CHAPTER 92

Inflation and Price Change in Economic Analysis

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1. INTRODUCTION TO INFLATION AND DEFLATION

The economic analysis of investment alternatives generally entails the estimation of cash flows and the application of some measure of worth, such as net present value or the internal rate of return, in order to make a decision. The estimation of these cash flows requires the estimation of prices, whether they be the price of goods sold to forecast revenues or the estimation of wages to forecast labor costs. Over time these prices change. An increase in price is known as inflation, while a decrease in price is termed deflation. These concepts and their measurement are explained in this chapter. Cash flow analysis methods are revisited under the assumption of price changes, as their effects can be significant (Fleischer 1994). This is especially true when one considers after-tax cash flow analysis, as the effects of depreciation and taxes represent one of the most important aspects of investment analysis (Park and Sharp-Bette 1990).

1.1. Inflation, Deflation, and Purchasing Power

Consider the price of something as simple as a stamp. In 1967, a typical first class postage stamp cost just 5 cents in the United States (Park 1997). In 1999, that same stamp (in that it provided the same service as in 1967) cost 33 cents. So if you had tucked away a nickel in 1967 to buy a stamp in 1999, you would be out of luck. This is not the case with all commodities, as competition and improved manufacturing efficiencies may actually lead to decreases in the price of a good or commodity over time. Consider the average price of a one-minute long distance (interstate) call. In 1970, it cost roughly $0.25 cents per minute to place a call between states in the United States. Now, prices average in the neighborhood of $0.08 per minute (Festa 1999). This effect is known as deflation.

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Although it is rare, it has happened with numerous commodities in the last decade, mainly due to deregulation and increased competition.

Changes in price affect one's purchasing power. When inflation occurs, the worth or value of money decreases in that one cannot buy as much commodity with the same amount of money as earlier. This is known as a decrease in purchasing power. In the case of deflation, the same amount of money purchases more commodity than previously possible and thus results in an increase in purchasing power.

Because investment alternatives are generally modeled as cash flows over some time horizon, it is important to include the effects of inflation in economic equivalence calculations (Thuesen and Fabrycky 1994). This chapter illustrates measures of inflation and how to include it in cash flow analysis calculations. For the remainder of this chapter, the term *inflation* is utilized exclusively and *deflation* refers to negative inflation.

1.2. Measures of Inflation

The movement of prices is tracked in a variety of ways for a variety of products, generally with the use of a price index. A price index is merely a measure of the change in price of a commodity relative to some baseline. This is generally taken as a ratio of the price at some point in time to a price at some earlier point in time. In the United States, the Consumer Price Index (CPI) is a popular method to measure inflation associated with the cost of living. The CPI tracks the movement in price of a variety of commodities and services, including food and beverages, housing, apparel, transportation, medical care, recreation, education and communication, and energy (Bureau of Labor Statistics 1999b). While the CPI tracks this bundle of commodities and services, indexes also exist for the individual commodities and services that comprise the CPI. For example, there is a conglomerate index for transportation that is composed of indexes for private transportation and public transportation. Private transportation is composed of indexes for new and used motor vehicles, gasoline, motor vehicle parts and equipment, and motor vehicle maintenance and repair. These indexes are generally available on a regional basis and/or seasonally adjusted.

For the CPI, the baseline is given at 100 for the year 1967. The CPI for the end of 1998 was 491.0, and at the end of November 1999 the index stood at 504.1 (Bureau of Labor Statistics 1999a).

While the CPI is very popular, it is not the only price index available to those in need of estimating inflation for a specific service or commodity. The Bureau of Labor Statistics publishes a number of price indexes that may or may not influence the CPI (Standard & Poor's Statistical Service 1999). To provide some semblance of the range of available data, the Bureau of Labor Statistics provides producer price indexes for selected lumber, including hardwoods, southern pine, oak, softwoods, and Douglas fir. Breakdowns to such fine detail are available for a variety of commodities. Indexes are also available for industry trade groups to provide information for specific estimates, such as construction costs and machinery.

1.3. Computing Periodic and Average Inflation

Price indexes provide a measure of the relative change in prices from year to year. Thus, the inflation rate for one period (generally a month or a year) may be computed from the relative change in the index over the corresponding period. We will use the CPI in the following calculations, but the relationships hold for any price index. We follow notation similar to (Thuesen and Fabrycky 1994).

Define the inflation for period n as f_n. This value is calculated as:

$$f_n = \frac{CPI_n - CPI_{n-1}}{CPI_{n-1}}$$

In the one-period case, the previous period ($n - 1$) represents the base period. This calculation is valid only for single period price changes. For changes over multiple periods, an average inflation rate may be calculated.

Define the average inflation rate over n periods (period t to period $t + n$), $\bar{f}$, as:

$$CPI_t(1 + \bar{f})^n = CPI_{t+n}$$

such that:

$$\bar{f} = \left[\frac{CPI_{t+n}}{CPI_t}\right]^{1/n} - 1$$

In this calculation, time t refers to the base period for calculations. Generally, the bar is dropped and this rate is referred to as f. Note that this interest rate includes the effects of compounding and is not merely an arithmetic average of CPI values.

Consider the change in the CPI from 1967. Given an index value of 100 at the end of 1967, rising to 491.0 at the end of 1998, the average annual inflation rate over those 31 years is calculated as:

$$f = \left[\frac{491.0}{100.0}\right]^{1/31} - 1 = 0.0527 = 5.27\%$$

Again, this is an average annual rate over the respective 31 years. While this may seem like a high annual rate when one considers the recent years of low inflation, one must consider the double-digit percentage inflation rates of the 1970s in the United States.

Consider the postage stamp example presented earlier. The two prices from 1967 and 1999 can be used to determine the average inflation rate for the postage stamp over the past 31 years as follows:

$$f = \left[\frac{0.33}{0.05}\right]^{1/31} - 1 = .0628 = 6.28\%$$

Thus, it can be concluded that the average annual increases in price for a postage stamp have outpaced the average increases in the cost of living over the past 31 years in the United States. This further illustrates the need to utilize specialized indexes to achieve more accurate inflation estimates.

2. INCORPORATING INFLATION INTO ECONOMIC ANALYSIS

The role of economic analysis in engineering is to determine whether engineering projects are economically viable. This generally entails determining whether a single project should be accepted or rejected, or choosing the best project from a set of feasible projects. The analysis may take on the form of cost minimization or profit maximization. Regardless of the application or situation, the procedure generally requires the estimation of relevant cash flows and their conversion to some common denominator, such as the net present value, in order to make a decision. In this section, we consider this decision process under the assumption that the cash flows are subject to inflation. After terminology is introduced, an analytic approach is outlined to handle these types of analyses, with or without inflation. Generalizations are made for cases where the inflation rate varies according to different cash flows and over time.

2.1. Inflation-Free and Market Interest Rates

The inflation rate has been defined which allows one to predict price changes. However, economic analysis requires the use of an interest rate for discounting or compounding procedures in order to reduce a set of cash flows to a common measure for analysis, such as net present value. Because cash flows may or may not be inflated, two interest rates must be defined for use in analysis.

Define the market interest rate i as the rate of interest that can be expected to be earned on investments in the marketplace. It is a function of the investments available on the market and their associated risks. Additionally, this rate includes the effect of inflation such that if there is an upward movement in prices (inflation), there is an upward movement in the market interest rate. Other terms that may be commonly used to refer to the market rate include the minimum attractive rate of return (MARR), nominal MARR, actual interest rate, effective rate, and inflated interest rate.

Define the inflation-free interest rate i' as the market interest rate with the effects of inflation removed. While this rate is fictitious on the market, because any interest rate available on the market includes the effects of inflation, it is useful in economic analysis. This rate is also termed the real interest rate.

In the previous section, we defined the inflation rate f as the periodic increase in the price of some good or service. This rate provides the link between inflation-free and market rates because the first rate excludes inflation while the latter rate includes the effects of inflation.

To convert from an inflation-free interest rate to a market interest rate, one must add inflation. This is accomplished as follows:

$$(1 + i) = (1 + i')(1 + f)$$

such that:

$$i = i' + f + i'f$$

Similarly, to convert a market interest rate to an inflation-free rate, one must remove inflation, as:

$$i' = \frac{(1 + i)}{(1 + f)} - 1$$

Thus, the periodic inflation rate provides the link between the inflation-free interest rate and the market rate.

2.2. Actual and Constant Dollar Cash Flows

As noted earlier, economic analysis may be performed on cash flows that are either inflated or not. Thus, two interest rates were defined. The market rate includes the effects of inflation, while these effects are removed from the real interest rate. Here, the corresponding cash flows are defined.

Define actual dollars as cash flows that incorporate the effects of inflation. These may be viewed as out-of-pocket dollars because they are the true expenses paid or revenues received in business transactions at any point in time. Unfortunately, a common terminology does not exist for differentiating dollars, as actual dollars are often referred to as current, nominal, future, or inflated dollars.

Constant dollar cash flows do not include the effects of inflation. As with the inflation-free interest rate, constant dollars are a fictitious concept that represents the change in purchasing power through inflation according to some baseline in time. Constant dollars are often referred to as real, deflated, or today's dollars.

The relationship between constant dollars and actual dollars lies in the inflation rate, as with the relationship between the market and inflation-free interest rates. Because constant dollars do not include the effects of inflation, they may be converted to actual dollars by adding inflation. To show this mathematically, define the actual dollar cash flow at time n as F_n and the constant dollar cash flow at time n as F'_n. To convert constant to actual dollars at the same time period, one must incorporate the effects of inflation, or:

$$F_n = F'_n(1 + f)^n$$

Similarly, to convert actual dollars to constant dollars, one must remove inflation, as follows:

$$F'_n = \frac{F_n}{(1 + f)^n}$$

Figure 1 illustrates the two realms of cash flows: actual dollars and constant dollars. As illustrated in the figure, discounting with actual dollars requires use of the market interest rate, while the

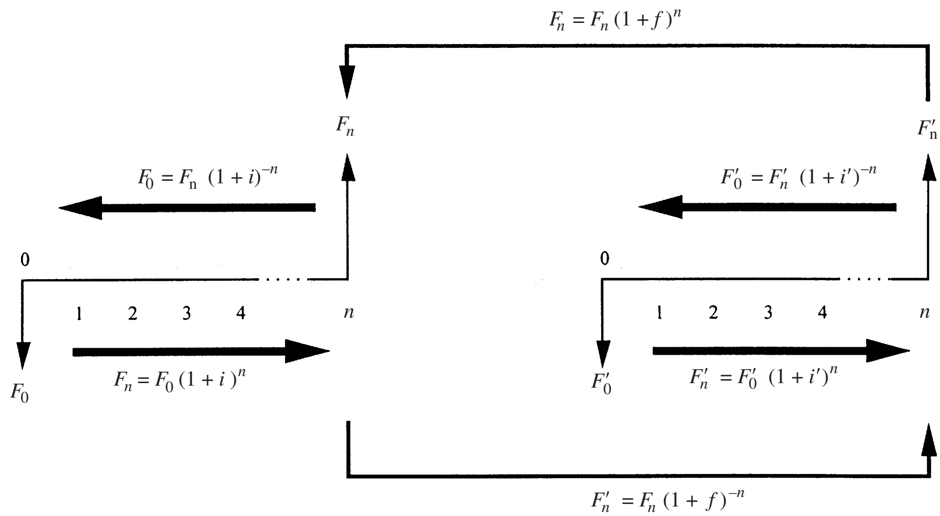

Figure 1 Time Value of Money Calculations for Actual and Constant Dollar Cash Flows and Conversions between the Two with the Inflation Rate.

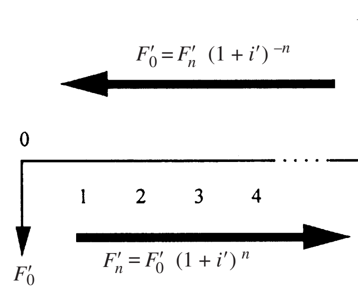

Figure 2 Time Value of Money Calculations for Constant Dolllar Cash Flows with the Inflation-free Interest Rate.

inflation-free rate is utilized with constant dollars. To convert between the interest rates or the dollar domains, the inflation rate is utilized. This is shown with the arrows that cross the domain boundaries.

2.3. Economic Equivalence Calculations with Inflation

Because there are two types of cash flows and two types of interest rates, one may perform economic analyses in one of two domains: (1) constant dollar cash flows with the inflation free interest rate or (2) actual dollar cash flows with the market interest rate. Figure 1 can be broken into the two different domains of cash flows for analysis. Figure 2 illustrates the computations in the constant-dollar domain. These calculations require the use of i'.

Similarly, Figure 3 breaks out the calculations in the actual dollar domain. As noted earlier and illustrated in Figure 1, the inflation rate provides the mathematical link between these environments because it allows one to transform constant-dollar cash flows to actual dollar-cash flows and inflation-free interest rates to market interest rates.

For a decision maker, it is critical that analysis with actual dollars be conducted with the market interest rate and analysis with constant dollars be evaluated at the inflation-free interest rate. If one is provided with constant dollars and a market interest rate or actual dollars and an inflation-free interest rate, the inflation rate must be used to convert either the dollars (from actual to constant or vice versa) or the interest rate (from market to inflation free or vice versa) to the appropriate combination for the ensuing analysis.

Example

Due to deterioration, an industrial firm replaces a piece of its machinery every year. The time zero price of the asset is \$25,000. The purchase price is expected to rise at a rate of 4.35% each year. What is the present value of the capital costs incurred by the firm over the next three years? Assume purchases occur at time zero and at the end of each of the three years and a market interest rate of 20%.

Constant Dollar Analysis Because inflationary effects on the cash flows can be ignored, the constant dollar cash flow diagram consists of four expenditures of \$25,000 each. The one caveat in

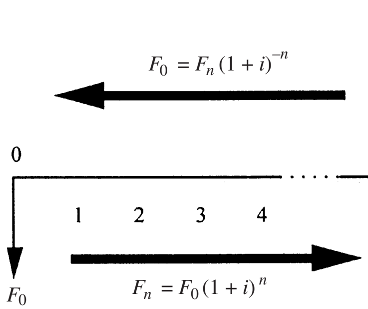

Figure 3 Time Value of Money Calculations for Actual Cash Flows with the Market Interest Rate.

this problem is that the discount rate is the market rate, not the inflation-free rate. Convert the rate as follows:

$$i' = \frac{(1 + i)}{(1 + f)} - 1 = \frac{(1 + 0.20)}{(1 + 0.0435)} - 1 = 0.14997 \simeq 15\%$$

With the inflation-free rate, the net present value (NPV) of the three expenditures in periods one through three can be brought back to time zero using the equal-payment series present-worth factor (Thuesen and Fabrycky 1994) and added to the time zero expenditure, as follows:

$$\begin{aligned} \text{NPV} &= \$25{,}000 + \$25{,}000(P/A, 15\%, 3) \\ &= \$25{,}000 + \$25{,}000 \left[\frac{(1 + 0.15)^3 - 1}{0.15(1 + 0.15)^3} \right] = \$82{,}080.63 \end{aligned}$$

An equivalent analysis can be conducted with actual dollar cash flows.

Actual Dollar Analysis The actual cash flows must be calculated in each period. To convert the constant dollar flows, the inflation rate must be utilized as follows:

$$F_n = F'_n(1 + f)^n$$

For time zero, the cash flows are the same, but for time period one:

$$F_1 = \$25{,}000(1 + .0435) = \$26{,}087.50$$

The cash flows have been calculated for each period and are listed in Table 1.

The NPV of the actual cash flows is found using the market interest rate of 20%. Because there is no pattern, each cash flow must be brought back to time zero individually as follows:

$$\text{NPV} = \$25{,}000 + \frac{\$26{,}087.50}{(1 + 0.20)} + \frac{\$27{,}222.31}{(1 + 0.20)^2} + \frac{\$28{,}406.48}{(1 + 0.20)^3} = \$82{,}082.90$$

Note that the constant dollar and actual dollar analysis lead to the same net present value of costs (within an acceptable rounding error of \$2.27). To illustrate that this holds in general, the NPV value for the actual dollar cash flow can be rewritten as follows:

$$\begin{aligned} \text{NPV} &= F_0 + \frac{F_1}{(1 + i)} + \frac{F_2}{(1 + i)^2} + \frac{F_3}{(1 + i)^3} \\ &= \$25{,}000 + \frac{\$26{,}087.50}{(1 + 0.20)} + \frac{\$27{,}222.31}{(1 + 0.20)^2} + \frac{\$28{,}406.48}{(1 + 0.20)^3} \\ &= \$25{,}000 + \frac{\$25{,}000(1 + 0.0435)}{(1 + 0.20)} + \frac{\$25{,}000(1 + 0.0435)^2}{(1 + 0.20)^2} + \frac{\$25{,}000(1 + 0.0435)^3}{(1 + 0.20)^3} \\ &= F'_0 + \frac{F'_1(1 + f)}{(1 + i)} + \frac{F'_2(1 + f)^2}{(1 + i)^2} + \frac{F'_3(1 + f)^3}{(1 + i)^3} \\ &= F'_0 + \frac{F'_1}{(1 + i')} + \frac{F'_2}{(1 + i')^2} + \frac{F'_3}{(1 + i')^3} \end{aligned}$$

Therefore, if one inflation rate is assumed for all cash flows, then actual and constant dollar analyses are equivalent. This allows one to choose the more straightforward analysis for calculations. In the

TABLE 1 Constant and Actual Dollar Cash Flows for Example

Year	Constant-Dollar Cash Flow	Actual Dollar Cash Flow
0	\$25,000.00	\$25,000.00
1	\$25,000.00	\$26,087.50
2	\$25,000.00	\$27,222.31
3	\$25,000.00	\$28,406.48

previous example, the constant dollar cash flow was straightforward because the cash flows resulted in an equal payment series while the actual dollar cash flows had no pattern. Unfortunately, most applications do not allow for the decision maker to select the analysis, as inflation rates may differ by the cash flow or time period or inflation may not affect certain cash flows, such as loan payments and the cash flow effects of depreciation (in the United States). These cases are examined in the following sections.

2.4. Differing Inflation Rates for Component Cash Flows

As noted earlier, numerous price indexes exist for estimating inflation rates for various commodities. Thus, for a given analysis, one may compute a variety of inflation rates for use in a single analysis. Unless a single inflation rate affects all the cash flows involved in calculations, each component cash flow must be converted individually at the appropriate rate for proper analysis (Oakford and Salazar 1982).

Define f_k as the inflation rate for cash flow component k. If the net cash flow at time n, F_n, is made up of a number (K) of component cash flows $F_{k,n}$, then the actual net cash flow is equivalent to the sum of the actual component cash flows:

$$F_n = \sum_{k=1}^{K} F_{k,n}$$

To convert constant dollar component cash flows to a net actual dollar cash flow, one sums the individually converted cash flows as follows:

$$F_n = \sum_{k=1}^{K} F'_{k,n}\,(1 + f_k)^n$$

This is illustrated in the following example.

Example. Labor costs are estimated at \$10,000 at time zero and are assumed to increase at a rate of 3.2% per year. Time zero material costs are estimated at \$20,000 and are assumed to increase at a rate of 5%. Finally, fuel and energy costs are expected to increase at a rate of 4.5% from a time zero value of \$12,000. What are the total expected costs at the end of period 3?

Constant Dollar Analysis As all costs are given in time zero dollars, a straightforward solution would be that the total costs are:

$$\$10{,}000 + \$20{,}000 + \$12{,}000 = \$42{,}000$$

in constant dollars (with time zero serving as the baseline).

Actual Dollar Analysis The individual component inflation rates must be used to convert this to an actual dollar cash flow at the end of period 3. This is accomplished as follows:

$$\$10{,}000(1 + 0.032)^3 + \$20{,}000(1 + 0.05)^3 + \$12{,}000(1 + 0.045)^3 = \$47{,}837.54$$

This represents the expected net expenses at the end of time period 3 in actual dollars.

2.5. Differing Inflation Rates per Time Period

An additional complication is an inflation rate that changes over time. This is generally more realistic, as inflation is not a constant, which is assumed when using the average rate f defined earlier. Examining various price indexes, such as the CPI, it is clear that the inflation rate changes periodically. To perform precise economic analysis, the inflation rate must also change periodically. This is not difficult, but it complicates the analysis as calculations must be repeated.

Consider the conversion of a constant dollar cash flow at time n, F'_n, to an actual dollar cash flow F_n at the same time period:

$$F_n = F'_n(1 + f)^n$$

Because there are n periods of inflation, we may generalize this to n different periodic inflation rates. Define f_t as the inflation rate for period t. The conversion can now be written as:

$$f_n = F'_n(1 + f_1)(1 + f_2) \ldots (1 + f_n)$$

We revisit the example of the previous section to illustrate the calculation.

Example. Labor costs are estimated at $10,000 at time zero and are assumed to increase at a rate of 2.0% the first year, 2.4% the second year, and 3.5% the third year. Time zero material costs are estimated at $20,000 and are assumed to increase at rates of 3.0, 4.0, and 5.5% for each of the three years, respectively. Finally, fuel and energy costs are expected to increase at rates of 4.5, 4.1, and 5.2%, respectively, from a time zero value of $12,000. What are the total expected costs at the end of period 3?

Constant Dollar Analysis As before, the constant dollar solution is as follows:

$$\$10{,}000 + \$20{,}000 + \$12{,}000 = \$42{,}000$$

in constant dollars (with time zero serving as the baseline).

Actual Dollar Analysis Again, the component inflation rates, with considerations for the change over time, must be used to convert this to an actual dollar cash flow at the end of period 3. This is accomplished as follows:

$$\$10{,}000(1 + 0.02)(1 + 0.024)(1 + 0.035) + \$20{,}000(1 + 0.03)(1 + 0.04)(1 + 0.055) + \$12{,}000(1 + 0.045)(1 + 0.041)(1 + 0.052) = \$47{,}145.64$$

As before, this represents the expected net expenses at the end of time period three in actual dollars.

2.6. Relationship between Inflation and Exchange Rates

Inflation is a general term that refers to changes in price. As illustrated in this chapter, it allows one to convert a constant dollar cash flow, which does not incorporate the effects of inflation, into an actual dollar cash flow. An exchange rate, which allows for the conversion from one form of currency to another, acts in a similar manner. Exchange rates allow one to convert dollars in a given currency to another currency. Furthermore, changes in the exchange rate between two currencies over time are analogous to changes in the general inflation rate because the relative purchasing power between the two currencies changes with the exchange rate (DeGarmo et al. 1997). We examine this with the following investment example.

Example. A multinational firm based in the United States is considering an investment in Mexico. The exchange rate is currently 10 Mexican pesos to 1 U.S. dollar. (There are a variety of sources available with exchange rate data, e.g., Tukiainen 1999.) Assume an investment cost of 10 million pesos with an expected annual return of 500,000 pesos per year over a five-year horizon.

Assuming a constant exchange rate over the five-year horizon, the analysis is straightforward. In pesos, a 10 million-peso investment at time zero results in five equal annual payments of 500,000 pesos. The exchange rate converts this to U.S. dollars, in which it is equivalent to a 1 million-dollar investment at time zero with annual profits of $50,000 per year. Because these are assumed to be constant dollars, the net present value can be computed given the appropriate inflation-free interest rate.

A more interesting question that is more directly related to the issue of inflation is whether the currency exchange rate is expected to change over time. For example, assume that the exchange rate between the U.S. and Mexico is expected to decline at 1% per year over the next five years. As the exchange rate is merely an index to convert currencies, the change in the rate corresponds to inflation. See DeGarmo et al. (1997) and Lee and Sullivan (1995) for more on this matter.

3. THE EFFECTS OF INFLATION IN ECONOMIC ANALYSIS

This section examines more complicated investment scenarios from both before-tax and after-tax cash flow perspectives. It is important to analyze these cases because inflation has a much more drastic impact on after-tax analysis due to the necessary inclusion of depreciation charges. Specifically, examples that include taxes and loans are examined to illustrate the different effects of inflation.

3.1. Before-Tax Cash Flow Analysis

The following example illustrates the effects of inflation on borrowing funds. This analysis is similar to any contract signed for future services because the cash flows are agreed upon and therefore represent out-of-pocket expenses. Thus, by definition, these expenses are actual dollar cash flows. The following example illustrates the required calculations to determine the net present value or rate of return for a project.

Example. A firm purchases a piece of machinery for $50,000. Half of the purchase price is borrowed at an interest rate of 7% per year. Two equal payments are made at the end of years 1 and 2 to pay off the loan. The asset is expected to generate revenues of $75,000 per year for three years,

TABLE 2 Actual Cash Flows for Investment Example

Year	0	1	2	3
Revenues		\$78,750.00	\$82,687.50	\$86,821.88
O&M expenses		(\$15,750.00)	(\$16,537.50)	(\$17,364.38)
Interest expenses		(\$1,750.00)	(\$904.59)	
Capital cost	(\$50,000)			\$11,576.25
Loan principal	\$25,000	(\$12,077.30)	(\$12,922.70)	
Before-tax cash flow	(\$25,000)	\$49,172.70	\$52,322.71	\$81,033.75

at which time it is sold for a salvage value of \$10,000. Operating and maintenance (O&M) expenses are assumed to be \$15,000 per year. All costs here are given in time zero dollars. Determine the net present value of the transaction assuming a market interest rate of 20%.

Because loan interest and principal payments are defined by a contractual agreement, they are actual dollar cash flows. Thus, we first analyze this problem from the perspective of the actual dollar domain.

Actual Dollar Analysis (*1*) In this first actual dollar analysis, we assume one (general) annual inflation rate of 5% for the revenues and expenses and the salvage value of the asset at the end of period 3. All of the actual dollar cash flows are given in Table 2. Negative cash flows are denoted in parentheses. Note that the interest and principal payments are unaffected by the inflation rate becasue they are dependent on a prior agreement.

The present value of the actual dollar before-tax cash flow is calculated using the market interest rate, as follows:

$$\text{NPV} = -\$25{,}000 + \frac{\$49{,}172.70}{(1 + 0.20)} + \frac{\$52{,}322.71}{(1 + 0.20)^2} + \frac{\$81{,}033.75}{(1 + 0.20)^3} = \$99{,}207.00$$

Constant Dollar Analysis In the original problem statement, all cash flows were presented in constant dollars. Thus, revenues, operating and maintenance expenses, and capital costs are straightforward. However, the loan principal and interest expenses are actual dollar cash flows, which means they include the effects of inflation. Thus, these cash flows must be deflated with the inflation rate of 5%. These calculated cash flows, along with the other constant cash flows, are provided in Table 3.

The present value of the constant dollar before-tax cash flow is calculated using the inflation free interest rate. The inflation-free rate is calculated from the market and inflation rates, as follows:

$$i' = \frac{1 + 0.20}{1 + 0.05} - 1 = 0.1429 = 14.29\%$$

This rate is then used to calculate the net present value with the constant dollar before-tax cash flow:

$$\text{NPV} = -\$25{,}000 + \frac{\$46{,}831.14}{(1 + 0.1429)} + \frac{\$47{,}458.24}{(1 + 0.1429)^2} + \frac{\$70{,}000.00}{(1 + 0.1429)^3} = \$99{,}197.46$$

As expected, the net present value is equivalent (within rounding error) of the actual dollar analysis.

TABLE 3 Constant Cash Flows for Investment Example

Year	0	1	2	3
Revenues		\$75,000.00	\$75,000.00	\$75,000.00
O&M expenses		(\$15,000.00)	(\$15,000.00)	(\$15,000.00)
Interest expenses		(\$1,666.67)	(\$820.49)	
Capital costs	(\$50,000)			\$10,000.00
Loan principal	\$25,000	(\$11,502.19)	(\$11,721.27)	
Before-tax cash flow	(\$25,000)	\$46,831.14	\$47,458.24	\$70,000.00

TABLE 4 Actual Cash Flows with Varying Inflation Rates for Investment Example

Year	0	1	2	3
Revenues		$78,750.00	$82,687.50	$86,821.88
O&M expenses		($15,450.00)	($15,913.50)	($16,390.91)
Interest expenses		($1,750.00)	($904.59)	
Capital costs	($50,000)			$12,597.12
Loan principal	$25,000	($12,077.30)	($12,922.70)	
Before-tax cash flow	($25,000)	$49,472.70	$52,946.71	$83,028.09

Actual Dollar Analysis (*2*) Because it is more realistic to assume inflation rates that vary for each cash flow component, we revisit the actual dollar analysis. Assume that the revenues grow at a rate of 5% per period, O&M expenses increase at a rate of 3% and the salvage value grows at a rate of 8% per period. The revised actual dollar cash flows for this example are given in Table 4. As before, interest expenses, the initial purchase cost, and the loan principal payments are unaffected.

Using the market interest rate of 20%, the net present value of the actual dollar before-tax cash flows is $101,044.46. Because O&M costs were projected lower and the salvage value higher than in the previous actual dollar cash flow analysis, this higher net present value was expected.

The calculations are not repeated here for the corresponding constant dollar analysis, as the cash flows are the same as before. However, there is one final point that must be addressed. In this problem instance, the market rate was provided. This allows for the discounting of actual dollar cash flows. Generally, with the use of an inflation rate, the market rate can be converted to an inflation-free rate. However, in this final example with component cash flows having different inflation rates, this analysis is not straightforward. In this case, a *general* inflation rate (Park 1997) is generally specified such that the inflation free rate can be calculated. This rate would generally correspond to a broader index, such as the CPI, whereas the component inflation rates would be derived from more specific indexes. If an inflation-free rate had been specified in the problem, then a general inflation rate would have also had to be specified to determine a single market interest rate for discounting.

3.2. After-Tax Cash Flow Analysis

As shown in the previous section, loans and agreed-upon prices can complicate analyses because not all cash flows are defined as either constant or actual dollars, but rather a mix. Performing after-tax analysis presents further complicating issues. However, it is generally recognized that after-tax analyses should be performed when economically validating engineering projects (Park and Sharp-Bette 1990).

When considering after-tax cash flows, some form of actual dollar analysis must be performed because taxes are paid on profits from actual revenues. This is not to say that constant dollar analyses cannot be performed, but the actual dollars must generally be estimated before conversion to constant dollars can take place.

The following after-tax analyses revisit the before-tax transaction from the previous section. It should be noted that the goal of this section is not to present current tax law, which is forever changing and in which rules are different according to the location where the asset is in use and the time the asset is both placed in and removed from service. Rather, the point here is to illustrate the effect of inflation on differing components of cash flows common to after-tax analysis.

Example. The example analyzed as a before-tax transaction is again examined here. It is assumed that the asset is depreciated using the straight-line method over three years. For simplicity, no adjustments to the amount of depreciation are made at the time of sale.

Because taxes are paid on actual revenues and expenses, the actual dollar cash flow is analyzed first. A market interest rate of 15% is assumed.

Actual Dollar Analysis Assume a single annual inflation rate of 5% for the revenues, expenses, and salvage value of the asset at the end of period 3. As before, the interest and principal payments are not affected by inflation. Table 5 provides the actual cash flows for this example. Note that depreciation expenses are provided in order to calculate taxes although they are not cash flows. For the sake of simplicity, we assume a marginal tax rate of 40%, which is also applied to capital gains.

The after-tax cash flow, which is generally found on a cash flow statement, adds the after-tax profit, purchase costs, and loan principal payments and also adds back in the depreciation expenses because depreciation is not a cash flow. The net present value of the actual dollar after-tax cash flow is then computed at the market rate of 15%. The NPV for this example is $63,474.15.

TABLE 5 Actual Cash Flows for After-Tax Investment Example

Year	0	1	2	3
Revenues		$78,750.00	$82,687.50	$86,821.88
O&M expenses		($15,750.00)	($16,537.50)	($17,364.38)
Interest expenses		($1,750.00)	($904.59)	
Depreciation expenses		($16,666.67)	($16,666.67)	($16,666.67)
Before-tax operating profit		$44,583.33	$48,578.74	$52,790.83
Salvage values				$11,576.25
Tax on capital gain				($4,630.50)
Total taxes		($17,833.33)	($19,431.50)	($25,746.83)
After-tax profit		$26,750.00	$29,147.24	$38,620.25
Purchase cost	($50,000)			
Loan principal	$25,000	($12,077.30)	($12,922.70)	
After-tax cash flow	($25,000)	$31,339.37	$32,891.21	$55,286.92

Constant Dollar Analysis Unlike previous examples presented in this chapter, the constant dollar analysis for after-tax cash flows requires the computation of actual dollar cash flows because taxes are actual dollar cash flows. Although the net actual dollar after-tax cash flow can be converted to a net constant dollar cash flow for analysis, we explicitly show each of the component cash flows in Table 6 because if the component inflation rates differ, each component cash flow must be analyzed with the appropriate rate. The cash flows given in Table 5 as depreciation expenses are no longer needed as the taxes have been calculated. The general 5% inflation rate is used here to deflate all actual dollar cash flows.

With a market interest rate of 15% and a general inflation rate of 5%, the inflation-free rate is 9.52%. This leads to a net present value of $63,480.63, which is equivalent to the actual dollar analysis (within rounding error). This is expected because all of the component cash flows were inflated at the same rate. As noted in the before-tax analysis, if the component inflation rates differ, then there may be a discrepancy in the final net present value analysis because the conversion from a market rate to an inflation-free rate (or vice versa) requires a single inflation rate.

This may lead one to ask the question of which analysis should be performed. There is no correct answer. However, as these examples illustrated, more complicated cash flow scenarios generally lead to actual dollar cash flow analysis. This is because the computation of one's tax liability is based on actual dollar cash flows. Also, contracted prices, such as loans and lease agreements, are based on actual dollar cash flows. In these situations, actual dollar cash flow analysis may be more straightforward because most of the data are also in the correct form.

Regardless of the method chosen, it is important to include inflation in analysis because it may have drastic effects. Consider the after-tax example analyzed in this section. Table 7 provides the cash flows ignoring inflation. Note that these flows are not equivalent to constant dollar cash flows, as the interest and principal payments remain as actual dollar flows. The present value, at the 15% market rate, is $54,765.84. In this example, ignoring inflation leads to a lower net present value and thus may lead to a different decision. Inflation can have various effects on project cash flows and thus greatly influence decisions. In general, inflation results in overstated income and higher taxes. This is sometimes referred to as tax bracket creep because inflation may push a company's revenues, and thus profits, into a higher tax bracket. Similarly, inflated salvage values result in larger capital gains and resulting taxes. As shown in the above example, inflation can actually reduce the cost of

TABLE 6 Constant Cash Flows for After-Tax Investment Example

Year	0	1	2	3
Revenues		$75,000.00	$75,000.00	$75,000.00
O&M expenses		($15,000.00)	($15,000.00)	($15,000.00)
Interest expenses		($1,666.67)	($820.49)	
Salvage values				$10,000.00
Total taxes		($16,984.12)	($17,624.94)	($22,241.08)
Purchase cost	($50,000)			
Loan principal	$25,000	($11,502.19)	($11,721.27)	
After-tax cash flow	($25,000)	$29,847.02	$29,833.30	$47,758.92

TABLE 7 Cash Flows Ignoring Inflation for After-Tax Investment Example

Year	0	1	2	3
Revenues		$75,000.00	$75,000.00	$75,000.00
O&M expenses		($15,000.00)	($15,000.00)	($15,000.00)
Interest expenses		($1,750.00)	($904.59)	
Salvage values				$10,000.00
Total taxes		($16,633.33)	($16,971.50)	($21,333.33)
Purchase cost	($50,000)			
Loan principal	$25,000	($12,077.30)	($12,922.70)	
After-tax cash flow	($25,000)	$29,539.37	$29,201.21	$48,666.67

financing because that cost is fixed at the agreed upon rate. In summary, the effects of inflation can alter cash flows significantly and thus strongly influence the decision to accept or reject a project. Therefore, they should not be ignored in any economic analysis.

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