Chapter 3

Basic Calculations

INTRODUCTION

This chapter provides a review of basic calculations and the fundamentals of measurement. Four topics receive treatment:

1 Units and Dimensions
2 Conversion of Units
3 The Gravitational Constant, g_c
4 Significant Figures and Scientific Notation

The reader is directed to the literature in the Reference section of this chapter[1–3] for additional information on these four topics.

UNITS AND DIMENSIONS

The units used in this text are consistent with those adopted by the engineering profession in the United States. For engineering work, SI (*Système International*) and *English* units are most often employed. In the United States, the *English engineering* units are generally used, although efforts are still underway to obtain universal adoption of SI units for all engineering and science applications. The SI units have the advantage of being based on the decimal system, which allows for more convenient conversion of units within the system. There are other systems of units; some of the more common of these are shown in Table 3.1. Although English engineering units will primarily be used, Tables 3.2 and 3.3 present units for both the English and SI systems, respectively. Some of the more common prefixes for SI units are given in Table 3.4 (see also Appendix A.5) and the decimal equivalents are provided in Table 3.5. Conversion factors between SI and English units and additional details on the SI system are provided in Appendices A and B.

Mass Transfer Operations for the Practicing Engineer. By Louis Theodore and Francesco Ricci

Table 3.1 Common Systems of Units

System	Length	Time	Mass	Force	Energy	Temperature
SI	meter	second	kilogram	Newton	Joule	Kelvin, degree Celsius
egs	centimeter	second	gram	dyne	erg, Joule, or calorie	Kelvin, degree Celsius
fps	foot	second	pound	poundal	foot poundal	degree Rankine, degree Fahrenheit
American Engineering	foot	second	pound	pound (force)	British thermal unit, horsepower · hour	degree Rankine, degree Fahrenheit
British Engineering	foot	second	slug	pound (force)	British thermal unit, foot pound (force)	degree Rankine, degree Fahrenheit

Table 3.2 English Engineering Units

Physical quantity	Name of unit	Symbol for unit
Length	foot	ft
Time	second, minute, hour	s, min, h
Mass	pound (mass)	lb
Temperature	degree Rankine	°R
Temperature (alternative)	degree Fahrenheit	°F
Moles	pound mole	lbmol
Energy	British thermal unit	Btu
Energy (alternative)	horsepower · hour	hp · h
Force	pound (force)	lb_f
Acceleration	foot per second square	ft/s^2
Velocity	foot per second	ft/s
Volume	cubic foot	ft^3
Area	square foot	ft^2
Frequency	cycles per second, Hertz	cycles/s, Hz
Power	horsepower, Btu per second	hp, Btu/s
Heat capacity	British thermal unit per (pound mass · degree Rankine)	Btu/lb · °R
Density	pound (mass) per cubic foot	lb/ft^3
Pressure	pound (force) per square inch	psi
	pound (force) per square foot	psf
	atmospheres	atm
	bar	bar

Table 3.3 SI Units

Physical unit	Name of unit	Symbol for unit
Length	meter	m
Mass	kilogram, gram	kg, g
Time	second	s
Temperature	Kelvin	K
Temperature (alternative)	degree Celsius	°C
Moles	gram mole	gmol
Energy	Joule	J, $kg \cdot m^2/s^2$
Force	Newton	N, $kg \cdot m/s^2$, J/m
Acceleration	meters per second squared	m/s^2
Pressure	Pascal, Newton per square meter	Pa, N/m^2
Pressure (alternative)	bar	bar
Velocity	meters per second	m/s
Volume	cubic meters, liters	m^3, L
Area	square meters	m^2
Frequency	Hertz	Hz, cycles/s
Power	Watt	W, $kg \cdot m^2 \cdot s^3$, J/s
Heat capacity	Joule per kilogram · Kelvin	J/kg · K
Density	kilogram per cubic meter	kg/m^3
Angular velocity	radians per second	rad/s

Table 3.4 Prefixes for SI Units

Multiplication factors	Prefix	Symbol
$1{,}000{,}000{,}000{,}000{,}000{,}000 = 10^{18}$	exa	E
$1{,}000{,}000{,}000{,}000{,}000 = 10^{15}$	peta	P
$1{,}000{,}000{,}000{,}000 = 10^{12}$	tera	T
$1{,}000{,}000{,}000 = 10^{9}$	giga	G
$1{,}000{,}000 = 10^{6}$	mega	M
$1{,}000 = 10^{3}$	kilo	k
$100 = 10^{2}$	hecto	h
$10 = 10^{1}$	deka	da
$0.1 = 10^{-1}$	deci	d
$0.01 = 10^{-2}$	centi	c
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001 = 10^{-9}$	nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	p
$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$	femto	f
$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$	atto	a

Table 3.5 Decimal Equivalents

Inch in fractions	Decimal equivalent	Millimeter equivalent
A. 4ths and 8ths		
1/8	0.125	3.175
1/4	0.250	6.350
3/8	0.375	9.525
1/2	0.500	12.700
5/8	0.625	15.875
3/4	0.750	19.050
7/8	0.875	22.225
B. 16ths		
1/16	0.0625	1.588
3/16	0.1875	4.763
5/16	0.3125	7.938
7/16	0.4375	11.113
9/16	0.5625	14.288
11/16	0.6875	17.463
13/16	0.8125	20.638
15/16	0.9375	23.813
C. 32nds		
1/32	0.03125	0.794
3/32	0.09375	2.381

(*Continued*)

TABLE 3.5 *Continued*

Inch in fractions	Decimal equivalent	Millimeter equivalent
5/32	0.15625	3.969
7/32	0.21875	5.556
9/32	0.28125	7.144
11/32	0.34375	8.731
13/32	0.40625	10.319
15/32	0.46875	11.906
17/32	0.53125	13.494
19/32	0.59375	15.081
21/32	0.65625	16.669
23/32	0.71875	18.256
25/32	0.78125	19.844
27/32	0.84375	21.431
29/32	0.90625	23.019
31/32	0.96875	24.606

Two units that appear in dated literature are the *poundal* and *slug*. By definition, one poundal force will give a 1 pound mass an acceleration of 1 ft/s^2. Alternatively, 1 slug is defined as the mass that will accelerate 1 ft/s^2 when acted upon by a 1 pound force; thus, a slug is equal to 32.2 pounds mass.

CONVERSION OF UNITS

Converting a measurement from one unit to another can conveniently be accomplished by using *unit conversion factors*; these factors are obtained from a simple equation that relates the two units numerically. For example, from

$$12 \text{ inches (in)} = 1 \text{ foot (ft)} \tag{3.1}$$

the following conversion factor can be obtained:

$$12 \text{ in}/1 \text{ ft} = 1 \tag{3.2}$$

Since this factor is equal to unity, multiplying some quantity (e.g., 18 ft) by this factor cannot alter its value. Hence

$$18 \text{ ft } (12 \text{ in}/1 \text{ ft}) = 216 \text{ in} \tag{3.3}$$

Note that in Equation (3.3), the old units of *feet* on the left-hand side cancel out leaving only the desired units of *inches*.

Physical equations must be dimensionally consistent. For the equality to hold, each additive term in the equation must have the same dimensions. This condition can be and should be checked when solving engineering problems. Throughout the text, great care is exercised in maintaining the dimensional formulas of all terms and the dimensional homogeneity of each equation. Equations will generally be developed in terms of specific units rather than general dimensions, e.g., feet, rather than length. This approach should help the reader to more easily attach physical significance to the equations presented in these chapters.

Consider now the example of calculating the perimeter, P, of a rectangle with length, L, and height, H. Mathematically, this may be expressed as $P = 2L + 2H$. This is about as simple a mathematical equation that one can find. However, it only applies when P, L, and H are expressed in the same units.

Terms in equations must be consistent from a "magnitude" viewpoint.[3] Differential terms cannot be equated with finite or integral terms. Care should also be exercised in solving differential equations. In order to solve differential equations to obtain a description of the pressure, temperature, composition, etc., of a system, it is necessary to specify boundary and/or initial conditions (B a/o IC) for the system. This information arises from a description of the problem or the physical situation. The number of boundary conditions (BC) that must be specified is the sum of the highest order derivative for each independent differential equation. A value of the solution on the boundary of the system is one type of boundary condition. The number of initial conditions (IC) that must be specified is the highest order time derivative appearing in the differential equation. The value for the solution at time equal to zero constitutes an initial condition. For example, the equation

$$\frac{d^2C_A}{dz^2} = 0; \quad C_A = \text{concentration} \tag{3.4}$$

requires 2 BCs (in terms of the position variable z). The equation

$$\frac{dC_A}{dt} = 0; \quad t = \text{time} \tag{3.5}$$

requires 1 IC. And finally, the equation

$$\frac{\partial C_A}{\partial t} = D\frac{\partial^2 C_A}{\partial y^2} \tag{3.6}$$

requires 1 IC and 2 BCs (in terms of the position variable y).

ILLUSTRATIVE EXAMPLE 3.1

Convert units of acceleration in cm/s^2 to miles/yr^2.

***SOLUTION*:** The procedure outlined on the previous page is applied to the units of cm/s^2.

$$\left(\frac{1\ \text{cm}}{\text{s}^2}\right)\left(\frac{3600^2\ \text{s}^2}{1\ \text{h}^2}\right)\left(\frac{24^2\ \text{h}^2}{1\ \text{day}^2}\right)\left(\frac{365^2\ \text{day}^2}{1\ \text{yr}^2}\right)\left(\frac{1\ \text{in}}{2.54\ \text{cm}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)\left(\frac{1\ \text{mile}}{5280\ \text{ft}}\right)$$
$$= 6.18 \times 10^9\ \text{miles/yr}^2$$

Thus, 1.0 cm/s^2 is equal to 6.18×10^9 miles/yr^2. ■

THE GRAVITATIONAL CONSTANT g_c

The momentum of a system is defined as the product of the mass and velocity of the system:

$$\text{Momentum} = (\text{mass})(\text{velocity}) \tag{3.7}$$

A commonly employed set of units for momentum are therefore $\text{lb} \cdot \text{ft/s}$. The units of the time rate of change of momentum (hereafter referred to as rate of momentum) are simply the units of momentum divided by time, i.e.,

$$\text{Rate of momentum} \equiv \frac{\text{lb} \cdot \text{ft}}{\text{s}^2} \tag{3.8}$$

The above units can be converted to units of pound force (lb_f) if multiplied by an appropriate constant. As noted earlier, a conversion constant is a term that is used to obtain units in a more convenient form; all conversion constants have magnitude and units in the term, but can also be shown to be equal to 1.0 (unity) with *no* units (i.e., dimensionless).

A defining equation is

$$1\,\text{lb}_\text{f} = 32.2\,\frac{\text{lb} \cdot \text{ft}}{\text{s}^2} \tag{3.9}$$

If this equation is divided by lb_f, one obtains

$$1.0 = 32.2\,\frac{\text{lb} \cdot \text{ft}}{\text{lb}_\text{f} \cdot \text{s}^2} \tag{3.10}$$

This serves to define the conversion constant g_c. If the rate of momentum is divided by g_c as $32.2\,\text{lb} \cdot \text{ft/lb}_\text{f} \cdot \text{s}^2$—this operation being equivalent to dividing by 1.0—the following units result:

$$\begin{aligned} \text{Rate of momentum} &\equiv \left(\frac{\text{lb} \cdot \text{ft}}{\text{s}^2}\right)\left(\frac{\text{lb}_\text{f} \cdot \text{s}^2}{\text{lb} \cdot \text{ft}}\right) \\ &\equiv \text{lb}_\text{f} \end{aligned} \tag{3.11}$$

One can conclude from the above dimensional analysis that a force is equivalent to a rate of momentum.

SIGNIFICANT FIGURES AND SCIENTIFIC NOTATION[3]

Significant figures provide an indication of the precision with which a quantity is measured or known. The last digit represents, in a qualitative sense, some degree of doubt. For example, a measurement of 8.32 inches implies that the actual quantity is somewhere between 8.315 and 8.325 inches. This applies to calculated and measured quantities; quantities that are known exactly (e.g., pure integers) have an infinite number of significant figures.

The significant digits of a number are the digits from the first nonzero digit on the left to either (a) the last digit (whether it is nonzero or zero) on the right if there is a decimal point, or (b) the last nonzero digit of the number if there is no decimal point. For example:

370	has 2 significant figures
370.	has 3 significant figures
370.0	has 4 significant figures
28,070	has 4 significant figures
0.037	has 2 significant figures
0.0370	has 3 significant figures
0.02807	has 4 significant figures

Whenever quantities are combined by multiplication and/or division, the number of significant figures in the result should equal the lowest number of significant figures of any of the quantities. In long calculations, the final result should be rounded off to the correct number of significant figures. When quantities are combined by addition and/or subtraction, the final result cannot be more precise than any of the quantities added or subtracted. Therefore, the position (relative to the decimal point) of the last significant digit in the number that has the lowest degree of precision is the position of the last permissible significant digit in the result. For example, the sum of 3702., 370, 0.037, 4, and 37. should be reported as 4110 (without a decimal). The least precise of the five numbers is 370, which has its last significant digit in the *tens* position. The answer should also have its last significant digit in the *tens* position.

Unfortunately, engineers and scientists rarely concern themselves with significant figures in their calculations. However, it is recommended—at least for this chapter—that the reader attempt to follow the calculational procedure set forth in this section.

In the process of performing engineering calculations, very large and very small numbers are often encountered. A convenient way to represent these numbers is to use *scientific notation*. Generally, a number represented in scientific notation is the product of a number (<10 but $>$ or $=1$) and 10 raised to an integer power. For example,

$$28{,}070{,}000{,}000 = 2.807 \times 10^{10}$$

$$0.000\,002\,807 = 2.807 \times 10^{-6}$$

A positive feature of using scientific notation is that only the significant figures need appear in the number.

REFERENCES

1. D. Green and R. Perry (eds), "*Perry's Chemical Engineers' Handbook*," 8th edition, McGraw-Hill, New York City, NY, 2008.
2. J. Reynolds, J. Jeris, and L. Theodore, "*Handbook of Chemical and Environmental Engineering Calculations*," John Wiley & Sons, Hoboken, NJ, 2004.
3. J. Santoleri, J. Reynolds, and L. Theodore, "*Introduction to Hazardous Waste Incineration*," 2nd edition, John Wiley & Sons, Hoboken, NJ, 2000.

NOTE: Additional problems are available for all readers at www.wiley.com. Follow links for this title. These problems may be used for additional review, homework, and/or exam purposes.