CHAPTER 70
Measurement Assurance

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Measurements play a major role in modern technology. Their purpose is typically threefold: to validate or invalidate a theory or model by comparing the measured value of a variable with a prediction; to determine whether a process or product meets specifications or quality requirements; and to enable closed-loop control of systems and processes. More generally, the purpose of measurement is to present an observer with a numerical value for the variable being measured. The input to a measurement system is the true value of the variable being measured, while the output is the measured value of this variable. In general, this measured value does not equal the true value of the variable. For example, a force sensor may read a value of 10.2 N when the true value of the force is 10.25 N; the speed of an engine as measured by a tachometer may be 3000 rpm when the true value is 3010 rpm; and so on. The problems involved in establishing the true value of the variable, an assessment of measurement error, a determination of the static and dynamic characteristics of measurement systems, and some remarks about noise in measurement systems constitute this chapter.

1. ELEMENTS OF A MEASUREMENT SYSTEM

A modern measurement system typically consists of the four distinct elements, as shown in Figure 1.

1. The *sensing element* probes the process, either in a contact or noncontact mode, and gives an output that depends on the variable being measured. Examples are:

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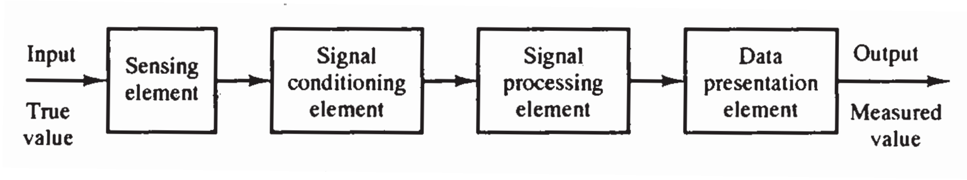

Figure 1 General Structure of Measurement System.

- Thermocouple, where an output voltage depends on temperature
- Piezoelectric force sensor, where an output charge depends on force
- Capacitive displacement transducer, where the capacitance output is a function of position or displacement

2. The *signal conditioning* element converts the output of the sensing element into a form more suitable for processing, usually a current, voltage, or frequency signal. Examples are:
   - Charge amplifier for a piezo force sensor, which converts a charge to a voltage.
   - Amplifier, which magnifies, say, a millivolt signal to a signal of several volts.
3. The *signal processing* element, which converts the output of the conditioning element into a form suitable for presentation. Typically, this element is an analog-to-digital (A/D) converter, which digitizes the analog signal output of the conditioning element.
4. The *data-presentation* element, which presents the measured value in an easily recognizable form. Examples are a chart recorder, graphical or numerical output on a computer terminal, and an oscilloscope.

Figure 2 shows a force-measurement system that incorporates each of these elements. Some measurement systems may have several of these elements.

2. CHARACTERIZATION OF MEASUREMENT SYSTEM ELEMENTS

In the previous section, we described the various elements that constitute a typical measurement system. The characteristics of the elements, as far as their input–output relationship impact the overall performance of the system, and as such it is important to describe these characteristics. In this section, we describe the static and dynamic characteristics of the system or system elements. We begin with a discussion of static or steady-state characteristics; these are the relationships that exist between the output (O) and input (I) of an element when I is either constant or changing slowly.

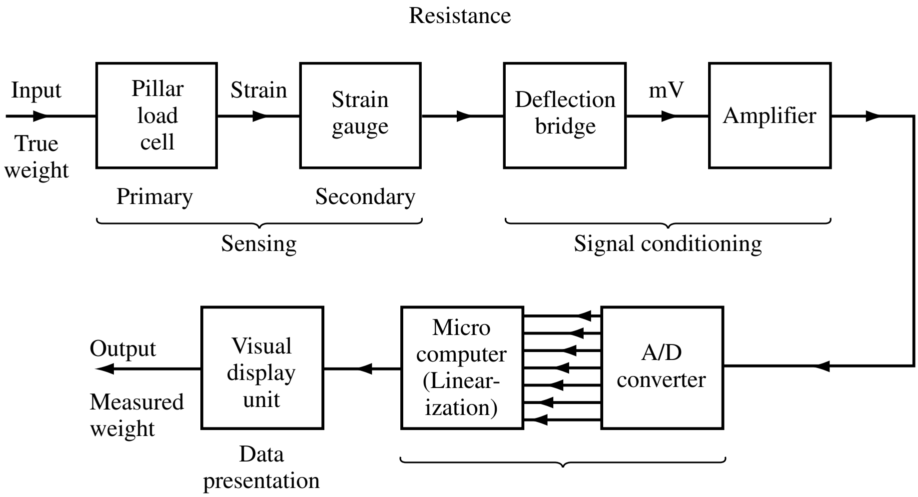

Figure 2 Weight-Measurement System.

2.1 Systematic Characteristics

Systematic characteristics are those that can be accurately quantified by analytical or empirical methods. These are to be distinguished from statistical characteristics, which will be considered later.

2.1.1. Range

This gives the range of values allowable for the input and output. For example, a piezo force sensor may have an input range of 0–10,000 N and a corresponding output range of 0–10 V.

2.1.2. Linearity

The operating ranges of most measurement systems are characterized by a linear relation between input (I) and output (O). In such instances:

$$O(I) = KI + a \tag{1}$$

where

$$K = \text{slope} = \frac{O_{\text{max}} - O_{\text{min}}}{I_{\text{max}} - I_{\text{min}}}$$

In practice, measurement systems show small deviations from linearity even within their operating range. These nonlinearities, which can be obtained from calibration experiments, may be ignored except in specific instances of measurement. In the presence of nonlinearity, equation (1) becomes:

$$O(I) = KI + a + N(I) \tag{2}$$

where $N(I)$ is the nonlinear term.

This nonlinearity is typically expressed as a percentage of the full-scale output range; for most measurement systems this value is less than 1% of the full-scale range.

If we look at the voltage output (E) of a chromel-alumel thermocouple in temperature (T) range of 0–200°C, where $E\,(T)$ is given by $E\,(T) = 40.1\,T + 2.09 \times 10^{-2}\,T^2 + 4.31 \times 10^{-4}\,T^4$ + higher-order terms

Then, with reference to Eq. (2):

$$K = 40.1,\ a = 0, \text{ and}$$
$$N(I) = 2.09 \times 10^{-2}\,T^2 + \ldots$$

2.1.3. Sensitivity(s)

Sensitivity(s) is the rate of change of output with input, that is:

$$S = \frac{dO}{dI}$$

A high value of S is often a common requirement for measurement systems in their operating range.

2.1.4. Environmental Effects, Wear, and Hysteresis

The output O is often influenced by fluctuations in the environment, such as changes in humidity, ambient temperature, and supply voltage. These fluctuations add additional terms to Eq. (2) or cause changes in the sensitivity-related parameter K. Aging of system elements is also a common source of changes in system sensitivity.

Hysteresis refers to the differing dependence of O on I, depending on whether the measurement is being made while I is increasing or decreasing. An example of hysteresis is backlash in gears and rack–pinion arrangements. The input–output relationship for a measurement system with hysteresis and non-linearities is shown in Figure 3.

Interfering and modifying inputs are environmental inputs that cause the linear sensitivity and intercept of the system, respectively, to change. An example of an interfering input is fluctuations in the reference junction temperature of a thermocouple.

We can now collect together the various characteristics of a measurement system, except for hysteresis, and express the general model for the system in the form of a block diagram. Such a block diagram representation is shown in Figure 4. The dynamic and static characteristics are shown decoupled in the diagram. The dynamic characteristics will be discussed shortly.

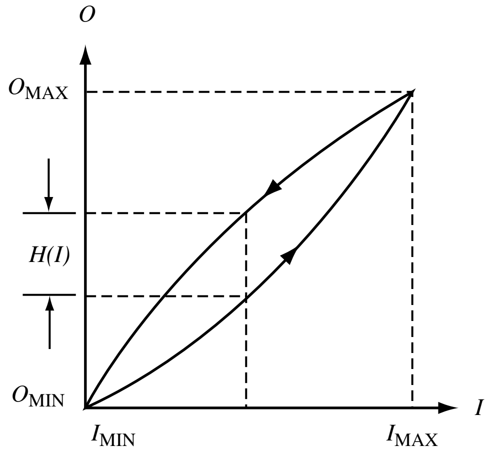

Figure 3 Hysteresis and Nonlinearity.

2.2. Statistical Characteristics

2.2.1. Repeatability

Suppose we apply a constant dead-weight load to a force-measurement system and keep the load applied for several hours. If the output O is monitored over this period of time, it is often likely that the output value will fluctuate about an expected value for the output. For example, if the load is 100 N, for which the expected output value of the sensing element should be 1 V, it is likely that over time the output will assume values such as 1.01, 0.98, 0.99, 0.98, 1.02, etc. This effect is termed as a lack of repeatability. Repeatability is the ability of a system to give the same output for the same input, when this input is repeatedly applied to it. The most common causes for lack of repeatability are random variations in the measurement system elements and their environment. By making reasonable assumptions about the fluctuations of the various inputs, including environment-related ones, it is possible to analytically characterize the fluctuations expected in the output.

Consider an output O that is a function of two types of inputs I and I_E. Further, let us assume, as is typically done, that the fluctuations in I and I_E can be described by normal distributions with standard deviations σ_1 and σ_2, respectively.

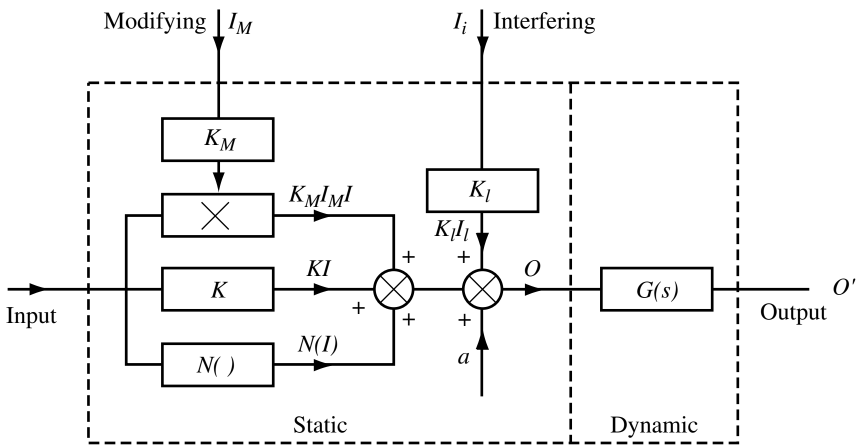

Figure 4 General Model of Measurement Element.

Then for

$$O = f(I, I_E)$$
$$\Delta O = \frac{\partial f}{\partial I}\Delta I + \frac{\partial f}{\partial I_E}\Delta I_E \tag{3}$$

and the standard deviation of O about its mean is given by

$$\sigma_o = \left(\frac{\partial f}{\partial I}\right)^2 \sigma_1^2 + \left(\frac{\partial f}{\partial I_E}\right)^2 \sigma_2^2 \tag{4}$$

Thus, σ_o can be calculated from a knowledge of σ_1 and σ_2. Alternatively, a calibration experiment can be done on the system, which allows for σ_o to be estimated directly from the experimental results. The mean value $\overline{O}$ for the output can be estimated from the experimental data. The corresponding probability density function for O can then be estimated as

$$P(O) = \frac{1}{\sigma_o\sqrt{2\pi}}\exp\left(\frac{-(O - \overline{O})^2}{2\sigma_o^2}\right) \tag{5}$$

3. IDENTIFICATION OF STATIC CHARACTERISTICS—CALIBRATION

The static characteristics of an element can be found experimentally by measuring corresponding values of the input I, the output O and the environmental inputs I_M, I_I, when I is either at a constant value or changing slowly. This type of experiment is referred to as calibration. The measurement of the variables I, O, I_M, I_I must be accurate if meaningful results are to be obtained. The instruments and techniques used to quantify these variables are referred to as standards.

The accuracy of measurement of a variable is the closeness of the measurement to the true value of the variable. It is quantified in terms of measurement error, that is, the difference between the measured value and the true value. Thus, the accuracy of a laboratory standard pressure gauge is the closeness of the reading to the true value of pressure. This brings us to the problem of how to establish the true value of a variable. We define the true value of a variable as the measured value obtained with a standard of ultimate accuracy. Thus, the accuracy of the above pressure gauge is quantified by the difference between the gauge reading for a given pressure and the reading given by the ultimate pressure standard. However, the manufacturer of the pressure gauge may not have access to the ultimate standard to measure the accuracy of his products. They can, however, measure the accuracy of the gauges relative to a portable intermediate or transfer standard, such as a dead-weight pressure tester. The accuracy of the transfer standard must be established by calibration against the ultimate pressure standard. This introduces the concept of a traceability ladder, which is shown in simplified form in Figure 5.

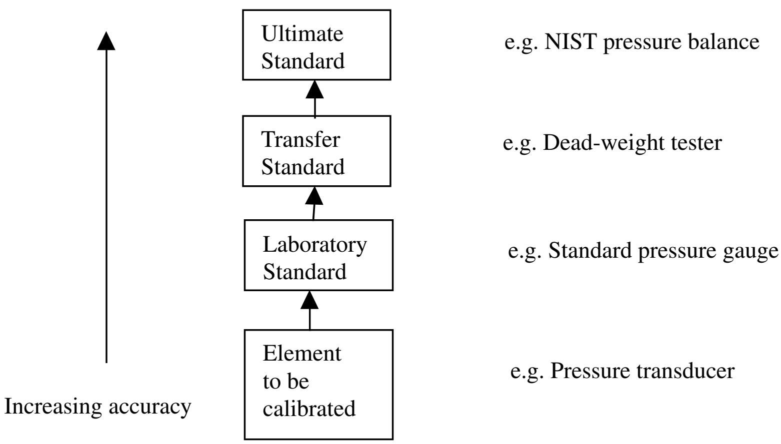

Figure 5 Schematic of Traceability Ladder.

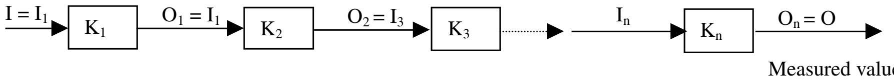

Figure 6 Measurement System Consisting of *n* Measuring Elements in Series. Input to the System is the True Value of the Variable to be Measured, and Output is the Measured Value of this Variable. The K_i's are the Gains of Each Measuring Element.

The measuring element is calibrated using the laboratory standard, which should itself be calibrated using the transfer standard, and this in turn should be calibrated using the ultimate standard. Each element in a traceability ladder should be significantly more accurate than the one below it.

4. ACCURACY OF MEASUREMENT SYSTEMS IN STEADY STATE

The input to a measurement system is the true value of the variable being measured, while the output corresponds to the measured value of the variable. Also, assuming the measurement system is complete, the system output is the measured value of the variable. Accuracy is defined in terms of measurement error, that is, the difference between the measured and true values of the variable. It follows, therefore, that accuracy is a property of a complete measurement system rather than a single element. Accuracy is quantified using measurement error E where:

$$\begin{aligned} E &= \text{measured value} - \text{true value} \\ &= \text{system output} - \text{system input} \end{aligned}$$

We can use the static model of a single element, developed previously, to calculate the output and thus the measurement error for a complete measurement system that is made up of several measuring elements. Consider the system shown in Figure 6 consisting of n elements in series. Suppose each element is ideal, that is, perfectly linear and not subject to environmental inputs. If it is also assumed that the intercept or bias is zero, that is, $a = 0$, then:

$$O_I = K_i I_I$$

for $i = 1, \ldots, n$, where K_i is the linear sensitivity or slope. It follows that $O_2 = K_2 I_2 = K_2 K_1 I$, $O_3 = K_3 I_3 = K_3 K_2 K_1 I$, and for the whole system:

$$O = O_n = K_1 K_2 K_3 \ldots K_i \ldots K_n I$$

If the measurement system is complete, then $E = O - I$, giving:

$$E = (K_1 K_2 K_3 \ldots K_n - 1) I$$

Thus, if

$$K_1 K_2 K_3 \ldots K_n = 1$$

we have $E = O$ and the system is perfectly accurate. The temperature measurement system shown in Figure 7 appears to satisfy the above condition. The indicator is simply a moving coil voltmeter with a scale marked in degrees Celsius so that an input change of 1 V to the indicator causes a change in deflection of 25°C. This system has $K_1 K_2 K_3 = 40 \times 10^{-6} \times 10^3 \times 25 = 1$ and thus appears to be perfectly accurate. The system is not accurate, however, because none of the three elements present is ideal. For instance, the thermocouple is nonlinear, so that as the input temperature

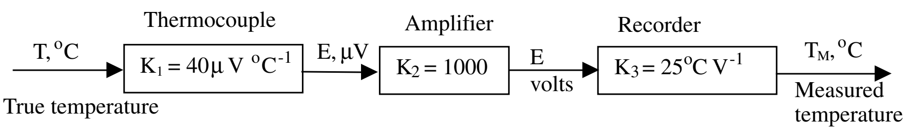

Figure 7 Schematic of a Temperature-Measurement System.

changes the sensitivity is no longer 40 $\mu V°C^{-1}$. Also, changes in reference junction temperature cause the thermocouple emf. to change. The output voltage of the amplifier may also be affected by changes in ambient temperature. The sensitivity K_3 of the indicator usually depends on the stiffness of the restoring spring in the moving coil assembly. This is affected by changes in environmental temperature and wear, causing K_3 to deviate from 25°C V^{-1}. Thus, the condition $K_1K_2K_3 = 1$ cannot be always satisfied and the system is in error.

In general, the error of any measurement system depends on the nonideal characteristics—for example. nonlinearity, environmental, and statistical effects—of every element in the system. Thus, in order to quantify this error as precisely as possible, it is necessary to use the general model for a single element.

The error probability density function for a measurement system comprising of n elements in series as in Figure 6 can be estimated from a knowledge of the corresponding density functions for each of the elements. This can be done analytically for a variety of density functions (see, for e.g., Bentley 1995). Alternatively, the density function for the entire system can be estimated from a calibration experiment.

4.1. Error-Reduction Methods

The error of a measurement system depends on the nonideal characteristics of each of the elements in the system. It is possible to identify which of the elements show significant nonideal behavior using calibration experiments. Compensation strategies may then be devised for each of these elements so that significant reductions can be accomplished for the overall system error. Some of these compensation methods are briefly highlighted, especially for nonlinear and environmental effects.

A common method for correcting a nonlinear element is to introduce a *compensating non-linear element* into the system such that the overall characteristic of the system is as linear as possible within the operating range of the system. This is illustrated in Figure 8 for a resistance thermometer. Depending on the degree of linearity desired, the bridge in Figure 8 can be suitably designed.

The most common method for reducing the effects of environmental inputs is *isolation*. Here, each of the measuring elements is effectively isolated from environmental changes. Examples are the placement of reference junction of a thermocouple in a temperature-controlled enclosure and the use of active vibration-isolation tables to isolate a measuring system (e.g., atomic force microscope) from external mechanical vibrations. Of course, it is possible to reduce environmental influences by selecting a transducer material that is completely insensitive to a specific environmental parameter. An example is the use of a metal alloy in strain gauges that has a zero coefficient of thermal expansion. But such an ideal material is often difficult to find and quite expensive.

A successful method for neutralizing environmental inputs is by providing an *opposing environmental input*. This is done by incorporating a second element(s) into the measuring system that is exposed to the same environmental input as the element of interest, such that the two effects tend to cancel out. An example is the use of two identically matched strain gauges in adjacent arms of a Wheatstone bridge to compensate for changes in the ambient temperature. In such an arrangement,

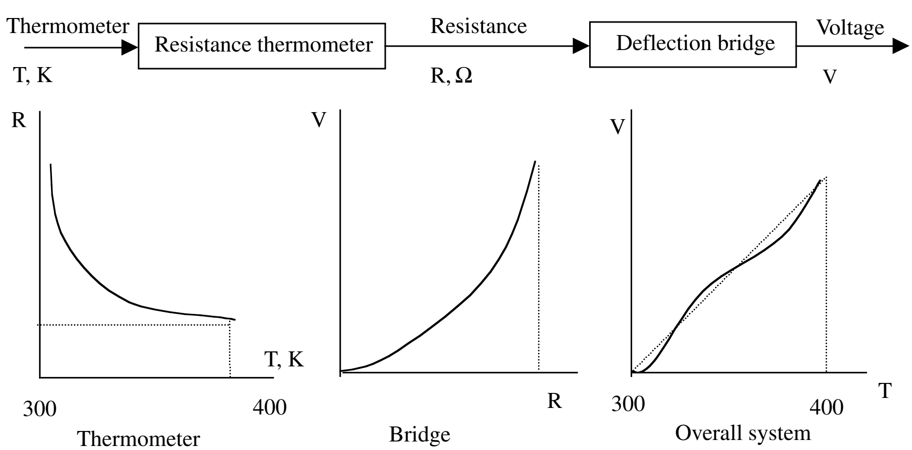

Figure 8 Nonlinearity Compensation in a Temperature-Measuring Element Using a Nonlinear Compensation Element.

one gauge is measuring a mechanical tensile strain and the other an equal compressive strain. However, because temperature-induced strains will have the same sign and magnitude in both the gauges, the bridge design effectively subtracts the resistances corresponding to the strains in the two gauges, thereby canceling out the thermal strain while doubling the mechanical strain being measured.

Other methods of compensation include the use of high-gain negative feedback and computer estimation of measured values. A discussion of these methods can be found in Doebbelin (1990).

5. DYNAMIC CHARACTERISTICS

The discussion thus far has centered on measurement system characteristics that are relevant to measurements of parameters that are either constant or changing very slowly with time. That is, the input (I) to the measurement system or a measuring element is varying rather slowly. By slow, we mean here that the variation in I with time is small over a duration equal to the response time of the measurement system or measuring element. However, in practice there are many instances where the input to a measurement system may change quite rapidly or even, in some cases, almost instantaneously. An example in this regard is the sudden change in the temperature input to a thermocouple from an ambient temperature of 30°C to 90°C when it is brought into contact with, say, an object at 90°C. In this case, the output of the thermocouple will change with time and reach a steady state output value that is equal or close to 90°C only after a finite amount of time. This finite amount may be a few microseconds or a few seconds depending on the type of thermocouple. The characteristics of the measurement system that describe its response to rapid changes in input are known as *dynamic characteristics*, and these are most conveniently described using the concept of a transfer function from linear systems theory.

5.1. Transfer Function of a Measurement System

Consider the measurement system comprising of n measuring elements in series (Figure 6) The input–output relationship for each element in this figure has been specified by an amplifier gain K_I, which means that the output can change instantaneously with time in the same manner as the input. In practice, this is not the case. The variation of $O(t)$ with $I(t)$ for a measuring element is often best described for most elements by a first-order or second-order linear differential equation. This description can be translated into an equivalent description in the form of a block diagram using the concept of a transfer function.

Consider a thermocouple that is brought into contact with a temperature source T. Then the output of the thermocouple, T_o, translated into a temperature reading, can often be described by the first-order differential equation

$$\tau \frac{dT_o(t)}{dt} + T_o(t) = T_i(t)$$

assuming that $T_o(0) = 0$ and $T_I(0) = 0$. The measured value of the variable T_i is T_o. The transfer function of this thermocouple element is given by

$$G(s) = \frac{T_o(s)}{T_i(s)} = \frac{1}{\tau s + 1}$$

The parameter τ is called the time constant of the element, and its value completely describes the response of the element. The smaller the value of τ, the faster the response of the element, which is an extremely desirable characteristic in measurement elements and systems.

Another common type of measurement system is a second-order, linear system described by a linear differential equation of the form

$$\frac{1}{\omega_n^2}\frac{d^2O}{dt^2} + \frac{2\xi}{\omega_n}\frac{dO}{dt} + O(t) = I(t)$$

where $O(t)$ and $I(t)$ are the output and input, respectively. The transfer function of this system is

$$G(s) = \frac{O(s)}{I(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

and the system behavior is completely determined by the two system parameters, ξ and ω_n, which are the damping ratio and natural frequency, respectively. Together, the two parameters determine the system response. A small value of ξ results in a fast response, but the overshoot in the response is also high. A high value of ω_n also results in a fast response. Furthermore, a high value for ω_n in a measurement element increases the range of signal frequencies over which measurements of the signal can be made accurately. Roughly speaking, signal frequencies up to a third of ω_n can be measured accurately without much attenuation. For further discussion of first- and second-order systems and their responses, see Franklin and Powell (1994).

We shall illustrate the computation of output and measurement error in a piezoelectric force sensor that is subjected to a time varying force. Figure 9 shows the block diagram of a piezoelectric force sensor or measurement system consisting of three elements. The charge amplifier is a first-order system with time constant of 0.1 sec, while the recording element is a second-order system with ξ and ω_n given in the figure. The measurement system is complete in the sense that the product of the elemental gains, $20 \times 10^{-3} \times 50$, is 1.

Let the system be subjected to a time varying input force,

$$F_i(t) = 100 \left[\sin 10t + \frac{1}{3} \sin 30t + \frac{1}{5} \sin 50t \right]$$

This is the true value of the force to be measured. The output $(F_o(t))$, which is the measured force, can be estimated in the usual way either analytically, or more easily using MATLAB. The error $E(t)$ is then obtained as $F_i(t) - F_o(t)$. An increase in the value of τ or ω_n can be used to reduce the magnitude of the error, as is easily verified by a calculation.

We will not discuss the calibration of dynamic characteristics of measurement systems here. Any one of the books under Additional Reading dealing with measurement systems can be referred to for this purpose.

6. NOISE IN MEASUREMENT SYSTEMS

Unwanted signals that interfere with the measurement process are often present in measurement systems. These signals can be deterministic or random; collectively, they are referred to as noise.

Random signals can be quantified using the following statistical functions: mean, standard deviation, probability density, power spectral density, and autocorrelation. Deterministic noise, such as in rotating machinery, can be quantified by their power spectrum. A variety of methods are available to reduce the effects of different types of noise signals. These include electromagnetic shielding, electrostatic screening, filtering, modulation, averaging, and modulation. See Additional Reading for detailed discussions of these issues.

7. SUMMARY

A brief tutorial on measurement systems has been presented. Description of measurement system elements and their characteristics, assessment of measurement error, and a short discussion of noise in measurement systems have constituted this tutorial. Additional Reading lists books related to these aspects where more detailed discussions of the topics reside. Included are references in which elements of measurement systems for sensing of specific parameters, such as force, temperature, and displacement, are discussed at length.

Acknowledgment

The author would like to thank Linda Weybright and Sridhar Kompella for their help in the preparation of the manuscript.

ADDITIONAL READING

Doebelin, E. O., *Measurement Systems,* McGraw-Hill, New York, 1990.

Bentley, J. P., *Principles of Measurement Systems,* Longman Scientific & Technical, Harlow, UK, and John Wiley & Sons, New York, 1995.

These two books provide a good overview and considerable detail about all aspects of measurement systems, including specific systems for force, temperature, displacement, pressure measurement, etc.

Lyons, L., *A Practical Guide to Data Analysis for Science Students,* Cambridge University Press, Cambridge, 1991.

Taylor, J. R., *An Introduction to Error Analysis,* University Science Books, Sausalito, CA, 1997.

These two books are useful guides for analysis of experimental data.

Franklin, G. F., Powell, J. D., and Emami-Naei, A., *Feedback Control of Dynamic Systems,* 3rd Ed., Addison-Wesley, Reading, MA, 1994.

Discusses linear systems and transfer functions.

Neubert, K. H. P., *Instrument Transducers: An Introduction to their Performance and Design,* 3rd Ed., Clarendon Press, Oxford, 1983.

Instrument Society of America (ISA), *ISA Transducer Compendium,* ISA, Pittsburgh.

Sydenham, P. H., *Mechanical Design of Instruments,* Instrument Society of America, Research Triangle Park, NC, 1986.

Compendium of transducers for various applications.

Whitehouse, D. J., *Handbook of Surface Metrology,* Institute of Physics Publishing, Bristol, UK, 1994.

A one-step solution for manufacturing metrology. Discusses surface and dimensional metrology.

Jones, R. V., *Instruments and Experiences,* John Wiley & Sons, New York, 1988.

Some perspective on the evolution of measuring elements written by a person who headed radar development in the United Kingdom during World War II.

APPENDIX
Measurement-Related Journals

1. *The Review of Scientific Instruments*
2. *Journal of Physics E: Scientific Instruments* (UK)
3. *Transactions of Instrument Society of America*
4. *Experimental Mechanics*
5. *Industrial Laboratory* (Russian; English translation)
6. *Instruments and Control Systems*
7. *Control Engineering*
8. *Journal of Instrument Society of America*
9. *Archiv für Technisches Messen* (Germany)
10. *Journal of Research of the National Bureau of Standards* (National Institute of Standards and Technology after 1988)
11. *Transactions of the Society of Instrument Technology* (Great Britain)
12. *ASME Journal of Dynamic Systems, Measurement and Control*
13. *Biomedical Engineering* (London)
14. *Medical Electronics and Data*
15. *Experimental Techniques*
16. *Optical Engineering*
17. *Sensors and Actuators*
18. *Transactions of Institute of Measurement and Control* (London)
19. *IEEE Transactions in Instruments and Measurements*
20. *Precision Engineering*
21. *Journal of Optical Sensors*
22. *Applied Optics*
23. *ASME Journal of Manufacturing Science and Engineering*
24. *Precision Engineering*
25. *Journal of Japan Society of Precision Engineering*