CHAPTER 72

Reliability and Maintainability

KAILASH C. KAPUR
The University of Washington

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1. INTRODUCTION

The ultimate objective of any system is the performance of some intended function. This function is frequently called the *mission.* The term often used to describe the overall capability of a system to accomplish its mission is *system effectiveness* For consumer products, system effectiveness is related to customer satisfaction, which is related to the overall concept of quality. Quality of products and services is defined and evaluated by the customers and users. Similarly, system effectiveness is defined and evaluated by the customers and users of the product. System effectiveness is defined as the probability that the system can successfully meet an operational demand within a given time when operating under specified conditions. Effectiveness is influenced by the way the system is designed, manufactured, used, and maintained. Thus, the effectiveness of a system is a function of several attributes, such as design adequacy, performance measures, safety, reliability, quality, manufacturability, and maintainability. The disciplines of assurance sciences help to increase the overall effectiveness of any system. The assurance sciences are engineering disciplines that have the common objective of attaining product integrity (product does what it says it is supposed to do) (Carrubba and Gordon 1978). The term *product assurance* is also popular in many companies. This chapter is concerned with the reliability, maintainability, serviceability, and availability aspects of the product assurance system.

Reliability is one of the major attributes determining system effectiveness. It is generally defined as the probability that a given system will perform its intended function satisfactorily, for its intended life, under specified operating conditions. With this definition the obvious problems are (1) the acceptance of the probabilistic notion of reliability; (2) the problems associated with defining adequate performance, particularly for system parameters that deteriorate slowly with time; and (3) the judgment required to determine the proper statement of operating conditions. Thus, reliability is a relative measure, or in terms of probability, it is conditional probability. It is relative to (1) definition of function from the viewpoint of the customer; (2) definition of failure from the viewpoint of the customer; (3) definition of intended life; and (4) customer's operating and environmental conditions.

Reliability is an inherent attribute of a system resulting from design, just as is the system's capacity, performance, or power rating. The reliability level is established at the design phase, and subsequent testing and production will not raise the reliability without a basic design change. Because reliability is an abstract concept that is difficult to grasp and to measure, many organizations find themselves unable to implement a comprehensive reliability program, primarily because of the lack of understanding on the part of both management and technical system design personnel. This is not to say that the system designers or managers in the organization are not interested in a reliable product; rather, the pressures on the design engineer, and very often on the organizational structure, impede the development of an effective reliability program (Kapur and Lamberson 1996).

With increasing system complexity, reliability becomes an elusive and difficult design parameter. It becomes more difficult not only to define and achieve as a design parameter, but to control and demonstrate in production and thus to ensure as an operational characteristic under the projected environmental conditions of use. However, past history has demonstrated that where reliability was recognized as a necessary program development component, with the practice of various reliability engineering methods throughout the evolutionary life cycle of the system, reliability can be quantified during the specification of design requirements, can be predicted by testing, can be controlled during production, and can be sustained in the field (Kapur 1996a). The purpose of this chapter is to present some of the reliability and maintainability methodologies and philosophies that are applicable throughout the life cycle of a system.

1.1. Reliability and Maintainability Activities during the System Life Cycle

Reliability and maintainability activities should span the entire life cycle of the system. Figure 1 shows the major points of reliability practice in a typical system life cycle (AMC 1968). The activities given in the exhibit are briefly explained here. There is a continuous feedback, and designs go through several cycles of the reliability program activities.

1.1.1. Step 1: The Need

The need for a reliability and maintainability program must be anticipated right from the beginning. The need for such programs cannot be overemphasized. These programs are justified based on specific system requirements in terms of life-cycle cost and other operational requirements. As already mentioned, the effectiveness of a system is determined by its reliability and maintainability characteristics.

1.1.2. Step 2: Goals and Definitions

All the requirements must be specified in terms of well-defined and quantitative goals. The goals and requirements are defined by some of the following measures:

1. *Reliability measures.* Mission reliability, a reliability function based on specified failure distribution, mean time to failure (MTTF), and failure rate.
2. *Maintainability measures.* Maintainability function based on time to repair distribution, mean time to repair (MTTR), percentile of time to repair, and maintenance ratio.

There are other measures as well, and the relationship among them is given in Figure 2 (Von Alven 1964).

The term *reliability* has already been defined; some of the other terms are defined as follows:

1. *Mission reliability* is the probability that the product and/or system will successfully complete a given mission with specified operating requirements and time duration.
2. *Maintainability* is defined as the probability that a failed system can be made operable in a specified interval of downtime. As shown in Figure 2, the downtime includes the failure detection time, the active repair time, the logistics time connected with the repairs of the product, and all the administrative time. The maintainability function describes probabilistically how long a system remains in the failed state.

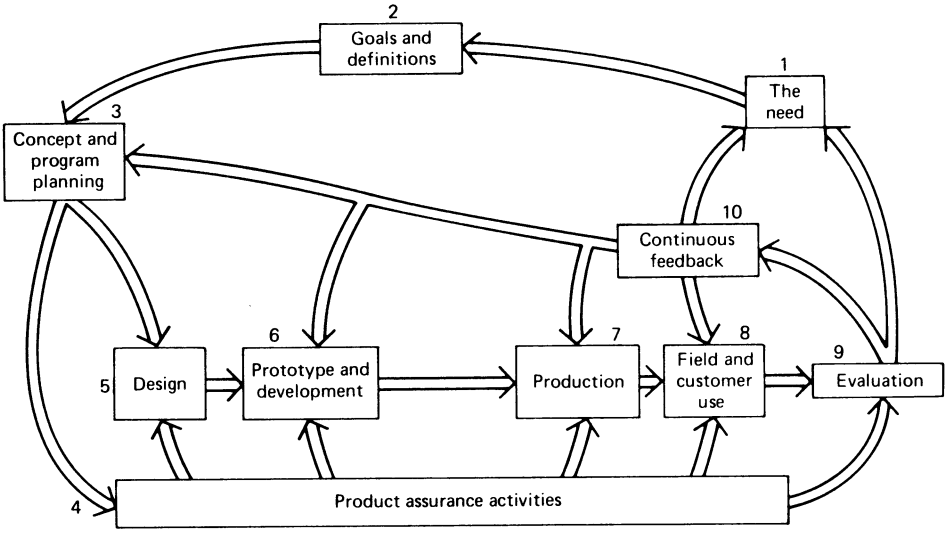


Figure 1 Reliability and Maintainability Activities During System Life Cycle.

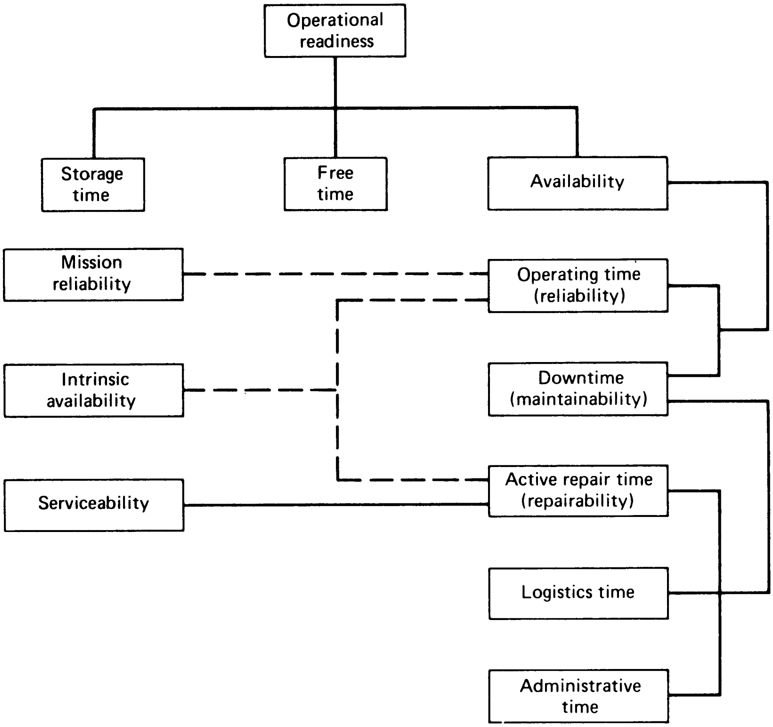


Figure 2 Relationship Among Various Product Assurance Measures.

3. *Repairability* deals only with the active repair time and can be defined by the time to actively repair random variable and the associated distribution. Repairability is defined as the probability that a failed system be restored to a satisfactory operating condition in a specified interval of active repair time. This measure is more valuable to the administration of the repair facility since it helps to quantify the workload for the facility and its workers.
4. *Serviceability* is defined as the ease with which a system can be repaired. Serviceability, like reliability, is a characteristic of the system design and must be planned at the design stage. Serviceability is difficult to measure on a ratio scale; however, it can easily be measured on an ordinal scale by a specifically developed rating and/or ranking procedure, which requires that systems be compared and ranked according to the ease of serviceability.
5. *Operational readiness* is defined as the probability that a system either is operating or can operate satisfactorily when the system is used under stated conditions. Operational readiness deals with all the time elements, including storage time, free time, operating time, and downtime.
6. *Availability* is defined as the probability that a system is operating satisfactorily at any point in time. Availability considers only the operating time and downtime, thus excluding the idle time. Therefore, it is a measure of the ratio of operating time of the system to the operating time plus the downtime. Availability is a function of both the reliability and maintainability of the system.
7. *Intrinsic availability* is more restrictive than availability because it is limited to operating and active repair time only.

These reliability, maintainability, and availability measures should be defined and system requirements specified using these quantitative measures. The effectiveness of the total product assurance program depends on these definitions.

1.1.3. Step 3: Concept and Program Planning

Based on the reliability and other operational requirements, various concepts are developed that can potentially meet these requirements. Also, at this stage, total product assurance program plans must be formulated and responsibilities assigned to different groups. The conceptual stage is an important part of the system life cycle because it has a major impact on the future system. Studies done by the U.S. Department of Defense indicate that 70% of the system life-cycle cost is determined by the decisions made at the concept stage. The detailed nature of the reliability programs will also determine the effectiveness of the total program.

1.1.4. Step 4: Product Assurance Activities

The plans developed in step 3 are implemented and the total program is continuously monitored, as indicated in Figure 1. An organization for the implementation of these plans must exist, with well-defined responsibilities

1.1.5. Step 5: Design

The conceptual system selected in step 3 is designed. Reliability and maintainability of this design are assessed. Various methodologies, such as design review, failure mode and effect analysis, fault tree analysis, and probabilistic design approach, can be applied at this step. Reliability is a design parameter and must be incorporated in the system at the design step.

1.1.6. Step 6: Prototype and Development

Prototypes are developed based on the design specifications. Reliability of the design is verified by testing. If the design has certain deficiencies, they are corrected by redesign. Reliability growth-management plans must be developed for this step in order to monitor continuously the growth and progress of the program. After the system has achieved the required level of reliability, the design is released for production.

1.1.7. Step 7: Production

The system is manufactured based on design specifications. Quality control methodologies are essential during this step. All the parts, materials, and processes are controlled based on methodologies discussed in previous chapters in the area of quality assurance. One of the objectives of the quality control program is to make sure that the inherent reliability of the design is not degraded.

1.1.8. Step 8: Field and Customer Use

Before the system is actually used in the field by its customers, it is very important to develop all the service and maintenance instructions, which are well documented. Just like reliability, maintainability is considered throughout the life cycle, and its purpose is to sustain required levels of reliability and availability in the field. Maintainability program plans are developed at the planning step.

1.1.9. Step 9: System Evaluation

The system in the field is continuously evaluated to determine whether the original reliability and maintainability goals are met by the system. For this purpose a reliability monitoring program and field data-collection program must be established.

1.1.10. Step 10: Continuous Feedback

There must be continuous feedback among all the steps in the system's life cycle. A proper communication system should be developed among all the groups responsible for the various steps. All the field deficiencies must be reported to the appropriate groups. This will help guide the system improvements.

The methodology related to some of the activities during the system life cycle is given in this chapter.

1.2. Reliability and Life Characteristic Curve

Reliability has sometimes been described as ‘‘quality in the time dimension’’ (RDG-376 1964) and a ‘‘time oriented quality characteristic’’ (Kapur 1986). The reliability characteristics of a product change with time. One of the characteristics is the concept of failure rate, which is defined mathematically later in this chapter. The failure rate, or the hazard rate, changes with the age or life of a product and has three distinct periods, as shown in Figures 3(*a*) and 3(*b*). These three periods are described here (Kececioglu 1991).

1.2.1. Infant Mortality Period

The total item population or a system generally exhibits a relatively high failure rate in the beginning, which decreases rapidly and stabilizes at some approximate time t_1. This initial period is generally called the ‘burn-in,’‘ ‘‘infant mortality,’‘ or ‘‘debugging’‘ period. The item population has ‘‘weak’‘ items, and these fail in the beginning. To understand the nature of these early failures, some of their causes are listed:

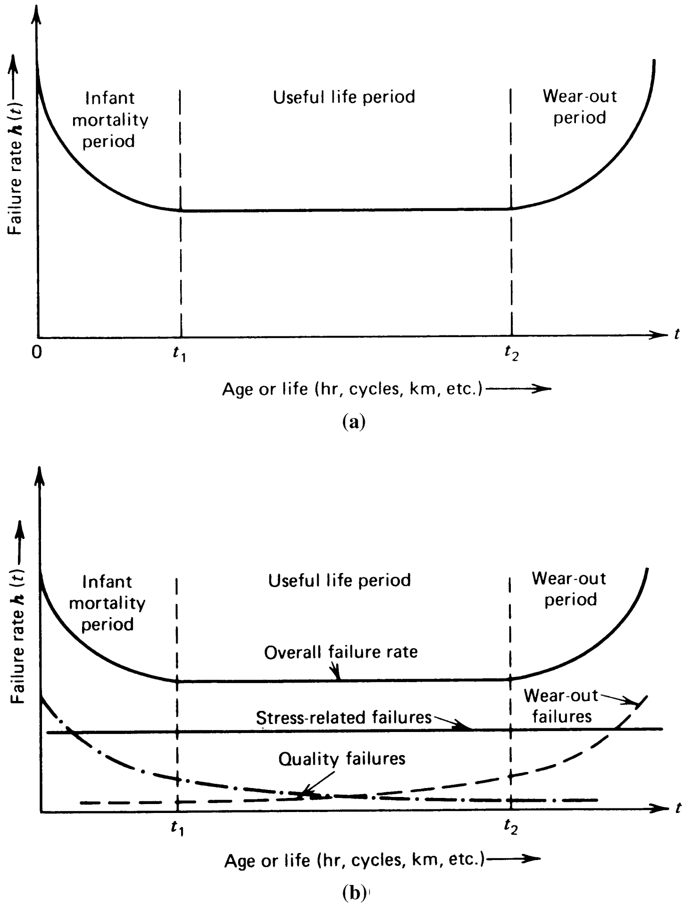


Figure 3 (*a*) Failure Rate-life Characteristic Curve. (*b*) Failure Rate Based on Components of Failure.

Substandard workmanship
Poor quality control
Substandard materials
Insufficient debugging
Poor manufacturing techniques
Poor processes and handling techniques
Problems due to assembly
Contamination
Improper installation
Improper start-up
Human error
Parts failure in storage and transit
Improper packaging and transportation practices

Fundamentally, these failures reflect the "manufacturability" of the product, and many are due to the lack of an effective quality control system in manufacturing. Thus, these early failures would show up during process audits, in-process or final tests, life tests, environmental tests, and so on.

Most manufacturers provide a burn-in period for their products so that the early or infant mortality failures occur in the plant and are not experienced by the customer in the field, where it is much more costly to fix these failures. The duration of the burn-in period determines what portion of the early failures is eliminated. Burn-in is a very expensive way to improve quality. A better way is to improve the process capability in terms of reducing variance. This will decrease the weak items in the population.

1.2.2. Useful Life Period

After having burned in, the item population reaches its lowest failure rate level, remaining relatively constant during this period. This failure rate is related to the inherent design reliability of the product and hence is given the most weight during the design reliability. This is also the most significant period for reliability prediction and assessment activities. Some of the causes of these failures are:

Low safety factors
Stress-related failures—higher-than-expected random loads
Lower random strength than expected
Defects that cannot be detected by the ‘‘best’‘ available inspection techniques
Abuse
Human errors
Failures that cannot be observed during debugging
Failures that cannot be prevented by the ‘‘best’’ preventive maintenance practices
Unexplainable causes
‘‘Act of God’’ failures

1.2.3. Wear-Out Period

Most of the products are designed to last for a specified period of useful life. The time t_2 in Figures 3(*a*) and 3(*b*) indicates the end of useful life or the start of the wear-out period. After this point, the failure rate increases rapidly. The wear-out, or deterioration, results from a number of familiar chemical, physical, or other causes, some of which are as follows:

Corrosion or oxidation
Frictional wear or fatigue
Aging and degradation
Creep
Poor maintenance or service practices
Improper overhaul practices
Short designed-in life
Shrinkage or cracking in plastics

A reliability program must consider all three of these distinct periods. It must also be pointed out that not all products have these three periods. The importance of the periods depends on the magnitudes of time t_1 and t_2, where $0 \leq t_1 \leq t_2 < \infty$. Thus, we can develop various types of life characteristic curves, depending on the values t_1 and t_2. The best way to reduce infant mortality failures is to reduce the variance of the manufacturing process and reduce variation from the target. Early failures can also be eliminated by systematic procedures of controlled screening, quality control, and burn-in tests. Stress-related failures during the useful life can be minimized by providing adequate design or safety margins. Wear-out failures can be minimized by preventive maintenance and replacement policies.

2. RELIABILITY MEASURES

Reliability has been defined as the probability that a given system will perform satisfactorily its intended function for its intended life under specified operating conditions. Thus, reliability is related to the probability of the successful performance of any system. It is clear that we must define what the successful performance of any system is or what we mean by the failure of the system; otherwise it is not possible to predict when any system will fail to perform its intended function. The time to failure or ‘‘life’’ of a system cannot be deterministically defined, and hence it is a random variable. Thus, we must quantify reliability by assigning a probability function to the time to failure random variable.

2.1. Mathematics of Reliability Measures

Let T denote the time to failure random variable. Then reliability at any time t, denoted by $R(t)$, is the probability that the system will not fail by time t, or mathematically:

$$R(t) = P[\mathrm{T} > t] \tag{1}$$

Let $f(t)$ be the probability density function for the failure random variable T. Then the cumulative distribution function $F(t)$ is given by

$$P[T \leq t] = F(t) = \int_0^t f(\tau)d\tau \tag{2}$$

Hence from Eqs. (1) and (2) we have the following fundamental relationships (Eq. 3) between

$$R(t) = 1 - P[T \leq t] = 1 - F(t) = 1 - \int_0^t f(\tau)d\tau \tag{3}$$

the reliability function, cumulative distribution function, and probability density function.

Figures 4(*a*), 4(*b*), and 4(*c*) show, respectively, the failure probability density function, the cumulative distribution function, and the reliability function for the well-known case when the time to failure is exponentially distributed. Here we have

$$f(t) = \lambda e^{-\lambda t} \qquad t \geq 0,\ \lambda > 0 \tag{4}$$

and

$$F(t) = \int_0^t \lambda e^{-\lambda\tau}\, d\tau = 1 \geqslant\mathrm{q}\ e^{-\lambda t} \qquad t \geq 0 \tag{5}$$

$$R(t) = e^{-\lambda t} \qquad t \geq 0 \tag{6}$$

These functions are all related, and selection of any one determines the others. This is obvious from Eqs. 1, 2, and 3 or 4, 5, and 6.

Obviously, the reliability function inherent in a system, by virtue of its design, dictates the probability of successful system operation during the system's fife. A natural question is then, "How does one know the shape of a reliability function for a particular systems?" There are basically three ways in which it can be determined:

1. Test many systems to failure using a mission profile that is identical to use conditions. This would allow one to develop empirically a curve such as that shown in Figure 4(*c*).
2. Test many subsystems and components to failure where use conditions are recreated in the test environment This allows one to develop empirically the component reliability functions and then to derive analytically the system reliability function.
3. Based on past experience with similar systems, the underlying failure distribution may be hypothesized. Then one can test fewer systems to determine the parameters needed to adapt the failure distribution to a particular situation. For example, the lifetime of many different kinds of electronic components follows the exponential distribution as previously given in Eq. 4. To apply this distribution, one must know the value of the parameter λ for a particular situation. Elaborate studies have been done, so that for a given environment and mission, parameter λ can be determined for most electronic components.
4. In some cases, the failure physics involved in a particular situation may lead one to hypothesize a particular distribution. For example, fatigue of certain metals tends to follow either the lognormal or Weibull distributions. Here again, once a distribution is selected, the parameters for a particular application must be ascertained (MIL-HDBK 1974, 1979; Klion 1992).

Another measure that is frequently used as an indirect indicator of system reliability is the MTTF, which is the expected or mean value of the time to failure random variable. Thus, the MTTF is theoretically defined as

$$\mathrm{MTTF} = E[T] = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt \tag{7}$$

Sometimes the term *mean time between failures* (MTBF) when the product can be repaired or renewed is also used to denote $E[T]$. The problem with using only the MTTF as an indicator of

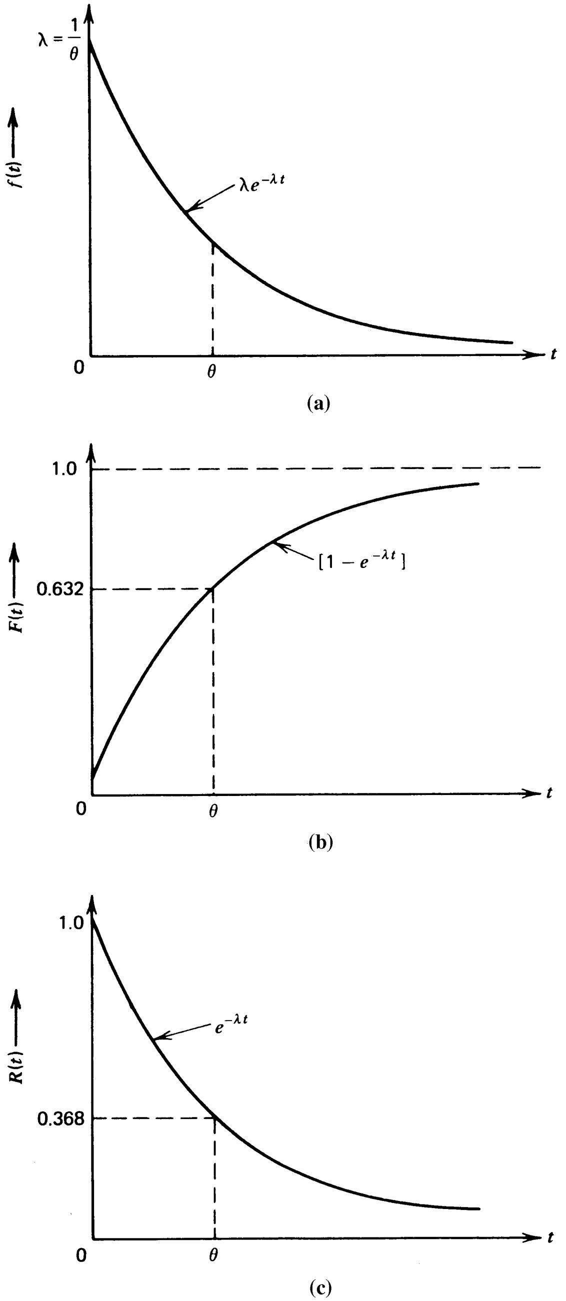


Figure 4 (*a*) Exponential Density Function. (*b*) Exponential Distribution Function. (*c*) Reliability Function for Exponential Distribution.

system reliability is that it uniquely determines reliability only if the underlying time to failure distribution is exponential. If the failure distribution is other than exponential, the MTTF can produce erroneous comparisons and we must develop other moments of the life distribution.

If we have a large population of the items whose reliability we are interested in studying, then for replacement and maintenance purposes we are interested in the rate at which the items in the population that have survived at any specific time will fail. This is the failure rate, or hazard rate, and is given by the following relationship:

$$h(t) = \frac{f(t)}{R(t)} \tag{8}$$

The failure rate for most components follows the curve shown in Figure 3(*a*), which is also called the life characteristic curve.

To help understand the notion of failure rate or hazard rate, basic mathematical relations are given here.

The hazard rate is defined as the limit of the instantaneous failure rate given no failure up to time *t* and is given by

$$\begin{aligned} h(t) &= \lim_{\Delta t \to 0} \frac{P[t < T \leq t + \Delta t | T > t]}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)} \\ &= \frac{1}{R(t)} \left[-\frac{d}{dt} R(t) \right] \\ &= \frac{f(t)}{R(t)} \end{aligned} \tag{9}$$

Also

$$f(t) = h(t) \exp \left[-\int_0^t h(\tau) d\tau \right] \tag{10}$$

and thus

$$R(t) = \exp \left[-\int_0^t h(\tau) d\tau \right] \tag{11}$$

2.2. Various Life Distributions

The properties of some life distributions that are used in reliability and maintainability discipline are given below.

2.2.1. Exponential Distribution

$$f(t) = \lambda e^{-\lambda t} \qquad t \geq 0 \tag{12}$$

$$F(t) = 1 - e^{-\lambda t} \qquad t \geq 0 \tag{13}$$

$$R(t) = e^{-\lambda t} \qquad t \geq 0 \tag{14}$$

$$h(t) = \lambda \tag{15}$$

$$\text{MTBF} = \theta = \frac{1}{\lambda} \tag{16}$$

Thus, the failure rate for the exponential distribution is always constant.

2.2.2. Normal Distribution

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right] \quad -\infty < t < \infty \tag{17}$$

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma}\right) \tag{18}$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right) \tag{19}$$

$$h(t) = \frac{\phi[(t-\mu)/\sigma]}{\sigma R(t)} \tag{20}$$

$$MTBF = \mu \tag{21}$$

Thus, $\Phi(z)$ is the cumulative distribution function and $\phi(z)$ is the probability density function, respectively, for the standard normal variable Z. The failure rate for the normal distribution is a monotonically increasing function. Normal distribution should be used as a life distribution when $\mu > 6\sigma$ because then the probability that T will be negative is exceedingly small. Otherwise, truncated normal distribution should be used.

2.2.3. Lognormal Distribution

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln t-\mu}{\sigma}\right)\right]^2 \quad t \geq 0 \tag{22}$$

$$F(t) = \Phi\left(\frac{\ln t-\mu}{\sigma}\right) \tag{23}$$

$$R(t) = 1 - \Phi\left(\frac{\ln t-\mu}{\sigma}\right) \tag{24}$$

$$h(t) = \phi\left(\frac{\ln t-\mu}{\sigma}\right)\Big/ t\sigma R(t) \tag{25}$$

$$\text{MTBF} = \exp\left(\mu + \frac{\sigma^2}{2}\right) \tag{26}$$

The failure rate for the lognormal distribution is neither always increasing nor always decreasing. It takes different shapes, depending on the parameters μ and σ.

2.2.4. Weibull Distribution (Three Parameters $\theta > \delta$)

$$f(t) = \frac{\beta(t-\delta)^{\beta-1}}{(\theta-\delta)^\beta} \exp\left[-\left(\frac{t-\delta}{\theta-\delta}\right)^\beta\right] \quad t \geq \delta \geq 0 \tag{27}$$

$$F(t) = 1 - \exp\left[-\left(\frac{t-\delta}{\theta-\delta}\right)^\beta\right] \tag{28}$$

$$R(t) = \exp\left[-\left(\frac{t-\delta}{\theta-\delta}\right)^\beta\right] \tag{29}$$

$$h(t) = \frac{\beta(t-\delta)^{\beta-1}}{(\theta-\delta)^\beta} \tag{30}$$

$$\text{MTBF} = \delta + (\theta-\delta)\Gamma\left(1 + \frac{1}{\beta}\right) \tag{31}$$

The failure rate for the Weibull distribution is decreasing when $\beta < 1$, constant when $\beta = 1$ (same as the exponential distribution), and increasing when $\beta > 1$.

2.2.5. Gamma Distribution

$$f(t) = \frac{\lambda^{\eta}}{\Gamma(\eta)} t^{\eta-1} e - \lambda t \qquad t \geq 0 \tag{32}$$

$$F(t) = \sum_{k=\eta}^{\infty} \frac{(\lambda t)^k \exp[-\lambda t]}{k!} \qquad \text{when } \eta \text{ is integer} \tag{33}$$

$$R(t) = \sum_{k=0}^{\eta-1} \frac{(\lambda t)^k \exp[-\lambda t]}{k!} \qquad \text{when } \eta \text{ is integer} \tag{34}$$

$$h(t) = \frac{f(t)}{R(t)} \qquad \text{[Using Eqs. (32) and (34)]} \tag{35}$$

$$MTBF = \frac{\eta}{\lambda} \tag{36}$$

The failure rate for the gamma distribution is decreasing when $\eta < 1$, constant when $\eta = 1$, and is increasing when $\eta > 1$.

2.3. Summary

Properties of extreme value distributions are given by Gumbel (Gumbel 1958). Other mathematical properties of the preceding distributions and their use in the reliability field have been discussed extensively by several authors, and results are available in the literature (Barlow and Proschan 1975; Kapur and Lamberson; 1977; Kececioglu 1991, 1993). To use these distributions in the product assurance field, one must understand their nature and properties and the conditions under which they are applicable to describe various physical phenomena (Bane and Engelhardt 1991; Elsayed 1996). The following discussion on the application of life distributions is given in Kapur (1996b).

Exponential distribution is a good model for the life of a complex system that has a large number of components. Because the exponential distribution has a constant failure rate, it is a good model for the useful life of many products after the end of the infant mortality period. Some applications for the exponential distribution are electrical and electronic systems, computer systems, and automobile transmissions. The *normal distribution* is used to model various physical, mechanical, electrical, or chemical properties of systems. Some examples are gas molecule velocity, wear, noise, chamber pressure from firing ammunition, tensile strength of aluminum alloy steel, capacity variation of electrical condensers, electrical power consumption in a given area, generator output voltage, and electrical resistance. The *lognormal distribution* is a positively skewed distribution and can be used to model situations where large occurrences are concentrated at the tail (left) end of the range. Some examples are amount of electricity used by different customers, downtime of systems, time to repair, light intensities of bulbs, concentration of chemical process residues, and automotive mileage accumulation by different customers. The *two-parameter Weibull distribution* can also be used to model skewed data. When $\beta < 1$, the failure rate for the Weibull distribution is decreasing and hence can be used to model infant mortality or debugging period or for situations when the reliability in terms of failure rate is improving or for reliability growth. When $\beta = 1$, the Weibull distribution is the same as the exponential distribution, and all of the previous comments for the exponential distribution are applicable. When $\beta > 1$, the failure rate is increasing, and hence it is a good model for determining wear out and end-of-useful life period. Some of the examples are corrosion life, fatigue life, life of antifriction bearings, transmission gears, and electronic tubes. The *three-parameter Weibull distribution* is a good model when we have a minimum life and the odds of the component failing before the minimum life are close to zero. Many strength characteristics of systems do have a minimum value significantly greater than zero. Some examples are electrical resistance, capacitance, and fatigue strength.

3. SYSTEM RELIABILITY MODELS

To analyze and measure the reliability and maintainability characteristics of a system, there must be a mathematical model of the system that shows the functional relationships among all the components, the subsystems, and the overall system. The reliability of the system is a function of the reliabilities of its components. A system reliability model consists of some combination of a reliability block diagram or cause–consequence chart, a definition of all equipment failure and repair distributions, and a statement of spare and repair strategies (Kapur 1996a). All reliability analyses and optimizations are made on these conceptual mathematical models of the system.

3.1. Reliability Block Diagram

A reliability block diagram is obtained from a careful analysis of the manner in which the system operates. An analysis has to be done of the effects on overall system performance of failures of the various components; the support environment and constraints, including such factors as the number and assignment of spare parts and repairpersons; and the mission for the system.

Engineering analysis on the system has to be done in order to develop a reliability model. The engineering analysis consists of the following steps:

1. Develop a functional block diagram of the system based on physical principles governing the operations of the system.
2. Develop the logical and topological relationships between functional elements of the system.
3. Performance-evaluation studies are used to determine the extent to which the system can operate in a degraded state.
4. Define the spares and repairs strategies (for maintenance systems).

Based on the preceding analysis, a reliability block diagram is developed, which is used for calculating various measures of reliability and maintainability. The reliability block diagram is a pictorial way of showing all the success or failure combinations for the system. Some of the guidelines for drawing these diagrams are as follows:

1. A group of components that are essential for the performance of the system and/or its mission are drawn in series [Figure 5(*a*)].
2. Components that can substitute for other components are drawn in parallel [Figure 5(*b*)].
3. Each block in the diagram is like a switch: it is closed when the component it represents is working and is opened when the component has failed. Any closed path through the diagram is a success path.

The failure behavior of all the redundant components must be specified. Some of the common types of redundancies are:

1. *Active redundancy* or *hot standby:* The component has the same failure rate as if it was operating in the system.
2. *Passive redundancy, spare,* or *cold standby:* The standby component cannot fail. This is generally assumed of spare or shelf items.
3. *Warm standby:* The standby component has a lower failure rate than the operating component. This is usually a realistic assumption.

Some mathematical relationships between the system reliability and the reliabilities of its components are given in the next subsections. In the following, R_s denotes the reliability of the system, and R_i denotes the reliability of the ith component, where $i = 1, 2, \ldots, n$ and the system has n components. In addition, in the following relationships, it is also assumed that all the components work or fail independently of each other.

3.1.1. Series Configuration [See Figure 5(*a*)]

For the static situation we have

$$R_s = \prod_{i=1}^{n} R_i \tag{37}$$

and for the dynamic situation

$$R_s(t) = \prod_{i=1}^{n} R_i(t) \tag{38}$$

The failure rate $h_s(t)$ for the system is given by

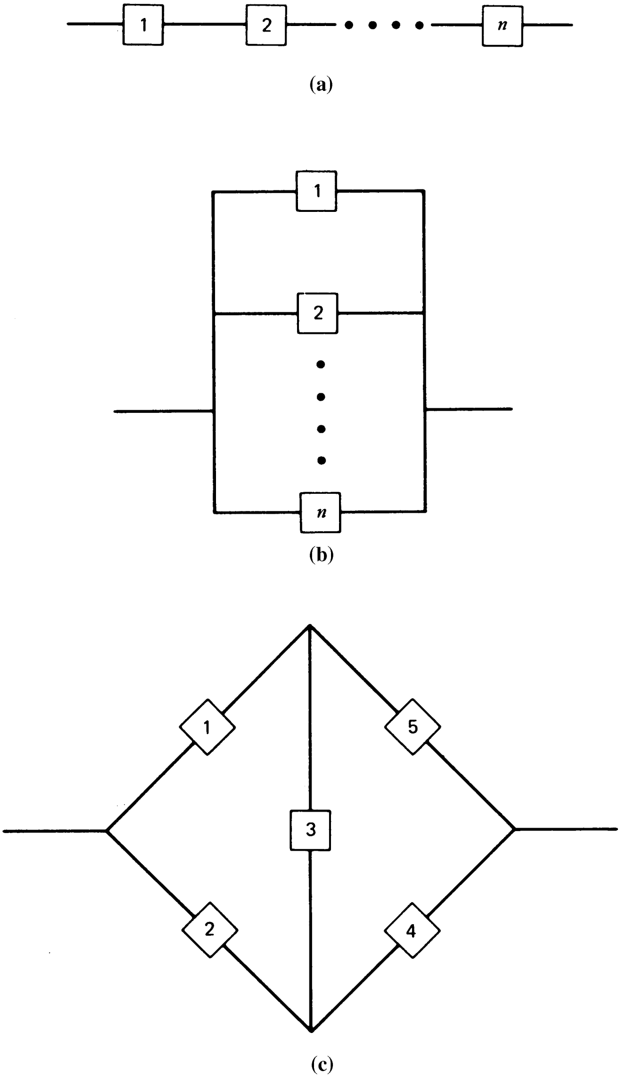


Figure 5 (*a*) Series Configuration. (*b*) Parallel configuration. (*c*) Bridge Structure.

$$h_s(t) = \sum_{i=1}^{n} h_i(t) \tag{39}$$

where $h_i(t)$ is the failure rate of ith component.

If all the components have an exponentially distributed time to failure, we have

$$h_s(t) = \sum_{i=1}^{n} \lambda_i \tag{40}$$

and MTBF for the system is given by

$$\frac{1}{\sum_{i=1}^{n} \lambda_i} \tag{41}$$

3.1.2. *Parallel Configuration* [See Figure 5(*b*)]

For the static case, we have

$$R_s = 1 - \prod_{i=1}^{n} (1 - R_i) \tag{42}$$

and for the dynamic case

$$R_s(t) = 1 - \prod_{i=1}^{n} [1 - R_i(t)] \tag{43}$$

If the time to failures for all the components is exponentially distributed with MTBF θ, then the MTBF for the system is given by

$$\sum_{i=1}^{n} \frac{\theta}{i} \tag{44}$$

where θ = MTBF for every component.

3.1.3. *The* **k**-*out-of-***n** *Configuration*

In this configuration the system works if and only if at least k components out of the n components work, $1 \leq k \leq n$. For this case, when $R_i = R(t)$ for all i, we have

$$R_s(t) = \sum_{i=k}^{n} \binom{n}{i} [R(t)]^i [1 - R(t)]^{n-i} \tag{45}$$

If $R(t) = e^{-t/\theta}$, for exponential case, MTBF for the system is given by

$$\sum_{i=k}^{n} \frac{\theta}{i} \tag{46}$$

3.1.4. *Coherent Systems*

The reliability block diagrams for many systems cannot be represented by the preceding three configurations. In general, the concept of coherent systems can be used to determine the reliability of any system (Barlow and Proschan 1975). The performance of each of the n components in the system is represented by a binary indicator variable, x_i, which takes the value 1 if the ith component functions and 0 if the ith component fails. Similarly, the binary variable ϕ indicates the state of the system, and ϕ is a function of $x = (x_1, \ldots, x_n)$.

The function $\phi(x)$ is called the "structure function" of the system. The structure function is represented by using the concept of minimal path and minimal cut. A minimal path is a minimal set of components whose functioning ensures the functioning of the system. A minimal cut is a minimal set of components whose failures cause the system to fail. Let $\alpha_j(x)$ be the jth minimal path series structure for path A_j, $j = 1, \ldots, p$ and $\beta_k(x)$ be the kth minimal parallel cut structure for cut B_k, $k = 1, \ldots s$. Then we have

$$\alpha_j(x) = \prod_{i \in A_j} x_i \tag{47}$$

$$\beta_k(x) = 1 - \prod_{i \in B_k} (1 - x_i) \tag{48}$$

and

$$\phi(x) = 1 - \prod_{j=1}^{p} [1 - \alpha_j(x)] \tag{49}$$

$$= \prod_{k=1}^{s} \beta_k(x) \tag{50}$$

For the bridge structure [Figure 5(c)] we have four minimal paths and four minimal cuts, and their structure functions are given as

$$\begin{aligned}
\alpha_1 &= x_1x_5 & \beta_1 &= 1 - (1 - x_1)(1 - x_2) \\
\alpha_2 &= x_2x_4 & \beta_2 &= 1 - (1 - x_4)(1 - x_5) \\
\alpha_3 &= x_1x_3x_4 & \beta_3 &= 1 - (1 - x_1)(1 - x_3)(1 - x_4) \\
\alpha_4 &= x_2x_3x_5 & \beta_4 &= 1 - (1 - x_2)(1 - x_3)(1 - x_5)
\end{aligned}$$

Then the reliability of the system is given by

$$R_s = P[\phi(x) = 1] = E[\phi(x)]$$

If R_i is the reliability of the ith component, we have for the bridge structure

$$\begin{aligned}
R_s = {} & R_1R_5 + R_1R_3R_4 + R_2R_3R_5 + R_2R_4 \\
& - R_1R_3R_4R_5 - R_1R_2R_3R_5 - R_1R_2R_4R_5 \\
& - R_1R_2R_3R_4 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5
\end{aligned}$$

If all $R_i = R = 0.9$, we have

$$\begin{aligned}
R_s &= 2R^2 + 2R^3 - 5R^4 + 2R^5 \\
&= 0.9785
\end{aligned}$$

The exact calculations for R_s are generally very tedious because the paths and the cuts are dependent, since they may contain the same component. Bounds on system reliability are given by

$$\prod_{k=1}^{s} P[\beta_k(x) = 1] \le P[\phi(x) = 1] \le - \prod_{j=1}^{p} \{1 - P[\alpha_j(x) = 1]\}$$

Using these bounds for the bridge structure, we have, when $R_i = R = 0.9$,

$$\begin{aligned}
\text{Upper bound on} \quad R_s &= 1 - (1 - R^2)^2(1 - R^3)^2 \\
&= 0.9973 \\
\text{Lower bound on} \quad R_s &= [1 - (1 - R)^2]^2[1 - (1 - R)^3]^2 \\
&= 0.9781
\end{aligned}$$

The bounds on system reliability using the concepts of minimum paths and cuts can be improved. Further details and derivations for coherent systems can also be found in Barlow and Proschan 1975.

4. FAULT TREE ANALYSIS

Fault tree analysis is one of the methods for system safety and reliability analysis (Henley et al. 1992). The concept was originated by Bell Telephone Laboratories as a technique for safety evaluation of the Minuteman Launch Control System. Many reliability techniques are inductive and are concerned primarily with ensuring that hardware will accomplish its intended functions. Fault tree analysis is a detailed deductive analysis that usually requires considerable information about the system. It is concerned with ensuring that all critical aspects of a system are identified and controlled. It is a graphical representation of Boolean logic associated with the development of a particular system failure (consequence), called the top event, to basic failures (causes), called primary events. These top events can be broad, all-encompassing events, such as release of radioactivity from a nuclear power plant or inadvertent launch of an ICBM missile, or they can be specific events, such as failure to insert control rods or energizing power available to ordnance ignition line.

Fault tree analysis is of value in:

1. Providing options for qualitative and quantitative reliability analysis
2. Helping the analyst to understand system failures deductively
3. Pointing out the aspects of a system that are important with respect to the failure of interest
4. Providing the analyst an insight into system behavior

A fault tree is a model that graphically and logically represents the various combinations of possible events, both fault and normal, occurring in a system that lead to the top event. A fault event is an abnormal system state. A normal event is an event that is expected to occur. The term *event* denotes a dynamic change of state that occurs in a system element. System elements include hardware, software, human, and environmental factors. Details about the construction of fault trees can be found in Henley et al. (1992).

5. ALLOCATION OF RELIABILITY REQUIREMENTS

Reliability and design engineers must translate overall system performance, including reliability, into component performance, including reliability. The process of assigning reliability requirements to individual components in order to attain the specified system reliability is called reliability allocation. There are many different ways in which reliability can be allocated in order to achieve this end.

The allocation problem is complex for several reasons, including: (1) the role a component plays for the functioning of the system; (2) the methods available for accomplishing this function; (3) the complexity of the component; and (4) the reliability of the component, which may change with the type of function to be performed. The problem is further complicated by the lack of detailed information on many of these factors early in the system design phase. However, a tentative reliability allocation must be accomplished in order to guide the design engineer. The typical decision process from a reliability allocation standpoint is illustrated in Figure 6. A process such as this attempts to force all concerned to make decisions in an orderly and knowledgeable fashion rather than on an ad hoc basis.

Some of the advantages of the reliability allocation process are:

1. The process forces system design and development personnel to understand and develop the relationships among component, subsystem, and system reliabilities. This leads to an understanding of the basic reliability problems inherent in the design.
2. The design engineer is obliged to consider reliability equally with other system parameters, such as weight, cost, and performance characteristics.
3. Reliability allocation ensures adequate design, manufacturing methods, and testing procedures.

The allocation process is approximate, and the system effectiveness parameters, such as reliability and maintainability apportioned to the subsystems, are used as guidelines to determine design feasibility. If the allocated parameters for a system cannot be achieved using the current technology, then the system must be modified and the allocations reassigned. This procedure is repeated until an allocation is achieved that satisfies the system requirements.

Various allocation algorithms for reliability and availability requirements are available (Von Alven 1964; Kapur and Lamberson 1977).

6. DESIGN FOR RELIABILITY

Reliability is a time-oriented quality characteristic (Kapur 1986) and is defined and evaluated by the customer just like any other quality characteristic. Inherent reliability is a function of system concept and its design. After a design has been completed and released for manufacturing, the maximum reliability level of the system has been determined by virtue of its design. Essentially, the reliability effort is over once the design is released, and all that quality control during manufacturing phase can do is to ensure that this reliability level does not degrade during manufacturing. From a reliability standpoint, quality control during manufacturing is after the fact and thus is too late to consider reliability. Thus, the reliability effort must be an integral part of system design and development because this is where the reliability level is established. Reliability activities that can be performed during system design and development are described briefly in the remainder of this section.

To ensure most economically and effectively the production of a reliable product, the reliability activities must start early in the product-development cycle. However, at this stage, the identification of reliability improvements depends heavily on the experience of the personnel studying the product from blueprints and preliminary system mock-ups because no hard data are available for a quantitative assessment of reliability. To consider reliability early in the design cycle, one must rely on a formalized design review procedure, which is now briefly explained.

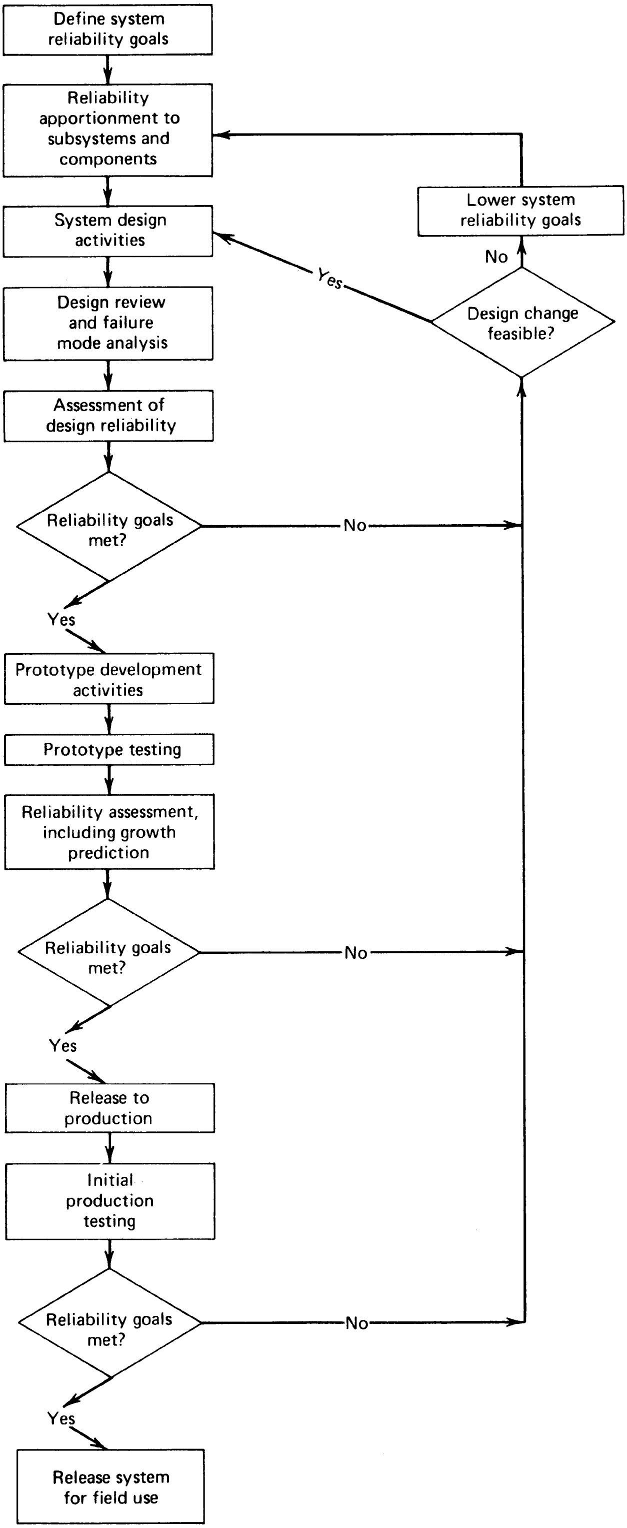


Figure 6 Reliability Allocation Process.

6.1. Design Review

The design review, a formal and documented review of a system design, is conducted by a committee of senior company personnel who are experienced in various pertinent aspects of product design, reliability, manufacturing, materials, stress analysis, human factors, safety, logistics, maintenance, and so on. The design review extends over all phases of product development, from conception to production. In each phase, previous work is updated, and the review is based on current information.

A mature design requires trade-offs between many conflicting factors, such as performance, manufacturability, reliability, and maintainability. These trade-offs depend heavily on experienced judgment and require continuous communication among experienced reviewers. The design review committee approach has been found to be extremely beneficial to this process. The committee adopts the system's point of view and considers all conceivable phases of design and system use to ensure that the best trade-offs have been made for the particular situation.

A complete design review procedure must be multiphased in order to follow the design cycle until the system is released for production. A typical example of a review committee, including personnel and their responsibilities, is shown in Table 1. Here the review process has been subdivided into three phases, and each phase is an update of more detailed analysis based on the latest knowledge.

Ultimately the design engineer has the responsibility for investigating and incorporating the ideas and suggestions posed by the design review committee. The committee's chairperson is responsible for adequately reporting all suggestions by way of a formal and documented summary. The design engineer then can accept or reject various points in the summary; however, he or she must formally report back to the committee, stating reasons for his or her actions.

It should be recognized that considerably more thought and detail than the basic philosophy presented here must go into developing the management structure and procedures for conduct in order to have a successful review procedure. It should be noted that this review procedure considers not only reliability, but all important factors in order to ensure that a mature design will result from the design effort.

In the next subsection, attention is focused on a technique that has been proven effective in identifying failure situations early in the design cycle and before product testing.

TABLE 1 Design Review Committee

	Review Phase			
Member	1	2	3	Responsibility
Chairperson	x	x	x	Ensure that review is conducted in an efficient fashion. Issue major reports and monitor follow-up.
Customer and/or marketing representative	x	x	x	Ensure that the customer's viewpoint is adequately presented (especially at the design trade-off stage)
Design engineer (of this product)	x	x	x	Prepare and present initial design with calculations and supporting data.
Design engineer (not of this product)	x	x	x	Review and verify adequacy of design.
Reliability engineer	x	x	x	Evaluate design for maximum reliability consistent with system goals.
Manufacturing engineer		x	x	Ensure manufacturability at reasonable cost. Check for tooling adequacy and assembly problems.
Materials engineer		x		Ensure optimum material usage considering application and environment.
Stress analyst		x		Review and verify stress calculations.
Quality control engineer		x	x	Review tolerancing problems, manufacturing capability, inspection strategies, and testing problems.
Human factors engineer		x		Ensure adequate consideration to human operator, identification of potential human-induced problems.
Safety engineer		x		Ensure safety to operating and auxiliary personnel.
Maintainability engineer		x	x	Analyze for ease of maintenance repair and field servicing problems.
Logistics engineer		x	x	Evaluate and specify logistic support. Identify logistics problems.

6.2. Failure Mode and Effects Analysis

Failure mode and effects analysis is a design-evaluation procedure used to identify all conceivable and potential failure modes and determine the effect of each failure mode on system performance. This procedure is accomplished by formal documentation, which serves (1) to standardize the procedure, (2) as a means of historical documentation, and (3) as a basis for future improvement.

The procedure consists of a sequence of logical steps, starting with the analysis of lower-level subsystems or components. The analysis assumes a failure point of view and identifies all potential modes of failure along with the causative agent, termed the "failure mechanism." The effect of each failure mode is then traced up to the systems level (MIL-STD-1969 1974).

A criticality rating is developed for each failure mode and resulting effect. The rating is based on the probability of occurrence, severity, and detectability. For failures scoring high on this rating, design changes to reduce criticality are recommended. This procedure is aimed at providing a better design from a reliability standpoint.

6.3. Probabilistic Approach to Design

Reliability is basically a design parameter and has to be incorporated in the system at the design stage. One way to quantify reliability during design and design for reliability is the probabilistic approach to design (Kececioglu and Cormier 1968; Kececioglu 1991, 1995; Haugen 1968). The design variables and parameters are random variables, and hence the design methodology must consider them as random variables. The reliability of any system is a function of the reliabilities of its components. To analyze the reliability of the system, we first have to understand how to compute the reliabilities of the components. The basic idea in reliability analysis from the probabilistic design methodology viewpoint is that a given component has certain strength that, if exceeded, will result in the failure of the component. The factors that determine the strength of the component are random variables, as are the factors that determine the stresses or loading acting on the component. "Stress" is used to indicate any agency that tends to induce failure, whereas "strength" indicates any agency resisting failure. "Failure" is taken to mean failure to function as intended; it occurs when the actual stress exceeds the actual strength for the first time.

Let $f(x)$ and $g(y)$ be the probability density functions for the stress random variable X and the strength random variable Y, respectively, for a certain mode of failure. Also, let $F(x)$ and $G(y)$ be the cumulative distribution functions for the random variables X and Y, respectively. Then the reliability R of the component for the failure mode under consideration, with the assumption that the stress and the strength are independent random variables, is given by

$$R = P\{Y > X\} \tag{51}$$

$$= \int_{-\infty}^{\infty} g(y) \left\{ \int_{-\infty}^{y} f(x)dx \right\} dy \tag{52}$$

$$= \int_{-\infty}^{\infty} g(y)F(y)dy \tag{53}$$

$$= \int_{-\infty}^{\infty} f(x) \left\{ \int_{x}^{\infty} g(y)dy \right\} dx \tag{54}$$

$$= \int_{-\infty}^{\infty} f(x)\{1 - G(x)\{dx \tag{55}$$

For example, suppose that the stress random variable X is normally distributed with mean value of μ_x, and standard deviation (SD) of σ_x, and that the strength random variable Y is also normally distributed with parameters μ_y and σ_y. The reliability R is then given by

$$R = \Phi \left(\frac{\mu_y - \mu_x}{\sqrt{\sigma_y^2 + \sigma_x^2}} \right) \tag{56}$$

where $\Phi(z)$ is the cumulative distribution function for the standard normal variate Z.

The reliability computations for other distributions, such as exponential, lognormal, gamma, Weibull, and extreme value distributions, have also been developed (Kapur and Lamberson 1977). In addition, the reliability analysis has been generalized when the stress and strength variables follow a known stochastic process. The references cited in this subsection also contain simple design examples illustrating the use of the probabilistic approach to design.

7. HUMAN FACTORS IN RELIABILITY

All systems are of, by, and for humans. Human factors therefore become actively important in the system design process and consequently must be weighed against safety, reliability, maintainability, and other system parameters in order to obtain trade-offs to increase system effectiveness. Human interaction with the system of interest consists of:

1. Design and production of systems
2. Operators and repairers of systems
3. Operators and repairers as decision elements

Man–machine interface consists of such aspects as allocation of functions (man vs. machine), automation, accessibility, human tasks, stress characteristics, information presented to the human, and the reliability of interfaces coupled with the decisions on the basis of such information. Both human and machine elements of a system can fail, and their failures have varying effects on the system's performance. Some human errors cause total system failure or increase the risk of such failure. Human factors exert a strong influence on the design and ultimate reliability of a system (Kirwan 1994).

Both reliability and human factors are concerned with predicting, measuring, and improving system effectiveness. When the man–machine interface is complex, the possibility of human error increases, which results in an increase in the probability of system failure. An interesting facet of the human factors–reliability–maintainability relationship is that the system's reliability–maintainability depends on the detection and correction of system malfunctions. This task is generally performed by humans. Thus, the system performance can be enhanced or degraded depending on the human response. The quantification of human reliability characteristics and the development of a methodology for quantifying human performance, error prediction, control, and measurement are given in many sources (Gertman and Blackman 1994; Meister 1996).

Reliability of a system is affected by the allocation of system functions to man, machine, or both. Characteristics tending to favor humans are:

1. Ability to detect certain forms of energy
2. Sensitivity to a wide variety of stimuli within a restricted range
3. Ability to detect signals and patterns in high-noise environments
4. Ability to store large amounts of information for long periods and remember relevant facts
5. Ability to profit from experience
6. Ability to use judgment
7. Ability to improvise and adopt flexible procedures
8. Ability to arrive at new and completely different solutions to problems
9. Ability to handle low-probability or unexpected events
10. Ability to perform fine manipulations
11. Ability to reason instinctively

Characteristics tending to favor machines are:

1. Computing ability
2. Performance of routine, repetitive, and precise tasks
3. Quick response to control signals
4. Ability to exert large amounts of force smoothly and precisely
5. Ability to store and recall large amounts of data for short periods
6. Ability to reason deductively
7. Insensitivity to extraneous factors
8. Ability to handle highly complex operations that involve doing several things at once

8. RELIABILITY MEASUREMENT

Reliability measurement techniques provide a common discipline that can be used to measure, predict, and evaluate system reliability throughout the system life cycles. The two major components of the reliability measurement system are the test program and the data system. Test programs have to be developed throughout the life cycle, and the test effort has to ensure that the reliability goals are met at different stages in the cycle. Procedures for gathering the data generated throughout all the phases must be documented in sufficient detail for complete identification and integration into the data-processing system.

8.1. Test Programs

Figure 7 shows a sequence of different types of tests that may be used throughout the life cycle, consisting of design, development, production, and service/field use phases. Brief descriptions of these tests follow.

8.1.1. Design Support Tests

These tests are used to determine the need for parts, materials, and component evaluation or qualification to meet system performance and other reliability design criteria. Some of the objectives are:

Parts application data
Parts evaluation
Parts qualification
Parts comparative evaluation
Vendor control

8.1.2. Design-Verification Tests

These tests are used to verify the functional adequacy of design and to corroborate preliminary predictions and failure mode and effects analysis that disclose high-risk areas and reliability problems in the proposed design. Design-verification tests fulfill the following essential design phase functions:

Analytical verification
Functional evaluation
Parts and materials definition
Preliminary reliability verification

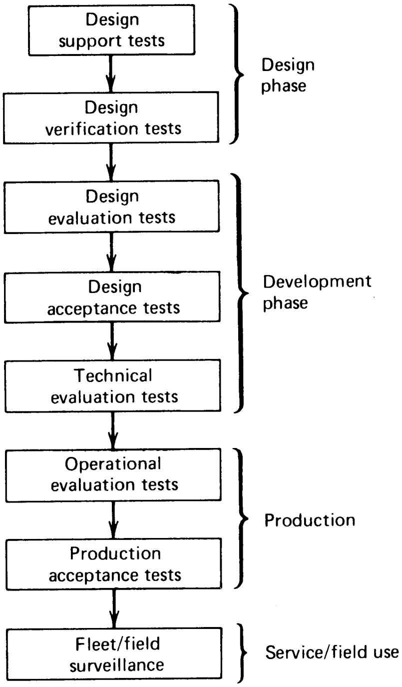


Figure 7 Integrated Test Flow Diagram.

8.1.3. Design-Evaluation Tests

These tests are used to evaluate the design under environmental conditions, verify the compatibility of subsystem interfaces, and review the design from the maintainability point of view. Some of the tests under this category are:

Environmental (evaluation tests)
Longevity (failure rates) tests
Operability tests
Engineering change-evaluation tests

8.1.4. Design-Acceptance Tests

These tests are used to demonstrate that design meets required levels of reliability. Thus, a reliability demonstration test is considered mandatory for design acceptance. Definitions of test requirements are:

1. Define acceptable levels of reliability and decision risks (type 1 and type 2 errors, confidence limits).
2. Define test conditions.
3. Define the specific test plan:
   - **(a)** MTBF tests
   - **(b)** Mission reliability test
   - **(c)** Availability tests
4. Define "failure" and scoring criteria:
   - **(a)** System failures
   - **(b)** Mission failures
   - **(c)** Maintenance actions (chargeable, nonchargeable)
   - **(d)** Criticality factors
5. Determine sample size.

8.1.5. Technical Evaluation Tests

These tests are used to evaluate the technical suitability of a prototype or a preproduction model. It is sometimes practical to integrate technical evaluation with operational evaluation when the earlier design-acceptance test demonstrates complete conformance.

8.1.6. Operational Evaluation Tests

These tests are used to evaluate operational suitability of the production model.

8.1.7. Production Acceptance Tests

These tests are used to determine the acceptability of individual production items in order to ensure production control and critical interfaces, parts, and material quality. Manufacturing operations that result in significant reliability degradation should be carefully studied.

8.1.8. Fleet/Field Surveillance Tests

These tests and evaluation programs during the field use of the product are for the continuing assessment of reliability and quality.

8.1.9. Test Procedures

Any test procedure must consider the following factors:

1. Purpose of test
2. Test items—description and sample selection
3. Test monitoring and review procedures
4. Test equipment requirements
5. Test equipment-calibration procedures
6. Test equipment proofing
7. Environmental conditions to be applied

8. Operating conditions
9. Test-point identification
10. Definition of failures and scoring criteria
11. Procedure for conducting tests
12. Test report procedures and documents

8.2. Reliability Estimation

Reliability measurement tests are used to make estimates of the reliability of a system or a population of items. Parametric and nonparametric estimates are used. Parametric estimates are based on a known or assumed distribution of the system characteristic of interest. The parameters are the constants that describe the shape of the distribution. Nonparametric estimates are used without assuming the nature of the underlying probability distribution. Generally, nonparametric estimates are not as efficient as parametric estimates. Nonparametric reliability estimates apply only to a specific test interval and cannot be extrapolated. Parametric estimates are described in this section when the underlying distribution is exponential and Weibull. The three types of parametric estimates that are frequently used are:

1. Point estimate: a single-valued estimate of a reliability measure
2. Interval estimate: an estimate of an interval that is believed to contain the true value of the parameter
3. Distribution estimate: an estimate of the parameters of a reliability distribution

8.2.1. Reliability Estimation: Exponential Distribution

The two types of test procedures considered here are:

1. Type 1 censored test: The items are tested for a specified time T, and then the testing is stopped.
2. Type 2 censored test: The test time is not specified, but the testing is stopped when a desired number of items fail.

Let us consider the situation when n items are placed on test and the test is stopped as soon as r failures are observed ($r \leq n$). This is type 2 censoring with nonreplacement of items. Let the observed failure times be, in order of magnitude,

$$0 = t_0 < t_1 < t_2 < \dots < t_{r-1} < t_r \tag{57}$$

Then, making the transformation,

$$u_i = \begin{cases} nt_i & i = 0 \\ (n - i)(t_{i-1} - t_i) & i = 1, 2, \dots, r - 1 \end{cases} \tag{58}$$

it is well known that the (u_i, $i = 0, \dots, r - 1$) are independently and identically distributed with common density function

$$\left(\frac{1}{\theta}\right) e^{-u/\theta}$$

It is clear that the total time on test is given by

$$\begin{aligned} V(t_r) &= \text{total time on test} \\ &= \sum_{i=0}^{r-1} u_i \\ &= \sum_{i=1}^{r} t_i + (n - r)t_r \end{aligned} \tag{59}$$

Then

$$\hat{\theta} = \frac{V(t_r)}{r} = \frac{1}{r}\left[\sum_{i=1}^{r} t_i + (n - r)t_r\right] \tag{60}$$

is the minimum variance unbiased estimator of θ. Since $V(t_r) = \Sigma_{i=0}^{r-1} u_i$ and the $\{u_i\}$ are independently distributed with a common exponential density function, it follows that $V(t_r)$ has a gamma distribution with parameters (θ, r). Hence $2V(t_r)/\theta = 2\hat{\theta}r/\theta$ is distributed as χ^2_{2r}.

The 100(1 − α)% confidence limits on θ are given by (Bane and Engelhardt 1991; Kececioglu 1993; Kapur and Lamberson 1977):

$$P\left[\chi^2_{1-(\alpha/2),2r} \le \frac{2\hat{\theta}r}{\theta} < \chi^2_{\alpha/2,2r}\right] = 1 - \alpha$$

or

$$\frac{2\hat{\theta}r}{\chi^2_{\alpha/2,2r}} \le \theta \le \frac{2\hat{\theta}r}{\chi^2_{1-(\alpha/2),2r}} \tag{61}$$

The life-testing procedures are often used in a quality control context in which we wish to detect the deviations of θ below some desired levels, say, θ_0. Then for a significance level of α, the probability of accepting H_0 is

$$P_a = P\left(\frac{2r\hat{\theta}}{\theta_0} \le \chi^2_{\alpha,2r} | \theta = \theta_0\right) = 1 - \alpha \tag{62}$$

The expected time to complete the test is given by

$$E(t_r) = \theta \sum_{i=1}^{r} \frac{1}{n - i + 1} \tag{63}$$

Let

θ_0 = desired reliability goal for MTBF
$1 - \alpha$ = probability of accepting items with true MTBF of θ_o
θ_1 = alternative MTBF ($\theta_1 < \theta_0$)
β = probability of accepting items with true MTBF of θ_1

With this information, reliability testing consists of putting *n* items on test and stopping the test when the number of failures is given by the smallest integer satisfying

$$\frac{\theta_1}{\theta_0} \le \frac{\chi^2_{\alpha,2r}}{\chi^2_{1-\beta,2r}} \tag{64}$$

Thus, when we know θ_0, θ_1, α, and β, we can compute the necessary value for r.

For the type 1 censored test, where *r* failures are observed on an interval of total test time T, the 100(1 − α)% confidence limits on θ are given by [a modification of Eq. (61)]

$$\frac{2T}{\chi^2_{\alpha/2,2(r+1)}} \le \theta \le \frac{2T}{\chi^2_{1-(\alpha/2),2r}} \tag{65}$$

8.2.2. Reliability Estimation: Weibull Distribution

Weibull distribution (Weibull 1961) is probably one of the most widely used distributions in life-testing applications. One of the reasons is the ease with which graphic procedures can be used to estimate the parameters of the Weibull distribution and thus the reliability of the product. Confidence limits can also be easily developed. In addition, various statistical estimation procedures have recently been developed, and these can also be easily used by reliability engineers (Nelson 1990; Abernethy 1996; Kapur and Lamberson 1977).

The density function for the Weibull time to failure random variable is given by [see Eq. (27)]

$$f(t) = \frac{\beta(t - \delta)^{\beta-1}}{(\theta - \delta)^{\beta}} \exp\left[-\left(\frac{t - \delta}{\theta - \delta}\right)^{\beta}\right] t \geq \delta > 0 \tag{66}$$

where

β = shape parameter or the Weibull slope, $\beta > 0$
θ = scale parameter or the characteristic life
δ = location parameter or the minimum life

If the minimum life $\delta = 0$, the cumulative distribution is given by

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right] \tag{67}$$

After rearranging and talking twice the natural logarithm, we have

$$\ln\left[\ln\frac{1}{1 - F(t)}\right] = \beta \ln t - \beta \ln \theta$$

or

$$\ln t = \frac{1}{\beta} \ln\left[\ln\frac{1}{1 - F(t)}\right] + \ln \theta \tag{68}$$

Weibull graph paper is constructed by plotting ln t as the horizontal axis vs. $\ln\{\ln(1/[1 - F(t)]\}$ as the vertical axis, and then β is the slope of the straight-line plot. Figure 8 shows such Weibull paper. Various plotting procedures as well as statistical estimation methods with tables are available (Mann et al. 1974). Mann et al. offer point and interval estimation procedures for various other distributions and also tables that can be used for statistical estimation of the parameters of the Weibull distribution. This source is also a good reference for testing reliability hypothesis. The statistical methods discussed in this section can also be applied for accelerated life testing, which consists of a variety of test methods for shortening the life of products or hastening degradation of their performance (Nelson 1990). The aim of such testing is to obtain data quickly that, properly modeled and analyzed, give the proper information on product life under normal operating conditions by the customer.

9. MAINTAINABILITY

Maintainability is one of the system design parameters that has a great impact on the effectiveness of the system (Kececioglu 1995). Failures will occur no matter how reliable a system is made to be. A system's ability to be maintained, that is, retained in or restored to effective usable condition, is often as important to system effectiveness as is its reliability. Maintainability is a characteristic of the system and its design just like reliability. It is concerned with such system attributes as accessibility to failed parts, diagnosis of failures, repairs, test points, test equipment and tools, maintenance manuals, displays, and safety. Maintainability may be defined as a characteristic of design and installation that imparts to a system a great inherent ability to be maintained, so as to lower the required maintenance person-hours, skill levels, tools, test equipment, facilities, and logistics costs and thus achieve greater availability.

9.1. Maintainability Measures

Maintainability is the probability that a system in need of maintenance will be retained in or restored to a specified operational condition within a given period. Thus, the underlying random variable is the maintenance time. Let T be the repair time random variable. Then the maintainability function $M(t)$ is given by

$$M(t) = P[T \leq t] \tag{69}$$

If the repair time T follows the exponential distribution with mean time to repair, MTTR, of $1/\mu$, where μ is the repair rate, then

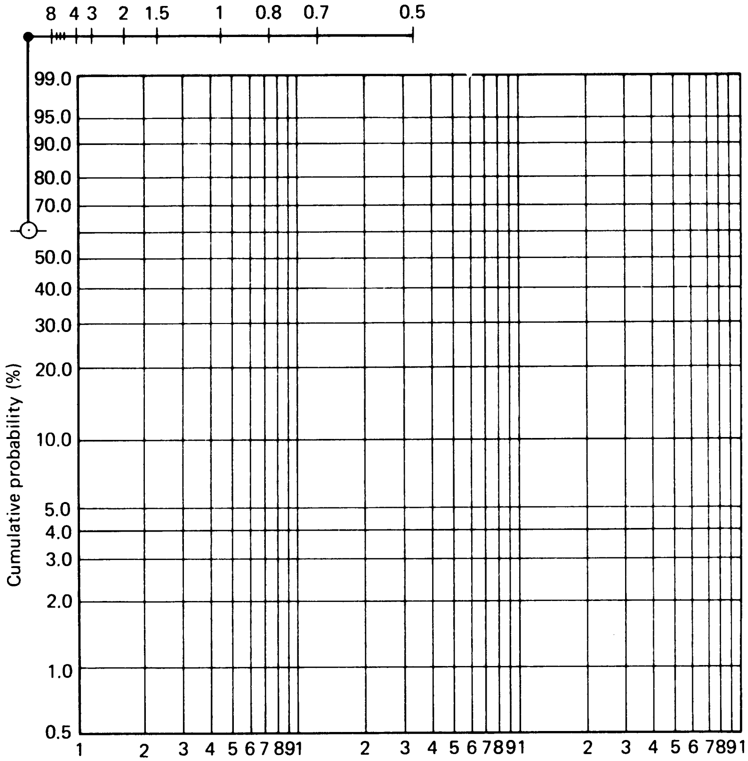


Figure 8 Weibull Probability Paper.

$$M(t) = 1 - \exp\left(-\frac{t}{\mathrm{MTTR}}\right) \tag{70}$$

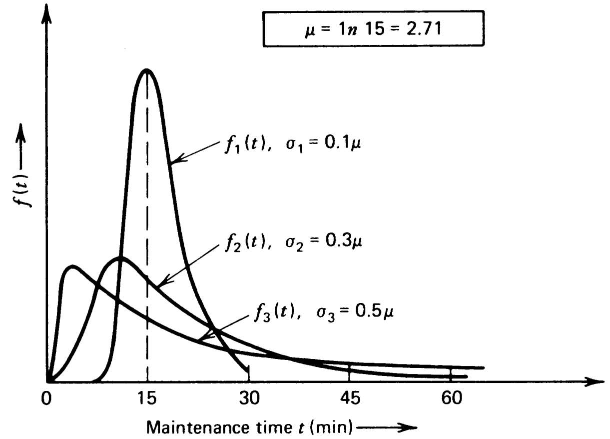
Various other distributions, such as lognormal, Weibull, and normal, are used to model the repair time. In addition, other time-related indices, such as median (50th percentile) and $\mathrm{M}_{\mathrm{MAX}}$ (90th or 95th percentile), are used as maintainability measures. The lognormal probability density functions with an MTTR of 15 min, but with different values for SD, are given in Figure 9(*a*), whereas the associated maintainability functions are shown in Figure 9(*b*). From the maintainability function plot, different percentiles, such as the 90th, can be easily read. In other instances, the maintenance person-hours per system operating hours or maintenance ratio MR may be specified and maintainability design goals then derived from such specifications.

The MTTR, which is the mean of the distribution of system repair time, may be evaluated by

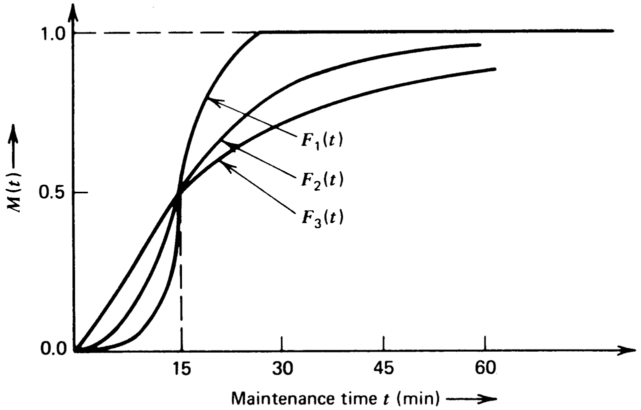
$$\mathrm{MTTR} = \frac{\sum_{i=1}^{n} \lambda_i t_i}{\sum_{i=1}^{n} \lambda_i} \tag{71}$$

where n = number of components in the system
λ_i = failure rate of ith repairable component
t_i = time required to repair the system when the ith component fails

In addition, other measures, such as mean active corrective maintenance time and mean active preventive maintenance time, are used to measure maintainability. Some of the components of the corrective maintenance tasks are:



(a)



(b)

Figure 9 (*a*) Lognormal Probability Density Functions. (*b*) Maintainability Function $M(t)$ Based on Lognormal Distribution with median equal to 15 min.

Localization: determining the location of a failure to the extent possible, without using accessory equipment

Isolation: determining the location of a failure by the use of accessory test equipment

Disassembly: disassembling the equipment to gain access to the item being replaced

Interchange: removing the failed item and installing the replacement

Alignment: performing any alignment, testing, and adjustment made necessary by the repair action

Checkout: performing checks or tests or verify that the equipment has been restored to a satisfactory operating condition

As an example of MTTR computation, assume that a communication system consists of five assemblies, with the data given in Table 2. Column 2 gives the number of units n_i for assembly i. Column 3 indicates the failure rate per thousand hours for each unit. Thus, column 4 gives us the total failure rate for an assembly i. Column 5 gives the average time to perform all the maintenance actions discussed previously. Then, MTTR is given by

$$\text{MTTR} = \frac{\sum n_i \lambda_i t_i}{\sum n_i \lambda_i} = \frac{63.5}{161} = 0.394 \text{ hr} \tag{72}$$

TABLE 2 Worksheet for MTTR Prediction

Assemblies	n_I	λ_i ($\times 10^3$)	$n_i\lambda_i$ ($\times 10^3$)	t_i (hr)	Repair Time per 10^3 hr ($n_i\lambda_i$ t_i)
1	4	10	40	0.10	4.0
2	6	5	30	0.20	6.0
3	2	8	16	1.00	16.0
4	1	15	15	0.50	7.5
5	5	12	60	0.50	30.0
			$\Sigma = 161$		$\Sigma = 63.5$

10. AVAILABILITY

Availability is the vehicle that translates measures of reliability and maintainability into a combined index of effectiveness for a system. It is based on the question “Is the equipment available in a working condition when it is needed?” Availability analysis can be used for trading between and establish requirements for reliability and maintainability.

10.1. Availability Measures

By its very nature, availability measures are time related. The breakdown of total time upon which the availability analyses are based was briefly described earlier in this chapter. The time elements are:

1. Storage, free, and off time
2. Operating time
3. Standby time—availability for operations
4. Downtime, which consists of corrective and preventive maintenance and is also due to administrative and logistics delays

Measures for operational readiness are based on all the time elements. However, the availability measures do not consider the off time, including storage and free time. Achieved availability (A_a) is used for development and initial production testing where the system is not operating in its intended operational environment and is equal to operating test time divided by operating test time plus total preventive and corrective maintenance time (clock time). Excluded are operator before-and-after operating checks and supply, administration, and waiting time. Standby time is excluded both by definition and by environment. Thus,

$$A_a = \frac{\text{OT}}{\text{OT} + \text{TPM} + \text{TCM}} \tag{73}$$

where OT = operating time
TPM = total preventive maintenance time
TCM = total corrective maintenance time

Operational availability (A_o) covers all segments of the time that the system should be operative. Thus, we must consider standby time (ST) as well as administrative and logistics delay time (ALDT). Hence,

$$A_o = \frac{\text{OT} + \text{ST}}{\text{OT} + \text{ST} + \text{TPM} + \text{TCM} + \text{ALDT}} \tag{74}$$

Sometimes there is a need to define the availability with respect to operating time and corrective maintenance when the system is operating in an ideal support environment. This form of availability, called inherent availability (A_I), is useful for determining certain figures of merit for the system per se, such as frequency and type of failure occurrence, repairability (active repair time), and analysis of maintenance actions. Thus, A_I is given by

$$A_I = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{75}$$

Standby time, delay times associated with scheduled or preventive maintenance, and administrative and logistics downtime are excluded.

10.2. Reliability–Maintainability–Availability Trade-Off

The system availability A_I, is a function of variables of reliability (MTBF) and maintainability (MTTR) as given by Eq. (75). Since MTBF = $1/\lambda$ where λ is the failure rate and MTTR = $1/\mu$ where μ is the repair rate (both valid when the underlying distribution is exponential), Eq. (75) may be rewritten as

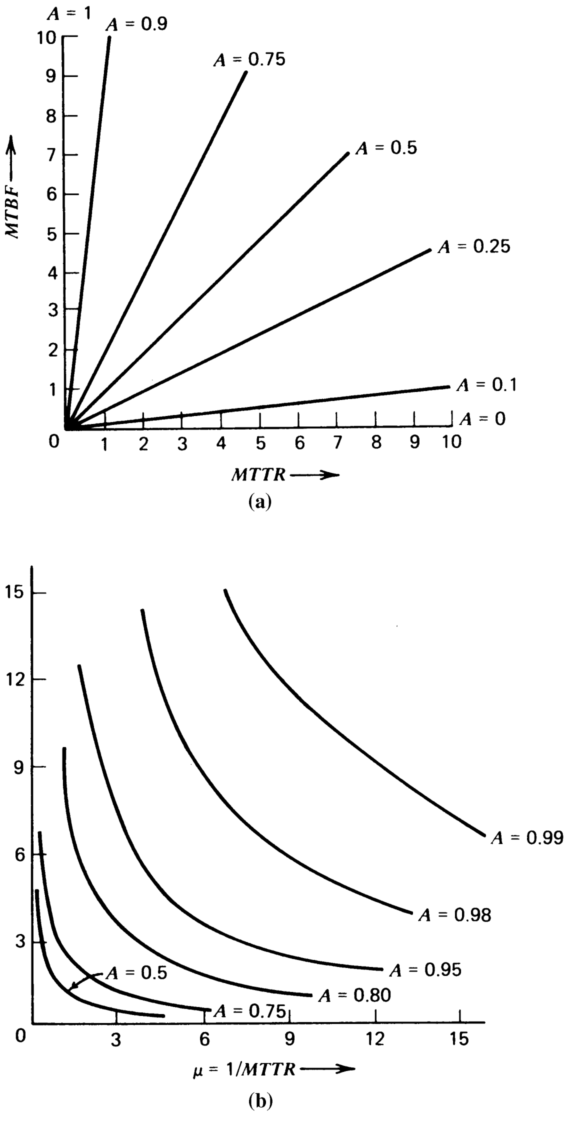


Figure 10 (*a*) Availability (*A*) as a Function of MTBF and MTTR. (*b*) Availability as a function of MTBF and Repair Rate.

$$A_I = A = \frac{\mu}{\mu + \lambda} \tag{76}$$

A generalized plot of Eq. (75) is given in Figure 10(*a*). This shows that to optimize availability, it is desirable to make the ratio of MTBF to MTTR as high as possible. Since increasing MTBF and decreasing MTTR is desirable, the equation for availability is plotted in terms of MTBF and $\mu = 1/\text{MTTR}$, as shown in Figure 10(*b*). Each of the curves representing the same availability is called an isoavailability contour; corresponding values of MTBF and MTTR give the same value of A, all other things being equal. Based on various physical, technological, and economic constraints, tradeoff optimization models can be developed. There are practical limits as to how high a value for MTBF can be achieved or how low MTTR can be made. Increasing MTBF may require the redundancy level to be so high that the desired reliability could not be realistically achieved within the state of the art or else the cost would be high. Low values for MTTR would require excellent maintainability design features, such as complete built-in test features, automatic fault isolation, and automatic switchover from a failed to a standby item.

11. RELIABILITY GROWTH

As the product goes through the various steps in the life cycle, its reliability should be estimated and predicted. These values, when plotted at selected points in the life cycle, result in a growth curve, as shown in Figure 11, that reflects the comparative levels of reliability.

Reliability growth represents the effort spent to achieve the reliability potential either during design and development or during production or subsequently during field and operational use. During early development the achieved reliability of a prototype is much lower than its predicted reliability because of initial design and engineering deficiencies as well as manufacturing flaws. Also, the reliability of a fielded system is much lower than its inherent or potential reliability predicted during design and development for the following reasons:

1. Reliability degradation due to manufacturing, assembly, and quality control errors as well as to ineffectiveness of some of the screening tests.
2. Reliability degradation due to interaction of man, machine, and environment. Degradation may be due to rough handling, extended duty cycles, or neglected maintenance.
3. There is degradation due to maintenance activities because excessive handling brought about by frequent preventive maintenance or poor maintenance practices reduces reliability.

However, during all phases there is also reliability growth due to the underlying learning process. There is reliability growth during design and development as a result of an iterative design process. The essential elements involved in achieving reliability growth are:

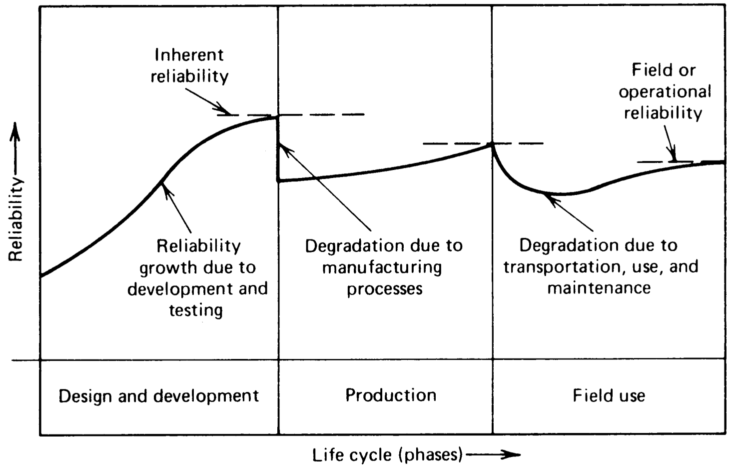


Figure 11 Reliability Growth and Degradation During System Life.

1. Detection of failure sources
2. Feedback of problems identified
3. Redesign effort based on problems identified that will potentially be corrected

If the failure sources are detected by testing, then we have the following:

4. Manufacturing or fabrication of prototype hardware.
5. Redesign, detection, and analysis of failure sources serve as verification of redesign effort.

Some of the benefits of reliability growth methodology are:

1. It enables us to take advantage of experience gained in similar programs in the past.
2. It enables us to evaluate better the progress being made by an ongoing program.
3. It enables us to evaluate possible courses of corrective actions if an ongoing program experiences problems.

11.1 Reliability Growth Models

In 1964, Duane of GE (Duane 1964) published a report describing his observations on failure data for five different types of systems during their development programs at GE. His analysis revealed that for these systems the observed cumulative failure rate followed a consistent pattern, approximately a straight line, when plotted on a log–log paper as a function of cumulative test hours. This can be expressed mathematically as

$$\Lambda(t) = kt^{-\beta} \tag{77}$$

where $\Lambda(t)$ = cumulative failure rate at time t
t = cumulative test time
k = a constant ($k > 0$)
β = growth factor ($\beta > 0$)

Let $N(t)$ be the total number of failures accumulated over the cumulative test time t. Then

$$\Lambda(t) = \frac{N(t)}{t} \tag{78}$$

Thus, the cumulative failure rate $\Lambda(t)$ will decrease as reliability grows as a result of the development of corrective effort because fewer failures will be observed. If we take logarithms of both sides of Eq. (77), we have

$$\log \Lambda(t) = \log k - \beta \log t \tag{79}$$

The value of constant k depends on system complexity, design margins, and design objectives for reliability, whereas the value of growth factor β depends on the development effort

Substituting Eq. (78) in Eq (77), we have

$$N(t) = kt^{1-\beta} \tag{80}$$

$$= kt^{\beta'} \qquad (\beta' = 1 - \beta) \tag{81}$$

Talking the logarithm of both sides of Eq. (81), we have

$$\log N(t) = \log k + \beta' \log t \tag{82}$$

Thus, if we plot the cumulative number of failures with respect to cumulative test time t on a log–log paper, we get a straight line with slope β'. It is clear from Eq. (77) that the higher the value of β, the more growth the system has.

The actual failure rate of the system if the design is released after test time t is given by

$$\begin{aligned}\lambda(t) &= \frac{dN(t)}{dt} \\ &= k(1 - \beta)t^{-\beta} \qquad (83)\end{aligned}$$

Thus, if we are given some goal for failure rate $\lambda(t)$, we can compute the total test time required during the development effort using Eq. (83).

12. DESIGN AND MANAGEMENT OF RELIABILITY PROGRAMS

The establishment of an effective product assurance program throughout the system life cycle requires a management with exceptional perception of the assurance sciences. This is because of the many conflicting factors involved when making decisions on the delegation or redistribution of responsibility and authority and also because reliability is at best a poorly understood discipline without a universal approach that applies to every product and every organization. A reliability program extends far beyond the estimation of reliability numbers: it must create an attitude of anticipation of reliability problems and initiate the preplanning necessary to eliminate or reduce the effects of unreliability to an acceptable and planned-for level. Because of the breadth of the subject, a product assurance program must span the total system life cycle; therefore, the program will infringe on several well-established technical management groups, such as design, testing, purchasing, quality control, manufacturing, sales, and service groups.

Unreliability is actually not the result of any one group but is due to the complexity of today's systems and of the organization required to create these systems. During system design, development, and production, anything that can contribute to unreliability should be identified and reviewed. A reliability-management structure must be created to accomplish this task. The question then is, ''Who should do this?'' In a ''perfect'' organization with complete flow of information, the unreliability problems might be taken care of by existing groups. However, such an organization rarely exists in practice; thus, over the years many organizations have found it necessary and advantageous to spend the extra effort to create a reliability group and integrate this group into the existing organization. The reliability group performs essentially an assurance function, facilitating and supporting the design and development process from a reliability point of view.

The establishment and maintenance of a viable reliability program must be done based on management foresight and intuition since the payback is not readily measured in dollars. Unreliability problems uncovered by a reliability group tend to be taken care of outside of the formal organizational communication channels and in general will be attributed to design engineering. Forcing a reliability group to point up all unreliability problems will mean that management is forcing it into a position of accusation, and this will create an intractable climate that may well hinder the product development cycle and the reliability improvement effort. Thus, it is very important to understand the role and the function of the reliability group in the total organization.

12.1. Elements of a Reliability Program

Management and control of system reliability must be based on a recognition of the system's life cycle, beginning at concept and extending through design, production, use, and discarding of the system. One of the objectives is to achieve acceptable levels of operational reliability and maintainability. Achievement of this objective requires numerous tasks. The activities of a reliability program have applications throughout the system life cycle. Some of these applications are: as follows:

Applications during design

Develop safety margins from reliability viewpoint.
Predict component reliability from the data bank of the failure rates.
Compute system reliability from component reliability.
Determine amount of redundancy needed to achieve a reliability goal.
Provide input to human engineering.
Interact with value engineering.
Evaluate design changes.
Perform trade-off analysis.
Compare two or more designs.
Provide guidelines for design review.
Work with cost-reduction programs.

Applications during development of testing

Establish reliability growth curves for the development and testing phase.
Develop guidelines for the amount of testing.

Develop bathtub curve based on the failure rate data and the test data.
Participate in the development of failure definition and scoring criteria document.
Participate in the scoring of the test failures.

Applications during manufacturing

Provide guidelines for manufacturing processes.
Provide input to quality control.
Provide input for guidelines to evaluate the suppliers and vendors.
Develop product burn-in or debugging time.

Applications during Field Use

Establish warranty cost and help reduce it.
Optimize the length of warranty.
Reduce inventory costs.
Develop maintenance procedures, both corrective and preventive.
Provide input to the spare parts-allocation models.
Participate in the collection and analysis of the field data.
Participate in the feedback process to report and correct the field failures.

The reliability group coordinates and directs the overall reliability effort to provide assurance that the optimum reliability has been achieved and that the consequences of unreliability have been considered in the overall plans. Obviously, the reliability group can be effective only if given proper authority, demanding formal sign-off during all critical stages of system development

An acceptable reliability level for a system is really a many-faceted problem that requires many complex trade-offs. Considerations in design, cost, and manufacturability, material availability, maintenance, and serviceability all enter into this problem. For example, a highly reliable design that cannot be manufactured effectively or cannot be maintained may represent an unacceptable situation from the total system point of view. The important thing is to plan for adequate overall system effectiveness, utilizing good knowledge on the actual reliability level of the system.

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