a thermal diffusivity α equal to 5×10^{-7} m^2/s. The energy piles are 0.4 m in diameter and water circulates up and down the piles to take advantage of the beneficial effect of the soil temperature (hotter in the winter and cooler in the summer). The energy piles operate 8 months out of the year, and for optimum operation performance, the increase in temperature in adjacent energy piles due to the operation of one energy pile must not exceed 10% of the initial temperature difference between pile and soil. Calculate the minimum spacing between energy piles.

16.3 A cylindrical soil sample ($D = 0.075$ m, $L = 0.150$ m) is put in an oven where the temperature is kept at $T_f = 45^{o}$C. The initial temperature of the soil sample is $T_i = 25^{o}$C. The soil sample thermal conductivity k is 1.2 W/m.K and the volumetric heat capacity C is 1.2×10^6 J/m^3.K. Using the literature, find the solution that gives the increase in temperature at the center of the cylindrical soil sample as a function of time and calculate how long it will take for the center of the sample to reach 30°C, 35°C, and 40°C.

16.4 A two-layer system is made of a concrete pavement overlaying a sandy subgrade. The thermal properties of the two layers are shown in Figure 16.1s.

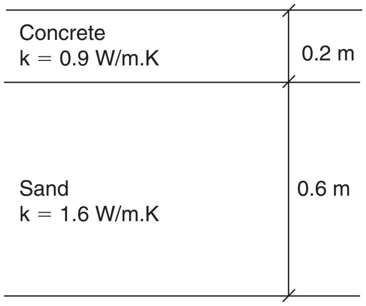

Figure 16.1s Two-layer system.

a. What is the equivalent thermal conductivity of the system if the heat flows horizontally?
b. What is the equivalent thermal conductivity of the system if the heat flows vertically?

16.5 Calculate the change in volume of a saturated soil with a water content of 30% and unit weight of 18 kN/m^3 if 90% of the water by weight becomes frozen.

16.6 Add a column to Table 16.1 dealing with electricity. Write Ohm's law and compare it to Darcy's and Fourier's laws.

16.7 A 0.3 m diameter, 10 m long probe is pushed into a clay. The clay has a thermal conductivity equal to 1.2 W/m.K and a thermal diffusivity equal to 2×10^{-6}m^2/s. The probe is at 2500°C and the intent is to bake the clay in place to create a baked-in-place pile with a wall thickness equal to 0.1 m. If the clay becomes permanently solidified at 1700°C and the initial clay temperature is 0°C, how long will it take before the pile is cooked and the probe can be removed to bake the next pile?

Problems and Solutions

Problem 16.1

A house is built on a frozen soil layer. The house generates heat such that it maintains a temperature of 20°C in the house. If the thermal conductivity of the frozen soil is $k_{frozen} = 1.3$ W/m.K, if the thermal conductivity of thawed-out soil is $k_{unfrozen} = 1.1$ W/m.K, and the temperature gradient in the frozen soil $i_{frozen} = -15$°C/m, what thickness of soil will thaw out?

Solution 16.1

Based on the principle of continuity of heat flow, and assuming that the surface temperature is T_s, the freezing temperature is T_f (equal to 0°C), and the thawing depth is x, we can write:

$$q_{unfrozen} = q_{frozen}$$

$$k_{unfrozen} \times A \times \frac{T_f - T_s}{x} = k_{frozen} \times A \times i_{frozen}$$

$$k_{unfrozen} \times A \times \frac{T_f - T_s}{x} = k_{frozen} \times A \times i_{frozen}$$

$$x = \frac{k_{unfrozen}}{k_{frozen}} \times \frac{T_f - T_s}{i_{frozen}}$$

The thawing depth is thus:

$$x = \frac{1.1}{1.3} \times \frac{0 - 20}{(-15)} = 1.13 \text{ m}$$

Problem 16.2

A building is to be built with a geothermal foundation in a soil with a thermal diffusivity α equal to 5×10^{-7} m^2/s. The energy piles are 0.4 m in diameter and water circulates up and down the piles to take advantage of the beneficial effect of the soil temperature (hotter in the winter and cooler in the summer). The energy piles operate 8 months out of the year, and for optimum operation performance, the increase in temperature in adjacent energy piles due to the operation of one energy pile must not exceed 10% of the initial temperature difference between pile and soil. Calculate the minimum spacing between energy piles.

Solution 16.2

From the problem data and using Figure 16.2 from the text, $T_m/T_o = 0.1$. The time factor T_F is calculated using Eq. 16.20:

$$T_F = \frac{t \times \alpha}{R_o^2} = \frac{8 \times 30 \times 24 \times 3600 \times 5 \times 10^{-7}}{0.2^2} = 259.2$$

From Figure 16.2, $\log_{10} R/R_0 = 1.3$; therefore, $R/R_0 = 19.95$ and the minimum distance between energy piles should be $R = R_o \times 19.95 = 4$ m.

Problem 16.3

A cylindrical soil sample (D = 0.075 m, L = 0.150 m) is put in an oven where the temperature is kept at $T_f = 45^oC$. The initial temperature of the soil sample is $T_i = 25^oC$. The soil sample thermal conductivity k is 1.2 W/m.K and the volumetric heat capacity C is 1.2×10^6 J/m^3.K. Using the literature, find the solution that gives the increase in temperature at the center of the cylindrical soil sample as a function of time and calculate how long it will take for the center of the sample to reach 30°C, 35°C, and 40°C.

Solution 16.3

Carslaw and Jaeger (1947) developed the solution for the temperature increase at the center of a cylindrical sample as a function of time. The percentage increase or decrease in soil sample temperature U can be plotted versus the normalized time factor T as shown in Figure 16.1s, for both a finite-length sample and an infinite-length sample.

$$U = \frac{T - T_{\min}}{T_{\max} - T_{\min}}$$

$$T = \frac{\alpha(m^2/s) \times t(s)}{D^2(m^2)}$$

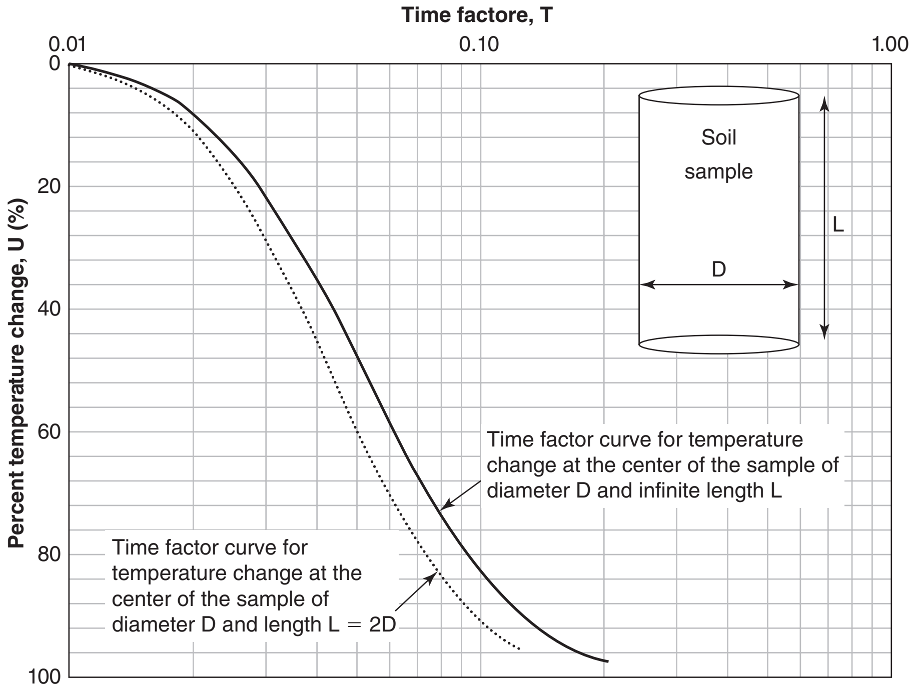

Figure 16.2s Percent temperature change U versus time factor T.

The thermal diffusivity α of the soil sample is:

$$\alpha(m^2/s) = \frac{k}{C} = \frac{1.2}{1.2 \times 10^6} = 10^{-6}$$

When the temperature reaches 30°C, $U = 25\%$; from Figure 16.1s, $T = 0.03$:

$$t(s) = \frac{T \times D^2}{\alpha} = \frac{0.03 \times 0.075^2}{10^{-6}} = 168 \text{ sec}$$

When the temperature reaches 35°C, $U = 50\%$; from Figure 16.1s, $T = 0.04$:

$$t(s) = \frac{T \times D^2}{\alpha} = \frac{0.04 \times 0.075^2}{10^{-6}} = 225 \text{ sec}$$

When the temperature reaches 40°C, $U = 75\%$; from Figure 16.1s, $T = 0.07$:

$$t(s) = \frac{T \times D^2}{\alpha} = \frac{0.07 \times 0.075^2}{10^{-6}} = 393 \text{ sec}$$

Problem 16.4

A two-layer system is made of a concrete pavement overlaying a sandy subgrade. The thermal properties of the two layers are shown in Figure 16.2s.

a. What is the equivalent thermal conductivity of the system if the heat flows horizontally?
b. What is the equivalent thermal conductivity of the system if the heat flows vertically?

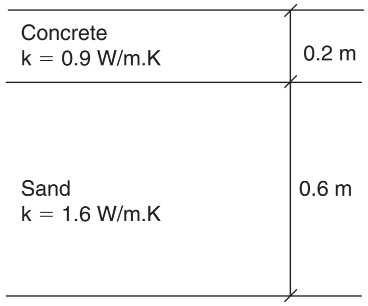

Figure 16.1s Two-layer system.

Solution 16.4

a. The equivalent thermal conductivity of the system if the heat flows horizontally can be calculated using Eq. 16.26:

$$k_e = \frac{k_1 h_1 + k_2 h_2}{h_1 + h_2} = \frac{0.9 \times 0.2 + 1.6 \times 0.6}{0.2 + 0.6} = 1.42 \text{ W/m.K}$$

b. The equivalent thermal conductivity of the system if the heat flows horizontally can be calculated using Eq. 16.32:

$$k_e = \frac{h_1 + h_2}{\dfrac{h_1}{k_1} + \dfrac{h_2}{k_2}} = \frac{0.2 + 0.6}{\dfrac{0.2}{0.9} + \dfrac{0.6}{1.6}} = 1.34 \text{ W/m.K}$$

Problem 16.5

Calculate the change in volume of a saturated soil with a water content of 30% and unit weight of 18 kN/m^3 if 90% of the water by weight becomes frozen.

Solution 16.5

First we have to calculate the weight of water in unfrozen conditions. The total unit weight γ is:

$$\gamma_t = \frac{W_w + W_s}{V_t} = 18 \text{ kN/m}^3$$

The water content is 30%; therefore, $W_w = 0.3W_s$. Assuming a soil unit volume of 1 m^3, the water weight W_w is 4.15 kN and the solid weight W_s is 13.85 kN.

The volume of water in the unfrozen condition is:

$$\gamma_w = \frac{W_w}{V_w} = 10 \text{ kN/m}^3 \quad \text{or} \quad \text{V}_w = 0.415 \text{ m}^3$$

If 90% of water weight becomes frozen, then the weight of ice W_i is 3.735 kN. The unit weight of ice γ_i is 10% less than the unit weight of water; therefore, the volume of ice is:

$$V_i = \frac{W_i}{\gamma_i} = \frac{3.735}{9} = 0.415 \text{ m}^3$$

The volume of remaining unfrozen water is:

$$V_w = \frac{W_w}{\gamma_w} = \frac{(4.15 - 3.735)}{10} = 0.0415 \text{ m}^3$$

The total volume of water and ice when 90% of water mass becomes frozen is 0.4565 m^3; therefore, the change in volume of the soil is 0.0415 m^3 or 4.15% of the original volume.

Problem 16.6

Add a column to Table 16.1 dealing with electricity. Write Ohm's law and compare it to Darcy's and Fourier's laws.

Solution 16.6

Parameter	Flow of water	Flow of heat	Flow of current
Quantity	Volume V (m^3)	Heat Q (J)	Electric charge (C)
Potential	Head h_t (m)	Temperature T (K)	Voltage (V)
Gradient	Hydraulic gradient i_h (unitless)	Temperature gradient i_t (K/m)	Electric field gradient E(V/m)
Flux	Flow rate Q (m^3/s)	Heat transfer rate H (J/s)	Electric current flow (C/s)
Flux density	Velocity v (m/s)	Heat flow q (J/s.m^2)	Electrical flux density (C/m^2)
Conductivity	Hydraulic conductivity k_h (m/s)	Thermal conductivity k_t (J/s.K.m)	Electric conductivity, σ (S/m)
Law	Darcy	Fourier	Ohm
Storage	Compressibility	Specific heat c (J/kg.K)	Capacitance
Decay coefficient	Coefficient of consolidation c_v (m^2/s)	Thermal diffusivity α (m^2/s)	Electrical diffusivity D(m^2/s)

Problem 16.7

A 0.3 m diameter, 10 m long probe is pushed into a clay. The clay has a thermal conductivity equal to 1.2 W/m.K and a thermal diffusivity equal to 2×10^{-6}m^2/s. The probe is at 2500°C and the intent is to bake the clay in place to create a baked-in-place pile with a wall thickness equal to 0.1 m. If the clay becomes permanently solidified at 1700°C and the initial clay temperature is 0°C, how long will it take before the pile is cooked and the probe can be removed to bake the next pile?

Solution 16.7

The probe can be considered an infinite cylindrical heat source, because the length-to-diameter ratio is very large. The increase in temperature that must be achieved at a radial distance $R = R_o + 0.1 = 0.25$ m is 1700°C. Using Figure 16.2:

$$T_m/T_o = 1700/2500 = 0.68$$
$$R/R_o = 0.25/0.15 = 1.67$$
$$\log(R/R_o) = 0.222$$

From Figure 16.2, $T_f = 1$. Therefore:

$$T_F = 1 = \frac{t \times \alpha}{R_o^2} \Rightarrow t = \frac{R_o^2}{\alpha} = \frac{0.15^2}{2 \times 10^{-6}} = 11250\,\text{sec.} = 3.125 \text{ hours}$$