Appendix **A**

SDP Trade Space Concepts

Each of the $n = 1, 2, \ldots N$ competing solutions created during an SDP application are evaluated against a set of $m = 1, 2, \ldots M$ value measures whose numerical estimate of value, $VM_{nm} = f_m(x_{nm}) \in [0, 100]$, results from translating data estimates (aka: scores) x_{nm} into a common unit of value via stakeholder assigned value functions f_m. Value functions can be discrete or continuous over their domains and easily accommodate both objective and subjective data estimates. These value functions are, in turn, aggregated into a value return estimate, V_n, for each of the n competing feasible solution alternatives, using the additive value model most commonly seen in multiobjective decision analysis [1]: $V_n = \sum_1^M w_m \cdot f_m(x_{nm})$, where $0 < w_m \leq 1$ and $\sum_1^M w_m = 1$. Infeasible alternatives are eliminated from consideration prior to this point because they fail to meet mandatory requirements (screening criteria) identified by stakeholders and accepted by the decision maker. Efforts to establish and maintain independence between value measures begin during the construction of the qualitative value model and continue throughout the SDP as data estimates and new information become available.

The individual weights w_m used as multipliers for the m value functions are normalized swing weights s_m such that $w_m = s_m / \sum_{m=1}^M s_m,\ 0 < s_m \leq UB$, where UB is an arbitrary upper bound [2]. These swing weights reflect a decision maker's assessment of relative importance of each of the m value measures combined with an estimate of the impact of value measure range swings ([3],[4]) on the decision. This latter estimate enhances the benefits afforded by weighting beyond simple preference ranking, especially for those value measures in which a small change in numerical value has a very large impact on the decision [5].

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This quantitative value model, while similar to the normative approach in utility models [6] from the standpoint of using a function to convert dissimilar units into a common unit of measure, differs from utility modeling in that these value functions are not required to be supported by certain equivalence relationships, are not assessed with lotteries, and are not affected by stakeholder risk attitude [7]. Thus, when properly constructed a value model simply translates and aggregates key system performance estimates into a stakeholder-weighted, value return estimate for each solution. This translational layer of value functions also distinguishes the SDP quantitative value model structure from data envelopment analysis (DEA) models that effectively employ ratios to eliminate disparate data units for aggregating measures [8].

Cost enters consideration neither as a value measure nor as screening criteria in the construction of tradeoff models. Estimates of cost for each feasible solution alternative, C_n, are developed in separate models incorporating cost estimating relationships, time value of money, system life cycle, reliability, and other considerations (see Chapter 5). While typically defined as life cycle monetary costs, C_n can be more generically defined in terms of resource expenditures such as time or effort, elements of risk such as loss of life or compromise of critical information, and so on, thereby affording a good deal of flexibility in applications.

> ***Definition 1.*** The Cartesian plot of each solution's life cycle cost estimate C_n versus value return estimate V_n defines a deterministic *decision trade space*, $D \subset \mathbb{R}^2$, in which each solution $A_n = (C_n, V_n) \in D$ strives to achieve a single, stakeholder expressed ideal, $A_{\text{ideal}} = (C_{\text{ideal}}, V_{\text{ideal}}) \in D$.

The ideal values in each dimension are characterized by an upper bound on total value: $V_{\text{ideal}} = V_{UB}$, and a lower bound on total life cycle cost: $C_{\text{ideal}} = C_{LB}$. In most applications, the chosen ideals are set to 100 and 0, respectively, predominantly because for ease of understanding on the part of stakeholders. The existence of a stakeholder ideal establishes a partial preference ordering on D.

> ***Definition 2.*** For any two solutions $A_k, A_n \in D$, with $A_k = (C_k, V_k)$ and $A_n = (C_n, V_n), k \neq n, A_k$ is preferred to A_n, denoted by $A_k \prec A_n$, whenever $C_k \leq C_n$ and $V_k \geq V_n$ (unless equality holds for both, which yields indifference).

The preference ordering underlying both axes in the decision trade space motivates the principle that a rational decision maker should not desire lower value for higher cost. This rationality, coupled with the properties of trade space efficiency and dominance defined in what follows, supports the construction of a *choice set* C^* of solutions from which a rational decision maker should select a solution.

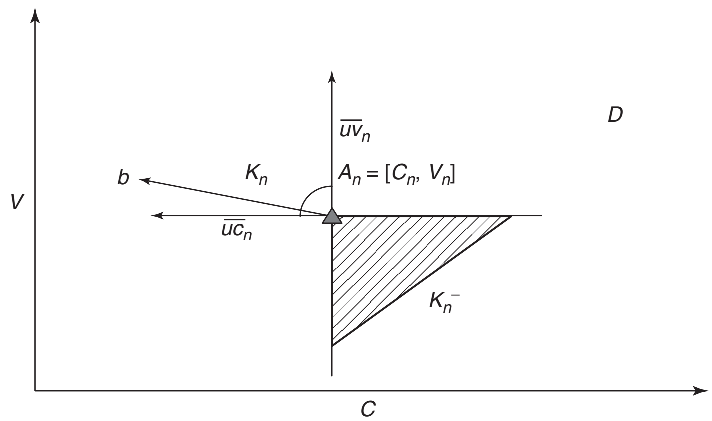

Figure A.1 The cone K_n and polar cone K_n^- of alternative A_n.

Let D be the deterministic decision trade space created via the SDP with preference ordering on C_n and V_n as defined. Let $A_n = (C_n, V_n) \in D, n = 1, \ldots N$ be a set of solutions with $A_{\text{ideal}} = (C_{LB}, V_{UB}) \prec A_n$ for all n, so that A_{ideal} is logically preferred over A_n. For each A_n, define two linearly independent *generating unit vectors:* $\overrightarrow{uc}_n = [-1, 0]$ and $\overrightarrow{uv}_n = [0, 1]$, originating from translated origins centered on each A_n. Figure A.1 illustrates these elements with respect to a single solution A_n.

> ***Definition 3.*** The *cone* $K_n \subseteq D$ of A_n is the set of all vectors (points) $b \in D$ such that $b = A_n + \lambda_1 \overrightarrow{uc}_n + \lambda_2 \overrightarrow{uv}_n$, with $\lambda_1, \lambda_2 \geq 0$.

Since all points in D correspond to solutions, Definition 3 states that any solution lying in the cone K_n of A_n is a nonnegative linear combination of the generating unit vectors $\overrightarrow{uc}_n$ and $\overrightarrow{uv}_n$. Figure A.2 shows Solution 4 as being contained in the cone of Solution 5 as $A_4 = [C_4, V_4] = A_5 + \lambda_1 \overrightarrow{uc}_5 + \lambda_2 \overrightarrow{uv}_5$ with $\lambda_i \geq 0, i = 1, 2$.

> ***Definition 4.*** Given a cone $K_n \subseteq D$ of A_n, the *polar cone* of $A_n, K_n^- \subseteq D$ is defined as the set of all points $l \in D$ such that $l = A_n - \lambda_1 \overrightarrow{uc}_n - \lambda_2 \overrightarrow{uv}_n$ with $\lambda_i \geq 0, i = 1, 2$.

Equivalently, the polar cone can be defined using the inner product $\langle l, b \rangle = \|l\| \|b\| \cos\theta$ as $K_n^- = \{l \in D : \langle l, b \rangle \leq 0 \forall b \in K_n\}$. Thus, the polar cone accounts for all points in D whose vectors extending from the translated origin of solution A_n make an angle $90 \leq \theta \leq 180$ degrees with any vector corresponding to points within or on the boundary of the cone K_n (See [9], or [10]).

Figure A.3 shows a set of solutions with superimposed generating unit vectors. Given the preference ordering on D, a solution A_k lying in the cone K_n of another

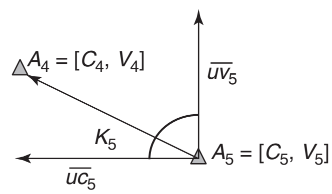

Figure A.2 Solution 4 is contained in the cone K_5 of Solution 5: A_4 strictly dominates A_5.

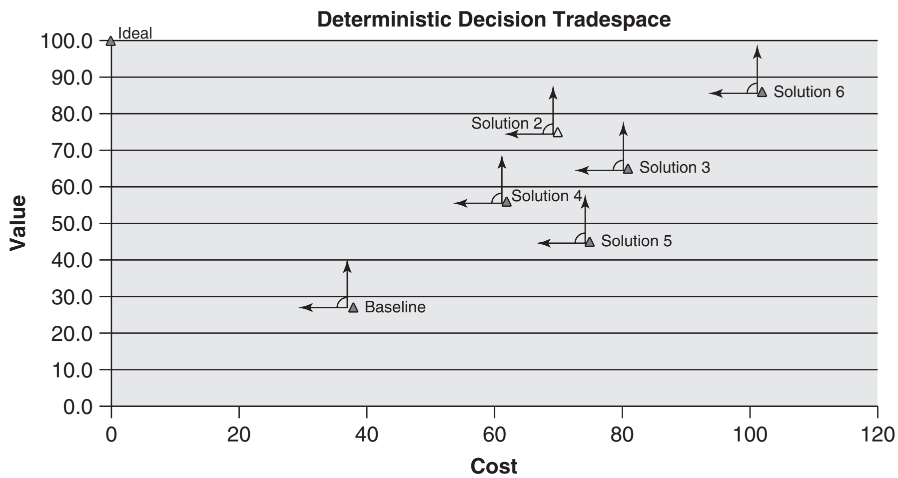

Figure A.3 Solutions with their cone generating unit vectors.

solution A_n has $C_k \leq C_n$ and $V_k \geq V_n$. It follows that there is a need to define equivalence between solutions in this trade space.

Definition 5. A solution $A_n \in D$ *dominates* solution $A_k \in D, k \neq n$, (equivalently, A_k is trade space inferior to A_n) if and only if $A_n \in K_k$ with $\lambda_1 > 0$ or $\lambda_2 > 0$, but not both.

Definition 6. A solution $A_n \in D$ *strictly dominates* solution $A_k \in D, k \neq n$, (equivalently, A_k is strictly trade space inferior to A_n) if and only if $A_n \in K_k$ with both $\lambda_1 > 0$ and $\lambda_2 > 0$.

Remark. Let $A_k, A_n \in D, k \neq n$. A_n dominates A_k if its value is greater ($V_n > V_k$) and its cost is at least a small ($C_n \leq C_k$), or its cost is less ($C_n < C_k$) and its value is at least as great ($V_n \geq V_k$). If both inequalities are strict, then A_n strictly dominates A_k.

Corollary 6.1. A solution A_n *dominates* A_k if $A_n \neq A_k$ and A_k lies in the polar cone of A_n.

Corollary 6.2. A solution A_n *strictly dominates* A_k if $A_k \in K_n^-$ but does not lie on its boundary.

Finally, we define the notion of trade space efficiency that is fundamental to decision support via the SDP.

Definition 7. A solution $A_n \in D$ is *trade space efficient* if no other solution $A_k, k \neq n$ lies in its cone except the ideal, A_{ideal}.

Definition 8. The *choice set*, C^*, is the set of all trade space efficient solutions.

This concept of trade space efficiency depends solely on the determination of dominance among solutions. In a systems setting, it is sometimes the case that the existing (baseline) system is efficient for returning value for cost. However, degraded value return over time due to maturing life cycle stages, changes in the environment, and other factors initiate a decision problem in which improved value return for potentially greater cost is sought. The goal of value modeling in decision support is to identify a trade space efficient set of solutions from which the decision maker should pick.

Remark. The choice set $C^* \in D$ constructed by quantitative modeling via the SDP is a trade space efficient (nondominated) set of solutions.

Trade space efficiency is for most value modeling applications not equivalent to the formal economic notion of Pareto efficiency [11] underscoring data envelopment analysis (DEA) models. This characterization of choice set membership via cones is a much simpler means of identifying the subset of trade space efficient solutions a related optimization-dependent *value free efficiency* calculation affords [12] for weighted aggregate data, largely due to the existence of A_{ideal} which is absent in a DEA setting.

The trade space cost modeling as described in this book will always conclude with a nonempty choice set of trade space efficient solutions. Once an appropriate uncertainty analysis is performed on the choice set solutions by the systems engineering team using Monte Carlo simulation, the decision maker should either select from these solutions, apply decision-focused thinking, or use value-focused thinking to develop creative solutions not previously identified (see Chapter 12).

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