CHAPTER 28

Industrial Engineering Applications in Financial Asset Management

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1. INTRODUCTION

Over the last decade, the challenges faced by asset managers have become significantly more complex. The array of investment choices has expanded tremendously in response to globalization and financial engineering. New products are now available such as collateralized debt, insurance-linked securities, and exotic structured products. At the same time, the need for investment planning has shifted more to individuals as companies and public entities relinquish responsibility for retirement programs.

Because of this incredible change in the investment world, institutions such as pension plan administrators, endowments, foundations, and asset managers are under greater pressure than ever to produce higher portfolio returns. This is the genesis for embracing new assets, which hopefully will

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generate superior investment performance than plain vanilla portfolios of stocks and bonds. Better returns reduce the need for future cash injections to fund obligations.

Unfortunately, market risks have increased as the array of available investment instruments has broadened. For example, the Mexican peso crisis in 1994, the Asian currency debacle and recession beginning in 1997, the Russian debt default and the unprecedented hedge fund bail-out coordinated by the Federal Reserve Bank in 1998, and a 30% drop in the price of technology shares early in 2000 all had major repercussions for financial markets. Where is an investor to find solace in such an unfriendly and disturbing environment?

The obvious answer to heightened complexity and uncertainty lies in utilizing financial engineering techniques to manage asset portfolios. This chapter reviews the current state of the art from a practitioner's perspective. The prime focus is on mean-variance optimization techniques, which remain the principal application tool. The key message is that while the methods employed by today's specialists are not especially onerous mathematically or computationally, there are major issues in problem formulation and structure. It is in this arena that imagination and inventiveness take center stage.

2. THE ASSET-MANAGEMENT PROBLEM

The job of investment managers is to create portfolios of assets that maximize investment returns consistent with risk tolerance. In the past, this required simply selecting a blend of stocks, bonds, and cash that matched the client's needs. Asset managers typically recommended portfolios heavily weighted in stocks for aggressive investors desiring to build wealth. They proffered portfolios overweight in bonds and cash for conservative investors bent on wealth preservation. Aggressive stock-heavy portfolios would be expected to yield higher returns over time but with considerable fluctuations in value. Concentrated cash and bond portfolios would be less volatile, thus preserving capital, but produce a lower return.

2.1. Origins

Until recently, a casual rule-of-thumb approach sufficed and was deemed adequate to produce reasonable performance for investors. For example, a portfolio consisting of 65% equities and 35% bonds generated returns and risk similar to a 75% equities and 25% bonds portfolio. However, following the inflation trough in the 1990s, bond returns declined and investors with low equity exposure suffered. In addition, those investors ignoring alternative assets such as hedge funds and venture capital found themselves with lagging performance and greater portfolio volatility.

Studies have consistently shown that selection of the asset mix is the most important determinant of investment performance. Early influential research by Brinson et al. (1986, 1991) and a more recent update by Ibbotson and Kaplan (1999) indicate that asset allocation explains up to 90% of portfolio returns. Security selection and other factors explain the remainder. Consequently, the asset blend is the key intellectual challenge for investment managers and should receive the most attention. Traditional rules of thumb no longer work in a dynamic world with many choices and unexpected risks.

2.2. Problem Structure and Overview

Markowitz was the first to propose an explicit quantification of the asset-allocation problem (Markowitz 1959). Three categorical inputs are required: the expected return for each asset in the portfolio, the risk or variance of each asset's return, and the correlation between asset returns. The objective is to select the optimal weights for each asset that maximizes total portfolio return for a given level of portfolio risk. The set of optimum portfolios over the risk spectrum traces out what is called the efficient frontier.

This approach, usually referred to as mean-variance (MV) analysis, lies at the heart of modern portfolio theory. The technique is now mainstream, regularly taught in investment strategy courses. A massive literature exists exploring the methodology, its intricacies, and variations. "Black box" MV optimization programs now reside on the desks of thousands of brokers, financial advisors, and research staff employed by major financial institutions.

The principal advantage of MV analysis is that it establishes portfolio construction as a process that explicitly incorporates risk in a probabilistic framework. In this way, the approach acknowledges that asset returns are uncertain and requires that precise estimates of uncertainty be incorporated in the problem specification.

On the surface, MV analysis is not especially difficult to implement. For example, it is very easy to guess at future stock and bond returns and use historical variances and correlations to produce an optimum portfolio. It is not so simple to create a multidimensional portfolio consisting of multiple equity and fixed income instruments combined with alternative assets such as private equity, venture capital, hedge funds, and other wonders. Sophisticated applications require a lot of groundwork, creativity, and rigor.

2.3. Implementation

Because the vast majority of MV users are neophytes who are not fully aware of its subtleties, nuances, and limitations, they are sometimes dismayed by the results obtained from MV optimization programs. The reason is that achieving sensible outcomes is highly dependent on quality input. Very often the return, risk, and correlations injected into MV models are empirically and theoretically inconsistent. This generates fallacious and highly distorted portfolios, leading many novices to reject the approach as capricious and unrealistic.

Mean-variance analysis must be integrated with a specific investment process if the results are to be useful. The steps required include:

1. Specifying the investment alternatives to be considered
2. Accurately forecasting expected asset returns, variances, and correlations
3. Executing the optimization
4. Choosing the appropriate implementation vehicles that deliver the performance characteristics embedded in the analysis

Optimization results must be carefully reviewed to ensure that assumptions satisfy consistency requirements and examined for solution sensitivity to changes in inputs.

3. AN ILLUSTRATION

3.1. The Efficient Frontier

To illustrate the essence of the asset-allocation problem, I begin with a simple example that includes the following assets: U.S. stocks, international stocks, U.S. bonds, international bonds, and cash. This constitutes a broad asset mix typical of that used by many professional managers. Expected returns are 11.0%, 12.6%, 6.0%, 6.9%, and 5.0%, respectively. The expected standard deviations (risk) of these returns are 12.5%, 14.7%, 4.3%, 7.8%, and 0.0%. Note that the standard deviation of cash returns is zero because this asset is presumed to consist of riskless U.S. Treasury securities. The correlation matrix for asset returns is:

$$\mathbf{R} = \begin{vmatrix} 1.0 & 0.55 & 0.30 & 0.00 & 0.00 \\ & 1.0 & 0.10 & 0.30 & 0.00 \\ & & 1.0 & 0.30 & 0.00 \\ & & & 1.0 & 0.00 \\ & & & & 1.0 \end{vmatrix}$$

Optimum portfolios are obtained by selecting the asset weights that maximize portfolio return for a specific portfolio risk. The problem constraints are that (1) the portfolio return is a linear combination of the separate asset returns, (2) portfolio variance is a quadratic combination of weights, asset risks, and asset correlations, and (3) all weights are positive and sum to one.

The results of producing MV portfolios are presented in Table 1. The corresponding efficient frontier is shown in Figure 1. The allocations obtained are typical for analyses of this type. That is, to obtain higher portfolio returns, the investor must take on more risk. This is accomplished by weighting equities more heavily. The most conservative portfolios have both low returns and low risk. They consist primarily of cash and bonds. For moderate levels of risk, investors should blend stocks and bonds together in more equal proportions. In addition, note that the most risky portfolios skew to higher weights for international equities, which are more risky than U.S. stocks.

It should be obvious that the production of MV portfolios is not extraordinarily input intensive. Efficient frontiers for five asset portfolios require only five predicted returns, five standard deviations, and a five-by-five symmetrical matrix of correlation coefficients. Yet the process yields indispensable information that allows investors to select suitable portfolios.

3.2. Investor Risk Preferences

Once the efficient frontier is established, the issue of investor risk preferences must be addressed. Individuals exhibit markedly different attitudes towards risk. Some are extremely risk averse, tolerating nothing but near certainty in life. Others relish risk taking. Most are somewhere between these two extremes.

Risk attitudes depend on personality, life experience, wealth, and other socioeconomic factors. By default, extremely risk-averse investors are not interested in building wealth via asset accumulation, because they are unwilling to tolerate the risk necessary to obtain high returns. The opposite is true for adventurous souls ready to gamble for large payoffs. In this regard, ‘‘old-money’’ investors

TABLE 1 Selected Optimal Mean-Variance Portfolios

Return	Risk	Sharpe Ratio[a]	U.S. Stocks	International Stocks	U.S. Bonds	International Bonds	Cash
			Portfolio Allocation				
12.6%	15.0%	0.84	—	100%	—	—	—
12.5%	14.0%	0.89	9%	91%	—	—	—
12.2%	13.0%	0.94	25%	75%	—	—	—
11.8%	12.0%	0.99	43%	55%	—	2%	—
11.3%	11.0%	1.03	42%	48%	—	10%	—
10.8%	10.0%	1.08	40%	40%	—	20%	—
10.3%	9.0%	1.14	37%	34%	6%	23%	—
9.7%	8.0%	1.21	32%	29%	17%	22%	—
9.1%	7.0%	1.30	27%	25%	19%	19%	9%
8.5%	6.0%	1.42	23%	22%	17%	17%	22%
7.9%	5.0%	1.59	19%	18%	14%	14%	35%
7.4%	4.0%	1.84	16%	15%	11%	11%	48%
6.8%	3.0%	2.26	12%	11%	8%	8%	61%
6.2%	2.0%	3.09	8%	7%	6%	6%	74%
5.6%	1.0%	5.59	4%	4%	3%	3%	87%

Source: Results of optimization analysis.
[a]The Sharpe ratio is the portfolio return less the risk-free Treasury rate divided by portfolio risk.

are usually more risk averse than "new-money" investors. They require portfolios heavily weighted in conservative assets. New-money investors are comfortable with inherently risky equities (Figure 2). Age often plays an important role. Older investors are commonly more risk averse because they are either retired or close to retirement and dissaving (Figure 3).

How does one ascertain risk tolerance to guarantee a relevant portfolio is matched to the investor's needs? There are a number of ways. One is to estimate the risk-aversion parameter based on the investor's response to a battery of questions designed to trace out their return preferences with different payoff probabilities.

Some financial managers simply use personal knowledge of the investor to deduce risk tolerance. For example, if an investor with a $10 million portfolio is distraught if the portfolio's value plunges 20% in a year, he or she is not a candidate for an all-equity portfolio. Yet another approach used by some managers is to work backwards and determine the risk necessary for the investor to reach specific wealth-accumulation goals.

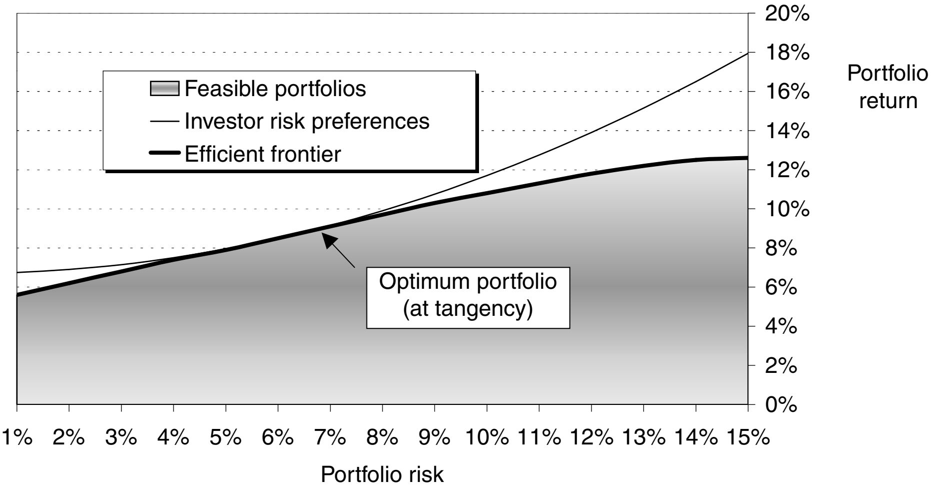

Figure 1 The Efficient Frontier and Investor Risk Preferences.

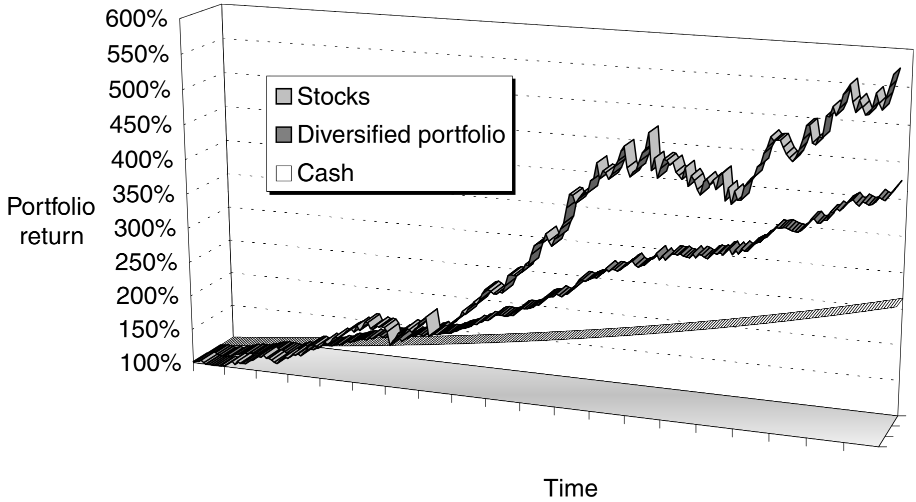

Figure 2 Wealth Creation vs. Wealth Preservation.

4. THE OPTIMIZATION PROBLEM

The specific mathematical formulation for the standard MV problem is straightforward:

$$\max \mathrm{U}(\mathbf{w}) = (r - \lambda\sigma^2) \quad \text{subject to} \tag{1}$$

$$r = \mathbf{w}^{\mathrm{T}}\mathbf{r} \tag{2}$$

$$\sigma = \mathbf{w}^{\mathbf{T}}\mathbf{S}^{\mathbf{T}}\mathbf{R}\mathbf{S}\mathbf{w} \tag{3}$$

$$\text{colsum } \mathbf{w} = 1 \tag{4}$$

$$w_j \geq 0 \tag{5}$$

where $\mathbf{w} = [w_1\ w_2\ .\ .\ .\ w_J]^{\mathrm{T}}$ = the portfolio weights for each asset
$\mathbf{r} = [r_1\ r_2\ .\ .\ .\ r_J]^{\mathrm{T}}$ = the return vector
$\mathbf{S}$ = the risk matrix with diagonal standard deviations $s_1\ s_2\ .\ .\ .\ s_J$
$\mathbf{R}$ = the $J \times J$ asset correlation matrix
λ = the investor's risk-aversion parameter

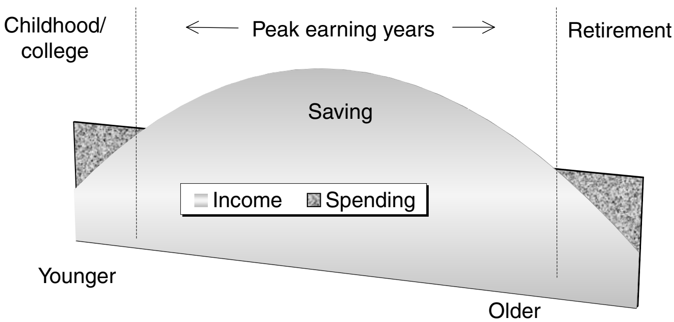

Figure 3 The Investor's Life Cycle.

Equation (1) incorporates the investor's attitude to risk via the objective function U. Equation (2) represents the portfolio return. Equation (3) is portfolio risk. Constraint (4) requires that the portfolio weights sum to 100%, while constraint (5) requires that weights be positive. This latter restriction can be dropped if short selling is allowed, but this is not usually the case for most investors.

This formulation is a standard quadratic programming problem for which an analytical solution exists from the corresponding Kuhn–Tucker conditions. Different versions of the objective function are sometimes used, but the quadratic version is appealing theoretically because it allows investor preferences to be convex.

The set of efficient portfolios can be produced more simply by solving

$$\max r = \mathbf{w}^{\mathrm{T}}\mathbf{r} \text{ subject to } \sigma = \mathbf{w}^{\mathbf{T}}\mathbf{S}^{\mathbf{T}}\mathbf{RSw}, \text{ colsum } \mathbf{w} = 1,\ w_j \geq 0 \text{ for target } \sigma \tag{6}$$

or alternatively,

$$\max \sigma = \mathbf{w}^{\mathbf{T}}\mathbf{S}^{\mathbf{T}}\mathbf{RSw} \text{ subject to } r = \mathbf{w}^{\mathrm{T}}\mathbf{r}, \text{ colsum } \mathbf{w} = 1,\ w_j \geq 0 \text{ for target } r \tag{7}$$

A vast array of alternative specifications is also possible. Practitioners often employ additional inequality constraints on portfolio weights, limiting them to maximums, minimums, or linear combinations. For example, total equity exposure may be restricted to a percentage of the portfolio or cash to a minimum required level. Another common variation is to add inequality constraints to force solutions close to benchmarks. This minimizes the risk of underperforming.

With respect to computation, for limited numbers of assets (small J), solutions are easily obtained (although not necessarily efficiently) using standard spreadsheet optimizers. This works for the vast majority of allocation problems because most applications typically include no more than a dozen assets. More specialized optimizers are sometimes necessary when there are many assets. For example, if MV is applied to select a stock portfolio, there may be hundreds of securities used as admissible "assets."

5. CAVEATS

5.1. Shortcomings of Mean-Variance Analysis

One must be cognizant that the MV approach has several important shortcomings that limit its effectiveness. First, model solutions are sometimes very sensitive to changes in the inputs. Second, the number of assets that can be included is generally bounded. Otherwise, collinearity problems can result that produce unstable allocations and extreme asset switching in the optimal portfolios. Third, the asset allocation is only as good as forecasts of prospective returns, risk, and correlation. Inaccurate forecasts produce very poorly performing portfolios. The first two limitations can be addressed through skillful model specification. The third requires that one have superlative forecasting ability.

A common mistake committed by naive users of MV analysis is to use recent returns, risk, and correlation as predictors of the future. Portfolios produced using such linear extrapolation methods normally exhibit poor performance. Table 2 shows historical returns and risk for various asset classes over the last decade. A variety of extraneous market developments, shocks, and one-time events produced these results. Rest assured that future time paths will not closely resemble those of the 1990s. For this reason, while one cannot ignore history, extending past performance into the future is a poor technique.

5.2. Dangers of Extrapolating from History

As an example of the extrapolation fallacy, consider portfolio performance over the last two decades. If one constructed an efficient portfolio in 1990 based on the 1980s history, large allocations would have been made to international equities. This is primarily due to the fact that Japanese stocks produced the best returns in the world up to 1989. Yet in the 1990s, Japanese equities fell by more than 50% from their 1989 peak, and the best asset allocation would have been to U.S. equities. Using the 1980s history to construct MV portfolios would have produced dismal portfolio returns (Table 3).

Empirical work by Chopra and Ziembra (1993) demonstrated that the most critical aspect of constructing optimal portfolios is the return forecast. For this reason, a shortcut employed by some practitioners is to concentrate on the return forecast and use historical risk and correlation to construct optimum portfolios. This may prove satisfactory because correlations and risk are more stable than returns and are therefore more easily predicted. However, this line of attack may be ineffective if return forecasts substantially deviate from history and are combined with historical risk and correlations. In this case, the optimal allocations can skew overwhelmingly to the high return assets

Whatever method is used to obtain forecasts, once the optimum portfolio is determined, the manager can momentarily relax and wait. Of course, actual outcomes will seldom match expectations.

TABLE 2 Annual Returns and Risk for Various Assets, 1990–99

Group	Asset	Return	Volatility	Sharpe Ratio
Equity	Large caps	17.7%	13.4%	0.94
	Mid caps	17.2%	16.0%	0.75
	Small caps	14.1%	17.3%	0.52
	Nasdaq	24.2%	20.6%	0.92
	International	8.6%	17.1%	0.20
	Europe	14.7%	14.6%	0.65
	Japan	2.6%	26.0%	−0.10
	Emerging markets	13.5%	23.8%	0.35
Fixed income	Treasury bonds	7.2%	4.0%	0.51
	Treasury notes	6.4%	1.7%	0.74
	Municipals	6.7%	4.2%	0.36
	International bonds hedged	8.1%	3.5%	0.83
	International bonds unhedged	8.4%	8.7%	0.37
	U.S. high yield debt	10.5%	5.5%	0.96
	Emerging market debt	16.3%	17.0%	0.65
Alternative	Passive commodities	−0.9%	7.5%	−0.81
	Active commodities (CTAs)	5.2%	11.7%	0.00
	Hedge funds	14.8%	4.3%	2.22
	Venture capital	28.1%	19.0%	1.21
	Private equity	15.7%	8.0%	1.32
	REITs	8.5%	12.1%	0.28
	Cash (3-month Treasuries)	5.2%	0.4%	0.00

Source: Deutsche Bank.

No investment manager possesses perfect foresight. Errors in forecasting returns, risk, and correlation will produce errors in portfolio composition. Obviously, managers with the best forecasting ability will reap superior results.

5.3. Asset Selection

If a manager does not forecast extremely well, it is possible to produce superior investment performance via asset selection. That is, choosing an exceptional array of candidate assets, can enhance portfolio returns due to diversification benefits that other managers miss.

For example, consider a simple portfolio of U.S. equities and bonds. Normally managers with the best forecasts will achieve better performance than other managers investing in the same assets. But another manager, who may not forecast U.S. equity and bond returns extremely well, can outperform by allocating funds to assets such as international equities and bonds. These assets possess different returns, risks, and correlations with each other and U.S. assets. Their inclusion shifts the efficient frontier upward beyond that resulting when only U.S. stocks and bonds are considered.

TABLE 3 Optimum Portfolios: The 1980s vs. the 1990s

		Asset				Portfolio	
Period	Measure	S&P	International Stocks	Bonds	Cash	Return	Risk
1980s	Weight	15%	47%	34%	4%		
	Returns	18%	22%	12%	9%	17.4%	
	Risk	16%	18%	12%	0%		10.5%
1990s	Weight	75%	0%	25%	0%		
	Returns	18%	7%	8%	5%	15.2%	
	Risk	14%	18%	4%	0%		10.5%
1980s portfolio performance in 1990s:						8.9%	10.8%

Source: Deutsche Bank.

The primary distinction between asset selection and asset allocation is that the thought processes differ. In asset selection, a manager focuses on defining the candidate universe broadly. In asset allocation, assets are typically viewed as given and the effort is on forecast accuracy.

A deep knowledge of markets and investment possibilities is necessary to identify the broadest possible asset universe. A manager who incorporates new assets with enticing features has a larger information set than a manager laboring in a narrowly defined world. This is why astute investment managers are constantly searching for new assets—their goal is to gain an edge over competitors by shifting the efficient frontier outward (Figure 4). Neglecting the opportunity to employ an ubiquitous asset domain is a common failure of beginners who rely on black box MV solutions they do not fully understand.

6. NEW ASSET CLASSES

A virtually endless stream of new investment products is paraded forth by financial service companies each year. Although promoted as innovative with superlative benefits, in reality most such creations are often narrowly focused with high risk or are reconfigured and redundant transformations of assets already available. Sorting through the muck and discovering innovative products that shift the efficient frontier outward is not an easy task

6.1. Desirable Asset Characteristics

In general, new assets that effectively shift the efficient frontier outward must possess differential return and risk profiles, and have low correlation with the existing assets in the portfolio. Incorporating an asset that has similar returns and risk and is collinear with an asset already in the portfolio is duplicative. One needs only one of the two.

Differential returns, risk, and correlation are not enough, however. A new asset that has a lower return and significantly higher volatility will not likely enter optimal portfolios even if it has a low correlation with existing assets. In contrast, a new asset with a higher return, lower risk, and modest correlation with existing assets will virtually always enter at least some of the optimal portfolios on the efficient frontier.

In decades past, adding real estate, commodities, and gold to stock and bond portfolios produced the desired advantages. Low returns for these assets in the 1980s and subsequent years led to their eventual purging from most portfolios by the mid-1990s. They were replaced with assets such as international stocks and bonds, emerging market securities, and high-yield debt.

Today, most managers employ multiple equity classes in their portfolios to achieve asset diversification. They usually include U.S. as well as international and emerging market stocks. Some managers split the U.S. equities category into large, midsized, and small capitalization securities, or alternatively into growth and value stocks. Some managers use microcap stocks. The fixed-income assets commonly used include U.S. bonds, international bonds, high-yield debt, and emerging market debt. Some managers further break U.S. bonds into short, medium, and long-duration "buckets." A

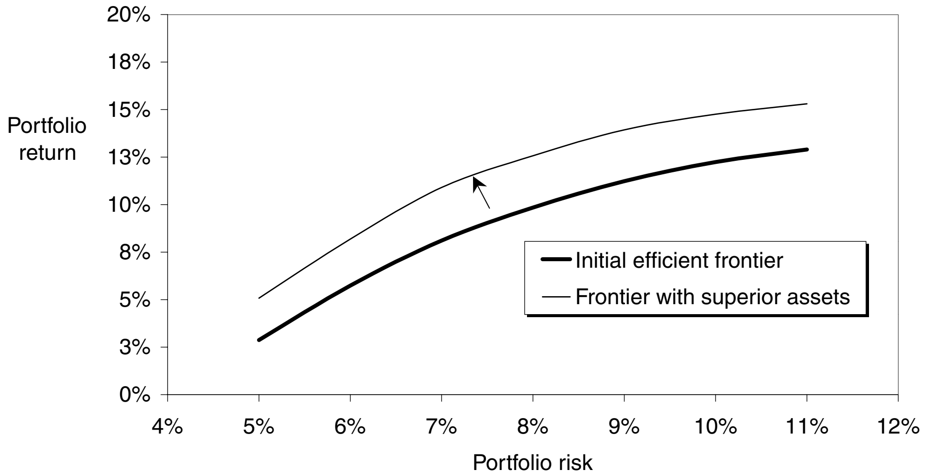

Figure 4 Improving Investment Performance via Asset Selection.

few investment managers retain real estate as an investable asset, while a smaller minority includes commodities as an inflation hedge.

One problem is that the correlation between different equity and fixed income subcategories is fairly high. Thus, the advantages gained from dividing equities and fixed income into components are not that great. In fact, cross-market correlation has increased recently due to globalization. For example, the performance of large, small, and midcap equities, as well as that of international equities, now moves much more in parallel than in the past.

Where are the new opportunities to improve portfolio performance by creatively expanding the investable asset domain? There are several assets that are now increasingly accepted and utilized in portfolios. The most important are hedge funds, private equity, and venture capital. Other new assets have shown the ability to improve portfolio performance but for various reasons have not yet made the leap to widespread recognition. These include inflation-protected securities and insurance-linked products.

6.2. Hedge Funds

Over the last several years, hedge funds have garnered tremendous attention as an attractive addition to classic stock and bond portfolios. A virtual avalanche of published MV studies indicates that hedge funds extend the efficient frontier. Examples include research by Agarwal and Naik (2000), Lamm and Ghaleb-Harter (2000b), Lamm (1999a), Purcell and Crowley (1999), Edwards and Lieu (1999), and Schneeweis and Spurgin (1999). Other work supporting the inclusion of hedge funds in portfolios was done by Tremont Partners and TASS Investment Research (1999), Goldman Sachs and Financial Risk Management (1999), Yago et al. (1999), Schneeweis and Spurgin (1998), and Fung and Hseih (1997). The general conclusions are that substantial hedge fund allocations are appropriate, even for conservative investors.

All of these studies presume that hedge fund exposure is accomplished via broad-based portfolios. That is, like equities, hedge fund investments need to be spread across a variety of managers using different investment strategies. This means at least a dozen or more. Investing in only one hedge fund is an extremely risky proposition, much like investing in only one stock.

The primary advantage of hedge fund portfolios is that they have provided double-digit returns going back to the 1980s with very low risk. Indeed, hedge fund portfolio volatility is close to that of bonds. With much higher returns and low correlation compared with traditional asset classes, they exhibit the necessary characteristics required to enhance overall portfolio performance.

Hedge funds are private partnerships and are less regulated than stocks. For this reason, a strict due diligence process is essential to safeguard against the rare occasion when unscrupulous managers surface. Because most investors cannot or do not want to perform comprehensive evaluations of each hedge fund, funds of hedge funds are increasingly used as implementation vehicles. Large investment banks or financial institutions usually sponsor these funds. They have professional staffs capable of performing a laborious investigative and review process.

Table 4 reproduces the results of a MV optimization by Lamm and Ghaleb-Harter to assess the impact of adding hedge funds to portfolios of traditional assets. Results are given for a conservative range of prospective hedge fund returns varying from 6.5–10.5%. The volatility of the hedge fund portfolio is close to 4%, not much different from history. The assumed correlation with stocks and bonds is 0.3, also close to historical experience. The other assumptions are similar to those presented in the Section 3 example.

The key findings are that (1) hedge funds primarily substitute for bonds and (2) substantial allocations are appropriate for practically all investors, even conservative ones. This is true even if relatively low hedge fund returns are assumed. The conclusion that hedge funds are suitable for conservative investors may be counterintuitive to many readers, who likely perceive hedge funds as very risky investments. However, this result is now strongly supported by MV analysis and occurs primarily because hedge fund managers practice diverse investment strategies that are independent of directional moves in stocks and bonds. These strategies include merger and acquisition arbitrage, yield curve arbitrage, convertible securities arbitrage, and market-neutral strategies in which long positions in equities are offset by short positions.

In response to recent MV studies, there has been a strong inflow of capital into hedge funds. Even some normally risk-averse institutions such as pension funds are now making substantial allocations to hedge funds. Swensen (2000) maintains that a significant allocation to hedge funds as well as other illiquid assets is one of the reasons for substantial outperformance by leading university endowment funds over the last decade.

6.3. Private Equity and Venture Capital

As in the case of hedge funds, managers are increasingly investing in private equity to improve portfolio efficiency. Although there is much less conclusive research on private equity and investing, the recent performance record of such investments is strong.

TABLE 4 Optimum Portfolios with Hedge Funds and Conventional Assets

Hedge Fund Return	Total Portfolio		Allocation					
			Equities		Bonds		Hedge Funds	Cash
	Risk	Return	U.S.	International	U.S.	International		
10.5%	12.5%	12.2%	—	83%	—	—	17%	—
	10.5%	11.9%	—	66%	—	—	34%	—
	8.5%	11.5%	—	50%	—	—	50%	—
	6.5%	11.2%	—	31%	—	—	69%	—
	4.5%	10.6%	—	4%	—	—	96%	—
9.5%	12.5%	12.1%	15%	76%	—	—	9%	—
	10.5%	11.6%	11%	62%	—	—	27%	—
	8.5%	11.0%	7%	46%	—	—	46%	—
	6.5%	10.5%	3%	29%	—	—	67%	—
	4.5%	9.6%	—	7%	—	—	90%	—
8.5%	12.5%	12.0%	33%	66%	—	—	1%	—
	10.5%	11.3%	26%	53%	—	—	21%	—
	8.5%	10.6%	19%	40%	—	—	41%	—
	6.5%	9.8%	11%	25%	—	—	64%	—
	4.5%	8.8%	5%	8%	—	10%	77%	—
7.5%	12.5%	12.4%	35%	65%	—	—	—	—
	10.5%	11.7%	34%	48%	—	—	18%	—
	8.5%	10.7%	26%	36%	—	3%	35%	—
	6.5%	9.7%	19%	23%	—	11%	48%	—
	4.5%	8.4%	10%	8%	—	19%	62%	—
6.5%	12.5%	12.4%	35%	65%	—	—	—	—
	10.5%	11.6%	41%	44%	—	15%	—	—
	8.5%	10.4%	34%	31%	6%	22%	6%	—
	6.5%	9.2%	23%	22%	15%	20%	20%	—
	4.5%	7.9%	16%	14%	12%	14%	17%	27%

Source: Lamm and Ghaleb-Harter 2000b.

There are varying definitions of private equity, but in general it consists of investments made by partnerships in venture capital (VC) leveraged buyout (LBO) and mezzanine finance funds. The investor usually agrees to commit a minimum of at least $1 million and often significantly more. The managing partner then draws against commitments as investment opportunities arise.

In LBO deals, the managing partner aggregates investor funds, borrows additional money, and purchases the stock of publicly traded companies, taking them private. The targeted companies are then restructured by selling off noncore holdings, combined with similar operating units from other companies, and costs reduced. Often existing management is replaced with better-qualified and experienced experts. After several years, the new company is presumably operating more profitably and is sold back to the public. Investors receive the returns on the portfolio of deals completed over the partnership's life.

The major issue with private equity is that the lock-up period is often as long as a decade. There is no liquidity if investors want to exit. Also, because the minimum investment is very high, only high-net-worth individuals and institutions can participate. In addition, investments are often concentrated in only a few deals, so that returns from different LBO funds can vary immensely. Efforts have been made to neutralize the overconcentration issue with institutions offering funds of private equity funds operated by different managing partners.

VC funds are virtually identical in structure to LBO funds except that the partnership invests in start-up companies. These partnerships profit when the start-up firm goes public via an initial public offering (IPO). The partnership often sells its shares during the IPO but may hold longer if the company's prospects are especially bright. VC funds received extraordinary attention in 1999 during the internet stock frenzy. During this period, many VC funds realized incredible returns practically overnight as intense public demand developed for dot-com IPOs and prices were bid up speculatively.

Because LBO and VC funds are illiquid, they are not typically included in MV portfolio optimizations. The reason is primarily time consistency of asset characteristics. That is, the risk associated with LBOs and VC funds is multiyear in nature. It cannot be equated directly with the risk of stocks, bonds, and hedge funds, for which there is ample liquidity. A less egregious comparison requires multiyear return, risk, and correlation calculations.

6.4. Inflation-Protected Securities

Treasury inflation-protected securities (TIPs) are one of the most innovative financial products to appear in recent years. They pay a real coupon, plus the return on the Consumer Price Index (CPI). Thus, they automatically protect investors 100% against rising inflation, a property no other security possesses. Further, in a rising inflation environment, returns on TIPs are negatively correlated with those of bonds and other assets whose prices tend to decline when inflation rises.

In an MV study of combining TIPs with conventional assets, Lamm (1998a, b) finds that an overweighting in these securities vs. traditional bonds is appropriate only when inflation is expected to rise. He suggests a 50/50 weighting vs. traditional bonds when inflation direction is uncertain and underweighting when inflation is expected to fall.

In years past, some investment managers touted gold, commodities, or real estate as inflation hedges. The availability of TIPs neutralizes arguments made in favor of these "real" assets because their correlation with inflation is less than perfect. TIPs provide a direct one-to-one hedge against rising inflation.

Despite the attractive characteristics of TIPs, they have met with limited market acceptance. This is a consequence partly of a lack of awareness but also because TIPs significantly underperformed conventional Treasury securities as inflation declined immediately after their introduction in 1997. However, for the first time in years, inflation is now rising and TIPs have substantially outperformed Treasuries over the last year. This will likely change investor attitudes and lead to greater acceptance of the asset class.

6.5. Other Assets

Another new asset proposed for inclusion in portfolios is insurance-linked products such as catastrophe bonds. These securities were evaluated in MV studies done by Lamm (1998b,1999b). They are issued by insurance companies as a way of protecting against heavy losses that arise as a consequence of hurricanes or earthquakes. The investor receives a large payoff if the specified catastrophe does not occur or a low return if they do. Because payoffs are linked to acts of nature, the correlation of insurance-linked securities with other assets is close to zero. Unfortunately, the market for insurance-linked securities has failed to develop sufficient liquidity to make them broadly accessible to investors.

Another asset occasionally employed by investment managers is currency. While currencies meet liquidity criteria, their expected returns are fairly low. In addition, exposure to foreign stocks and bonds contains implicit currency risk. For this reason, currencies are not widely viewed as a distinct standalone asset. Even managers that do allocate to currencies tend to view positions more as short-term and tactical in nature.

Finally, there are a number of structured derivative products often marketed as new asset classes. Such products are usually perturbations of existing assets. For example, enhanced index products are typically a weighted combination of exposure to a specific equities or bond class, augmented with exposure to a particular hedge fund strategy. Similarly, yield-enhanced cash substitutes with principal guarantee features are composed of zero coupon Treasuries with the residual cash invested in options or other derivatives. Such products are part hedge fund and part primary asset from an allocation perspective and are better viewed as implementation vehicles rather than incorporated explicitly in the asset allocation.

7. THE FORECASTING PROBLEM

Beyond asset selection, the key to successful investment performance via MV asset allocation depends largely on the accuracy of return, risk, and correlation forecasts. These forecasts may be subjective or quantitative. A subjective or purely judgmental approach allows one the luxury of considering any number of factors that can influence returns. The disadvantage of pure judgment is that it is sometimes nebulous, not always easily explained, and may sometimes be theoretically inconsistent with macroeconomic constraints.

Model forecasts are rigorous and explicit. They derive straightforwardly from a particular variable set. Most quantitative approaches to forecasting are based on time series methods, explored in great depth in the finance literature over the years. Beckers (1996) provides a good review of such methods used to forecast returns, while Alexander (1996) surveys risk and correlation forecasting. In addition, Lummer et al. (1994) describe an extrapolative method as a basis for long-term asset allocation. An alternative to pure time series is causal models. For example, Connor (1996) explored macroeconomic factor models to explain equity returns. Lamm (2000) proposes a modified bivariate Garch model as one way of improving MV forecast accuracy.

In the case of a two-asset stock and bond portfolio, the modified Garch model proposed by Lamm is simply:

$$r_{1t} = \varphi_{11} + \varphi_{12} r_{1,t-1} + \Sigma \gamma_k x_{k,t-1} + \varepsilon_{1t} \tag{8}$$

$$r_{2t} = \varphi_{21} + \varphi_{22} r_{2,t-1} + \Sigma \gamma_k x_{k,t-1} + \varepsilon_{2t} \tag{9}$$

$$\sigma^2_{1t} = \alpha_{10} + \alpha_{11} \varepsilon^2_{1,t-1} + \beta_{11} \sigma^2_{1,t-1} \tag{10}$$

$$\sigma^2_{2t} = \alpha_{20} + \alpha_{21} \varepsilon^2_{2,t-1} + \beta_{21} \sigma^2_{2,t-1} \tag{11}$$

$$\sigma_{12t} = \alpha_{30} + \alpha_{31} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + \beta_{31} \sigma_{12,t-1} \tag{12}$$

where r_{it} = the returns on equities and bonds, respectively
σ^2_{it} = the corresponding variances
ε_{it} = residuals
σ_{12t} = the covariance between stocks and bonds
x_k = exogenous factors (e.g., inflation, industrial production, corporate earnings, and interest rates) that determine market returns

The other symbols are estimated parameters. The first two equations predict returns, while the second two forecast associated risk. The correlation between stocks and bonds in any period is simply $\rho = \sigma_{12t} / \sigma_{1t}\sigma_{2t}$, which is derived from the last three equations. This model postulates that when extraneous changes occur that are not reflected in economic variables, the resulting prediction errors push up risk and reduce correlation—exactly the pattern observed in response to market shocks through time.

Lamm reports that augmenting Garch with exogenous variables significantly improves forecast accuracy. These findings have important implications for portfolio management. Critically, augmented Garch provides a more logical and systematic basis for reallocation decisions through the economic cycle (Figure 5) and changing inflation scenarios, which shift efficient frontiers (Figure 6). The augmented Garch process also allows one to distinguish economic influences from purely unexpected shocks, which are often event driven. A pure time series approach provides no such delineation.

If one desires to focus only on return forecasting, a useful approach is vector autoregression (VAR). Although largely atheoretic, except regarding system specification, such models have been shown to have superior predictive capability. In particular, VAR forecasts are more accurate the longer the periodicity of the data. For example, monthly VAR models typically explain 5–10% of the variation in returns, while annual VAR models often explain 90% or more. Quarterly forecasting models produce results in between. VAR models thus provide a reasonably good method for annual asset allocation while providing only a slight edge for monthly allocation.

VAR models are fairly simple and are specified as:

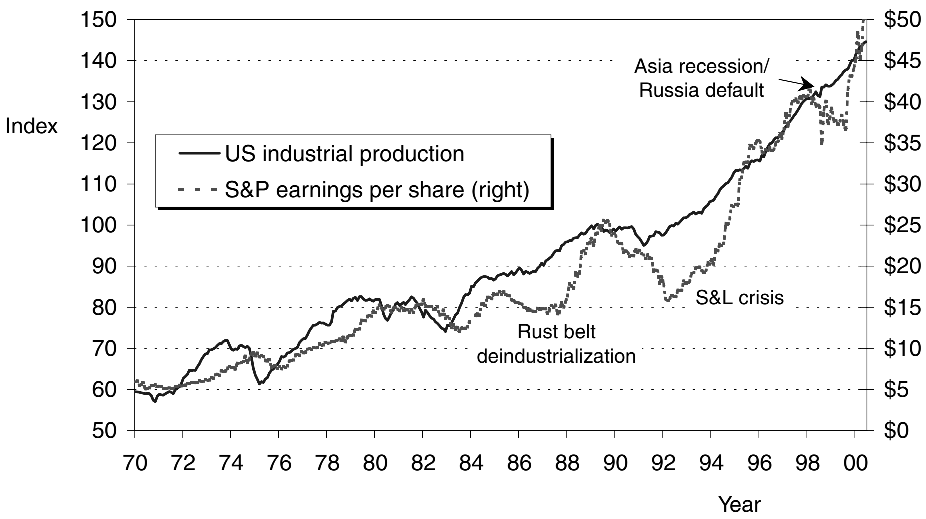

Figure 5 Corporate Earnings and Stock Prices Follow Economic Growth Cycles.

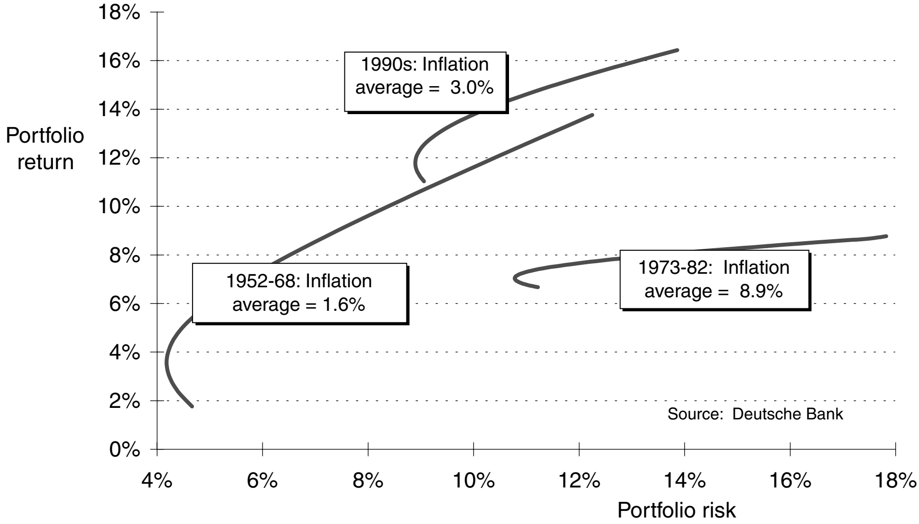

Figure 6 Efficient Frontiers in Different Macroeconomic Environments.

$$\mathbf{y}_t = \mathbf{A}L(\mathbf{y}_{t-1}) + \mathbf{e}_t \tag{13}$$

where $\mathbf{y}_t$ = the vector of system variables (including returns) that are to be predicted
$L(\ldots)$ = the lag operator
$\mathbf{e}_t$ = a vector of errors
$\mathbf{A}$ = the parameter matrix.

If some of the system variables are policy variables (such as Federal Reserve interest rate targets), they can be endogenized for making forecasts and evaluating scenarios. The forecasting system then becomes:

$$\mathbf{z}_{t+1} = \mathbf{B}L(\mathbf{z}_t) + \mathbf{C}L(\mathbf{x}_t) \tag{14}$$

where $\mathbf{z}$ contains the system variables to be forecast from $\mathbf{y} = [\mathbf{z}|\mathbf{x}]^{\mathrm{T}}$
$\mathbf{x}$ contains the expected policy values
$\mathbf{B}$ and $\mathbf{C}$ are the relevant coefficient submatrices contained in $\mathbf{A}$

8. ENGINEERING CLIENT-TAILORED SOLUTIONS: APPLYING PORTFOLIO RESTRICTIONS

The discussion so far has focused on asset allocation as generally applicable to a broad cross-section of investors. In reality, the vast majority of individual investors face special restrictions that limit their flexibility to implement optimal portfolios. For example, many investors hold real estate, concentrated stock holdings, VC or LBO partnerships, restricted stock, or incentive stock options that for various reasons cannot be sold. For these investors, standard MV optimization is still appropriate and they are well advised to target the prescribed optimum portfolio. They should then utilize derivative products such as swaps to achieve a synthetic replication.

Synthetic rebalancing cannot always be done, however. This is likely to be the case for illiquid assets and those with legal covenants limiting transfer. For example, an investor may own a partnership or hold a concentrated stock position in a trust whose position cannot be swapped away. In these situations, MV optimization must be amended to include these assets with their weights restricted to the prescribed levels. The returns, risk, and correlation forecasts for the restricted assets must then be incorporated explicitly in the analysis to take account of their interaction with other assets. The resulting constrained optimum portfolios will comprise a second-best efficient frontier but may not be too far off the unconstrained version.

An interesting problem arises when investors own contingent assets. For example, incentive stock options have no liquid market value if they are not yet exercisable, but they are nonetheless worth something to the investor. Because banks will not lend against such options and owners cannot realize value until exercise, it can be argued that they should be excluded from asset allocation, at least until the asset can be sold and income received. Proceeds should then be invested consistent with the investor's MV portfolio. An alternative is to probabilistically discount the potential value of the option to the present and include the delta-adjusted stock position in the analysis. To be complete, the analysis must also discount the value of other contingent assets such as income flow after taxes and living expenses.

9. TAXATION

Taxes are largely immaterial for pension funds, foundations, endowments, and offshore investors because of their exempt status. Mean-variance asset allocation can be applied directly for these investors as already described. However, for domestic investors, taxes are a critical issue that must be considered in modeling investment choices.

9.1. Tax-Efficient Optimization

Interest, dividends, and short-term capital gains are taxed at "ordinary" income rates in the United States. The combined federal, state, and local marginal tax rate on such income approaches 50% in some jurisdictions. In contrast, long-term capital gains are taxed at a maximum of 20%, which can be postponed until realization upon sale. For this reason, equities are tax advantaged compared with bonds and investments that deliver ordinary income.

In an unadulterated world, MV analysis would simply be applied to after-tax return, risk, and correlations to produce tax-efficient portfolios. Regrettably, tax law is complex and there are a variety of alternative tax structures for holding financial assets. In addition, there are tax-free versions of some instruments such as municipal bonds. Further complicating the analysis is the fact that one must know the split between ordinary income and long-term gains to forecast MV model inputs. This muddles what would otherwise be a fairly straightforward portfolio-allocation problem and requires that tax structure subtleties be built into the MV framework to perform tax-efficient portfolio optimization.

9.2. Alternative Tax Structures

As one might imagine, modeling the features of the tax system is not a trivial exercise. Each structure available to investors possesses its own unique advantages and disadvantages. For example, one can obtain equity exposure in a number of ways. Stocks can be purchased outright and held. Such a "buy-and-hold" strategy has the advantage that gains may be realized at the investor's discretion and the lower capital gains rate applied.

Alternatively, investors may obtain equity exposure by purchasing mutual funds. Mutual fund managers tend to trade their portfolios aggressively, realizing substantially more short-term and long-term gains than may be tax efficient. These gains are distributed annually, forcing investors to pay taxes at higher rates and sacrificing some of the benefits of tax deferral. If portfolio trading produces higher returns than a simple buy-and-hold strategy, such turnover may be justified. But this is not necessarily the case.

A third option for equity exposure is to purchase stocks via retirement fund structures using pretax contributions. Tax vehicles such as 401k, IRA, Keogh, and deferred compensation programs fall in this category. The major benefit of this approach is that it allows a larger initial investment to compound. However, upon withdrawal, returns are taxed at ordinary income tax rates.

A fourth alternative for achieving equity exposure is through annuity contracts offered by insurance companies. These are often referred to as wrappers. This structure allows investors to purchase equity mutual funds with after-tax dollars and pay no taxes until distribution. Thus, gains are sheltered. Even so, all income above basis is taxed at ordinary rates upon distribution. Investors receive deferral benefits but lose the advantage of paying lower rates on returns arising from long-term capital gains.

Although I have described four alternative tax vehicles for equity exposure, the same general conclusions apply for other assets. The one difference is that consideration of ordinary vs. capital gains differentials must explicitly be considered for each type of asset. Table 5 summarizes the tax treatment of investment income through currently available tax structures.

9.3. Results of Tax-Efficient MV Optimizations

Another perplexing feature of the tax system is that after-tax returns are affected by the length of the holding period. For example, it is well known that the advantage of owning equities directly increases the longer is the holding period. This is because deferring capital gains produces benefits via tax-free compounding (Figure 7). Similarly, assets in insurance wrappers must be held over very long periods before the positive effects of tax-free compounding offset the potential negatives of converting long-term gains to ordinary tax rates.

TABLE 5 Alternative Tax Structures for Asset Holding

Structure	Annual Flow Taxation: Dividends and Interest	Annual Flow Taxation: Capital Gains	Annual Pre-Tax Return	Annualized After-Tax Return
Direct ownership	Ordinary rates	Deferred	$b_t = d_t + (r_T)^{1/T} - 1$	$\tau d_t + [\kappa(r_T)^{1/T} - 1]$
Mutual fund	Ordinary rates	Paid as gains realized	$f_t = d_t + r_t$	$\tau d_t + [\tau\alpha + \kappa(1 - \alpha)]r_t$
Retirement fund structure	Deferred	Deferred	$s_t = (2 - \tau)f_t$ or $s_t = (2 - \tau)b_t$	$(2 - \tau)\tau(d_t + r_t)$
Wrapper	Deferred	Deferred	$w_t = d_t + r_t$	$\tau(d_t + r_t)$

Source: Lamm and Ghaleb-Harter 2000c.
d_t = dividend return, r_T = gain from period 0 to T equal to $(p_T - p_0)/p_0$; r_t = gain from prior period, τ = one minus ordinary income tax rate, κ = one minus capital gains tax rate, α = percentage of gains that are short term, and T = investment horizon.

When it comes to applying the MV approach to taxable investing, the methodology is the same as that used for tax-free portfolios, except that four different after-tax return streams, risk measures, and four-square correlations are required for each asset. Table 6 compares the pre-tax and after-tax returns for assets typically considered in taxable portfolio allocation studies.

A complete review of after-tax-efficient frontiers would be beyond the scope of this chapter. However, there are four general conclusions. First, a basic conclusion of most after-tax allocation studies is that investors are well advised to hold the maximum possible portion of their investable wealth in tax-deferred vehicles such as 401k programs and deferred compensation plans. The 401k structure is particularly attractive because employers often match employee contributions.

Second, after maximizing contributions to tax-deferred vehicles, investors should utilize buy-and-hold strategies for equities and hold municipal bonds as the appropriate fixed income investment. Third, hedge funds enter most optimal allocations directly even though as much as 75% of the income is taxed at ordinary rates. Optimal allocations to hedge funds are lower than for tax-exempt investors, however. Fourth, for long horizons of at least a decade and more, wrapped assets often enter optimum portfolios, particularly for hedge funds.

Figure 7 The Benefit of Deferring Capital Gains Taxes.

TABLE 6 After-Tax Returns

Asset	Pre-Tax Return	5-Year Horizon Buy/Hold	5-Year Horizon Fund	Wrap	10-Year Horizon Buy/Hold	10-Year Horizon Fund	Wrap	20-Year Horizon Buy/Hold	20-Year Horizon Fund	Wrap
U.S. stocks	11.5%	8.6%	7.8%	5.9%	9.0%	7.8%	6.6%	9.6%	7.8%	7.6%
International stocks	13.1%	9.8%	8.8%	6.9%	10.3%	8.8%	7.8%	11.0%	8.8%	9.1%
U.S. bonds	6.5%	3.5%	3.5%	2.7%	3.5%	3.5%	3.0%	3.5%	3.5%	3.4%
International bonds	7.4%	4.0%	4.0%	3.3%	4.0%	4.0%	3.6%	4.0%	4.0%	4.1%
Muni bonds	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
REITs	8.25%	4.8%	4.8%	4.1%	4.8%	4.8%	4.5%	4.8%	4.8%	5.3%
Hedge funds	10.5%	6.2%	6.2%	5.2%	6.2%	6.2%	5.8%	6.2%	6.2%	6.8%
Cash	5.5%	3.0%	3.0%	2.1%	3.0%	3.0%	2.3%	3.0%	3.0%	2.6%
TE cash	3.2%	3.2%	3.2%	3.2%	3.2%	3.2%	3.2%	3.2%	3.2%	3.2%

Source: Lamm and Ghaleb-Harter 2000c.
Ordinary tax rates are assumed to be 46% (state and federal). Incremental costs of 1% annually are deducted for the wrap structure.

These generalizations presume investors are in the top marginal tax bracket (federal, state, and local) both at the beginning of the horizon and at the end. The extent to which investors move into lower brackets via diminished post-retirement income or by migrating to lower tax jurisdictions make the benefits of deferral even greater.

10. TIME HORIZON

Beyond the taxation issue, a key component of MV problem specification is properly defining the horizon over which the optimization is applied. In particular, risk and correlation can differ significantly, depending on the time frame considered. For example, equities are much less risky over five-year horizons than over one year (Figure 8). This has led some analysts to argue for larger equity allocations for long-term investors because stock risk declines relative to bonds over lengthy horizons.

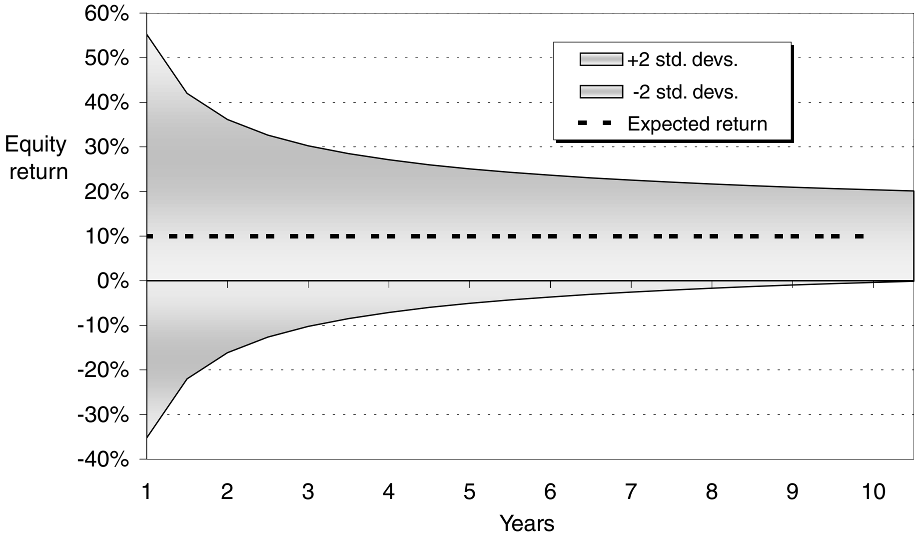

Figure 8 Equity Risk Is Lower over Long Time Horizons.

This issue is especially pertinent if illiquid assets such as private equity are to be included in the portfolio. Using annual volatility for private equity makes no sense—10-year risk must be considered.

In general, the longer the time horizon, the lower the risk for assets that appear to be risky when evaluated based on short-term data. As a consequence, long-horizon investors will allocate more to assets such as stocks and private equity. The reason for this is that annual risk is related to risk over other intervals by the identity $\sigma_a = \sigma_T T^{1/2}$, where T is the periodicity of the data. Note that T = 4, 12, or 52 for quarterly, monthly, and weekly data and T = 1/2, 1/5, or 1/10 for 2-, 5-, and 10-year intervals. Thus, assets with high annual risk have exponentially declining risk over lengthy time intervals. This is why long-horizon portfolios often have relatively small allocations to fixed-income instruments and higher allocations to what are perceived as riskier assets (Figure 9). Some managers go as far as to suggest that a long-horizon investor hold 100% equities if they desire liquidity and the ability to exit portfolios at a time of their choosing.

11. EXTENSIONS AND NEW FRONTIERS

There are numerous extensions of the basic MV model beyond those already described. For example, by dropping the constraint that portfolio weights be positive, the model can be used to ascertain short and long positions that should be held for various assets. Similarly, incorporating borrowing rates allows MV models to be used to design optimal leverageable portfolios. Furthermore, the MV approach can be applied to specific asset classes to build portfolios of individual securities such as equities or bonds. Beyond this, MV analysis has even broader strategic uses. For example, it can be applied by corporations to design portfolios of businesses.

One additional application of MV analysis is to use the technique to reengineer implied market returns. This requires the problem be reformulated to select a set of returns given asset weights, risk, and correlations. The weights are derived from current market capitalizations for equities, bonds, and other assets. The presumption is that today's market values reflect the collective portfolio optimizations of all investors. Thus, the set of returns that minimizes risk is the market's forecast of future expected returns. This information can be compared with the user's own views as a reliability check. If the user's views differ significantly, they may need to be modified. Otherwise the user can establish the portfolio positions reflecting his or her outlook under the premise his or her forecasts are superior.

Other variants of MV optimization have been proposed to address some of its shortcomings. For example, MV analysis presumes normal distributions for asset returns. In actuality, financial asset return distributions sometimes possess "fat tails." Alternative distributions can be used. In addition, the MV definition of risk as the standard deviation of returns is arbitrary. Why not define risk as first- or third-order deviation instead of second? Why not use absolute or downside deviation?

Recently, Duarte (1999) has suggested that value-at-risk approaches be used to derive efficient frontiers. This approach differs from MV analysis in that it uses Monte Carlo simulation to determine the dollar value of the portfolio that is at risk with a particular degree of statistical confidence. That

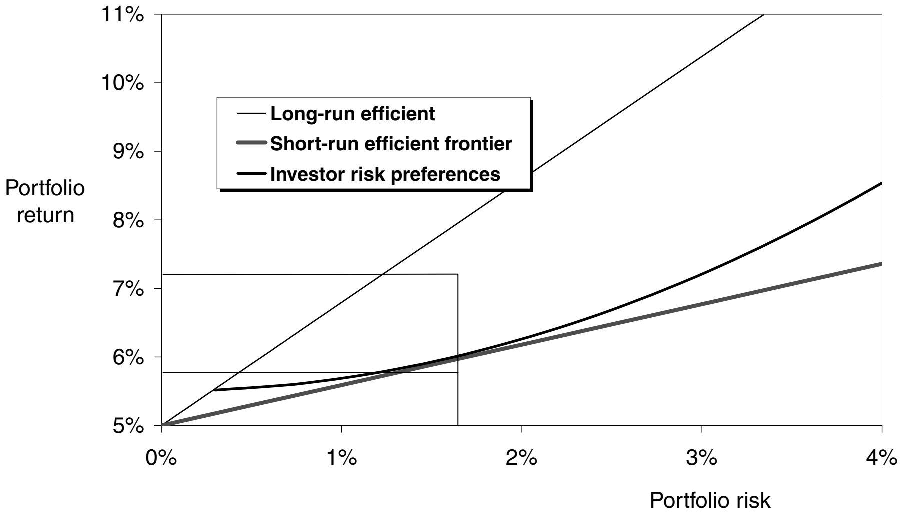

Figure 9 Long-Run vs. Short-Run Time Horizons.

is, a \$10 million portfolio might be found to have a value at risk on a specific day of \$1.1 million with 95% confidence. This means the value of the portfolio would be expected to fall more than this only 5% of the time. Changing the composition of the portfolio allows a value-at-risk efficient frontier to be traced out.

Duarte also proposes a generalized approach to asset allocation that includes mean semi-variance, mean absolute deviation, MV, and value-at-risk as special cases. While cleverly broad, Duarte's method relies on simulation techniques and is computationally burdensome. In addition, because simulation approaches do not have explicit analytical solutions, the technique loses some of the precision of MV analysis. For example, one can examine solution sensitivities from the second-order conditions of MV problems, but this is not so easy with the simulation. It remains to be seen whether simulation approaches receive widespread acceptance for solving portfolio problems. Nonetheless, simulation techniques offer tremendous potential for future applications.

12. COMBINING MEAN-VARIANCE ANALYSIS WITH OTHER TECHNIQUES—CONSTRUCTING OPTIMAL HEDGE FUND PORTFOLIOS

Because of the difficulty of identifying new assets that shift the efficient frontier upward, some investment managers have taken to creating their own. These assets are often ''bottoms-up'' constructions that in aggregate possess the desired return, risk, and correlation characteristics

In the case of funds of hedge funds, the current industry practice is simply to assemble judgmentally a montage consisting of well-known managers with established track records. Although some effort is expended towards obtaining diversification, a quantitative approach is seldom employed. As a consequence, most portfolios of hedge funds exhibit positive correlation with other asset classes such as stocks. This became obvious in 1998 when many hedge fund portfolios produced negative performance at the same time equity prices fell following the Russian debt default.

Lamm and Ghaleb-Harter (2000a) have proposed a methodology for constructing portfolios of hedge funds that have zero correlation with other asset classes. Their approach brings more rigor to the design of hedge fund portfolios by using an MV optimization format under the constraint that aggregate correlation with traditional assets is zero.

The steps in this process are as follows. First, hedge fund returns are decomposed into two components using a reformulation of Sharpe's (1992) style analysis. Specifically:

$$h_j = \alpha_j + \Sigma\beta_{jk}r_k \qquad j = 1, \ldots, J;\ k = 1, \ldots, K \tag{15}$$

where h_j = the return for the jth hedge fund in a portfolio of J funds
α_j = the skill performance of the fund
β_{jk} = the exposure of the fund to the kth ''traditional'' asset class
r_k = the traditional asset return

Time subscripts, which apply to h_j and r_k, are dropped.

Equation (15) can be written more simply as:

$$\mathbf{h} = \mathbf{a} + \mathbf{Br} \tag{16}$$

where $\mathbf{h} = [h_1\ h_2 \ldots h_J]^{\mathrm{T}}$
$\mathbf{a} = [\alpha_1\ \alpha_2 \ldots \alpha_J]^{\mathrm{T}}$
$\mathbf{r} = [r_1\ r_2 \ldots r_k]^{\mathrm{T}}$

and the asset exposure matrix is:

$$\mathbf{B} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1K} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2K} \\ \vdots & \vdots & \cdots & \vdots \\ \beta_{J1} & \beta_{J2} & \cdots & \beta_{JK} \end{bmatrix} \tag{17}$$

The return for the hedge fund portfolio is:

$$h = \mathbf{w}^{\mathrm{T}}\mathbf{h} = \mathbf{w}^{\mathrm{T}}(\mathbf{a} + \mathbf{Br}) \tag{18}$$

where $\mathbf{w} = [w_1\ w_2 \ldots w_J]^{\mathrm{T}}$ are the portfolio weights assigned to each hedge fund.

The variance of the hedge fund portfolio is:

$$\sigma = \mathbf{w}^{\mathrm{T}}\mathbf{S}^{\mathrm{T}}\mathbf{R}\mathbf{S}\mathbf{w} \tag{19}$$

where $\mathbf{S}$ = the risk matrix with diagonal standard deviations $s_1\ s_2 \ldots s_J$
$\mathbf{R}$ = the $J \times J$ hedge fund correlation matrix.

To select the optimum MV hedge fund portfolio that produces the highest return with a target zero beta exposure to other assets, the problem is to:

$$\max \mathrm{U}(\mathbf{w}) = (h - \lambda\sigma^2) \quad \text{subject to} \tag{20}$$

$$h = \mathbf{w}^{\mathrm{T}}(\mathbf{a} + \mathbf{B}\mathbf{r}) \tag{21}$$

$$\sigma = \mathbf{w}^{\mathrm{T}}\mathbf{S}^{\mathrm{T}}\mathbf{R}\mathbf{S}\mathbf{w} \tag{22}$$

$$\text{colsum } \mathbf{w}^{\mathrm{T}}\mathbf{B} = \mathbf{0} \tag{23}$$

$$\text{colsum } \mathbf{w} = 1 \tag{24}$$

$$w_j \geq 0 \tag{25}$$

$$w_j \leq 1/n \tag{26}$$

where $1/n$ = the maximum weight allowed for any hedge fund
λ = the risk aversion parameter.

This is the same as the standard MV model except the portfolio return takes a more complex form (21) and an additional constraint (23) is added that forces net exposure to traditional assets to zero. Also, constraint (26) forces sufficient diversification in the number of hedge funds.

Obtaining a feasible solution for this problem requires a diverse range of beta exposures across the set of hedge funds considered for inclusion. For example, if all equity betas for admissible hedge funds are greater than zero, a feasible solution is impossible to attain. Fortunately, this problem is alleviated by the availability of hundreds of hedge funds in which to invest.

A significant amount of additional information is required to solve this problem compared to that needed for most MV optimizations. Not only are individual hedge fund returns, risk, and correlations necessary, but returns must also be decomposed into skill and style components. This requires a series of regressions that link each hedge fund's return to those of traditional assets. If 200 hedge funds are candidates, then the same number of regressions is necessary to estimate the α and β parameters.

Table 7 presents optimum hedge fund portfolios that maximize returns for given risk levels while constraining beta exposure to traditional assets to zero. The optimization results are shown for (1) no constraints; (2) restricting net portfolio beta exposure to traditional assets to zero; (3) restricting the weight on any hedge fund to a maximum of 10% to assure adequate diversification; and (4) imposing the maximum weight constraint and the requirement that net beta exposure equals zero. Alternatives are derived for hedge fund portfolios with 3% and 4% risk.

Adding more constraints shifts the hedge fund efficient frontier backward as one progressively tightens restrictions. For example, with total portfolio risk set at 3%, hedge fund portfolio returns fall from 27.4% to 24.4% when zero beta exposure to traditional assets is required. Returns decline to 23.0% when hedge fund weights are restricted to no more than 10%. And returns decrease to 20.4% when net beta exposure is forced to zero and hedge fund weights can be no more than 10%.

13. CONCLUSION

This chapter has presented a brief introduction to asset management, focusing on primary applications. The basic analytical tool for portfolio analysis has been and remains MV analysis and variants of the technique. Mean-variance analysis is intellectually deep, has an intuitive theoretical foundation, and is mathematically efficient. Virtually all asset-management problems are solved using the approach or modified versions of it.

The MV methodology is not without shortcomings. It must be used cautiously with great care paid to problem formulation. The optimum portfolios it produces are only as good as the forecasts used in their derivation and the asset universe from which the solutions are derived. Mean-variance analysis without a concerted effort directed to forecasting and asset selection is not likely to add significant value to the quality of portfolio decisions and may even produce worse results than would otherwise be the case.

While MV analysis still represents the current paradigm, other approaches to portfolio optimization exist and may eventually displace it. Value-at-risk simulation methodologies may ultimately prove more than tangential. Even so, for many practitioners there is still a long way to go before forecasting techniques, asset identification, and time horizon considerations are satisfactorily inte-

TABLE 7 Optimized Portfolios—Hedge Fund Allocations under Various Constraints

		$\sigma = 3\%$				$\sigma = 4\%$			
Fund Strategy	Manager ID	None	$\beta_k = 0$	$w_i = 0.1$	$w_i < 0.1$ $\beta_k = 0$	None	$\beta_k = 0$	$w_i = 0.1$	$w_i < 0.1$ $\beta_k = 0$
Relative value	L/S #1	6%	—	10%	10%	—	—	10%	10%
	CA #1	—	4%	—	—	—	—	—	—
	CA #2	—	—	2%	—	—	—	2%	—
	BH #1	—	5%	—	—	—	3%	—	—
	RO #1	58%	53%	10%	10%	75%	60%	10%	10%
	AG #1	—	—	10%	10%	—	—	10%	10%
	AG #2	—	—	10%	7%	—	—	10%	7%
Event driven	DA #1	3%	4%	8%	9%	3%	2%	10%	10%
	DS #1	15%	—	10%	10%	2%	—	2%	—
	ME #1	—	—	2%	—	—	—	3%	—
Equity hedge	DM #1	3%	3%	7%	8%	—	—	5%	8%
	DM #2	—	3%	4%	10%	—	2%	5%	10%
	DM #3	2%	—	5%	—	3%	—	8%	—
	DM #4	—	—	2%	—	—	—	4%	—
	OP #1	—	—	—	—	—	—	—	3%
	GI #1	—	3%	—	—	—	—	—	2%
	GI #2	—	—	—	—	—	3%	—	—
Global asset allocation	Sy #1	5%	7%	3%	5%	6%	11%	5%	10%
	Sy #2	—	—	2%	—	—	2%	4%	7%
	Sy #3	—	2%	—	4%	—	2%	—	10%
	Sy #4	—	—	3%	—	—	—	5%	—
Short-sellers	SS #1	2%	4%	—	—	—	—	—	—
	SS #2	—	3%	6%	7%	2%	6%	6%	6%
	SS #3	3%		—	—	3%	—	—	—
	Others	3%	9%	7%	7%	6%	8%	6%	6%
Total		100%	100%	100%	100%	100%	100%	100%	100%
	Funds	16	19	24	19	14	15	19	17
Portfolio	Return	27.4	24.4	23.0	20.4	30.6	26.6	23.5	21.2

Source: Lamm and Ghaleb-Harter 2000b.
L/S = long/short; CA = convertible arb; BH = bond hedge; RO = rotational; AG = aggressive; DA = deal arb; DS = distressed; ME = multi-event; DM = domestic; GI = global/international; Sy = systematic; SS = short sellers.

grated with core MV applications. Greater rewards are likely to come from efforts in this direction rather than from devising new methodologies that will also require accurate forecasting and asset selection in order to be successful.

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