

## CHAPTER 8

# Elements of Geophysics

### 8.1 GENERAL

*Geophysics* is an area of science dealing with the physics of the Earth. In its broadest sense it includes seismology, geodesy, atmospheric science, geomagnetometry, geothermometry, hydrology, oceanography, tectonophysics, geodynamics, glaciology, petrophysics, mineral physics, and exploration and engineering geophysics. This chapter is an introduction to the last topic: exploration geophysics for civil engineering applications. This exploration relies on a number of nondestructive geophysics tests aimed at obtaining soil and rock properties and soil and rock stratigraphy from the surface. Borehole geophysics and remote sensing are also parts of geophysical methods.

Geophysical methods include seismic techniques, gravity techniques, magnetic techniques, electrical techniques, electromagnetic techniques, borehole techniques, and remote sensing techniques. Gravity and magnetic techniques are not used very often in geotechnical engineering and thus are not covered here. They essentially consist of measuring the gravity field and the magnetic field to infer stratigraphy. Geophysical techniques differ from geotechnical techniques in that they tend to give average soil and rock properties of large masses (many cubic meters) nondestructively, whereas geotechnical techniques give soil and rock properties at a much smaller scale (a few cubic decimeters) through mechanical testing. Geophysical methods are extremely useful in geotechnical engineering because they allow the engineer to infer the large-scale properties between sites of geotechnical measurements and because some of them give parameters that are directly useful in design. Engineering geology contributes to the geotechnical engineering knowledge of a site at an even larger scale.

### 8.2 SEISMIC TECHNIQUES

#### 8.2.1 Seismic Waves

*Seismic waves* are waves of energy (particle motion) that travel through soil, rock, or water. They may be created by

a natural event (for example, an earthquake) or an artificial impact (as in seismic testing). Seismic waves propagate because the disturbance created by a shock at a point A influences the particles at point B next to point A, which influence the particles at point C next to point B, and so on. The disturbance in this case is the motion of particles. The velocity of the particle is  $u$  and the velocity of the wave is  $v$ . The particle shakes at a frequency  $f$  when the wave passes by the particle location. After the wave has passed the particle location, the particle stops shaking. If the wave propagation in one direction is frozen at a given time, it shows a wave crest followed by a wave trough, followed by a wave crest and so on (Figure 8.1).

Waves are defined by a number of parameters. The wave velocity  $v$  is the speed at which the particle motion is propagated from one particle to the next. The particle velocity  $u$  is

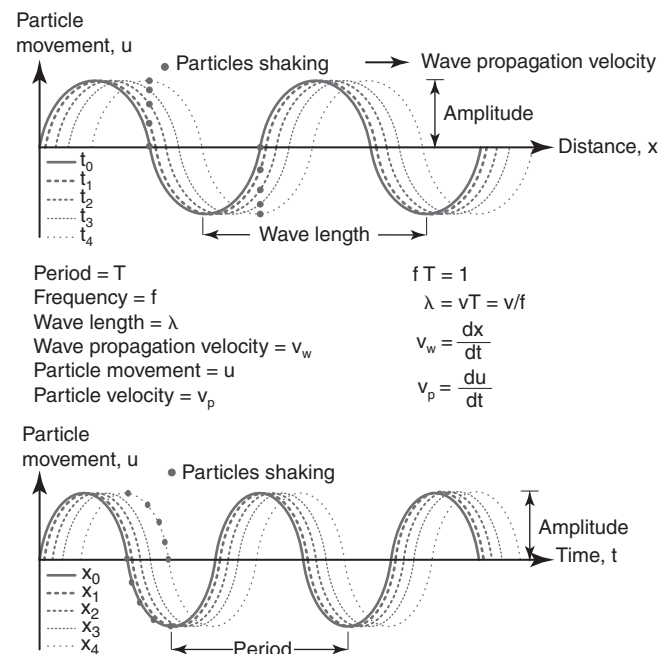


Figure 8.1 Propagation of waves.

the speed at which the particle is moving around its own location. The wave amplitude,  $a$ , is the maximum displacement of the particle from its equilibrium position. The period  $T$  of a wave is the time between the arrival of two consecutive crests (or troughs) at a given location. The wave frequency  $f$  is the number of periods per unit time (frequency with which the particle shakes). The wavelength  $\lambda$  is the distance between two adjacent crests (or troughs) at a given time. The frequency is set by whatever creates the initial shock; the wave speed is set by the medium through which it propagates. The following relationships exist between these parameters:

$$f = 1/T \tag{8.1}$$

where  $f$  is the wave frequency and  $T$  is the wave period.

$$\lambda = v T = v/f \tag{8.2}$$

where  $\lambda$  is the wave length and  $v$  is the wave velocity. For sinusoidal waves, the displacement of a particle  $u(t)$  is linked to time by:

$$\begin{aligned} u(x, t) &= a(x, t) \sin(kx \pm \omega t + \phi) \\ &= a(x, t) \sin\left(\frac{2\pi}{\lambda} (x \pm vt) + \phi\right) \end{aligned} \tag{8.3}$$

where  $u$  is the displacement of the particle,  $t$  the time,  $a$  the amplitude of motion, and  $\omega$  the angular frequency. The *phase* of a wave refers to the point in the cycle of a waveform, measured as an angle:

$$\varphi = \omega t \tag{8.4}$$

The period  $T$  corresponds to a phase equal to 360 degrees or  $2\pi$ :

$$2\pi = \omega T = \omega/f \tag{8.5}$$

Two categories of waves are identified: body waves and surface waves. Body waves propagate throughout the soil mass, whereas surface waves propagate along the ground surface. Body waves are of two types: compression waves or longitudinal waves called *P waves* (primary waves or pressure waves) and shear waves or transverse waves called *S waves* (secondary waves or shear waves) (Figure 8.2). P waves propagate by displacing a particle in the same direction as the direction of wave propagation; S waves propagate by displacing a particle perpendicular to the direction of wave propagation. In air, P waves are called *sound waves* and propagate at the speed of sound or  $v_p = 330$  m/s. In water they propagate at  $v_p = 1450$  m/s; in ordinary concrete at about 4000 m/s; and in granite at up to  $v_p = 6000$  m/s. Table 8.1 gives some estimates of wave velocities in earth materials.

The wave speed is related to the ratio of a modulus over the density of the material through which the wave propagates:

$$v_p = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} = \sqrt{\frac{E}{\rho} \frac{(1-\nu)}{(1+\nu)(1-2\nu)}} \tag{8.6}$$

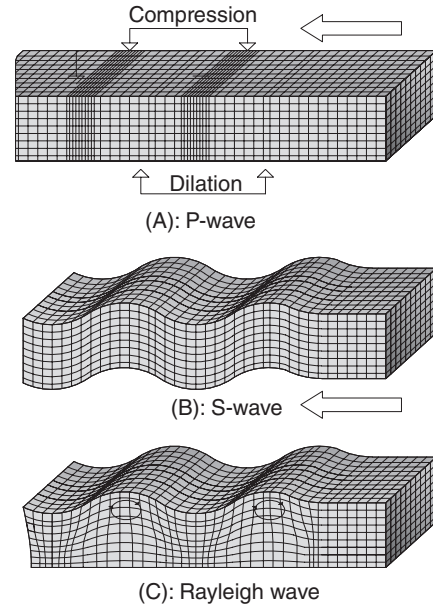


Figure 8.2 Propagation of seismic body waves and surface waves.

Table 8.1 Approximate Soil and Rock Wave Velocities

Material	P-Wave Velocity (m/s)	S-Wave Velocity (m/s)	Density (kg/m <sup>3</sup> )
Organic soil	300–700	100–300	1400–1700
Dry sand/gravel	400–1500	100–600	1500–1800
Saturated sand	1000–2000	350–600	1900–2100
Saturated clay	1000–2000	200–600	2000–2200
Shale	2000–3500	700–1500	2100–2500
Marl	2000–3000	750–1500	2100–2600
Sandstone	2000–3500	800–1800	2100–2400
Chalk	2300–2600	1100–1300	2100–2600
Limestone	3500–6000	2000–3300	2400–2800
Granite	4500–6000	2500–3500	2500–2700
Water	1450–1500	—	1000
Ice	3400–3800	1700–1900	900

(After ASTM D7128.)

Where  $v_p$  is the P wave velocity;  $M$ ,  $K$ ,  $G$ , and  $E$  are the constrained modulus, bulk modulus, shear modulus, and Young’s modulus, respectively (see Chapter 12 on soil constitutive models); and  $\rho$  is the mass density. Note that in soils, these moduli correspond to a strain level associated with the particle motion during the wave propagation. This strain level is typically extremely small. The higher the ratio in equation 8.6 is, the faster the wave propagates in the material.

S waves propagate in solids more slowly than do P waves ( $v_s \sim 0.6 v_p$ ), so they arrive at the detector after the P waves:

$$v_s = \sqrt{\frac{G}{\rho}} \quad (8.7)$$

where  $G$  is the shear modulus (see Chapter 12) and  $\rho$  is the mass density. For a homogeneous, isotropic, and elastic soil, the P wave velocity  $v_p$  and the S wave velocity  $v_s$  are related as follows:

$$v_p = v_s \sqrt{\frac{1-\nu}{0.5-\nu}} \quad (8.8)$$

where  $\nu$  is the Poisson's ratio.

In soils, P waves propagate both through the soil skeleton and through the water. S waves propagate through the soil skeleton only, as water cannot transmit shear waves. In solving geotechnical engineering problems, knowing the stiffness of the soil skeleton is often much more useful than knowing the stiffness of the water or the combined skeleton and water; therefore, shear waves are more useful than P waves in most cases except when trying to detect the depth of the ground water level.

Surface waves are also of two types: Rayleigh waves and Love waves. A large earthquake can create surface waves that travel around the Earth surface several times before dissipating. *Rayleigh waves*, sometimes called *ground rolls*, were discovered by Lord Rayleigh in the UK in 1885. Their propagation is analogous to the propagation you see when you drop a pebble into calm water. The wave displaces the particle along an ellipse in a plane perpendicular to the surface and in the direction of the wave as it passes through the soil. Rayleigh waves are slower than body waves ( $v_R \sim 0.9 v_s$ ). A good approximation of  $v_R$  is given by:

$$v_R \approx v_s \frac{0.87 + 1.12\nu}{1 + \nu} \quad (8.9)$$

where  $v_R$  and  $v_s$  are the Rayleigh wave and shear-wave velocities respectively and  $\nu$  is the Poisson's ratio. Rayleigh waves have large amplitude, large wave length, and long duration, and propagate further than shear waves and P waves along the surface. Because their wave length is related to the depth being affected by the waves, different frequencies can be used to investigate the variation of soil properties with depth. *Love waves* are slightly faster than Rayleigh waves, and are named after Augustus Love in the UK who discovered them in 1911.

### 8.2.2 Seismic Reflection

Waves will reflect back to the surface (Figure 8.3) anytime they encounter a boundary separating two layers with a contrast in acoustic impedance  $I_a$ . *Acoustic impedance* is the product of the density  $\rho$  and the wave speed  $v$ :

$$I_a = \rho v \quad (8.10)$$

The higher the impedance contrast is, the better the chance that it will be detected by seismic reflection. The acoustic impedance ratio  $R$  is defined as the ratio of the acoustic impedance of the lower layer over the acoustic impedance of the upper layer.

Seismic reflection consists of sending seismic waves down into the soil, receiving the reflected wave at a receiver, and identifying the time that it takes for the wave to travel down to the boundary and back to the surface (Figure 8.3).

The depth of the reflector or boundary is given by:

$$D = \frac{1}{2} \sqrt{(vt)^2 - L^2} \quad (8.11)$$

where  $D$  is the depth of the boundary reflecting the wave,  $t$  is the measured travel time of the wave,  $v$  is the wave velocity, and  $L$  is the distance between the shock point and the geophone. The shock wave is usually created by hitting the ground and the receivers are usually geophones (instruments

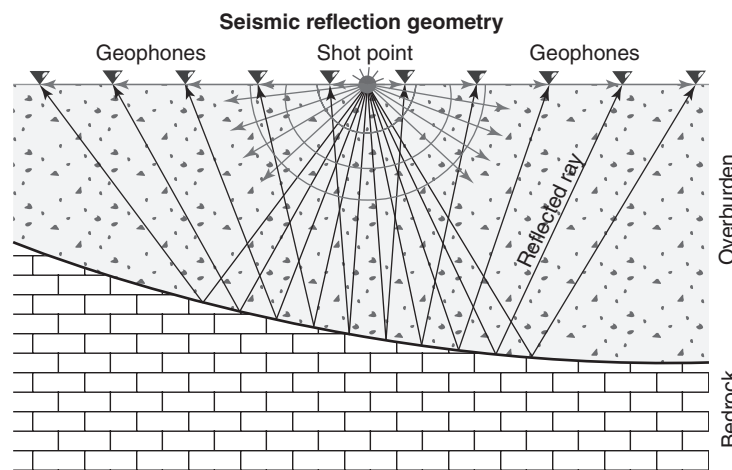


Figure 8.3 Seismic reflection test. (Courtesy of Timothy Bechtel, Envirosan, Inc.)

that measure velocity of the point where they are located). The geophones are arranged in a line over a length related to the width and depth of the soil or rock boundaries to be tested. The distance between geophones is related to the required horizontal resolution: the closer they are, the higher the resolution is.

Seismic reflection typically makes use of P waves and has the following characteristics. The depth to be studied should be more than about 10 m; indeed, at shallower depths the surface waves arrive at about the same time as and with larger amplitude than the reflecting waves, making it difficult to distinguish them. At greater depths, the reflected waves arrive after the surface waves and thus can be detected more easily. Seismic reflection does not require a very long array of geophones because the waves simply reflect back to the surface. Nevertheless, seismic reflection tends to be 3 to 5 times more expensive than seismic refraction because the inversion and interpretation are more complex. The vertical resolution is between 5 and 10% of the depth, while the horizontal resolution is about 50% of the geophone spacing. The applications are the delineation of layer boundaries (such as finding the depth to bedrock), the discovery of fractures and faults, determination of water level, detection of cavities like tunnels or sinkholes, and determination of elastic modulus for soils and rocks.

**8.2.3 Seismic Refraction**

When a wave comes to a boundary with a distinct change of acoustic impedance (see section 8.2.2), part of the wave will be reflected (going back to the surface) and part of the wave will be refracted (going through to the next layer). The direction of the refracted wave will be at the angle of

refraction, which follows Snell’s law (Figure 8.4):

$$n_i \sin \alpha_i = n_r \sin \alpha_r \tag{8.12}$$

where  $n_i$  is the refractive index of the layer the wave is leaving,  $\alpha_i$  is the incident angle between the wave direction and the normal to the boundary between the two layers,  $n_r$  is the refractive index of the layer the wave is entering, and  $\alpha_r$  is the refractive angle between the wave direction and the normal to the two layers. Willebrord Snell was a Dutch physicist who made this contribution in 1621.

The *refractive index* is the ratio between the wave velocity in a reference medium and the wave velocity in the soil considered. Therefore, for seismic wave propagation at interfaces, Snell’s law becomes:

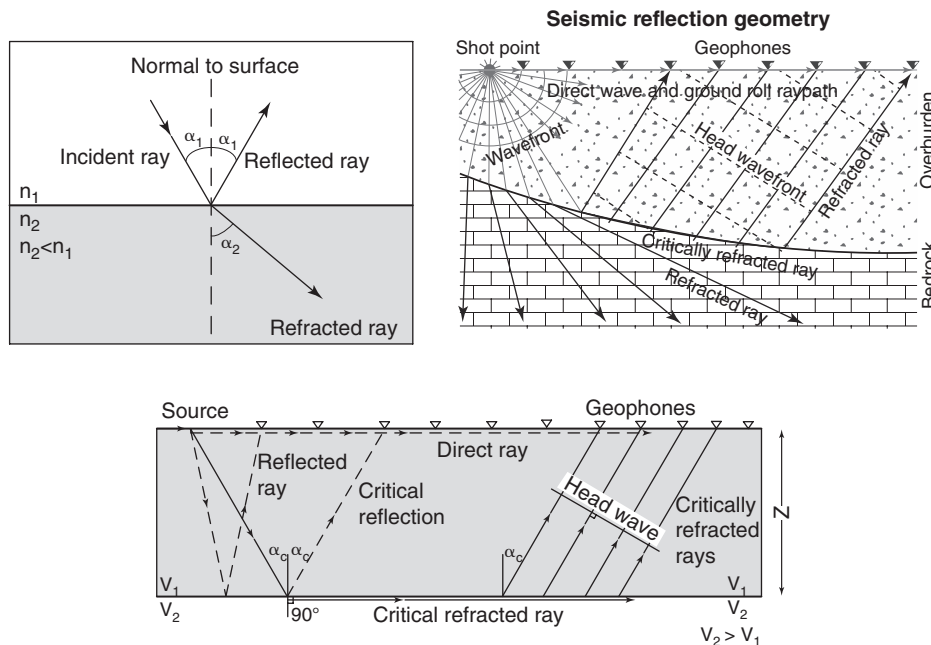
$$\frac{\sin \alpha_1}{v_1} = \frac{\sin \alpha_2}{v_2} \tag{8.13}$$

where  $v_1$  is the wave velocity in the upper layer and  $v_2$  the wave velocity in the lower layer. Note that there is no change in wave frequency as the wave enters the next layer, only a change in wave direction, wave velocity, and wave length.

If  $v_2$  is larger than  $v_1$ , there is an angle  $\alpha_c$  such that:

$$\frac{\sin \alpha_c}{v_1} = \frac{\sin 90^\circ}{v_2} \tag{8.14}$$

The angle  $\alpha_c$  is the critical angle at which the refracted wave propagates along the top of the second layer where the velocity is higher (Figure 8.4). At any time, the critically refracted wave that travels along the interface refracts back into the upper layer and strikes a geophone on the surface



**Figure 8.4** Seismic refraction test. (b: Courtesy of Timothy Bechtel, Enviroscan, Inc.)

which senses its arrival (Figure 8.4). The wave can travel directly along the surface from the source to the geophone or down to the lower layer and back to the surface. At the beginning of the recording, the waves travelling directly in the upper layer arrive first at a given geophone. After a while, the refracted waves arrive at the geophone before the reflected waves because the waves go faster in the lower layer if the lower layer has higher impedance (stiffer). The time at which this change occurs is called the crossover time  $t_c$  and corresponds to the cross-over distance  $x_c$  (Figure 8.5).

A plot of time of arrival versus distance between the detecting geophone and the source (Figure 8.5) shows two lines. The first is the arrival of the wave coming from direct propagation, which has a slope of  $1/v_1$ ; the second line is the arrival of the refracted wave, which has a slope of  $1/v_2$  (see problem solutions in this chapter for derivations). The intersection of these two lines gives the crossover time  $t_c$  and crossover distance  $X_c$ . The crossover time  $t_c$  can be used to obtain the depth of the lower layer:

$$Z = \frac{t_c v_1}{2} \frac{\sqrt{V_2 - V_1}}{\sqrt{V_2 + V_1}} \quad (8.15)$$

where  $Z$  is the depth to the lower layer and  $V_1$  and  $V_2$  are the velocities in the upper and lower layers, respectively. Therefore, seismic refraction can give the velocities  $V_1$  and  $V_2$  from the slope of the lines and the depth of the interface from  $t_c$ .

Seismic refraction typically makes use of P waves and has the following characteristics. The depth to be studied is typically up to 30 m; the length of the geophone array is on the order of 4 to 5 times the depth of the boundary to be detected. Although detection depths beyond 30 m are possible, they require very long geophone arrays and very large shock sources for the wave to be detected far away. Seismic refraction tends to be much less expensive than seismic reflection. The vertical resolution is about 15% of the depth studied and the horizontal resolution is about 50% of the geophone spacing.

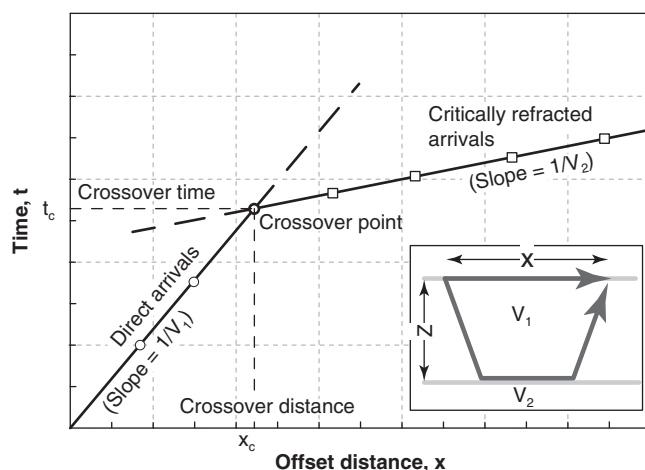


Figure 8.5 Interpreted signal from seismic refraction.

Seismic refraction is used primarily for determining the stratigraphy of soil layers, including depth to bedrock, as well as wave propagation characteristics of the layers penetrated. The critically refracted waves exist only if the soil or rock becomes stiffer or denser with depth, which is the most common case. However, a strong layer underlain by a weak layer will not produce critically refracted waves. The seismic refraction technique has been successfully applied to mapping depth to base of backfilled quarries, landfills, thickness of overburden, and the topography of groundwater.

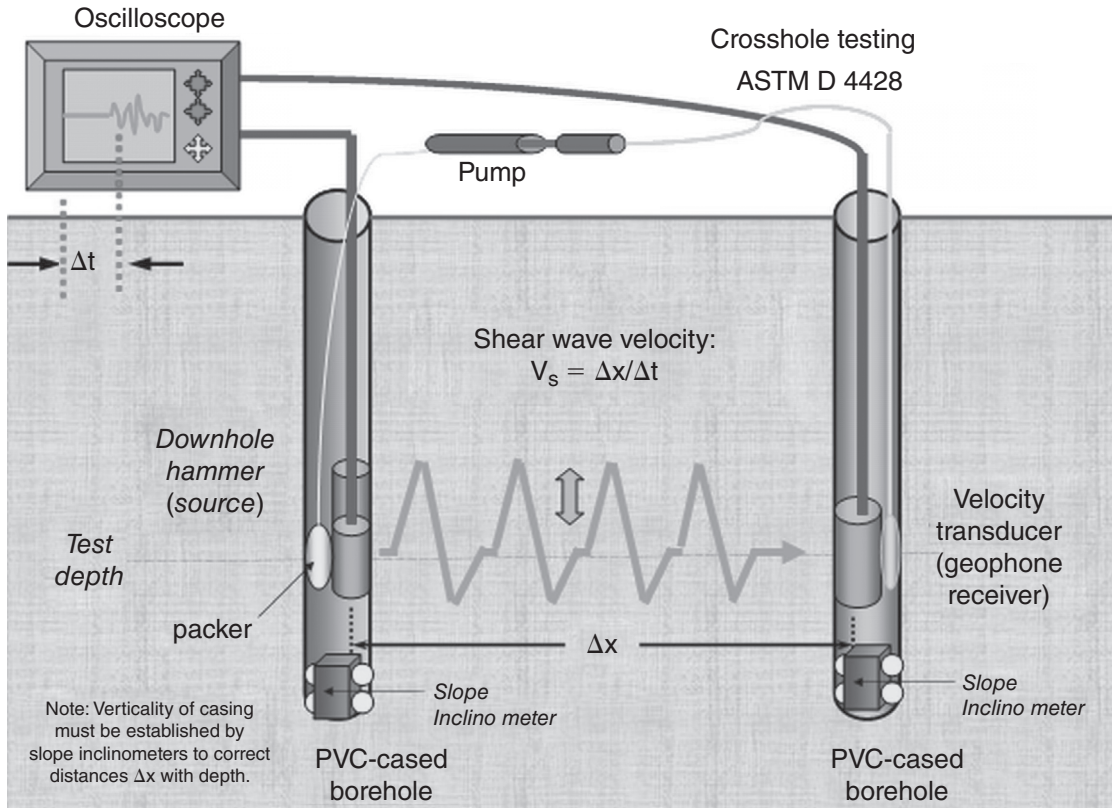
#### 8.2.4 Cross Hole Test, Seismic Cone Test, and Seismic Dilatometer Test

The cross hole test (CHT; ASTM D4428) (Figure 8.6) requires 2 borings separated by a distance  $L$ . This distance varies, but is typically between 3 to 6 m for geotechnical applications. Geophones are placed in boring 2 while the impact generator is placed in boring 1. Because shear waves isolate the behavior of the soil skeleton, they are more useful in geotechnical engineering than compression waves. Therefore, the source in boring 1 is usually one that generates a shear wave; this can be done by dropping an upper wedge on a lower wedge, for example. The time  $t$  required for the wave to travel from boring 1 to boring 2 is recorded and the shear-wave velocity is calculated as:

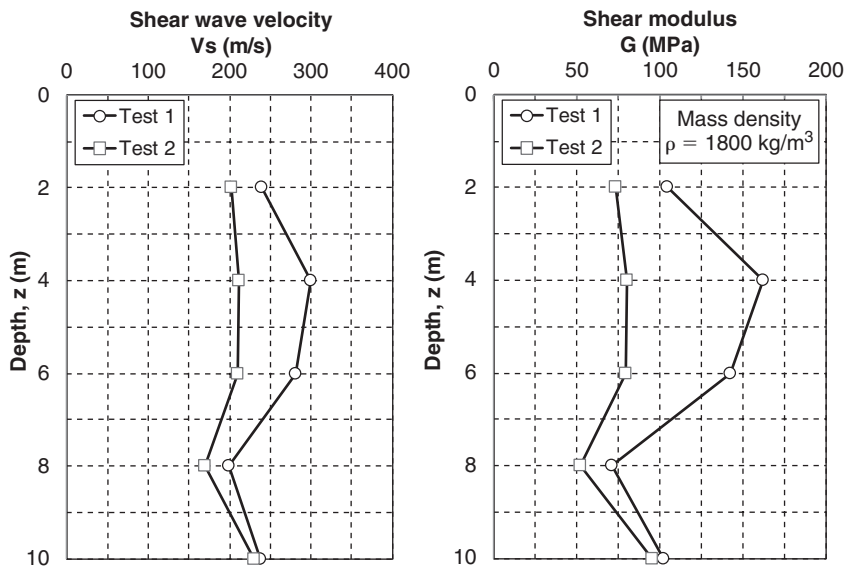
$$v_s = L/t \quad (8.16)$$

The distance  $L$  between borings may vary with depth, as the borings may not be perfectly parallel. For increased precision of  $L$ , it is desirable to run inclinometers in the borings to know the horizontal distance between borings at any depth with more accuracy. After the first CHT test, the depth of the source in boring 1 and the depth of the geophones in boring 2 are increased and the test is performed at each depth to obtain a shear-wave velocity profile (Figure 8.7). It is often desirable to use 3 borings, with the source in boring 1 and geophones in borings 2 and 3, because measurement of the wave travel time is easier to make in this case. The test can also be performed with the seismic cone penetrometer test by creating the shear-wave shock at the surface and recording the arrival of the shear wave at the depth where the cone penetrometer point (equipped with a geophone) is located. A similar test can be performed with the seismic dilatometer test.

The downhole technique consists of inserting a long probe with a source and a receiver on the same probe (Figure 8.8). The probe is inflated so that the source and receiver are in good contact with the wall of the borehole and the surrounding soil. With some probes, there is no direct contact; instead, transmission of the wave takes place through the liquid-filled borehole. The length of probe separating the source from the receiver is very flexible so that the wave propagating through the soil will arrive well before the one through the probe does. The wave travelling through the probe is purposely attenuated by the damping characteristic of that part



**Figure 8.6** Cross hole test. (Courtesy of Professor Paul Mayne, Georgia Institute of Technology, USA.)

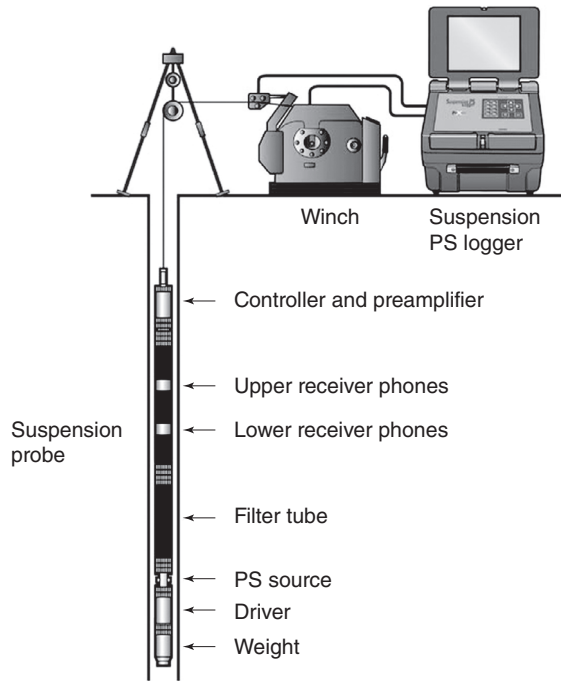


**Figure 8.7** Profile of shear-wave velocities from a cross hole test.

of the probe. This type of equipment is used primarily in deep boreholes or for offshore investigations. This type of equipment is also used in the oil well industry for electrical resistivity logging, neutron logging, gamma logging, and caliper logging.

### 8.2.5 Spectral Analysis of Surface Waves

The technique of *spectral analysis of surface waves (SASW)* has evolved over the years, but it seems appropriate to give credit to Ken Stokoe in the USA for a major part of its early development during the 1970s (Stokoe, Joh, and Woods,



**Figure 8.8** Downhole seismic test. (Courtesy of the OYO Corporation.)

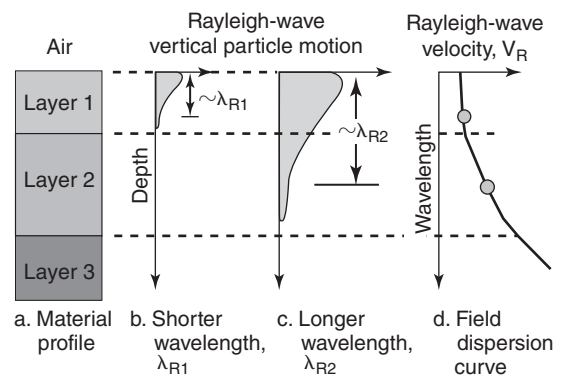
2004). If the velocity of a wave travelling in a material depends only on the physical properties of the material, then the wave velocity is constant and independent of frequency. Such a material is called a *nondispersive material* and waves traveling through this medium will maintain a constant shape (light propagates in a nondispersive and nondissipating way; this is why it can propagate over astronomical distances). This would be the case of a wave travelling in a soil that has uniform properties independent of depth. However, almost all soils have properties that vary with depth, because of differences such as layering and variations in effective stress; therefore, soils are dispersive materials. As a result, when many waves with different frequencies travel through the material, the wave train contains a lot of waves with individual frequencies, and the shape of the wave train changes as the wave travels. Some waves within the wave train travel faster than the wave train (longer wave length) and die out as they approach the leading edge. Some waves within the wave train travel slower than the wave train (shorter wave length) and die out as they approach the trailing edge.

The *group velocity*  $v_g$  is the speed with which the wave train or wave envelope propagates; it is the travel speed of the energy carried by the wave. The *phase velocity*  $v_{ph}$ , in contrast, is the speed with which an individual wave of the wave train travels. The phase velocity depends on the frequency of the individual wave contributing to the overall wave train. In the case of nondispersive material,  $v_g$  and  $v_{ph}$  are the same and independent of frequency. In the case of dispersive material like soil,  $v_g$  and  $v_{ph}$  are different and  $v_{ph}$

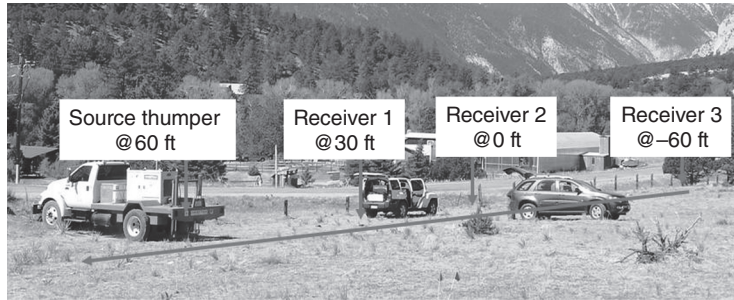
depends on frequency. See <http://physics.usask.ca/~hirose/ep225/animation/dispersion/anim-dispersion.html> or <http://paws.kettering.edu/~drussell/Demos/Dispersion/dispersion.html> for an animation of the difference between dispersive and nondispersive waves and between group and phase velocities.

The SASW makes use of Rayleigh waves because they travel along the ground surface and because they attenuate a lot less than body waves:  $1/\sqrt{r}$  instead of  $1/r^2$ . In fact, about two thirds of the seismic energy at shallow depth is made of Rayleigh waves. The SASW takes advantage of these concepts to link the frequency content of the wave train to the shear-wave velocity profile of the soil at a site. The high-frequency waves have short wave lengths and only penetrate the shallow layers of the soil deposit (Figure 8.9). Thus, they only give the shear-wave velocity of the shallow layers. The low-frequency waves have long wave lengths and penetrate much deeper in the soil deposit. Hence, they give the shear-wave velocity of the deeper layers. In the field, the test consists of placing receivers (geophones or accelerometers) on the ground surface at regular intervals away from where the shock is generated during the test. The receivers are placed along a single radial path from the impact location. These instruments measure the vertical movement of the soil as the waves pass by. A first set of data is collected at shallow depth by placing the receivers close to each other (Figure 8.10), generating the shock (by hammer blow, weight drop, explosive), and collecting the data at each receiver. A second set of data is collected after repositioning the receivers and doubling the distance between them to test the response of deeper layers. A third set of data is collected after again doubling that distance, and so on. This test is usually repeated about 6 to 8 times to obtain a shear-wave velocity versus depth profile. As the spacing between receivers increases, the impact source must generate larger amplitude and lower frequencies.

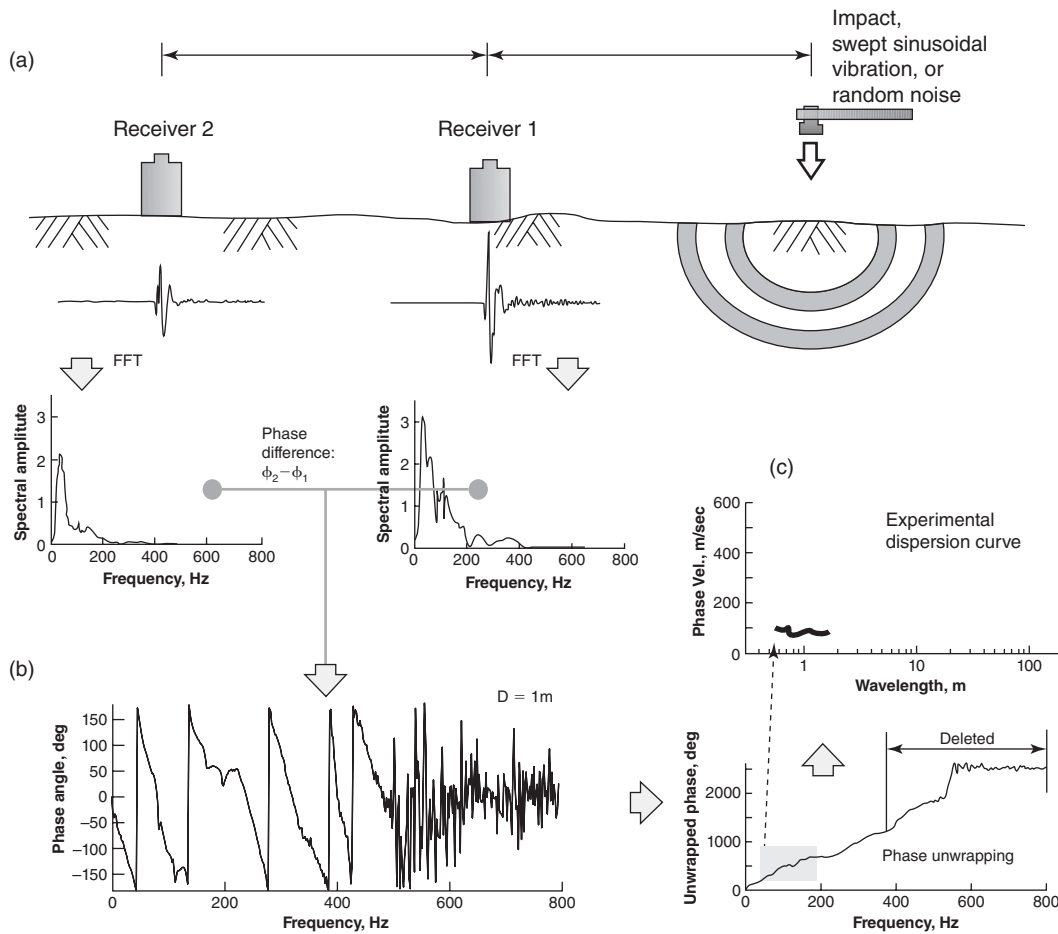
The data reduction involves the following sequence. First the amplitude versus time signal of the wave train, recorded at sequential receivers, is transformed from the time domain to



**Figure 8.9** Principle of the SASW method. (Courtesy of Professor Kenneth Stokoe, University of Texas, USA)



**Figure 8.10** Field SASW test. (Courtesy of Professor Kenneth Stokoe, University of Texas, USA)



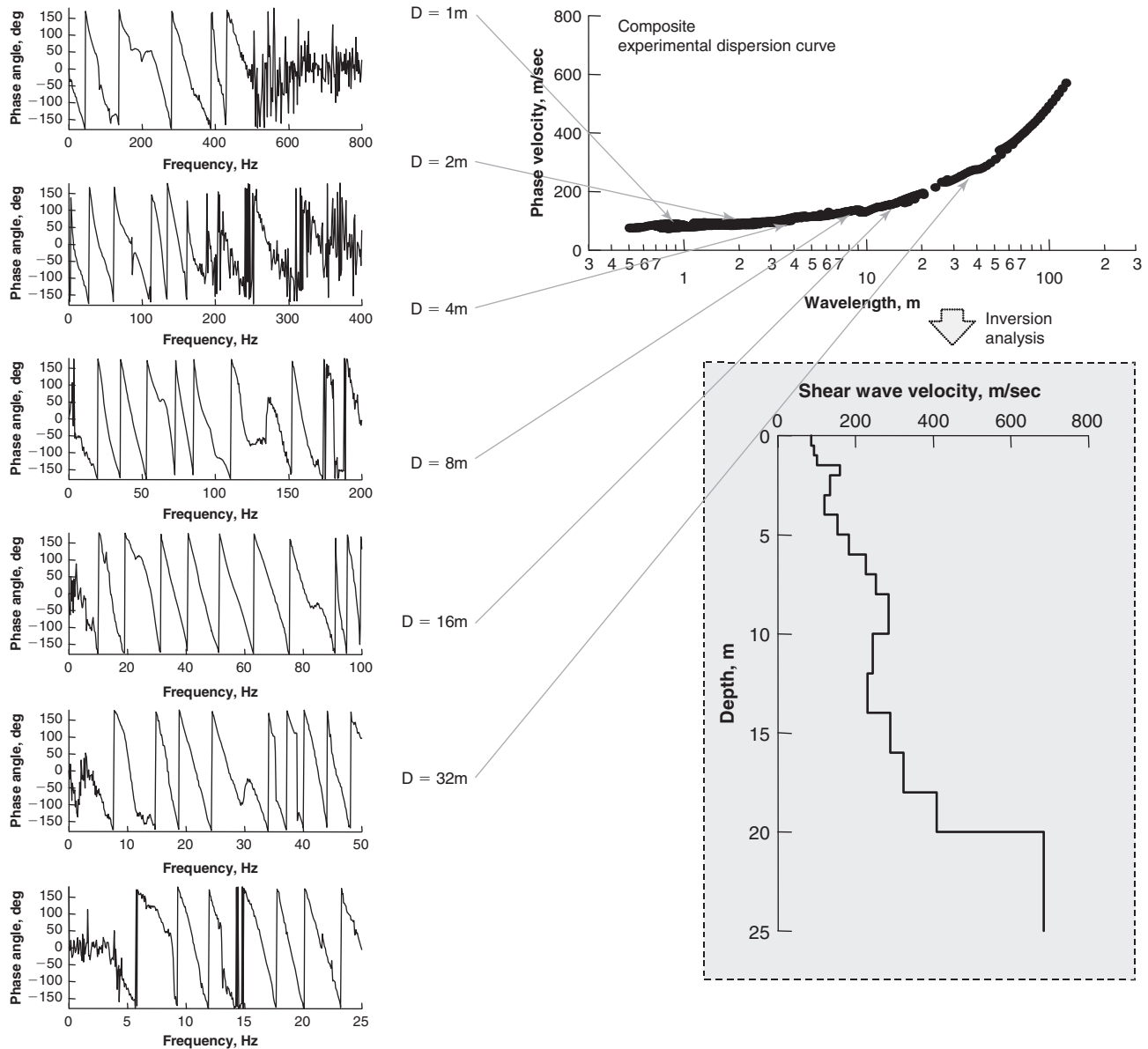
**Figure 8.11** Calculation of phase velocity for the SASW. (Courtesy of Professor Kenneth Stokoe, University of Texas, USA.)

the frequency domain by performing Fourier transformation (Fourier 1822). The amplitude  $a$  versus frequency  $f$  plots and the phase versus frequency plots are obtained in this fashion (Figure 8.11). Then the phase angle versus frequency diagram is transformed into an unwrapped phase angle  $\varphi$  versus frequency  $f$  diagram. This means that rather than keeping the phase angle between 0 and 360 degrees, the phase angle keeps increasing instead of being re-zeroed at 360 degrees. Then the phase velocity  $v$  versus wave length  $\lambda$

diagram is obtained from each phase versus frequency plot, as the tester knows the frequency, the phase angle, and the distance between receivers. The phase velocity  $v$  is obtained from the distance  $s$  between receivers and the elapsed time  $t$ , while the wave length  $\lambda$  is obtained from the unwrapped phase angle  $\varphi$ . In simple terms, it consists of writing the following equations:

$$T = 1/f \tag{8.17}$$





**Figure 8.12** Determination of a dispersion curve and shear-wave velocity for the SASW. (Courtesy of Professor Kenneth Stokoe, University of Texas, USA.)

More generally:

$$t = \varphi / 2\pi f \quad (8.18)$$

But

$$v = \lambda f = s/t \quad (8.19)$$

Therefore,

$$\lambda = 2\pi s / \varphi \quad (8.20)$$

where  $T$  is the period of the wave,  $f$  is the frequency,  $t$  is the time elapsed between the arrival of the wave at the first and second receivers,  $\varphi$  is the phase difference between the first and second receivers,  $v$  is the phase velocity,  $\lambda$  is the wave

length, and  $s$  is the distance between the two receivers. This indicates that when the phase  $\varphi$  is known, the wave length  $\lambda$  can be calculated. The plot of phase velocity  $v$  versus wave length  $\lambda$  is the *dispersion curve* for a given receiver spacing (Figure 8.11). This procedure is repeated for all receiver spacings and the individual dispersion curves for each spacing are assembled into a single composite dispersion curve (Figure 8.12). Once the composite dispersion curve is generated for the site, an iterative forward modeling procedure or an inversion analysis algorithm is used to determine a shear-wave velocity profile by matching the field dispersion curve with the theoretically determined dispersion curve (Figure 8.9).

**8.3 ELECTRICAL RESISTIVITY TECHNIQUES**

**8.3.1 Background on Electricity**

Electricity is related to the organized movement of electrons (electronic conduction) or of ions (electrolytic conduction) in a medium. Electricity in metals, for example, is electronic conduction; in wet soils or human flesh, it is electrolytic conduction. Metals often have electrons that can be moved when subjected to a potential difference. Electrolytes (e.g., fluids) contain atoms that either have more protons than electrons or more electrons than protons. These atoms are charged and are called *ions*. When subjected to a potential difference, the positive ions move in one direction and the negative ions move in the opposite direction. The speed of this movement of electrons or ions is very low, but because the material is full of electrons or ions, when the first one moves the last one also begins moving almost immediately. Under an alternating current, the electrons or ions shake in place, but again the ones far away shake as well, as the shaking is transmitted very quickly because the material is packed with electrons or ions ready to move. In soils, the main conduction is electrolytic, although electronic conduction can also occur (e.g., iron ore). The amount of readily moving electrons or ions is called the electric charge  $Q$ . The current  $I$  is the amount of charge passing at a location per unit of time. Voltage or potential relates to the difference in energy per unit charge between two points. In simple terms, the electrons or ions are pushing to go from one place to another and the difference in “pressure” is the voltage or potential. The resistance  $R$  is the resistance to flow of the electrons or ions and depends on how strongly the electrons or ions are bound. The power  $P$  is the rate of energy consumed per unit of time. If an analogy is drawn to hydraulics,  $Q$  would be the volume of water,  $V$  would be the difference in pressure between two points,  $I$  would be the flow rate, and  $R$  would be a constriction in the pipe.

$$I = Q/t \tag{8.21}$$

$$V = E/Q \tag{8.22}$$

$$R = V/I \tag{8.23}$$

$$P = E/t = VI \tag{8.24}$$

where  $I$  is the current (amperes),  $Q$  the charge (coulombs),  $t$  the time (seconds),  $V$  the voltage (volts),  $E$  the energy (joules),  $R$  the resistance (ohms), and  $P$  the power (watts). When electricity goes through a wire, the resistance can be written as:

$$R = \rho L/A \tag{8.25}$$

where  $\rho$  is the resistivity (ohm.m),  $A$  is the cross-sectional area of the wire, and  $L$  the length of the wire. The resistivity  $\rho$  and its inverse, the conductivity ( $\sigma = 1/\rho$ ), are properties of the material and independent of the dimensions. A low resistivity means very little resistance to an electrical current. Although soil and rock deposits are not wires, they also have

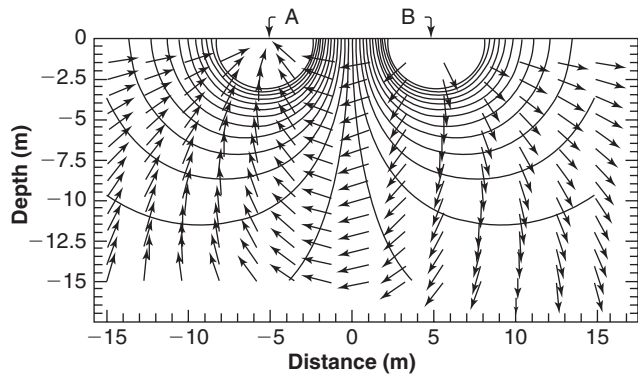
**Table 8.2 Example Values of Resistivity  $\rho$  for Soils and Rocks**

Soil or Rock	Low Value (ohm.m)	High Value (ohm.m)
Groundwater	1	200
Seawater	0.2	1
Sea ice	20	1000
Permafrost	500	10,000
Intact igneous and metamorphic rocks	1000	100,000
Weathered igneous and metamorphic rocks	1	1000
Porous limestone	50	2000
Dense limestone	1000	100,000
Sandstone	50	5000
Shale	5	2000
Clay and silt	2	100
Sand	50	2000
Gravel	400	10,000

resistivity values; Table 8.2 gives some of those values. The range is due in part to the significant influence of the water content in the soil or rock. Water has a very low resistivity, and a saturated soil will have a much lower resistivity than the same soil in the dry state. Porosity, degree of saturation, cation exchange capacity, temperature, and concentration of dissolved salts are other parameters that influence the resistivity of a soil.

**8.3.2 Resistivity Tomography**

Figure 8.13 shows two electrodes placed at the surface of a homogeneous soil deposit. The current flows from electrode B to electrode A along the electrical flow lines. Perpendicular to the flow lines are the electrical equipotential lines.



**Figure 8.13** Current lines and equipotential lines for an electrical resistivity test in a homogeneous soil deposit. (From Herman, 2001, Courtesy of American Association of Physics Teachers.)

These two sets of lines are the graphical solution to the Poisson's differential equation that governs electrical flow in a homogeneous material.

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0 \quad (8.26)$$

where  $V$  is the voltage or potential and  $x, y, z$  are the Cartesian coordinates in three dimensions. In simple terms, from Eqs. 8.23 and 8.25 comes:

$$\rho_a = \frac{\Delta V}{I} \frac{A}{L} = RK \quad (8.27)$$

where  $\Delta V/I$  is the resistance  $R$ , and  $A/L$  is the geometry coefficient  $K$ . The general solution to Eq. 8.26 depends on the placement of the electrodes and on the material in which they are placed, but the general form of Eq. 8.27 is maintained as:

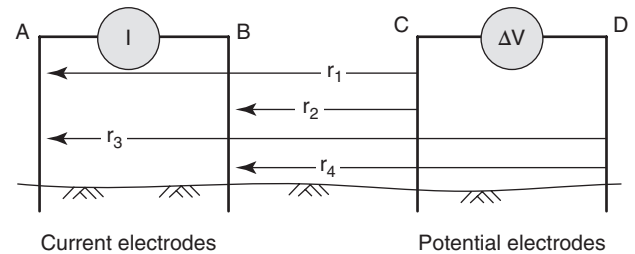
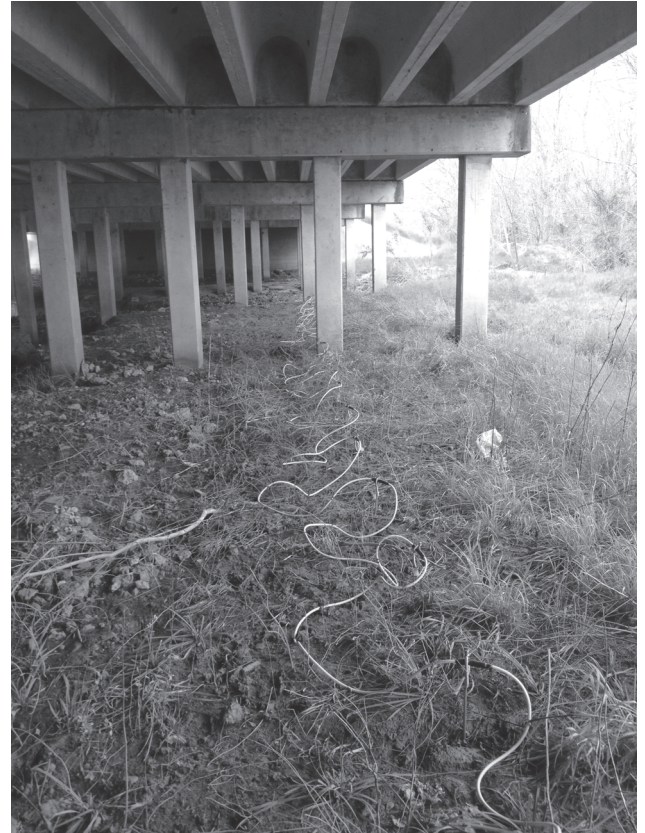
$$\rho_a = \frac{\Delta V}{I} K \quad (8.28)$$

In the field, electrodes can be placed on the ground surface in a line, as shown in Figure 8.14. In this case, Eq. 8.28 becomes:

$$\rho_a = \frac{\Delta V}{I} \frac{2\pi}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \left(\frac{1}{r_3} - \frac{1}{r_4}\right)} \quad (8.29)$$

where  $\Delta V$  is the difference of potential or voltage between the potential electrodes;  $I$  is the current existing between the current electrodes; and  $r_1, r_2, r_3,$  and  $r_4$  are the distances between electrodes as shown in Figure 8.14. Different arrays have been proposed to optimize the arrangement of the electrodes. For the Wenner array shown in Figure 8.15, the geometric factor  $K$  becomes  $2\pi a$  where  $a$  is the distance between electrodes. Other electrode spacings are being used in practice, such as the Schlumberger array and the dipole-dipole array. The best arrays for a field survey depend on the subsurface feature to be mapped, the sensitivity of the resistivity meter, and the background noise.

If the soil is made of two layers, with the lower layer having a lower electrical resistivity than the top layer, then the flow lines and equipotential lines are affected as shown in Figure 8.16. Furthermore, the resistivity obtained from the measurements is an equivalent or apparent electrical resistivity, as both layers are involved in the electrical response to the potential difference. The field test is generalized and many electrodes are placed on the ground surface at regular intervals. Alternatively, measurements may be made by using two electrodes as current electrodes and many others as potential electrodes. A mathematical inversion process is then used to back-calculate the electrical resistivity map that best fits the series of measurements. Such electrical resistivity maps are used for geotechnical engineering issues such as stratigraphy mapping, finding the depth of the water table, inferring the presence of leachates, and determining the depth of a landfill,



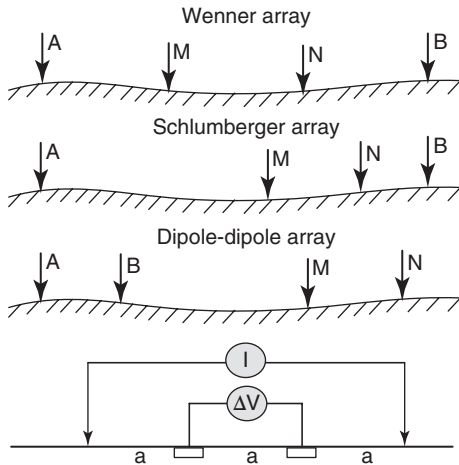
**Figure 8.14** Current and potential electrodes placement. (Bottom: After Cardimona, 1993.)

the presence of cavities, and the depth of bedrock. The depth of investigation for resistivity tomography is about 20% of the length of the string of electrodes placed on the ground.

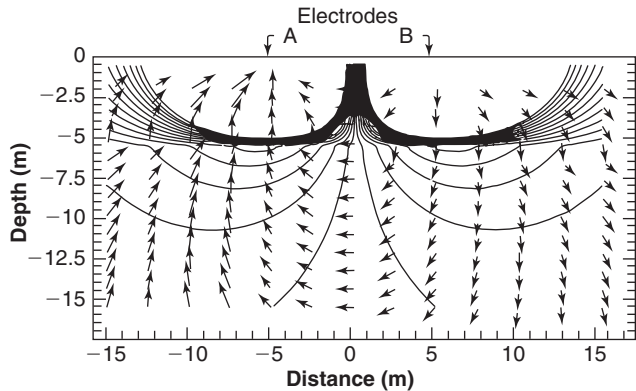
## 8.4 ELECTROMAGNETIC METHODS

### 8.4.1 Electromagnetic Waves

There are basically two main types of waves: mechanical waves and electromagnetic waves. *Mechanical waves* can only propagate through a material; they cannot propagate in vacuum. Seismic waves are mechanical waves. *Electromagnetic waves* can propagate in both a material and a vacuum. Light is one example of an electromagnetic wave. You can create an electromagnetic wave by shaking an electron; the electron will create a wave that propagates as ripples across



**Figure 8.15** Common electrode arrays for field resistivity test. (After American Association of Physics Teachers.)



**Figure 8.16** Current lines and equipotential lines for an electrical resistivity test in a two-layer soil deposit; the deeper layer has a lower resistivity than the top layer. (From Herman, 2001, Courtesy of American Association of Physics Teachers.)

the vacuum of space. When you shake the electron, the electromagnetic wave propagates as a *transverse wave*, a wave where the motion is perpendicular to the direction of propagation. What moves is a photon described as an electric field in the vertical direction and a magnetic field in the horizontal direction. Photons represent bundles of energy that can be equally considered as particles with zero mass or waves.

The elements of wave propagation described for seismic waves in section 8.2.1 (period, frequency, wave length, wave speed) also apply to electromagnetic waves. The speed of propagation of an electromagnetic wave is the speed of light. In a vacuum, that speed is approximately 300,000 km/s, which represents an accepted upper speed limit for our universe. The wave length  $\lambda$  of electromagnetic waves varies significantly from one end to the other of the spectrum (Figure 8.17). Radio waves ( $\lambda = 3000$  to  $0.3$  m) are used in broadcasting, microwaves ( $\lambda = 0.3$  to  $3 \times 10^{-4}$  m) are

used to heat food and in communications, infrared light ( $\lambda = 3 \times 10^{-4}$  to  $4 \times 10^{-7}$  m) is used for night vision and muscle therapy, visible light ( $\lambda = 4 \times 10^{-7}$  to  $7 \times 10^{-7}$  m) is a very small range of the electromagnetic wave length spectrum, ultraviolet light ( $7 \times 10^{-7}$  to  $3 \times 10^{-9}$  m) is used to detect forgery of paintings and in tanning salons, X-rays ( $\lambda = 3 \times 10^{-9}$  to  $3 \times 10^{-11}$  m) are used to see through the human body and through sampling tubes, and gamma rays ( $\lambda = 3 \times 10^{-11}$  to  $3 \times 10^{-13}$  m) are used to kill cancer cells in the human body.

### 8.4.2 Ground-Penetrating Radar

Ground-penetrating radar (GPR) uses electromagnetic waves in the radio-to-microwave range to penetrate the soil and give an image of the subsurface. The waves are generated by a source antenna that is in contact with the ground surface; the waves propagate in the soil, reflect from anomalies such as layer interfaces, cavities, and buried objects, and travel back to the surface where they are detected by a receiver antenna that is also in contact with the ground surface. *Antennas* are devices that transform electric current into electromagnetic waves and vice versa. Figure 8.18 shows a GPR test and a typical result. Note that the travel time from source to receiver is extremely short: Electromagnetic waves travel extremely fast, so this time is measured in nanoseconds. The electromagnetic waves are reflected any time they encounter a boundary between two materials with different dielectric constants. A dielectric material is a poor conductor of electricity and the *dielectric constant* is a measure of this property. Materials with relatively low dielectric constants, like air, glass, and ceramic, are good electric insulators. Materials with relatively high dielectric constants, like metal oxides, are good electric conductors. Soils having high electrical conductivity rapidly attenuate radar energy.

The depth of penetration of the GPR varies significantly depending on the soil type and on the frequency of the wave generated. The frequency used in GPR testing varies from as low as 20 MHz to as high as 2000 MHz. The user is often faced with a compromise between using a low frequency to penetrate deeply and a high frequency to obtain good definition. Indeed, as with mechanical waves, long wave lengths lead to deeper penetration, but short wave lengths lead to more precise definition of the objects encountered. Regarding the soil type, GPR works best in dry sand and gravel, where depths of tens of meters are possible with low-frequency antennas. However, in wet clays and saline soils the penetration is less than one meter. Figure 8.19 is a map of potential penetration with GPR in the USA. GPR is used for detecting pipes, tunnels, cavities, and unexploded ordinance, among other things.

### 8.4.3 Time Domain Reflectometry

Time domain reflectometry (TDR) makes use of the propagation of an electromagnetic wave in a cable. It was first used to

Wave Length, $\lambda$	Names	Frequency, f(Hz)	Period, T(s)	Color
$10^{-6}$ nm	Gamma rays	$3 \times 10^{23}$	$3.33 \times 10^{-24}$	
$10^{-5}$ nm		$3 \times 10^{22}$	$3.33 \times 10^{-23}$	
$10^{-4}$ nm		$3 \times 10^{21}$	$3.33 \times 10^{-22}$	
$10^{-3}$ nm		$3 \times 10^{20}$	$3.33 \times 10^{-21}$	
$10^{-2}$ nm	X rays	$3 \times 10^{19}$	$3.33 \times 10^{-20}$	
$10^{-1}$ nm		$3 \times 10^{18}$	$3.33 \times 10^{-19}$	
1 nm	Ultraviolet visible	$3 \times 10^{17}$	$3.33 \times 10^{-18}$	Violet
10 nm		$3 \times 10^{16}$	$3.33 \times 10^{-17}$	Blue
100 nm		$3 \times 10^{15}$	$3.33 \times 10^{-16}$	Green
1 $\mu\text{m}$		$3 \times 10^{14}$	$3.33 \times 10^{-15}$	Yellow
10 $\mu\text{m}$	Infrared	$3 \times 10^{13}$	$3.33 \times 10^{-14}$	Orange
100 $\mu\text{m}$		$3 \times 10^{12}$	$3.33 \times 10^{-13}$	Red
1 mm	Microwaves	$3 \times 10^{11}$	$3.33 \times 10^{-12}$	
1 cm		$3 \times 10^{10}$	$3.33 \times 10^{-11}$	
10 cm		$3 \times 10^9$	$3.33 \times 10^{-10}$	
1 m	Radio waves	$3 \times 10^8$	$3.33 \times 10^{-9}$	
10 m		$3 \times 10^7$	$3.33 \times 10^{-8}$	
100 m		$3 \times 10^6$	$3.33 \times 10^{-7}$	
1 km		$3 \times 10^5$	$3.33 \times 10^{-6}$	
10 km		$3 \times 10^4$	$3.33 \times 10^{-5}$	
100 km		$3 \times 10^3$	$3.33 \times 10^{-4}$	

Figure 8.17 Electromagnetic wave length spectrum.

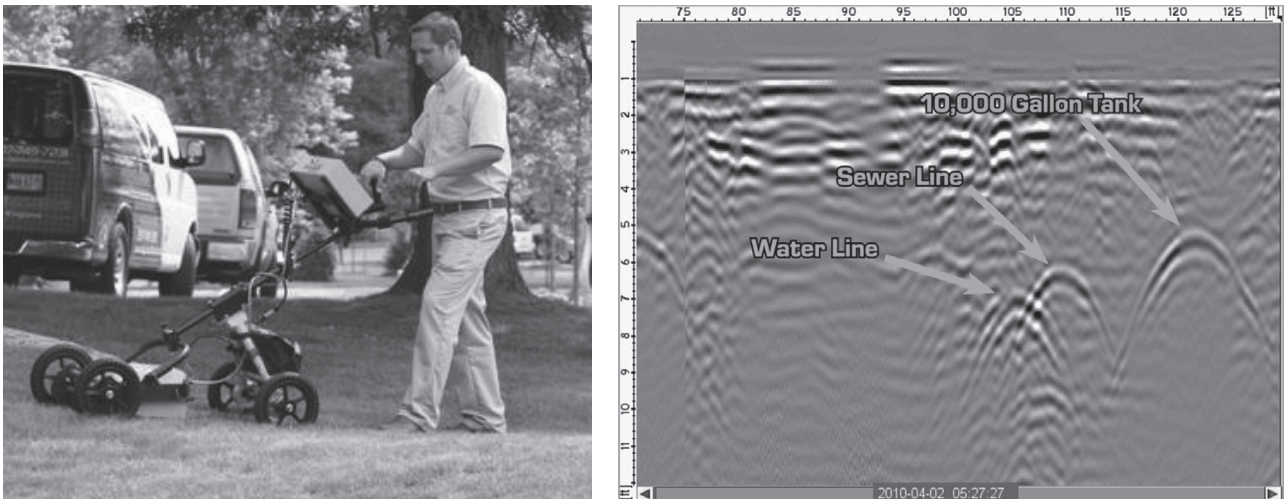
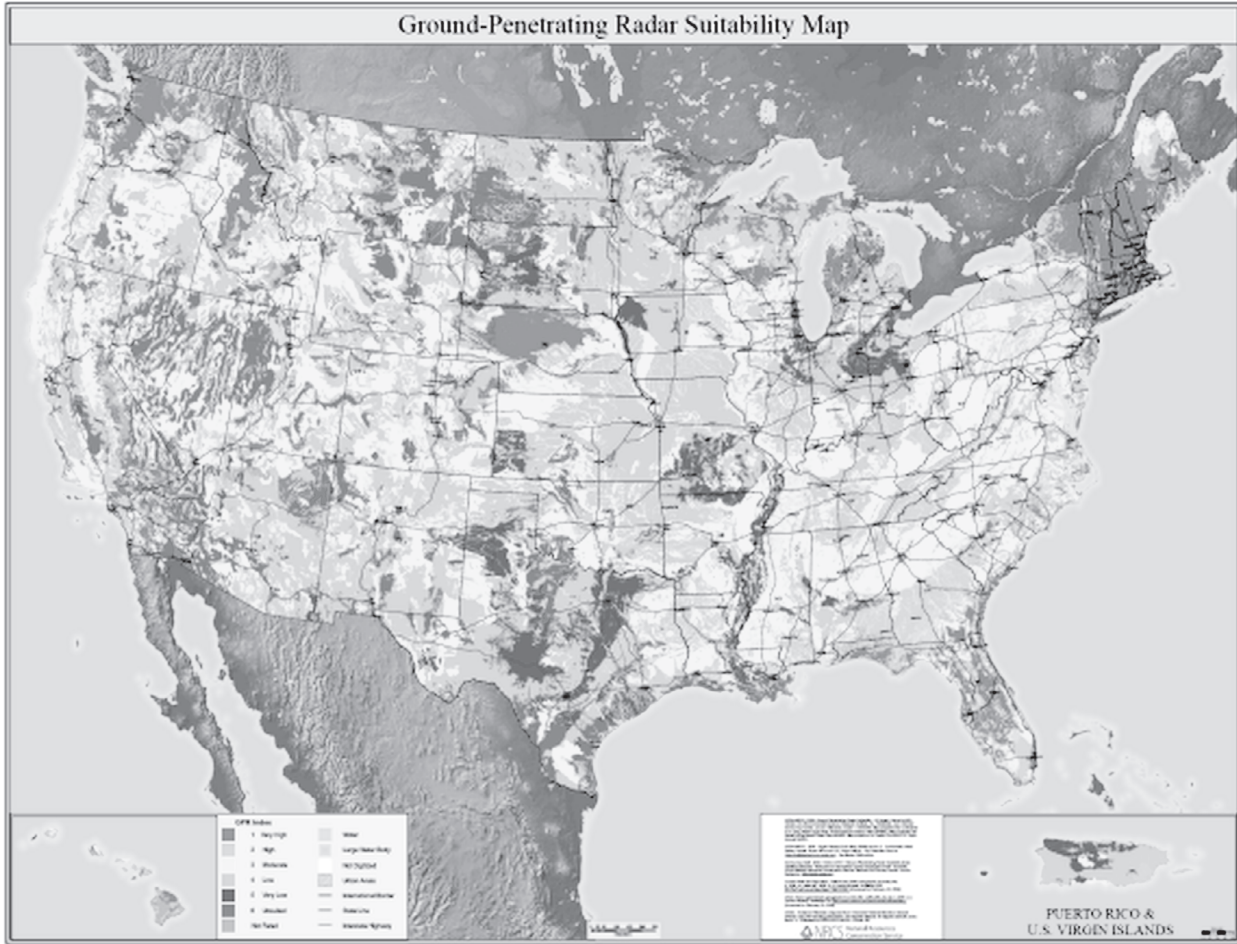


Figure 8.18 GPR testing and results. (Courtesy of Dig Smart of Maine)

find breaks in cables by measuring the travel time of the wave to the defect and back and using the travel speed to find out where the cable break was located. Although an electromagnetic wave travels at the speed of light in a vacuum, it travels at only a fraction of that value in a cable. Nevertheless, the time measurements must still be in nanoseconds or even

picoseconds. TDR was extended to soil water content and soil density measurements by using two rods pushed into the soil surface (Figure 8.20).

Materials are classified as conductors or insulators, also called dielectric, depending on their ability to conduct electricity. The dielectric permittivity  $\epsilon$  of a soil is a measure



**Figure 8.19** Suitability map for GPR testing. (Courtesy of NRCS [National Resources Conservation Services]).



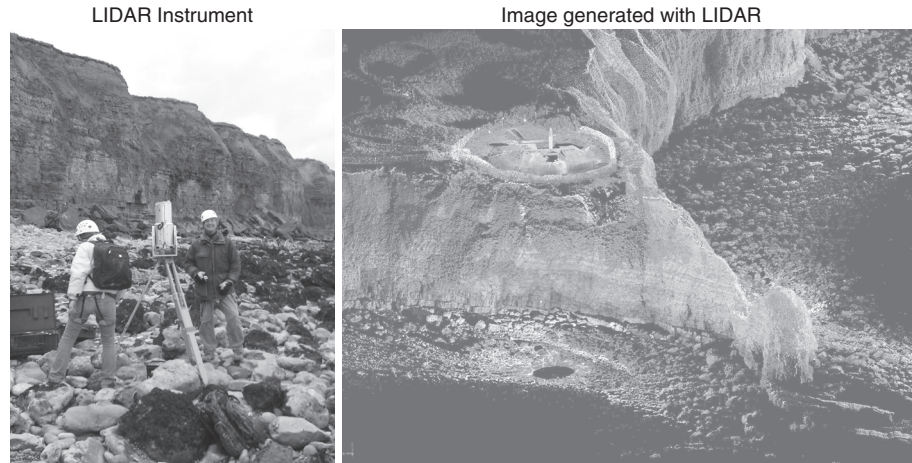
**Figure 8.20** Example of time domain reflectometry field probe. (Courtesy of Professor Vincent Drnevich, Purdue University)

of how fast an electromagnetic wave propagates through the soil:

$$\epsilon = (c/v)^2 \tag{8.30}$$

where  $c$  and  $v$  are the velocity of the electromagnetic wave in the soil and in a vacuum (300,000 km/s) respectively. If a

rod is embedded in a soil mass, the velocity of the electromagnetic wave propagating in the rod will be affected by the permittivity of the soil surrounding the rod. This velocity can be obtained by measuring the length of the rods embedded in the soil and the time required for the wave to travel down the rod and back; the soil dielectric permittivity can be obtained



**Figure 8.21** Example of LIDAR instrument and result: (a) LIDAR instrument. (b) Image generated with LIDAR. (Courtesy of Professor Robert Warden, Texas A&M University)

from this measurement by use of equation 8.30. For example, a dry soil has a dielectric permittivity value of around 4, moist soils around 30, water about 80, and air close to 1. So water impacts the dielectric permittivity significantly, and testers take advantage of this fact to relate soil dielectric permittivity to the soil water content. Calibrations are necessary to obtain the best correlation equation. This technique is used to obtaining the soil density as well as the soil water content. Such measurements are particularly useful on compaction projects, such as the field performance of landfill covers.

## 8.5 REMOTE SENSING TECHNIQUES

### 8.5.1 LIDAR

*LIDAR* stands for light detection and ranging and is sometimes called *laser radar*. The LIDAR test consists of sending a laser (light amplification by stimulated emission of radiation) beam of electromagnetic waves (infrared, visible, ultraviolet) at an object and detecting the time required for that beam to reflect from the object and come back to the LIDAR receiver. The beam is sent in a series of wave pulses at a very high frequency. Because the beam travels at the speed of light in air (close to 300,000 km/s), the time involved is measured in nanoseconds or picoseconds. These very short times can be measured with instruments such as optoelectronic streak cameras. Knowing the time of flight and the speed of the wave, the tester can back-calculate the distance. LIDAR works like a camera, as it sweeps through the landscape it is aimed at and records the distance of all objects it is “seeing.” The exact location of the instrument is obtained through the global positioning system (GPS), and the distances measured can be connected to elevations and coordinates. The result of a LIDAR test is a three-dimensional image of the landscape swept by the LIDAR equipment in which all points recorded are documented with coordinates.

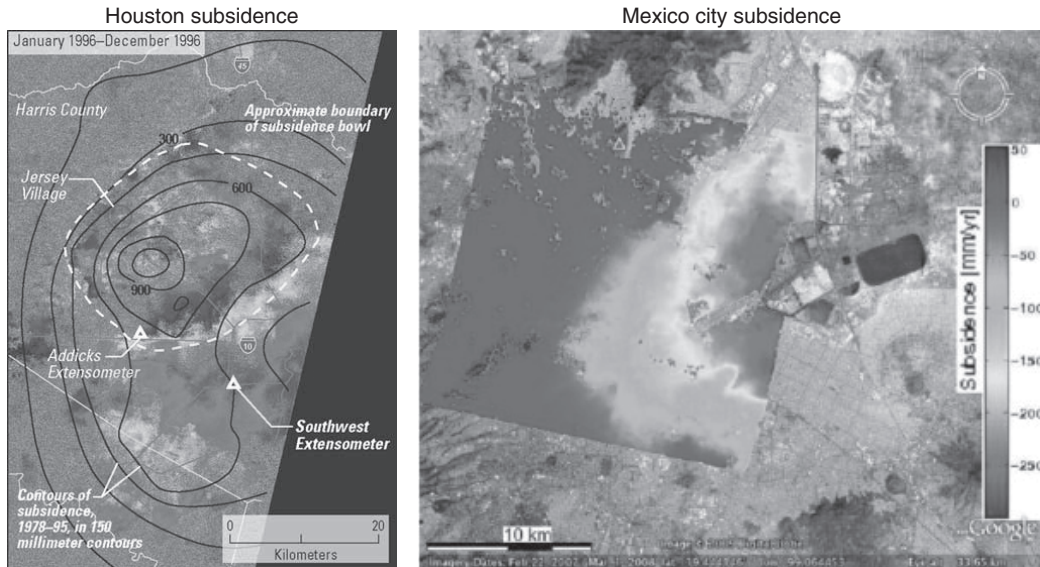
LIDAR uses short wavelengths of the electromagnetic spectrum, typically in the ultraviolet, visible, or near-infrared range. This allows the LIDAR equipment to define objects within a few millimeters (or at least centimeters), as it is possible to image a feature or object only about the same size as the wavelength, or larger. This makes it difficult for LIDAR to see through aerosol, rain, snow, mist, fog, and smoke; LIDAR works best when the sky is clear without clouds, rain, or haze, and functions equally well day or night.

For applications from the ground, a LIDAR system (Figure 8.21) is composed of a laser scanning system and a global positioning system. GPS. A GPS is a space-based global navigation satellite system (GNSS) that provides reliable location and time information in all weather, at all times, and anywhere on or near the Earth when and where there is an unobstructed line of sight to four or more GPS satellites. It was established in 1973 by the United States government, which maintains it, and is freely accessible to anyone with a GPS receiver.

In addition, for LIDAR used from an airplane (airborne LIDAR), an inertial measuring unit (IMU) is required to take into account the speed of the airplane when calculating the coordinates of the points recorded. An IMU is an electronic device that measures the airplane’s velocity and orientation using a combination of accelerometers and gyroscopes.

### 8.5.2 Satellite Imaging

*Satellite imaging*, also known as *radar satellite*, is based on the same principle as LIDAR but gives a picture of a much larger area than LIDAR. LIDAR is more applicable to smaller areas, whereas satellite imaging is more applicable to larger areas (Figure 8.22). For example, if the problem is to record the site contours of a levee breach after a flood, LIDAR is more applicable; if the problem is to record the subsidence over time of a large city due to water pumping, satellite imaging is faster and less time-consuming. The satellite imaging system



**Figure 8.22** Example of satellite imaging results. (a) Houston subsidence. (b) Mexico City subsidence. (a: Courtesy of USGS. b: The ESA Envisat ASAR data is made available through the GEO Geohazards Supersite.)

requires the following parts: the optical system in the satellite, which views the area targeted; the internal processor, which collects and stores the data; the data transiting system; and

the ground analysis and postprocessor. Satellite imaging is about as precise as LIDAR. Google Earth is a system based on satellite imaging.

## PROBLEMS

- 8.1 Explain the difference between wave velocity and particle velocity.
- 8.2 If the shear-wave velocity in a soil is 250 m/s and the unit weight is  $20 \text{ kN/m}^3$ , what is the small-strain shear modulus of that soil?
- 8.3 If sound propagates in water at 5702 km/h, what is the constrained modulus of elasticity of water?
- 8.4 If sound propagates at 22000 km/h in steel, how long does it take for the wave to propagate down to the bottom of a 30 m long H pile, and what is the modulus of elasticity of steel? (Density of steel is  $7850 \text{ kg/m}^3$  and the Poisson's ratio is 0.3.)
- 8.5 A wave has a wave length of 600 nm and a frequency of  $5 \times 10^{14} \text{ Hz}$ . What kind of wave is it?
- 8.6 What is the difference in particle motion between a shear wave and a Rayleigh wave?
- 8.7 Explain the difference between seismic reflection and seismic refraction techniques.
- 8.8 Derive the crossover time equation for seismic refraction.
- 8.9 What is a dispersion curve?
- 8.10 Describe the basic concept of the SASW technique.
- 8.11 Describe the basic concept of the electrical tomography technique.
- 8.12 What is an electromagnetic wave, and what are its main properties?
- 8.13 Describe the basic concept of the GPR.
- 8.14 Describe the basic concept of the TDR.
- 8.15 Describe the basic concept of LIDAR.

## Problems and Solutions

### Problem 8.1

Explain the difference between wave velocity and particle velocity.

### Solution 8.1

The wave velocity  $v$  is the speed at which the particle motion is propagated from one particle to the next. The particle velocity  $u$  is the speed at which the particle is moving around its own location.



**Problem 8.2**

If the shear-wave velocity in a soil is 250 m/s and the unit weight is 20 kN/m<sup>3</sup>, what is the small-strain shear modulus of that soil?

**Solution 8.2**

$$\begin{aligned}\gamma &= 20 \text{ kN/m}^3 = \rho g = \rho \times 9.81 \text{ m/s}^2 \\ \rho &= \frac{20000 \text{ N/m}^3}{9.81 \text{ m/s}^2} = 2039 \frac{\text{kg}}{\text{m}^3} \\ G &= \rho \times v_s^2 = 2039 \times 250^2 = 127 \text{ MPa}\end{aligned}$$

**Problem 8.3**

If sound propagates in water at 5702 km/h, what is the constrained modulus of elasticity of water?

**Solution 8.3**

$$v_p = 5702 \frac{\text{km}}{\text{hr}} = 5702 \frac{1000 \text{ m}}{3600 \text{ sec}} = 1584 \frac{\text{m}}{\text{s}}$$

$$v_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \left. \begin{array}{l} \Rightarrow K = v_p^2 \rho \\ \text{water} \rightarrow G = 0 \end{array} \right\}$$

$$\left. \begin{array}{l} v_p = 1584 \frac{\text{m}}{\text{s}} \\ \rho = 1000 \frac{\text{kg}}{\text{m}^3} \end{array} \right\} K = 2.51 \times 10^9 \frac{\text{N}}{\text{m}^2} = 2.51 \text{ GPa}$$

**Problem 8.4**

If sound propagates at 22000 km/h in steel, how long does it take for the wave to propagate down to the bottom of a 30 m long H pile, and what is the modulus of elasticity of steel? (Density of steel is 7850 kg/m<sup>3</sup> and the Poisson's ratio is 0.3.)

**Solution 8.4**

$$v_p = 22000 \frac{\text{km}}{\text{hr}} = 22000 \frac{1000 \text{ m}}{3600 \text{ sec}} = 6111 \frac{\text{m}}{\text{s}} \left. \begin{array}{l} t = \frac{L}{v} = \frac{30}{6111} = 4.91 \times 10^{-3} \text{ sec} \\ \text{Pile Length } L = 30 \text{ m} \end{array} \right\}$$

$$v_p = \sqrt{\frac{E}{\rho} \frac{(1-\nu)}{(1+\nu)(1-2\nu)}} \Rightarrow E = v_p^2 \rho \frac{(1+\nu)(1-2\nu)}{(1-\nu)}$$

$$\left. \begin{array}{l} v_p = 6111 \frac{\text{m}}{\text{s}} \\ \nu = 0.3 \\ \rho = 7850 \frac{\text{kg}}{\text{m}^3} \end{array} \right\} E = 6111^2 \times 7850 \frac{(1+0.3)(1-2 \times 0.3)}{(1-0.3)} = E = 2.17 \times 10^{11} \frac{\text{N}}{\text{m}^2} = 217 \text{ GPa}$$

**Problem 8.5**

A wave has a wave length of 600 nm and a frequency of  $5 \times 10^{14}$  Hz. What kind of wave is it?

**Solution 8.5**

Using the fundamental equation  $\lambda = v/f$ , the velocity of the wave is:

$$v = \lambda \times f = (600 \times 10^{-9}) \times (5 \times 10^{14}) = 3 \times 10^8 \text{ m/s}$$

$3 \times 10^8$  m/s is the speed of light. Therefore the wave is an electromagnetic wave.

**Problem 8.6**

What is the difference in particle motion between a shear wave and a Rayleigh wave?

**Solution 8.6**

In shear waves, the wave displaces the particle along a line perpendicular to the direction of the wave. In Rayleigh waves, which are surface waves, the wave displaces the particle along an ellipse in a plane that is in the direction of the wave and perpendicular to the surface.

**Problem 8.7**

Explain the difference between seismic reflection and seismic refraction techniques.

**Solution 8.7**

Seismic reflection consists of sending seismic waves down into the soil, receiving the reflected wave at a receiver, and identifying the time that it took for the wave to travel down to the boundary and back to the surface. It typically makes use of P waves.

Seismic refraction involves measuring the travel time of the component of seismic energy that travels down to the top of a layer boundary, is refracted along that boundary, and returns to the surface as a reflected wave. Seismic refraction typically makes use of P waves.

**Problem 8.8**

Derive the crossover time equation for seismic refraction.

**Solution 8.8**

$$t_1 = \frac{\overline{af}}{V_1} = \frac{X}{V_1}$$

$$t_2 = \frac{\overline{ac}}{V_1} + \frac{\overline{cd}}{V_2} + \frac{\overline{df}}{V_1}$$

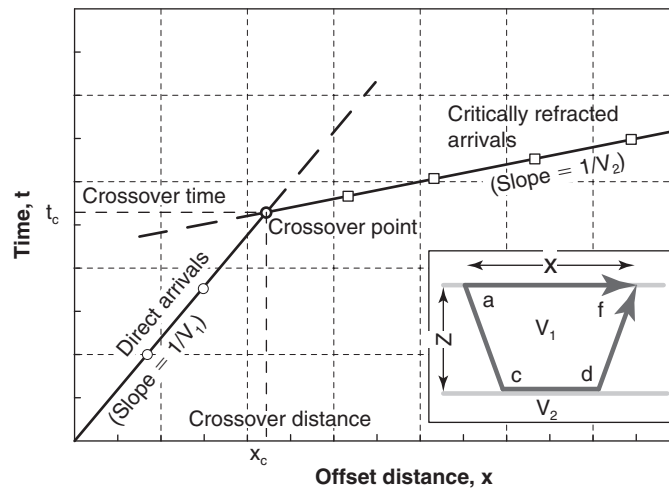
$$\text{Here, } \overline{ac} = \overline{df} = \frac{Z}{\cos \alpha_c}$$

$$\text{and } \overline{cd} = X - 2Z \tan \alpha_c$$

Therefore,

$$t_2 = \frac{2Z}{V_1 \cos \alpha_c} + \frac{X - 2Z \tan \alpha_c}{V_2} = \frac{2Z}{V_1 \cos \alpha_c} - \frac{2Z \tan \alpha_c}{V_2} + \frac{X}{V_2}$$

$$= 2Z \left( \frac{1}{V_1 \cos \alpha_c} - \frac{\tan \alpha_c}{V_2} \right) + \frac{X}{V_2} = 2Z \left( \frac{V_2 - V_1 \sin \alpha_c}{V_1 V_2 \cos \alpha_c} \right) + \frac{X}{V_2}$$



**Figure 8.1s** Illustration for seismic refraction test.

Based on Snell's law,

$$\sin \alpha_c = \frac{V_1}{V_2},$$

$$\cos \alpha_c = \sqrt{1 - \sin^2 \alpha_c} = \sqrt{1 - \left(\frac{V_1}{V_2}\right)^2}$$

$$t_2 = 2Z \left( \frac{V_2 - V_1^2/V_2}{V_1 V_2 \sqrt{1 - \left(\frac{V_1}{V_2}\right)^2}} \right) + \frac{Z}{V_2} = 2Z \left( \frac{V_2^2 - V_1^2}{V_1 V_2^2 \sqrt{1 - \left(\frac{V_1}{V_2}\right)^2}} \right) + \frac{Z}{V_2} = 2Z \left( \frac{\sqrt{V_2^2 - V_1^2}}{V_1 V_2} \right) + \frac{X}{V_2}$$

Based on the definition of crossover time for seismic refraction,

$$t_1 = t_2 = t_c, \text{ meanwhile, } X = X_c$$

where  $t_c$  is the crossover time and  $x_c$  is the crossover distance.

Therefore,

$$\frac{X_c}{V_1} = 2Z \left( \frac{\sqrt{V_2^2 - V_1^2}}{V_1 V_2} \right) + \frac{X_c}{V_2},$$

$$X_c = 2Z \frac{\sqrt{V_2 + V_1}}{\sqrt{V_2 - V_1}},$$

$$t_c = \frac{X_c}{V_1} = \frac{2Z \sqrt{V_2 + V_1}}{V_1 \sqrt{V_2 - V_1}}$$

$$\text{and } Z = \frac{t_c V_1}{2} \frac{\sqrt{V_2 - V_1}}{\sqrt{V_2 + V_1}}$$

### Problem 8.9

What is a dispersion curve?

**Solution 8.9**

The plot of phase velocity  $v$  versus wave length  $\lambda$  is the dispersion curve for a given receiver spacing. This plot is obtained for all receivers at different spacing and the individual dispersion curves for each spacing are assembled into a single composite dispersion curve. Once the composite dispersion curve is generated for the site, an iterative forward modeling procedure or an inversion analysis algorithm is used to determine a shear-wave velocity versus depth profile by matching the field dispersion curve with the theoretically determined dispersion curve.

**Problem 8.10**

Describe the basic concept of the SASW technique.

**Solution 8.10**

The spectral analysis of surface waves technique is based on the concept that if the velocity of a wave travelling in a material depends only on the physical properties of the material, then the wave velocity is constant and independent of frequency. Waves traveling through such nondispersive material will maintain a constant shape. This would be the case of a wave travelling in a soil that had uniform properties independent of depth. However, soils have properties that vary with depth because of differences such as layering and variations in effective stress; therefore, soils are dispersive materials. As a result, when many waves with different frequencies travel through the material, the wave train contains a lot of waves with individual frequencies, and the shape of the wave train changes as the wave travels. Some waves within the wave train travel faster than the wave train (longer wave length) and die out as they approach the leading edge. Some waves within the wave train are slower than the wave train (shorter wave length) and die out as they approach the trailing edge. SASW makes use of Rayleigh waves because they travel along the ground surface and they attenuate a lot less than body waves. SASW links the frequency content of the wave train to the shear-wave velocity profile of the soil at a site. In the field, the test consists of placing receivers on the ground surface at regular intervals away from where the shock is generated during the test. The receivers are placed along a single radial path from the impact location. These instruments measure the vertical movement of the soil as the waves pass by. This procedure is repeated about 6 to 8 times to obtain a shear-wave velocity versus depth profile.

**Problem 8.11**

Describe the basic concept of the electrical tomography technique.

**Solution 8.11**

Electrical tomography is a geophysical technique in which a current is passed between metal electrodes inserted into the ground. For soils composed of different layers having different electrical resistivity, the resistivity obtained from the measurements is an equivalent or apparent electrical resistivity, depending on the layers involved in the electrical response to the potential difference. Because these measurements are made at different depths and in different directions, a three-dimensional image of the site can be obtained through an inversion process.

**Problem 8.12**

What is an electromagnetic wave, and what are its main properties?

**Solution 8.12**

Electromagnetic waves can propagate in both a material and a vacuum. Light is one example of an electromagnetic wave. You can create an electromagnetic wave by shaking an electron; the electron will create a wave that will propagate as ripples across the vacuum of space. When you shake the electron, the electromagnetic wave propagates as a transverse wave, a wave where the motion is perpendicular to the direction of propagation. What moves is an electric field in the vertical direction and a magnetic field in the horizontal direction called a photon. Photons represent bundles of energy that can be equally considered as particles with zero mass or waves.

Some of the properties are:

- a. The direction of motion is perpendicular to the propagation of the wave.
- b. The speed of propagation of an electromagnetic wave is the speed of light (300,000 km/s in vacuum)
- c. The wave length  $\lambda$  of electromagnetic waves varies significantly. For example, the radio waves used in broadcasting have a wave length varying from 3000 to 0.3 m, the microwaves used to heat food have a wave length varying from 0.3 to  $3 \times 10^{-4}$  m, and the infrared light waves used for night vision and muscle therapy have a wave length varying from  $3 \times 10^{-4}$  to  $4 \times 10^{-7}$  m.

**Problem 8.13**

Describe the basic concept of the GPR.

**Solution 8.13**

Ground-penetrating radar is a nondestructive geophysical method that uses electromagnetic waves in the microwave range to penetrate the soil and give an image of the subsoil. The waves are generated by a source antenna in contact with the ground surface; the waves propagate in the soil, reflect from anomalies such as layer interfaces, cavities, and buried objects; and travel back to the surface, where they are detected by a receiver antenna also in contact with the ground surface. GPR can be used in a variety of media, including soil, rock, ice, pavement, and structures. It can also be used to detect changes in material properties and the presence of voids or cracks, among other things.

**Problem 8.14**

Describe the basic concept of the TDR.

**Solution 8.14**

Materials are classified as conductors or insulators, also called dielectric, depending on their ability to conduct electricity. The dielectric permittivity  $\epsilon$  of a soil is a measure of how fast an electromagnetic wave propagates through the soil:

$$\epsilon = (c/v)^2 \quad (\text{see equation 8.30})$$

where  $c$  and  $v$  are the velocity of the electromagnetic wave in the soil and in a vacuum (300,000 km/s) respectively. If a rod is embedded in a soil mass, the velocity of the electromagnetic wave propagating in the rod will be affected by the permittivity of the soil surrounding the rod. This velocity can be obtained by measuring the length of the rods embedded in the soil and the time required for the wave to travel down the rod and back; then the soil dielectric permittivity can be obtained from this measurement. For example, a dry soil has a dielectric permittivity value of around 4, moist soils of around 30, water of about 80, and air close to 1. Obviously, water significantly affects the dielectric permittivity and advantage is taken of this fact to relate soil dielectric permittivity to the soil water content. Calibrations are necessary to obtain the best correlation equation. This technique is used to obtain the soil density as well as the soil water content. Such measurements are particularly useful on compaction projects, such as the field performance of landfill covers.

**Problem 8.15**

Describe the basic concept of LIDAR.

**Solution 8.15**

LIDAR stands for light detection and ranging and is sometimes called laser radar. The LIDAR test consist of sending a laser beam of electromagnetic waves (light) at an object and detecting the time required for that beam to reflect on the object and come back to the LIDAR receiver. The beam is sent in a series of wave pulses at a very high frequency. Because the beam travels at the speed of light in air (close to 300,000 km/s), the time involved is measured in nanoseconds or picoseconds. These very short times can be measured with instruments such as optoelectronic streak cameras. Knowing the time of flight and the speed of the wave, the distance can be back-calculated. LIDAR works like a camera as it sweeps through the landscape it is aimed at and records the distance of all objects it is seeing. The result of a LIDAR test is a three-dimensional image of the landscape swept by the LIDAR where all points recorded are documented with coordinates. LIDAR works equally well during the daytime and during the night.