CHAPTER 68
Understanding Variation

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1. VARIATION IN DATA

Data from observations or measures will vary over time or location, and analysis of this variation is often used as a basis for action on the process. Sometimes this action is inappropriate or counterproductive because of a lack of understanding of the concept of common and special causes of variation.

One approach to analyze the performance of a process is to compare measures to an established standard, set of specifications, or customer requirements. The outcomes of the process can then be classified as acceptable or unacceptable. The unacceptable product is then scrapped, reworked, repaired, blended, or sold at a lower price. An unacceptable service usually requires rework, as well as management of an unhappy customer. This approach to the analysis of a process is an application of inspection. Inspection is useful to sort the good product or service from the bad, but without further analysis of the data, it provides no help in determining what should be done to improve the performance of the process.

A fundamental concept for the study and improvement of processes, due to Walter Shewhart (1931), is that variation in a measure of quality has its origins in one of two types of causes:

1. *Common causes:* those causes that are inherent in the process over time, affect everyone working in the process, and affect all outcomes of the process

Handbook of Industrial Engineering: Technology and Operations Management, Third Edition.
Edited by Gavriel Salvendy

2. *Special causes:* those causes that are not part of the process all the time or do not affect everyone but arise because of specific circumstances*

For example, the variation in cycle time in an assembly process is affected by causes common to the process and to all the workers in the process. Some possible examples of common causes of variation in cycle time are line speed, equipment reliability, staffing levels, complexity of orders, and supplier performance. If a high cycle time is due to these common causes, changes in the system by management will be required to reduce the times. If the high cycle times are due to special causes (e.g., broken belt, an absent worker, a fire in a supplier's plant), reduced cycle time will require specific actions by process workers and managers to remedy these issues. This example illustrates the importance of knowing whether the process is dominated by common or special causes before assigning responsibility for improvement. This example is used only to illustrate the concept. In practice, the distinction between common and special causes must be made with the aid of a control chart.

A process that has only common causes affecting the outcomes is called a *stable process,* or one that is in a state of statistical control. A stable process is one in which the cause system for the variation remains essentially constant over time. This does not mean there is no variation in the outcomes of the process, the variation is small, or the outcomes meet the requirements. A stable

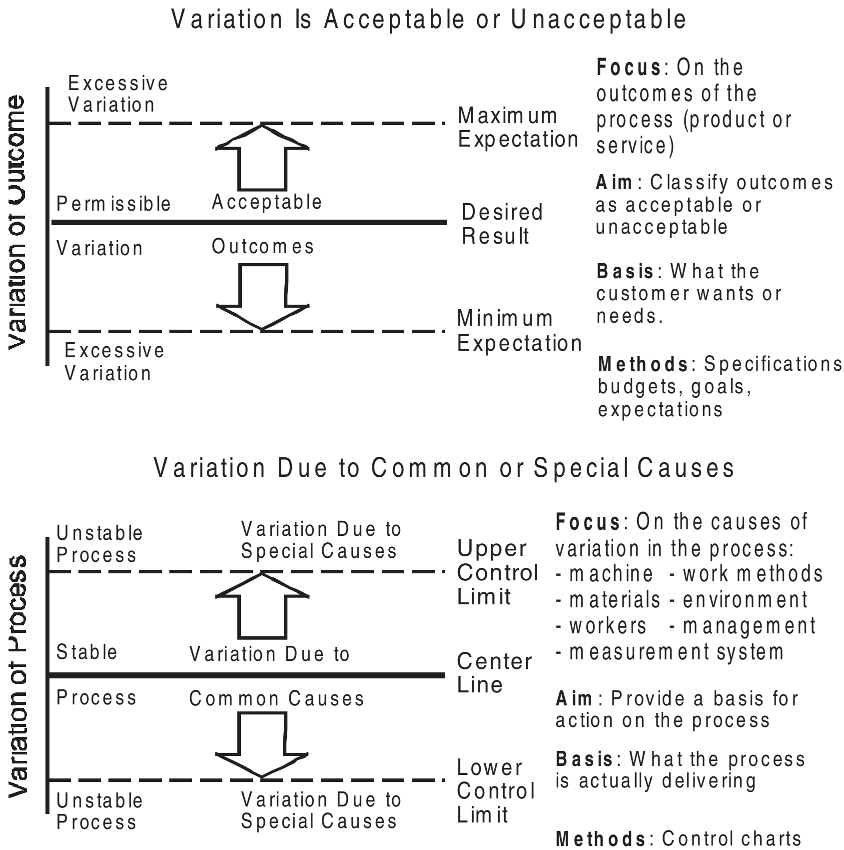


Figure 1 Two Views of Variation. (Copyright 1980–1998 Associates in Process Improvement)

* Shewhart (1931) used the terms *assignable* and *chance* rather than *special* and *common* to describe these two types of causes. Deming (1986) popularized the latter two terms.

process implies only that the variation is predictable within statistically established limits. A process whose outcomes are affected by both common causes and special causes is called an *unstable process*. An unstable process does not necessarily mean one with large variation. It means that the magnitude of the variation from one time period to the next is unpredictable. The two views of variation are contrasted in Figure 1.

As special causes are identified and removed, the process becomes stable. Deming (1986, p. 340) gives several benefits of a stable process. Some of them are:

- The process has an identity; its performance is predictable.
- Costs and quality are predictable.
- Productivity is at a maximum and costs at a minimum under the present system.
- It is relatively easy to evaluate the effect of changes in the process.
- Stability provides a solid basis for altering specifications that cannot be met economically.

Besides providing the basic concepts, Shewhart also provided a tool, the Shewhart control chart, for determining whether a process is dominated by common or special causes. The control chart is the means to operationally define the concept of a stable process. There are many different types of control charts. The appropriate chart to use in a particular application depends in part on the type of data obtained from the process or product.

2. APPLICATION OF THE CONCEPTS OF COMMON AND SPECIAL CAUSES

Although Shewhart focused his initial work on manufacturing processes, the concepts of common and special causes and of stable and unstable processes have implications in many areas, including:

- Operation of processes
- Management of processes
- Improvement of quality
- Supervision and leadership

2.1. Operation of Processes

In manufacturing processes, it is often easy to make adjustments to the average of a process. It is a mistake to make these adjustments on the basis of inspection results without the aid of a control chart. For example, if a dimension of a machined part is inspected and is found to exceed the upper specification, an adjustment is made to the machine so that the average dimension of future parts is lowered. If a batch of a particular chemical is outside of specifications, an adjustment is made by changing the amount of catalyst added to the next batch. In both of these cases, there are circumstances in which the adjustments described will improve the performance of the process and circumstances in which the adjustment will result in even worse performance. It is vital that both managers and operators be able to distinguish between these two sets of circumstances. Fortunately, there is a simple way to do this.

Adjustment to reduce the variability of a stable process, that is, one whose output is dominated by common causes, will make the performance worse. Improvement of a stable process is achieved through a fundamental change in the process that results in the removal of some of the common causes. If a special cause is found and will persist for some time, for example a lot of raw material, an adjustment of the process to counteract the special cause may be helpful in the short term. The control chart is an important tool to help the operator know when an adjustment to the process is needed.

2.2. Management of Processes

Tools such as specifications, standards, forecasts, and budgets are useful for planning, pricing of product, and other functions of management. They are used to communicate what the customer or manager expects or wants from the process. It is important to keep in mind that they do not communicate reality, that is, they do not communicate how the process is doing or what it is capable of doing.

A control chart of important measures such as costs, material usage, volume of production, sales and profit, and an analysis of the capability of the process (if the process is stable) communicates a realistic view of the performance of the process. Without the aid of a control chart and an understanding of the concept of common and special causes of variation, the tools for planning are mistaken for reality or the capability of the process. Workers or other managers are often asked to conform to that "reality." If the salesman does not meet the forecast, his performance is unacceptable. When the production worker does not achieve the production standard, his performance is unacceptable.

When a manager compares a measure of performance of the process, such as costs or sales to a planning tool such as a forecast or standard, and uses this comparison as a basis for action, his actions are analogous to the operator adjusting the machine on the basis of specifications. Sometimes his actions will be appropriate, other times they will not. Just as in the case of the operator, there is a simple way to know which set of actions is appropriate.

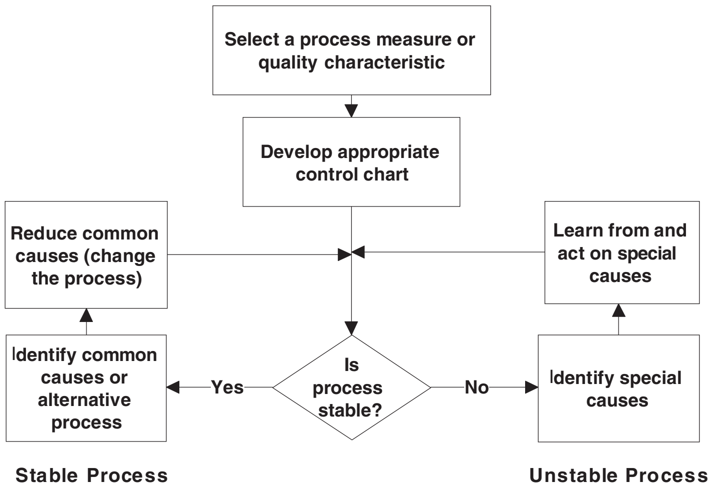
If the process is stable with respect to a particular measure of performance such as costs, then a fundamental change in the process, the responsibility of management, will be needed to reduce cost. Exhortations to lower-level managers or workers in the process to meet the forecast or standard will make things worse. Deming (1986) calls this type of action "tampering." Webster's dictionary defines "tamper" as follows:

- To interfere so as to weaken or change for the worse
- To alter for an improper purpose or in an improper way

It is vital in managing processes that planning tools are kept in their proper place and tools such as control charts and capability analysis are used as a basis for action on the process.

2.3. Improvement of Quality

To improve the quality of a process, it is useful to recognize whether the process is dominated by common causes or special causes. This will determine who is responsible for specific steps of improvement, what resources are needed, and what statistical tools will be useful (Figure 2). Since



Primary Methods for Learning

Stable Process	Unstable Process
1. Planned experimentation	1. Control charts
2. Subgrouping/stratification	2. Subgrouping/stratification
3. Control charts	3. Planned experimentation

Responsibility for Identification

Stable Process	Unstable Process
1. Technical experts	1. Workers in the process
2. Supervisors	2. Supervisors
3. Workers in the process	3. Technical experts

Responsibility for Improvement

Stable Process	Unstable Process
1. Management	1. Supervisors
2. Technical experts	2. Technical experts
3. Supervisors	3. Management
4. Workers in the process	4. Workers in the process

Figure 2 Using the Concepts of Variation to Guide Improvement. (Copyright 1980–1998 Associates in Process Improvement)

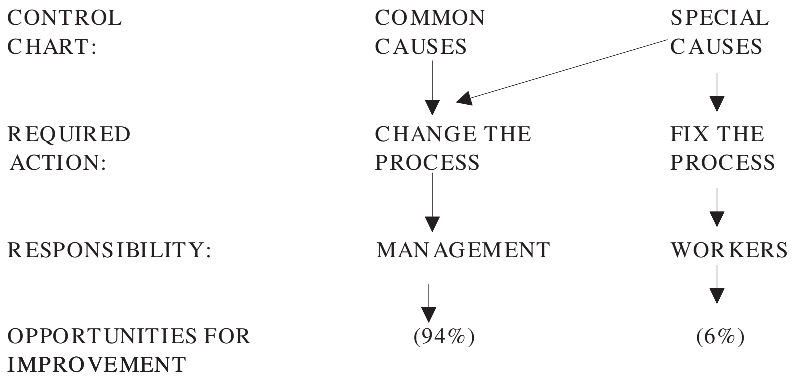


Figure 3 Opportunities for Improvement. (Copyright 1980–1998 Associates in Process Improvement)

unacceptable product or service can result from either common or special causes, the comparison of quality characteristics to requirements (product inspection) is not a basis for action on the process. Product inspection is useful to sort good products or services from bad and to set priorities on which processes to improve.

Activities to improve quality include the assignment of various people in the organization to work on common causes and special causes. The appropriate people to identify special causes are usually different than those needed to identify common causes. The same is true of those needed to remove causes. Removal of common causes is the responsibility of management, often with the aid of experts in the process such as engineers, chemists, and systems analysts. Special causes can frequently be handled at a local level by those working in the process, such as supervisors and operators. Without a knowledge of the concepts of common and special causes, it is difficult to allocate human resources efficiently to improve quality.

Many leaders of quality improvement have emphasized that most of the improvements in quality will take action by management. For example, Deming (1986) states that in almost all cases the removal of common causes will take a fundamental change in the process initiated by management. Some special causes can be removed by operators or supervisors. Others will require action by management in another process, possibly one of management or administration. For example, a special cause of variation in a production process may result when there is a change from one supplier's material to another. To prevent the special cause from occuring in the particular production process or other production processes, a change in the way the organization chooses and works with suppliers is needed. Figure 3 contains a summary of these concepts.

2.4. Supervision and Leadership

Another area in which the knowledge of common and special causes of variation is vital is in the supervision and leadership of people. A frequently made mistake is the assignment of faults of the process (common causes) to those working in the process, such as operators and clerks, rather than to those in charge of the process, management. It is obviously important for a supervisor or manager to know whether problems, mistakes, or rejected material are a result of common causes, special causes related to the system, or special causes related to the people under his or her supervision. Again, the use of a control chart will help the supervisor and manager accomplish this. For a thorough explanation of the role of statistical thinking in supervision, see Deming (1986).

3. TOOLS FOR LEARNING FROM VARIATION IN DATA

Some tools for learning from the variation in data are presented in Chapter 67. The primary tools are shown in Figure 4. Each of these tools looks at a particular aspect of the variation:

1. *Run charts:* View variation in data over time; study the impact of changes on measures.
2. *Control charts:* Distinguish between special and common causes of variation.
3. *Pareto charts:* Focus on areas of improvement with greatest impact.
4. *Frequency plots:* Understand location, spread, shape, and patterns of data.
5. *Scatterplots:* Analyze the associations or relationship between two variables.

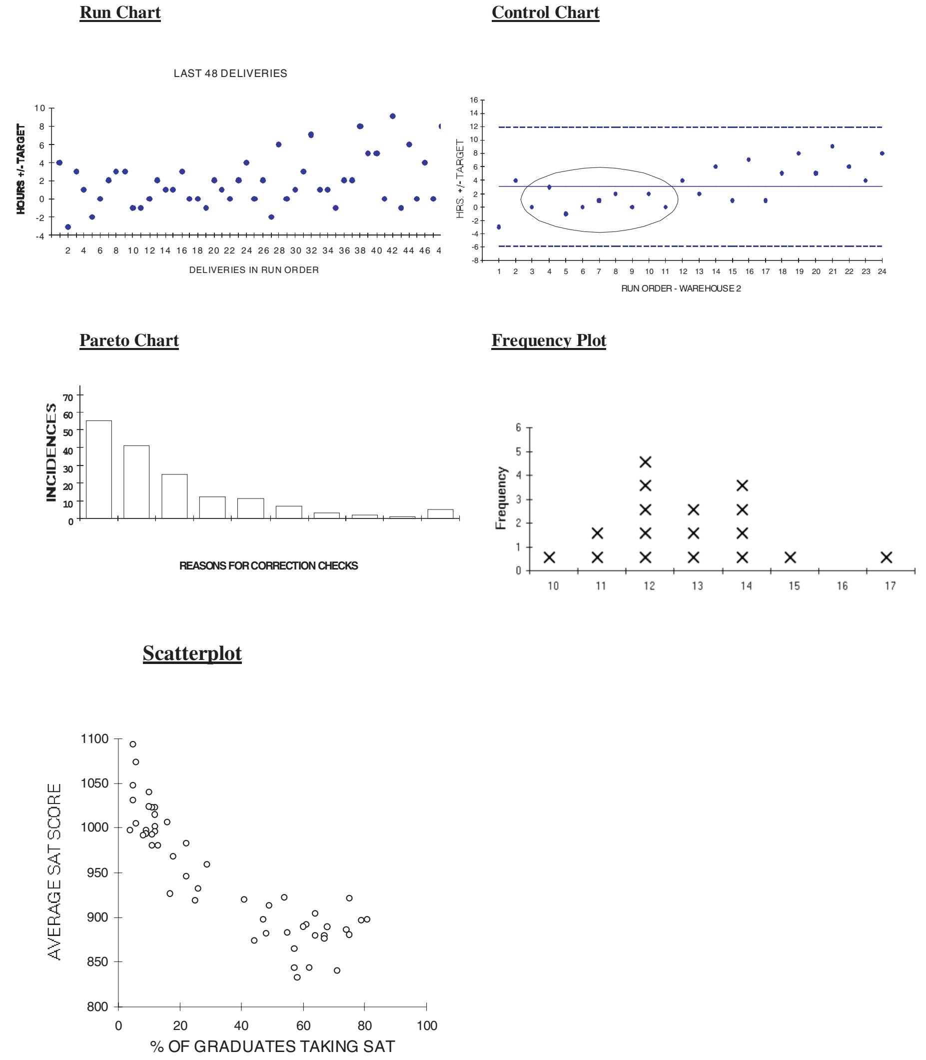


Figure 4 Tools to Learn from Variation. (Copyright 1980–1998 Associates in Process Improvement)

The run chart is simply a graphical record of a measure or characteristic plotted over time. Some type of run chart should always be a part of the study of variation in a process or system. The run chart focuses on dynamic complexity in a system (complexity over time) as well as the detail complexity of specific measures. The very simplicity of the chart is what makes it so powerful (Deming 1986). Everyone connected with the process can use and understand a run chart. Run charts are commonly used in business and economic documents.

The control chart (discussed extensively below) is an extension of the run chart. The control chart method provides a more formal way to learn from variation and guide the development of changes for improvement. The Shewhart control chart is a fundamental tool to guide improvement of processes.

The Pareto chart is a tool to help focus quality improvement efforts. It is useful whenever general classifications of problems, errors, defects, customer feedback, and so on can be classified for further study and actions. Often a few (the ‘‘vital few’’) classifications dominate the problem of interest while all the rest (the ‘‘useful many’’) contribute only a small proportion. To improve a process, it is important to find out which are the vital few problem areas.

A frequency plot is a tool to display data. It presents to the user basic information about the location, shape, and spread of a set of data. The frequency plot is widely used as a tool to help one understand variability. The frequency plot should only be used with adequate knowledge of the stability of the characteristic being measured. If the process is stable, the frequency plot serves as a prediction of the performance of the process in the future. If the process is unstable, then the frequency plot is simply a summary of what the process has done in the past. A basic type of frequency plot is the histogram, which is constructed by putting the scale for the characteristic of interest on the horizontal axis and the number of occurrences on the vertical axis.

A scatterplot is a tool used to study such relationships between possible causes and effects. It can also be used to study the association (or correlation) between different quality characteristics. A scatterplot is a graphic representation of the association between pairs of data. This pairing of data is the result of associating different measurements of a certain cause (e.g., pressure) with the corresponding measurement of the quality characteristic (e.g., paint thickness). The paired data could also be the measurements of two causes (e.g., pressure and temperature) or two quality characteristics (thickness and glossiness). Each pair becomes one point of the scatterplot.

4. SHEWHART CONTROL CHARTS

The control chart method provides an operational definition of the two types of causes of variation in a measure: common and special causes. Besides providing a new theory of variation, Shewhart also provided the method, the Shewhart control chart, for determining whether a system is dominated by common or special causes. The control chart is a statistical tool used to distinguish between variation in a measure of quality due to common causes and variation due to special causes. The name used to describe the chart (‘‘control’’) is misleading because the most common uses of these charts are to learn about variation and to evaluate the impact of changes. A better name might be ‘‘learning charts.’’ But Shewhart’s name has persisted. Figure 5 shows an example of a typical control chart.

The construction of a control chart typically involves:

- Plotting the data or some summary of the data in a run order (time is the most common order)
- Determining some measure of the central tendency of the data (such as the average)
- Determining some measure of the common cause variation of the data
- Calculating a centerline and upper and lower control limits (see Figure 5)

In developing the control chart method, Shewhart emphasized the importance of the economic balance between looking for special causes when they do not exist and overlooking special causes that do exist. It is also necesssary to develop rules that will give an acceptable economic balance for all types of measures in a variety of systems, processes, and products. Figure 6 illustrates the impact of these two mistakes.

4.1. Rationale for Shewhart Control Limits

Shewhart called the control limits three-sigma control limits and gave a general formula to calculate the limits for any statistic. Let T be the statistic to be charted, then:

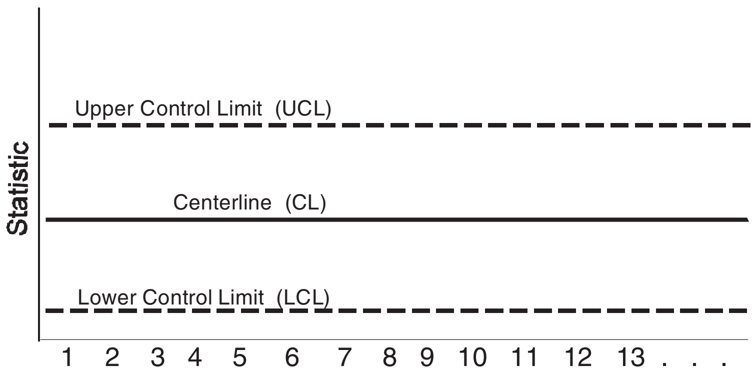


Figure 5 Illustration of a Control Chart. (Copyright 1980–1998 Associates in Process Improvement)

MISTAKE 1: To react to an outcome as if it came from a special cause, when actually it came from common causes of variation.

MISTAKE 2: To treat an outcome as if it came from common causes of variation, when actually it came from a special cause.

ACTION TAKEN	**ACTUAL SITUATION** **NO CHANGE**	**CHANGE**
Take action on individual outcome (special)	**− $**	**+ $**
Treat outcome as part of system; work on changing the system (common)	**+ $**	**− $**

Figure 6 Mistakes Made in Attempts to Improve. (Copyright 1980–1998 Associates in Process Improvement)

$$\text{The centerline: CL} = \mathrm{U}_t$$

$$\text{The upper control limit: UCL} = \mathrm{U} + 3 * \sigma_t$$

$$\text{The lower control limit: LCL} = \mathrm{U} - 3 * \sigma_t$$

where U is the expected value of the statistic and σ_t is the standard deviation of the statistic. Shewhart emphasized that statistical theory can furnish the expected value and standard deviation of the statistic, but empirical evidence justifies the width of the limits (the use of "3" in the control limit calculation).

The challenge for any particular situation is to develop appropriate estimates of the expected value and standard deviation of the statistic to be plotted. Appropriate statistics have been developed for control charts for a wide variety of applications.

The rationale for the use of Shewhart's three-sigma limits is:

- The limits have a basis in statistical theory.
- The limits have proven in practice to distinguish between special and common causes of variation.
- In most cases, use of the limits will approximately minimize the total cost due to overreaction and underreaction to variation in the process.
- The limits protect the morale of workers in the process by defining the magnitude of the variation that has been built into the process.

4.2. Interpretation of a Control Chart

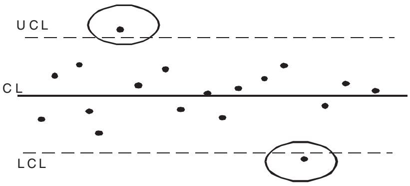
The control chart provides a basis for taking action to improve a process. A process is considered to be stable when there is a random distribution of the plotted points within the control limits. For a stable process, action should be directed at identifying the important causes of variation common to all of the points. If the distribution (or pattern) of points is not random, the process is considered to be unstable and action should be taken to learn about the special causes of variation.

There is general agreement among users of control charts that a single point outside the control limits is an indication of a special cause of variation. However, there have been many suggestions for systems of rules to identify special causes that appear as nonrandom patterns within the control limits. Figure 7 contains five rules that are recommended for general use with control charts. These rules are consistent in the sense that the chance of occurrence of rules #2 through #5 in a stable process is close to the chance of rule #1 occurring in a stable process.

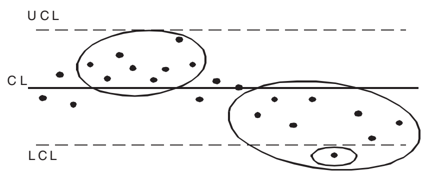
When applying the rules, the following guidelines will help with consistent interpretation of charts:

- Ties between two consecutive points do not cancel or add to a trend (rule 3).
- A point exactly on a control limit is not considered outside the limit (rule 1).

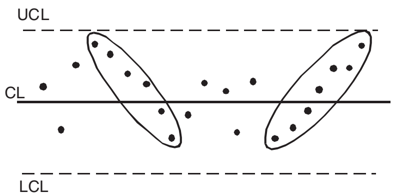
1. A single point outside the control limits.



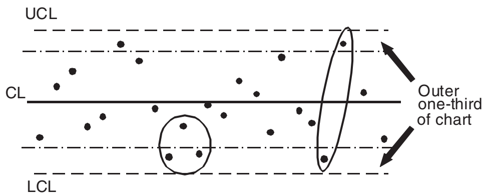
2. A run of eight or more points in a row above (or below) the centerline



3. Six consecutive points increasing (trend up) or decreasing (trend down).



4. Two out of three consecutive points near (outer one-third) a control limit.



5. Fifteen consecutive points close (inner one-third of the chart) to the centerline

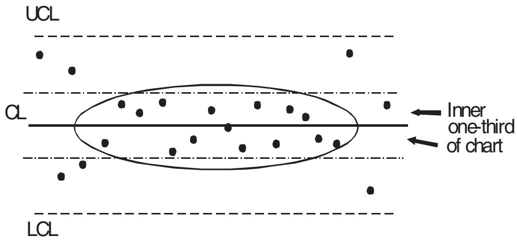


Figure 7 Rules for Determining a Special Cause. (Copyright 1980–1998 Associates in Process Improvement)

- When control charts have varying limits due to varying numbers of measurements within subgroups, rule 3 should not be applied.
- A point exactly on the centerline does not cancel or count towards a run (rule 2).
- When there is not a lower or upper control limit (for example, on a range chart with less than seven measures in a subgroup or on a P chart with 100% as a possible result for the process), rules 1 and 4 do not apply to the missing limit.

Rule 5 is especially useful in detecting a reduction of variation with an individual chart or for detecting improper subgrouping with an X-bar chart. Special circumstances may warrant use of some additional tests given by Nelson (1984). Deming (1986) emphasizes that the most important issue is the necessity to state in advance what rules to apply to a given situation.

4.3. Control Charts for Different Data Types

The different kinds of control charts are based on two groupings of types of data: attribute data and variable data. Attribute data includes classification, count, and rank data. Variable data refers primarily to continuous data, but rank data are often analyzed using a variable-control chart (realizing that the arithmetic functions are not theoretically valid). Otherwise the ranks can be converted to classification data and analyzed using attribute charts. Figure 8 contains examples of each of these categories of data.

Type of Data	Quality Characteristic	Recorded Data
Classification	Delivery Performance	On-time/late delivery
	Rework	OK the first time/rework
	Scratches	OK/excessive scratches
Count	Changes	Number of changes/design
	Accidents	Number of accidents/month
	Scratches	Scratches/surface
Continuous	Time	Minutes early or late
	Weight	Grams using a laboratory scale
	Scratches	Length in cm of each scratch

Figure 8 Examples of Types of Data. (Copyright 1980–1998 Associates in Process Improvement)

As can be seen from Figure 8, data for some characteristics can be recorded as any one of three types. For example, for a part with a large number of dimensional characteristics, the data could be recorded in the following ways:

- Classification: part meets or does not meet specification
- Count: number of dimensions not meeting specification
- Continuous: measured value for selected dimensions

Continuous data can be converted to attribute data by applying an operational definition for the count or classification. A recorded dimension can be classified as meeting or not meeting the specification; however, this conversion does not work in reverse. The measured dimensions are unknown for a part that is recorded as not meeting specifications.

For classification data, quality attributes are recorded in one of two classes. Example of these classes are conforming units/nonconforming units, go/no-go, and good/bad. To obtain count data, the number of incidences of a particular type is recorded: number of mistakes, number of accidents, or number of sales leads. For continuous data, a measured numerical value is recorded: a dimension, physical attribute, or calculated number.

Examples of continuous data include height, weight, density, elapsed time, viscosity, and costs. Continuous data can be converted to attribute data by applying an operational definition for the count or classification. In general, data should be collected as continuous data whenever possible because learning can occur with many fewer measurements compared to attribute classifications or counts. The control charts for continuous data require fewer measurements in each subgroup than the attribute control charts. Typical subgroup sizes for charts for continuous data range from 1 to 10, while subgroup sizes for attribute data range from 30 to 1000.

Figure 9 contains a summary of the frequently used control charts and the type of data to which they apply.

4.4. Subgrouping and Stratification

The concept of subgrouping is one of the most important components of the control chart method. Shewhart said the following about subgrouping (Shewhart 1931, p. 299):

> Obviously, the ultimate object is not only to detect trouble but also to find it, and such discovery naturally involves classification. The engineer who is sucessful in dividing his data into rational subgroups based upon rational hypotheses is therefore inherently better off in the long run than the one who is not thus successful.

Shewhart's concept was to organize data from the process in a way that is likely to give the greatest chance for the data in each subgroup to be alike and the greatest chance for data in other subgroups to be different. The aim of rational subgrouping is to include only common causes of variation within a subgroup, with all special causes of variation occurring between subgroups.

The most common method to obtain rational subgroups is to hold time constant within a subgroup. Only data taken at the same time (or for some selected time period) are included in a subgroup. Data from different time periods will be other subgroups. This use of time as the basis of subgrouping allows the detection of causes of variation that come and go with time.

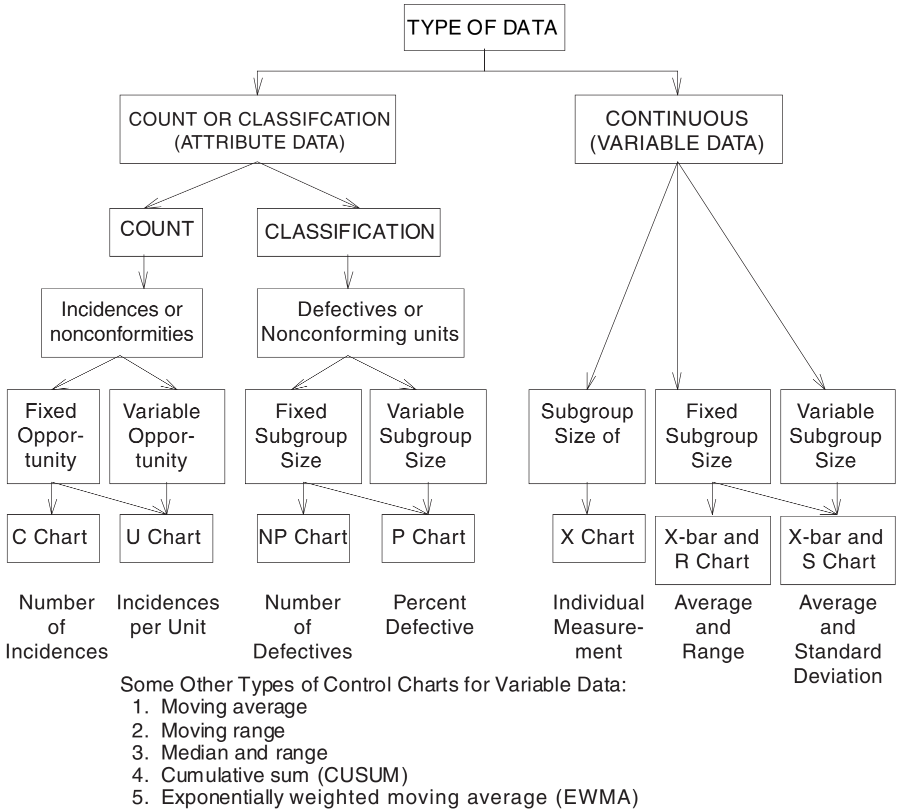


Figure 9 Selection of Particular Type of Control Chart. (Copyright 1980–1998 Associates in Process Improvement)

As an example of subgrouping, consider a study planned to reduce late payments. Historical data from the accounting files will be used to study the variation in late payments. What is a good way to subgroup the historical data on late payments? The data could be grouped by billing month, receiving month, major account, product line, or account manager. Knowledge or theories about the process should be used to develop rational subgroups. Some combination of time (either receiving or billing month) and one or more of the other variables in the process would be a reasonable way to develop the first chart.

After selecting a method of subgrouping, the user of the control chart should be able to state which sources of variation in the process will be present within subgroups and which sources will occur between subgroups. The specific objective of the control chart will often help determine the strategy for subgrouping the data. For example, if the objective is to evaluate differences between raw material suppliers, then only material from a single supplier should be included in data within a subgroup.

Since there is no grouping of the measurements for X charts, the power of rational subgrouping is not available. The use of stratification and rational ordering of the measurements with X charts provides an alternative to rational subgrouping for individual charts. Stratification is the separation and classification of data according to selected variables or factors. Stratification on a control chart is done in two different ways.

1. Plotting a symbol (instead of the usual • or x) to indicate a classification for the measurement or statistic being plotted. For example, plot the symbol A, B, or C to indicate which of the three offices the measurements came from.
2. Ordering the measurements, or subgroups of measurements, by stratification variables such as laboratory, classroom, material type, supplier, shift, programmer, part position, etc., to investigate the importance of these factors.

4.5. Planning a Control Chart

Constructing a control chart is a relatively simple process. Anyone can get started by selecting a measure of quality and plotting it in order of time. When enough data become available, a centerline and control limits can be calculated (e.g., using the individual control limit formulas). Many useful control charts have been developed with this minimal amount of planning.

In other cases, lack of planning and preparation has made attempts to use control charts unsuccessful. In these more complex situations, the effective use of control charts requires careful planning to develop and maintain the chart. Figure 10 contains a planning form that can be used to guide the planning of a control chart.

Every control chart should be associated with one or more specific objectives. The objective might be to improve the yield of the process, identify and remove special causes from a process, or establish statistical control so that the capability of the process can be determined. The objectives should be summarized on the control chart form. After a period of time, the objective may be met. The control chart should be discontinued at that time, or a new objective developed.

A number of issues related to measurement and sampling must be resolved prior to beginning a control chart. The type of data for each variable to be charted will determine the type of chart to

1. **OBJECTIVE OF THE CHART:**

2. **SAMPLING, MEASUREMENT, AND SUBGROUPING:**

   Measure to be charted:

   Type of data:

   Type of control chart:

   Method of measurement:

   Quality of measurement process:

   Location of sampling:

   Strategy for subgrouping:

   Frequency of subgroups:

3. **MOST LIKELY SPECIAL CAUSES:**

4. **NOTES REQUIRED:**

Note	Responsibility

5. **REACTION PLAN FOR OUT OF CONTROL POINTS:** (attach copy)

6. **ADMINISTRATION:**

Task	Responsibility
Making measurements:	
Recording data on charts:	
Computing statistics:	
Plotting statistics:	
Extending/changing control limits:	
Filing:	

7. **SCHEDULE FOR ANALYSIS:**

Figure 10 Form for Planning a Control Chart. (Copyright 1980–1998 Associates in Process Improvement) use. Information about the variability of the measurement system to be used should be documented. If the variability is not known, or if the stability of the measurement process is not documented, an effort to develop that information should be planned.

Important issues of sampling for control charts include the location in the process for measuring or sampling, the frequency of sampling, the number of samples, and the strategy for subgrouping mesurements (see section 4.4).

The documentation of information about the process is a most important part of many control charts. This documentation includes changes in the process, identification of special causes, investigations of special causes, and other relevant process data. Flow charts and cause-and-effect diagrams can be used to identify particular notes that should be recorded. Responsibility for recording this critical information should be clearly stated.

A plan for reaction to special causes on the chart should be established. Often a checklist of items to evaluate or a flow chart of the steps to follow is useful. The reaction plan should state the transfer of responsibility for identification of the special cause if it cannot be done at the local level. A plan for reaction to special causes on the chart should be established. Often a checklist of items to evaluate or a flow chart of the steps to follow is useful. The reaction plan should state the transfer of responsibility for identification of the special cause if it cannot be done at the local level. As an example, a reaction plan for a control chart in a laboratory to monitor a measurement system might have the following reaction plan:

1. Run the quality control standard.
2. Notify operations of a potential problem.
3. Review the log book for any recent changes in instrumentation.
4. Prepare a new QC standard and test it.
5. Replace the column in the instrument.
6. Notify the supervisor and call instrument repair.
7. Document the results of these investigations on the control chart.

There are a number of administrative duties required to maintain an effective control chart. Responsibility for measurement, recording data, calculating statistics, and plotting the statistics on the chart must be delineated. Proper revision and extension of control limits is an important consideration.

Control limits for the chart should be established using 20–30 subgroups from a period when the process is stable. If it is desirable to extend the control limits, any points affected by special causes should be removed and the control limits recalculated. The limits should only be extended when they are calculated using data without special causes.

Revision of the control limits should only be done when the existing limits are no longer appropriate. There are four circumstances when the original control limits should be recalculated:

1. When the initial control chart has special causes and there is a desire to use the calculated limits for analysis of data to be collected in the future. In this case, control limits should be recalculated after removing the data associated with the special causes.
2. When "trial" control limits have been calculated with fewer than 20–30 subgroups (note: trial limits should be calculated with fewer than 12 subgroups). In this case, the limits should be recalculated when 20–30 subgroups become available.
3. When improvements have been made to the process and the improvements result in special causes on the control chart. Control limits should than be calculated for the new process.
4. When the control chart remains out of control for an extended period of time (20 or more subgroups) and approaches to identify and remove the special cause(s) have been exhausted. Control limits should be recalculated to determine if the process has stabilized at a different operating level.

The date the control limits were last calculated should be a part of the ongoing record for the control chart. Some notation (such as vertical lines on the chart) should be used to indicate subgroups used to calculate control limits.

The form to record the data and to plot the control chart is another important consideration. The form should allow for a continuing record and not have to be restarted every day or week. The control chart form should include space to document the important decisions and information about the process from the planning form. The recorded data should include the time and place and the person making the measurements as well as the results of the measurements. The scale on the charts should be established to give a clear visual interpretation of the variation in the process. With the control limits centered on the chart, about one-half of the scale should be included inside the control limits.

A schedule for analysis should be established for every active control chart. The frequency of analysis will vary depending on the objective of the chart. For example, the quality improvement team might meet to analyze the chart weekly to assist in their improvement effort, while the department manager might be interested in a monthly review of the chart for planning purposes. The production vice president might review the charts with the department manager at the end of each quarter for planning and evaluation purposes.

Figure 11 shows an example of a completed planning form for a control chart maintained by an accounting group. Taking the time to plan a control chart before data collection is begun will help ensure that the chart leads to learning about the process or system.

4.6. Control Chart for Individual Measurements

One of the most useful control charts is the control for individual measurements, or the X chart. This control chart is a simple extension of the run chart. The control chart for individuals is useful when:

1. OBJECTIVE OF THE CHART: *To learn about the causes of returned invoices in order to reduce the number of returned invoices that have to be billed again.*

2. SAMPLING, MEASUREMENT, AND SUBGROUPING:
Measure of quality to be charted: *Percent of invoices returned that are not paid*
Type of data: *Classification*
Method of measurement: *Accounting supervisor records number of invoices sent each week and number returned unpaid.*
Quality of measurement process: *Complete, accurate counts can be made. The totals can be validated.*
Location of sampling: *Master list and returns that cross the supervisor's desk.*
Strategy for subgrouping: *Subgroup will be all invoices mailed in a given week (historically 35-90 invoices)*
Frequency of subgroups: *One per week—100% of invoices for that week.*
Type of control chart: *P chart*

3. MOST LIKELY SPECIAL CAUSES:
New customers, price changes, computer program updates, new employees in the Accounting Department.

4. NOTES REQUIRED:

Note	Responsibility
Number of new customers each week	*Supervisor*
New employees	*Supervisor*
Changes in computer program	*Systems*

5. REACTION PLAN FOR OUT OF CONTROL POINTS:
Supervisor will call meeting of Department to discuss all special causes.

6. ADMINISTRATION:

Task	Responsibility
Making measurements:	*Supervisor*
Recording data on charts:	*Supervisor*
Computing statistics:	*Supervisor*
Plotting statistics:	*Supervisor*
Extending/changing control limits:	*Dept. QI Team*
Filing:	*Supervisor*

7. SCHEDULE FOR ANALYSIS: QI team review each month

Figure 11 Example of a Completed Control Chart Planning Form. (Copyright 1980–1998 Associates in Process Improvement)

- There is no rational way to organize the data into subgroups (see later section on X-bar and R charts for a detailed discussion of control charts).
- Measures of performance of the process can only be obtained infrequently.
- The variation at any one time (within a subgroup) is insignificant relative to the between subgroup variation.

Examples of situations and data where a control chart for individuals can be useful include batch processes, accounting data, maintenance records, shipment data, yields, efficiencies, sales, costs, and forecast or budget variances. Often the frequency of data collection cannot be controlled for these situations and types of data.

Instrument readings such as temperatures, flows, and pressures often have minimal variation at any one time but will change over time. The study of tool wear is another example of insignificant short-term variation relative to variation over time. Control charts of the individual measurements can often be useful in these cases.

Some advantages of the control chart for individuals (compared to other types of control charts) are:

- The chart is an extension of the familiar run chart.
- No calculations are required when plotting on the chart.
- Plotting is done each time a measurement is made, providing fast feedback. Study of the process does not have to wait for additional measurements.
- Because only one chart is required for each measure of quality, charts for multiple measures of performance can be grouped on one form for presentation purposes to facilitate evaluation of a process.
- The capability of a process can be evaluated directly from the control limits on the chart.

Because of these advantages, the control chart for individuals is sometimes used when another type of control chart is more appropriate. The X chart is somewhat less sensitive than other variable control charts with larger subgroup sizes in its ability to detect the presence of a special cause. Sometimes data analyzed with an X chart will indicate a stable process, but the same data analyzed with a more appropriate chart (P chart, C chart, or X-bar and R chart, discussed in later sections) will clearly indicate the presence of special causes.

Besides this lesser sensitivity, there are some other disadvantages to using an X chart to study variation in data:

- Because each individual measure is plotted on the chart, there is no opportunity to focus on different sources of variation through subgrouping.
- All sources of variation are combined on one chart, sometimes making identification of the important sources of variation difficult.
- The X chart is sensitive to a nonsymmetric distribution of data and may require data transformation to be used effectively.

To develop a control chart for individuals, 20–30 measurements are required. The symbol for the number of measures used to calculate control limits is "*k*." The individual measurements are plotted on the X chart and the average of the individual measurements is used for the centerline of the chart. The moving ranges of consecutive measurements are used to estimate the variation of the process and develop control limits for the X chart.

The moving range is calculated by pairing consecutive measurements. The range is calculated for each set of two measurements by subtracting the low value from the high value. Each individual measurement is considered twice in the calculation of the moving ranges. Because a previous measurement is not available for the first measurement in the set, only $k - 1$ moving ranges can be calculated. The average of the moving ranges ($\overline{MR}$) is used for control limit calculations. Because the X chart of individual measurements contains all the information available in the data, it is not necesssary to plot the moving ranges.

A example of an X chart concerns a chemical product that is shipped in hopper cars with a sample taken from each car during loading. Laboratory tests are made on each sample for product certification, and this test becomes one dot on the chart. The cars are loaded from storage bins that are filled on an intermittent basis from a process unit. Laboratory results for the concentration of an additive for the last 25 cars loaded are used to develop control charts for the product shipped.

The control chart for the additive is shown in Figure 12, and the calculations of control limits are shown in Figure 13. As can be seen in the two figures, the moving ranges for car numbers 14 and 15 are greater than the moving range upper control limit of 12.8. These two values are removed

Chart133	Chart Name Additive at Hopper Car Loading
Objective Study variation and density special causes	Subgrouped by: Hopper Car
Process Hopper car loading	Product Q100 through Q209 · Target 225 +/- 25 · date 9/93
Chart Responsibility: Lab Post #2	Characteristic: Additive · Measurement Method: GC · Unit: PPM · zero=0

car	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
time																									
measure	215	218	222	217	216	214	219	221	216	220	218	218	221	236	222	221	216	218	223	217	218	221	220	219	215
MR	-	3	4	5	1	2	5	2	5	4	2	0	3	15	14	1	5	2	5	6	7	3	1	1	4

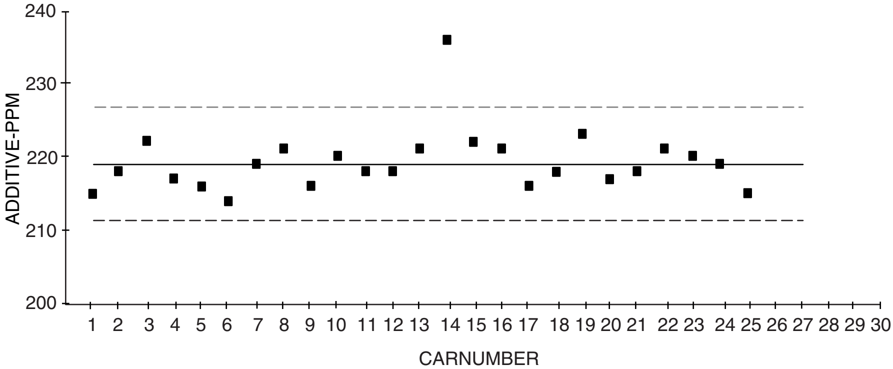


Figure 12 Individual Control Chart for Additive. (Copyright 1980–1998 Associates in Process Improvement)

NAME __Additive________________________________ DATE ____9/93________________

PROCESS ___Hopper Car Loading __________ SAMPLE DESCRIPTION __Composite_____

NUMBER OF SUBGROUPS (k) ____ 25______ BETWEEN (dates) Car 1 - 25

$\overline{X} = \frac{\Sigma X}{k} = \frac{5481}{25} = 219.2$ $\qquad \overline{MR} = \frac{\Sigma MR}{k-1} = \frac{94}{24} = 3.92$

X CHART **MR Calculation**

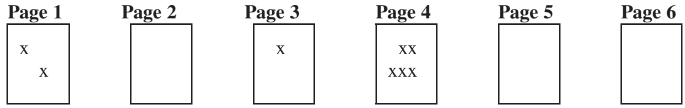
UCL = $\overline{X}$ + (2.66 ∗ $\overline{MR}$)
UCL = 219.2 + (2.66 ∗ 2.95)
UCL = 219.2 + 7.8
UCL = 227.0

UCL_{MR} = 3.27 ∗ $\overline{MR}$
UCL_{MR} = 3.27 ∗ 3.92
UCL_{MR} = _12.8____

LCL = $\overline{X}$ – (2.66 ∗ $\overline{MR}$)
LCL = 219.2 – (2.66 ∗ 2.95)
LCL = 219.2 – 7.8
LCL = 211.4

Recalculate $\overline{MR}$ after removing MR's greater than UCL_{MR}
$\overline{MR}$ = 3 MR / k - ?
$\overline{MR}$ = _65__ / _22__
$\overline{MR}$ = 2.95___

Figure 13 Control Chart Calculations for Additive X Chart. (Copyright 1980–1998 Associates in Process Improvement)



Classification	**bad**	**good**	**bad**	**bad**	**good**	**good**
Count (mistakes)	**2**	**0**	**1**	**5**	**0**	**0**

Summary: Classification—3/6 bad = 50% "bad" pages
Count— 8 mistakes in six pages = 1.33 mistakes per page

Figure 14 Two Types of Attribute Data. (Copyright 1980–1998 Associates in Process Improvement)

and the average moving range recalculated. The revised $\overline{MR} = 2.95$ was used to calculate the control limits for the X chart. The control chart indicates there is a special cause present for car number 14. Note that the 236 ppm concentration for car 14 is associated with the two moving ranges that were above the upper control limit.

4.7. Control Charts for Attribute Data

The two basic types of attribute data were discussed in Section 4.3:

1. *Classifications of units:* conforming units/nonconforming units, blue/not blue, go/no-go, etc.
2. *Count of incidence:* number of nonconformities, defects, accidents, trips, calls, etc.

Often data can be collected as either type. For example, in evaluating spelling errors in a manuscript (see Figure 14), each page could be classified as (1) having one or more spelling mistakes or (2) having none. This would be classification of units, with each page as a unit. Alternatively, the number of spelling mistakes on each page could be counted.

To develop an attribute control chart, a subgrouping strategy must first be determined. The subgroup size (n) is the number of units tested for classification data, or the area of opportunity for the incidence to occur for count data. There are four commonly used control charts for attribute data, depending on the type of attribute data and the constancy of the subgroup size. Table 1 summarizes these charts.

4.7.1. The P Chart for Classification Data

The P chart is appropriate whenever classifications are made in two categories, such as good parts and scrap parts. The P chart is usually preferred over the NP chart because percentages are more easily interpreted than counts in most applications and the P chart can be used with either a constant or variable subgroup size. The percentage of units in one of the categories (either the positive or the negative one, i.e., percent good product or percent scrap) is then calculated and graphed to develop the chart. Twenty to 30 subgroups are desirable for calculating the control limits, with at least 30 units in each subgroup.

Many times it is desirable to construct and use a P chart when the subgroup size is variable. This is usually the case when a set time period, such as a day or week, rather than a specific number of units, is used to define the subgroup. However, it is not necessary for each subgroup to contain exactly the same number of units to be considered constant. If the maximum and minimum subgroup sizes are within 20% of the average subgroup size, there will be an insignificant effect on the control limits if the average subgroup size is used for all calculations. If this is not the case, the subgroup

TABLE 1 Types of Attribute Control Charts

Chart Name	Type of Attribute Data	Statistic Charted	Subgroup Size
NP Chart	classification	number of nonconforming units (D)	constant
P Chart	classification	percent nonconforming units (P)	may vary
C Chart	count	number of incidents (C)	constant
U Chart	count	incidents per unit (U)	may vary

size must be considered variable and different sets of control limits must then be calculated for each subgroup size (or for sets of subgroup sizes with the individual subgroups within 20% of the average for the set). Methods for accommodating variable subgroup sizes are given under Additional Reading.

Once a subgrouping strategy has been determined, the following steps should be followed when constructing a P chart (see Figure 15 for calculation form):

1. Calculate p $\{p$ = (number in a certain category/number in subgroup) $*$ 100$\}$ for each subgroup.
2. Calculate $\overline{\text{P}}$, the average of the p's and the centerline for the chart
3. Determine the control limits for the P chart.
4. Figure and draw a scale on appropriate graph paper so the upper control limit is placed approximately one quarter of the way from the top. If there is a lower control limit, it should be placed 10–25% above the bottom of the chart. (Note: the scale should begin with zero for most situations.)
5. Plot the p's on the chart and draw in the control limits and centerline.

d = Nonconforming sample units per subgroup
n = Number of sample units per subgroup
k = Number of subgroups
p = Percent nonconforming units = 100*d/n

Control limits when subgroup size (n) is constant:

$$\overline{\text{P}} = \frac{\Sigma p}{k} = \frac{\_\_\_\_\_\_}{\;} = \_\_\_\_\_\_\_\_ \quad \text{(centerline)}$$

$$\hat{\sigma}p = \sqrt{\frac{\bar{p} * (100 - \bar{p})}{\text{n}}} = \sqrt{\frac{\_\_\_ * (100 - \_\_\_)}{\_\_\_}} = \_\_\_\_\_\_$$

UCL = $\bar{p}$ + (3 * $\hat{\sigma}p$) LCL = $\bar{p}$ − (3 * $\hat{\sigma}p$)

UCL = _____ + (3 * _____) LCL = _____ − (3 * _____)

UCL = _____ + ________ LCL = _____ − ________

UCL = ________ LCL = ________

Control limits when subgroup size (n) is variable:

$$\bar{p} = \frac{\Sigma d}{\Sigma n} * 100 = \frac{\_\_\_\_\_\_}{\;} * 100 = \_\_\_\_\_\_ \quad \text{(centerline)}$$

$$\hat{\sigma}p = \frac{\sqrt{\bar{p} * (100 - \bar{p})}}{\sqrt{n}} = \frac{\sqrt{\_\_\_ * (100 - \_\_\_)}}{\sqrt{n}} = \frac{\_\_\_\_\_\_}{\sqrt{n}}$$

UCL = $\bar{p}$ + (3 * $\hat{\sigma}p$) LCL = $\bar{p}$ − (3 * $\hat{\sigma}p$)

UCL = ___ + (3 * _____ / $\sqrt{\text{n}}$) LCL = ___ − (3 * _____ / $\sqrt{\text{n}}$)

UCL = ___ + (_____ / $\sqrt{\text{n}}$) LCL = ___ − (_____ / $\sqrt{\text{n}}$)

n:	______	______	______	______	______
$\sqrt{\text{n}}$:	______	______	______	______	______
3* $\hat{\sigma}$p:	______	______	______	______	______
UCL:	______	______	______	______	______
LCL:	______	______	______	______	______

Figure 15 P Chart Calculation Form. (Copyright 1980–1998 Associates in Process Improvement)

TABLE 2 Data on Absenteeism in the Accounting Department

Absenteeism (90 Employees)				
Day	Total Absences	*p*	Unexcused Absences	*p*
1	10	11.1	2	2.2
2	8	8.9	3	3.3
3	14	15.6	1	1.1
4	6	6.7	1	1.1
5	8	8.9	1	1.1
6	7	7.8	2	2.2
7	16	17.8	0	0.0
8	12	13.3	3	3.3
9	10	11.1	1	1.0
10	9	10.0	8	8.8
11	12	13.3	1	1.1
12	10	11.1	2	2.2
13	14	15.6	0	0.0
14	4	4.4	4	4.4
15	8	8.9	3	3.3
16	12	13.3	1	1.1
17	9	10.0	0	0.0
18	5	5.6	2	2.2
19	14	15.6	1	1.1
20	10	11.1	0	0.0
	$\Sigma d = 198$	$\Sigma p = 220.0$	$\Sigma d = 36$	$\Sigma p = 40.0$

A P chart example deals with a situation where a constant subgroup size is appropriate. The manager of the accounting department of a company decided to gather information on the absenteeism of her 90 employees. Each day for one month, the number of employees who were absent and whether their absence was unexcused was noted. Table 2 contains the data collected during that month.

Part/Product: Absenteeism										Operation: Accounting							Date of limits: 2/15/95						
Operator: Accounting Manager									Characteristic Inspected: Total Absent														
Date																							
Time																							
Total Inspected	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
Total Defective	10	8	14	6	8	7	16	12	10	9	12	10	14	4	8	12	9	5	14	10			
% Defective	11.1	8.9	15.6	6.7	8.9	7.8	17.8	13.3	11.1	10.0	13.3	11.1	15.6	4.4	8.9	13.3	10.0	5.6	15.6	11.1			
Notes:																							

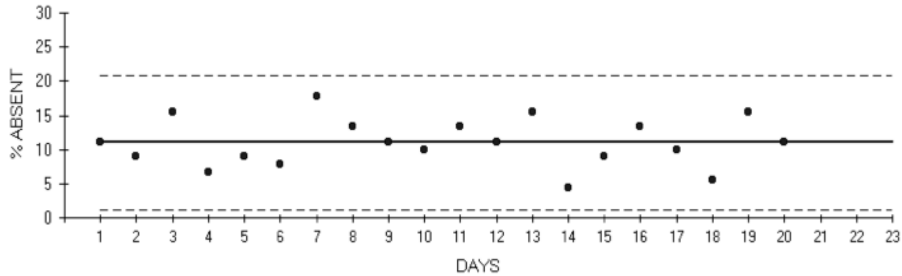


Figure 16 P Chart for Total Absences. (Copyright 1980–1998 Associates in Process Improvement)

d = nonconforming sample units per subgroup
n = number of sample units per subgroup
k = number of subgroups
p = percent nonconforming units = 100*d/n

Control limits when subgroup size (n) is constant:

Total Absent $\bar{p} = \frac{\Sigma p}{k} = \frac{220.0}{20} = \underline{11.0}$ (centerline)

$$\hat{\sigma}p = \sqrt{\frac{\bar{p} * (100 - \bar{p})}{n}} = \sqrt{\frac{11.0 * (100 - 11.0)}{90}} = \underline{3.3}$$

$$\text{UCL} = \bar{p} + (3 * \hat{\sigma}p) \qquad \text{LCL} = \bar{p} - (3 * \hat{\sigma}p)$$

$$\text{UCL} = 11.0 + (3 * 3.3) \qquad \text{LCL} = 11.0 - (3 * 3.3)$$

$$\text{UCL} = 11.0 + 9.9 \qquad \text{LCL} = 11.0 - 9.9$$

$$\text{UCL} = 20.9 \qquad \text{LCL} = 1.1$$

Figure 17 Calculations for Total Absence. (Copyright 1980–1998 Associates in Process Improvement)

Figure 16 shows the control chart for total absences. Figure 17 shows the calculations for total absences. Since there were 90 employees, calculations for a constant subgroup size were utilized. Figure 18 shows the chart and calculations for unexcused absences. The calculations resulted in no lower control limit for the chart on unexcused absences. The P chart for total absences is stable. A fundamental change to the system is required in order to reduce the average daily absenteeism of 11%. The P chart for unexcused absences indicates a special cause on day 10. Reasons for this special cause should be investigated and used to help develop a strategy to reduce unexcused absenteeism.

4.7.2. Control Charts for Count Data

When actual counts of incidence (often nonconformities) rather than classification of units are made, either a C chart or a U chart is usually the appropriate control chart. Figure 14 illustrated the difference between counts and classifications. Since the subgrouping method for counts is not always based on the selection of a certain number of units, a subgroup is defined as an *area of opportunity,* when working with count data.

An area of opportunity is simply the region selected for the count and could be of the following forms:

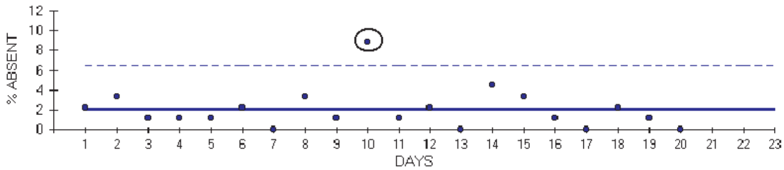
- Number of units (e.g., five television sets, requisitions per day)
- Space (e.g., 200 feet of yarn, 15 square yards of coated paper, one-quart sample of a product)
- Time (e.g., three months, one shift)

The decision whether to use a C chart or a U chart is made by determining whether the area of opportunity will be constant or will vary for each group of counts. For example, an area of opportunity could be the number of bills received in an office each week. If the number of errors on these bills is counted, the count will be distorted if the number of bills received from week to week is different. How to deal with this situation will be included in the discussion of when and how to use the C chart or U chart in the remaining part of this section. Table 3 lists examples of applications of C and U charts.

A C chart is used when the area of opportunity is constant for each subgroup. This would be the case in the example given above if 50 bills were received in the office each week. The statistic plotted for a C chart is simply the number of incidents (errors) in each area of opportunity (a week or 50 bills). It is not necessary for the area of opportunity to be exactly the same for each subgroup in order to use a C chart. The area of opportunity in any analysis can be considered constant if each region (number of units, time, or space) on which the counts are taken is within 20% of the overall average.

Part/Product: Absenteeism	Operation: Accounting	Date of limits: 2/15/95
Operator: Accounting Manager	Characteristic Inspected: Unexcused Absence	

Date																							
Time																							
Total Inspected	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
Total Defective	2	3	1	1	1	2	0	3	1	8	1	2	0	4	3	1	0	2	1	0			
% Defective	2.2	3.3	1.1	1.1	1.1	2.2	0.0	3.3	1.1	8.9	1.1	2.2	0.0	4.4	3.3	1.1	0.0	2.2	1.1	0.0			
Notes:																							



Notes:

d = nonconforming sample units per subgroup
n = number of sample units per subgroup
k = number of subgroups
p = percent nonconforming units = 100*d/n

Control limits when subgroup size (n) is constant:

Total Absent $$\bar{p} = \frac{\Sigma p}{k} = \frac{220.0}{20} = \underline{11.0} \quad \text{(centerline)}$$

$$\hat{\sigma}p = \sqrt{\frac{\bar{p} * (100 - \bar{p})}{n}} = \sqrt{\frac{11.0 * (100 - 11.0)}{90}} = \underline{3.3}$$

$$\text{UCL} = \bar{p} + (3 * \hat{\sigma}p) \qquad \text{LCL} = \bar{p} - (3 * \hat{\sigma}p)$$

$$\text{UCL} = \underline{11.0} + (3 * \underline{3.3}) \qquad \text{LCL} = \underline{11.0} - (3 * \underline{3.3})$$

$$\text{UCL} = \underline{11.0} + \underline{9.9} \qquad \text{LCL} = \underline{11.0} - 9.9$$

$$\text{UCL} = \underline{20.9} \qquad \text{LCL} = \underline{1.1}$$

Figure 18 C and U Chart Calculation Form. (Copyright 1980–1998 Associates in Process Improvement)

Once it has been determined that the area of opportunity will be constant for each subgroup, the following steps should be followed to construct a C chart:

1. Record the count c for 20 to 30 subgroups.
2. Compute $\bar{c}$, the centerline for the C chart.
3. Compute the control limits for the C chart.
4. Calculate and draw a scale on the charting form such that the upper control limit is 25% below the top of the chart. Plot the individual c's, the centerline, and the control limits.

The example that follows illustrates some of the important points concerning construction of a C chart. In an effort to improve safety in their factory, a company decided to chart the number of injuries that required first aid each month. Since approximately the same number of hours were worked each month, the area of opportunity (total man-hours worked in one month) was constant and a C chart was utilized. Table 4 contains the data collected over a two-year period.

Figure 19 shows the control chart and Figure 20 shows the calculations of the control limits. In July 1998, the reporting of 23 injuries resulted in a point above the upper control limit. This special cause was the result of a large amount of vacation leave taken during July. Untrained people and excessive overtime were needed to achieve the normal number of hours worked for a month. There

TABLE 3 C Chart and U Chart Situations

Area of Opportunity	Use a C Chart if	*c* Statistic	Use a U Chart if	*u* Statistic
Three documents (number of units)	Number of total pages is the same	Number of errors in the three documents	Number of total pages is different	Number of errors is per 50 pages (or other number)
Roll of carpet (space)	Square yards on each roll are the same	Number of visual defects per roll	Square yards on each roll are different	Number of visual defects per 25 square yards
One month (time)	Number of hours worked each month is the same	Number of accidents per month	Number of hours worked each month is different	Number of accidents per 100,000 hours

TABLE 4 Injury Data for C Chart

Month / Year	Number of Injuries [c]
Jan. 1998	6
Feb.	2
March	3
April	8
May	5
June	4
July	23
Aug.	7
Sept.	3
Oct.	5
Nov.	12
Dec.	7
Jan. 1999	10
Feb.	5
March	9
April	4
May	3
June	2
July	2
Aug.	1
Sept.	3
Oct.	4
Nov.	3
Dec.	1
	$\Sigma c = 133$

was also a run of nine points in a row below the centerline, starting in April 1999. This indicated that the average number of reported first aid cases per month had been reduced. This reduction was attributed to a switch from wire to plastic baskets for the carrying and storing of parts and tools, which greatly reduced the number of injuries due to cuts. If this trend continues, the control limits should be recalculated when sufficient data is available.

It should be noted that there is no lower control limit in the control chart of the previous example. Therefore, a run of eight or more points is required to demonstrate improvement. Combining data by quarter (three months) is one way to increase c and thus obtain a LCL.

Part/Product: Safety | Operation: Entire plant—300K hours/month | Date of limits: 1/95

Operator: Entire plant | Characteristic Inspected: Injuries requiring first aid

	1993												1994											
Date	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D
Time																								
Total Injuries	6	2	4	8	5	4	23	7	3	5	12	7	10	5	9	4	3	2	2	1	3	4	3	1
Notes:																								

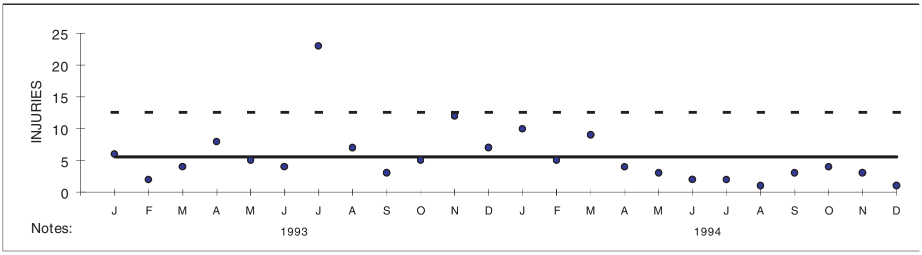


Figure 19 C Chart—Injury Data. (Copyright 1980–1998 Associates in Process Improvement)

C CHART CONTROL LIMITS (area of opportunity constant)

c = number of incidences per subgroup
k = number of subgroups
note: The subgroup size is defined by the "area of opportunity" for incidences and must be constant.

$\bar{c} = \frac{\Sigma c}{k} = \frac{133}{24} = \underline{\quad 5.5 \quad}$ (centerline)

$UCL = \bar{c} + (3 * \sqrt{\bar{c}})$ LCL = $\bar{c} - (3 * \sqrt{\bar{c}})$

UCL = 5.5 + (3 * 2.3) LCL = 5.5 – (3 * 2.3)

UCL = 5.5 + 6.9 LCL = 5.5 – 6.9

UCL = 12.4 LCL = ——

U-CHART CONTROL LIMITS (area of opportunity may vary)

c - number of incidences per subgroup
n - number of "standard areas of opportunity" in a subgroup (n may vary)
u - incidences per standard area of opportunity = c/n
k - number of subgroups
note: The standard area of opportunity will be defined by the people planning the control chart in units such as man-hours, miles driven, per ten invoices, etc.

$\bar{u} = \frac{\Sigma c}{k}$ = ________ = ________ (Center Line)

$UCL = \bar{u} + (3 * \sqrt{\bar{u}}) / \sqrt{n}$ LCL = $\bar{u} - (3 * \sqrt{\bar{u}})$

UCL = ___ + (3 * ___) / $\sqrt{n}$ LCL = ___ - (3 * ____) / $\sqrt{n}$

UCL = ___ + ______ / $\sqrt{n}$ LCL = ___ - _______ / $\sqrt{n}$

UCL = _______ LCL = ______

n:	_____	_____	_____	_____	_____
$\sqrt{n}$	_____	_____	_____	_____	_____
UCL:	_____	_____	_____	_____	_____
LCL:	_____	_____	_____	_____	_____

Figure 20 C Chart Calculations—Injury Data. (Copyright 1980–1998 Associates in Process Improvement)

4.8. X-Bar and R Control Charts

When continuous data are obtained from a process, it is sometimes of interest to learn about both the average level of the process and the variation about the average level. In these cases, two control charts are often used to study the process: the X-bar chart and the R chart.

An important aspect of the collection of data for the construction of X-bar and R control charts is that the collection is done in subgroups. A subgroup for continuous data is a set (usually three to six) of measurements of some characteristic in a process, which were obtained under similar conditions or at about the same time. The X-bar chart contains the averages and the R chart the ranges calculated from the measurements in each subgroup. These averages and ranges are usually plotted over time.

Figure 21 contains a form used to calculate the appropriate control limits. There are a number of symbols associated with X-bar and R charts:

X = individual measurement of quality characteristic
n = subgroup size (number of measurements per subgroup)
k = number of subgroups used to develop control limits

NAME ____ DATE ________________

PROCESS _______________________ SAMPLE DESCRIPTION ____________________

NUMBER OF SUBGROUPS (k) ________ BETWEEN (DATES)_______-

NUMBER OF SAMPLES OR MEASUREMENTS PER SUBGROUP (n) _____________

$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{k}$ = ________ = ________ $\bar{R} = \frac{\Sigma R}{k}$ = ________ = _____

$\bar{X}$ CHART

UCL = $\bar{\bar{X}}$ + (A_2 * $\bar{R}$)

UCL = + (*)

UCL = +

UCL = ___________

LCL = $\bar{\bar{X}}$ - (A_2 * $\bar{R}$)

LCL = - (*)

LCL = -

LCL = ___________

R CHART

UCL = D_4 * $\bar{R}$

UCL = *

UCL = __________

LCL = D_3 * $\bar{R}$

LCL = *

LCL = __________

—

FACTORS FOR CONTROL LIMITS

n	A_2	D_3	D_4	d_2
2	1.88	-	3.27	1.128
3	1.02	-	2.57	1.693
4	0.73	-	2.28	2.059
5	0.58	-	2.11	2.326
6	0.48	-	2.00	2.534
7	0.42	0.08	1.92	2.704
8	0.37	0.14	1.86	2.847
9	0.34	0.18	1.82	2.970
10	0.31	0.22	1.78	3.087

PROCESS CAPABILITY

If the process is in statistical control, the standard deviation is:

$\hat{\sigma}$ = $\bar{R}$ / d_2

$\hat{\sigma}$ = /

$\hat{\sigma}$ = _____________

the process capability is:

$\bar{\bar{X}}$ - 3 * $\hat{\sigma}$ to $\bar{\bar{X}}$ + 3 * $\hat{\sigma}$

- to +

_________ to _______

Figure 21 X-bar and R Control Chart Calculation Form. (Copyright 1980–1998 Associates in Process Improvement)

Σ = summation symbol

$\overline{X}$ = (X-bar) subgroup average

$\overline{\overline{X}}$ = (X-double bar) average of the averages of all the subgroups

R = subgroup range (largest–smallest)

$\overline{R}$ = (R-bar) average of the ranges of all the subgroups

$A2$, $D3$, $D4$, d_2 = factors for computing control limits and process capability

$*$ = multiplication symbol

The steps for developing X-bar and R control charts follow. All averages that are calculated should be rounded to one more decimal place (significant figure) than the values being averaged.

1. Calculate $\overline{X}$ ($\overline{X} = \Sigma\, \overline{X}/n$) for each subgroup.
2. Calculate R (largest − smallest value) for each subgroup.
3. Calculate $\overline{\overline{X}}$ ($\overline{\overline{X}} = \overline{\Sigma X}/k$), the centerline of the X-bar chart.
4. Calculate $\overline{R}$ ($\overline{R} = \Sigma R/k$), the centerline of the R chart.
5. Calculate the control limits for the X-bar chart using:

$$\mathrm{UCL} = \overline{\overline{X}} + (A2 * \overline{R})$$

$$\mathrm{LCL} = \overline{\overline{X}} - (A2 * \overline{R})$$

Note: $A2$ is a constant based on n obtained from Figure 21.

‹AR and R Calculation Sheet										Impurity in Plastic Pellets														
rpose- Maintaining control of Process																								
aracteristic - Impurity										Measurement Method - Gas Chromatograph (ppm)														
									Subgroups															
	6/5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
easure	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1	172	199	188	216	190	184	195	198	181	197	199	182	259	199	187	193	158	145	161	183	143	151	190	15
2	172	213	191	205	189	197	179	181	188	191	214	162	197	166	206	217	163	176	174	167	175	161	155	15
3	174	199	203	191	182	221	192	205	179	194	197	189	235	185	209	202	150	145	178	197	168	151	177	15
4	196	182	172	207	190	191	194	189	169	210	215	177	212	174	144	175	171	197	158	163	151	163	168	15
5	192	206	176	235	216	212	198	184	192	183	183	213	247	154	204	185	169	152	177	178	196	175	158	14
6																								
erage	181.2	199.8	186.0	210.8	193.4	201.0	191.6	191.4	181.8	195.0	201.6	184.6	230.0	175.6	190.0	194.4	162.2	163.0	169.6	177.6	166.6	160.2	169.6	153.
ige	24.0	31.0	31.0	44.0	34.0	37.0	19.0	24.0	23.0	27.0	32.0	51.0	62.0	45.0	65.0	42.0	21.0	52.0	20.0	34.0	53.0	24.0	35.0	14.

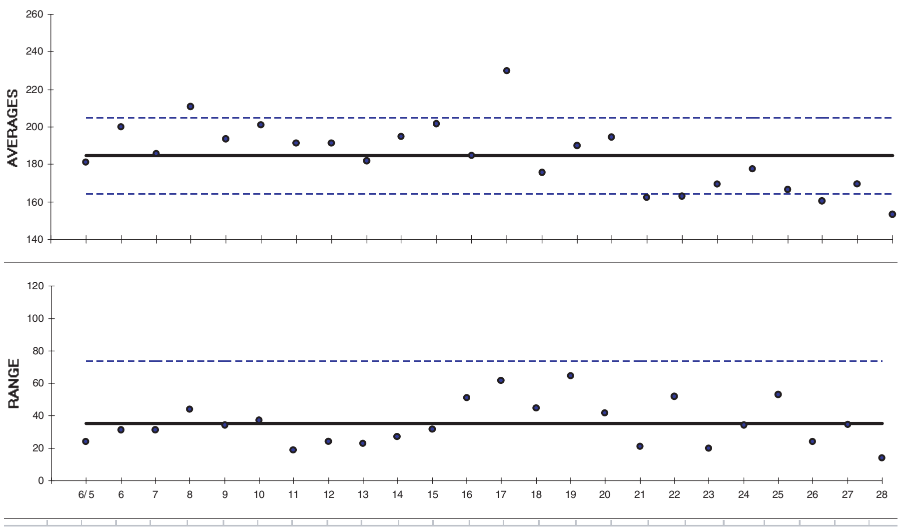


Figure 22 X-bar and R chart for Chemical Process. (Copyright 1980–1998 Associates in Process Improvement)

NAME Impurity in Pellets DATE 6/28

PROCESS Chemical SAMPLE DESCRIPTION Grab sample about every 5 Hours

NUMBER OF SUBGROUPS (k) 24 BETWEEN (DATES) 6/25 and 6/28

NUMBER OF SAMPLES OR MEASUREMENTS PER SUBGROUP (n) 5

$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{k} = \frac{4430.3}{24} = 184.59$

$\bar{R} = \frac{\Sigma R}{k} = \frac{844}{24} = 35.4$

XCHART	R CHART
UCL = $\bar{\bar{X}}$ + (A_2 * R)	UCL = D_4 * R
UCL = 184.59 + (.58 * 35.4)	UCL = 2.11 * 35.4
UCL = 184.59 + 20.53	UCL = 74.7
UCL = 205.12	
LCL = $\bar{\bar{X}}$ - (A_2 * R)	LCL = D_3 * R
LCL = 184.59 - (.58 * 35.4)	LCL = *
LCL = 184.59 - 20.53	LCL = -
LCL = 164.06	

Figure 23 Calculations for X-bar and R chart for Chemical Process. (Copyright 1980–1998 Associates in Process Improvement)

6. Calculate the control limits for the R chart using:

$$UCL = D4 * \overline{R}$$
$$LCL = D3 * \overline{R}$$

   Note: $D3$ and $D4$ are factors that depend on the size of the subgroup and can be obtained from Figure 21. Note that there is no lower control limit for R when n is less than 7.
7. Calculate a scale for the X-bar chart such that the control limits enclose the inner 50% of the charting area. Calculate the scale for the R chart such that the upper control limit is placed 25–35% below the top of the chart.
8. Plot the $\overline{X}$'s on the X-bar chart and the R's on the R chart.
9. Draw the control limits and centerline on the X-bar chart.
10. Draw the control limits and centerline on the R chart.

The following example illustrates some of the important aspects concerning X-bar and R control charts. In a chemical process, a control chart was to be constructed to monitor the concentration of an impurity in finished pellets. Customers wanted the impurity to be stable below 200 ppm. Five grab samples were selected from the continuous process each day (approximately one every five hours). Data were collected for 24 days before control limits were calculated. Therefore, 24 subgroups were used in the calculations. Figure 22 contains the control chart and Figure 23 shows the calculations of the control limits. Since each subgroup contains five measurements, there is no lower control limit for the R chart.

After review of the control chart, the process was determined to be unstable. On June 8 and 17, points were above the upper control limit on the X-bar chart. Beginning on June 21, four points were below the lower control limit on the X-bar chart and there was a run of eight points in a row below the average. Since the process was unstable, action was taken to eliminate the special causes of variation. The special cause detected on June 8 was associated with poor color of the feedstock supplied by the Quality Chemical Company. Discussions with this supplier were initiated immediately and the problem was corrected. Material with better color was introduced into the process on June

21. The lower values of impurity that resulted were detected as a special cause on the X-bar chart. The special cause detected on June 17 was the result of a temporary 10% drop in the production rate. The production planning department was notified to make them aware of the effect of rates on impurity levels.

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