CHAPTER 15

Shear Strength Properties

15.1 GENERAL

Three strengths are usually considered for a material: compressive strength, tensile strength, and shear strength. *Compressive strength* is tested by applying an all-around pressure (hydrostatic loading) on a sample and recording the pressure at which the sample fails. In general, soils are very strong in all-around compression. Exceptions include soils with a very loose structure and a slight cementation such as calcareous sands; under such loading, these soils can collapse on themselves and crush with a drastic reduction in volume. (For comparison purposes, other materials that are weak in compression are puffed rice and marshmallow.)

Tension strength is tested by pulling on a sample at both ends. In general, soils are very weak in tension. This mode of failure does not often control the behavior of soils, however, because tensile stresses between the grains are rare, due in part to gravity stresses that impose a natural prestressing in the deposit. If tensile stresses develop between the grains, they first correspond to a decrease in compression rather than true tension. Tensile cracks do develop at the top of failing slopes or in shrinking soils near the ground surface.

Shear strength can be tested by moving the top part of a sample with respect to the bottom part of a sample in the direction of the plane separating the top from the bottom. Most often, the shear strength is what controls the ultimate loads in geotechnical engineering projects. This is why it is so important to the geotechnical engineer. As an added example to convince you, think of the unconfined compression test. The soil sample is loaded vertically and has no lateral pressure applied. When the vertical stress becomes too high, the sample fails along a diagonal where the shear stress reaches the shear strength. So, even though the loading is compression, the failure is in shear. By comparison with concrete and steel, the strength of soil is very small (Table 15.1).

Where does the shear resistance come from in a soil mass? It cannot be from the shearing resistance of the air or the water, because these shear resistances are negligible. In fact, it comes from the shearing resistance at the particle-to-particle contacts. The particles are pressed against each other by normal forces and the shear resistance is due in large part to the friction at the contacts. The normal stress between particles is quantified by the effective stress, and therefore one component of the shear strength is the product of the effective stress on the plane of failure times the coefficient of friction of the interface. The second component of the shearing resistance at the contacts is the glue that may exist at the contacts. This glue may be real, as in the case of calcium cementation, or apparent, as in the case of water tension between the particles that pulls the particles together when the soil dries. The apparent cohesion is in fact a part of the friction resistance, as the effective stress is enhanced by the tension in the water.

Table 15.1 Strength of Soils, Concrete, and Steel

Material	Shear Strength	Unconfined Compression Strength	Tensile Strength
Soil	5 kPa to 500 kPa	10 to 1000 kPa	0 to 100 kPa
Concrete	750* to 1100* kPa	20,000 to 40,000 kPa	2000 to 4000 kPa
Steel	230,000 kPa	250,000 kPa	400,000 kPa

*This low value is explained in section 15.3.

As will be seen, many factors can affect the shear strength of a soil. The best way to obtain the shear strength of a soil is to measure it directly by laboratory test or in situ test and by reproducing in the test the same stress conditions as those anticipated in the field. Any shear strength parameter should be quoted by explaining how it was measured and over what stress range the soil was tested.

15.2 BASIC EXPERIMENTS

15.2.1 Experiment 1

If a block of concrete with a weight N is placed on a concrete floor (Figure 15.1), the force F necessary to initiate motion

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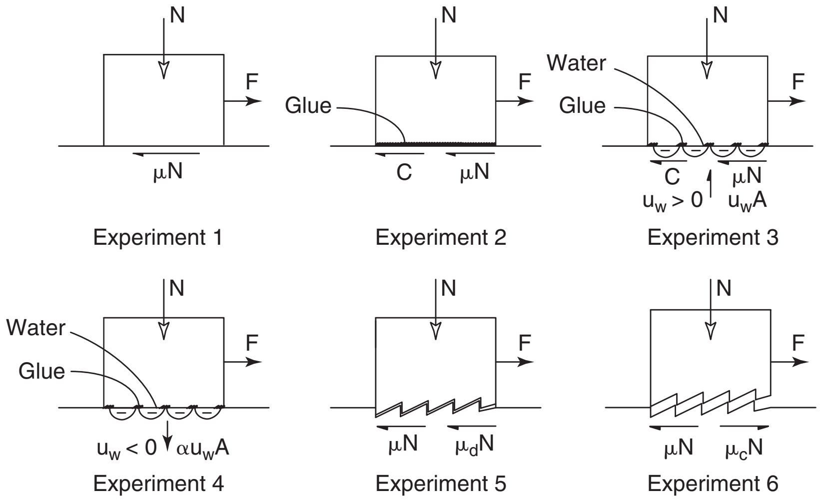


Figure 15.1 Basic experiments.

by dragging the concrete block on the concrete floor is given by:

$$F = \mu N \quad (15.1)$$

where μ is the coefficient of friction of the concrete-to-concrete interface, F is the shear force, and N is the normal force. By dividing both sides of Eq. 15.1 by the interface contact area A, and replacing μ by $\tan\varphi$, the equation becomes:

$$\frac{F}{A} = \frac{N}{A}\tan\varphi \qquad \text{or} \qquad \tau_f = \sigma\tan\varphi \quad (15.2)$$

15.2.2 Experiment 2

Imagine that before I place the concrete block on the concrete floor, I paint a layer of glue on the concrete floor (Figure 15.1) and then I place the concrete block on the glue and I let it set. Now I repeat the experiment and exert a force F, higher than in the first case because of the glue, to drag the block. Then Eq. 15.1 becomes:

$$F = C + \mu N \quad (15.3)$$

where C is the force required to break the glue. If I divide again by the total area A and use $\tan\varphi$ instead of μ, I get:

$$\frac{F}{A} = \frac{C}{A} + \frac{N}{A}\tan\varphi \qquad \text{or} \qquad \tau_f = c + \sigma\tan\varphi \quad (15.4)$$

15.2.3 Experiment 3

Imagine now that I make some small holes on the concrete floor, that I paint the glue only on the top of the bumps between holes, and that I flood the holes with water before I place the block (Figure 15.1). When I place the concrete block on top of the concrete floor, two things happen: the glue sets and the water is squeezed between the two surfaces. If the water saturates the holes and if the water cannot escape, there will be a water compression stress $u_w (u_w > 0)$ under the block and an associated uplift force $u_w \times A$, which decreases the normal force on the sliding plane. Equation 15.1 then becomes:

$$F = C + \mu(N - u_w A) \quad (15.5)$$

If I divide by the total area A, I get:

$$\frac{F}{A} = \frac{C}{A} + \frac{(N - u_w A)}{A}\tan\varphi \quad \text{or} \quad \tau_f = c + (\sigma - u_w)\tan\varphi \quad (15.6)$$

15.2.4 Experiment 4

Let's repeat that last experiment, but this time, before I place the concrete block, I dry up some of the water in the holes such that the little amount of water that is left is held in the holes by tension in the water ($u_w < 0$) (Figure 15.1). This creates a suction force between the block and the concrete floor that increases the force F necessary to move the block. This force is equal to the water tension times the area over which the water exists. This area is a fraction α of the total area A and is represented by αA where α is less than one. Equation 15.1 becomes:

$$F = C + \mu(N - \alpha u_w A) \quad (15.7)$$

If I divide by the total contact area A, I get

$$\frac{F}{A} = \frac{C}{A} + \frac{(N - \alpha u_w A)}{A}\tan\varphi$$
$$\text{or} \quad \tau_f = c + (\sigma - \alpha u_w)\tan\varphi \quad (15.8)$$

15.2.5 Experiment 5

Let's go back to experiment 1, but this time we design some special grooves in the concrete floor and matching grooves at the bottom of the concrete block. These grooves are inclined as shown in Figure 15.1, such that to move the concrete block,

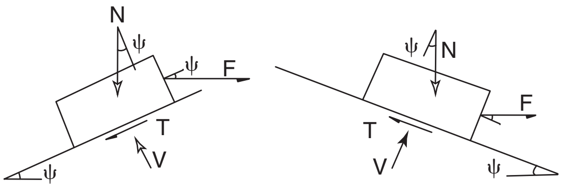


Figure 15.2 Concrete block on slopes.

the block has to be pushed sideways and upward. This type of interface increases the shear force F necessary to move the block as follows. The friction force μN is still necessary, but a force $\mu_d N$ has to be added to overcome the roughness of the upward grooves. The subscript d is for dilation. The coefficient μ_d is equal to $\tan\psi$ where ψ is the angle of the groove with the horizontal. Here is why. Referring to Figure 15.2, the friction force T is equal to the normal force $N\cos\psi$ times the coefficient of friction $\tan\varphi$; this is the constitutive law:

$$T = N\cos\psi\tan\varphi \tag{15.9}$$

Then equilibrium in the direction of the force T can be written; this is the fundamental law:

$$T + N\sin\psi = F\cos\psi \tag{15.10}$$

By combining Equations 15.9 and 15.10, we get:

$$F = N\tan\varphi + N\tan\psi \tag{15.11}$$

If we divide by the total contact area, we get:

$$\begin{aligned}\tau_f &= c + \sigma\tan\varphi + \sigma\tan\psi \\ &= c + \sigma\tan(\varphi+\psi)(1-\tan\varphi\tan\psi)\end{aligned} \tag{15.12}$$

Note that if ψ is relatively small, the term $(1-\tan\varphi\tan\psi)$ is close to 1.

15.2.6 Experiment 6

Let's repeat experiment 5, but now with the grooves slanted in the other direction (Figure 15.1). This time the downward slope creates a force that decreases the value of F. Equation 15.11 becomes:

$$F = N\tan\varphi - N\tan\psi \tag{15.13}$$

If we divide by the total contact area, we get:

$$\begin{aligned}\tau_f &= c + \sigma\tan\varphi - \sigma\tan\psi \\ &= c + \sigma\tan(\varphi-\psi)(1+\tan\varphi\tan\psi)\end{aligned} \tag{15.14}$$

Again, if ψ is relatively small, the term $(1+\tan\varphi\tan\psi)$ is close to 1.

15.3 STRESS-STRAIN CURVE, WATER STRESS RESPONSE, AND STRESS PATH

The stress-strain curve of a soil depends on a number of factors, including the soil stress history, the current stress level, the structure of the soil, and others. Two types of curves are usually encountered. The first exhibits a peak followed by a strain softening region; the second does not exhibit a peak but simply an increase toward a plateau at large strains (Figure 15.3).

Overconsolidated soils, hard soils, and dense soils have curves exhibiting peaks (brittle), whereas normally consolidated soils, soft soils, and loose soils have curves exhibiting no peak (ductile). For the same soil, under the same confinement, but for an overconsolidated and normally consolidated case, both curves tend to reach a common strength at large strain (Figure 15.3). This point is called the *critical state*. At that point the soil does not change volume while shearing.

The water stress exhibits two different types of behavior for these two distinct types of curves. In the case of the curve with no peak, the soil compresses throughout the shearing process and the water goes into compression, thereby reducing the effective stress. In the case of the curve with a peak, the water goes into compression initially (reduction in effective stress) and then the soil starts to dilate; the associated increase in volume creates a decrease in water stress that ends up as tension. As a result, the effective stress increases. Note that water stress is not always measured during such tests. Nevertheless, the water stress is necessary for proper reduction of the data in terms of effective stress.

The stress path in two dimensions is the path described by the top of the Mohr circle. It describes the evolution of certain stresses throughout the loading of the sample. Specifically, it tracks the path described by the points with p, q stress coordinates where p and q are defined as:

$$\mathrm{p} = \frac{\sigma_1+\sigma_3}{2} \quad \text{or} \quad \mathrm{p} = \frac{\sigma_v+\sigma_h}{2} \tag{15.15}$$

$$\mathrm{q} = \frac{\sigma_1-\sigma_3}{2} \quad \text{or} \quad \mathrm{q} = \frac{\sigma_v-\sigma_h}{2} \tag{15.16}$$

where σ_v and σ_h are the vertical and horizontal total stresses in a triaxial test, for example. The most useful stress paths are plotted in terms of effective stresses (p′ and q′)::

$$\mathrm{p}' = \frac{\sigma_1'+\sigma_3'}{2} \quad \text{or} \quad \mathrm{p}' = \frac{\sigma_v'+\sigma_h'}{2} \tag{15.17}$$

$$\mathrm{q}' = \frac{\sigma_1'-\sigma_3'}{2} \quad \text{or} \quad \mathrm{q}' = \frac{\sigma_v'-\sigma_h'}{2} \tag{15.18}$$

where σ_v' and σ_h' are the vertical and horizontal total stresses in a triaxial test, for example. Examples of effective stress paths for different types of tests are shown in Figure 15.4. In any lab test, it is most desirable to match the effective stress path followed by the soil in the field during the project construction and the project life. Examples of field stress

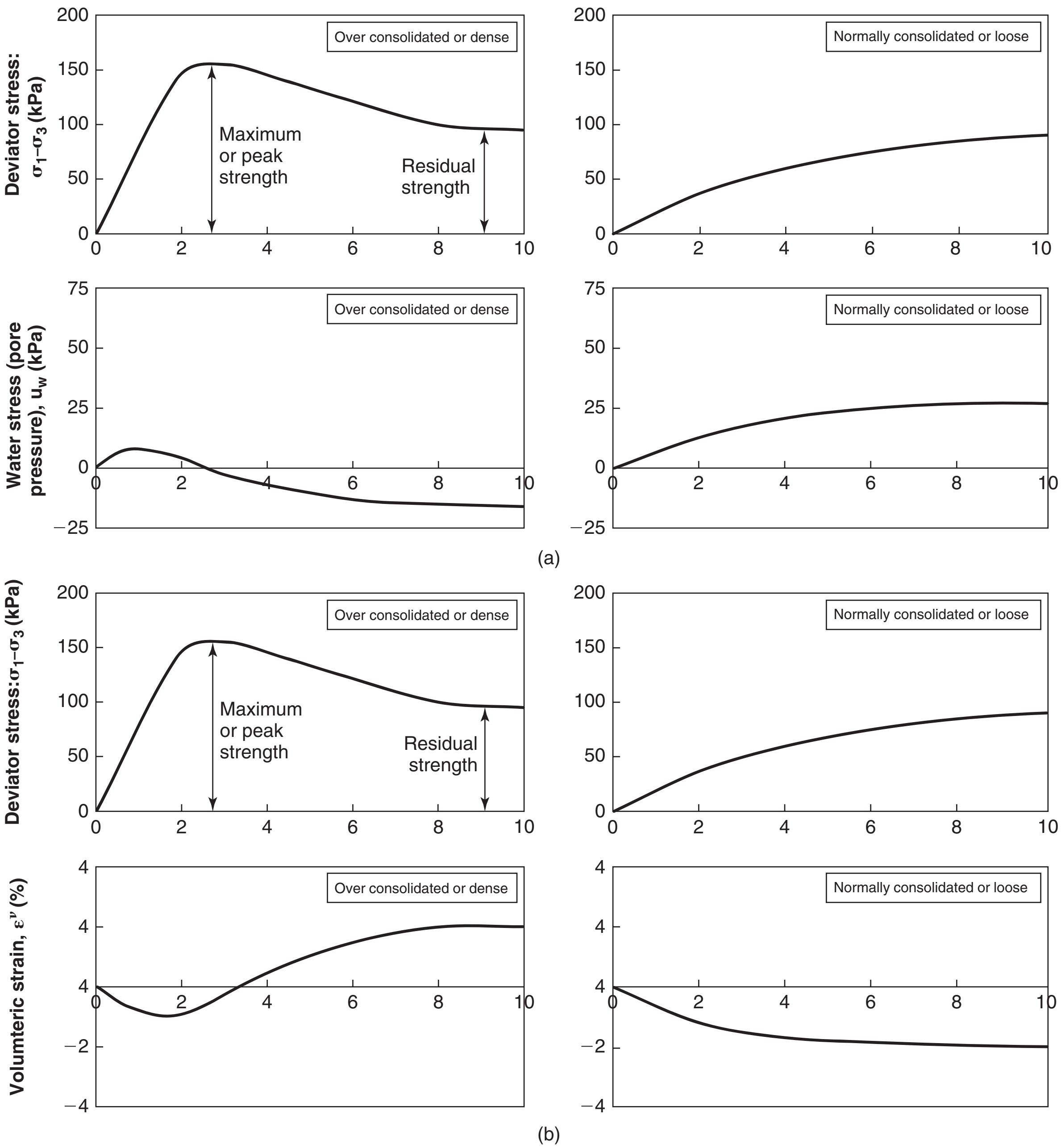


Figure 15.3 Stress-strain curves in soils: *(a)* Consolidated undrained test. *(b)* Consolidated drained test.

paths are shown in Figure 15.5. Stress path OA would be the case of the wetting of an unsaturated soil or the filling of an earth dam reservoir. Stress path OB might be associated with a slow excavation process. Stress path OC would correspond to a rapid embankment construction. Stress path OD would be the case of a slow embankment construction. As can be seen from Figure 15.5, stress paths OC and OB are those that will approach the failure envelope the fastest, because they go toward the strength envelope with the shortest distance.

The *shear strength* τ_f of a soil is defined as the highest shear stress the soil can resist. For the curve with a peak, it will be the shear stress corresponding to the peak of the curve, known as the *peak shear strength*. For the curve with no peak, it is the shear stress at large strain; a value of 10% strain is often used when no obvious plateau is reached. The residual shear strength of a soil is defined only when the curve has a peak. In this case, the value of the shear stress corresponding to the post-peak plateau is the *residual shear strength*. The *remolded shear strength* is the shear strength of the remolded soil. The remolded shear strength can be equal to the residual shear strength, but more often it is less than the residual shear strength.

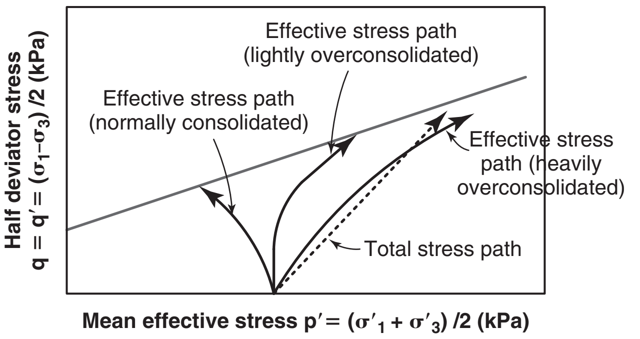


Figure 15.4 Stress paths for an overconsolidated and a normally consolidated soil.

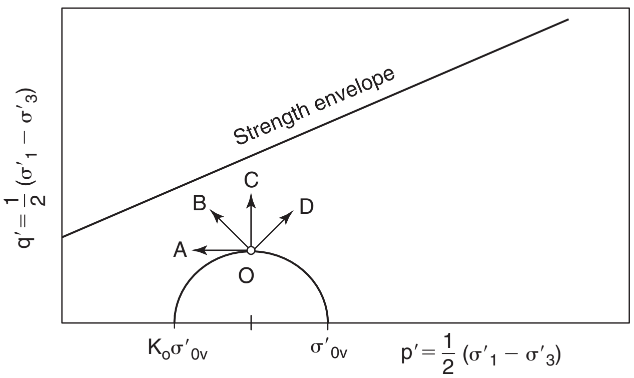


Figure 15.5 Example of field stress paths.

15.4 SHEAR STRENGTH ENVELOPE

15.4.1 General Case

Each one of the experiments described in section 15.2 has a parallel for soils. Imagine now that the interface, instead of concrete on concrete, is a plane in a soil with no water. The shear strength that can be generated by the soil will definitely have a component due to friction, as explained in experiment 1:

$$\tau_f = \sigma \tan\varphi \qquad (15.19)$$

where τ_f is the soil shear strength, σ is the normal stress on the plane of failure, and φ is the friction angle. Recall that $\tan\varphi$ is a coefficient of friction and as such is often between 0 and 1, although we will see later that it could actually be higher than 1. The glue added in experiment 2 refers to any cohesion that may exist between the soil particles. This cohesion is relatively rare, and when it is not zero, it is quite small (5 to 20 kPa). The cohesion plus the friction give:

$$\tau_f = c + \sigma \tan\varphi \qquad (15.20)$$

The water added in experiment 3 refers to the case where the voids between the soil particles are full of water or 100% saturation. In this case the water is under a certain amount of pressure u_w (compression below the groundwater level, GWL, or tension above the GWL) that changes the effect of the normal stress. The normal stress σ becomes the effective normal stress σ': the difference between the total normal stress σ and the water stress u_w. Also, the cohesion becomes the effective stress cohesion c′ and the friction angle becomes the effective stress friction angle φ':

$$\tau_f = c' + (\sigma - u_w)\tan\varphi' = c' + \sigma'\tan\varphi' \qquad (15.21)$$

In experiment 4, the water no longer filled the voids and covered a fraction α of the total area. As a result, Eq. 15.21 is modified because the expression of the effective stress σ' has changed:

$$\tau_f = c' + (\sigma - \alpha u_w)\tan\varphi' = c' + \sigma'\tan\varphi' \qquad (15.22)$$

Experiment 5 conveys an important message regarding soil shear strength: the concept of dilatancy. When a very dense soil is sheared, it tends to increase in volume or dilate. This is due to each particle having to climb over the one in front of it during shearing. This increase in volume is associated with a lifting effect similar to that of the concrete block and increases the shear strength compared to a no-volume-change situation. The shear strength equation becomes:

$$\tau_f = c' + \sigma'\tan\varphi' + \sigma'\tan\psi' \qquad (15.23)$$

where ψ' is the effective stress dilatancy angle. If ψ' is small, Eq. 15.23 can be rewritten as:

$$\tau_f = c' + \sigma'\tan(\varphi' + \psi') \qquad (15.24)$$

In experiment 6, the problem of dilatancy became a problem of compression and the term $\sigma'\tan\psi'$ had to be subtracted rather than added. In geotechnical engineering, it is common instead to use a negative value of ψ' and keep Eq. 15.24 the same. In the general case, the shear strength of soils is measured and the effects of dilatancy or compression are absorbed in the value of φ'. The general equation for the shear strength of soils is therefore:

$$\tau_f = c' + \sigma'\tan\varphi' \qquad (15.25)$$

where τ_f is the shear strength, c′ is the effective stress cohesion intercept, σ' is the effective stress normal to the plane of failure ($\sigma - \alpha u_w$), and φ' is the effective stress friction angle. This equation works for all soils in all situations, including saturated or unsaturated, drained or undrained, dilative or compressive. If c′ and φ' are considered to be constants, then Eq. 15.25 is a straight line on the τ vs. σ' set of axes (Figure 15.6) and is referred to as the *strength envelope*. Any stress point below or on the envelope is possible, but it is not possible for any stress point to plot above that line. Thus, any failure Mohr circle will have to be tangent to the failure envelope.

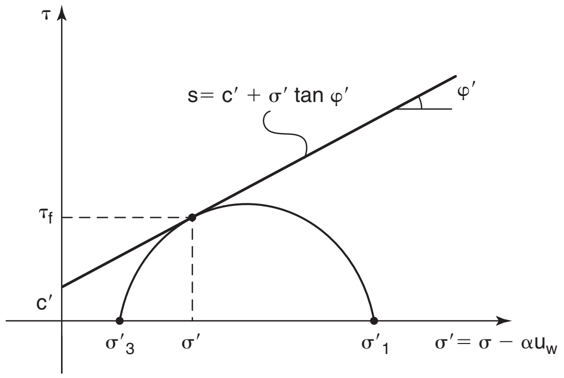


Figure 15.6 Shear strength envelope for soils.

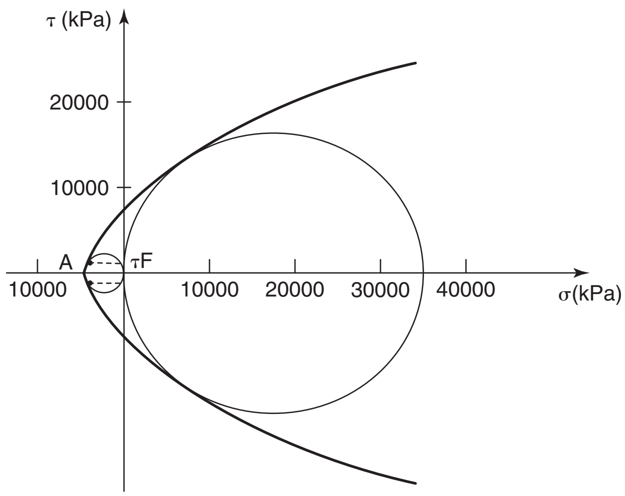


Figure 15.7 Strength envelope for concrete.

15.4.2 The Case of Concrete

Concrete has a very large cohesion intercept compared to that of soils. Figure 15.7 shows the Mohr circle for an unconfined compression test on concrete ($f_c = 35000$ kPa) on one side and for an unconfined tension test on concrete ($f_t = 3500$ kPa) on the other. The shear strength envelope for concrete is also shown conceptually. The value of the shear strength used in the code is given by the equation s (kPa) $= 5.25(f_c(\text{kPa})^{0.5}) = 982$ kPa. This value is shown on the strength envelope (Figure 15.7) and is associated with a significant tension. The reason is that in concrete beam design, shear typically occurs in sections near the supports where tension is large. It would be like using Eq. 15.25 with a large tension for the normal stress; this would decrease the shear strength significantly. This is why the shear strength of concrete in Table 15.1 is quite small—much less than one-half the unconfined compression strength.

15.4.3 Overconsolidated Fine-Grained Soils

A special case occurs with overconsolidated soils where the shear strength envelope does not quite follow the straight line of Eq. 15.25. These soils exhibit a preconsolidation pressure σ'_p as measured in the consolidation test. For stresses less than σ'_p, deformations are small; for stresses higher than σ'_p, deformations are much larger for the same increase in effective stress. The preconsolidation pressure can be thought of as a *yield stress* on the consolidation stress-strain curve. This yield stress also affects the shear strength envelope. Indeed, when the effective stress σ' on the plane of failure is less than σ'_p, the cohesion intercept found in many overconsolidated clays is apparent. However, when the effective stress σ' on the plane of failure is larger than σ'_p, the cohesion intercept is destroyed by the stress level that destructures the soil, and the envelope goes through the origin (Figure 15.8). Others have proposed that the envelope be curved as shown in Figure 15.8. Mesri and Abdelghafar (1993) proposed an empirical equation, for stresses less than σ'_p, that takes into account the overconsolidation ratio on the drained shear strength, as follows:

$$\tau_f = \sigma' \tan\varphi' \left(\frac{\sigma'_p}{\sigma'}\right)^{1-m} \tag{15.26}$$

where m is a shear strength coefficient given in Table 15.2.

15.4.4 Coarse-Grained Soils

A special case also arises for coarse-grained soils where the shear strength envelope does not quite follow the straight line of Eq. 15.25. These soils tend to dilate during shear at

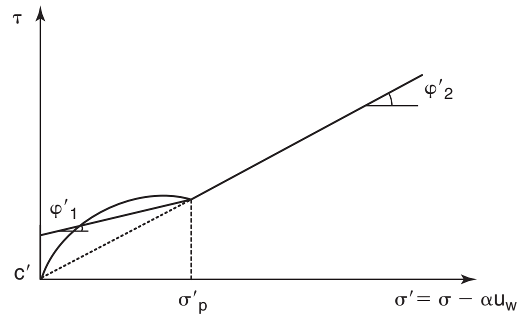


Figure 15.8 Strength envelope for overconsolidated fine-grained soils.

Table 15.2 Parameter m for Equation 15.22

Soil	m for intact soil	m for destructured soil
Cemented soft clays	0.4–0.5	0.5–0.7
Stiff clays and shales	0.5–0.6	0.6–0.8
Soft clays	0.6–0.7	0.7–0.9

(After Terzaghi et al. 1996.)

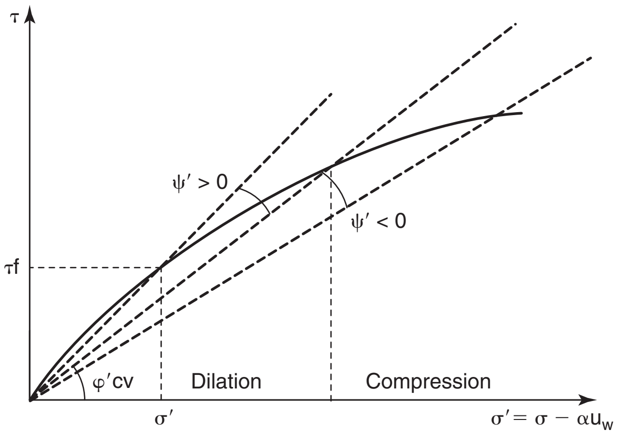


Figure 15.9 Strength envelope for coarse-grained soil.

a low confinement level and compress at higher confinement levels. The stress level at which the change between dilation and compression occurs depends on the relative density of the coarse-grained soil: the higher the density, the larger the stress range over which the soil dilates. All soils end up compressing during shear at some level of stress. Recall the simplified equation including the dilation angle ψ':

$$\tau_f = c' + \sigma' \tan(\varphi' + \psi') \quad (15.27)$$

Because ψ' is positive at smaller stresses (dilation) and becomes negative at higher stresses (compression), the sum $\varphi' + \psi'$ is larger at smaller effective stresses than it is at higher effective stresses, and the shear strength envelope is curved (Figure 15.9).

When the soil dilates, a distinction is made between the friction angle φ'_{peak} associated with the peak of the stress-strain curve and the post-peak large strain friction angle φ'_{cv} at which the soil reaches a point where shearing takes place at constant volume. The difference between the two is the dilation angle ψ':

$$\varphi'_{peak} = \varphi'_{cv} + \psi' \quad (15.28)$$

For dense soils, φ'_{peak} is larger than φ'_{cv} and ψ' is positive; for loose soils, φ'_{peak} is smaller than or equal to φ'_{cv} and ψ' can be negative. In most tests, the angle φ'_{peak} is the one measured. The angle ψ' can be inferred from the post-peak large strain shear strength that gives φ'_{cv} and then using Eq. 15.28.

15.5 UNSATURATED SOILS

For unsaturated soils, the effective stress can be calculated as explained in section 10.13. The most general expression for the effective stress σ' is:

$$\sigma' = \sigma - \alpha u_w - \beta u_a \quad (15.29)$$

Therefore, the general equation for the shear strength of a soil (unsaturated or saturated) is:

$$\tau_f = c' + \sigma' \tan\varphi' = c' + (\sigma - \alpha u_w - \beta u_a) \tan\varphi' \quad (15.30)$$

where σ' is the normal effective stress on the plane of failure, σ is the normal total stress on the plane of failure, α is the fraction of the total area of the failure plane covered by the water, β is the fraction of the plane covered by the air, u_w is the water stress, u_a is the air stress, and φ' is the effective stress friction angle. As explained in section 10.13, when the soil is saturated or when the air is occluded, Eq. 15.29 becomes:

$$\sigma' = \sigma - u_w \quad (15.31)$$

If the air is not occluded, there is a path for the air to be connected directly to the atmosphere and the air stress is atmospheric or zero gauge pressure. Then the most general expression of the effective stress in soils covering all real cases is:

$$\sigma' = \sigma - \alpha u_w \quad (15.32)$$

Therefore, in all real cases for unsaturated soils ($u_a = 0$ or $u_a = u_w$) and saturated soils, the equation for the shear strength τ_f is:

$$\tau_f = c' + (\sigma - \alpha u_w) \tan\varphi' \quad (15.33)$$

The parameter α can be estimated by taking it equal to the degree of saturation S (Figure 10.16) or by using a slightly modified version of the Khalili and Khabbaz (1998) equation (Figure 10.17):

$$\alpha = S \quad (15.34)$$

$$\alpha = \sqrt{\frac{u_{wae}}{u_w}} \quad (15.35)$$

where u_{wae} is the air entry value of the water tension and u_w is the water tension.

Shear strength equations other than Eq. 15.30 have been proposed, such as the one of Fredlund and Rahardjo (1993):

$$\tau_f = c' + (\sigma - u_a) \tan\varphi' + (u_a - u_w) \tan\varphi^b \quad (15.36)$$

where φ^b is an angle indicating the rate of increase in shear strength relative to the matric suction $u_a - u_w$. Equation 15.36 can be reorganized as follows:

$$\tau_f = c' + \left(\sigma - \frac{\tan\varphi^b}{\tan\varphi'} u_w - \left(1 - \frac{\tan\varphi^b}{\tan\varphi'}\right) u_a\right) \tan\varphi' \quad (15.37)$$

Comparison of Eq. 15.37 with Eq. 15.30 shows that the two equations are identical if:

$$\alpha = \frac{\tan\varphi^b}{\tan\varphi'} \quad \text{and} \quad \beta = 1 - \frac{\tan\varphi^b}{\tan\varphi'} \quad (15.38)$$

You may recall from section 10.13, Eq. 10.52, that $\alpha + \beta = 1$; therefore, both conditions are satisfied automatically and the ratio $\tan\varphi^b/\tan\varphi'$ can be estimated through Eqs. 15.34 and 15.35.

15.6 EXPERIMENTAL DETERMINATION OF SHEAR STRENGTH (LAB TESTS, IN SITU TESTS)

There are many ways to determine the effective stress shear strength parameters of soils. Because many factors influence the shear strength, it is best to aim at reproducing the initial stress conditions and the stress path during loading, while matching the drainage conditions to be encountered in the field. In the laboratory, the most common tests are the unconsolidated undrained triaxial test (UUT), the consolidated undrained triaxial test (CUT), the unconsolidated undrained direct shear test (UUDS), the consolidated undrained direct shear test (CUDS), the unconsolidated undrained simple shear test (UUSS), and the consolidated undrained simple shear test (CUSS). *Unconsolidated* means that no drainage is allowed when the confining pressure is applied; *consolidated* means that drainage is allowed during application of the confining pressure until the excess water stress has come back down to zero. The second letter in the acronym refers to the loading process; for example, a consolidated undrained test means that the loading process is done while allowing no drainage. Table 15.3 shows which test and test requirements are applicable to determining which shear strength parameters for saturated and unsaturated soils. Note that if the water in the soil voids is in tension (saturated or unsaturated), additional

Table 15.3 Laboratory Tests for Shear Strength Determination of Saturated and Unsaturated Soils

Test	Measurements	Shear Strength	Comments
Direct shear test, Unconsolidated Undrained	Normal stress, shear stress	s_u	Effective stress $\sigma' =$ existing σ' in sample
Direct shear test, Consolidated Undrained	Normal stress, shear stress	s_u	Effective stress $\sigma' =$ chosen σ' for confinement
Direct shear test, Consolidated Drained	Normal stress, shear stress	c', φ'	Estimate of dilatancy angle ψ' if horizontal and vertical displacements measured. If water is in tension, measurements of water tension, air entry water tension, and water content are also necessary.
Simple shear test, Unconsolidated Undrained	Normal stress, shear stress, displacement	s_u and complete stress-strain curve	Effective stress $\sigma' =$ existing σ' in sample
Simple shear test, Consolidated Undrained	Normal stress, shear stress, displacement	s_u and complete stress-strain curve	Effective stress $\sigma' =$ chosen σ' for confinement
Simple shear test, Consolidated Drained	Normal stress, shear stress, displacement	c', φ', and complete stress-strain curve	Estimate of dilatancy angle ψ' if horizontal and vertical displacements measured. If water is in tension, measurements of water tension, air entry water tension, and water content are also necessary.
Triaxial test, Unconsolidated Undrained	Vertical stress, confinement stress, displacement	s_u, complete stress-strain curve, and c', φ' if water stress measured	Effective stress $\sigma' =$ existing σ' in sample
Triaxial test, Consolidated Undrained	Vertical stress, confinement stress, displacement	s_u, complete stress-strain curve, and c', φ' if water stress measured	Effective stress $\sigma' =$ chosen σ' for confinement
Triaxial test, Consolidated Drained	Vertical stress, shear stress, displacement	c', φ' and complete stress-strain curve	Estimate of dilatancy angle ψ' if volume change measured. If water is in tension, measurements of water tension, air entry water tension, and water content are also necessary.

measurements are necessary to obtain the effective stress shear strength parameters. These additional measurements include the measurement of the water tension u_w, the air entry water tension u_{wae}, and the water content w or degree of saturation S. The reason is that the equation for the shear strength is:

$$\tau_f = c' + (\sigma - \alpha u_w) \tan \varphi' \tag{15.39}$$

which requires estimating α as S or $\sqrt{u_{wae}/u_w}$

The undrained shear strength s_u is simply read as the peak shear stress reached during an undrained test. The effective stress shear strength parameters (c', φ') require plotting the results on the shear stress τ vs. effective normal stress σ', as shown in sections 9.9, 9.10, and 9.12.

In situ tests (see Chapter 7) can also be used to obtain the shear strength of soils. The most direct tests are the vane shear test (VST) and the borehole shear test (BHST). The VST is simple and can be used to obtain the undrained shear strength of fine-grained soils. The BHST is a bit more complicated, but can be used to obtain the effective stress friction angle of coarse-grained soils. The BHST can also be used for the undrained shear strength of saturated fine-grained soils by conducting a rapid test, and the effective stress shear strength parameters of saturated soils by conducting a test slow enough not to generate water stress. Water stress is not typically measured during the BHST or the VST. Other tests such as the standard penetration test (SPT) and the cone penetration test (CPT) can be used to obtain shear strength parameters through correlations. For example, the blow count N of the SPT and the point resistance q_c of the CPT have been used to estimate the friction angle of coarse-grained soils, as well as the undrained shear strength of fine-grained soils.

15.7 ESTIMATING EFFECTIVE STRESS SHEAR STRENGTH PARAMETERS

The parameters referred to in this section are the effective stress cohesion intercept c', the effective stress friction angle φ', and the effective stress dilation/compression angle ψ'.

15.7.1 Coarse-Grained Soils

For coarse-grained soils, the *effective stress cohesion intercept* c′ is considered to be equal to zero, which often leads to coarse-grained soils being called cohesionless soils. The parameter φ' controls the shear strength of these soils, along with the normal effective stress on the plane of failure. The friction angle φ' for coarse-grained soils varies between 25 and 50 degrees. Recall that $\tan \varphi'$ is the coefficient of friction μ, which varies correspondingly between 0.5 and 1.2. A coefficient of friction higher than 1 is possible in soils because of the dilatancy effect, which combines friction and lifting. Tables 15.4 and 15.5 as well as Figure 15.10 give suggested values of the friction angle for coarse-grained soils. These are values of φ' typically obtained in a triaxial test or a direct shear test. Note that the value of φ' obtained in a plane strain test is about 10% higher than the one obtained in a triaxial compression test. The reason is that in the plane strain deformation process, the particles are forced to move in a restricted two-dimensional domain and cannot find the path of least resistance. Thus, the resistance is slightly higher and so is the friction angle. An application of this observation is in the difference between the friction angle for a strip footing and for a circular or square footing:

$$\varphi'_{plane\ strain} \simeq 1.1 \times \varphi'_{triaxial\ compression} \tag{15.40}$$

The dilation/compression angle ψ' is typically included in the measurement of the friction angle φ'. Therefore, it should not be added to the measured value of φ'. The following relationship between the two angles has been used:

$$\psi' = \varphi' - 30 \tag{15.41}$$

Houlsby (1991) presents a plot (Figure 15.11) indicating that Eq. 15.41 should be modified to:

$$\psi' = \varphi' - 34 \tag{15.42}$$

In any case, the angle ψ' varies between −5 for very loose soils to +15 degrees for very dense soils.

Table 15.4 Range of Values for φ'

	Friction angle in degrees		Coefficient of friction	
Soil	Loose	Dense	Loose	Dense
Gravel with sand	35	50	0.7	1.2
Sand, angular grains, well graded	33	45	0.65	1.0
Sand, round grain, uniform	27.5	34	0.52	0.67
Silty sand	27 to 33	30 to 34	0.51 to 0.65	0.58 to 0.67
Inorganic silt	27 to 30	30 to 35	0.51 to 0.58	0.58 to 0.7

(After Terzaghi and Peck 1967)

Table 15.5 Guide for Values for φ'

Gravels and Sands Strength			
Description	φ'°	N (bpf)	Simple field test*
Very loose	$<28^{\circ}$	<4	12 mm diameter rebar pushed in 0.3 m by hand. Shows definite marks of footsteps; hard to walk on.
Loose	28°–30°	4–10	12 mm diameter rebar pushed in 0.1 m by hand. Shows footsteps.
Medium or compact	30°–36°	10–30	12 mm diameter rebar driven 0.3 m with carpenter hammer. Footsteps barely noticeable.
Dense	36°–41°	30–50	12 mm diameter rebar driven 0.1 m with carpenter hammer. No marks of footsteps.
Very dense	$>41^{\circ}$	>50	12 mm diameter rebar driven 0.03 m with carpenter hammer. No marks of footsteps.

*Note that these tests are performed at the ground surface of the gravel-sand deposit, not on a sample.

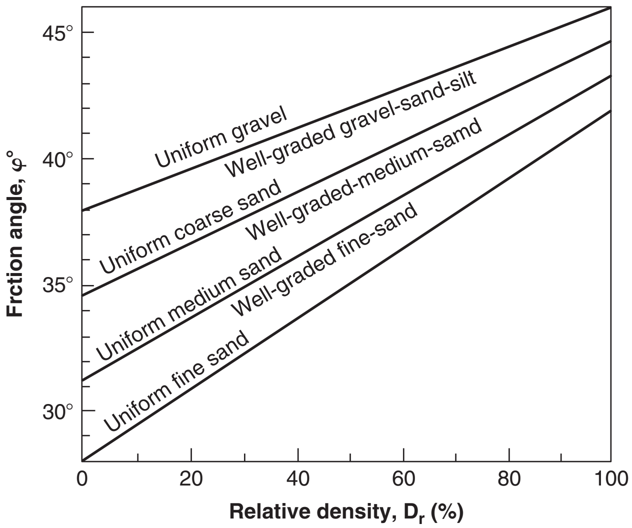


Figure 15.10 Friction angle vs. relative density. (From Schmertmann 1975)

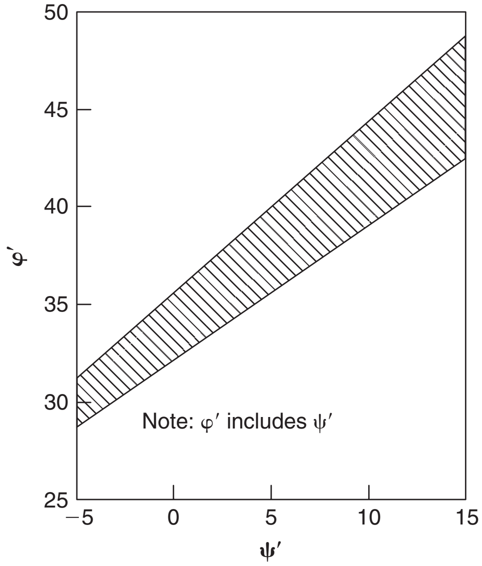


Figure 15.11 Peak friction angle vs. dilation/compression angle. (After Houlsby 1991)

The parameter φ' can be measured directly in situ by using the BHST. The BHST may be the only tool that can give a direct measure of φ' for coarse-grained soils (see section 7.6). The parameter φ' has also been correlated with in situ test results including the SPT blow count N and the CPT point resistance q_c. It is not recommended to use the PMT limit pressure p_L to obtain the friction angle. Using N or q_c to obtain φ' requires understanding the following. The shear strength of a coarse-grained soil is expressed as:

$$s = \sigma' \tan \varphi' \tag{15.43}$$

Therefore, there are two components involved in the soil response to the SPT or CPT: the effective stress level σ' at the depth of the test and the frictional characteristics of the soil $\tan \varphi'$. Hence, it is important to extract the influence of σ' from N and q_c before correlating them with φ'. The corrections for the influence of σ' on N were discussed in section 7.2:

$$N_1 = N_{measured} \times \left(\frac{\sigma'_{ov}}{p_a}\right)^{-0.5} \tag{15.44}$$

where N_1 and $N_{measured}$ are the corrected and uncorrected values of the SPT blow count respectively, σ'_{ov} is the vertical effective stress at the depth of the test, and p_a is the atmospheric pressure used for normalization.

There are other ways to include the influence of the stress level in the correlation. The following is a correlation between N and φ' that incorporates the stress level influence separately; it was proposed by Schmertmann (1975) and formulated into an equation by Kulhawy and Mayne (1990):

$$\tan\varphi' = \left(\frac{N}{12.2 + 20.3\frac{\sigma'_{ov}}{p_a}}\right)^{0.34} \tag{15.45}$$

Terzaghi and Peck (1967) proposed the simple correlation shown in Figure 15.12.

The cone penetrometer point resistance q_c should also be corrected for the stress level before attempting correlation with the friction angle φ' (Figure 15.13). Much like the correction for N, the correction for q_c is:

$$q_{c1} = q_{c\,measured} \times \left(\frac{\sigma'_{ov}}{p_a}\right)^{-0.5} \tag{15.46}$$

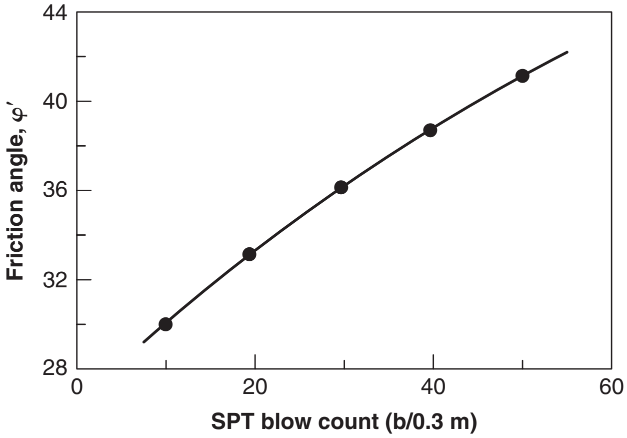


Figure 15.12 Correlation between the SPT blow count N and the friction angle φ' for coarse-grained soils. (After Terzaghi and Peck 1967)

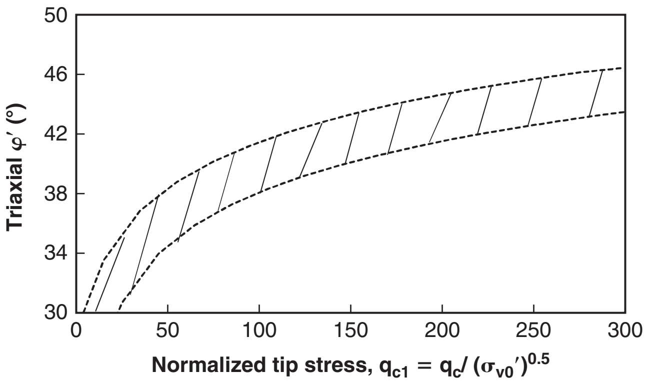


Figure 15.13 Correlation between the CPT point resistance q_c and the friction angle φ' for coarse-grained soils. (After Mayne 2007a, 2007b)

Then the following correlations exist between q_{c1} and φ' (Mayne 2007a, 2007b):

$$\varphi'_{\text{deg}} = 17.6 + 11 \times \log\left(\frac{q_{c1}}{p_a}\right) = 17.6 + 11 \times \log\left(\frac{q_{c\,measured}}{\sqrt{\sigma'_{ov}p_a}}\right) \tag{15.47}$$

15.7.2 Fine-Grained Soils

Normally consolidated fine-grained soils have no cohesion, but some overconsolidated fine-grained soils do exhibit true cohesion c'. It is obtained by drawing a straight line (the shear strength envelope) through the failure points from shear strength tests and determining the intercept at $\sigma' = 0$. Sometimes fine-grained soils are called *cohesive soils*, but this is misleading, as the friction component of the shear strength still dominates. In fact, it is safe to ignore the cohesion c′ for most geotechnical problems. What creates the c′ value ? The phenomenological reason for any "glue" between particles can be attributed to electrical forces between fine particles and to cementation that may develop through chemical reaction. These bonds are sometimes called *diagenetic bonds*. This parameter c′ is called *true cohesion* and is not to be confused with the apparent cohesion c_{app} which comes from water tension in the voids. In fact, c_{app} is part of the friction term in Eq. 15.33:

$$c_{app} = -\alpha u_w \tan\varphi' \tag{15.48}$$

where α is the water area ratio, u_w is the water tension, and φ' is the effective stress friction angle. Because u_w has a negative value, c_{app} is positive and can be significant if the soil dries enough to generate significant water tension. This water tension can reach 10,000 kPa; therefore, c_{app} can reach hundreds of kPa. The value of c', in comparison, is rarely higher than 25 kPa. Table 15.6 gives some possible values for different soils.

The friction angle φ' corresponding to the peak shear strength for overconsolidated fine-grained soils and to the large strain strength for normally consolidated fine-grained

Table 15.6 Range of Possible Values for the Effective Stress Cohesion c′ of Fine-Grained Soils

Soil	Cohesion c′ in kPa
Coarse-grained soils	0
Silts, low plasticity	0
Silts, high plasticity, overconsolidated	5 to 10
Clays, normally consolidated	0
Clays, overconsolidated, low plasticity	10 to 15
Clays, overconsolidated, high plasticity	15 to 20

Table 15.7 Range of Possible Values for the Effective Stress Friction Angle φ' of Fine-Grained Soils

Soil	Friction Angle φ' in degrees
Silts, low plasticity	30 to 38
Silts, high plasticity	18 to 30
Clays, low plasticity	23 to 31
Clays, high plasticity	16 to 26

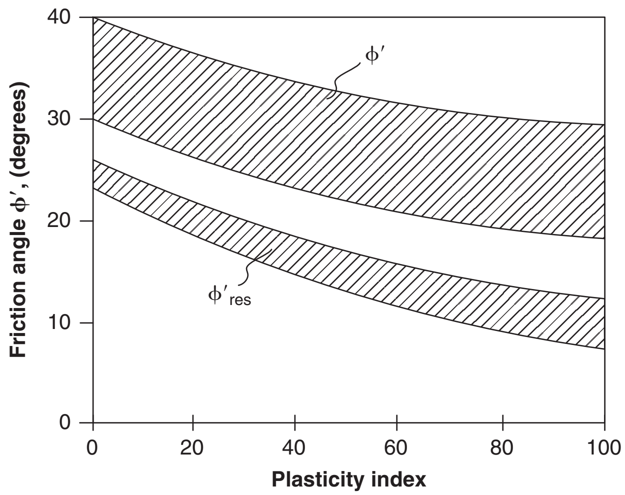


Figure 15.14 Effective stress friction angle φ' versus plasticity index for fine-grained soils.

soils is lower than the one for coarse-grained soils and varies from 20 to 35 degrees. Table 15.7 shows some possible values of φ' for various fine-grained soils. In general, the friction angle φ' decreases when the plasticity index increases. You will realize this if you wash your hands after handling a kaolinite clay (baby powder) and then after handling a bentonite clay. The bentonite will feel a lot more slippery than the kaolinite. Figure 15.14 shows general trends of φ' with the plasticity index I_p. The effective stress parameters for fine-grained soils are not obtained from in situ tests because it is difficult to ensure that the test is a drained test. One exception to this statement is the use of the borehole shear test, which is essentially a direct shear tests on the wall of the borehole; in this case the test must be performed slowly enough during the consolidation phase and the shearing phase that the assumption of no excess water stress can be made.

15.8 UNDRAINED SHEAR STRENGTH OF SATURATED FINE-GRAINED SOILS

A particular case arises when a soil is loaded fast enough that the water does not have time to drain during the loading time or if drainage is prevented in a laboratory test. In this case the shear strength is called the *undrained shear strength* and designated as s_u. This undrained case occurs rarely for most construction problems concerning coarse-grained soils, but it is often encountered with construction problems involving fine-grained soils. For example, if it takes a month to build an embankment, a clean sand would have time to fully drain, but a high-plasticity clay would not. As will be shown next, during the undrained loading of a fine-grained soil, the effective stress does not increase significantly and therefore the shear strength does not increase significantly either. Instead, the water stress increases significantly. So the controlling design case for loading on a fine-grained soil is often the *undrained case*, also called the *short-term case*. Indeed, at that time the load is maximum and the shear strength is minimum. As time goes by, the water stress decreases due to water drainage, the shear strength increases accordingly, and the factor of safety against failure increases. The critical time in the case of a fine-grained soil is immediately after loading. This is why the undrained shear strength of fine-grained soils is so important: It controls the stability design of many geotechnical structures.

As pointed out before, the general equation (Eq. 15.25) applies in all cases, including the undrained case, and s_u can be expressed as:

$$s_u = c' + \sigma' \tan\varphi' \qquad (15.49)$$

The problem is that it is often difficult to obtain the effective stress on the plane of failure σ'. One of the important factors in this case is how compressible the soil skeleton is compared to water (Figure 15.15).

15.8.1 Weak Soil Skeleton: Soft, Normally Consolidated Soils

When a load is applied rapidly to a soft, normally consolidated soil, the water picks up the entire load because the soil skeleton is too weak to contribute. Therefore, the increase in normal stress $\Delta\sigma$ on the soil due to loading is equal to the increase in water stress Δu_w. The effective stress before loading σ_b' is equal to:

$$\sigma_b' = \sigma_b - u_{wb} \qquad (15.50)$$

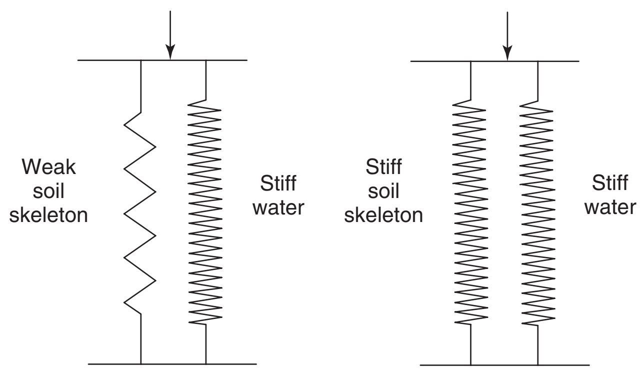


Figure 15.15 Model of saturated soil skeleton and water.

Where σ_b is the total stress before loading and u_{wb} is the water stress before loading. The effective stress immediately after loading σ'_a is equal to:

$$\begin{aligned}\sigma'_a &= \sigma_b + \Delta\sigma - (u_{wb} + \Delta u_w)\\ &= \sigma_b + \Delta\sigma - (u_{wb} + \Delta\sigma)\\ &= \sigma_b - u_{wb} = \sigma'_b \end{aligned} \tag{15.51}$$

As can be seen, the effective stress has not increased and therefore the shear strength has not increased. The undrained shear strength of a saturated, fine-grained soil with a weak skeleton is a constant s_u. This statement must be qualified by adding the following: provided that the stress level (confinement) is the same, the stress history is the same (OCR), and the stress path followed to go from the initial state to failure is the same. Indeed, all three factors can influence s_u and selecting the correct s_u is more complex than often thought (Ladd, 1991).

When a fine-grained soil with a weak skeleton is loaded in an undrained test, the Mohr circle is as shown in Figure 15.16*a* in the effective stress set of axes (Mohr circle 1) and as shown in Figure 15.16*b* in the total stress set of axes (Mohr circle 2). The difference between the total stress and the effective stress is the water stress u_w. If a second undrained test is performed on the same soil but after increasing the confining pressure by $\Delta\sigma$ (Mohr circle 3), then the water stress also increases by $\Delta\sigma$, the effective stresses do not change, and the effective stress Mohr circle does not change (still Mohr circle 1 on Figure 15.16*a*). The reason why the undrained shear strength is a constant independent of the total stress is because the effective stress Mohr circle remains the same regardless of the total stress.

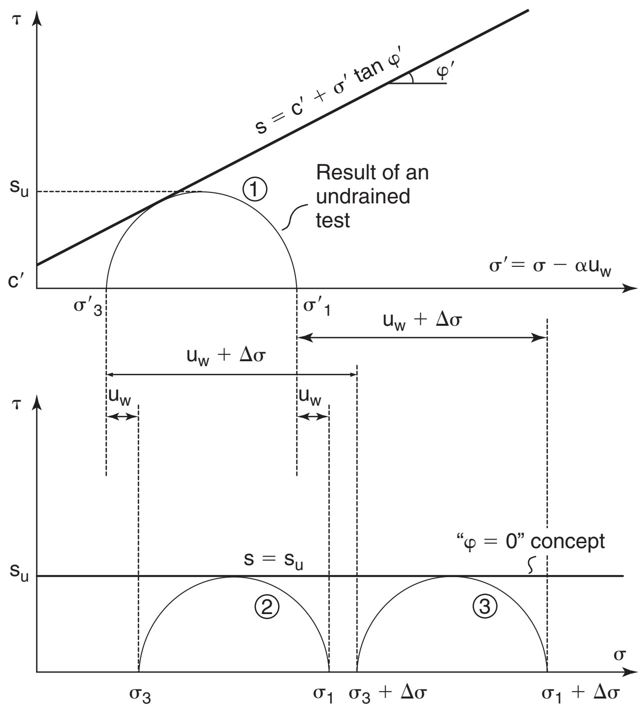


Figure 15.16 Undrained shear strength for weak soil skeleton.

This is often called the $\varphi = 0$ concept because the envelope on the shear strength vs. total stress set of axes is horizontal. This is not to say that such a soil is frictionless ($\mu = \tan\varphi = 0$). It simply means that the envelope is horizontal. In reality, the soil always has a nonzero friction angle, but it shows up in the shear strength vs. effective stress set of axes only ($\mu' = \tan\varphi' \neq 0$). Recall that the intergranular stress is represented by the effective stress, and in that set of axes, the soil friction is identified.

15.8.2 Strong Soil Skeleton: Overconsolidated Soils

In the case of an overconsolidated soil with a strong skeleton, when the load is applied rapidly, the soil skeleton is able to resist part of the load and the water picks up the rest of the load. The increase in water stress Δu_w is not as large as the increase in normal stress $\Delta\sigma$ on the soil, and is equal to f × $\Delta\sigma$ where f is smaller than 1. The effective stress before loading σ'_b is equal to:

$$\sigma'_b = \sigma_b - u_{wb} \tag{15.52}$$

Where σ_b is the total stress before loading and u_{wb} is the water stress before loading. The effective stress immediately after loading σ'_a is equal to:

$$\begin{aligned}\sigma'_a &= \sigma_b + \Delta\sigma - (u_{wb} + \Delta u_w)\\ &= \sigma_b + \Delta\sigma - (u_{wb} + f\Delta\sigma)\\ &= \sigma_b - u_{wb} + (1-f)\Delta\sigma > \sigma'_b \end{aligned} \tag{15.53}$$

As can be seen, the effective stress has increased and therefore the shear strength has increased. The undrained shear strength s_u of a saturated, fine-grained soil with a strong skeleton increases somewhat with the total stress because the effective stress increases somewhat. Again, factors like stress level reached under drained conditions (confinement), the stress history (OCR), and the stress path followed to go from the initial state to failure influence the value of s_u (Ladd 1991).

When a fine-grained soil with a strong skeleton is loaded in an undrained test, the Mohr circle is as shown in Figure 15.17*a* in the effective stress set of axes (Mohr circle 1) and as shown in Figure 15.17*b* in the total stress set of axes (Mohr circle 2). The difference between the total stress and the effective stress is the water stress u_w. If a second undrained test is performed on the same soil, but after increasing the confining pressure by $\Delta\sigma$ (Mohr circle 4 in the total stress set of axes), then the water stress increases by a fraction f x $\Delta\sigma$ of $\Delta\sigma$, the effective stress increases somewhat, and the effective stress Mohr circle moves (Mohr circle 3 on Figure 15.17*a*). The reason why the undrained shear strength increases slightly with an increase in total stress is that the effective stress increases slightly.

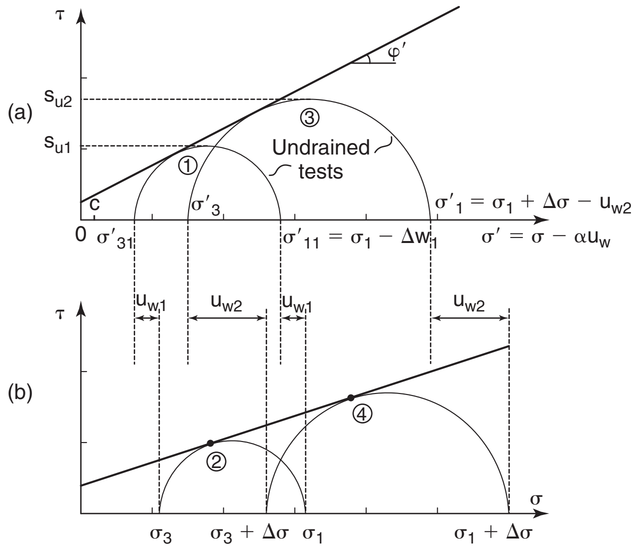


Figure 15.17 Undrained shear strength for strong soil skeleton.

15.8.3 Rate of Loading Effect on the Undrained Strength

Soils, like many other materials, are viscous: They increase in strength when the loading rate increases. The reason is attributed to the difference in water stress being developed at slower rates and at higher rates. At higher rates, the soil grains do not have time to move by finding the path of least resistance and more dilation is generated, thereby inducing higher effective stresses and shear strength. Also, the water in the voids has viscosity of its own. Indeed, water and air are viscous as well. They are Newtonian fluids and therefore are linearly viscous. They obey the following law:

$$\tau = \eta \dot{\gamma} \tag{15.54}$$

where τ is the shear stress, η is the dynamic viscosity of the material, and $\dot{\gamma}$ is the strain rate. The dynamic viscosity of water at 20°C is 10^{-6} kPa.s and the dynamic viscosity of air at 20°C is 1.8×10^{-8} kPa.s. The kinematic viscosity ν takes units of m^2/s and is defined as:

$$\nu = \frac{\eta}{\rho} \tag{15.55}$$

where ρ is the mass density of the material. For water, ρ is 1000 kg/m^3 and for air it is 1.2 kg/m^3 at the Earth's surface.

Soils are much less viscous than water and air. Equation 15.54 states that if the strain rate is doubled, the shear stress resistance will also double. In soils, if the strain rate is doubled, the shear stress resistance will be increased by a few percentage points. You might think: "Then why worry about it?" The issue is that sometimes the strain rate can be multiplied by factors of 1000 or more, and in such cases the increase or decrease can be significant. Briaud and Garland (1985) proposed the following model for the undrained shear strength of fine-grained soils:

$$\frac{s_{u1}}{s_{u2}} = \left(\frac{t_1}{t_2}\right)^{-n} \tag{15.56}$$

where s_{u1} and s_{u2} are the undrained shear strengths measured in time to failure t_1 and t_2 respectively and n is the viscous exponent for the fine-grained soil. This exponent was correlated to the reference undrained shear strength s_{uref} (Figure 15.18). The exponent n was also correlated with other soil parameters as follows:

$$n = 0.044\left(\frac{s_{u\,ref}}{p_a}\right)^{-0.22} \tag{15.57}$$

$$n = 0.028 + 0.00060\, w \tag{15.58}$$

$$n = 0.035 + 0.00066\, PI \tag{15.59}$$

$$n = 0.036 + 0.046\, LI \tag{15.60}$$

where n is the soil viscous exponent in Eq. 15.56, s_{uref} is the reference undrained shear strength taken as the one obtained with a time to failure equal to one hour, p_a is the atmospheric pressure, w is the natural water content in percent, PI is the plasticity index in percent, and LI is the liquidity index (as a fraction, not a percent). The scatter in those correlations is significant, as shown in Figure 15.18. All in all, the most common values of the exponent n vary from 0.03 to 0.06, with 0.03 occurring for a high-strength, low-plasticity clay and 0.06 for a low-strength, high-*PI* clay.

15.9 THE RATIO S_U/σ'_{OV} AND THE SHANSEP METHOD

The undrained shear strength, like any soil shear strength, depends on the effective stress on the failure plane at the time of failure. A measure of this effective stress level is the vertical effective stress at rest σ'_{ov} in the field at the depth z considered. The ratio s_u/σ'_{ov} has been used to try to normalize the variation of s_u with depth and with effective stress level. For normally consolidated, saturated, fine-grained soils, the ratio has been found to vary between 0.2 and 0.35, increasing slightly with the plasticity index. Holtz et al. (2011) propose that for normally consolidated, saturated, fine-grained soils:

$$\left(\frac{s_u}{\sigma'_{ov}}\right)_{NC} = 0.23 \pm 0.04 \tag{15.61}$$

When the overconsolidation ratio (OCR) increases above 1, the ratio s_u/σ'_{ov} of the overconsolidated soil becomes higher than the ratio s_u/σ'_{ov} of the normally consolidated soil. This increase is not linear, and the following relationship has been proposed (Ladd et al., 1977)

For overconsolidated, saturated, fine-grained soils:

$$\left(\frac{s_u}{\sigma'_{ov}}\right)_{OC} = S(OCR)^m \tag{15.62}$$

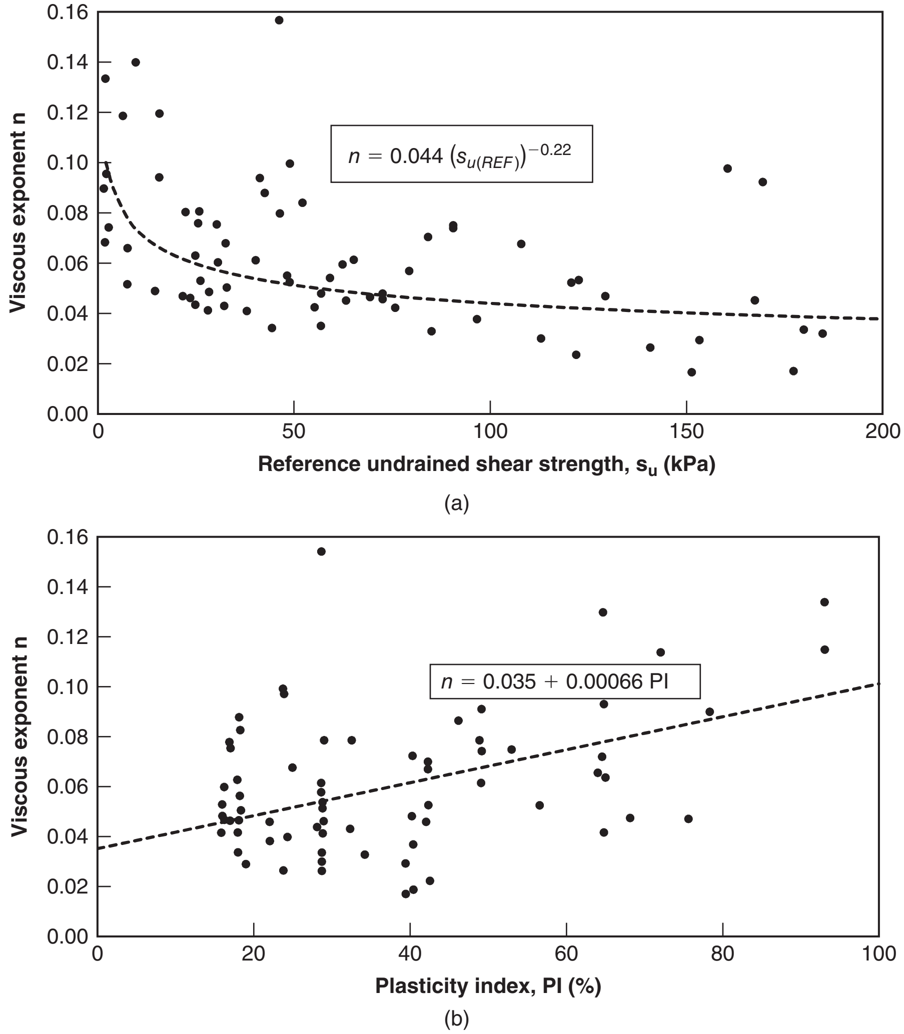


Figure 15.18 Viscous exponent n for fine-grained soils: *(a)* Influence of strength. *(b)* Influence of plasticity.

where S is the ratio for a normally consolidated soil (Eq. 15.61) and m is estimated to be 0.8.

For overconsolidated saturated fine grained soils:

$$\left(\frac{s_u}{\sigma'_{ov}}\right)_{OC} = 0.23 \times (OCR)^{0.8} \tag{15.63}$$

Several factors influence the value of the undrained shear strength, one of which is the disturbance of the sample. Several methods have been proposed for "healing" a sample from its disturbance. One is to do a drained recompression of the sample to the in situ effective stress σ'_{ov}. This approach tends to give too high an s_u value, as the recompression decreases the sample volume and water content below its natural state. Another is the stress history and normalized soil engineering properties (SHANSEP) method developed by Ladd and Foott (1974). The method consists of four steps:

1. Determine the preconsolidation pressure σ'_p from consolidation tests.
2. Test samples of the soil in consolidated undrained tests (preferably under K_o consolidation) at confining pressures well beyond σ'_p to destructure the clay and obtain the normally consolidated behavior. These tests give the value of S in Eq. 15.62.
3. Obtain the influence of OCR by overconsolidating the sample, reducing the vertical stress, and measuring s_u at that point. These tests give the value of m in Eq. 15.62 and both S and m are therefore known.
4. Use Eq. 15.62 to develop the s_u profile for the consolidation pressure σ'_{vc} to be encountered under the structure (e.g., foundation or embankment):

$$\frac{s_u}{\sigma'_{vc}} = S\left(\frac{\sigma'_p}{\sigma'_{vc}}\right)^m \tag{15.64}$$

As was shown, many factors affect the undrained shear strength of a soil. Therefore, any undrained shear strength value should be quoted by explaining how it was measured and over what stress range the soil was tested.

15.10 UNDRAINED SHEAR STRENGTH FOR UNSATURATED SOILS

The undrained shear strength for unsaturated soils is obtained by shearing the soil while preventing any drainage of air or water during the test. A distinction must be made among four categories of soils:

1. Soils where the water is in tension and the air has a continuous path to the ground surface or the boundary (typical degree of saturation $S < 0.85$)
2. Soils where the water is in tension and the air is occluded (typical degree of saturation $0.85 < S < 1$)
3. Soils where the water is in tension and the soil is saturated
4. Soils where the water is in compression and the soil is saturated

Sections 15.8 and 15.9 discussed results applicable to categories 2, 3, and 4 in the preceding list. This section discusses the undrained shear strength of soils in category 1: soils that are unsaturated and where the air has a continuous path to the boundary. In this case, the model in Figure 15.19 shows that part of the total stress applied to the soil will be transferred to the soil skeleton (effective stress) because the air spring is very compressible and must be compressed before stress is transferred to the water spring. The amount of total stress transferred to the water depends on the degree of saturation of the soil. For soils with very low degrees of saturation, most of the total stress will be transferred to the soil skeleton, whereas for soils with degrees of saturation close to about 0.85, most of the total stress will be transferred to the water.

This has a big impact on the undrained shear strength. Indeed, if most of the total stress imposed is carried by the soil skeleton (low degree of saturation S), then the effective stress increases nearly as much as the total stress imposed and the shear strength increases with the total stress (Figure 15.20*a*). If, in contrast, most of the total stress imposed is carried by the water (S approaching 0.85), then the effective stress does not

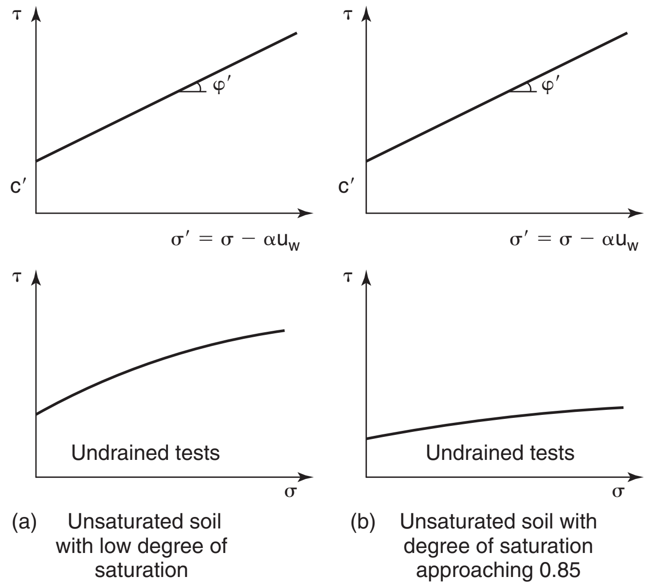


Figure 15.20 Strength envelopes for unsaturated soils.

increase much and the shear strength is nearly independent of the total stress (Figure 15.20*b*). If the degree of saturation is low but the confining stress is high enough to compress the air, including bringing it into solution, then the initially low-saturation soil will start behaving more like a saturated soil (Figure 15.20*a*). As a result, the undrained shear strength can be highly variable for unsaturated soils, depending on the degree of saturation and the total stress level. Note that as the fine-grained soil becomes drier, the water tension that is generated thereby increases the effective stress between particles and therefore the undrained shear strength.

In the field, cases in which an unsaturated soil would be loaded in an undrained fashion are rare, and are limited to high-rate dynamic loading. The concept of undrained shear strength should be handled with care for unsaturated soils, as the total stress level influences the value, especially for soils with a low degree of saturation. For these soils, the undrained shear strength case does not support the simplifying assumption that it offers for soft saturated soils, where the undrained shear strength can be considered independent of the total stress.

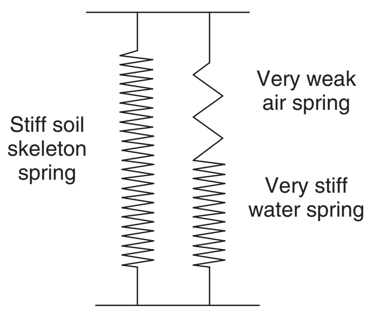


Figure 15.19 Model of unsaturated soil skeleton, air, and water.

15.11 PORE-PRESSURE PARAMETERS A AND B

Pore-pressure parameters have been found convenient to quantify the variation of the water stress in response to undrained loading. Skempton (1954) and Bishop and Henkel (1962) proposed the following equation linking the change in water stress Δu_w due to a variation in the major principal stress $\Delta\sigma_1$ and a variation in the minor principal stress $\Delta\sigma_3$:

$$\Delta u_w = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \quad (15.65)$$

where B is the pore-pressure parameter associated with an increase in confining stress $\Delta\sigma_3$ and A is the pore-pressure parameter associated with an increase in deviator stress $\Delta\sigma_1 - \Delta\sigma_3$. For saturated soils, B is close to one and A depends on the overconsolidation ratio. At failure, A_f is about 1 for normally consolidated soils, decreases with OCR, and can be negative for heavily overconsolidated soils. In practice, the coefficient $\overline{B}$ is sometimes used:

$$\Delta u_w = \overline{B}\ \Delta\sigma_v \tag{15.66}$$

where $\Delta\sigma_v$ is the increase in vertical stress. The coefficient $\overline{B}$ can be assumed in the design calculations, say 0.5, and then construction is monitored with piezometers to ensure that the water stress does not rise above $\overline{B}\ \Delta\sigma_v$. If it does, construction is halted until the water stress recedes sufficiently below that value.

For unsaturated soils, there is a need to distinguish between the response of the water and that of the air. Fredlund and Rahardjo (1993) propose:

$$du_w = B_w\ (d\sigma_3 + A_w d(\sigma_1 - \sigma_3)) \tag{15.67}$$

$$du_a = B_a\ (d\sigma_3 + A_a d(\sigma_1 - \sigma_3)) \tag{15.68}$$

Note that B_w, A_w, B_a, andA_a all depend on the degree of saturation of the soil. Also note that all pore-pressure parameters are like moduli, in that they depend on the strain level and strain rate at which they are defined.

15.12 ESTIMATING UNDRAINED SHEAR STRENGTH VALUES

There are many ways to estimate the undrained shear strength of fine-grained soils. The problem is that the value of s_u is not unique and depends on many factors. Nevertheless, fine-grained soils are often categorized by their undrained strength, as shown in Table 15.8.

The best way to determine s_u is to test the fine-grained soil in the laboratory using high-quality samples and to reproduce during the tests the initial stress conditions and the stress path during loading, while assuring no drainage. As discussed earlier, common laboratory tests available to obtain s_u include the:

1. unconsolidated undrained triaxial test (UUT)
2. consolidated undrained triaxial test (CUT)
3. unconsolidated undrained direct shear test (UUDS)
4. consolidated undrained direct shear test (CUDS)
5. unconsolidated undrained simple shear test (UUSS)
6. consolidated undrained simple shear test (CUSS)

In situ tests can also be used, including the vane shear test (VST), the borehole shear test (BHST), the cone penetrometer test (CPT), the pressuremeter test (PMT), and the standard penetration test (SPT). The VST (see Section 7.5) is the best in situ test to obtain s_u and is particularly useful offshore, where sample decompression upon retrieval from deep-water boreholes can decrease the undrained shear strength by up to 40% (Denk et al. 1981). Bjerrum (1972) used 14 case histories to back-calculate the full-scale undrained shear strength s_u (field) from embankment failures and compare it to s_u (VST) obtained from the VST performed at the sites. Because the values did not correspond, Bjerrum proposed a correction factor μ (Figure 15.21) as a function of the plasticity index I_p:

$$s_{u(Field)} = \mu\, s_{u(VST)} \tag{15.69}$$

Ladd et al. (1977) collected additional failure case histories and confirmed the trend. As can be seen, the correction factor indicates that s_u (VST) is larger than s_u (field); this is attributed to the facts that the rate of shearing is much higher in the VST than in the failure of the embankment and that this rate effect is more prominent in high-plasticity clays than in low-plasticity clays. Differences in the influence of anisotropy and plane strain conditions between the VST and the embankment are also contributing factors.

Table 15.8 Classification of Fine-Grained Soils by Undrained Shear Strength

Silts and Clays Strength			
Description	s_u (kPa)	N (bpf)	Simple field test*
Very soft	<12	<2	Squeezes between your fingers
Soft	12–25	2–4	Easily penetrated by light thumb pressure
Medium or firm	25–50	4–8	Penetrated by strong thumb pressure
Stiff	50–100	8–15	Indented by strong thumb pressure
Very stiff	100–200	15–30	Slightly indented by strong thumb pressure
Hard	200–400	30–50	Slightly indented by thumbnail
Very hard	>400	>50	Not indented by thumbnail

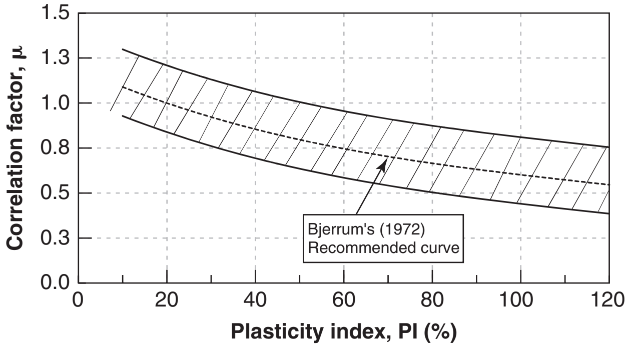


Figure 15.21 Bjerrum correction factor for undrained shear strength from vane test. (After Bjerrum 1972; Ladd et al. 1977)

The borehole shear test (see section 7.6) can be used to obtain a value of s_u in situ by direct measurement as long as the shearing is performed rapidly to ensure undrained behavior. Because the normal total stress (horizontal) on the plane of failure (vertical) can be varied in the BHST, the influence of the total normal stress on s_u (as discussed in sections 15.8 to 15.10) can be evaluated.

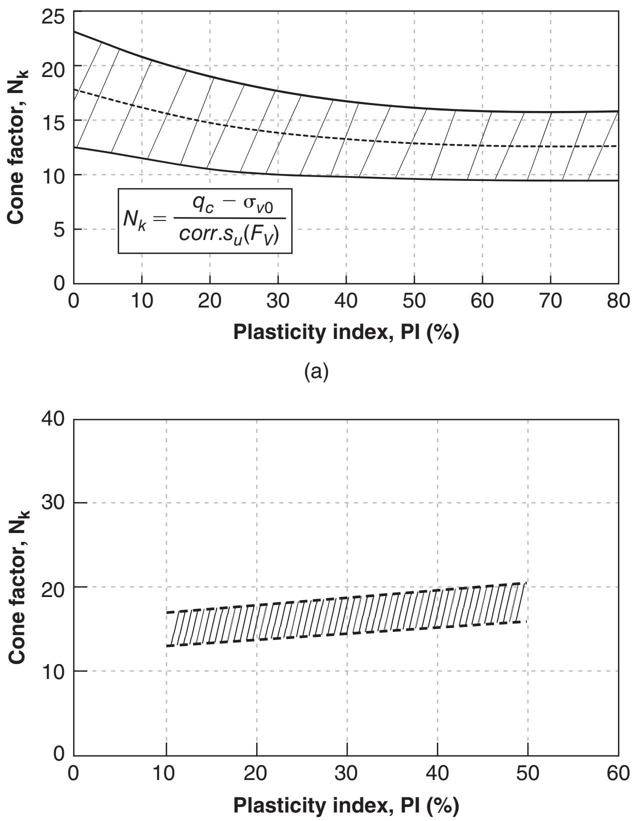
The cone penetrometer test (see section 7.2) has also been used to obtain s_u. The equation used is:

$$s_{u(CPT)} = \frac{q_c - \sigma_{vo}}{N_k} \tag{15.70}$$

where q_c is the point resistance of the CPT, σ_{vo} is the vertical total stress at the depth where q_c is measured, and N_k is the cone factor. This equation comes from the ultimate bearing pressure p_u under a pile point:

$$p_u = N_c s_u + \sigma_{vo} \tag{15.71}$$

where N_c is a bearing capacity factor usually taken equal to 9 for deep localized failure. One would therefore expect that N_k would be 9. However, many differences between a pile point and the CPT lead N_k to be quite different from 9 and quite variable. The differences include the rate of loading effect, the scale effect, and the installation procedure. The penetration of the CPT goes much faster than the pile penetration during a typical load test ($N_k > 9$). The cone is much smaller in size than the pile; as a result, the cone detects thinner layers than the pile, which averages the soil resistance over a larger zone ($N_k > 9$); also, the cone is pushed in, whereas the pile is either driven or drilled in place. All in all, the value of N_k seems to average 14 ± 5 for s_u being determined from Eq. 15.70 (Figure 15.22), but correlations have led to values varying from 5 to 70. The main problem is that, as discussed earlier, s_u is not unique, so no general correlation can be proposed. The best way to approach the problem is to run a few lab tests to obtain the right s_u value needed for the project, run parallel CPT soundings, generate a local correlation to obtain a site value of N_k from s_u and q_c, and then extend the results by running additional CPTs.



(b)

Figure 15.22 N_k factor for obtaining s_u from CPT q_c value: *(a)* based on data from Baligh et al. 1980; Lunne and Kleven 1981; s_u mostly from vane shear tests. *(b)* based on data from Aas et al. 1986; s_u mostly from vane shear tests.

The undrained shear strength s_u can also be obtained from a pressuremeter test (PMT; see section 7.3). In this case the limit pressure p_L is used as follows:

$$s_{u(PMT)} = \frac{p_L}{N_p} \tag{15.72}$$

where N_p is the pressuremeter factor. This factor can be taken as 7.5 in first approximation, but the relationship is nonlinear and Briaud (1992) proposed:

$$s_{u(PMT)}(\text{kPa}) = 0.67(p_L(\text{kPa}))^{0.75} \tag{15.73}$$

Figure 15.23 shows this relationship compared to two databases. The likely reason for this nonlinearity is that for lower values of s_u, the fine-grained soils tend to have stress-strain curves exhibiting no peak (strain hardening behavior), whereas at higher s_u values the fine-grained soils tend to exhibit peak strength and post-peak softening down to a residual strength. Because the limit pressure involves large strains near the cavity and smaller strain at some distance from the cavity, the strength mobilized is an average between the two. This average will tend to be higher for strain hardening

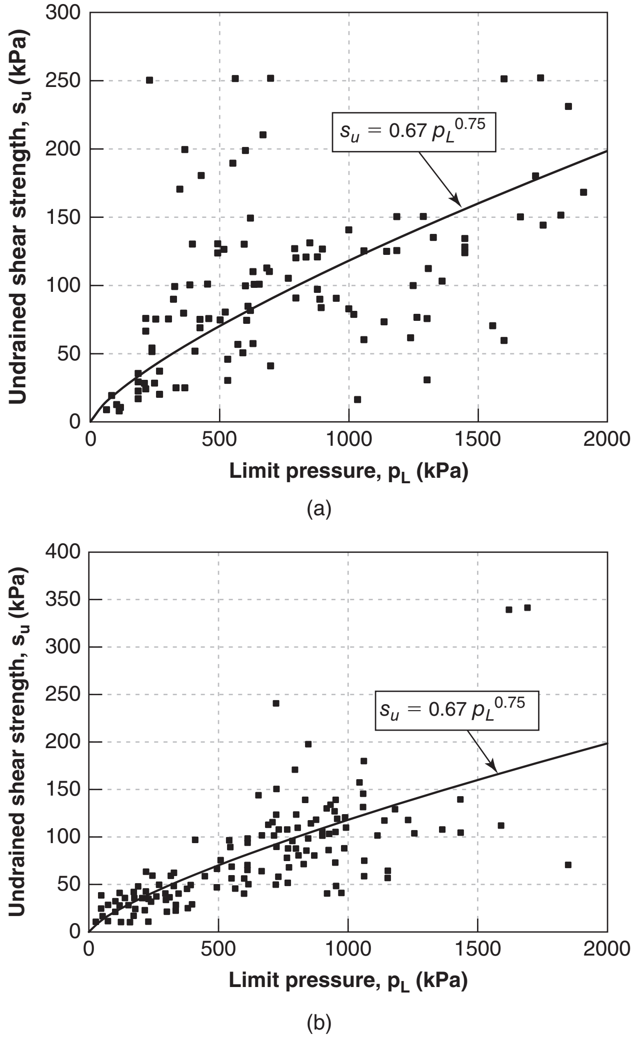


Figure 15.23 Correlation between s_u and the pressuremeter limit pressure: (a) based on data from Briaud 1992; s_u mostly from unconfined compression tests. (b) based on data from Baguelin et al. 1978; s_u from laboratory tests and vane tests.

soils (low s_u) than for strain softening soils (high s_u). The advantage of using p_L to get s_u is that the PMT involves a larger mass of soil than most other tests in the response to the expansion; as such, it can bridge over microfissures and other small-scale features and is more representative of the mass strength. The drawback is that the test is typically more expensive than the vane test, for example.

The standard penetration test (SPT; see section 7.1) and its blow count N have also been used to obtain the undrained shear strength s_u. Such correlations should be used as a last resort, however. Terzaghi et al. (1996) propose the following relationship to obtain a relatively conservative value of s_u:

$$s_{u(SPT)}(\text{kPa}) = 4.4N_{60} \qquad (15.74)$$

where N_{60} is the blow count (blows per foot) corrected to 60% of maximum energy (see section 7.1). Terzaghi et al.

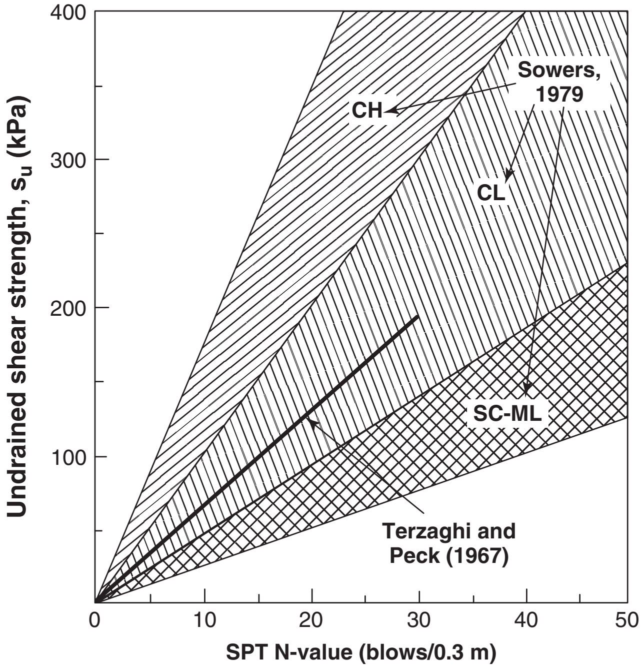


Figure 15.24 Correlation between s_u and the SPT blow count N. (After Sowers 1979; Terzaghi and Peck 1967)

point out that for low-plasticity, fine-grained soils, the factor 4.4 in Eq. 15.74 can go up to 7. Terzaghi and Peck (1967) proposed:

$$s_{u(SPT)}(\text{kPa}) = 6.7N \qquad (15.75)$$

Sowers (1979) presents his experience in a figure relating N and s_u (Figure 15.24).

15.13 RESIDUAL STRENGTH PARAMETERS AND SENSITIVITY

The *residual strength* of a soil is the strength at very large strains long after the peak strength. It exists for the effective stress shear strength and for the undrained shear strength. The residual effective stress cohesion can be taken as zero and the residual effective stress friction angle is reduced:

$$\tau_{f\ res} = \sigma' \tan \varphi'_{res} \qquad (15.76)$$

where $\tau_{f\ res}$ is the residual shear strength, σ' is the effective normal stress on the plane of failure, and φ'_{res} is the residual friction angle. The amount of reduction from φ' to φ'_{res} depends on the soil type. Loose coarse-grained soils and normally consolidated, saturated, low-plasticity, fine-grained soils do not exhibit much reduction between the friction angle and the residual friction angle. The reduction for soils with higher plasticity is more significant, as exemplified by Figure 15.25 based on data from Stark and Eid (1994) and Lupini et al. (1981).

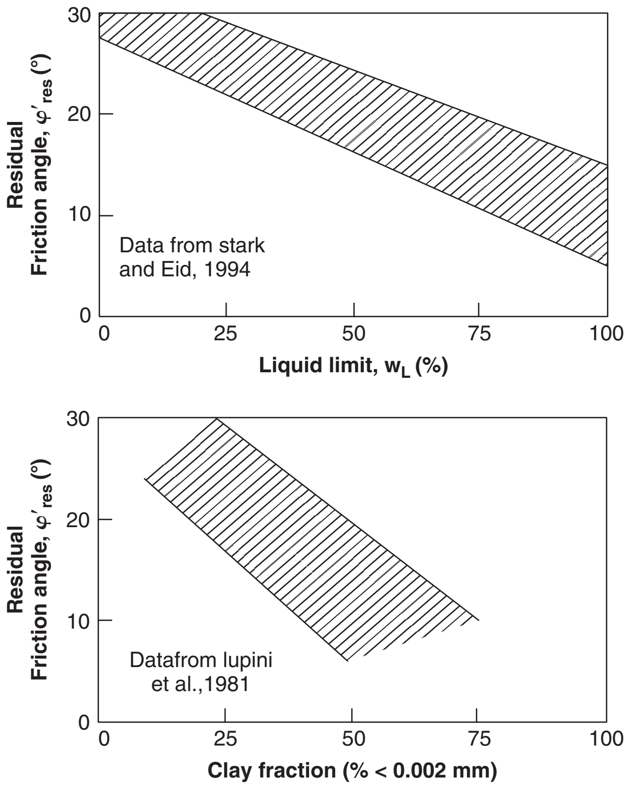


Figure 15.25 Correlation between effective stress residual friction angle and soil properties. (After Stark and Eid 1994; Lupini et al. 1981.)

The residual undrained shear strength $s_{u\,res}$ is best measured directly, either in the laboratory or in the field. In the laboratory, the best apparatus is the ring shear apparatus, which consists of a split-donut type of device. In this apparatus the top half of the donut is rotated one way while the bottom half is held in place. In this fashion, very large strains can be reached until the shear strength reaches the residual strength plateau. In the field, the vane shear test can be used. The vane is rotated until the peak shear strength s_{upeak} is obtained and then rotation continues while recording the torque. When the torque stabilizes, the residual undrained shear strength $s_{u\,res}$ is reached. ASTM recommends that after reaching the residual shear strength, the remolded shear strength be obtained by rapidly rotating the vane 5 to 10 times. The remolded undrained shear strength $s_{u\,rem}$ is then obtained by repeating the vane test immediately after the rapid rotations.

The sensitivity S_t of a fine-grained soil is defined as:

$$S_t = \frac{s_{u\ peak}}{s_{u\ rem}} \tag{15.77}$$

where $s_{u\ peak}$ is the peak undrained shear strength and $s_{u\ rem}$ is the remolded undrained shear strength. Some clays are not sensitive and some are very sensitive. For example, a low-plasticity, soft kaolinite clay is unlikely to be very sensitive ($S_t < 2$), but a quick clay may have a sensitivity in excess of 20. These quick clays do have some strength when undisturbed, say $s_u = 25$ kPa, but become a thick liquid when disturbed (see section.13.2.9). A soil with a sensitivity of less than 4 would be qualified as a low-sensitivity soil; from 4 to 10 would be medium sensitivity; 10 to 20 would be highly sensitive; and above 20 would be quick.

15.14 STRENGTH PROFILES

The strength profile of a soil deposit can give a lot of information about the deposit. This strength can be measured by the CPT point resistance q_c, or by the SPT blow count N, or by the PMT limit pressure p_L, or by the undrained shear strength s_u for a fine-grained soil. If the profile shows a linear increase with depth with a zero value at the surface, the deposit could be a normally consolidated, soft, fine-grained soil, as would be expected in a city like New Orleans (Figure 15.26*a*). If the profile goes through zero at the surface but increases nonlinearly with depth with a downward curvature, then the deposit could be a dry sand deposit (Figure 15.26*b*). If the profile increased linearly with depth but had a definite nonzero value at the surface, the deposit could be a fine-grained soil overconsolidated by overburden removal through erosion or through the melting of a glacier (Figure 15.26*c*). If the profile indicated a constant strength with depth (Figure 15.26*d*), the deposit could be an unsaturated silty sand where the total stress increased with depth but the water tension decreased with depth, thereby maintaining the effective stress constant and the shear strength constant with depth. It could also be an underconsolidated soft clay. If the profile showed a curved decrease with depth near the surface followed by

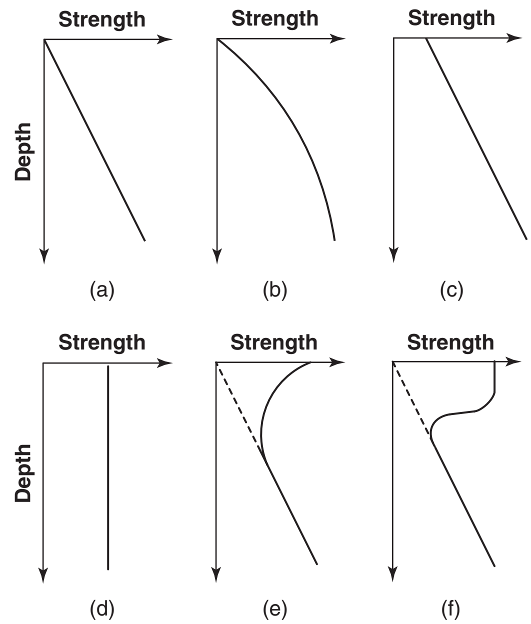


Figure 15.26 Soil strength profiles.

an increase at larger depths, the deposit could be a high-plasticity, fine-grained soil overconsolidated by desiccation near the surface but becoming normally consolidated at depth where the seasonal shrink-swell cycles no longer have an influence (Figure 15.26*e*). If the profile shows a strong layer near the surface and a softer layer at some depth, it could indicate the presence of a crust of the softer layer below (Figure 15.26*f*).

Note that these strength profiles, if rotated 90 degrees counterclockwise, represent shear strength envelopes in the case of uniform soil. Indeed, after rotation, the vertical axis represents a measure of the strength and the horizontal axis a measure of the total stress. The horizontal axis can be transformed in a measure of the effective stress if the water stresses are known (such as a hydrostatic condition, for example).

15.15 TYPES OF ANALYSES

In this chapter we have talked about effective stress, total stress, undrained strength, and drained strength. Each strength is associated with a type of analysis in design and it is important to understand which analysis is used for what strength. The types of strength analyses typically performed in geotechnical engineering include: effective stress analysis, total stress analysis, undrained analysis, drained analysis, short-term analysis, and long-term analysis.

Effective stress analysis: an analysis in which the soil is considered to be made of particles, water, and air. It is the most theoretically sound analysis, but it is also the most complicated analysis, because it requires knowledge of the total stress, the water stress, and the air stress (unless it can be assumed to be zero). It is applicable to all design cases.

Total stress analysis: an analysis in which the soil is considered to be made of one material, without distinguishing between particles, water, and air. It is the easiest of the analyses because the number of variables is decreased significantly. It is also the most likely to be erroneous, because the fundamental principles are not respected, except in a few cases like the undrained behavior of fine-grained soils where the undrained strength can be considered constant.

Undrained analysis: an analysis in which the water and air are not allowed to drain during loading. This analysis can be performed by using effective stress models and in a few specific cases total stress models. One of the difficulties in using this analysis together with an effective stress model is prediction of the water stress and possibly the air stress for unsaturated soils.

Drained analysis: an analysis in which the water and air are allowed to drain until any excess water stress and any excess air stress have gone back to zero. It is one of the simplest of all effective stress analyses, but its usefulness is limited because it only applies to long loading times.

Short-term analysis: an analysis of the behavior of the soil in the short term. A short-term analysis can be a drained analysis for a clean, coarse-grained soil and an undrained analysis for a fine-grained soil. It tends to control the design of structures that will load fine-grained soils.

Long-term analysis: an analysis of the behavior of the soil in the long term. A long-term analysis is similar to a drained analysis because in the long term—sometimes in the very long term—the soil will drain and excess water and air stresses will vanish. This analysis tends to control the design of excavations.

15.16 TRANSFORMATION FROM EFFECTIVE STRESS SOLUTION TO UNDRAINED STRENGTH SOLUTION

The results of an effective stress analysis can be transformed into the results of an undrained analysis when the undrained strength is constant and the $\varphi = 0$ concept applies. In this case the transformation consists of using the following correspondence principles:

1. Effective unit weight becomes total unit weight

$$\gamma_{eff} \rightarrow \gamma_t \tag{15.78}$$

2. Effective stress becomes total stress

$$\sigma' \rightarrow \sigma \tag{15.79}$$

3. Effective stress cohesion becomes undrained shear strength

$$c' \rightarrow s_u \tag{15.80}$$

4. Effective stress friction angle becomes zero

$$\varphi' \rightarrow 0 \tag{15.81}$$

In this fashion, for example, the shear strength changes as follows:

$$s = c' + \sigma' \tan\varphi' \rightarrow s = s_u \tag{15.82}$$

The ultimate bearing pressure changes as follows (see Chapter 17):

$$p_u = c' N_c + \frac{1}{2}\gamma_{eff} B N_\gamma + \gamma D N_q \rightarrow p_u = N_c s_u + \gamma D \tag{15.83}$$

because for $\varphi = 0$, $N_\gamma = 0$, and $N_q = 1$.

The passive earth pressure equation changes as follows (see Chapter 21):

$$\sigma'_{ph} = K_p \sigma'_{ov} + 2c'\sqrt{K_p} \rightarrow \sigma_{ph} = \sigma_{ov} + 2s_u \tag{15.84}$$

because for $\varphi = 0$, $K_p = 1$.

PROBLEMS

15.1 It is well known that a car with wider tires can take corners faster than the same car with narrower tires. That is to say, the shearing resistance of the car with wider tires is larger than the shearing resistance of the car with narrower tires. This seems counterintuitive when one considers that in both cases the weight of the car is the same, and therefore the friction should be the same regardless of the width of the tires. Explain why the car with wider tires develops more resistance to shear in the corners than the car with narrower tires.

15.2 A medium dense sand deposit has a dry unit weight of 17 kN/m^3, a saturated unit weight of 20 kN/m^3, and a friction angle of 32 degrees. Calculate the shear strength on a horizontal plane at a depth of 10 m if:

a. The groundwater level is much deeper than 10 m and the sand has no water.
b. The groundwater level is at the ground surface.
c. The groundwater level is at 12 m and the sand is saturated by capillary action.

15.3 In a simple shear test on a dense sand with no water, the normal stress is 100 kPa and the shear stress at failure is 80 kPa. At failure also, the vertical displacement is 0.5 mm and the horizontal displacement is 5 mm.

a. Calculate the friction angle φ' and the dilation angle ψ'.
b. Calculate the shear strength of the sand if the normal stress increases to 200 kPa and the angles φ' and ψ' remain the same.

15.4 A soft clay has formed a crust near the ground surface due to drying under the sun. At the ground surface the relative humidity has been 40% for a long time. A sample of the surface clay gives a unit weight of 17.5 kN/m^3 and a water content of 10%. Estimate the shear strength of the clay at the ground surface if the effective stress friction angle is 27 degrees and G_s is 2.7. What is the apparent cohesion of that clay?

15.5 A medium-stiff clay is tested in an undrained triaxial test. At failure, the effective stress on the failure plane is 230 kPa and the shear stress on the failure plane is 122 kPa. Calculate the undrained shear strength of this clay.

15.6 A soft, saturated clay is tested in an unconsolidated undrained direct shear test with a normal stress of 50 kPa; the shear strength obtained is 20 kPa. An identical sample is tested, also in an unconsolidated undrained direct shear test, but this time the normal stress is 100 kPa. What would you expect the shear strength to be?

15.7 A sand layer has an SPT blow count of 27 bpf and a CPT point resistance of 13.5 MPa. Both measurements come from a depth of 12 m. The groundwater level is at a depth of 5 m. What is your best estimate of the friction angle for this sand at that depth?

15.8 The undrained shear strength of a medium-stiff clay is 46 kPa when sheared in a time to failure equal to 3 minutes in a vane shear test. The medium-stiff clay has a water content of 35% and a plasticity index of 30% Solve the following two problems:

a. A guardrail post is placed in this clay on the side of the road to arrest cars upon impact. The rise time of the force during the impact is anticipated to be 20 milliseconds. What shear strength value should you use?
b. An embankment is placed on that clay. In the design process it is assumed that if a failure occurs, the failure of the embankment would be very slow and take place in about 6 hours. What undrained shear strength should be used in calculating the factor of safety against embankment failure?

15.9 Use average and associated ranges of rate effect viscous exponent to generate a curve similar to the Bjerrum correction factor for the vane shear test, undrained strength. Assume that the vane reaches the peak undrained shear strength in 3 minutes and that the embankment reaches failure in half a day.

15.10 A clay has an overconsolidation ratio equal to 2.5. Use the SHANSEP method and reasonable values of the parameters to estimate the undrained shear strength of that clay at a depth of 20 m. The clay is offshore at the bottom of the North Sea in 300 m of water.

15.11 An unsaturated sample of clay is tested in a simple shear test. At failure the total normal stress on the failure plane is 70 kPa and the shear stress is 175 kPa.

a. Is that possible?
b. After testing, the water content on the plane of failure is measured and the soil water retention curve gives a water tension of 1450 kPa. The water content coupled with the measurement of the unit weight and the assumption that Gs is 2.7 leads to a degree of saturation of 20%. If the clay has no effective stress cohesion, calculate the effective stress friction angle.

15.12 A lightly overconsolidated clay has a CPT point resistance of 1100 kPa, an OCR of 1.7, a PMT limit pressure of 590 kPa, an SPT blow count N of 13 bpf, and a unit weight of 18 kN/m^3. Estimate the undrained shear strength of that clay if the data comes from a depth of 6 m with the groundwater level being at a depth of 2 m.

15.13 You are at the beach lying on dry uniform sand. You take a handful of sand and let it fall from your hand onto a 0.3 m by 0.3 m wide plate. The sand pile on the plate has the shape of a pyramid and the angle of the pyramid with the horizontal is β. Demonstrate that β is equal to the friction angle φ'. You then take that same pile of sand and add a bit of water. Now you are able to mold the sand pile into a cylinder standing vertically. Where does the sand strength come from? Is it cohesion or friction?

Problems and Solutions

Problem 15.1

It is well known that a car with wider tires can take corners faster than the same car with narrower tires. That is to say, the shearing resistance of the car with wider tires is larger than the shearing resistance of the car with narrower tires. This seems counterintuitive when one considers that in both cases the weight of the car is the same, and therefore the friction should be the same regardless of the width of the tires. Explain why the car with wider tires develops more resistance to shear in the corners than the car with narrower tires.

Solution 15.1

The weight of the car is the same in both cases, so the friction should be the same (in theory). However, we need to consider the force generated by the cohesion or "glue" between the tire and the asphalt. In equation form, $F = \mu N + C$, where μ is the friction coefficient, N is the normal force, and C is the cohesion force. The cohesion force C depends on the contact area, whereas the normal force N does not. The area of a wide tire is larger than the area of a narrow tire. A wide tire will thus provide more area to resist the force between the tire and the pavement to turn around a corner. Direct shear tests between a piece of pavement and a piece of rubber from a tire would be required to demonstrate this possible explanation.

Problem 15.2

A medium dense sand deposit has a dry unit weight of 17 kN/m^3, a saturated unit weight of 20 kN/m^3, and a friction angle of 32 degrees. Calculate the shear strength on a horizontal plane at a depth of 10 m if:

a. The groundwater level is much deeper than 10 m and the sand has no water.
b. The groundwater level is at the ground surface.
c. The groundwater level is at 12 m and the sand is saturated by capillary action.

Solution 15.2

a. The groundwater level is much deeper than 10 m and the sand has no water:

$$\tau_f = c' + (\sigma - \alpha u_w) \tan \varphi'$$
$$\tau_f = 0 + (10 \times 17 - 0 \times 0) \tan 32 = 106.2 \text{ kPa}$$

b. The ground-water level is at the ground surface:

$$\tau_f = c' + (\sigma - \alpha u_w) \tan \varphi'$$
$$\tau_f = 0 + (10 \times 20 - 1 \times 10 \times 9.81) \tan 32 = 63.7 \text{ kPa}$$

c. The ground-water level is at 12 m and the sand is saturated by capillary action.

In this case, there is suction in the soil. Equation 15.8 is used and the pore water pressure is negative:

$$\tau_f = c' + (\sigma - \alpha u_w) \tan \varphi'$$
$$\tau_f = 0 + (10 \times 20 - 1(-2 \times 9.81)) \tan 32 = 137.2 \text{ kPa}$$

Problem 15.3

In a simple shear test on a dense sand with no water, the normal stress is 100 kPa and the shear stress at failure is 80 kPa. At failure also, the vertical displacement is 0.5 mm upward and the horizontal displacement is 5 mm.

a. Calculate the friction angle φ' and the dilation angle ψ'.
b. Calculate the shear strength of the sand if the normal stress increases to 200 kPa and the angles φ' and ψ' remain the same.

Solution 15.3

a. Using the shear strength equation and knowing that the effective stress cohesion of the dense sand is zero:

$$\tau_f = c' + (\sigma - \alpha u_w)\tan\varphi'$$
$$80 = 0 + (100 - 0 \times 0)\tan\varphi' \quad \text{or} \quad \varphi' = 38.66 \text{ degrees}$$

The tangent of the dilation angle is given by the ratio of vertical to horizontal displacement:

$$\tan\psi' = \frac{0.5}{5} \quad \text{or} \quad \psi' = 5.71 \text{ degrees}$$

b. Again using the shear strength equation:

$$\tau_f = 0 + (200 - 0 \times 0)\tan 38.66 = 160 \text{ kPa}$$

The dilation angle is not used because it is included in the friction angle φ'.

Problem 15.4

A soft clay has formed a crust near the ground surface due to drying under the sun. At the ground surface the relative humidity has been 40% for a long time. A sample of the surface clay gives a unit weight of 17.5 kN/m^3 and a water content of 10%. Estimate the shear strength of the clay at the ground surface if the effective stress friction angle is 27 degrees and G_s is 2.7. What is the apparent cohesion of that clay?

Solution 15.4

Based on the Kelvin equation (Eq. 10.69; see Chapter 10), we can calculate the water tension at the ground surface:

$$u(\text{kPa}) = 135000 \times \ln(RH) = 135022 \times \ln(0.4) = -123719 \text{ kPa} \tag{15.1s}$$

Based on the three-phase soil relationships, the void ratio is linked to the unit weight of solids, the water content, and the soil unit weight by:

$$e = \frac{\gamma_s(1+\omega)}{\gamma} - 1$$

Given

$$G_s = 2.7$$

and

$$\omega = 10\%, \gamma = 17.5 \text{ kN/m}^3$$

we can obtain:

$$e = \frac{\gamma_s(1+\omega)}{\gamma} - 1 = \frac{2.7 \times 9.81 \times (1+0.1)}{17.5} - 1 = 0.66$$

Another useful equation links the degree of saturation to Gs, w, and e:

$$S = \frac{\omega G_s}{e}$$

We can obtain:

$$S = \frac{\omega\gamma_s}{e\gamma_w} = \frac{0.1 \times 2.7}{0.66} = 0.41$$

Therefore, *a* can be estimated as S (i.e., 0.41). At the ground surface, the shear strength of the clay can now be calculated as:

$$\tau = c' + (\sigma - \alpha u_w)\tan\varphi' = 0 + (0 - 0.41 \times (-123719)) \times \tan 27^\circ + 0 = 25846 \text{ kPa}$$

The apparent cohesion of the clay is also 25,846 kPa, because both c′ and σ are zero.

Problem 15.5

A medium-stiff clay is tested in an undrained triaxial test. At failure, the effective stress on the failure plane is 230 kPa and the shear stress on the failure plane is 122 kPa. Calculate the undrained shear strength of this clay.

Solution 15.5

$$s_u = 122 \text{ kPa}$$

Problem 15.6

A soft, saturated clay is tested in an unconsolidated undrained direct shear test with a normal stress of 50 kPa; the shear strength obtained is 20 kPa. An identical sample is tested, also in an unconsolidated undrained direct shear test, but this time the normal stress is 100 kPa. What would you expect the shear strength to be?

Solution 15.6

Because the soil sample is being tested in an undrained condition, and because the soil skeleton is weak (soft clay), the increase in normal stress is taken up by the water and there is no increase in effective stress. Therefore, the expected undrained shear strength is the same as in the first test: 20 kPa.

Problem 15.7

A sand layer has an SPT blow count of 27 bpf and a CPT point resistance of 13.5 MPa. Both measurements come from a depth of 12 m. The groundwater level is at a depth of 5 m. What is your best estimate of the friction angle for this sand at that depth?

Solution 15.7

The vertical effective stress at the point of measurement of the SPT and the CPT is computed as:

$$\sigma'_{ov} = 20 \times 12 - 9.81 \times 7 = 171.3 \text{ kPa}$$

The SPT blow count is corrected for stress level:

$$N_1 = N_{measured} \times \left(\frac{\sigma'_{ov}}{p_a}\right)^{-0.5} = 27 \times \left(\frac{171.3}{101.3}\right)^{-0.5} = 20.8 \text{ blows/0.3 m}$$

Then the CPT point resistance is corrected for stress level:

$$q_{c1} = q_{c\,measured} \times \left(\frac{\sigma'_{ov}}{p_a}\right)^{-0.5} = 13.5 \times \left(\frac{171.3}{101.3}\right)^{-0.5} = 10.4 \text{ MPa}$$

The friction angle φ' can be evaluated in a number of ways. Using Mayne's recommendation:

$$\varphi' = 17.6 + 11 \times \log\left(\frac{q_{c1}}{p_a}\right) = 17.6 + 11 \times \log\left(\frac{10400}{101.3}\right) = 39.7 \text{ degrees}$$

Using an equation from Schmertmann (1975) and Kulhawy and Mayne (1990):

$$\tan\varphi' = \left(\frac{N}{12.2 + 20.3\dfrac{\sigma'_{ov}}{p_a}}\right)^{0.34} = \left(\frac{27}{12.2 + 20.3 \times \dfrac{171.3}{101.3}}\right)^{0.34} \quad \text{and} \quad \varphi' = 39.7 \text{ degrees}$$

Using the Terzaghi and Peck (1967) figure relating the friction angle to the blow count, we get:

$$\Phi' = 35 \text{ degrees}$$

Considering all the values collected, a cautiously conservative estimate of the friction angle might be 36 degrees.

Problem 15.8

The undrained shear strength of a medium-stiff clay is 46 kPa when sheared in a time to failure equal to 3 minutes in a vane shear test. The medium-stiff clay has a water content of 35% and a plasticity index of 30%. Solve the following two problems:

a. A guardrail post is placed in this clay on the side of the road to arrest cars upon impact. The rise time of the force during the impact is anticipated to be 20 milliseconds. What shear strength value should you use?
b. An embankment is placed on that clay. In the design process it is assumed that if a failure occurs, the failure of the embankment would be very slow and take place in about 6 hours. What undrained shear strength should be used in calculating the factor of safety against embankment failure?

Solution 15.8

The rate effect equation for the undrained shear strength of a clay is:

$$\frac{s_{u1}}{s_{u2}} = \left(\frac{t_1}{t_2}\right)^{-n}$$

The viscous exponent n is related to the water content by:

$$\text{n} = 0.028 + 0.0006\, w\% \text{ so n} = 0.049$$

The viscous exponent n is related to the plasticity index by:

$$\text{n} = 0.035 + 0.00066 \text{ PI\%} \text{ so n} = 0.0548$$

Use an average n value of $\text{n}_{\text{Avg}} = 0.0519$

a. In this case $s_{u1} = 46$ kPa, $t_1 = 180$ sec, $t_2 = 0.02$ sec, $s_{u2} =?$

$$s_{u2} = s_{u1}\left(\frac{t_1}{t_2}\right)^n$$

$$s_{u2} = 46\left(\frac{180}{0.02}\right)^{0.0519} = 73.8 \text{ kPa}$$

b. In that case $s_{u1} = 46$ kPa, $\text{t}_1 = 180$ sec, $\text{t}_2 = 21600$ sec, $s_{u2} =?$

$$s_{u2} = 46\left(\frac{180}{21600}\right)^{0.0519} = 35.9 \text{ kPa}$$

Problem 15.9

Use average and associated ranges of rate effect viscous exponent to generate a curve similar to the Bjerrum correction factor for the vane shear test, undrained strength. Assume that the vane reaches the peak undrained shear strength in 3 minutes and that the embankment reaches failure in half a day.

Solution 15.9

Use is made of the rate effect equation and of the correlation between the rate exponent n and the plasticity index PI in percent:

$$\frac{s_{u1}}{s_{u2}} = \left(\frac{t_1}{t_2}\right)^{-n}$$

$$\text{n} = 0.035 + 0.00066 \times \text{PI\%}$$

The strength s_{u1} is the field value of s_u, the strength s_{u2} is the vane test value of s_u, the time t_1 is the time for the embankment failure or half a day (720 min), the time t_2 is the time for the vane test or 3 min. Therefore, the equation for the Bjerrum factor is:

$$\mu = \frac{s_{u,field}}{s_{u,VST}} = 240^{-(0.035+0.00066\ \text{PI\%})}$$

To take into account the scatter in the n vs. PI correlation, the calculated μ value is bracketed between the following two expressions.

$$\mu = \frac{s_{u,field}}{s_{u,VST}} = 240^{-(0.01+0.00066\ \text{PI\%})}$$

$$\mu = \frac{s_{u,field}}{s_{u,VST}} = 240^{-(0.065+0.00066\ \text{PI\%})}$$

Figure 15.1s shows the range of the function μ vs. PI based on the rate effect model. It appears that the rate effect model explains much of the correction factor except at low PI values:

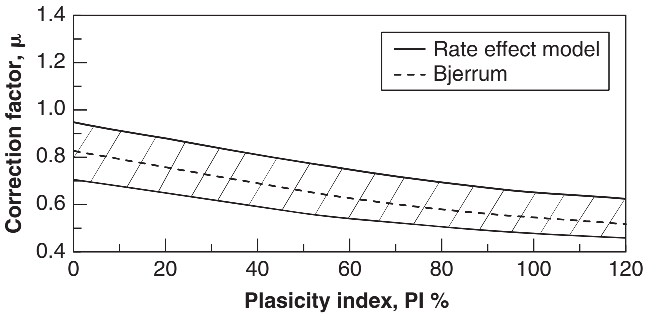


Figure 15.1s Correction factor vs. plasticity index.

Problem 15.10

A clay has an overconsolidation ratio equal to 2.5. Use the SHANSEP method and reasonable values of the parameters to estimate the undrained shear strength of that clay at a depth of 20 m. The clay is offshore at the bottom of the North Sea in 300 m of water.

Solution 15.10

$$\gamma_{sat} = 19\ \text{kN/m}^3$$

at depth 20 m below the sea floor in 300 m of water:

$$\sigma'_{ov} = 300 \times 9.81 + 20 \times 19 - 320 \times 9.81 = 183.8\ \text{kN/m}^2$$

For normally consolidated saturated fine grained soil:

$$\left(\frac{S_u}{\sigma'_{ov}}\right)_{NC} = 0.23$$

and for an overconsolidated fine grained soils with OCR = 2.5

$$\left(\frac{S_u}{\sigma'_{ov}}\right)_{OC} = \left(\frac{S_u}{\sigma'_{ov}}\right)_{NC} (OCR)^{0.8} = 0.23 \times 2.5^{0.8} = 0.48$$

Therefore

$$S_u = 0.48 \times 183.8 = 88.2\ \text{kN/m}^2$$

Problem 15.11

An unsaturated sample of clay is tested in a simple shear test. At failure the total normal stress on the failure plane is 70 kPa and the shear stress is 175 kPa.

a. Is that possible?
b. After testing, the water content on the plane of failure is measured and the soil water retention curve gives a water tension of 1450 kPa. The water content coupled with the measurement of the unit weight and the assumption that G_s is 2.7 leads to a degree of saturation of 20%. If the clay has no effective stress cohesion, calculate the effective stress friction angle.

Solution 15.11

a. Yes, it is possible. In unsaturated soils, the shear strength can be higher than the total normal stress because of the water tension increases the effective stress.
b.

$$\sigma = 70 \text{ kPa}$$
$$\tau_f = 175 \text{ kPa}$$
$$u_w = -1450 \text{ kPa}$$
$$S = 20\% \Rightarrow \alpha = 0.2$$
$$c' = 0$$
$$\tau_f = c + (\sigma - \alpha u_w) \tan \phi' = 0 + (70 - 0.2 \times (-1450)) \tan \phi' = 175 \text{ kPa}$$
$$\tan \phi' = 0.486 \Rightarrow \phi' = 25.9^\circ$$

Problem 15.12

A lightly overconsolidated clay has a CPT point resistance of 1100 kPa, an OCR of 1.7, a PMT limit pressure of 590 kPa, an SPT blow count N of 13 bpf, and a unit weight of 18 kN/m^3. Estimate the undrained shear strength of that clay if the data comes from a depth of 6 m with the groundwater level being at a depth of 2 m.

Solution 15.12

$$\gamma_T = 18 \text{ kN/m}^3$$
$$\sigma_{ov} = 18 \times 6 = 108 \text{ kPa}$$
$$\sigma'_{ov} = \sigma_{ov} - \alpha u_w = 108 - 1 \times (6 - 2) \times 9.81 = 69 \text{ kPa}$$

The CPT data equation gives:

$$s_{u(CPT)} = \frac{q_c - \sigma_{ov}}{N_k}$$

Assuming an average value of N_k equal to 14, the equation becomes:

$$s_{u(CPT)} = \frac{1100 - 108}{14} = 70.9 \text{ kPa}$$

The PMT data equation gives:

$$s_{u(PMT)} = \frac{p_L}{N_P}$$

Assuming an average value of N_p equal to 7.5:

$$s_{u(PMT)} = \frac{590}{7.5} = 79 \text{ kPa}$$

The recommendations also give:

$$\frac{s_{u(PMT)}}{p_a} = 0.21 \times \left(\frac{p_L}{p_a}\right)^{0.75}$$

Using 101.3 kPa for the atmospheric pressure:

$$s_{u(PMT)} = 0.21 \times \left(\frac{p_L}{p_a}\right)^{0.75} \times p_a = 0.21 \times \left(\frac{590}{101.3}\right)^{0.75} \times 101.3 = 79.8 \text{ kPa}$$

The SPT data equation gives:

$$s_{u(SPT)} = 4.4N_{60}$$

Assuming that $N = N_{60}$, then:

$$s_{u(SPT)} = 4.4 \times 13 = 57.2 \text{ kPa}$$

Given all the data, a cautiously conservative estimate of the undrained shear strength is 65 kPa.

Problem 15.13

You are at the beach lying on dry uniform sand. You take a handful of sand and let it fall from your hand onto a 0.3 m by 0.3 m wide plate. The sand pile on the plate has the shape of a pyramid and the angle of the pyramid with the horizontal is β. Demonstrate that β is equal to the friction angle φ'. You then take that same pile of sand and add a bit of water. Now you are able to mold the sand pile into a cylinder standing vertically. Where does the sand strength come from? Is it cohesion or friction?

Solution 15.13

The angle of the pyramid with the horizontal, β, is known as the *angle of repose*. When the sand falls, it comes to rest at the maximum angle possible; therefore, the slope of the sand pile is at impending failure. You can check that by tilting the plate slightly to one side: The side slope of the dry sand pyramid will fail to retain the same angle with the horizontal. If the slope is at impending failure, an element of soil as shown in Figure 15.2s is subjected to shear strength τ_f. The equilibrium of the element leads to the following equations, which show that the slope angle is the friction angle of the sand:

The shear force on the failure plane is:

$$T = W \sin\beta$$

The normal force on the failure plane is:

$$N = W \cos\beta$$

Therefore:

$$T = N \tan\beta$$

The maximum resisting force on the failure plane is:

$$S = N \tan\varphi'$$

At failure:

$$T = S \quad \text{therefore} \quad \beta = \varphi'$$

If we add a bit of water to the sand, water tension develops in the voids of the fine sand. This water tension pulls the particles against each other and creates an effective stress equal and opposite to the water tension. The shear strength of the sand is due to the friction related to the effective stress created by the water tension. This shear strength is often called *apparent cohesion* because the sand "sticks" together—yet the real mechanism is friction.

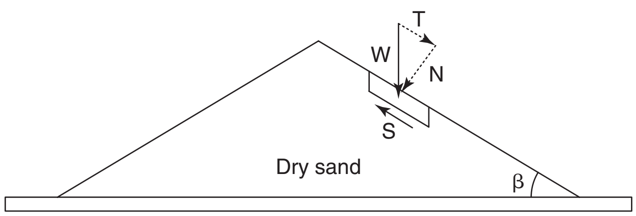


Figure 15.2s Dry sand pile and angle of repose.