CHAPTER 4

Soil Classification

To classify a soil, tests are performed according to the American Society for Testing and Materials (ASTM) standards, and the results of these tests are used in a classification system recommended by the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The tests are the sieve analysis, the hydrometer analysis, and the Atterberg limits. The classification system is called the Unified Soil Classification System (USCS).

4.1 SIEVE ANALYSIS

Sieve analysis is used for the classification of gravels and sands, which are coarse-grained soils. It consists of taking a given weight of dry soil, breaking the clumps of soil down to individual particles (using a mortar and rubber-tipped pestle), washing the soil through the smallest sieve (sieve #200), drying what remains on the sieve #200, and then sieving that remainder by shaking it through a stack of sieves of decreasing openings (Figure 4.1), the last one being a retaining pan. Recording all the weights involved during this process leads to the percent of soil finer than a given particle size by weight versus the particle size; this is the particle size distribution curve (Figure 4.2).

A typical set of sieve numbers and sieve openings is given in Table 4.1. The sieve number corresponds to the number of openings per 25 mm. For example, the no. 200 sieve—the smallest sieve commonly used—has 200 openings per 25 mm; however, each opening is not equal to 25 mm divided by 200 because of the thickness of the wire between openings. In fact, the opening of the no. 200 sieve is 0.075 mm. This opening corresponds to the boundary between sand- and silt-size particles; this is why sieve analysis is limited to the classification of gravels and sands.

The sieve analysis proceeds as follows. First, each sieve is weighed empty. Then the dry soil sample is weighed, soil clumps are broken down, and the soil sample is placed on a sieve #200. The sample is washed under a gentle stream of water and the soil left on the sieve #200 is dried in the oven. The purpose is to wash out the fine particles that may adhere to the larger particles or form clumps. Sieves are stacked in order of increasing opening, with the largest-opening sieve at the top. The dry soil is placed on the top sieve, which is then covered so that no soil is ejected during shaking. The stack is shaken in a vibrator for a given period of time. At the end of shaking, each sieve is weighed with the soil retained on it.

Because the total weight of the dry sample is known, the proportion of the soil sample on each sieve is calculated as the weight of that sieve plus soil minus the weight of the empty sieve divided by the total weight of the sample. With this data, the particle size distribution curve can be obtained. This curve is a plot of the percent finer by weight (sum of the weight of soil passing a certain sieve divided by the total weight of the sample, expressed as a percentage) on the vertical axis and the sieve opening taken as the particle size on the horizontal axis (log scale). Figure 4.3 shows the sieves and the dry weight retained on each sieve. The sieve analysis calculations are shown in Table 4.2. Figure 4.4 gives examples of particle size distribution curves. Note that the particle size determined by sieving through a given sieve is the second largest dimension of the particle that can pass through the sieve opening.

Figure 4.1 Stack of sieves and shaker.

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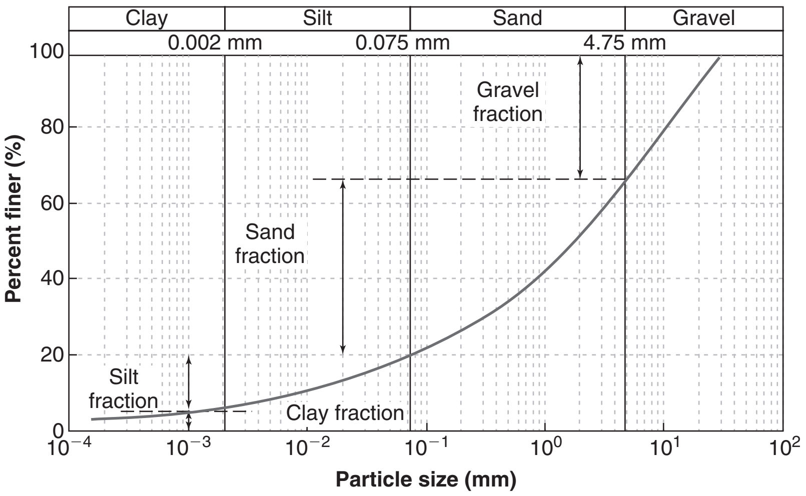

Figure 4.2 Particle size distribution curve.

Table 4.1 Sieve Numbers and Sieve Openings

Sieve Number	Sieve Opening
#1	25.4 mm
#4	4.75 mm
#10	2 mm
#20	0.85 mm
#40	0.425 mm
#80	0.18 mm
#200	0.075 mm

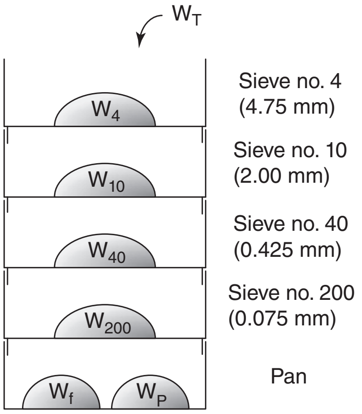

Figure 4.3 Dry weight retained on a stack of sieves.

4.2 HYDROMETER ANALYSIS

A *hydrometer* is an instrument made of glass (Figure 4.5) with a graduated stem on top of a bulb ballasted with lead beads so that it can float upright. It is used to measure the ratio of the density of the liquid in which it is immersed over the density of water. This ratio is read at the liquid surface (level of flotation of the hydrometer) on the graduated scale placed on the stem of the hydrometer. If the liquid being tested is very dense, the hydrometer does not sink very deep into the liquid, and vice versa. Therefore, the higher ratios are at the bottom of the stem.

Hydrometer analysis is used to obtain the particle size distribution curve of fine-grained soils: silts and clays. The

Table 4.2 Sieve Analysis Calculations

Initial weight of dry soil	W_t
Weight of dry soil retained on #200 after washing through #200	W_t (washed)
Weight of dry soil washed through #200	$W_{\text{fines}} = W_t - W_t$ (washed)
Dry weight retained on #4	W_4
Dry weight retained on #10	W_{10}
Dry weight retained on #40	W_{40}
Dry weight retained on #200	W_{200}
Dry weight retained on bottom pan	W_{p}
Percent finer than #4 (4.75 mm)	$((W_{10} + W_{40} + W_{200} + W_{\text{p}} + W_{\text{fines}})/W_{\text{t}}) \times 100$
Percent finer than #10 (2 mm)	$((\text{W}_{40} + W_{200} + W_p + W_{\text{fines}})/W_t) \times 100$
Percent finer than #40 (0.425 mm)	$((\text{W}_{200} + W_p + W_{\text{fines}})/W_t) \times 100$
Percent finer than #200 (0.075 mm)	$((W_p + W_{\text{fines}})/W_t) \times 100$

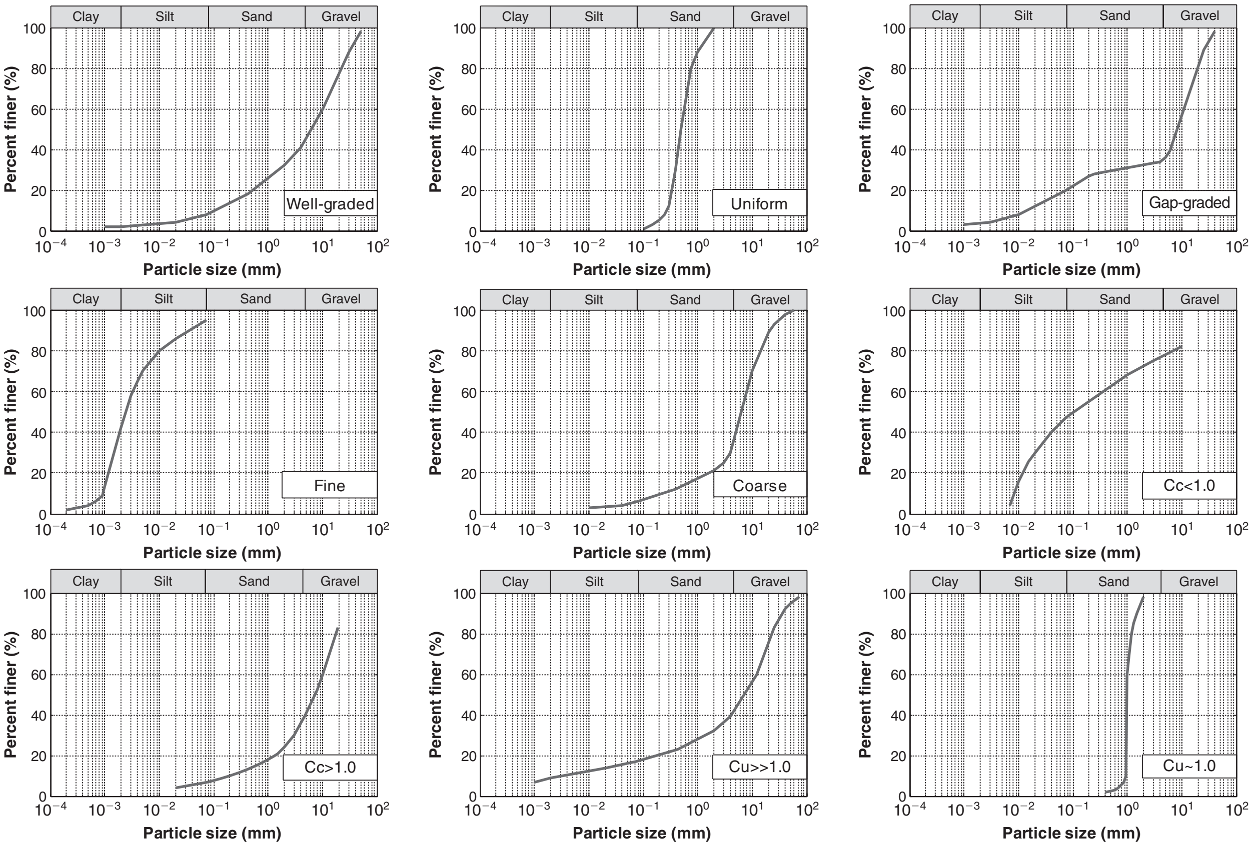

Figure 4.4 Examples of particle size distribution curves.

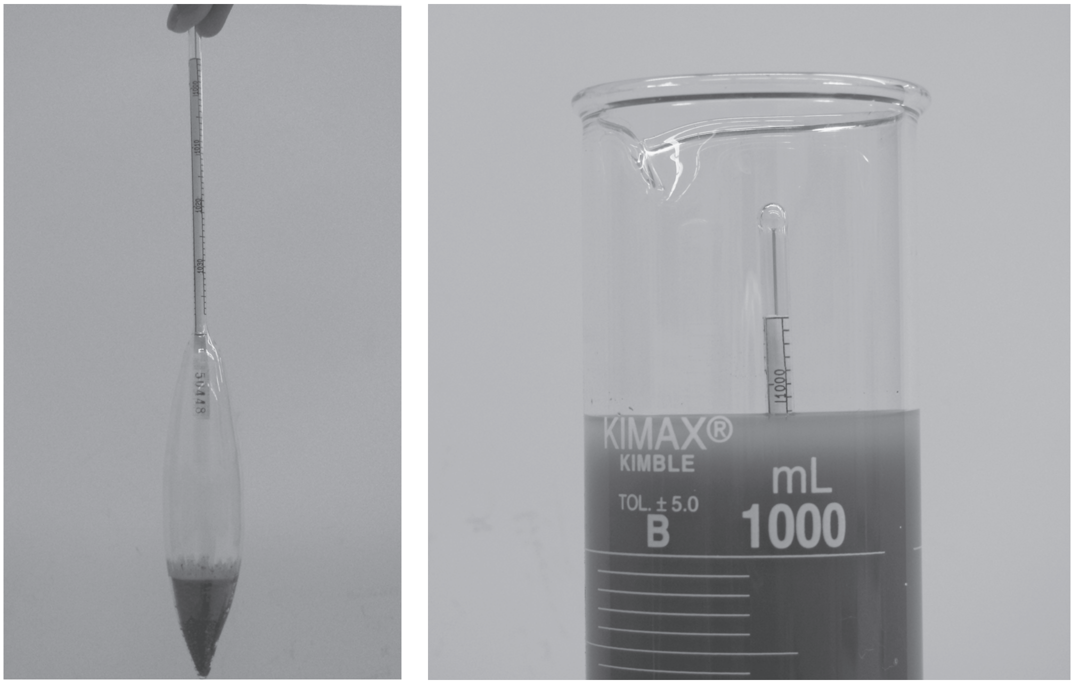

Figure 4.5 Hydrometer and hydrometer reading.

test consists of taking a given weight of dry soil, breaking it down into individual particles if clumps exist, mixing it with a dispersing agent (liquid), placing the wet mixture in a graduated cylinder, filling the container with water up to a known volume, shaking the cylinder to reach a uniform mixture, letting the soil particles settle, and recording the fall velocity at which the particles settle. The dispersing agent is used to ensure that the fine particles remain individually separated and do not form clusters. The fall velocity is obtained by measuring the unit weight of the soil-water mixture at a given depth z and at a given time t with a hydrometer. This unit weight decreases with time as the particles settle to the bottom of the container (Figure 4.6).

George Stokes was a British mathematician and physicist who made important contributions in fluid dynamics in the mid-1800s. Stokes's law relates the diameter of a sphere to its fall velocity in a liquid:

$$v = \left(\frac{\gamma_s - \gamma_f}{18\mu}\right) D^2 \tag{4.1}$$

where v is the fall velocity of the sphere, γ_s is the unit weight of the sphere, γ_f is the unit weight of the fluid (soil plus water), μ is the viscosity of the liquid, and D is the sphere diameter.

The depth z below the surface corresponding to the hydrometer reading ($r = \gamma_f/\gamma_w$) is the depth to the center of gravity of the hydrometer. At a time t after the beginning of the test, the smallest particles (equivalent spheres) which just passed the depth z have fallen at the velocity $v = z/t$. Knowing this velocity, plus the viscosity of water at the right temperature (e.g., 10^{-3} N.s/m^2 at 20°C), the unit weight of the sphere, and the unit weight of the liquid at time t as measured by the hydrometer, one can obtain the diameter D of this smallest particle (equivalent sphere) from Equation 4.1. The unit weight of the sphere is the unit weight of the soil particle (~26 kN/m^3 for mineral particles). The particle size determined by the hydrometer analysis is therefore the diameter D of a sphere made of the same material as the particle and falling at the same velocity as the particle.

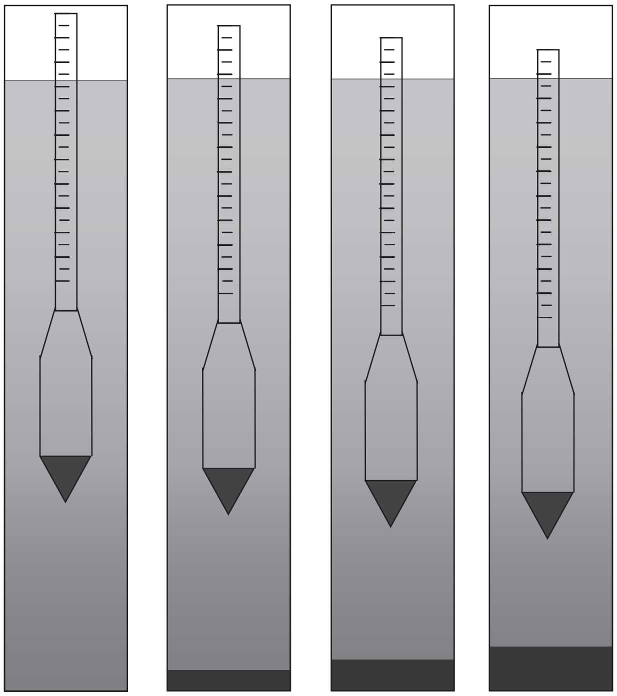

Figure 4.6 Different stages of the hydrometer analysis.

Because the purpose of hydrometer analysis is to obtain the particle distribution curve, it is now necessary to obtain the percent finer P associated with the sphere diameter D. The unit weight measured by the hydrometer γ_f can be expressed as follows:

$$\gamma_f = \frac{W_{s(<D)} + \gamma_w(V - V_{s(<D)})}{V} \tag{4.2}$$

where $W_{s(<D)}$ is the weight of particles finer than the particle size D, γ_w is the unit weight of water, V is the total volume involved in the hydrometer measurement, and $V_{s(<D)}$ is the volume of particles finer than the particle size D within the volume V.

But

$$V_{s(<D)} = \frac{W_{s(<D)}}{\gamma_s} = \frac{PW_s}{G_s\gamma_w} \tag{4.3}$$

where γ_s is the unit weight of solids (~26 kN/m^3 for mineral particles), P the percentage by weight of particles finer than the particle size D, and G_s is the specific gravity of the particles.

Therefore

$$\gamma_f = \gamma_w + \frac{(G_s - 1)W_sP}{G_sV} \tag{4.4}$$

and

$$P = \frac{G_sV}{(G_s - 1)W_s}\gamma_w(r - 1) \tag{4.5}$$

where V is taken as the volume of water in the graduated cylinder (usually 1000 cubic centimeters), W_s is the total weight of dry soil placed into the cylinder, and r is the hydrometer reading ($r = \gamma_f/\gamma_w$). Hydrometer readings are taken at various times as the particles fall through the water column; the particle size D and associated percent finer P are calculated from these readings. This gives several points on the particle distribution curve.

Using the results of the hydrometer analysis, the particle size distribution curve can be obtained for particles ranging from 0.075 mm down to at least 0.001 mm (Figure 4.7). The hydrometer curve can be added to the sieve analysis curve so as to generate a curve from 0.001 mm up to 10 mm particle size. Note that there may be a discontinuity at the 0.075 mm size, as the sieve analysis and the hydrometer analysis do not strictly measure the same particle size (as explained earlier). The second largest dimension is measured in the sieve

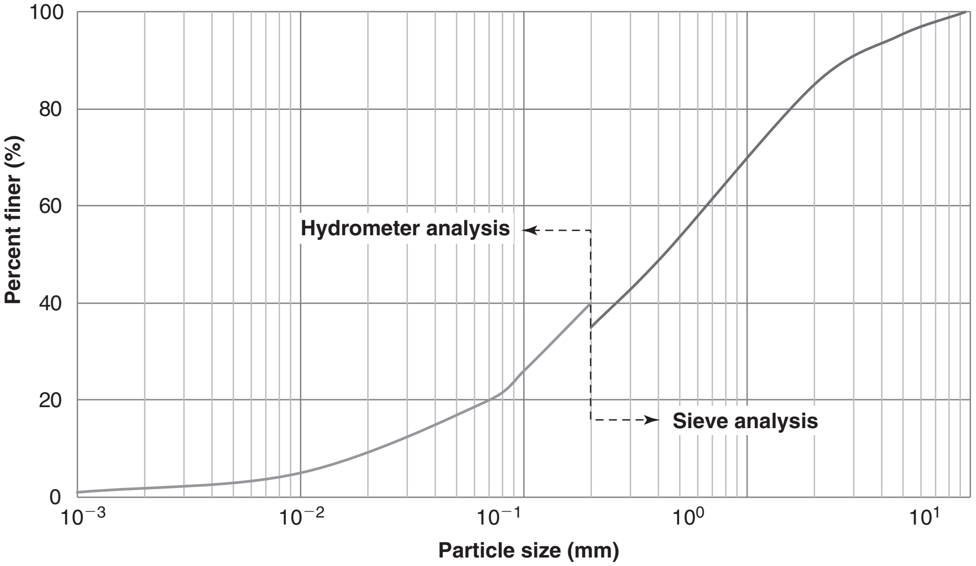

Figure 4.7 Combined sieve analysis and hydrometer analysis results.

analysis while the equivalent sphere diameter is measured in the hydrometer analysis. A discontinuity could also be due to other factors, such as the dispersing agent not working and fine particles clustering, thereby yielding a higher percentage of larger particles.

4.3 ATTERBERG LIMITS AND OTHER LIMITS

As mentioned earlier, particle size is not the main factor controlling the behavior of silts and clays. Instead, the behavior and therefore the classification are based on the ability of the soil to be deformed and stay together (*consistency*). This is measured by the *Atterberg limits*. Albert Atterberg was a Swedish chemist who worked in the field of agricultural science; he came up with what is now known as the Atterberg limits around 1910 as a means of classifying fine-grained soils. The Atterberg limits are water contents of remolded fine-grained soil. The limit tests are performed on remolded samples of silts or clays or more generally on the portion of a sample finer than sieve #40 (0.425 mm opening). These limits indicate the points at which the consistency of a fine-grained soil (Figure 4.8) changes from a liquid state to a plastic state (liquid limit), from a plastic state to a semisolid state (plastic limit), and from a semisolid state to a solid state (shrinkage limit).

The liquid limit w_L has a precise ASTM definition (ASTM D4318; ASTM 2004a). In short, it is the water content at which the remolded soil behaves like a soft paste (toothpaste consistency). This particular water content varies significantly depending on how fine the particles are. For example, very fine clay particles can have liquid limits approaching 100%, whereas silt particles may have liquid limits of around 30%. More precisely (Figure 4.9), the *liquid limit* is defined as the water content at which the two sides of a small amount of soil placed in a standard cup and grooved by a standard tool will flow together over a distance of 12.5 mm when hit by 25 blows in a standard liquid limit apparatus.

The plastic limit w_P also has a precise ASTM definition (ASTM D4318; ASTM 2004a). In short, it is the water content at which the remolded soil behaves like a hard paste (soft caramel). More precisely (Figure 4.10), the *plastic limit* is the water content at which a soil will begin to crumble when rolled into a thread 3.2 mm in diameter. The difference between the liquid limit and the plastic limit is the *plasticity index* or I_p. The plasticity index has been found to be related to a number of useful soil properties.

The shrinkage limit w_S is defined in ASTM D4943 (ASTM 2004b). The *shrinkage limit* is the water content corresponding to the amount of water necessary to fill all the voids of the dry soil after shrinkage. It is close to the lowest water content at which the remolded soil is still saturated during a drying process; any further drying leads to a degree of saturation less than 100%. The test consists of remolding the soil to a water

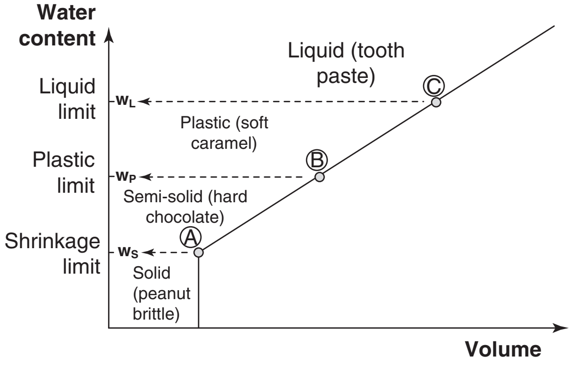

Figure 4.8 States of consistency and Atterberg limits.

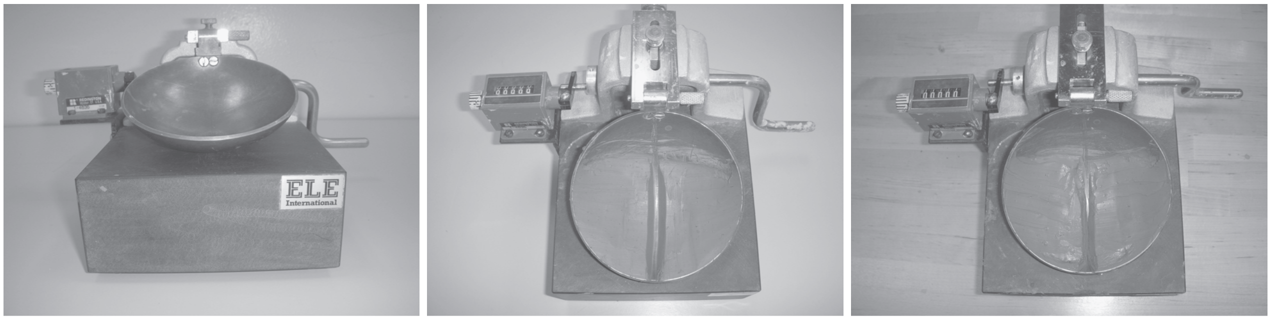

Figure 4.9 Liquid limit apparatus and test.

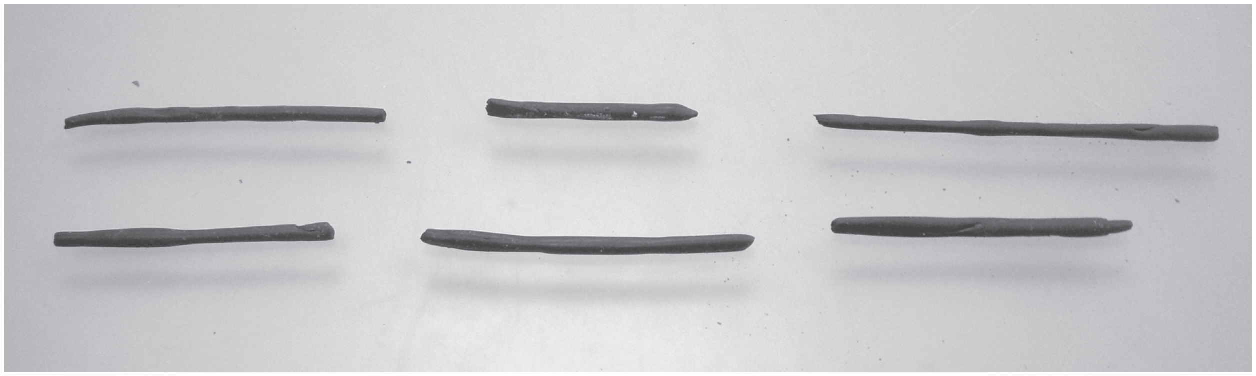

Figure 4.10 Plastic limit test and soil threads.

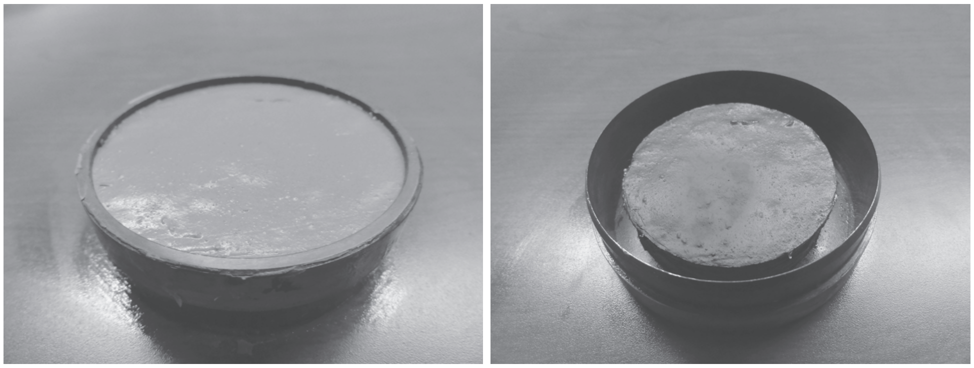

Figure 4.11 Shrinkage limit equipment and test.

content, w_o, above the shrinkage limit and filling a small cup of known volume, V_o, with the soil paste (Figure 4.11). The cup is weighed empty and then with the wet soil in it. The soil in the cup is then left to dry until it no longer shrinks. At this point, the cup plus dry soil is weighed and the weight of the dry soil is obtained (W_d). The dry sample is attached to a thread, dipped in hot wax, and pulled out. Once the film of wax now covering the sample has hardened, the sample is plunged into a graduated cylinder with water in it. The volume of the sample plus wax is measured by water displacement (V_{d+w}). The wax is removed and weighed; knowing the unit weight of the wax, the volume of wax V_w is calculated. The shrinkage limit is then:

$$w_S = w_o - \frac{(V_o - V_{d+w} + V_w)\gamma_w}{W_d}$$

where γ_w is the unit weight of water.

The liquid limit can also be determined by the fall-cone method (Figure 4.12). This method, developed in the early 1900 in Sweden, is now used in other countries such as France and the UK. In this test, a standard cone is brought to barely touch the surface of the soil and is released suddenly. The cone with a mass M and an apex angle θ penetrates into the soil a distance d. The liquid limit is reached when a

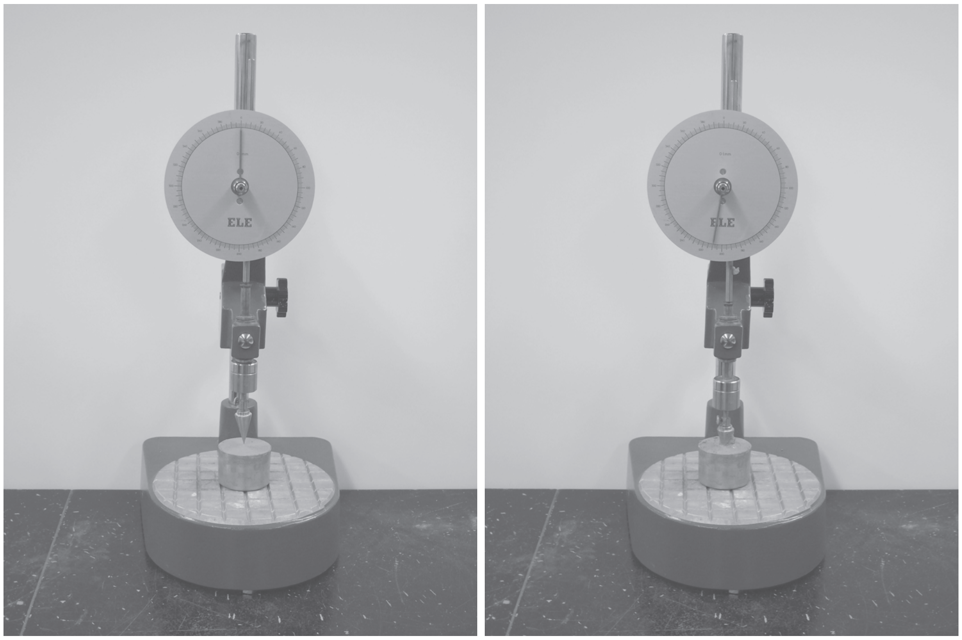

Figure 4.12 Fall-cone test for Atterberg limits.

Table 4.3 Fall-Cone Parameters for Liquid Limit Determination

Country	Cone mass	Cone apex angle	Cone penetration
Sweden	60 g	60°	10 mm
United Kingdom	80 g	30°	20 mm
France	80 g	30°	17 mm

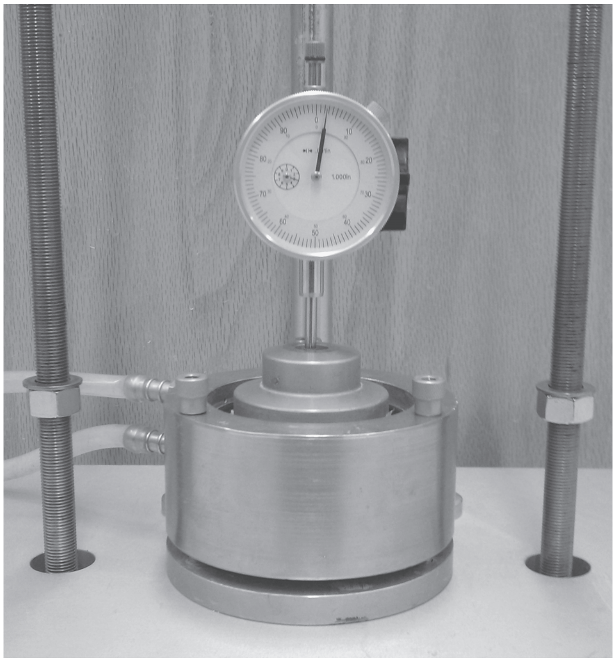

Figure 4.13 Free swell test for swell limit of undisturbed sample.

chosen value of d is obtained. Table 4.3 shows the values of M, θ, and d in different countries. There is as yet no ASTM standard for the fall-cone test.

The plastic limit can also be determined by the fall-cone method, but in this case the distance d is much smaller and a value of 2.2 mm seems to be appropriate.

Although not associated with the Atterberg limits, the swell limit w_{SW} is important as well. It is defined as the water content at which a soil submerged in water can no longer absorb water. The test consists of placing a soil sample in a snug-fitting cylindrical container (Figure 4.13), inundating the soil, and measuring the vertical swell movement as a function of time. When the swelling stops, the water content is measured; this gives the soil's swell limit. This test is called a *free swell test* because no pressure is applied on top of the sample. Note that in this case the sample is undisturbed, whereas the Atterberg limits are performed on remolded samples.

Associated with the undisturbed-sample swell limit w_{SW} is the undisturbed-sample shrinkage limit w_{SH}. This shrinkage limit is obtained by performing a free shrink test (Figure 4.14). A sample of soil is trimmed in a cylinder, its dimensions are measured, and it is weighed. The initial volume V_o and the initial weight W_o are recorded. The sample is then left to dry while the dimensions and the weight are measured as a function of time. This gives the volume V(t) and weight W(t). When the sample is air-dried, it is placed in the oven to obtain the oven dry weight W_s. The average water content

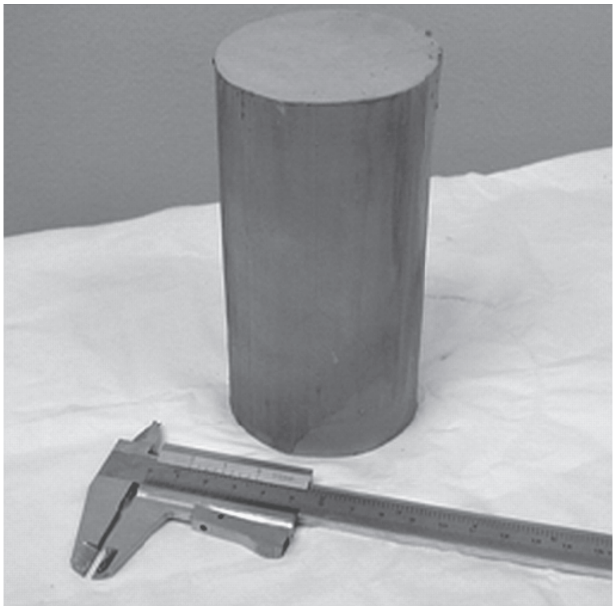

Figure 4.14 Free shrink test for shrinkage limit of undisturbed sample.

Table 4.4 Summary of Water Content Limits

Atterberg liquid limit	w_L	Remolded soil, toothpaste consistency
Atterberg plastic limit	w_P	Remolded soil, soft toffee consistency
Atterberg shrinkage limit	w_S	Remolded soil, hard chocolate consistency
Swell limit	w_{SW}	Undisturbed soil, maximum natural water content
Shrinkage limit	w_{SH}	Undisturbed soil, highest water content at which further drying yields no more shrinkage

of the sample at any time during the test is (W(t)–Ws)/Ws. A graph of the water content versus relative change in volume is plotted. The undisturbed-sample shrinkage limit w_{SH} is the water content corresponding to the point where the sample first stops decreasing in volume (point A on Figure 4.8). Sometimes point A is not clearly definable, particularly for high-plasticity soils (Figure 4.15), but there is always a distinct change in slope around the shrinkage limit. Table 4.4 summarizes the water content limits for soils.

4.4 CLASSIFICATION PARAMETERS

A number of reference particle sizes are determined from the particle size distribution curve. The parameter D_{50} is the particle size corresponding to a percent finer equal to

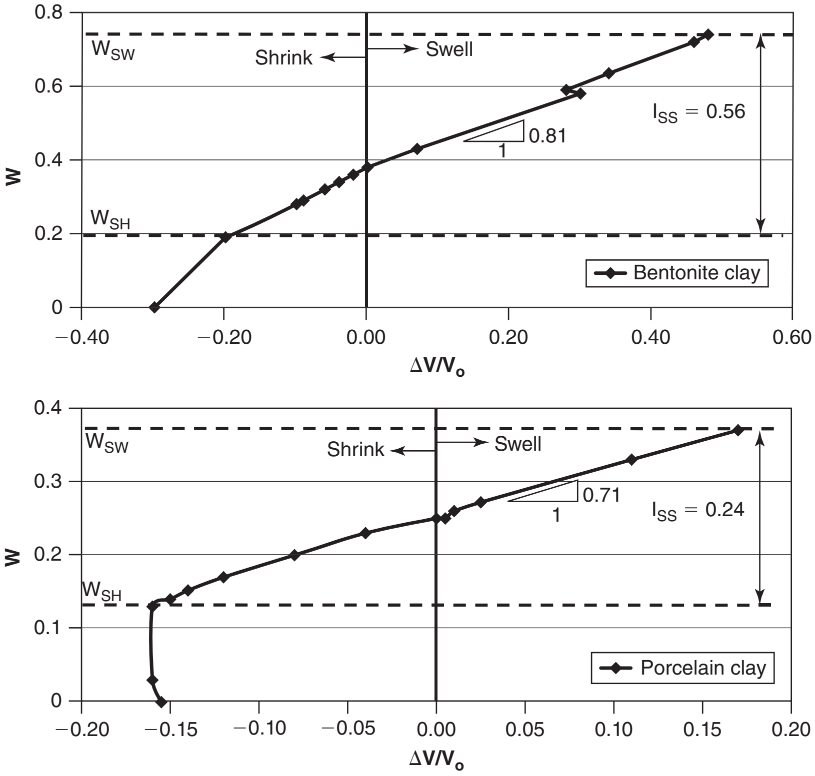

Figure 4.15 Shrink-swell test for porcelain clay and bentonite clay.

50 percent, D_{10} is the particle size corresponding to a percent finer equal to 10 percent, and so on. D_{10}, D_{30}, and D_{60} are used to calculate the coefficient of uniformity C_u and the coefficient of curvature C_c, as shown in Table 4.5.

Figure 4.4 illustrates some of these parameters. A coefficient of uniformity C_u close to 1 indicates that most of the particles in the soil have the same size. If C_u is large—say, larger than 6—then the soil contains particles that cover a wide range of sizes. If C_u is small, then the soil may be quite uniform with many particles of similar sizes. If the coefficient of curvature C_c is less than 1, then the particle size distribution has a downward curvature; if it is more than 3, then the particle size distribution has an upward curvature. If C_c is between 1 and 3, the particle size distribution curve will be reasonably straight and the soil is likely to contain particles with a wide range of sizes. Both C_u and C_c are used to classify coarse-grained soils. The size D_{50} is used extensively in erosion studies as a parameter that correlates well with the velocity at which a coarse-grained soil starts to erode. D_{15} and D_{85} are used in filter design for earth dams and other water-retaining structures. The particle size distribution curve is simple to obtain but very useful in geotechnical engineering as shown by these various applications.

A number of indices are determined from the Atterberg limits. The *plasticity index* I_P is the difference between the liquid limit w_L (quoted as a percent) and the plastic limit w_P (also quoted as a percent). The *shrinkage index* I_S is the difference between the plastic limit and the shrinkage limit, both in percent. The *liquidity index*, I_L, is defined in Table 4.5 and is quoted as a ratio or a percent; it indicates the relative position of the natural water content between the plastic limit and the liquid limit. The *shrink-swell index*, I_{SS}, is the difference between the undisturbed swell limit w_{SW} and the undisturbed shrinkage limit w_{SH}. It is very useful as an indicator of the shrink-swell potential of a soil. Other but less used indices are defined in Table 4.5.

Activity, A_c, is another parameter that helps describe a soil. This parameter is used for fine-grained soils, and is defined as the ratio between the plasticity index and the percent finer than 0.002 mm:

$$A_c = \frac{I_P}{\%\ finer\ than\ 0.002\ mm} \tag{4.6}$$

The values of A_c vary from less than 0.75 for relatively inactive soils (kaolinite) to more than 1.25 for very active soils (montmorillonite).

Table 4.5 Classification Parameters Definitions

Parameter Symbol	Name	Definition	Applications
D_X		Particle size corresponding to X% finer	Filter design, erosion of coarse-grained soils
C_u	Coefficient of uniformity	$C_u = \frac{D_{60}}{D_{10}}$	Classification of soils
C_c	Coefficient of curvature	$C_c = \frac{(D_{30})^2}{D_{60}D_{10}}$	Classification of soils
I_P	Plasticity index	$I_P = w_L - w_P$	Shrink-swell soil, fill specifications, correlations
I_{SS}	Shrink-swell index	$I_{SS} = w_{SW} - w_{SH}$	Shrink-swell potential
I_S	Shrinkage index	$I_S = w_P - w_S$	
I_L	Liquidity index	$I_L = \frac{(w - w_P)}{(w_L - w_P)}$	Correlations
I_C	Consistency index	$I_C = \frac{(w_L - w)}{(w_L - w_P)}$	
I_F	Flow index, slope of the water content vs. lg of number of blows in the liquid limit test	$I_F = \frac{(w_1 - w_2)}{(\lg N_1 - \lg N_2)}$	
I_T	Toughness index	$I_T = \frac{I_P}{I_F}$	
Ac	Activity	$A_c = \frac{I_P}{\%\ finer\ than\ 0.002\ mm}$	

4.5 ENGINEERING SIGNIFICANCE OF CLASSIFICATION PARAMETERS AND PLASTICITY CHART

The plasticity index I_P is definitely the index most used in practice, with the liquidity index a distant second. The others are rarely used. The I_P essentially relates to how small the clay particles are in the soil: the higher I_p is, the smaller the clay particles are. Table 4.6 shows the range of values that can be expected for common soils. Very high I_P values (60 or more) are associated with a predominance of very small clay particles, such as in montmorillonite; low I_P values (20 or less) are associated with a predominance of larger clay particles, such as in kaolinite. Thus, the I_P value gives an indication of some important properties of a soil. For example, a high I_P value indicates a soil that will be very difficult to compact, has a high shrink-swell potential, and has low permeability. A low I_P value is often required for fill material when good drainage is important, such as for pavement layers and retaining walls backfill.

From the point of view of soil strength, the friction between particles decreases with increasing I_P. Also, a comparison between the natural water content and the limits can give an indication of possible soil behavior. For example, if the natural water content is higher than the liquid limit, the soil is likely to be sensitive (it may lose significant strength when remolded). If the soil has a water content close to the shrinkage limit and a high shrink-swell index, beware of swelling problems if the soil can get wet.

The plasticity chart was developed by Arthur Cassagrande, an Austrian-born American civil engineer, around 1932. The plasticity chart is a plot of the plasticity index versus the liquid limit of a soil (Figure 4.16), and is used for the purpose of classifying fine-grained soils according to their plasticity. The A line is an empirically chosen line that splits the chart between clays above the A line and silts below the A line. The vertical line, corresponding to a liquid limit equal to 50%, separates high-plasticity fine-grained soils ($w_L > 50$) from low-plasticity fine-grained soils ($w_L < 50$). To classify a soil, the plasticity index and liquid limit of that soil are plotted on the chart; the region in which the point falls indicates what type of fine-grained soil it is or what kind of fines are encountered in a coarse-grained soil. The plasticity chart is the basis for the classification of fine-grained soils and of the fines fraction of coarse-grained soils.

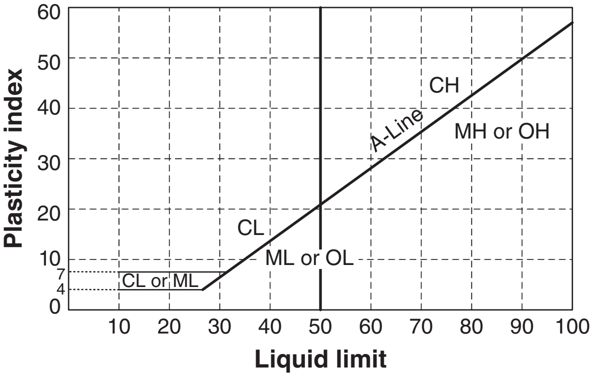

Figure 4.16 Plasticity chart.

Table 4.6 Range of Values for Atterberg Limits and Some Indices

Parameter	Low	Medium	High
Liquid limit	10–40	40–80	>80
Plastic limit	10–20	20–30	>30
Shrinkage limit	5–15	10–20	>20
Plasticity index	0–20	20–50	>50
Shrink-swell index	0–25	25–60	>60

4.6 UNIFIED SOIL CLASSIFICATION SYSTEM

The Unified Soil Classification System, or USCS, is the system used internationally to classify soils. Most commonly, it employs a two-letter symbol. The first letter indicates whether the soil is gravel (G), sand (S), silt (M), or clay (C). The letter for silt could not be S, as that letter was already used for sand, so the letter M was chosen; in Swedish *mjäla* means silt. The second letter gives additional information on the soil. For coarse-grained soils, the second letter can be M or C, indicating that the gravel or sand has a significant amount of silt or clay particles in it. For coarse-grained soils, the second letter can be W or P. W indicates that the gravel or sand is clean and well graded, meaning that all particle sizes are more or less represented. P indicates that the gravel or sand is clean and poorly graded, meaning that not all particle sizes are represented. For fine-grained soils, the second letter can be H, meaning high plasticity (high liquid limit and high I_p), or L for low plasticity (low liquid limit and low I_p).

An SC would be a soil with the majority of its particles in the sand-size range and Atterberg limits of the portion smaller than 0.425 mm, consistent with the Atterberg limits of clay. A GP would be a soil with the majority of its particles in the gravel-size range and poorly graded. An ML would be a low-plasticity silt, based on its Atterberg limits, and a CH would be a high-plasticity clay, again based on its Atterberg limits. The USCS two-letter symbols are understood throughout the world and help geotechnical engineers communicate with each other regardless of their native languages.

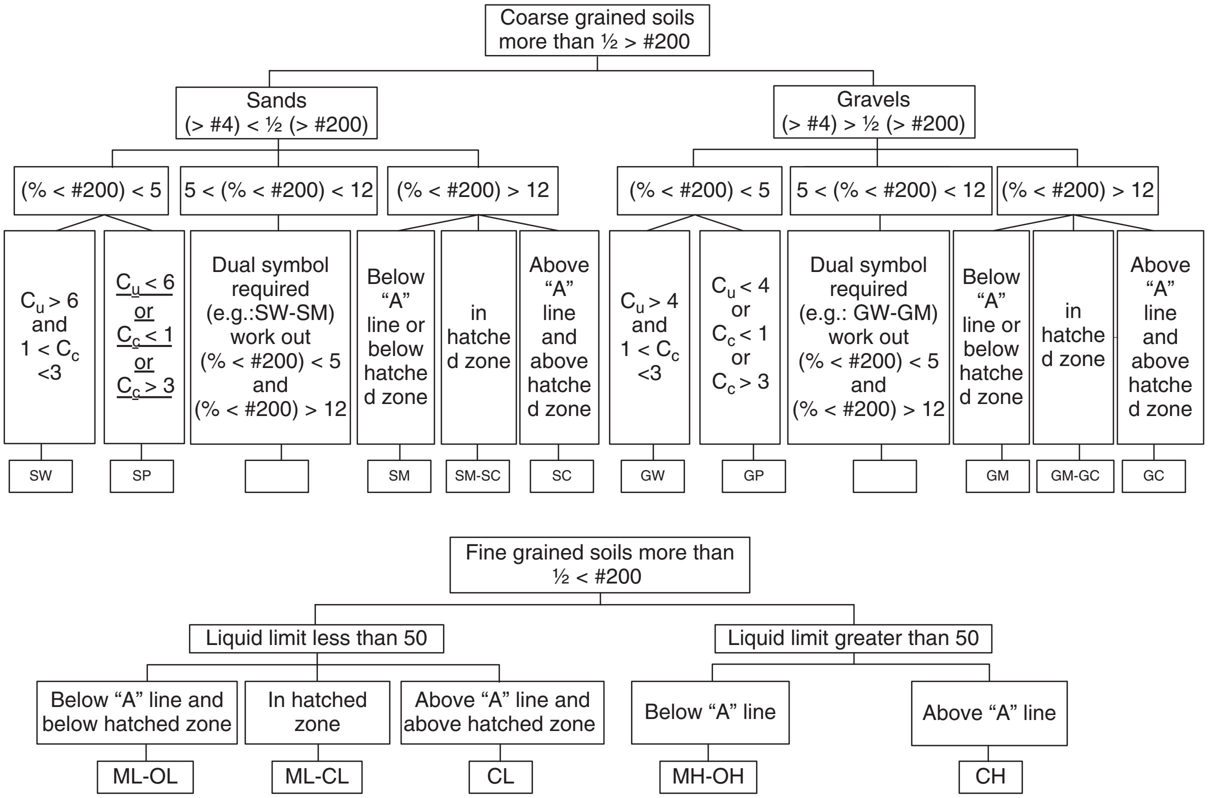

Figure 4.17 Flowchart to classify a soil by the USCS.

The exact process for classifying a soil consists of a series of steps organized in a decision tree as shown in Figure 4.17. The first decision is based on the percent passing the no. 200 sieve (#200), which has an opening of 0.075 mm. If the soil has more than 50% particles by weight larger than 0.075 mm (#200), the soil is a coarse-grained soil. If the soil has more than 50% by weight smaller than 0.075 mm (#200), the soil is a fine-grained soil. For coarse-grained soils, if the percent by weight of the gravel-size particles is larger than the percent by weight of the sand-size particles, the soil is a gravel and the first letter is G. If not, the soil is a sand and the first letter is S.

The second letter for a coarse-grained soil is W, P, M, or C. If the soil has less than 5% passing #200, it is clean and the second letter will be W or P, depending on the coefficient of uniformity C_u and the coefficient of curvature C_c obtained from the particle size distribution curve. If the coarse-grained soil has more than 12% passing #200, the soil is dirty and the second letter will be M or C, depending on the Atterberg limits of the portion smaller than 0.425 mm; M will be selected if the soil plots below the A line on the plasticity chart and C if it plots above. If the percent passing #200 is between 5% and 12%, then a dual symbol will be required, as the soil is intermediate between clean and dirty. In this instance, the classification for the <5% case and the >12% case are obtained and the soil ends up with a dual symbol (e.g., GP-GC or SW-SM). For fine-grained soils, the plasticity index and the liquid limit are plotted on the plasticity chart and the dual symbol is read from the quadrant of the chart where the point is situated.

PROBLEMS

4.1 Calculate the thickness of the wire in the no. 200 sieve.

4.2 A dry sample of soil weighs 5 N. It is shaken on a set of sieves: No. 4 (4.75 mm), No. 40 (0.425 mm), No. 200 (0.075 mm), and a pan. The weight retained on No. 4 is 2 N, on No. 40 is 1.5 N, and on No. 200 is 1 N. Calculate:

- The percent of coarse grain size particles by weight
- The percent of gravel-size particles by weight
- The percent of sand-size particles by weight
- The percent of fine grain size particles by weight
- The coefficient of uniformity and the coefficient of curvature

Based on these results, what would you call the soil?

4.3 Why is the particle size of the particle size curve plotted on a log scale? Plot the particle size curve of problem 2 as percent finer vs. particle size on a log scale and then as percent finer vs. log of particle size. Determine by calculations the position of a particle size equal to 0.075 mm and 4.75 mm on the particle size (log scale) axis and on the log of particle size axis.

4.4 Calculate how fast a particle of soil will settle in water if its equivalent diameter is 0.075 mm and then if its equivalent diameter is 0.002 mm.

4.5 A cylindrical hydrometer has a radius of 20 mm and weighs 2 N. It is lowered into water mixed with fine soil particles. If the hydrometer sinks and comes to floating equilibrium when it is 100 mm in the liquid, calculate the ratio of soil solids by volume that exists in the liquid. Assume that Gs = 2.65 if needed.

4.6 Explain the hydrometer analysis in your own words. Develop the equations necessary.

4.7 A soil has a natural water content of 22% and the following limits.

- Shrinkage limit = 13%
- Plastic limit = 25%
- Swell limit = 36%
- Liquid limit = 55%

Calculate the

- Plasticity index
- Liquidity index
- Shrink-swell index

4.8 Classify the following soils:

	S1 (% finer)	S2 (% finer)	S3 (% finer)	S4 (% finer)	S5 (% finer)
#4	52	52	63	98	100
#10	38	38	56	90	97
#40	18	18	42	47	82
#200	8	2	4	20	70
W_L	17	NP	NP	32	48
W_P	11	NP	NP	26	34

Problems and Solutions

Problem 4.1

Calculate the thickness of the wire in the no. 200 sieve.

Solution 4.1

The sieve number corresponds to the number of openings per 25 mm. For the sieve #200, the width of any opening is 0.075 mm; therefore, the total width of the openings in 25 mm of the #200 mesh is $200 \times 0.075 = 15$ mm. The total thickness of the wires in 25 mm of the #200 mesh is $(25 - 15) = 10$ mm, so the thickness of the wires in a sieve #200 is $10/200 = 0.05$ mm (about the diameter of a human hair).

Problem 4.2

A dry sample of soil weighs 5 N. It is shaken on a set of sieves: No. 4 (4.75 mm), No. 40 (0.425 mm), No. 200 (0.075 mm), and a pan. The weight retained on No. 4 is 2 N, on No. 40 is 1.5 N, and on No. 200 is 1 N. Calculate:

- The percent of coarse grain size particles by weight
- The percent of gravel-size particles by weight

- The percent of sand-size particles by weight
- The percent of fine grain size particles by weight
- The coefficient of uniformity and the coefficient of curvature

Based on these results, what would you call the soil?

Solution 4.2

The percent of coarse grain size particles is $= \dfrac{2+1.5+1}{5} \times 100 = 90\%$

The percent of gravel-size particles is $= \dfrac{2}{5} \times 100 = 40\%$

The percent of sand-size particles is $= \dfrac{1.5+1}{5} \times 100 = 50\%$

The percent of fine grain size particles is $= \dfrac{0.5}{5} \times 100 = 10\%$

	Retained soil on sieve			Passing through sieve	
	Weight (N)	Accumulated weight (N)	Accumulated weight (%)	Weight (N)	Accumulated weight (%)
No. 4 (4.75 mm)	2	2	40	3	60
No. 40 (0.425 mm)	1.5	3.5	70	1.5	30
No. 200 (0.075 mm)	1	4.5	90	0.5	10
Pan	0.5	5	100	0	0

From these results, $D_{60} = 4.75$ mm, $D_{30} = 0.425$ mm, and $D_{10} = 0.075$ mm.

$$C_u = \frac{D_{60}}{D_{10}} = \frac{4.75}{0.075} = 63$$

$$C_c = \frac{D^2{}_{30}}{D_{10} \times D_{60}} = \frac{0.425^2}{0.075 \times 4.75} = 0.5$$

Based on these results, the soil has 90% coarse fraction, therefore the soil is a coarse-grained soil; furthermore, 50% of the soil is retained between sieves #40 and #200, so the soil is sand.

Problem 4.3

Why is the particle size of the particle size curve plotted on a log scale? Plot the particle size curve of problem 2 as percent finer vs. particle size on a log scale and then as percent finer vs. log of particle size. Determine by calculations the position of a particle size equal to 0.075 mm and 4.75 mm on the particle size (log scale) axis and on the log of particle size axis.

Solution 4.3

The range of particle sizes in soils is very large, so we use the logarithmic scale because this scale stretches out the particle size distribution in the very small range. This allows us to distinguish the small sizes as well as the large sizes. Figure 4.1s shows the particle size curve as percent finer vs. particle size on a log scale. Figure 4.2s shows the particle size curve as percent finer vs. log of particle size. For the 0.075 mm particle, $\log\ 0.075 = -1.125$; this point can easily be found on the linear scale of Figure 4.2s. The position of this point is the same on the scale of Figure 4.1s. The same approach applies to the 4.75 mm particle: $\log\ 4.75 = 0.677$.

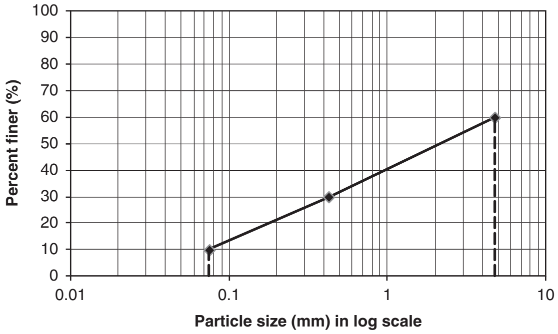

Figure 4.1s Percent finer vs. particle size on a log scale.

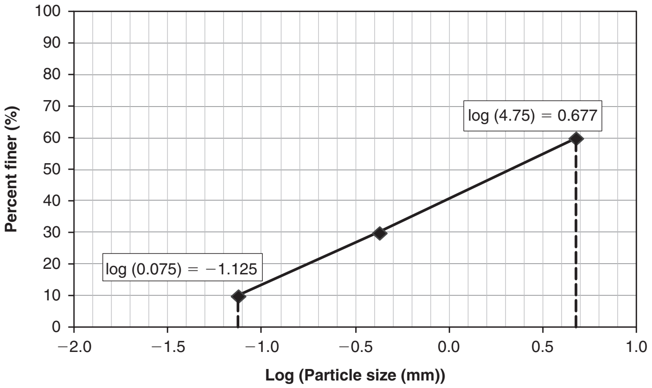

Figure 4.2s Percent finer vs. log of particle size on a normal scale.

Problem 4.4

Calculate how fast a particle of soil will settle in water if its equivalent diameter is 0.075 mm and then if its equivalent diameter is 0.002 mm.

Solution 4.4

Assume that:

- Water temperature = 20°C
- Specific gravity of particles is 2.65
- Viscosity of water is $\mu = 10^{-3}\ \mathrm{N \cdot s/m^2}$
- Unit weight of water is $\gamma_w = 9.79\ \mathrm{kN/m^3}$
- Unit weight of soil particles $\gamma_s = 2.65 \times 9.79\ \mathrm{kN/m^3} = 25.95\ \mathrm{kN/m^3}$

The fall velocity of a soil particle in water can be calculated using Stokes's law:

$$v = \left(\frac{\gamma_s - \gamma_f}{18\mu}\right) D^2$$

where $\gamma_s = 25.95\ \mathrm{kN/m^3}$ and $\gamma_f = \gamma_w = 9.79\ \mathrm{kN/m^3}$. For particles with D = 0.075 mm

$$v = \left(\frac{25.95 - 9.79}{18 \times 10^{-6}}\right) \times \left(\frac{0.075}{1000}\right)^2 = 0.0051(\mathrm{m/\,sec}) = 5.1(\mathrm{mm/\,sec})$$

For particles with D = 0.002 mm:

$$v = \left(\frac{25.95 - 9.79}{18 \times 10^{-6}}\right) \times \left(\frac{0.002}{1000}\right)^2 = 3.59 \times 10^{-6}(\mathrm{m/\,sec}) = 0.00359\,(\mathrm{mm/\,sec})$$

Problem 4.5

A cylindrical hydrometer has a radius of 20 mm and weighs 2 N. It is lowered into water mixed with fine soil particles. If the hydrometer sinks and comes to floating equilibrium when it is 100 mm in the liquid, calculate the ratio of soil solids by volume that exists in the liquid. Assume that Gs = 2.65 if needed.

Solution 4.5

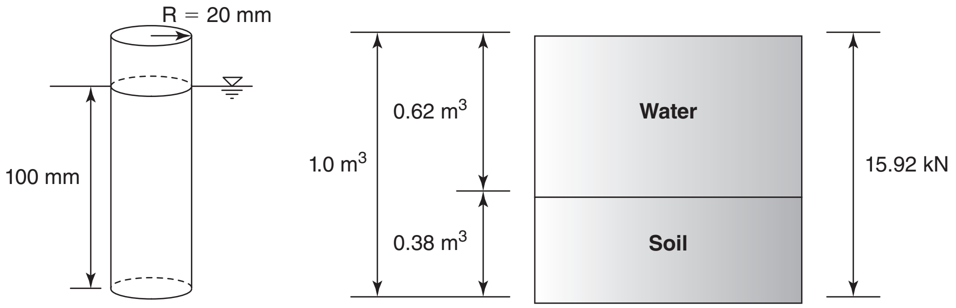

Figure 4.3s Hydrometer and three-phase diagram.

$$F_{\text{Buoyancy}} = W$$

$$V \times \gamma_{\text{mixture}} = W$$

$$\frac{\pi \times 0.04^2}{4} \times 0.1 \times \gamma_{\text{mixture}} = 2$$

$$\gamma_{\text{mixture}} = 15.92 \times 10^3 \text{ N/m}^3 = 15.92 \text{ kN/m}^3$$

Assuming 1 m^3 of the mixture and $G_s = 2.65$:

$$W_W + W_S = 15.92 \text{ kN}$$

$$V_W + V_S = 1 \text{ m}^3$$

$$\gamma_W V_W + \gamma_S V_S = 15.92 \text{ kN}$$

$$9.81 \times (1 - V_S) + (2.65 \times 9.81) \times V_S = 15.92 \text{ kN}$$

$$\therefore \quad V_S = 0.38 \text{ m}^3, \quad V_W = 0.62 \text{ m}^3$$

The volumetric percent of solids in the mixture is 38%.

Problem 4.6

Explain the hydrometer analysis in your own words. Develop the equations necessary.

Solution 4.6

See Section 4.2 in this chapter.

Problem 4.7

A soil has a natural water content of 22% and the following limits.

- Shrinkage limit = 13%
- Plastic limit = 25%
- Swell limit = 36%
- Liquid limit = 55%

Calculate the

- Plasticity index
- Liquidity index
- Shrink-swell index

Solution 4.7

- Plasticity index: PI = LL − PL = 55 − 25 = 30
- Liquidity index: LI = (*w* − PL)/PI = (22 − 25)/30 = −0.1
- Shrink-swell index: Iss = swell limit−shrinkage limit = 36 − 13 = 23

Problem 4.8

Classify the following soils:

	S1 (% finer)	S2 (% finer)	S3 (% finer)	S4 (% finer)	S5 (% finer)
#4	52	52	63	98	100
#10	38	38	56	90	97
#40	18	18	42	47	82
#200	8	2	4	20	70
w_L	17	NP	NP	32	48
w_P	11	NP	NP	26	34

Solution 4.8

The soils are classified based on the following criteria:

- Coarse grain size particles: retained on the no. 200 sieve (0.075 mm)
- Gravel-size particles: retained on the no. 4 sieve (4.75 mm)
- Sand-size particles: passing no. 4 sieve, retained on the no. 200
- Fine grain size particles: passing no. 200
- Plastic and liquid limit:

$$\text{Coefficient of uniformity } C_u = \frac{D_{60}}{D_{10}}$$

$$\text{Coefficient of curvature } C_c = \frac{D^2{}_{30}}{D_{10} \times D_{60}}$$

The particle size distribution curves are drawn on Figures 4.4s to 4.8s and theclassification of the 5 soils is presented in the Table below.

	S1 (% finer)	S2 (% finer)	S3 (% finer)	S4 (% finer)	S5 (% finer)
Sieve Opening (mm)			**Percent Finer**		
10	80	80	80	—	—
4.75	52	52	63	98	100
2	38	38	56	90	97
0.425	18	18	42	47	82
0.075	8	2	4	20	70
0.03 from hydrometer	—	—	—	—	9
Other properties					
w_L	17	NP	NP	32	48
w_P	11	NP	NP	26	34
I_p	6	NP	NP	6	14
Coarse fraction (%)	92	98	96	80	30
Fine fraction (%)	8	2	4	20	70
Gravel fraction (%)	48	48	37	2	0
Sand fraction (%)	44	50	59	78	30
D_{10} (mm)	0.11	0.19	0.098	0.036	
D_{30} (mm)	1.05	1.05	0.23	0.16	
D_{60} (mm)	7	7	3.2	0.7	
C_u	63.6	36.8	32.7	19.4	
C_c	1.4	0.8	0.2	1.0	
Classification	GW-(GC-GM)	SP	SP	SM	ML

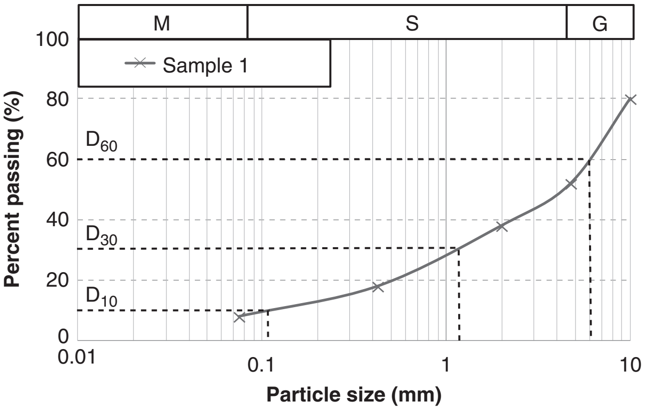

Figure 4.4s Percent finer vs. log of particle size of sample S1.

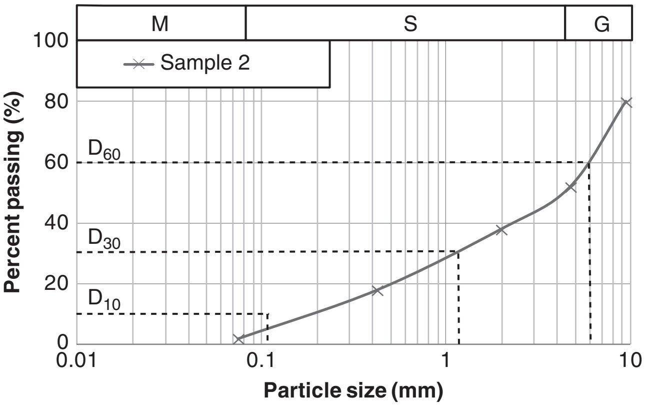

Figure 4.5s Percent finer vs. log of particle size of sample S2.

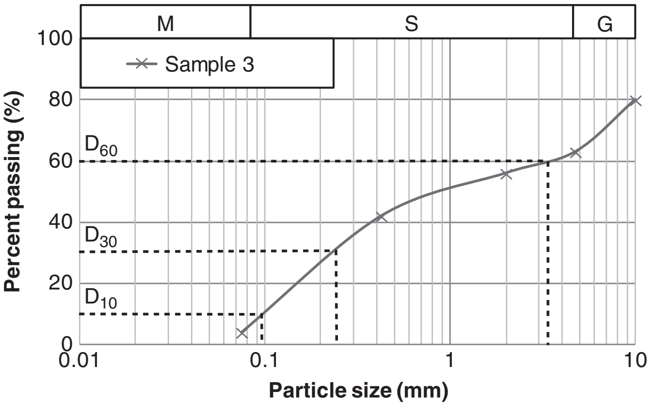

Figure 4.6s Percent finer vs. log of particle size of sample S3.

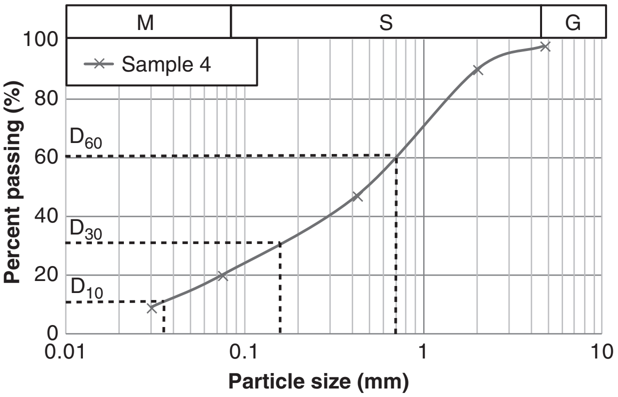

Figure 4.7s Percent finer vs. log of particle size of sample S4.

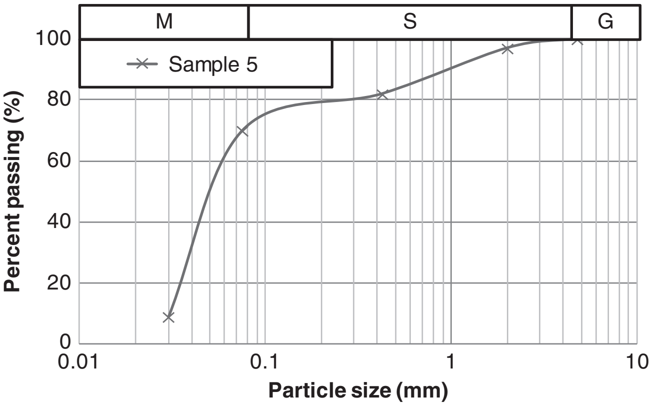

Figure 4.8s Percent finer vs. log of particle size of sample S5.