كل ما يحتاجه الطالب في جميع الصفوف من أوراق عمل واختبارات ومذكرات، يجده هنا في الروابط التالية لأفضل مواقع تعليمي إماراتي 100 %

تطبيق المناهج الإماراتية	الاجتماعيات	الرياضيات
الصفحة الرسمية على التلغرام	الاسلامية	العلوم
الصفحة الرسمية على الفيسبوك	الانجليزية	
التربية الاخلاقية لجميع الصفوف	اللغة العربية	
التربية الرياضية		

مجموعات التلغرام.	مجموعات الفيسبوك	قنوات تلغرام	قنوات الفيسبوك
الصف الأول	الصف الأول	الصف الأول	الصف الأول
الصف الثاني	الصف الثاني	الصف الثاني	الصف الثاني
الصف الثالث	الصف الثالث	الصف الثالث	الصف الثالث
الصف الرابع	الصف الرابع	الصف الرابع	الصف الرابع
الصف الخامس	الصف الخامس	الصف الخامس	الصف الخامس
الصف السادس	الصف السادس	الصف السادس	الصف السادس
الصف السابع	الصف السابع	الصف السابع	الصف السابع
الصف الثامن	الصف الثامن	الصف الثامن	الصف الثامن
الصف التاسع عام	الصف التاسع عام	الصف التاسع عام	الصف التاسع عام
الصف التاسع متقدم	الصف التاسع متقدم	الصف التاسع متقدم	تاسع متقدم
الصف العاشر عام	الصف العاشر عام	الصف العاشر عام	عاشر عام
الصف العاشر متقدم	الصف العاشر متقدم	الصف العاشر متقدم	عاشر متقدم
الحادي عشر عام	الحادي عشر عام	الحادي عشر عام	حادي عشر عام
الحادي عشر متقدم	الحادي عشر متقدم	الحادي عشر متقدم	حادي عشر متقدم
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ثاني عشر متقدم	الثاني عشر متقدم	ثاني عشر متقدم	ثاني عشر متقدم

UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF TOLERANCE

TEACHER EDITION 2018 - 2019

McGraw-Hill Education

Mathematics

8

General Stream

United Arab Emirates Edition

Interactive Student Guide

مجموعات فخر الوطن وعام زايد

McGraw Hill Education

Answer Key

McGraw-Hill Education

Mathematics

General Stream

United Arab Emirates Edition

Interactive Student Guide

مجموعات فخر الوطن وعام زايد

2018 - 2019

8

McGraw Hill Education

Inquiry Lab 1: Scatter Plots

Vocabulary Support: Sentence Frames

As students work through the Hands-On and Investigate activities, display sentence frames to help them communicate information and answers with their partners:

x is ____ . y is ____ . The trend is [positive/negative]. The arm span is about ____ centimeters.

The [diameter/circumference] is ____ . The coordinates are ____ . The trend is ____. The circumference would be about ____ .

When ____ increases, ____ increases. The trend ____ because ____ .

Student page

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Inquiry Lab 1 Guided Writing

Scatter Plots

HOW can I use a graph to investigate the relationship or trends between two sets of data?

Use the exercises below to help answer the Inquiry Question. Write the correct word or phrase on the lines provided. **Sample answers are given.**

1. Rewrite the question in your own words.
   See students' work.

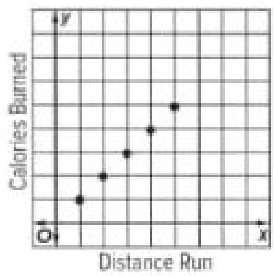
2. What key words do you see in the question?
   graph, relationship, trends, data

3. A pair of numbers used to locate a point in the coordinate plane is called an **ordered pair**.

4. Write a synonym for the word *trend*. **pattern**

5. Write the following data in the table as ordered pairs. Then graph the ordered pairs on the coordinate plane.

Distance Run (km)	Calories Burned	Ordered Pair
0.5	49	**(0.5, 49)**
1	98	**(1, 98)**
1.5	147	**(1.5, 147)**
2	196	**(2, 196)**
2.5	245	**(2.5, 245)**



6. Does the graph show a trend in the data? **yes**
   If yes, describe the trend. **The points form a line.**

HOW can I use a graph to investigate the relationship or trends between two sets of data?
Write the data as ordered pairs. Graph the data on a coordinate plane to see if there is a trend in the data.

NAME ______________________ DATE ____________ PERIOD ________

Lesson 1 Vocabulary

Scatter Plots

Use the word cards to define each vocabulary word or phrase and give an example. Sample answers are given.

Word Cards

bivariate data

Definition

data with two variables, or pairs of numerical observations

Example Sentence

The data for the number of students in school for each day of a week is bivariate data.

Word Cards

scatter plot

Definition

a graph that shows the relationship between a data set with two variables graphed on a coordinate plane

Example Sentence

I can graph the points (number of students in school, day of the week) as a scatter plot.

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Inquiry Lab 2 Guided Writing

Lines of Best Fit

HOW can I use a data model to predict an outcome?

Use the exercises below to help answer the Inquiry Question. Write the correct word or phrase on the lines provided. Sample answers are given.

1. Rewrite the question in your own words.
   See students' work.

2. What key words do you see in the question?
   data model, predict, outcome

3. **Predict** means to tell what you think will happen.

4. Write a synonym for the word *outcome*. **result**

5. Complete the steps on how to use a data model to predict an outcome.

   a. Conduct research to collect a set of **data**.

   b. Write the data in the form of **ordered** pairs.

   c. Create a **graph** by marking the points in a coordinate plane.

   d. Draw a **line** that goes through most of the data points.

   e. Make a **prediction** based on the line you drew.

HOW can I use a data model to predict an outcome?

Construct a scatter plot of the data. If the scatter plot suggests a positive or negative association, draw a line that goes through most of the data points. Use this line to make the prediction.

Student page

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Lesson 2 Vocabulary

Lines of Best Fit

Use the definition map to list qualities about the vocabulary word or phrase. Sample answers are given.

Vocabulary

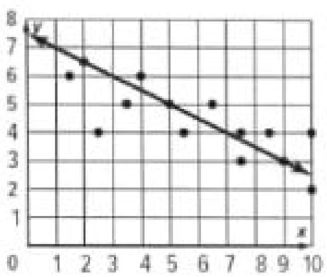
line of best fit

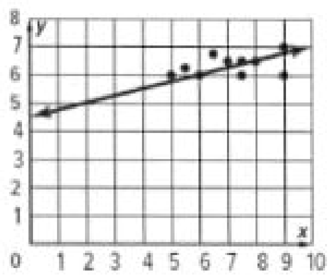
Description

a line that is very close to most of the data points in a scatter plot

Characteristics

- **a line that best represents the data**
- **using line of best fit makes it easier to make a conjecture**
- **approximates a linear relationship**





Draw the lines of best fit.

Chapter 9 Scatter Plots and Data Analysis **95**

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Inquiry Lab 3 Guided Writing

Graphing Technology: Linear and Nonlinear Association

HOW can you use technology to describe associations in scatter plots?

Use the exercises below to help answer the Inquiry Question. Write the correct word or phrase on the lines provided. Sample answers are given.

1. Rewrite the question in your own words.
   See students' work.

2. What key words do you see in the question?
   technology, associations, scatter plots

3. A **scatter plot** shows two sets of related data as ordered pairs on the same graph.

4. A **graphing calculator** is an electronic tool you can use to create a scatter plot of data.

5. A line that is very close to most of the data points is called **line of best fit**.

6. Write a synonym for the word *associations*. **relationships**.

7. The graph of a linear association is a **straight** line.

8. The correlation coefficient tells the **strength** of the association between two sets of data.

9. If data is clustered closely around the line of best fit, the strength of the association is **strong**.

10. If the data is not clustered closely around the line of best fit, the association is **weak**.

HOW can you use technology to describe associations in scatter plots?
You can use a graphing calculator to create the scatter plot. If the association is linear, you can find the equation for the line of best fit, and describe the strength of the association between the two sets of data.

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Lesson 3 Vocabulary

Two-Way Tables

Use the word cards to define each vocabulary word or phrase and give an example. Sample answers are given.

Word Cards

relative frequency

Definition

the ratio of the number of successes to the total number of attempts in an experiment

Example Sentence

The relative frequency of the number of students in the eighth grade that play an instrument to all of the students in the school is $\frac{67}{158}$

Word Cards

two-way table

Definition

a table that shows data that pertain to two different categories

Example Sentence

The two-way table shows that students that play an instrument usually take art classes.

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Lesson 4 Vocabulary

Descriptive Statistics

Use the two column chart to organize the vocabulary in this lesson. Then write the definition of each wo Sample answers are given.

Term	Definition
univariate data	data with one variable
quantitative data	data that cannot be given a numerical value
five-number summary	a way of characterizing a set of data that includes the minimum, first quartile, median, third quartile, and the maximum
measures of center	Numbers that are used to describe the center of a set of data; these measures include the mean, median, and mode.
quartiles	values that divide a set of data into four equal parts

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Lesson 5 Vocabulary
Measures of Variation

Use the word cards to define each vocabulary word or phrase and give an example. **Sample answers are given.**

Word Cards

mean absolute deviation

Definition
the average of the absolute values of differences between the mean and each value in a data set

Example Sentence
The mean absolute deviation can tell me how spread out the data is.

Word Cards

standard deviation

Definition
a measure of variation that describes how the data deviates from the mean of the data

Example Sentence
The standard deviation can tell me how spread out the data is numerically.

Student page

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Lesson 6 Vocabulary

Analyze Data Distributions

Use the definition map to list qualities about the vocabulary word or phrase.
Sample answers are given.

Vocabulary

distribution

Characteristics of Distributions

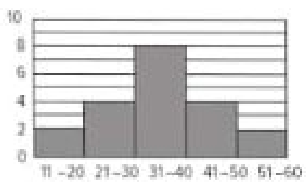
Distributions can be described by their centers, spreads, and overall shapes.

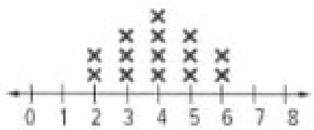
Description

shows the arrangement of data values

A distribution is symmetric if the left side of the distribution looks like the right side.

A distribution is non-symmetric if the left side of the distribution does not look like the right side.





Draw Examples of Symmetric Distributions

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Lesson 1 Multi-Step Problem Solving

Multi-Step Example

The table shows the 40 m dash times in seconds for athletes at varying weights in kilograms. Which describes the association between speed and weight as shown by a scatter plot of the data? MP 7

Speed (s)	Weight (kg)
4.24	89
4.28	88
4.29	78
4.29	93
4.24	99
4.29	94
4.29	97
4.28	91
4.29	92
4.29	100

Ⓐ negative linear association

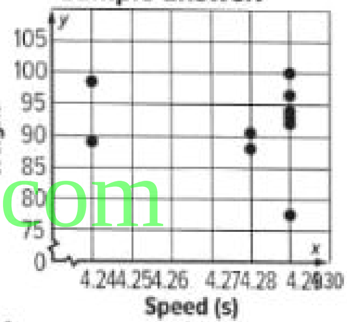
Ⓑ positive linear association

Ⓒ non-linear association

Ⓓ no association

Use a problem-solving model to solve this problem.

Sample answer:



1 Analyze

Read the problem. Circle the information you know.
Underline what the problem is asking you to find.

2 Plan

What will you need to do to solve the problem? Write your plan.

Step 1 Construct a **scatterplot** of the data on a separate sheet of grid paper.

Step 2 Determine the **association**, if any, among the observed data.

Read to Succeed!
A graph's scales can change its appearance. Choose scales for the x- and y-axes that will accurately show relationships among sets of data.

3 Solve

Use your plan to solve the problem. Show your steps.

The graph shows that weights for specific speeds vary greatly. For example, the weights for a speed of 4.29 seconds range from **78** to **100**. Since there is no obvious **pattern** the correct answer is **D**.

4 Justify and Evaluate

How do you know your solution is accurate?

Sample answer: I also checked the weights for a speed of 4.24 seconds, 89 and 99 kg. There does not appear to be an association among these data either.

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Lesson 1 *(continued)*

Use a problem-solving model to solve each problem.

1 The tables below show the average monthly temperatures in degrees Fahrenheit for a certain city for one year, with January representing month 1 and December representing month 12. Which describes the association among the data? MP 7

Month	1	2	3	4	5	6
°C	31	37	39	49	60	74

Month	7	8	9	10	11	12
°C	78	80	73	58	50	35

Ⓐ negative linear association
Ⓑ positive linear association
Ⓒ non-linear association
Ⓓ no association

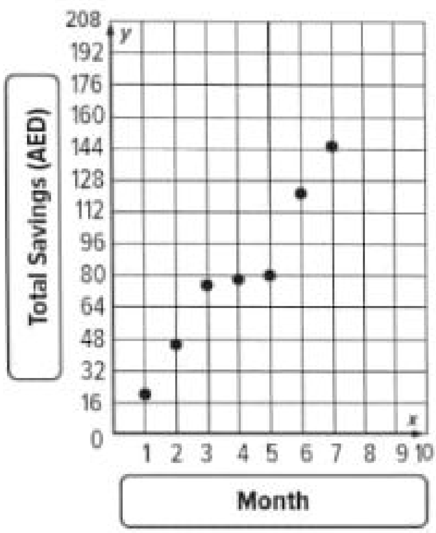
2 The table shows the number of liters in thousands of water in a swimming pool after each hour. What conjecture can be made from the data about the number of liters of water in the pool after 9 hours? MP 2

Time (h)	Water (1,000 L)
1	27
2	24
3	22
4	18
5	15
6	13

Sample answer: between 2,000 and 4,000 liters

3 **H.O.T. Problem** The table shows Osama's savings for seven months. Construct a scatterplot of the data. Analyze the scatterplot for patterns of association, outliers, and clusters. If a relationship exists, make a conjecture about how much money Osama will have saved after 10 months. MP 7

Osama's Savings							
Month	1	2	3	4	5	6	7
Total Savings (dirhams)	20	45	75	78	80	121	145



See students' work for graphs. Sample answer: The data have a positive association and a cluster between Weeks 3–5 at around 80. There are no outliers. Osama will have saved AED 195 after 10 months.

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Lesson 2 **Multi-Step** Problem Solving

Multi-Step Example

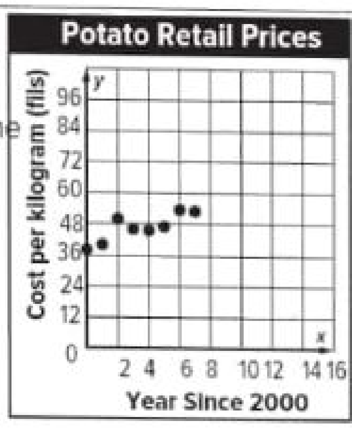
The scatterplot at the right shows the cost per kilogram of potatoes from 2000 to 2007. Use a trend line to determine the best estimate for the cost of a kilogram of potatoes in 2016. MP

Ⓐ 62 fils

Ⓑ 72 fils

Ⓒ 82 fils

Ⓓ 92 fils



Use a problem-solving model to solve this problem.

1 Analyze

Read the problem. Circle the information you know.
Underline what the problem is asking you to find.

2 Plan

What will you need to do to solve the problem? Write your plan in steps.

Step 1 Draw a **trend line** that represents the data.

Step 2 Write an equation for the **trend line**. Then use the equation to make a prediction of the **cost of potatoes in 2016**.

Read to Succeed!
Pay close attention to the scale when determining the slope and y-intercept.

3 Solve

Use your plan to solve the problem. Show your steps.

From my trend line, I found potatoes cost about **40** fils per kilogram in 2000 and increased about **2** fils per kilogram in following years. I then replaced x with **16** in the equation y = **2**x + **40**. Potatoes will cost about **72** fils per kilogram in 2016. So, **B** is the correct answer.

4 Justify and Evaluate

How do you know your solution is accurate?

Sample answer: I graphed the point (16, 72) on the scatterplot. My trend line passes through the point. So, I know my solution is accurate.

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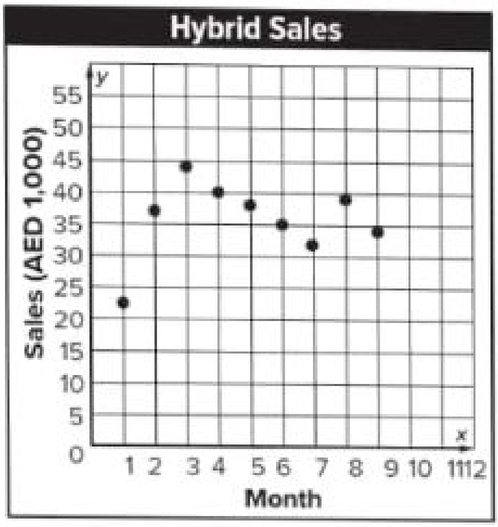
Lesson 2 *(continued)*

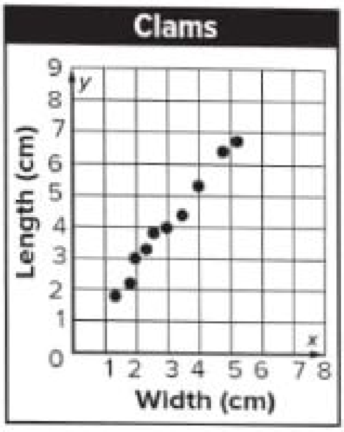
Use a problem-solving model to solve each problem.

1 The scatterplot below shows hybrid car sales, in thousands of dirhams, for the first 9 months of a certain year. What is the best estimate of hybrid sales in Month 11? MP 4



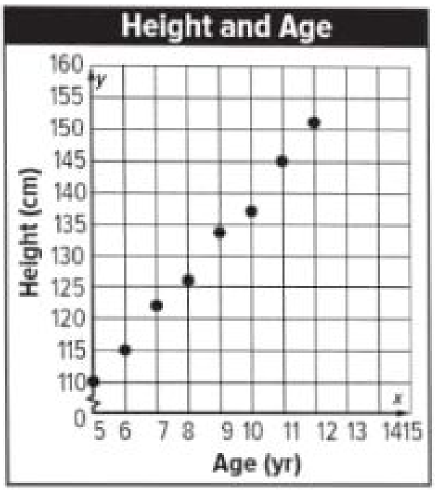
Ⓐ AED 32,000
Ⓑ AED 38,000
Ⓒ AED 44,000
Ⓓ AED 50,000

2 The scatterplot below shows the length and width of clams obtained in a sample from a certain body of water. Write an equation of a trend line that represents the data. MP 2



Sample answer: $y = x + 1$

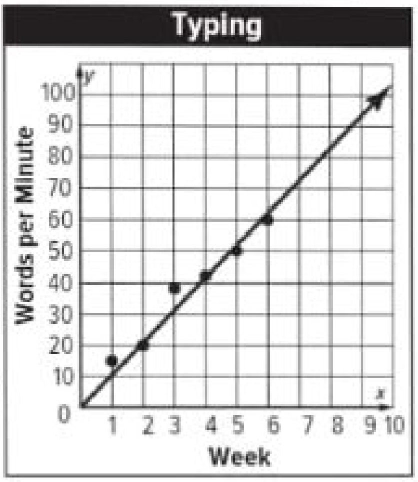
3 The scatterplot below shows the height of a young lady at various ages. Write an equation of a trend line that represents the data. MP 2



Sample answer: $y = 6x + 108$

4 **H.O.T. Problem** The table below shows the progress of a typing student. Construct a scatterplot and draw a trend line. Predict the number of words per minute typed after the 9th week. MP 7

Week	1	2	3	4	5	6
Word Per Minute	15	20	38	42	50	60



92 words

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Lesson 3 Multi-Step Problem Solving

Multi-Step Example

A group of males and females were surveyed about what color of car they owned. The data are shown in the two-way table. Which statement is true about males and females who own a black car? MP 7

Car Color	Males	Females
Red	14	15
Black	12	12
White	15	12

Ⓐ The same percentage of males and females own black cars.

Ⓑ A larger percentage of males than females own black cars.

Ⓒ A larger percentage of females than males own black cars.

Ⓓ There is not enough information in this table to make a comparison.

Use a problem-solving model to solve this problem.

Read to Succeed!
A two-way table shows data of one sample group as it relates to two different categories.

1 Analyze

Read the problem. Circle the information you know.
Underline what the problem is asking you to find.

2 Plan

What will you need to do to solve the problem? Write your steps.

Step 1 Find the total number of **males** and the total number of **females**.

Step 2 Use the totals to find the relative frequencies of **males and females** who own black cars.

Step 3 Compare the percentages and choose the correct statement.

3 Solve

Use your plan to solve the problem. Show your steps.

Total males: **41** Total females: **39**

The **relative frequency** of a male who owns a black car is **0.29**, and the **relative frequency** of a female who owns a black car is **0.31**.

The percentage of females who own black cars is **greater** than the percentage of males who own black cars.

The correct answer is **C**. Fill in that answer choice.

4 Justify and Evaluate

How do you know your solution is accurate?

Sample answer: Since there are the same number of males and females who own a black car and there are more total males, I know that the percentage of females should be greater.

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Lesson 3 *(continued)*

Use a problem-solving model to solve each problem.

1 A group of 21-year-olds were surveyed about whether they live with their parents and if they are in college. The results are shown in the two-way table. Which statement is true about the 21-year-olds? MP 7

	Attends College	Does Not Attend College
Lives with Parents	30	30
Does Not Live with Parents	55	60

Ⓐ The percentage of students who attend college is the same for those who do and do not live at home.

Ⓑ A larger percentage of those who attend college live with their parents than those who do not.

Ⓒ A larger percentage of those not in college live with their parents than those who do not.

Ⓓ There is not enough information in this table to make a comparison.

2 There are 203 male and 175 female students at Rashid Middle School. A survey showed that 117 males and 97 females ride the bus. What is the difference between the relative frequency of males who ride the bus and the relative frequency of females who do not ride the bus, rounded to the nearest hundredth? MP 1

	Rides Bus	Does Not Ride Bus	Total
Males	117	86	203
Females	97	78	175
Total	214	164	378

0.13

3 Amer surveyed 150 tenth-grade students to find out if they have a part-time job. There are 94 students who have a part-time job, including 57 honor roll students. Half of the students who do not have a job are on the honor roll. Complete the two-way table. What is the relative frequency of an honor roll student with no job rounded to the nearest hundredth? MP 2

	Honor Roll	No Honor Roll	Total
Job	57	**37**	94
No Job	**28**	**28**	**56**
Total	**85**	**65**	150

0.33

4 **H.O.T. Problem** Lamees is interpreting survey data about people who own a truck. Out of 100 males surveyed, 37 own a truck. Lamees makes the statement that of the people who own a truck, 37% are male. Is her statement accurate? Why or why not? MP 3

Sample answer: No; Lamees should have said 37% of the males surveyed own a truck.

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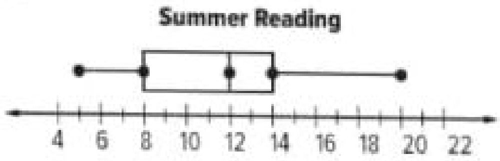
Lesson 4 **Multi-Step** Problem Solving

Multi-Step Example

The box plot shows the number of books read by students during the summer. How much greater is the range than the interquartile range?
Preparation for MP 4



Use a problem-solving model to solve this problem.

1 Analyze

Read the problem. Circle the information you know.
Underline what the problem is asking you to find.

2 Plan

What will you need to do to solve the problem? Write your plan in steps.

Step 1 Use the box plot to determine the **difference** between the range and interquartile range.

Step 2 **Subtract** the lesser value from the greater value.

3 Solve

Use your plan to solve the problem. Show your steps.

The range is **20** – **5**, or **15**. The interquartile range is **14** – **8**, or **6**. The range is **15** – **6**, or **9** units greater.

The answer is **9**.

Read to Succeed!
Remember the range is the difference between the maximum and minimum values and the interquartile range is the difference between the third and first quartiles.

4 Justify and Evaluate

How do you know your solution is accurate?

Sample answer: I used the box plot to count units to verify my range and interquartile range values. Then I added 6 and 9 to check my subtraction.

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Lesson 4 *(continued)*

Use a problem-solving model to solve each problem.

1 The heights of the girls on a basketball team are shown in the table below. How many centimeters greater is the range than the interquartile range? *Preparation for* MP 4

Heights (cm)				
162.5	175	165	182.5	167.5
177.5	162.5	170	175	172.5

10

2 The table below shows the amount of time that an eighth grader spent exercising. Which is greater: the mean or median? How much greater? *Preparation for* MP 4

Exercise Times (min)			
63	58	55	67
75	70	60	60

mean; 2 min

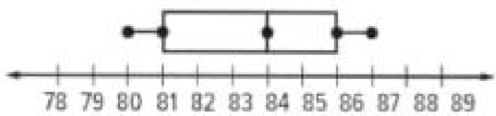
3 A player's score in a golf tournament is determined by the number of total strokes needed to play a golf course over four days. The table below shows six players' scores at a recent tournament. How much closer is the mode to the median than to the mean? *Preparation for* MP 4

Golf Scores		
267	270	265
273	275	267

1 stroke

4 **H.O.T. Problem** The table below shows the scores of a student's recent science tests. Construct a box plot of the data. What percent of the data is between 81 and 86? Explain. *Preparation for* MP 3

Science Test Scores						
80	81	84	84	87	86	86



50%; Sample answer: The interquartile range represents 50% of the data. Since 81 is the lower quartile and 86 is the upper quartile, the data between 81 and 86 is 50% of the data.

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Lesson 5 Multi-Step Problem Solving

Chapter 9

Multi-Step Example

The table shows the total points scored in men's and women's basketball games. The men's scores have a standard deviation of 15.1, and the women's scores have a standard deviation of 6.9. Make a comparison of the variation between the data sets, and use the standard deviations to support your answer. *Preparation for* MP 3

Women	57	69	73	79	62	65	59	54
Men	76	62	103	85	75	97	110	80

Use a problem-solving model to solve this problem.

Read to Succeed!
The mean absolute deviation is the average distance of each value from the mean.

1 Analyze

Read the problem. Circle the information you know. Underline what the problem is asking you to find.

2 Plan

What will you need to do to solve the problem? Write your steps.

Step 1 Find the mean absolute deviation **men's** scores and mean absolute deviation of the **women's** scores.

Step 2 Compare the **variations** of the scores and use the **standard deviations** to support your comparison.

3 Solve

Use your plan to solve the problem. Show your steps.

The mean absolute deviation of the **men's** scores **13** and of the **women's** scores is **6.75**. The **men's** scores have a greater variation than the **women's** scores.

The **standard deviations** support this because the majority of the scores for the men's team are between **70.9 and 101.1** and the majority of the scores of the women's team are between **57.85 and 71.65**.

4 Justify and Evaluate

How do you know your solution is accurate?

Sample answer: The men's mean absolute deviation is greater, so their scores have a greater variation. After applying the standard deviation, I know that the men's score have a greater range of variability and my answer is supported.

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Lesson 5 *(continued)*

Use a problem-solving model to solve each problem.

1 The table shows the lengths of ribbons used in different craft projects. The standard deviation of the lengths is 2.5 cm. If the mean of the data is round to the nearest tenth, which statement describes the data values that are within one standard deviation of the mean? *Preparation for* MP 3

Length of Ribbons (cm)			
7	5	6	10
10	7	9	3
9	11	12	7

Ⓐ The mean absolute variation is greater than the standard deviation.

Ⓑ The majority of the lengths will be shorter than 10.5 cm.

Ⓒ The majority of the lengths will be longer than 5.5 cm.

Ⓓ The majority of the lengths will be between 5.5 cm and 10.5 cm.

2 The standard deviation of test scores is 13.5. What are the test scores within two standard deviations of the mean? *Preparation for* MP 2

Test Scores			
79	63	59	86
88	92	100	53
72	76	70	69

Sample answer: Test scores within two standard deviations are between 48.6 and 102.6.

3 The speeds of cars ticketed in a school zone are listed in the table. What is the difference between the standard deviation of 4.85 and the mean absolute deviation of the data? *Preparation for* MP 2

Speeds of Cars (km/h)			
38	42	39	45
30	37	43	46

0.85

4 **H.O.T. Problem** Create a data set of 5 numbers with a range of 50. What is the mean absolute deviation? Will every data set with a range of 50 have the same mean absolute deviation? Why or why not? *Preparation for* MP 3

Sample answer: 10, 25, 40, 55, 60; 15.6; No, because there are many different numbers with a range of 50.

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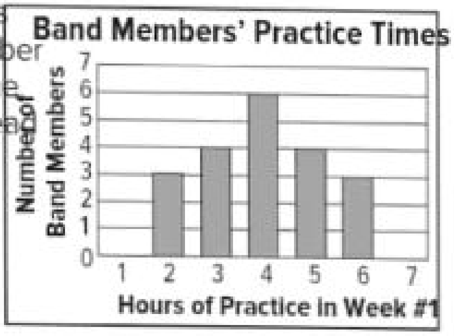
Lesson 6 Multi-Step Problem Solving

Multi-Step Example

From Week 1 to Week 2, the number of band members who practiced 3 hours increased by 75% and the number who practiced 4 hours decreased by 50%. Which of the following shows the best measures of center and spread for Week 2 data? *Preparation for* MP 1



Ⓐ median = 3.5, interquartile range = 2

Ⓑ median = 4, interquartile range = 2

Ⓒ mean = 3.85, mean average deviation = 1

Ⓓ mean = 4, mean average deviation = 1

Use a problem-solving model to solve this problem.

1 Analyze

Read the problem. Circle the information you know.
Underline what the problem is asking you to find.

2 Plan

What will you need to do to solve the problem? Write your plan in steps.

Step 1 Use the given **percentages** to construct the **Week 2** graph.

Step 2 Determine which measure of center and spread to use based on the **shape** of the **Week 2** graph.

3 Solve

Use your plan to solve the problem. Show your steps.

Construct the **Week 2** graph. Since the graph is not symmetric, the **median** will describe the center and **interquartile range** will describe the spread.

Since the **median** is **3.5** and the **interquartile range** is **2**, the correct answer is **A**.

Read to Succeed!

If the data distribution is symmetric, use the mean to describe the center and the mean absolute deviation to describe the spread.

If the data distribution is not symmetric, use the median to describe the center and the interquartile range to describe the spread.

4 Justify and Evaluate

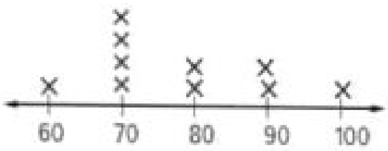
How do you know your solution is accurate?

Sample answer: I confirmed which measure of center and spread to use. Then I checked my median and interquartile range values.

Lesson 6 *(continued)*

Use a problem-solving model to solve each problem.

1 The line plot shows scores for the first of two quizzes. From Quiz 1 to Quiz 2, the number of scores in the 70s decreased by 50% and the number of scores in the 80s increased by 100%. Which option shows the best measures of center and spread for Quiz 2 data? *Preparation for* MP 1



Ⓐ median = 75, interquartile range = 20

Ⓑ median = 80, interquartile range = 20

Ⓒ mean = 78, mean average deviation = 8

Ⓓ mean = 80, mean average deviation = 8

2 Laila participated in a flying disc game. The ages of the players are shown below. What measure of spread should Laila use for the data? What is that number? *Preparation for* MP 2

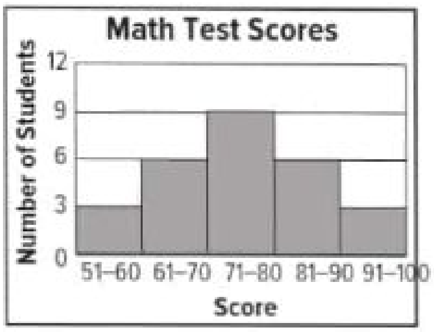
Players' Ages									
23	19	30	23	16	27	23	19	23	27

mean absolute deviation; 3

3 Manal recorded these low temperatures, in degrees Celsius, in her city on 10 consecutive days: 3, 2, 2, 1, −3, 1, 2, 2, 3, 7. What measure of spread did Manal use? What is the measure of spread? *Preparation for* MP 2

mean absolute deviation; 1.4

4 **H.O.T. Problem** Each test score shown in the histogram below is a multiple of 5. In each interval $\frac{2}{3}$ of the scores are multiples of 10. What are the measures of center and spread? Defend your answers. *Preparation for* MP 3



median = 80, *IQR* = 20; Sample answer: I used the median and *IQR*; I determined Q_1 = 70, the median = Q_2 = 80, Q_3 = 90, and *IQR* = $Q_3 - Q_1$ = 90 − 70 = 20.

10 Chapter Focus

Using the Interactive Student Guide

The *Interactive Student Guide* (ISG) can be used in conjunction with *Integrated Math 8*.

Teaching Tip SMP

The preview question for Lesson 10.2 would encourage the students to continue to discuss SMP 1 **(Make sense of problems and persevere in solving them).** Ask students to label the graph with the given information to help them understand the question and write everything they know based on the given information. For example, because $\angle QSR$ and $\angle RST$ are complementary angles, they know that $\angle QSR + \angle RST = 90$. Encourage students to assess their answers to check their validity. This can help them develop mathematical abilities.

Teaching Tip SMP

The preview question of lesson 10.4 represents a starting point for SMP 4 (Using Mathematical Forms). Students should interpret the given information and draw a geometrical figure that matches the given description. After they finish drawing the figure, they will have to find the area of the disc surface. Some students may be able to find the area of the surface without drawing the figure. Therefore, emphasize the importance of drawing figures to check the understanding of the question.

10 Tools of Geometry

CHAPTER FOCUS Learn about what you will explore in this chapter. Answer the preview questions. As you complete each lesson, return to these pages to check your work.

What You Will Learn	Preview Question
Lesson 10.2 Linear Measure	
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Point B is between point A and point C. If $AC = 10$ and $AB = 6$, how can you determine BC? **Because B is between A and C, $AB + BC = AC$. $6 + BC = 10$. So, $BC = 10 - 6 = 4$.**
Lesson 10.4 Proving Theorems about Line Segments	
Prove theorems about lines and angles. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)	Write a paragraph proof. **Given:** $XY = 2WX$ and $WX = YZ$ **Prove:** $2XY = WZ$  **Since $xy = 2wx$ and $wx = yz$. By the Segment Addition Postulate, $wx + xy + yz = wz$. Substituting, $wx + xy + wx = wz$ and $2wx + xy = wz$. Substituting again, $xy + xy = wz$ and then $2xy = wz$.**

10.2 Linear Measure

STANDARDS

Standards for Mathematical Practice: 2, 3, 5, 6, 7, 8

PREREQUISITES

- Recognize undefined terms
- Apply properties of square roots

MATERIALS

- Dynamic geometry software

EXAMPLE 1

Teaching Tip SMP 8

Students should recognize that any point selected on the circle will provide a segment of the same length as the initial segment.

Scaffolding Question

- How would you complete this construction without software? **Draw a segment; then set a compass to the length of the segment and draw a circle.**

10.2 Linear Measure

Objectives

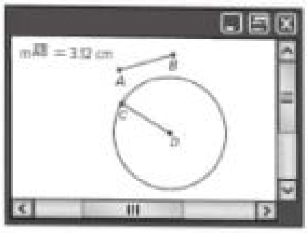
- Find the length of a segment.
- Construct a congruent segment.
- Find the distance between two points on a coordinate plane.
- Find the coordinates of a point on a directed line segment.

A part of a line consisting of two endpoints and all the points between them is called a **line segment**. The segment with endpoints P and Q is called $\overline{PQ}$ or $\overline{QP}$, and it has a measurable **length** designated by PQ. The length always includes a unit of measure. **Congruent** segments have the same length. Many tools can be used to construct a segment congruent to a given segment.

EXAMPLE 1 **Create a Congruent Segment**

EXPLORE Use the Geometer's Sketchpad to construct congruent line segments.

a. **USE TOOLS** Construct a line segment and label the endpoints A and B. Use **Measure Length** to find the length of $\overline{AB}$. Plot and label a point C some distance away from $\overline{AB}$. Construct a circle by selecting **Circle by Center + Radius** under the **Construct** menu and using center C and radius $\overline{AB}$. Then label a point D on the circle, construct $\overline{CD}$, and find CD.



Sample answer: $AB = 3.12$ cm; $CD = 3.12$ cm

b. **REASON ABSTRACTLY** What is the relationship of the two segments? If another point E is selected somewhere else on the circle, will $\overline{CE}$ have the same relationship with $\overline{AB}$?

Sample answer: The two segments have the same length, so they are congruent. As long as E is a point on the circle, the segments will be congruent.

120 CHAPTER 10 Tools of Geometry

Math Background

The concept underlying copying a segment is congruence. Two geometric figures are defined to be congruent if one can be obtained from the other by rotations, reflections, and translations. Students will encounter these transformations later in CHAPTER 10, which is why the formal definition is not given here.

The key mathematical idea in the construction is that any radius of the circle must be a copy of (congruent to) the line segment used to construct it. The construction allows a copy of the line segment to be constructed anywhere, at any angle.

A point lies on a segment if the point is between the endpoints of the segment.

Point C is **between** points A and B if and only if A, B, and C are collinear and $AC + CB = AB$. This definition allows us to write and solve equations to find the length of a segment.

EXAMPLE 2 **Write and Solve Equations to Find Measurements**

a. **REASON QUANTITATIVELY** Point D is between points C and E. Find CE.

$CD + DE = CE$

$1\frac{1}{4}$ cm $+ 2\frac{1}{2}$ cm $= CE$

$3\frac{3}{4}$ cm $= CE$

b. **REASON ABSTRACTLY** If $JK = 2x - 3$ and $KL = x - 1$, find the value of x and the lengths of $\overline{JK}$ and $\overline{KL}$.

$JK + KL = JL$

$2x - 3 + x - 1 = 5.3$

$3x - 4 = 5.3$

$3x = 9.3$

$x = 3.1$

$JK = 2(3.1) - 3$ or 3.2 cm

$KL = 3.1 - 1$ or 2.1 cm

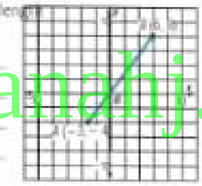
c. **MAKE A CONJECTURE** Is there a point B on $\overline{AC}$ for which $2(AB) = AC$? Explain.

Yes; Sample answer: When B is the midpoint of $\overline{AC}$, then $AB = BC$. Substituting, $AB + BC = AC$ becomes $AB + AB = AC$ and simplifying $2(AB) = AC$.

When line segments are used to show a movement, they are often shown as a directed line segment on a coordinate plane. Recall that the length of the segment on a coordinate plane is found using the distance formula. That is, if M has coordinates (x_1, y_1) and N has coordinates (x_2, y_2), then $MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. While a line segment has two endpoints, a **directed** line segment has a starting endpoint and a terminal endpoint. To find the coordinates of a point that partitions a directed line segment into a given ratio, add the fraction of the horizontal and vertical movement to the coordinates of the starting point.

EXAMPLE 3 **Find Points on a Segment**

a. **CALCULATE ACCURATELY** Use the distance formula to find the exact length of $\overline{AB}$.



$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(6 - (-3))^2 + (8 - (-4))^2}$

$AB = \sqrt{9^2 + 12^2}$

$AB = \sqrt{225}$ or 15

10.2 Linear Measure 1?

EXAMPLE 2

Teaching Tip SMP 3

For some students, it might be useful to emphasize the connections between solving for unknown line segments and solving for unknown variables as they did in Algebra 1. If students are struggling to set up the equations in order to solve **parts a** and **b**, then encourage them to think of lengths such as CD and DE as variables such as x or y.

Scaffolding Question

- How can you confirm your answers for **parts a** and **b**? **Use a ruler construct and measure the segments as shown for each part.**

Differentiating Instruction

In **Example 2**, students are working informally with the Segment Addition Postulate, expressed in **part a** as: if D lies on $\overline{CE}$, then $CD + DE = CE$. This is a critical idea for later constructions. Kinesthetic learners may benefit from working with a measuring tape to visualize the addition, subtraction, and division ideas developed here.

Part c prepares students for the idea of the midpoint of a line segment and the relationship of this concept to length or distance. For visual learners who may need help following the language of **part c**, unpack the idea with a sketch.

EXAMPLE 3

Teaching Tip SMP 7

For **part d,** have students recognize that they have just calculated the mean for the x- and y-coordinates; then, look at the structure of the mean of two distinct numbers, to understand that it must lie midway between the two numbers on a number line.

Scaffolding Questions

- How can you check your work to see if point D divides $\overline{AB}$ such that $AD = DB$? **Sample answer: If you use the distance formula to calculate AD and DB, the values should be equal.**
- How is the y-coordinate of the midpoint related to the y-coordinates of A and B? **It is the mean of the y-coordinates of A and B.**

b. COMMUNICATE PRECISELY Find the coordinates of point C on directed line segment $\overline{AB}$ that partitions it into two segments in a ratio of 2 to 1. Explain your solution.

(3, 4); If $AC:BC = 2:1$, then $AC:AB = \frac{2}{3}$. From A go 9 units to the right and 12 units up to get to B. Then to find C, add $\frac{2}{3}$ of the horizontal and vertical movements to the coordinates of point A. So, $\left(-3 + \frac{2}{3}(9), -4 + \frac{2}{3}(12)\right) = (-3 + 6, -4 + 8) = (3, 4)$.

c. CONSTRUCT ARGUMENTS Use the distance formula to verify that the ratio of $AC:CB$ is equal to 2:1.

$AC = \sqrt{(3-(-3))^2 + (4-(-4))^2} = \sqrt{100}$ or 10; $BC = \sqrt{(6-3)^2 + (8-4)^2} = \sqrt{25}$ or 5; $\frac{AC}{BC} = \frac{10}{5}$ or $\frac{2}{1}$

d. COMMUNICATE PRECISELY Find the coordinates of point D on $\overline{AB}$ that divides it into a 1 to 1 ratio. Show your work.

(1.5, 2); Sample answer: Point D is $\frac{1}{2}$ of the way from A to B. $\left(-3 + \frac{1}{2}(9), -4 + \frac{1}{2}(12)\right) = (-3 + 4.5, -4 + 6)$. The coordinates of point D are (1.5, 2).

e. EVALUATE REASONABLENESS Find the midpoint of $\overline{AC}$ using the formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ where (x_1, y_1) and (x_2, y_2) are the endpoints of the segment. How does this compare to the coordinates for point D in **part d**? What conclusion can you make?

(1.5, 2); The points have the same coordinate. Sample answer: The midpoint of a segment is the point that partitions it into two segments in a 1:1 ratio.

PRACTICE

1. a. **REASON QUANTITATIVELY** For line segment $\overline{AC}$, write and solve an equation to find AB.

$AB + BC = AC$

$AB + 1.5 \text{ cm} = 3.7 \text{ cm}$

$AB = 2.2 \text{ cm}$

b. **REASON QUANTITATIVELY** What length would EF need to be for $\overline{DE}$ to be congruent to $\overline{AB}$?

For $\overline{DE}$ to be congruent to $\overline{AB}$, we need $DE = AB$. So, $DE = 2.2$ cm. Because $DE + EF = DF$, and $DF = 3.5$ cm, we have that $EF = DF - DE = 3.5 \text{ cm} - 2.2 \text{ cm} = 1.3 \text{ cm}$.

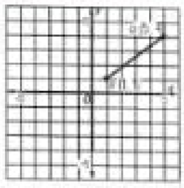
Emphasizing the Standards for Mathematical Practice

Example 3 provides an opportunity to address both the calculation and the communication aspects of **SMP 6 (Attend to precision)**. Emphasize to students the importance of not only calculating correctly, but also of explaining their process in a way that allows others to understand what is being done.

2. **USE STRUCTURE** Consider a rectangle $QRST$ with $QR = ST = 4$ cm and $RS = QT = 2$ cm. If point U is on $\overline{QR}$ such at $QU = UR$ and point V is on $\overline{RS}$ such that $RV = VS$, then is $\overline{QU}$ congruent to $\overline{RV}$? Explain your reasoning.

No; we know that $QU + UR = QR = 4$ and $QU = UR$, so $QU = 2$. Further, we know that $RV + VS = RS = 2$, and $RV = VS$, so $RV = 1$. Because QU is not equal to RV, we know that $\overline{QU}$ is not congruent $\overline{RV}$.

3. **CALCULATE ACCURATELY** What is the exact length of $\overline{RQ}$ shown at right?



$RQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$RQ = \sqrt{(5 - 1)^2 + (4 - 1)^2}$

$RQ = \sqrt{4^2 + 3^2}$

$RQ = \sqrt{25} = 5$ cm

4. a. **REASON QUANTITATIVELY** If you were to add a point T to $\overline{RQ}$ from **Exercise 3** such that the ratio $\overline{RT}$ to $\overline{TQ}$ is 3 to 2, what would be the coordinates of T?

$\left(\frac{17}{5}, \frac{14}{5}\right)$; point T is $\frac{3}{5}$ the way from R to Q. So, the coordinates of T are $\left(1 + \frac{3}{5}(4), 1 + \frac{3}{5}(3)\right) = \left(1 + \frac{12}{5}, 1 + \frac{9}{5}\right)$, which makes the coordinates of $T\left(\frac{17}{5}, \frac{14}{5}\right)$.

b. **COMMUNICATE PRECISELY** Find the midpoint M of $\overline{RQ}$. Without using the distance formula, calculate MT. Explain your reasoning.

$M = \left(\frac{1+5}{2}, \frac{1+4}{2}\right)$. Thus, we have that $M = (3, 2.5)$. Because M is the midpoint of $\overline{RQ}$, we know that $RM = \frac{1}{2}(5) = 2.5$ cm. Further, we know that $RT = \frac{3}{5}(5) = 3$ cm. Thus, $MT = RT - RM = 3$ cm $- 2.5$ cm $= 0.5$ cm.

c. Use the distance formula to confirm your result from **part b**.

$MT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$MT = \sqrt{\left(\frac{17}{5} - 3\right)^2 + \left(\frac{14}{5} - 2.5\right)^2}$

$MT = \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{3}{10}\right)^2}$

$MT = \sqrt{\frac{4}{25} + \frac{9}{100}}$

$MT = \sqrt{\frac{25}{100}}$

$MT = \frac{5}{10} = \frac{1}{2}$

PRACTICE

Exercise 1 requires students to use the definition of line segment as they determine the length of segments.

In **Exercise 2**, students must use given information about a rectangle to determine the truth of a statement about parts of the rectangle.

In **Exercise 3**, students use the distance formula to determine the length of a segment.

Exercise 4 requires students to locate the midpoint of a segment, dividing it into segments with ratio 1:1, or to locate a point that divides a given segment into a ratio other than 1:1.

Addressing the Standards

Exercise	SMP
1	2
2	7
3	6
4	2, 6

Emphasizing the Standards for Mathematical Practice

Exercise 4 can be used to address **SMP 2 (Reason abstractly and quantitatively)**. Students need, in **part a**, to translate information about a line segment divided in a certain ratio into a practical method of solution. The key is to recognize that a 3:2 ratio divides RT in the fractions $\frac{3}{5}$ and $\frac{2}{5}$. To do this students need to appreciate that the pattern of proportions is thus $3:2 = \frac{3}{5} = \frac{2}{5} = RT:TQ$.

10.4 Proving Theorems about Line Segments

STANDARDS

Standards for Mathematical Practice: 1, 3, 5, 6

PREREQUISITES

- Know and apply the concept of congruence
- Construct two-column and paragraph proofs

EXAMPLE 1

Teaching Tip SMP 6

You may want to briefly discuss definitions as students work on this construction. Be sure students understand that *bisect* means to divide into two equal parts. Point out that the prefix *bi-* means *two* (bicycle, bilingual, biweekly, etc.)

Scaffolding Questions

- In **step a**, does the exact amount of the compass opening matter? Explain. **No; the exact amount of the opening does not matter as long as the opening is more than half the length of $\overline{AB}$.**
- Does using a different compass setting produce a different outcome? Explain. **No, using a different compass setting will produce a larger or smaller pairs of arcs, but the location of the midpoint will always be the same.**

10.4 Proving Theorems about Line Segments

Objectives

- Construct the bisector of a segment.
- Prove theorems about line segments using the Segment Addition Postulate.

EXAMPLE 1 Bisect a Segment

EXPLORE Follow Steps a–c to use a compass and straightedge to bisect $\overline{AB}$. Analyze your construction in steps d–f.

a. Open the compass to a little more than half the length of $\overline{AB}$. Place the tip of the compass on point A and make an arc as shown below.

b. Without changing the compass setting, place the tip of the compass on point B. Make an arc that intersects the first arc at points P and Q, as shown.

c. Use the straightedge to draw $\overleftrightarrow{PQ}$. Label the intersection of $\overline{AB}$ and $\overleftrightarrow{PQ}$ as point M.

d. **USE TOOLS** Is M the midpoint of $\overline{AB}$? Explain.
Yes; M is the point of intersection of $\overline{AB}$ and its bisector $\overleftrightarrow{PQ}$. By the definition of bisector, $AM = MB$, so M is the midpoint of $\overline{AB}$.

e. **CONSTRUCT ARGUMENTS** In **step a**, why do you need to open the compass to more than half the length of $\overline{AB}$?
This ensures that the two arcs will intersect.

f. **CONSTRUCT ARGUMENTS** How are the lengths AM, MB, and AB related? Write one or more equations to express the relationships.
Sample answers: $AM + MB = AB$; $AM = \frac{1}{2}AB$; $MB = \frac{1}{2}AB$; $AM = MB$

124 CHAPTER 10 Tools of Geometry

Math Background

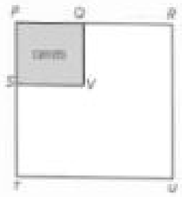
In this lesson, students work with line segments as they begin to read and write more complex proofs. Along the way, students will create and interpret geometric figures. This is a good time to address the facts that can and cannot be assumed from a given figure. In general, it can be assumed that lines that appear to be straight are indeed straight and that points that lie along a line are collinear. A point that appears to be a midpoint cannot be assumed to be a midpoint simply because it is near the middle of the segment. Similarly, when students turn their attention to angles in the next few lessons, they should not assume an angle is a right angle unless it is specifically marked as such in the figure.

In the exploration, you may have noticed a relationship among the lengths of the original segment and the lengths of the two shorter segments you created. The Segment Addition Postulate states that this relationship is true.

KEY CONCEPT **Segment Addition Postulate**

Complete the following statement.
If A, B, and C are collinear, then point B is between A and C if and only if **AB** + **BC** = **AC**.

EXAMPLE 2 **Use the Segment Addition Postulate**



Aisha has a square vegetable plot in her garden. She wants to set aside a corner of the plot for planting carrots, as shown. Based on her measurements, she knows that $\overline{PR}$ $\overline{PT}$ and $\overline{QR}$ $\overline{ST}$. She wants to know if she can conclude that $\overline{PQ}$ $\overline{PS}$.

a. **CONSTRUCT ARGUMENTS** Complete the two-column proof below.

Given: $\overline{PR} \cong \overline{PT}$ and $\overline{QR} \cong \overline{ST}$

Prove: $\overline{PQ} \cong \overline{PS}$

Statements	Reasons
1. $\overline{PR} \cong \overline{PT}$ and $\overline{QR} \cong \overline{ST}$	1. **Given**
2. **PR PT and QR ST**	2. Congruent segments have equal lengths.
3. $PR = PQ + QR$ and $PT = PS + ST$	3. **Segment Addition Postulate**
4. $PQ + QR = PS + ST$	4. **Substitution Property of Equality**
5. $PQ + QR = PS + QR$	5. **Substitution Property of Equality**
6. $PQ = PS$	6. **Subtraction Property of Equality**
7. **$\overline{PQ}$ $\overline{PS}$**	7. Segments with equal lengths are congruent.

b. **COMMUNICATE PRECISELY** In the second step of the proof, why is it necessary to change the given statements about congruent segments into statements about lengths of segments?

The proof depends upon using the Segment Addition Postulate, but the postulate involves lengths of segments, so it is necessary to convert the congruence statements into length statements.

c. **COMMUNICATE PRECISELY** Extend the line segment $\overline{SV}$ to the line segment $\overline{RU}$ and let W be the point where they intersect. Write a paragraph proof to prove that if $\overline{PR} \cong \overline{SW}$ and $\overline{QR} \cong \overline{VW}$, then $\overline{PQ} \cong \overline{SV}$.

Since $\overline{PR}$ $\overline{SW}$ and $\overline{QR}$ $\overline{VW}$, PR SW and QR ST because congruent segments have equal lengths. By the Segment Addition Postulate, PR PQ QR and SW SV VW. Because PR SW, PQ QR SV VW by the Substitution Property of Equality. Since QR VW, the equation becomes PQ VW SV VW by the Substitution Property of Equality. By the Subtraction Property of Equality, PQ SV, and segments with equal lengths are congruent, so $\overline{PQ}$ $\overline{SV}$.

Emphasizing the Standards for Mathematical Practice

The Segment Addition Postulate is an instance of **SMP 7 (Look for and make use of structure)**. Beginning with this lesson, students should get in the habit of seeing a line segment as a whole or as a "sum" of its parts. In the figure in the Key Concept, $\overline{AC}$ is a line segment, but it can also be viewed as a segment composed of the two shorter segments $\overline{AB}$ and $\overline{BC}$, which have exactly one point in common (point B). Working back and forth between the two ways of looking at a segment is a valuable skill when seeking a logical pathway to develop a proof.

EXAMPLE 2

Teaching Tip — SMP 3

Some students may have trouble following the reasoning in the two-column proof, especially in the steps where substitution is used. You may want to have students use a highlighter to mark any parts of expressions or equations that change from one step of the proof to the next. This may help them focus on the substitution that was made.

Scaffolding Questions

- What changes do you see from **step 4** to **step 5** of the proof? Explain how the Substitution Property of Equality justifies these statements. ***ST* is replaced with *QR*; we know that *QR* = *ST* (from step 2) and the Substitution Property of Equality states that you can therefore replace *ST* with *QR* in any expression or equation.**
- What do you do to the equation to get from **step 5** to **step 6**? **Subtract *QR* from both sides of the equation; Subtraction Property of Equality.**

PRACTICE

Exercises 1 and **2** give students additional practice in using a compass and straightedge to bisect a line segment. **Exercises 3** and **4** add a new dimension of reasoning to students' work with bisecting a line segment.

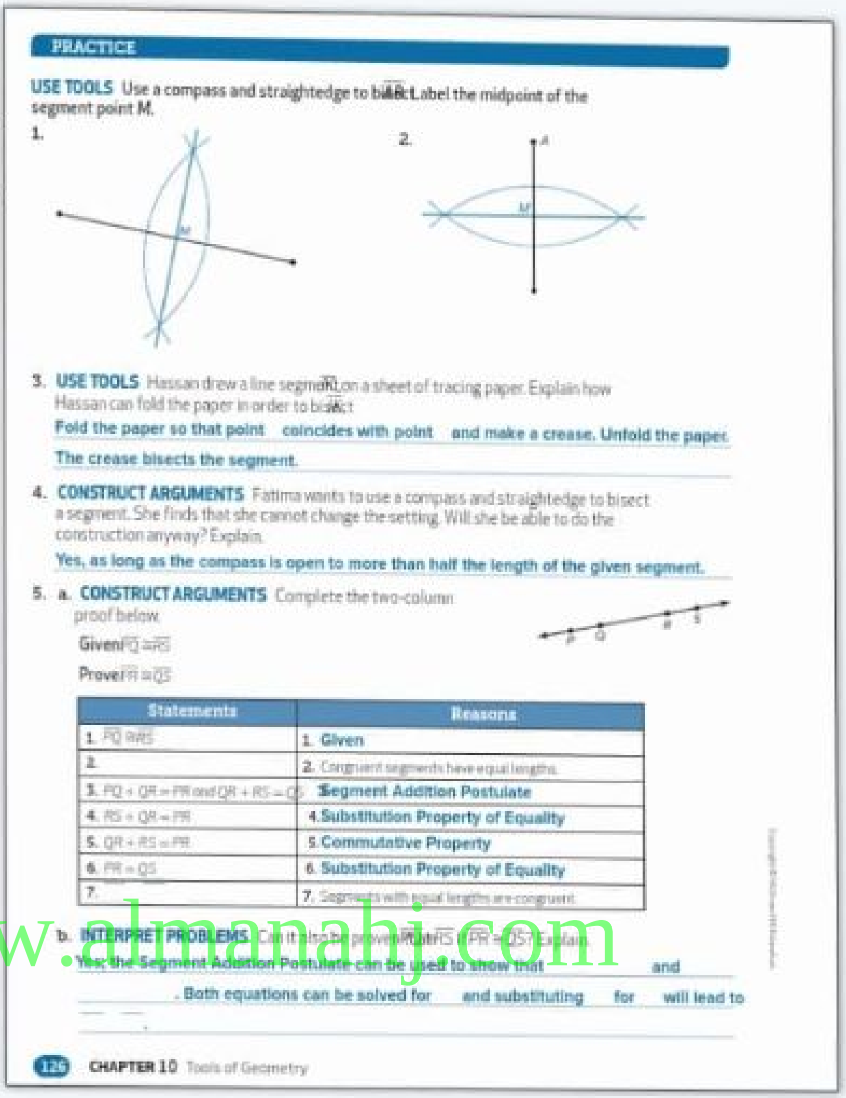
In **Exercises 5** and **6**, students complete a two-column proof.

In **Exercise 7**, students are asked to critique the reasoning of a proof.

Exercise 8 asks students to write a paragraph proof involving line segments.

Addressing the Standards

Exercise	SMP
1–3	5
4	3
5	3
6	1, 3
7–8	3


PRACTICE

USE TOOLS Use a compass and straightedge to bisect. Label the midpoint of the segment point *M*.

1.

2.

3. USE TOOLS Hassan drew a line segment on a sheet of tracing paper. Explain how Hassan can fold the paper in order to bisect.
Fold the paper so that point coincides with point and make a crease. Unfold the paper. The crease bisects the segment.

4. CONSTRUCT ARGUMENTS Fatima wants to use a compass and straightedge to bisect a segment. She finds that she cannot change the setting. Will she be able to do the construction anyway? Explain.
Yes, as long as the compass is open to more than half the length of the given segment.

5. a. CONSTRUCT ARGUMENTS Complete the two-column proof below.
Given: $\overline{PQ} \cong \overline{RS}$
Prove: $\overline{PR} \cong \overline{QS}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2.	2. Congruent segments have equal lengths.
3. $PQ + QR = PR$ and $QR + RS = QS$	3. Segment Addition Postulate
4. $RS + QR = PR$	4. Substitution Property of Equality
5. $QR + RS = PR$	5. Commutative Property
6. $PR = QS$	6. Substitution Property of Equality
7.	7. Segments with equal lengths are congruent.

b. INTERPRET PROBLEMS Can it also be proven that if $\overline{PR} \cong \overline{QS}$... Explain.
Yes, the Segment Addition Postulate can be used to show that and . Both equations can be solved for and substituting for will lead to .

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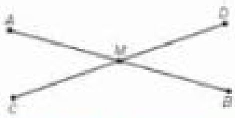
Common Errors

In **Exercise 5**, students may have difficulty identifying the reason for **step 4** of the proof. Because the statement $RS + QR = PR$ looks like the Segment Addition Postulate, students may cite this as the reason for this step. Point out that they have already shown that $PQ = RS$ **(step 2)** and $PQ + QR = PR$ **(step 3)**. Substituting RS for PQ in the latter expression gives $RS + QR = PR$, so the Substitution Property of Equality is the correct reason.

6. A city planner is designing a new park. The park has two straight paths, $\overline{AB}$ and $\overline{CD}$, which are the same length. A monument, M, is located at the midpoint of both paths.



a. **INTERPRET PROBLEMS** The city planner thinks that the length of $\overline{AM}$ will be the same as the length $\overline{CM}$. Explain why this makes sense.

Both segments are half the length of two congruent segments, so the lengths of these shorter segments must be the same.

b. **CONSTRUCT ARGUMENTS** Complete the two-column proof.

Given: $\overline{AB} \cong \overline{CD}$, M is the midpoint $\overline{AB}$ and $\overline{CD}$.

Prove: $\overline{AM} \cong \overline{CM}$

Statements	Reasons
1. **$\overline{AB} \cong \overline{CD}$; M is the midpoint of $\overline{AB}$ and $\overline{CD}$.**	1. Given
2. $AB = CD$	2. **Congruent segments have equal lengths.**
3. $\overline{AM} \cong \overline{MB}$; $\overline{CM} \cong \overline{MD}$	3. **Definition of midpoint**
4. **$AM = MB$; $CM = MD$**	4. Congruent segments have equal lengths.
5. $AM + MB = AB$; $CM + MD = CD$	5. **Segment Addition Postulate**
6. $AM + MB = CM + MD$	6. **Substitution Property of Equality**
7. $AM + AM = CM + CM$	7. **Substitution Property of Equality**
8. $2AM = 2CM$	8. **Distributive Property**
9. **$AM = CM$**	9. Division Property of Equality
10. **$\overline{AM} \cong \overline{CM}$**	10. Segments with equal lengths are congruent.

7. **CRITIQUE REASONING** Naser knows that point R is the midpoint of $\overline{QS}$, and he knows that this means $QR = RS$. He says that $PR = PQ + QR$ by the Segment Addition Postulate. So $PR = PQ + RS$ by substitution. Do you agree with Naser's reasoning? Explain.

No; the Segment Addition Postulate only applies to points that are collinear, but points P, Q, and R are not collinear.

8. **CONSTRUCT ARGUMENTS** Write a paragraph proof to prove that if P, Q, R, and S are collinear, $\overline{PQ} \cong \overline{RS}$ and Q is the midpoint of $\overline{PR}$, then R is the midpoint $\overline{QS}$.

Since $\overline{PQ} \cong \overline{RS}$ and congruent segments have equal lengths, $PQ = RS$. Because Q is the midpoint of $\overline{PR}$, $PQ = QR$. By the Substitution Property of Equality, $QR = RS$ so R is the midpoint of

Common Errors

In **Exercise 7**, students may state that Justin's reasoning is correct because he used substitution correctly. Students who make this error may not have looked at the figure carefully or considered whether the Segment Addition Postulate can be applied in this situation. Tell students that Justin's error is a common one that they should watch out for: since points P, Q, and R are not collinear, the Segment Addition Postulate does not apply.

Emphasizing the Standards for Mathematical Practice

Part a of **Exercise 6** is an important connection to **SMP 1 (Make sense of problems and persevere in solving them)**. In particular, when students are asked to write a proof, they should first make sense of the problem by asking themselves whether the statement they are trying to prove seems reasonable, and why. Students who are able to convince themselves that the statement to be proved is reasonable are usually in a better position to put together a convincing argument in the form of a proof.

10 Performance Task

Picture Perfect

Students use a grid to find dimensions, perimeters, and areas in reference to an art canvas.

STANDARDS

Standards for Mathematical Practice: This CHAPTER 10 Performance Task reinforces Mathematical Practices **SMP 1** and **SMP 2.**

Jump Start

To introduce the task, it may be helpful to demonstrate that coordinates can be assigned to the vertices of a figure on a grid by first assigning one vertex the coordinates (0, 0) and determining the other coordinates based on that.

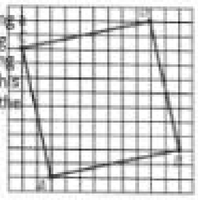
- If one of the vertices is to be assigned the coordinates (0, 0), does it matter which of the vertices is chosen? **No; any of the vertices can be chosen to have coordinates (0, 0).**
- What lengths do you need in order to find the perimeter of the canvas? ***AB*, *BC*, *CD*, and *AD*, the lengths of the sides of the canvas.**
- Vertically, point *C* lies 9 tiles above point *A*. How many centimeters does this correspond to? **Each tile has height 15 centimeters, so 9(15) = 135 centimeters.**

Performance Task

Picture Perfect

Provide a clear solution to the problem. Be sure to show all of your work, include all relevant drawings, and justify your answers.

Bilal plans to buy some artwork at a local gallery. He is considering a particular canvas displayed high on a wall. The canvas is not hung straight and Bilal must determine if the canvas will fit on his living room wall before he makes his purchase. The space available on his wall extends from the ceiling 1.8 m and horizontally 2.4 m from the adjoining wall.



Part A

Bilal plans to use the wall tiles behind the canvas to estimate its dimensions. If each square tile is 15 cm wide, what are the dimensions of the canvas? Justify your solution.

Emphasizing the Standards for Mathematical Practice

This Performance Task aligns primarily with **SMP 1 (Make sense of problems and persevere in solving them)**. The task requires students to determine what information they need in order to find quantities such as length, perimeter, and area based on an imposed grid. Students must assign coordinates to vertices and interpret results in a real-world situation as each part of the task builds on the parts before it.

Part B
Before hanging the canvas, Bilal wants to mat and frame the artwork. If he wants a 5 cm mat border around the perimeter of the canvas, what is the area of the mat border he must order? What is the total length of the inside perimeter of the frame that he must order to fit the canvas plus mat border? If the frame is 10 cm wide, what is the new area of the framed artwork?

Part C
The gallery recommended that Bilal install hardware on the back of the framed canvas that will support the artwork at both ends and then at points $\frac{1}{3}$ and $\frac{2}{3}$ along the length of the frame. Where should Bilal install the hardware? Justify your answer.

Part D
Bilal wants to center the canvas horizontally on his wall. He draws a line that runs the entire length of the available space. At what distance from the adjoining wall should Bilal place holes to accommodate the four supports that he added in Part C? Justify your answer.

Teaching Tip SMP 2

Parts C and D connect to **SMP 2 (Reason abstractly and quantitatively)** by asking students to divide a segment into three equal parts and transferring them to a point at which the canvas is centered on the wall. Have students first find where the left and right edges of canvas should be placed on the wall and then determine the two remaining points.

Common Errors

Students may mistakenly assign coordinates to the vertices of *ABCD* by regarding each grid line as 1 cm rather than 15 cm. Students may also make the mistake of placing the mat on the canvas so that the mat aligns with the outside edge of the canvas and extends 5 cm inward rather than outward.

Scoring Rubric

Part	Max Points	Full Credit Response
A	2	55.3 cm by 55.3 cm or 4.6 m by 4.6 m; Let the coordinates of *A* be (0, 0), then the rest of the coordinates are *B*(9, 2), *C*(−2, 9), and *D*(7, 11). The height of the canvas is $\sqrt{(11-9)^2+(7-(-2))^2} = \sqrt{85}$. The width of the canvas is $w = \sqrt{(9-0)^2+(2-0)^2} = \sqrt{85}$. Each tile measures 15 cm, so $15(\sqrt{85}) \approx 138.3$ cm or 0.138 m.
B	2	$(\sqrt{85}\times 15+10)^2-(\sqrt{85}\times 15)^2 \approx 2865$ cm²; $4(\sqrt{85}\times 15+10) \approx 30$ cm; $(\sqrt{85}\times 15+30)^2 \approx 28{,}322.5$ cm²
C	2	At each end, then at 56 cm from one end and 112 cm from the same end. The total length of the framed canvas is approximately 168 cm; $\frac{1}{3}(168) \approx 56$ cm; $\frac{2}{3}(168) \approx 112$ cm
D	2	36 cm, 92 cm, 148 cm, and 204 cm. The center of the 2.4-m section of wall is 120 cm from the adjoining wall. The canvas is approximately 168 cm long, so 84 cm should be on one side of the 120-cm mark and 84 cm should be on the other. So the first hole should be drilled at (120 − 84) = 36. The next support on the canvas is at (36 + 56) = 92 cm. The next support is at (36 + 112) = 148 cm. And the final support is at (36 + 168) = 204 cm.
Total	8	

10 Performance Task

Triangle Designs

Students explore different plans for paths across a city park, one of which involves angle bisectors and perpendicular bisectors.

STANDARDS

Standards for Mathematical Practice: This CHAPTER 10 Performance Task reinforces Mathematical Practices **SMP 1, SMP 2, SMP 5, SMP 6,** and **SMP 7**.

Materials

dynamic geometry software or compass and straightedge

Jump Start

Some students may be unsure about how to construct an angle bisector or a perpendicular bisector.

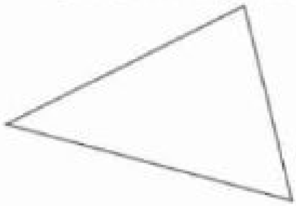
- How do you use compass and straightedge to create a perpendicular bisector? **Sample answer: Make an arc a little more than half the length of the line. Without changing the compass, repeat from the other point. Use the straightedge to draw a line segment between the two intersections of the arcs. That is the perpendicular bisector.**

Performance Task

Triangle Designs

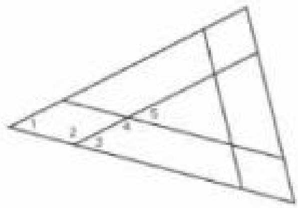
Provide a clear solution to the problem. Be sure to show all of your work, include all relevant drawings, and justify your answers.

Suhaila, a landscape architect, is designing a set of paths for a park in front of a new government building. The shape of the triangular park is as shown.



Part A

In her first design, Suhaila decides to place three paths in the park, each of which is parallel to a side of the triangle. Write a paragraph proof to prove that $\angle 1 \cong \angle 5$ given that $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 4$ are supplementary.



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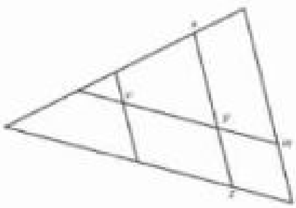
Emphasizing the Standards for Mathematical Practice

This Performance Task aligns primarily with **SMP 3 (Construct viable arguments and critique the reasoning of others).** The task requires students apply **SMP 5 (Use appropriate tools strategically)** to make the paper-and-pencil constructions identified in **Part C**. **Part C** asks students to implement **SMP 2 (Reason abstractly and quantitatively)** and **SMP 6 (Attend to precision)** to deduce that a circumscribed circle would result from the construction using perpendicular bisectors.

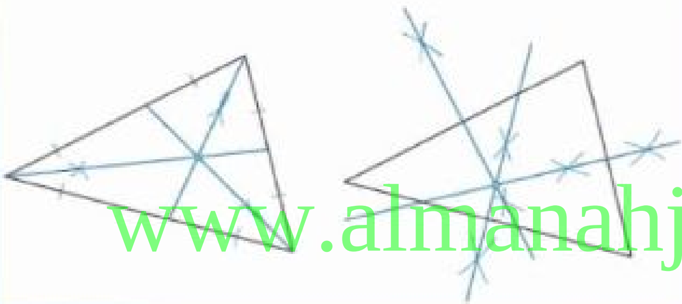
Part B

In her second design, Suhaila decides to place three paths in the park as shown in the diagram. Write a paragraph proof to show that if $VW = XZ$ and $YW = YZ$, then $VY = XY$.



Part C

Suhaila wants to draw two different designs of three paths for the park. In each design, a trash receptacle will be placed at the point where the three paths intersect. In one design the paths will be the angle bisectors of the triangle and in the other design the paths will be the perpendicular bisectors of the triangle. Explain the advantage of the placement of the trash receptacle in each design.



Jump Start (*continued*)

- How do you use compass and straightedge to create a perpendicular bisector? **Sample answer: Draw an arc that intersects the sides of the angle, using the vertex as the center. Using the same setting, put the compass on one of the intersections, and draw an arc within the angle. Repeat with the other intersection. Use the straightedge to draw a line from the vertex through the points where the arcs intersect. That is your angle bisector.**

Common Errors

Some students may incorrectly treat a construction as a sketch. Stress accuracy and proper use of the tools while the students are making their constructions. Careful alignment of the straightedge and setting the compass so that it does not expand during a rotation are critical skills for receiving the expected results.

Scoring Rubric

Part	Max Points	Full Credit Response
A	2	Because ∠1 is supplementary to ∠2 and ∠2 is supplementary to ∠3, ∠1 ≅ ∠3. Also, ∠3 ≅ ∠1 and ∠3 supplementary to ∠4 means that ∠1 must be supplementary to ∠4. Finally, ∠4 is supplementary to ∠1 and ∠5 so ∠1 ≅ ∠5.
B	2	$VW = VY + YW$ by the Seg. Add. Post. so $VY = VW - YW$ by Sub. Prop. of Eq. Similarly, $XZ = XY + YZ$ by the Seg. Add. Post. so $XY = XZ - YZ$ by Sub. Prop. of Eq. Using substitution, $VY = XZ - YZ = XY$.
C	4	See *Interactive Student Guide* for drawing. If Suhaila places the receptacle at the intersection of the angle bisectors, it will be equidistant from the sides of the triangle. If she places the receptacle at the intersection of the perpendicular bisectors, it will be equidistant from the vertices of the triangle.
Total	8	

Standardized Test Practice

Diagnosing Errors

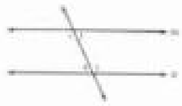
Students who answer **Item 3** incorrectly may not be fluent with terms used in this chapter. Make a list of common terms, such as linear pair, supplementary angles, and vertical angles, as well as mathematical properties such as the Transitive, Addition, and Subtraction properties of equality. For each, ask students to explain the term or property and draw or write an example.

Standardized Test Practice

1. An angle is a figure formed by two **rays** with a common **endpoint**.

2. A line segment is a part of a **line** that is formed by two **endpoints** and all points between them.

3. Complete the steps in the following proof of the Vertical Angles Theorem.



Prove: $\angle 1 \cong \angle 3$

$\angle 1$ and $\angle 2$ form **linear pair**. Therefore, by the Linear Pair Theorem, they are **supplementary**.
This means that $m\angle 1 + m\angle 2 =$ **180**.
Similarly, $m\angle 2 + m\angle 3 =$ **180**. By the **Transitive** Property of Equality, $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$, and by the **Subtraction** Property of Equality, **$\angle 1 \cong \angle 3$**.

4. If $AD = 3BC$, C is the midpoint of $\overline{BD}$, and $AB = 3$, what is the length of $\overline{AD}$?

$AD =$ **9**

5. Consider the following diagram.

Name three points shown in the diagram.
A, **B**, and **C**
Give three names for the line shown in the diagram.
m, **$\overleftrightarrow{AB}$**, and **$\overleftrightarrow{BA}$**

6. Quadrilateral $ABCD$ has vertices at $A(-2, 5)$, $B(-1, 12)$, $C(8, 3)$, and $D(14, -13)$.
   a. What is the perimeter of $ABCD$?
   $AB = 5\sqrt{2}$, $BC = 9\sqrt{2}$, $CD = 2\sqrt{73}$, and $AD = 2\sqrt{145}$ so the perimeter is $14\sqrt{2} + 2\sqrt{73} + 2\sqrt{145}$ or about 61.0 units.
   b. If $ABCD$ is translated along $\langle -3, 4 \rangle$ and reflected in the x-axis, find the vertices of the image.
   $(-5, -9)$, $(-4, -16)$, $(5, -7)$, $(11, 9)$
   c. Is the perimeter of $ABCD$ the same as the perimeter of its image?
   Yes; the side lengths of the image are $5\sqrt{2}$, $9\sqrt{2}$, $2\sqrt{73}$, and $2\sqrt{145}$; the sides are congruent to the sides of $ABCD$, so the perimeters are the same.

7. Complete the following proof.

Given: $\angle A$ is supplementary to $\angle B$

$\angle C$ is supplementary to $\angle B$

Prove: $\angle A \cong \angle C$

Statement	Reason
$\angle A$ is supplementary to $\angle B$	**Given**
$m\angle A + m\angle B = 180$	**Definition of Supplementary Angles**
$\angle C$ is supplementary to $\angle B$	Given
$m\angle C + m\angle B = 180$	Definition of Supplementary Angles
$m\angle A + m\angle B = m\angle C + m\angle B$	**Substitution**
$m\angle A = m\angle C$	Subtraction Property of Equality
$\angle A \cong \angle C$	**Definition of Congruent Angles**

8. The steps to create $\angle XYZ$ as a copy of $\angle A$ are listed below. In the first column, place the order for each step.

Order	Step
4	Without changing the compass width, move the compass point to X and draw a similar arc, creating
2	Draw $\overrightarrow{XY}$
7	Draw $\overrightarrow{XZ}$ containing T.
5	Set the compass point at B and set its width to BC.
1	Mark point X, which will be the vertex of the new angle.
6	Without changing the compass width, move the compass point to S and draw an arc crossing the first arc to create point T.
3	Set the compass point on A and draw an arc across the angle, creating points B and C.

9. Ahmad is constructing $\overline{YZ}$ to be congruent $\overline{AB}$. Describe the steps that Ahmad should take.

Set the compass at *A* and adjust compass width so that the tip is at *B*. Without changing the width, move the compass point to *Y* and make an arc. Place *Z* on the arc, and connect *Y* and *Z*.

10. The Converse of the Angle Bisector Theorem states that if a point in the interior of an angle is equidistant from both sides of the angle, that point lies on the angle bisector. Explain how this theorem justifies the method used to construct an angle bisector.

To construct an angle bisector, make an arc creating points on each ray that are equidistant from the vertex. Then create arcs from both of these points using the same compass width. The point where those arcs cross is equidistant from both sides of the angle, so it lies on the angle bisector.

Test-Taking Strategy

Some students may struggle to visualize the steps described in **Item 8.** Encourage students to perform the construction on scrap paper, being sure to use the same point names as are given in the problem. As they complete each step in the construction, have them find that step in the list and number it.

Diagnosing Errors

In **Item 7**, students who struggle to fill in the reasons may benefit from a diagram. Have these students draw a diagram of angles *A*, *B*, and *C*.

Students who incorrectly identify the third step in the list as step 2 for **Item 8** may not have read through all of the steps before ordering them. The second step listed is a better step 2, because further down the list there is a step that creates point *T*.

Item 14

[2] Answer includes measuring *AB* with a compass, making an arc with compass point on *Y*, and placing point *Z* on the arc.

[1] Answer includes 1 or 2 of the correct steps

[0] no response OR incorrect answers and reasoning

Item 15

[3] Correct answer for all parts

[2] Minor errors in calculating a perimeter or vertex of image but correct interpretation for **part c** OR one part incorrect

[1] At least one correct component

[0] no response OR incorrect answers and reasoning

11 Chapter Focus

Using the *Interactive Student Guide*

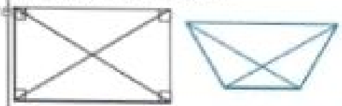
The *Interactive Student Guide* (ISG) can be used in conjunction with *Integrated Math 8*.

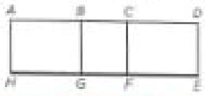
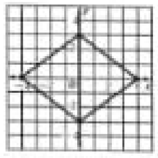
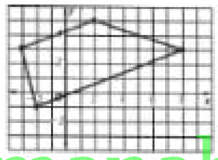
Teaching Tip SMP 2

The preview question for Lesson 11.2 provides practice for **SMP 2 (Reason abstractly and quantitatively)**. Ask students to graph the three given points and use their graph to determine the other possible vertices. Remind students that they must use algebra to prove each point is a vertex.

11 Quadrilaterals

CHAPTER FOCUS Learn about what you will explore in this chapter. Answer the preview question. As you complete each lesson, return to these pages to check your work.

What You Will Learn	Preview Question
Lesson 11.2 Parallelograms	
Use coordinates to prove simple geometric theorems algebraically. Prove theorems about parallelograms.	Three vertices of a parallelogram are (0, 4), (5, 0), and (10, 4). List all possible locations of the fourth vertex. **(15, 0), (−5, 0), and (5, 8)**
Lesson 11.3 Tests for Parallelograms	
Prove theorems about parallelograms. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Use coordinates to prove simple geometric theorems algebraically.	Karima drew the following figure to prove that if the diagonals of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Draw a counterexample to show that Karima is incorrect. What mistake did Karima make?  **You cannot prove a general statement with an example.** The vertices of quadrilateral *ABCD* are A(−2, 3), B(1, 6), C(7, 3), and D(5, 1). Find the slope of each side and determine if *ABCD* is a parallelogram. Explain your reasoning. **Slopes: $\overline{AB}$ has slope $\frac{6-3}{1-(-2)} = 1$, $\overline{BC}$ has slope $\frac{3-6}{7-1} = -\frac{1}{2}$, $\overline{CD}$ has slope $\frac{3-1}{7-5} = 1$, and $\overline{AD}$ has slope $\frac{1-3}{5-(-2)} = -\frac{2}{7}$. ABCD is not a parallelogram because it does not have two pairs of parallel opposite sides.** How could point *C* be moved so that *ABCD* is a parallelogram? **Sample answer: Move point C to C(8, 4); then the slope of $\overline{BC}$ is $\frac{4-6}{8-1} = -\frac{2}{7}$ and the slope of $\overline{CD}$ is unchanged. Then ABCD has two pairs of parallel opposite sides, so it is a parallelogram.**

What You Will Learn	Preview Question
Lesson 11.4 Rectangles	
Prove theorems about parallelograms. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Use coordinates to prove simple geometric theorems algebraically.	*ABGH* is a rectangle and *CDEF* is a parallelogram. Can you conclude *BCFG* is a rectangle? Explain.  **No; you know that *BCFG* has two right angles, but you do not know whether *BCFG* is a parallelogram.**
Lesson 11.5 Rhombi and Squares	
Prove theorems about parallelograms. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Use coordinates to prove simple geometric theorems algebraically.	Classify the quadrilateral shown on the coordinate grid. Explain.  **Rhombus; the side lengths are 5, so they are congruent. It is not a square because the diagonals are not congruent. The diagonals have lengths 6 and 8.**
Lesson 11.6 Trapezoids and Kites	
Use coordinates to prove simple geometric theorems algebraically.	What are the coordinates of the endpoints of the midsegment of the trapezoid?  **(−[illegible], 1) and (5, 4)**

Teaching Tip SMP 7

The preview question for Lesson 11.3 addresses **SMP 7 (Look for and make use of structure)**. One approach for solving the problem is to draw the three quadrilaterals mentioned in the problem statement separately. Have students mark everything they know before they separate the rectangles. Use this problem to reinforce that you cannot assume a figure is a rectangle simply because it looks like a rectangle. The theorems and definitions of geometry must be used to prove it.

Teaching Tip SMP 6

The preview question for Lesson 11.4 can prompt a discussion of **SMP 6 (Attend to precision)**. Classifying the quadrilateral requires that students be precise in their language and their thinking. Determining that the sides have the same length is enough to determine that the figure is a rhombus, but not enough to determine whether or not the figure is a square. Students must also calculate accurately, as they determine the side lengths and the lengths of the diagonals.

11.2 Parallelograms

STANDARDS

Standards for Mathematical Practice: 1, 2, 3, 5, 6, 7, 8

PREREQUISITES

- Use relationships of angle pairs formed by two parallel lines crossed by a transversal
- Prove that triangles are congruent

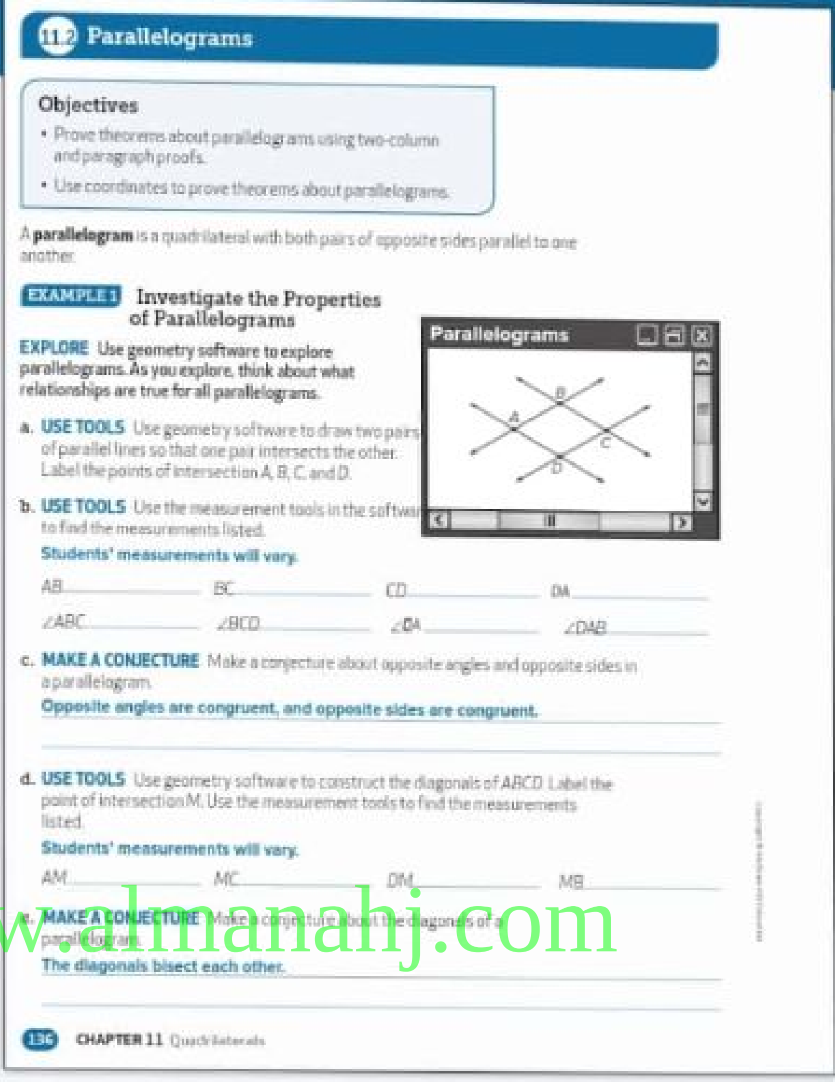
EXAMPLE 1

Teaching Tip SMP 7

Part f offers an opportunity to address **SMP 7 (Look for and make use of structure).** As students analyze the measurements they found, they should look for a pattern that shows which parts of a parallelogram are congruent.

Scaffolding Questions

- Are any of the sides ≅? If so, which sides? **Opposite sides are ≅.** Is this true for all pairs of opposite sides in your parallelograms? **Yes.** Do you think that opposite sides are ≅ in all parallelograms? Explain. **Students' responses will vary.**
- What do you notice about the diagonals of the parallelograms? **They bisect each other.** Does this mean the diagonals are ≅? Explain. **No, they can be different lengths and still bisect each other.**



11.2 Parallelograms

Objectives

- Prove theorems about parallelograms using two-column and paragraph proofs.
- Use coordinates to prove theorems about parallelograms.

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel to one another.

EXAMPLE 1 Investigate the Properties of Parallelograms

EXPLORE Use geometry software to explore parallelograms. As you explore, think about what relationships are true for all parallelograms.

a. **USE TOOLS** Use geometry software to draw two pairs of parallel lines so that one pair intersects the other. Label the points of intersection A, B, C and D.

b. **USE TOOLS** Use the measurement tools in the software to find the measurements listed.

Students' measurements will vary.

AB ____ BC ____ CD ____ DA ____

∠ABC ____ ∠BCD ____ ∠DA ____ ∠DAB ____

c. **MAKE A CONJECTURE** Make a conjecture about opposite angles and opposite sides in a parallelogram.

Opposite angles are congruent, and opposite sides are congruent.

d. **USE TOOLS** Use geometry software to construct the diagonals of ABCD. Label the point of intersection M. Use the measurement tools to find the measurements listed.

Students' measurements will vary.

AM ____ MC ____ DM ____ MB ____

e. **MAKE A CONJECTURE** Make a conjecture about the diagonals of a parallelogram.

The diagonals bisect each other.

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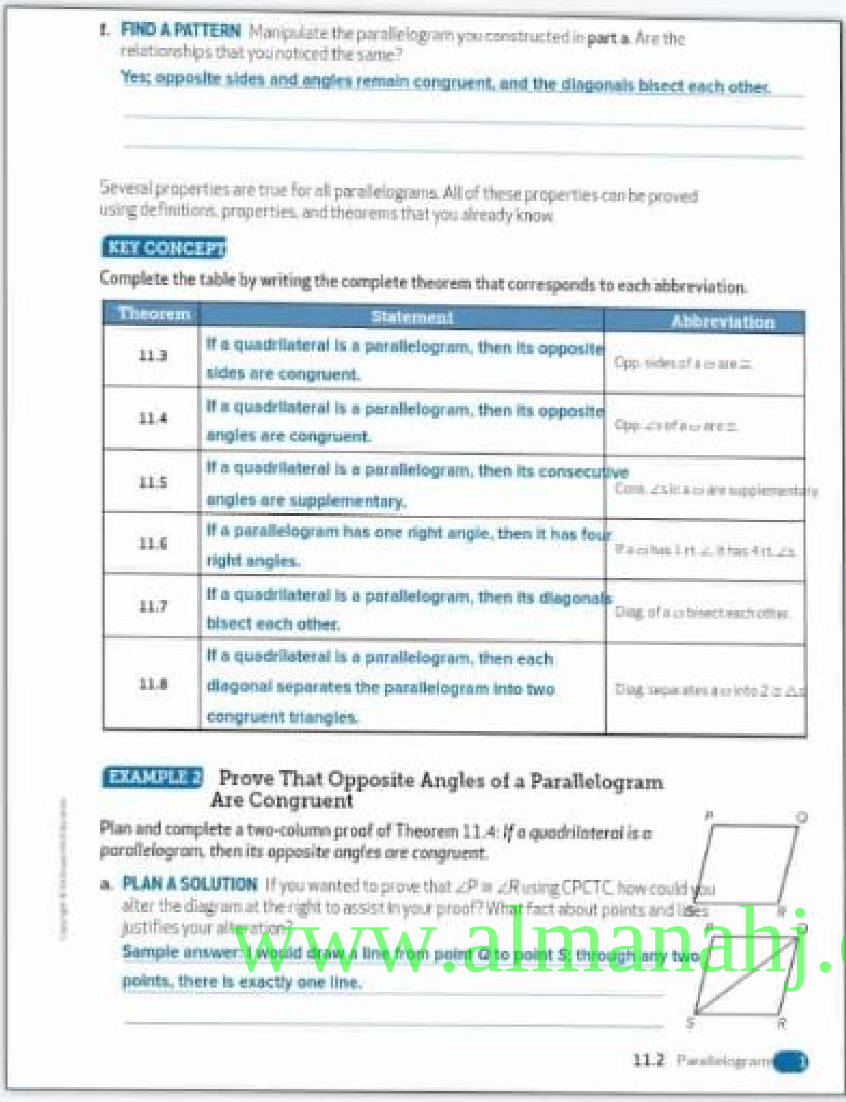
Math Background

A parallelogram is a type of quadrilateral that has both pairs of opposite sides parallel to one another. Parallelograms have the following properties.

- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Consecutive angles of a parallelogram are supplementary.
- Diagonals of a parallelogram bisect each other.

Each of these properties can be proved using the definition of a parallelogram and congruent triangles. These properties can by applied to any quadrilateral that is identified as a parallelogram.



f. **FIND A PATTERN** Manipulate the parallelogram you constructed in **part a**. Are the relationships that you noticed the same?

Yes; opposite sides and angles remain congruent, and the diagonals bisect each other.

Several properties are true for all parallelograms. All of these properties can be proved using definitions, properties, and theorems that you already know.

KEY CONCEPT

Complete the table by writing the complete theorem that corresponds to each abbreviation.

Theorem	Statement	Abbreviation
11.3	If a quadrilateral is a parallelogram, then its opposite sides are congruent.	Opp. sides of a ▱ are ≅.
11.4	If a quadrilateral is a parallelogram, then its opposite angles are congruent.	Opp. ∠s of a ▱ are ≅.
11.5	If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.	Cons. ∠s in a ▱ are supplementary.
11.6	If a parallelogram has one right angle, then it has four right angles.	If a ▱ has 1 rt. ∠, it has 4 rt. ∠s.
11.7	If a quadrilateral is a parallelogram, then its diagonals bisect each other.	Diag. of a ▱ bisect each other.
11.8	If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.	Diag. separates a ▱ into 2 ≅ △s.

EXAMPLE 2 Prove That Opposite Angles of a Parallelogram Are Congruent

Plan and complete a two-column proof of Theorem 11.4: *If a quadrilateral is a parallelogram, then its opposite angles are congruent.*

a. **PLAN A SOLUTION** If you wanted to prove that ∠P ≅ ∠R using CPCTC, how could you alter the diagram at the right to assist in your proof? What fact about points and lines justifies your alteration?

Sample answer: I would draw a line from point Q to point S; through any two points, there is exactly one line.

11.2 Parallelograms

EXAMPLE 2

Teaching Tip SMP 1

In **part a**, students must determine how to alter the given diagram to prove Theorem 11.4 in a specific way. Students can plan their solution by working backwards from the result they want (prove Theorem 11.4 using CPCTC), which uses **SMP 1 (Make sense of problems and persevere in solving them).**

Scaffolding Questions

- What are you trying to prove? **Opposite angles of a parallelogram are congruent.**
- What are the opposite angles in the diagram? **∠P ≅ ∠R and ∠Q ≅ ∠S**
- Why are triangles useful when trying to prove parts are congruent? **There are many ways to prove that triangles are ≅. You can divide parallelograms into triangles, and once you prove two triangles are ≅, you can use corresponding parts of congruent triangles to prove elements in the parallelograms congruent.**

EXAMPLE 3

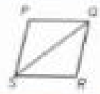
Teaching Tip SMP 3

Review the difference between a paragraph proof and a two-column proof. Emphasize that in both types of proofs, students must attend to **SMP 3 (Construct viable arguments and critique the reasoning of others).**

Scaffolding Questions

- Why are theorems about transversals helpful in proving theorems about parallelograms? **Since the opposite sides of a parallelogram are parallel, the adjacent sides form transversals. Therefore, we can use theorems about traversals to make statements about parallelograms.**
- To prove that $\angle J$ and $\angle K$ are supplementary, which segment is the transversal and which are the parallel lines? ***JM* and *KL* are the parallel segments. *JK* is the transversal.**

b. CONSTRUCT ARGUMENTS Fill in the missing statements and reasons to complete the proof.



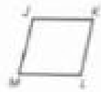
Given: Parallelogram *PQRS*
Prove: $\angle P \cong \angle R$

Statements	Reasons
1. *PQRS* is a parallelogram	1. Given
2. $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{SP}$	2. Definition of parallelogram
3. $\angle PSQ \cong \angle RQS$ and $\angle PQS \cong \angle RSQ$	3. Alt. Int. ∠s Thm.
4. $\overline{SQ} \cong \overline{SQ}$	4. Reflexive Property of Congruence
5. $\triangle PQS \cong \triangle RSQ$	5. ASA
6. $\angle P \cong \angle R$	6. CPCTC

c. DESCRIBE A METHOD How could you continue the proof to prove $\angle Q \cong \angle S$?
Sample answer: I could draw diagonal $\overline{PR}$ and then prove that $\triangle SPR \cong \triangle QRP$. Then I could use CPCTC to prove that $\angle Q \cong \angle S$. Corresponding parts of ≅ △s are ≅.

EXAMPLE 3 **Prove that Consecutive Angles of a Parallelogram are Supplementary**

Plan and write a paragraph proof of Theorem 11.5: *If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.*



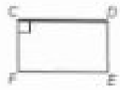
a. CONSTRUCT ARGUMENTS Complete the paragraph proof.

Given: *JKLM* is a parallelogram.
Prove: $\angle J$ and $\angle K$, $\angle K$ and $\angle L$, $\angle L$ and $\angle M$, and $\angle M$ and $\angle J$ are supplementary.

It is given that *JKLM* is a parallelogram, so $\overline{JK} \parallel \overline{ML}$ and $\overline{JM} \parallel \overline{KL}$. When two parallel lines are cut by a transversal, consecutive interior angles are supplementary. Therefore $\angle J$ and $\angle K$, $\angle K$ and $\angle L$, $\angle L$ and $\angle M$, and $\angle M$ and $\angle J$ are supplementary.

EXAMPLE 4 **Prove Right Angles in Parallelograms**

Write a paragraph proof of Theorem 11.6: *If a parallelogram has one right angle, then it has four right angles.*



a. CONSTRUCT ARGUMENTS Write a paragraph proof.

Given: Parallelogram *CDEF*; $\angle C$ is a right angle.
Prove: $\angle D$, $\angle E$, and $\angle F$ are right angles

Sample answer: It is given that *CDEF* is a ▱ and $\angle C$ is a right ∠. Thm. 11.4 says that opp. ∠s of a ▱ are ≅, so $\angle C \cong \angle E$; therefore $\angle E$ must also be a rt. ∠. Thm. 11.5 says that consecutive ∠s of a ▱ are supplementary, so $\angle C$ must be supplementary to both $\angle F$ and $\angle D$. Then $m\angle C + m\angle D = m\angle C + m\angle F = 180$ by the definition of supplementary. So, $m\angle D = m\angle F = 180 - 90 = 90$. Therefore, $\angle F$ and $\angle D$ are also rt. ∠s.

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Differentiating Instruction

Some students have difficulty remembering all the information needed to prove the properties of parallelograms. Before students begin the proofs, have them create a graphic organizer that summarizes information pertinent to the proofs.

∠s and Parallel Lines	Proving △s Congruent	Properties of Parallelograms

Have students brainstorm what information they should include in the first two columns and record it in a way that is useful to them. Have them fill out the third column as they work through the lesson. Encourage them to use the graphic organizer throughout the lesson.

EXAMPLE 5 **Prove That Diagonals of a Parallelogram Bisect Each Other**

Use algebra to prove Theorem 11.7: *If a quadrilateral is a parallelogram, then its diagonals bisect each other.*

a. **REASON ABSTRACTLY** You know that opposite sides of a parallelogram are parallel and that parallel lines have equal slopes. How can this information help you find the coordinates of point C in parallelogram ABCD?

Sample answer: Point B is y units above and x units to the right of point A. Since $\overline{AB}$ and $\overline{DC}$ have the same slope and $\overline{AD}$ and $\overline{BC}$ have the same slope, point C is also y units above and x units to the right of point D. The coordinates of point C are $(z + x, y)$.

b. **CALCULATE ACCURATELY** What are the midpoints of $\overline{AC}$ and $\overline{BD}$?

The midpoint of $\overline{AC}$ is $\left(\frac{x+z}{2}, \frac{y}{2}\right)$. The midpoint of $\overline{BD}$ is $\left(\frac{x+z}{2}, \frac{y}{2}\right)$.

c. **COMMUNICATE PRECISELY** $\overline{AC}$ and $\overline{BD}$ have the same midpoint, how does that show that the diagonals bisect each other?

Sample answer: By the definition of bisects, any segment, line, or plane that intersects a segment at its midpoint bisects the segment. Since $\overline{AC}$ intersects $\overline{BD}$ at the midpoint of $\overline{BD}$, $\overline{AC}$ bisects $\overline{BD}$. Similarly, since $\overline{BD}$ intersects $\overline{AC}$ at the midpoint of $\overline{AC}$, $\overline{BD}$ bisects $\overline{AC}$. So, the diagonals of a parallelogram bisect each other.

PRACTICE

1. **CONSTRUCT ARGUMENTS** Prove Theorem 11.3: *If a quadrilateral is a parallelogram, then its opposite sides are congruent.*

a. Fill in the missing statements and reasons.

Given: Parallelogram EFGH
Prove: $\overline{EF} \cong \overline{GH}$ and $\overline{HE} \cong \overline{FG}$

Statements	Reasons
1. **EFGH is a parallelogram.**	1. Given
2. $\overline{EF} \parallel \overline{GH}$ and $\overline{HE} \parallel \overline{FG}$	2. **Definition of parallelogram**
3. **$\angle EFH \cong \angle GHF$ and $\angle EHF \cong \angle GFH$**	3. Alt. Int. ∠s Thm.
4. $\overline{FH} \cong \overline{FH}$	4. **Refl. Prop. of ≅**
5. **$\triangle EFH \cong \triangle GHF$**	5. ASA
6. $\overline{EF} \cong \overline{GH}$ and $\overline{HE} \cong \overline{FG}$	6. **CPCTC**

b. Explain why this proof is true for all parallelograms.

Sample answer: All parallelograms have 2 pairs of parallel lines. When a diagonal is drawn, you can use the || lines to prove that alt. int. ∠s are ≅ and that the △s formed by the diagonal are ≅ △s.

11.2 Parallelograms

EXAMPLE 4

Teaching Tip SMP 3

Students should know that they can use past reasoning as they apply **SMP 3 (Construct viable arguments and critique the reasoning of others)** to prove new relationships. Discuss that once a theorem has been proved, it can be used as a reason in another proof without proving it all over again.

Scaffolding Questions

- What have you already proved about parallelograms in this lesson? **Thm. 11.4: Opposite ∠s of a parallelogram are ≅. Thm. 11.5: Consecutive ∠s in a parallelogram are supplementary.**
- How can you apply these theorems to the diagram for the proof? **∠C ≅ ∠E and ∠D ≅ ∠F; the following pairs of angles are supplementary: ∠C and ∠D, ∠D and ∠E, ∠E and ∠F, ∠F and ∠C.**

Emphasizing the Standards for Mathematical Practice

Example 4 requires students to develop their own plan for a proof that begins with the given and ends with what is supposed to be proved. They receive no help with the steps involved.

To help students think through the problem, have them tell you everything they already know about parallelograms and right angles. Then discuss possible methods they could use to prove that angles are right angles. Lead a discussion where students connect what they already know with what they are trying to prove. Once they have an overview of the thinking, have them complete the example.

EXAMPLE 5

Teaching Tip SMP 2

There are many ways to address **SMP 2 (Reason abstractly and quantitatively).** One is to use algebra to show relationships. As students work through the algebraic proof, they must connect the information they get using algebra with the geometric figure and what they are trying to prove about it.

Scaffolding Questions

- How are a midpoint and a bisector related? **Any bisector passes through the midpoint.**
- How does knowing the midpoints of the diagonals help you show they bisect each other? **If a diagonal passes through the other diagonal's midpoint, it bisects it.**

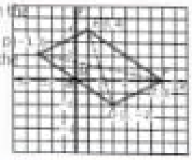
2. **CONSTRUCT ARGUMENTS** Write a paragraph proof of Theorem 11.8. *If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.*

Given: Parallelogram $KLMN$

Prove: $\triangle KLM \cong \triangle MNK$ and $\triangle NKL \cong \triangle LMN$

Sample answer: Diagonal [$\overline{KM}$] divides $KLMN$ into 2 $\triangle$s. It is given that $KLMN$ is a parallelogram so $\overline{KL} \parallel \overline{MN}$ and $\overline{KN} \parallel \overline{LM}$. Because they are alt. int. $\angle$s, $\angle NKM \cong \angle LMK$ and $\angle NMK \cong \angle LKM$. By the Refl. Prop. of $\cong$, $\overline{KM} \cong \overline{KM}$. Using the ASA Theorem, $\triangle NKM \cong \triangle LMK$. Using the diag[onal] $\overline{NL}$, the same reasoning can be used to prove $\triangle NKL \cong \triangle LMN$.

3. Yasmin sketched a parallelogram on a coordinate plane as shown in the diagram.



a. **USE STRUCTURE** Show how she can use algebra to show that the opposite sides of the parallelogram are congruent.

Sample answer: She can use the distance formula.

$DE = \sqrt{(-3-1)^2 + (2-4)^2} = \sqrt{20} = 2\sqrt{5}$;

$FG = \sqrt{(7-3)^2 + (0-(-2))^2} = \sqrt{20} = 2\sqrt{5}$;

$EF = \sqrt{(1-7)^2 + (4-0)^2} = \sqrt{52} = 2\sqrt{13}$; $GD = \sqrt{(3-(-3))^2 + (-2-2)^2} = \sqrt{52} = 2\sqrt{13}$

b. **USE STRUCTURE** Show how she can use algebra to show that the diagonals bisect each other?

Sample answer: She can use the midpoint formula.

midpoint of $\overline{DF}$: $\left(\frac{-3+7}{2}, \frac{2+0}{2}\right) = (2, 1)$ and midpoint of $\overline{EG}$: $\left(\frac{1+3}{2}, \frac{4-2}{2}\right) = (2, 1)$

The diagonals intersect at each other's midpoints, so they bisect each other.

c. **CRITIQUE REASONING** Raghad suggests to Yasmin that she has found alternative proofs to Theorems 11.3 and 11.7 using algebra. Is he correct? Why or why not?

Sample answer: She is incorrect. Part a does not prove Theorem 11.3, and part b does not prove Theorem 11.7. These demonstrations only verify that the theorem holds for this particular parallelogram. To create a proper proof, Yasmin would have to use a general parallelogram.

d. **PLAN A SOLUTION** How could Yasmin alter her $DEFG$ so that parts a and b are valid proofs of Theorems 11.3 and 11.7?

Sample answer: She could construct $DEFG$ so [$\overline{DE} \parallel \overline{FG}$] and $\overline{DG} \parallel \overline{EF}$, and so that the coordinates are in terms of variables instead of any number. This would make $DEFG$ a general parallelogram. Yasmin could then use the Distance Formula as in part a and the midpoint formula in part b to show that Theorems 11.3 and 11.7 are true for the general parallelogram.

Common Errors

Students may attempt to use imprecise methods for proving theorems. They may give reasons that involve visual impressions from the diagrams or measurements made with a ruler or protractor. Emphasize that all the reasons given must be mathematically sound and must be true for all parallelograms and not just the one represented by a diagram. Note that their proofs must use definitions, properties, postulates, theorems, and formulas as their reasons.

4. Below is a two-column proof of Theorem 11.7: *If a quadrilateral is a parallelogram, then its diagonals bisect each other.*

Given: Parallelogram $WXZY$

Prove: $\overline{WM} \cong \overline{MZ}$ and $\overline{XM} \cong \overline{MY}$

Statements	Reasons
1. $WXZY$ is a parallelogram	1. Given
2. $\overline{WX} \parallel \overline{ZY}$ and $\overline{WY} \parallel \overline{XZ}$	2. Definition of parallelogram
3. $\angle XWZ \cong \angle YZW$ and $\angle XYZ \cong \angle WXM$	3. Alternate Interior Angles Theorem
4. $\angle WMX \cong \angle YMZ$	4. Vertical angles are congruent
5. $\triangle WXM \cong \triangle ZYM$	5. AAA
6. $\overline{WM} \cong \overline{MZ}$ and $\overline{XM} \cong \overline{MY}$	6. CPCTC

a. **CRITIQUE REASONING** What is the error in the proof?
AAA is not a valid test for triangle congruence.

b. **CONSTRUCT ARGUMENTS** How would you correct the error?
Sample answer: I would show that opposite sides of the parallelogram are ≅ and then use ASA.

c. Rewrite the proof with your edits.

Statements	Reasons
1. **$WXYZ$ is a parallelogram.**	1. Given
2. **$\overline{WX} \parallel \overline{YZ}$ and $\overline{WY} \parallel \overline{XZ}$**	2. Definition of parallelogram
3. **$\angle XWZ \cong \angle YZW$ and $\angle XYZ \cong \angle W$[illegible]**	3. Alternate Interior Angles Theorem
4. **$\overline{WX} \cong \overline{YZ}$**	4. Theorem 11.3
5. **$\triangle WXM \cong \triangle ZYM$**	5. **ASA**
6. **$\overline{WM} \cong \overline{MZ}$ and $\overline{XM} \cong \overline{MY}$**	6. CPCTC

5. **FIND A PATTERN** Khalifa Street and Aloroubah Street are parallel. Jadat Alzouhor and Jadat Alkramah Avenue are parallel. Maher works at a pizza restaurant on the corner of Khalifa and Jadat Al Zouhor. He needs to deliver a pizza to a house on the corner of Aloroubah and Jadat Alkaramah. Maher is deciding whether to travel on Khalifa and Jadat Alkramah Avenue or on Aloroubah and Jadat Alzouhor. If he wants to travel the shortest distance, which route should he choose? Explain your reasoning.

Sample answer: Both routes are the same distance. Because Alkhalifah St is parallel to Aloroubah St and Jadat Alzouhor Ave is parallel to Jadat Alkaramah Ave, the figure formed by the four roads is a parallelogram by the definition of a parallelogram. By Theorem 11.3, opposite sides of a parallelogram are congruent. Therefore, the section of Khalifa St has the same length as the section of Oroubah St and the section of Jadat Alkaramah Ave has the same length as the section of Jadat Alzouhor Ave. Therefore, both routes are the same distance.

11.2 Parallelograms

PRACTICE

In **Exercises 1** and **2**, students must prove theorems about parallelograms.

Exercise 3 allows students to verify relationships in a parallelogram, using the coordinates of its vertices.

Exercise 4 requires students to analyze and reconstruct a proof about a parallelogram.

In **Exercise 5**, students practice by proving a statement about a parallelogram in a real-world situation.

Addressing the Standards

Exercise	SMP
1–2	3
3	1, 3, 7
4	3
5	7

Differentiating Instruction

Visual clues often help students think about a problem in a more concrete manner. Give each student a red and a blue pencil. Review the marks used to show parallel lines, congruent lines, and congruent angles. Have students mark the given using a red pencil and what they are trying to prove with a blue pencil. Each time they complete a step in the proof, have them mark the information on the diagram in red. Encourage students to use the color-coded diagram as they discuss and analyze what they know and what they are trying to prove.

11.3 Tests for Parallelograms

STANDARDS

Standards for Mathematical Practice: 1, 2, 3, 5, 7

PREREQUISITES

- Recognize and apply properties of parallelograms

MATERIALS

- The Geometer's Sketchpad

EXAMPLE 1

Teaching Tip SMP 5

Help students make conjectures about conditions of parallelograms by challenging them to use the dynamic geometry software to modify quadrilateral *EFGH* so that its opposite sides are *not* parallel. Discuss why this cannot be done.

Scaffolding Questions

- When changing the shape of *EFGH*, how can you explore relationships between the lengths of its sides? **Select the segment for each side. Use the Measure command to display the segment's length. The lengths will be updated automatically as *EFGH* changes.**
- How are the lengths of $\overline{EF}$ and $\overline{FG}$ related? **The length of $\overline{EF}$ does not affect the length of $\overline{FG}$, and vice versa. The lengths of adjacent sides of parallelograms are not related.**

11.3 Tests for Parallelograms

Objectives

- Prove theorems about parallelograms by making formal geometric constructions.
- Use coordinates to prove theorems about parallelograms.

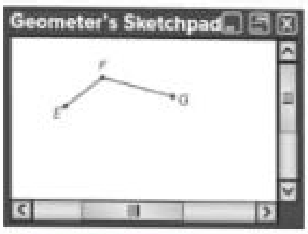
By the definition of a parallelogram, if both pairs of opposite sides of a quadrilateral are parallel, then it is a parallelogram. So, to prove that a quadrilateral is a parallelogram, show that both pairs of opposite sides are parallel.

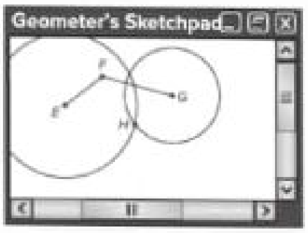
EXAMPLE 1 Investigate Conditions of Parallelograms

EXPLORE **Use dynamic geometry software to explore parallelograms. As you do so, think about different ways to use opposite sides to prove that a quadrilateral is a parallelogram.**

a. **USE TOOLS** Use The Geometer's Sketchpad to draw two segments that share an endpoint. Label these $\overline{EF}$ and $\overline{FG}$, as shown below on the left.

b. **USE TOOLS** Draw a circle using the "Circle by Center + Radius" tool with center E and radius FG. Draw a second circle in the same way with center G and radius EF, as shown below on the right. Label the point of intersection of the circles H. Construct $\overline{GH}$ and $\overline{HE}$. Then select and hide the circles.





c. **CONSTRUCT ARGUMENTS** Explain why $\overline{EF}$ and $\overline{GH}$ are congruent and why $\overline{FG}$ and $\overline{HE}$ are congruent.

Sample answer: Since the arc of the circle with center G was length EF, $\overline{EF} \cong \overline{GH}$; similarly, since the arc of the circle with center E was length FG, $\overline{FG} \cong \overline{HE}$.

d. **USE TOOLS** Use the slope tool to find the slopes of $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, and $\overline{HE}$. What can you conclude about the opposite sides of EFGH?

Sample answer: The slopes of opposite sides are the same, so the opposite sides are parallel.

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Math Background

When students use dynamic geometry software to construct the quadrilateral, they may assume that the figure will be a parallelogram and take shortcuts by simply drawing segments that appear to be parallel. Remind them that they must start with opposite sides congruent before making any other assumptions.

By using the measurement tools available with dynamic geometry software, students can explore what they have previously learned about properties of sides, angles, and diagonals of parallelograms. As they explore, encourage them to make conjectures about the conditions that ensure a quadrilateral is a parallelogram and to think about ways to prove these conjectures.

e. **MAKE A CONJECTURE** What is a reasonable conjecture about parallelograms based on your exploration of quadrilateral *EFGH*?
Sample answer: If both pairs of opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram

Showing that opposite sides are parallel is just one way to prove that a quadrilateral is a parallelogram. There are other conditions that ensure a quadrilateral is a parallelogram as well. Remember that only one condition needs to be satisfied to complete a proof.

KEY CONCEPT

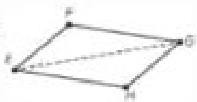
Complete the table by writing the complete theorem that corresponds to each abbreviation.

Theorem	Statement	Abbreviation
11.9	**If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.**	If both pairs of opp. sides are ≅, then quad. is a ▱.
11.10	**If both pairs of opposite angles of a quadrilateral are congruent, then it is a parallelogram.**	If both pairs of opp. ∠s are ≅, then quad. is a ▱.
11.11	**If the diagonals of a quadrilateral bisect each other, then it is a parallelogram**	If diag. bisect each other, then quad. is a ▱.
11.12	**If one pair of opposite sides of a quadrilateral is both congruent and parallel, then it is a parallelogram**	If one pair of opp. sides is ≅ and ∥, then quad. is a ▱.

EXAMPLE 2 Prove That a Quadrilateral Is a Parallelogram

Complete the two-column proof to show that if both pairs of opposite sides are congruent, then a quadrilateral is a parallelogram.

a. **CONSTRUCT ARGUMENTS** Fill in the missing statements and reasons to complete the proof.



Given: $\overline{EF} \cong \overline{GH}$, $\overline{FG} \cong \overline{EH}$
Prove: *EFGH* is a parallelogram.

Statements	Reasons
1. Draw $\overline{EG}$.	1. Through any two points there is exactly one line.
2. $\overline{EF} \cong \overline{GH}$, $\overline{FG} \cong \overline{EH}$	2. Given
3. **$\overline{EG} \cong \overline{GE}$**	3. Reflexive Property of Congruence
4. $\triangle EFG \cong \triangle GHE$	4. **SSS**
5. **$\angle FGE \cong \angle HEG$, $\angle FEG \cong \angle HGE$**	5. CPCTC
6. $\overline{EF} \parallel \overline{GH}$, $\overline{FG} \parallel \overline{EH}$	6. **Alternate Interior Angles Converse**
7. *EFGH* is a parallelogram.	7. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

b. **CRITIQUE REASONING** A student said that because $\angle F \cong \angle H$, it can also be shown that $\triangle EFG \cong \triangle GHE$ by SAS. Do you agree? Justify your answer.
Sample answer: No; it is not given that $\angle F \cong \angle H$ nor has it been proven.

11.3 Tests for Parallelograms

EXAMPLE 2

Teaching Tip SMP 3

If students have trouble understanding the first step of the proof, review the use of an auxiliary line.

Scaffolding Questions

- Why is an auxiliary line drawn in Statement 1? **Drawing the extra line helps you to analyze the geometric relationships between the two triangles formed by drawing a diagonal of the parallelogram.**
- Could an auxiliary line be drawn through points *F* and *H* instead? If so, how would the proof be affected? **Yes; the congruent triangles would be $\triangle EFH$ and $\triangle GHF$; all segments and angles in the proof would have to be revised accordingly.**
- Why is the Alternate Interior Angles Converse given as the reason for Statement 6 instead of the Alternate Interior Angles Theorem? **The theorem states that if lines are parallel, then alternate interior angles are congruent, while the converse states that if the alternate interior angles are congruent, then the lines are parallel. The latter is being shown in Statement 6.**

Emphasizing the Standards for Mathematical Practice

Use what students have learned about writing proofs to address **SMP 3 (Construct viable arguments and critique the reasoning of others)**. In **part c** of **Example 2**, students may incorrectly assume that opposite angles of quadrilateral *EFGH* are congruent. Remind students that proving that a given quadrilateral is a parallelogram is different from proving that a parallelogram has certain properties. If it has not been proven that a given quadrilateral is a parallelogram, then properties of parallelograms cannot be assumed but must be proven as well.

EXAMPLE 3

Teaching Tip **SMP 1**

If students need help planning the proof, suggest they start with the definition of a parallelogram. With this example, students will begin planning the proof of Theorem 11.11 that will be completed in **Exercise 5.**

Scaffolding Questions

- What information is given in the problem that could help you to begin to prove that opposite sides of quadrilateral *ABCD* are parallel? **Point *E* is the midpoint of each of the diagonals of quadrilateral *ABCD*.**
- How does knowing that point *E* is the midpoint of the diagonals of *ABCD* help you to plan your proof? **Since a midpoint divides a segment into two congruent segments, you will be able to identify congruent segments and use these relationships in the proof.**
- When you plan your proof, what do the intersection of the diagonals tell you about angles? **The diagonal forms two pairs of vertical angles, and vertical angles are congruent.**

c. **CONSTRUCT ARGUMENTS** Explain why Statement 3 is necessary.
Sample answer: The next step in the proof is to state that $\triangle EFG \cong \triangle GHE$ by SSS. Therefore, it must be explicitly stated that the sides in each corresponding pair, including the shared side represented as $\overline{EG}$ and $\overline{GE}$, are congruent.

d. **CONSTRUCT ARGUMENTS** Describe a general strategy for proving that opposite sides are parallel once it has been shown that the triangles formed by drawing the diagonal are congruent.
Sample answer: Use CPCTC to identify congruent corresponding angles that are also congruent angle pairs when parallel lines are intersected by a transversal. Then, use the converse for that type of angle pair to prove the opposite sides are parallel.

You can solve real-world problems by proving that quadrilaterals are parallelograms.

EXAMPLE 3 **Solve a Real-World Problem**

Jamila is assembling an accordion drying rack that can be folded flat or opened up to various heights as shown. In the figure, *E* is the midpoint of $\overline{AC}$ and $\overline{BD}$. Jamila wants to show that figure *ABCD* is a parallelogram.

a. **PLAN A SOLUTION** What must Jamila show in order to prove that *ABCD* is a parallelogram? Explain.
$\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$; If a quadrilateral has each pair of opposite sides parallel, it is a parallelogram.

b. **USE STRUCTURE** Which segments in the figure have point *E* as an endpoint? How are these segments related? Explain.
$\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$; $\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$; a midpoint divides a line segment into two equal segments.

c. **USE STRUCTURE** Which angles in the figure have point *E* as a vertex? How are these angles related? Explain.
$\angle AEB$, $\angle BEC$, $\angle CED$, and $\angle DEA$; $\angle AEB \cong \angle CED$, $\angle BEC \cong \angle DEA$; vertical angles are congruent.

You can also prove that a quadrilateral in the coordinate plane is a parallelogram.

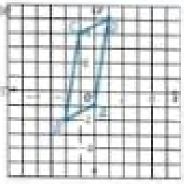
EXAMPLE 4 **Use Coordinates to Prove a Parallelogram**

Coordinates for three of the four vertices of parallelogram *ABCD* are given in the table. Note that the coordinates of point *A* are missing.

Point	A	B	C	D
x	?	−2	−1	1
y	?	−2	4	5

a. **CONSTRUCT ARGUMENTS** What strategy could you use to identify the coordinates of point *A*? Explain.
Sample answer: Graph *B*, *C*, and *D*. Sketch a ray from *D* parallel to $\overline{BC}$ and a ray from *B* parallel to $\overline{CD}$. Label the point of intersection of the rays as *A*. Use the slope formula to confirm that opposite sides are parallel.

b. **USE STRUCTURE** Draw parallelogram *ABCD* in the coordinate plane at the right. What are the coordinates of point *A*?



The coordinates of *A* are (0, −1).

c. **REASON QUANTITATIVELY** Find the slope of each side. The first has been done for you. What does this tell you about the quadrilateral?

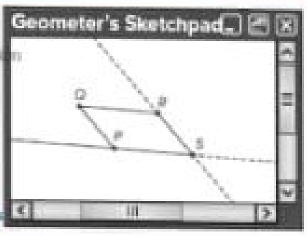
slope of $\overline{AB} = \frac{-2-(-1)}{-2-0}$ or $\frac{1}{2}$ slope of $\overline{DC} = \frac{4-5}{-1-1}$ or $\frac{1}{2}$

slope of $\overline{AD} = \frac{5-(1)}{1-0}$ or 6 slope of $\overline{CB} = \frac{-2-4}{-2-(-1)}$ or 6

Sample answer: Lines that have the same slope are parallel. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{CB}$. Since opposite sides are parallel, *ABCD* is a parallelogram by definition.

PRACTICE

1. **USE TOOLS** Use dynamic geometry software to construct parallelogram *PQRS* as shown. Remember to select and hide the parallel lines when the construction is complete.



a. **USE TOOLS** Use the measurement tools in the software to measure ∠P, ∠Q, ∠R, and ∠S. What do you notice? Change the shape or location of quadrilateral *PQRS*. Does this relationship remain the same?

∠P ≅ ∠R, ∠Q ≅ ∠S; these relationships always remain the same.

b. **MAKE A CONJECTURE** What can you conclude about quadrilateral *PQRS*?

Sample answer: Both pairs of opposite angles of *PQRS* are congruent, so *PQRS* is a parallelogram.

2. **CRITIQUE REASONING** A student wrote the paragraph proof below to prove that *PQRS* is a parallelogram. The proof contains a critical error. Find that error and correct it. Explain.

a. **Given:** ∠P ≅ ∠R, ∠Q ≅ ∠S
Prove: *PQRS* is a parallelogram.

Draw $\overline{PR}$ to form two triangles. Because the sum of the angles of one triangle is 180, the sum is 360 for two triangles. So, m∠P + m∠Q + m∠R + m∠S = 360. Since ∠P ≅ ∠R and ∠Q ≅ ∠S, m∠P = m∠R and m∠Q = m∠S. By substitution, m∠P + m∠P + m∠Q + m∠Q = 360. 2(m∠P) + 2(m∠Q) = 360, and dividing by 2 gives m∠P + m∠Q = 180. Likewise, 2(m∠P) + 2(m∠S) = 360, and dividing by 2 gives m∠P + m∠S = 180. Consecutive angles are congruent, so $\overline{PS} \parallel \overline{QR}$ and $\overline{PQ} \parallel \overline{SR}$. Opposite sides are parallel, so *PQRS* is a parallelogram.

Sample answer: The proof should say that consecutive angles are supplementary, not congruent. This proof relies on the condition that if consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.

Differentiating Instruction

Visual and kinesthetic learners may benefit from drawing "rise-over-run" arrows on the coordinate plane to help them visually confirm their calculations using the Slope Formula. For example, they could draw an arrow 1 unit up and 2 units to the right from point *C* to point *D* to confirm that the slope $\overline{DC}$ is $\frac{1}{2}$.

EXAMPLE 4

Teaching Tip SMP 2

Point out to students that although Example 4 requires using coordinates to prove that *ABCD* is a parallelogram, the basic strategy is still the same: Show that each pair of opposite sides is parallel. In the coordinate plane, parallel lines have the same slope. So, students can perform calculations using the Slope Formula to show that the slopes of the opposite sides are equal.

Scaffolding Questions

- Suppose a student incorrectly identified the coordinates of point A. How would it be determined that *ABCD* is not a parallelogram? **When the slope of each side is found, at least one pair of opposite sides would have different slopes.**
- Could the Midpoint Formula be used to prove that *ABCD* is a parallelogram? Explain. **Yes; if the diagonals of *ABCD* bisect each other, then *ABCD* is a parallelogram. So, use the Midpoint Formula to find the midpoint of each diagonal. If the diagonals have the same midpoint, then *ABCD* is a parallelogram.**

PRACTICE

Exercise 1 requires students to use dynamic geometry software to construct and make conjectures about parallelograms.

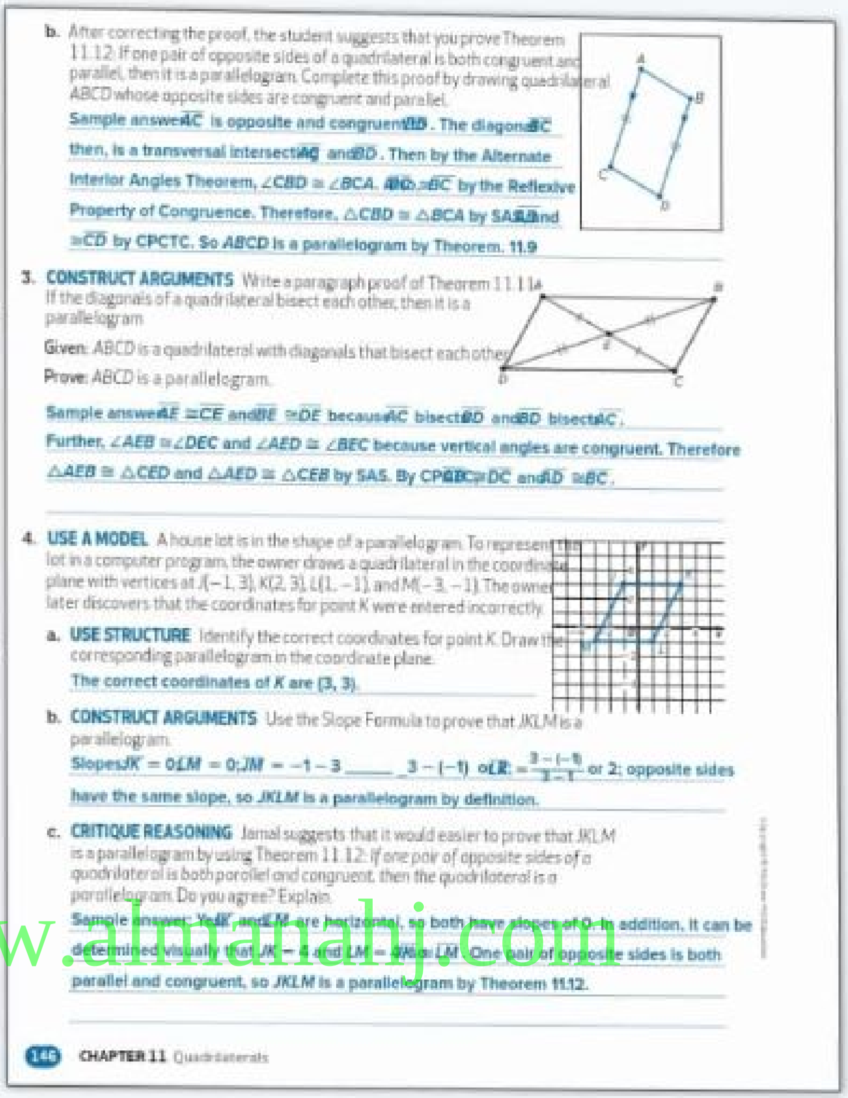
In **Exercise 2**, students critique an attempt at a proof and then construct an argument and create a paragraph proof about a theorem.

In **Exercise 3** students need to construct an argument and create a paragraph proof about a parallelogram.

In **Exercises 4** and **7**, students prove a theorem algebraically, by using coordinates to prove that a quadrilateral is a parallelogram.

In **Exercise 5**, students compete the proof that was planned in **Example 3.** While proving a theorem about parallelograms, students must make use of structure.

Exercise 6 asks students to solve a real-world problem by using coordinates to prove that a quadrilateral is a parallelogram.



b. After correcting the proof, the student suggests that you prove Theorem 11.12: If one pair of opposite sides of a quadrilateral is both congruent and parallel, then it is a parallelogram. Complete this proof by drawing quadrilateral ABCD whose opposite sides are congruent and parallel.

Sample answer: $\overline{AC}$ is opposite and congruent to $\overline{BD}$. The diagonal $\overline{BC}$ then, is a transversal intersecting $\overline{AC}$ and $\overline{BD}$. Then by the Alternate Interior Angles Theorem, $\angle CBD \cong \angle BCA$. $\overline{BC} \cong \overline{BC}$ by the Reflexive Property of Congruence. Therefore, $\triangle CBD \cong \triangle BCA$ by SAS, and $\overline{AB} \cong \overline{CD}$ by CPCTC. So $ABCD$ is a parallelogram by Theorem 11.9

3. **CONSTRUCT ARGUMENTS** Write a paragraph proof of Theorem 11.11: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given: ABCD is a quadrilateral with diagonals that bisect each other.

Prove: ABCD is a parallelogram.

Sample answer: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$ because $\overline{AC}$ bisects $\overline{BD}$ and $\overline{BD}$ bisects $\overline{AC}$. Further, $\angle AEB \cong \angle DEC$ and $\angle AED \cong \angle BEC$ because vertical angles are congruent. Therefore $\triangle AEB \cong \triangle CED$ and $\triangle AED \cong \triangle CEB$ by SAS. By CPCTC, $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$.

4. **USE A MODEL** A house lot is in the shape of a parallelogram. To represent the lot in a computer program, the owner draws a quadrilateral in the coordinate plane with vertices at J(−1, 3), K(2, 3), L(1, −1), and M(−3, −1). The owner later discovers that the coordinates for point K were entered incorrectly.

a. **USE STRUCTURE** Identify the correct coordinates for point K. Draw the corresponding parallelogram in the coordinate plane.

The correct coordinates of K are (3, 3).

b. **CONSTRUCT ARGUMENTS** Use the Slope Formula to prove that JKLM is a parallelogram.

Slopes: $JK = 0$; $LM = 0$; $JM = \frac{-1-3}{-3-(-1)}$ or 2; $LK = \frac{3-(-1)}{3-1}$ or 2; opposite sides have the same slope, so $JKLM$ is a parallelogram by definition.

c. **CRITIQUE REASONING** Jamal suggests that it would easier to prove that JKLM is a parallelogram by using Theorem 11.12: *If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.* Do you agree? Explain.

Sample answer: Yes; $\overline{JK}$ and $\overline{LM}$ are horizontal, so both have slopes of 0. In addition, it can be determined visually that $JK = 4$ and $LM = 4$, so $\overline{JK} \cong \overline{LM}$. One pair of opposite sides is both parallel and congruent, so $JKLM$ is a parallelogram by Theorem 11.12.

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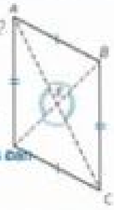
Common Errors

In the construction for **Exercise 1,** students may take shortcuts by simply drawing segments that appear to be parallel. Remind them that using the Construct Parallel Line command ensures that if the shape and location of a parallelogram are changed, then its opposite sides will remain parallel. In **Exercise 1 part a,** students may have problems using the available tools in the dynamic geometry software to identify and measure angles. A common error is having extra segments or points selected when the Measure command is selected. In this case, the option for measuring an angle might be grayed out on the menu, making it unselectable. Also advise students to double-check that the named angles that are displayed with their measurements correspond to the angles that students think they are measuring.

5. **USE STRUCTURE** Using your answers from **Example 3 parts b** and **c**, mark the diagram at right for congruency. How are $\triangle AEB$, $\triangle BEC$, $\triangle CED$, and $\triangle DEA$ related? Explain.



$\triangle AEB \cong \triangle CED$, $\triangle BEC \cong \triangle DEA$; the triangles are congruent by SAS.

a. **USE REASONING** How can congruent triangles be used to show that opposite sides of ABCD are parallel? Explain.

Sample answer: BY CPCTC, congruent angles of corresponding triangles can be identified. In ABCD, $\angle BAE \cong \angle DCE$ and $\angle DAE \cong \angle BCE$. Then $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$ can be shown by the Alternate Interior Angles Converse.

b. **CONSTRUCT AN ARGUMENT** Write a paragraph proof showing that ABCD is a parallelogram.

Sample answer: We are given $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$ because a midpoint divides a line segment into two equal segments. Also, $\angle AEB \cong \angle CED$ and $\angle BEC \cong \angle DEA$ because vertical angles are congruent. So, $\triangle AEB \cong \triangle CED$ and $\triangle BEC \cong \triangle DEA$ by SAS. By CPCTC, $\angle BAE \cong \angle DCE$ and $\angle DAE \cong \angle BCE$. This means that $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$ by the Alternate Interior Angles Converse. Opposite sides of ABCD are parallel, so by definition ABCD is a parallelogram.

6. **CRITIQUE REASONING** A student said that another way to prove that the quadrilateral ABCD from **Example 4** is a parallelogram is to use the Distance Formula. Do you agree? Justify your answer. If you agree, complete the proof.

Sample answer: Yes; if opposite sides of a quadrilateral are congruent, then it is a parallelogram. So, use the Distance Formula to show that $AB = DC$ and $AD = CB$.

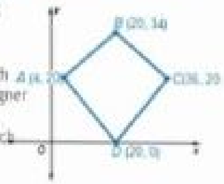
$AB = \sqrt{((-2)-0)^2 + ((-2)-(-1))^2} = \sqrt{5}$; $AD = \sqrt{(1-0)^2 + (5-(-1))^2} = \sqrt{37}$;

$CB = \sqrt{((-2)-(-1))^2 + ((-2)-4)^2} = \sqrt{37}$; $DC = \sqrt{((-1)-1)^2 + (4-5)^2} = \sqrt{5}$;

opposite sides of ABCD are congruent, so ABCD is a parallelogram by Theorem 11.9.

7. **REASON QUANTITATIVELY** A kite manufacturer is experimenting with different designs. The designer wants to modify a current design layout.

a. A current kite design is represented in the coordinate plane with vertices at $A(4, 20)$, $B(20, 34)$, $C(36, 20)$, and $D(20, 0)$. The designer wants to modify the design by shortening the length of the kite. Draw the kite design in the coordinate plane and determine which point should be moved to modify the kite. What are the new coordinates if the kite is to be in the shape of a parallelogram?



Point D should be moved to modify the design; (20, 6).

b. Prove that the new kite design is in fact in the shape of a parallelogram.

Sample answer: The slope of $\overline{AB}$ is $\frac{34-20}{20-4} = \frac{7}{8}$ and the slope of $\overline{DC}$ is $\frac{20-6}{36-20} = \frac{7}{8}$.

Using the distance formula, $AB = \sqrt{(20-4)^2 + (34-20)^2} = \sqrt{[illegible]}$ and $DC = \sqrt{(36-20)^2 + (20-6)^2} = \sqrt{[illegible]}$. By Theorem 11.12, ABCD is a parallelogram.

11.3 Tests for Parallelograms

Addressing the Standards

Exercise	SMP
1	3, 5
2	3
3	3
4	3, 7
5	3
6	3
7	2

Common Errors

In **Exercise 2**, students review a proof that uses opposite angles to prove that a quadrilateral is a parallelogram. Students are asked to identify a critical error and correct it. Identifying the error correctly depends upon the student's correct understanding of Theorem 11.5: *If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.* If students are having difficulty with the exercise, ask questions to make sure that they are not confusing consecutive and opposite angles or supplementary and congruent angles.

Emphasizing the Standards for Mathematical Practice

You may want to use **Exercise 4** to address **SMP 7 (Look for and make use of structure)**. Guide students to make connections between the visual appearance of a parallelogram in the coordinate plane and corresponding values found using the Slope, Midpoint, and Distance Formulas. For example, students might determine slope visually by graphing *JKLM* and counting squares to find rise over run. Have them confirm their visual observations by entering coordinates for each pair of vertices into the Slope Formula and comparing the results of their calculations to the slopes they determined visually.

11.4 Rectangles

STANDARDS

Standards for Mathematical Practice: 1, 2, 3, 5, 6

PREREQUISITES

- Know and apply properties of parallelograms
- Use slope and distance formulas

MATERIALS

- Compass
- Ruler

EXAMPLE 1

Teaching Tip SMP 5

Part a offers an opportunity to address **SMP 5 (Use appropriate tools strategically)**. As students construct a rectangle, encourage them to make connections between the steps in the constructions and why they work to create the desired figure.

11.4 Rectangles

Objectives

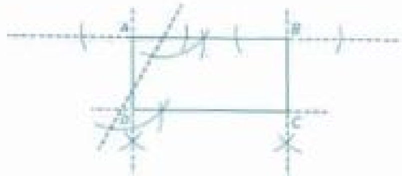
- Prove theorems about rectangles using two-column proofs.
- Use coordinates to prove theorems about rectangles.
- Make formal geometric constructions to understand theorems about rectangles.

A **rectangle** is a parallelogram with four right angles. Because a rectangle is a parallelogram, all the properties of parallelograms apply to rectangles.

EXAMPLE 1 Investigate Properties of Rectangles

EXPLORE Use a compass and straightedge to explore rectangles and their properties.

a. **USE TOOLS** Construct rectangle *ABCD* using the constructions of parallel and perpendicular lines.



b. **CONSTRUCT ARGUMENTS** Use the definition of a rectangle to explain how you know that *ABCD* is a rectangle.

Sample answer: A rectangle is a parallelogram with 4 rt. ∠s. A parallelogram has 2 pairs of || sides. $\overline{AC} \parallel \overline{BD}$ because both are $\perp \overline{AB}$, and if 2 lines are ⊥ to the same line, they are || to each other. $\overline{AB} \parallel \overline{CD}$ since both are $\perp \overline{AC}$. Each ∠ was drawn as a rt. ∠. Therefore, the figure *ABCD* is a rectangle by the definition of rectangle.

c. **MAKE A CONJECTURE** Use a ruler to find *AC* and *BD*. What do you notice? What hypothesis can you make about the diagonals of a rectangle? Can you assume your hypothesis is true based on examples?

Sample answer: $AC = BD$; Hypothesis: The diagonals of a rectangle are congruent. No; an example is not a proof. Proofs must be done using logic.

Theorem 11.13: If a parallelogram is a rectangle, then its diagonals are congruent.

Because Theorem 11.13 holds for all rectangles, we may add congruent diagonals to the list of properties of a rectangle.

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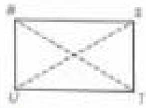
Math Background

A rectangle is a parallelogram with four right angles. Because it is a parallelogram, all the properties of parallelograms are also true of rectangles. In addition, the diagonals of a rectangle are congruent.

Proofs about rectangles can be done as two-column proofs using the properties of parallelograms and congruent triangles. Algebraic proofs on the coordinate plane are also possible. The distance formula can be used to show congruent sides and congruent diagonals. The slope formula can be used to prove that sides are perpendicular or parallel.

EXAMPLE 2 Prove that the Diagonals of a Rectangle Are Congruent

a. CONSTRUCT ARGUMENTS Fill in the missing reasons to complete the proof.
Given: *RSTU* is a rectangle.
Prove: $\overline{RT} \cong \overline{SU}$



Statement	Reason
1. *RSTU* is a rectangle.	1. Given
2. *RSTU* is a parallelogram.	2. Definition of a rectangle.
3. $\overline{RU} \cong \overline{ST}$	3. Opposite sides of a parallelogram are ≅.
4. $\overline{UT} \cong \overline{UT}$	4. Reflexive Property of Congruence
5. $\angle RUT$ and $\angle STU$ are right angles.	5. Definition of a rectangle.
6. $\angle RUT \cong \angle STU$	6. All right angles are ≅.
7. $\triangle RUT \cong \triangle STU$	7. SAS
8. $\overline{RT} \cong \overline{SU}$	8. CPCTC

b. REASON ABSTRACTLY Explain why this proof is true for all rectangles.

Sample answer: The only information that is given is that *RSTU* is a rectangle and you could use the same reasoning for any rectangle no matter how its vertices are labeled.

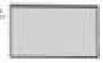
The converse of Theorem 11.13 is true as well.

Theorem 11.14: If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Finding congruent diagonals is a valuable tool for proving that a parallelogram is a rectangle.

EXAMPLE 3 Apply Properties of Rectangles

PLAN A SOLUTION Naela was asked to prove that the figure at the right is a rectangle. She has a ruler but no protractor or other tool to measure angles. How can she prove that the figure is a rectangle?



a. State the theorem that can be used to prove that the figure above is a parallelogram using only a ruler.

Sample answer: Theorem 11.9 states that if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

b. State the theorem that can be used to prove that a parallelogram is a rectangle using only a ruler.

Sample answer: Theorem 11.14 states that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

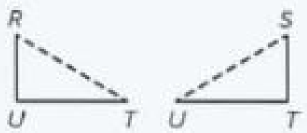
c. Using the theorems found in parts a and b, describe how Naela could show that the figure is a rectangle.

Sample answer: Naela could measure all 4 sides. If opposite sides are ≅, it is a parallelogram. She could then measure the diagonals. If they are ≅, it is a rectangle.

11.4 Rectangles

Differentiating Instruction

In **Example 2**, students may have difficulty visualizing the triangles that must be congruent in order for $\overline{RT} \cong \overline{SU}$. First ask students to locate $\overline{RT}$ and $\overline{SU}$ in the diagram. Help them separate the overlapping triangles so they can follow the reasoning in the proof more easily. Make sure they recognize that the same segment, $\overline{UT}$, is the base of each triangle. Have them mark congruent parts of the triangle as they work through the proof.



Scaffolding Questions

- What are two characteristics of a rectangle that are not true of all parallelograms? **The angles must be right angles and the diagonals must be equal.**
- Can we assume theorems about rectangles apply to parallelograms? **No. Theorems about rectangles do not necessarily apply to parallelograms.**

EXAMPLE 2

Teaching Tip SMP 3

In this example, remind students that they must use definitions, properties, postulates, and theorems that have already been proved as reasons to complete the proof.

Scaffolding Questions

- Why must *RSTU* be a parallelogram? **The definition of a rectangle says that it is a parallelogram.**
- Find $\overline{RU}$ and $\overline{ST}$ on the diagram. What parts of a parallelogram are they? **Opposite sides.** What theorem relates to opposite sides of a parallelogram? **Opposite sides of a parallelogram are congruent.**
- Your goal is to prove that $\overline{RT} \cong \overline{SU}$. Why is it useful to know that $\triangle RUT \cong \triangle STU$? **$\overline{RT}$ and $\overline{SU}$ are the hypotenuses of $\triangle RUT$ and $\triangle STU$ so they are corresponding sides.**

EXAMPLE 3

Teaching Tip SMP 1

This problem requires students to apply the properties of rectangles to solve a problem. They must figure out how to prove a quadrilateral is a rectangle only by considering lengths.

Scaffolding Questions

- Using the theorems you know, what can we prove about the figure by only measuring lengths? **If both pairs of opposite sides are congruent, then the figure is a parallelogram.**
- Why is it important to show that the figure is a parallelogram? **We need this information in order to use Theorem 11.14.**

EXAMPLE 4

Teaching Tip **SMP 1**

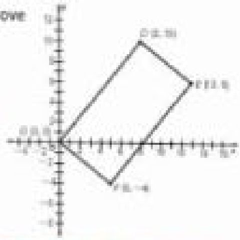
In **Example 4,** students must rely on algebra rather than measuring tools to prove that a figure is a rectangle. Challenge students to draw comparisons between measuring tools and algebraic formulas. For example, the distance formula may be used like a ruler to measure the length of a side.

Scaffolding Questions

- What formulas can you use given the endpoints of a line segment? **The distance formula gives the length of the segment and the slope formula gives its slope.**
- How do lengths help you prove that a figure is a rectangle? **If both pairs of opposite sides are congruent, it is a parallelogram. If it is a parallelogram with congruent diagonals, it is a rectangle.**
- How do slopes help you prove that a figure is a rectangle? **If both pairs of opposite sides have the same slope, they are parallel and it is a parallelogram. If the slopes of each pair of consecutive sides are negative reciprocals, they are perpendicular and the parallelogram is a rectangle.**

EXAMPLE 4 **Proving Rectangles on a Coordinate Plane**

The coordinates of a quadrilateral are shown. Use algebra to prove that it is a rectangle.



a. PLAN A SOLUTION Describe how you could construct an argument to prove that $DEFG$ is a rectangle?

Sample answer: If opposite sides are equal, $DEFG$ is a parallelogram. Find the slopes of the sides to see if consecutive sides are perpendicular.

b. REASON QUANTITATIVELY Show that $DEFG$ is a rectangle. Explain.

Sample answer: $DE = \sqrt{(8-13)^2 + (10-6)^2} = \sqrt{41}$ and $FG = \sqrt{(5-0)^2 + (-4-0)^2} = \sqrt{41}$;
$EF = \sqrt{(13-5)^2 + (6-(-4))^2} = \sqrt{164} = 2\sqrt{41}$ and $GD = \sqrt{(8-0)^2 + (10-0)^2} = \sqrt{164} = 2\sqrt{41}$.
The lengths of the opposite sides are equal, so $DEFG$ is a parallelogram.
$EF = \sqrt{(1-7)^2 + (4-0)^2} = \sqrt{52} = 2\sqrt{13}$ and $DF = \sqrt{(-3-3)^2 + (2-(-2))^2} = \sqrt{52} = 2\sqrt{13}$;
slope $\overline{DE} = -\frac{4}{5}$; slope $\overline{EF} = \frac{5}{4}$; slope $\overline{FG} = -\frac{4}{5}$; and slope $\overline{GD} = \frac{5}{4}$; the slopes of consecutive sides are negative reciprocals, so all angles are right angles. Since $DEFG$ is a parallelogram and all of the angles are rt. angles, $DEFG$ is a rectangle.

PRACTICE

1. **CRITIQUE REASONING** Omar argues that to prove a quadrilateral is a rectangle, it is sufficient to prove that its diagonals are congruent. Do you agree? If so, explain why. If not, explain and draw a counterexample.

No; a rectangle must be a parallelogram in addition to having ≅ diagonals.
Sample answer: An isosceles trapezoid is a counterexample.

a. How can you alter Omar's argument to make it correct?
Sample answer: To prove a parallelogram is a rectangle, it is sufficient to prove that its diagonals are congruent

b. Omar also says that to show that two diagonals of a quadrilateral are congruent, it is sufficient to show that all four angles of the quadrilateral are right angles. Is he correct? Explain.
Yes; If all four angles of a quadrilateral are right angles, then both pairs of opposite angles are congruent. So, by Theorem 11.10 the quadrilateral is a parallelogram. If a parallelogram has four right angles, then it is a rectangle. By Theorem 11.13, if a parallelogram is a rectangle, then its diagonals are congruent.

150 **CHAPTER 11** Quadrilaterals

Emphasizing the Standards for Mathematical Practice

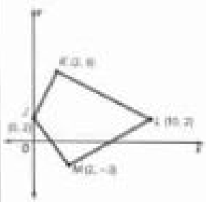
When students must prove geometric properties instead of memorizing them, they are more likely to internalize the concepts and apply them in a variety of situations. By emphasizing **SMP 3 (Construct viable arguments and critique the reasoning of others)**, you are having students take an active part in learning the properties of various types of parallelograms in a way that builds understanding.

As students work through the proofs, give them time to discuss the problems in small groups or as a whole class. Ask questions such as, "How do you know this?", "Why is this true?", and "Does this make sense?" Encourage students to develop explanations that use mathematical terminology and reasoning.

2. **CONSTRUCT ARGUMENTS** Fill in the missing parts to complete the proof.
Given: Parallelogram $KLMN$, $\overline{KM} \cong \overline{LN}$
Prove: $KLMN$ is a rectangle.

Statement	Reason
1. $KLMN$ is a parallelogram. $\overline{KM} \cong \overline{LN}$	1. **Given**
2. $\overline{KN} \cong \overline{LM}$	2. **Opp. sides of a parallelogram are ≅.**
3. $\overline{NM} \cong \overline{NM}$	3. **Reflexive Property of Congruence**
4. **$\triangle KNM \cong \triangle LMN$**	4. SSS Theorem
5. $\angle KNM \cong \angle LMN$	5. **Corresponding parts of ≅ △s are ≅.**
6. $\angle KNM$ and $\angle LMN$ are supplementary.	6. **Cons. ∠s of a parallelogram are supplementary.**
7. $\angle KNM$ and $\angle LMN$ are right angles.	7. **If 2 ∠s are ≅ and supplementary, they are rt. ∠s.**
8. **$\angle NKL$ and $\angle MLK$ are right angles.**	8. If a parallelogram has 1 rt. ∠, then it has 4 rt. ∠s.
9. $KLMN$ is a rectangle.	9. **Definition of rectangle**

3. Students were asked to find whether the quadrilateral formed by connecting $J(0, 2)$, $K(2, 6)$, $L(10, 2)$, and $M(2, -2)$ is a rectangle. Two students solutions are shown below.

Reem	Salma
I found the lengths of the sides. $JK = MJ = \sqrt{20}$ and $KL = LM = \sqrt{80}$, so pairs of sides are equal, and it is a parallelogram. I found the slope of each side: $\overline{JK} = 2$, $\overline{KL} = -\frac{1}{2}$, $\overline{LM} = \frac{1}{2}$, $\overline{MJ} = -2$. That shows that $\overline{JK} \perp \overline{KL}$ and $\overline{LM} \perp \overline{MJ}$. If 1 ∠ of a parallelogram is a rt. ∠, then all 4 angles are rt. ∠s. That makes it a rectangle.	I graphed it, and I could see that it is not a rectangle. 

a. **REASON ABSTRACTLY** Evaluate each students' solution.
Sample answer: Reem: Incorrect; consecutive sides (not opposite sides) are congruent, so it is not necessarily a parallelogram. Salma: correct; although she did not prove his answer

b. **CONSTRUCT ARGUMENTS** Explain how you would solve the problem.
Sample answer: I would find the slopes and lengths like Reem but would analyze the results with a graph like Salma used so I could visualize the sides and angles I was analyzing.

PRACTICE

In **Exercises 1** and **2**, students practice reasoning to prove a theorem about parallelograms and rectangles.

In **Exercise 3**, students must use coordinates and algebra to show whether or not the quadrilateral is a rectangle.

Addressing the Standards

Exercise	SMP
1	2
2	3
3	2, 3

Common Errors

Students may have difficulty citing the correct reason for some of the steps in **Exercise 2.** If they do not remember the theorem, take time to go over it again. For example, students may not remember that if two angles are both supplementary and congruent, they are right angles. Take time to revisit the reasoning behind the statement so that students understand why it is true.

In **Exercise 3,** students may think that they must choose one solution as the correct one. Help them see that Reem attempted to approach it analytically, but didn't apply the information from the formulas correctly. Salma's solution showed that she made sense of the problem. However, she did not use mathematics to back up her statement. Students' solution should combine the better parts of each solution shown.

11.5 Rhombi and Squares

STANDARDS

Standards for Mathematical Practice: 1, 2, 3, 5, 6, 7

PREREQUISITES

- Use distance and slope formulas to solve problems
- Use properties of parallelograms

MATERIALS

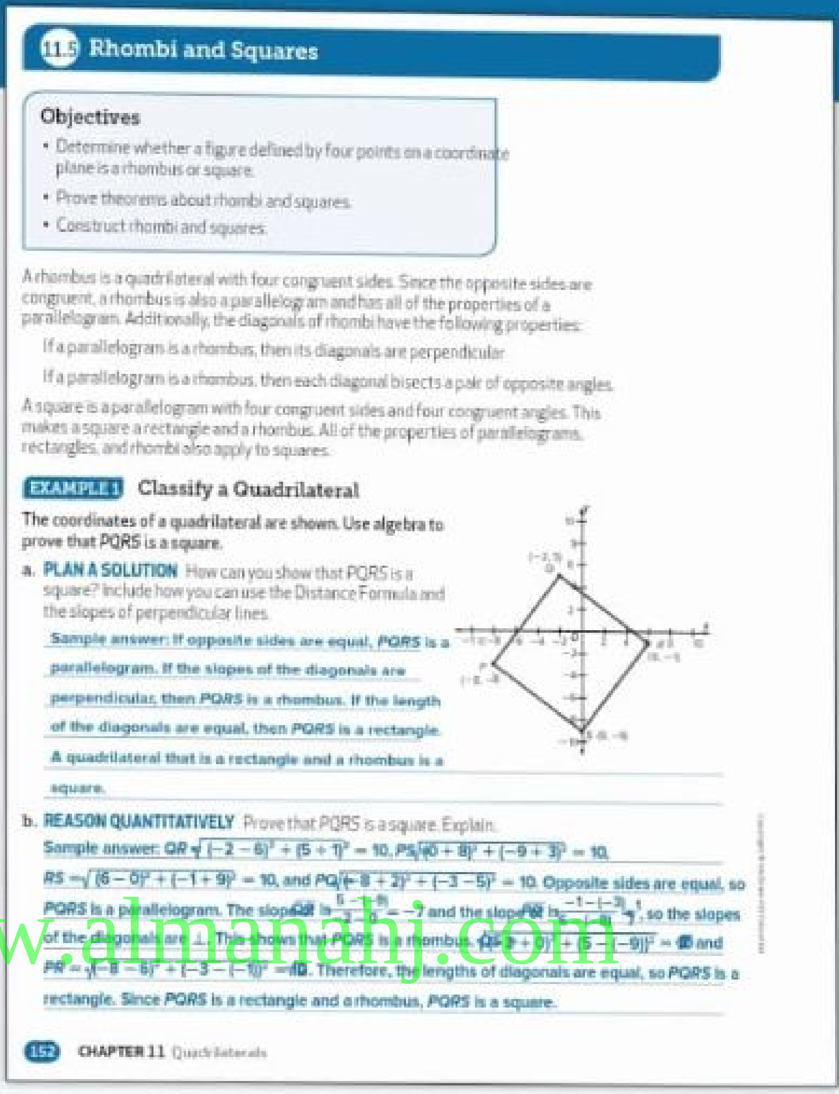
- Compass
- Straightedge
- Patty paper

EXAMPLE 1

Teaching Tip

SMP 1

Encourage students to be clear about the classification of the figure that they are working with. Remind them that to prove a figure is a rhombus or a square, they may first need to show that the figure is a parallelogram.



11.5 Rhombi and Squares

Objectives

- Determine whether a figure defined by four points on a coordinate plane is a rhombus or square.
- Prove theorems about rhombi and squares.
- Construct rhombi and squares.

A rhombus is a quadrilateral with four congruent sides. Since the opposite sides are congruent, a rhombus is also a parallelogram and has all of the properties of a parallelogram. Additionally, the diagonals of rhombi have the following properties:

If a parallelogram is a rhombus, then its diagonals are perpendicular.

If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

A square is a parallelogram with four congruent sides and four congruent angles. This makes a square a rectangle and a rhombus. All of the properties of parallelograms, rectangles, and rhombi also apply to squares.

EXAMPLE 1 **Classify a Quadrilateral**

The coordinates of a quadrilateral are shown. Use algebra to prove that PQRS is a square.

a. **PLAN A SOLUTION** How can you show that PQRS is a square? Include how you can use the Distance Formula and the slopes of perpendicular lines.

Sample answer: If opposite sides are equal, PQRS is a parallelogram. If the slopes of the diagonals are perpendicular, then PQRS is a rhombus. If the length of the diagonals are equal, then PQRS is a rectangle. A quadrilateral that is a rectangle and a rhombus is a square.

b. **REASON QUANTITATIVELY** Prove that PQRS is a square. Explain.

Sample answer: $QR = \sqrt{(-2-6)^2 + (5+1)^2} = 10$, $PS = \sqrt{(0+8)^2 + (-9+3)^2} = 10$, $RS = \sqrt{(6-0)^2 + (-1+9)^2} = 10$, and $PQ = \sqrt{(-8+2)^2 + (-3-5)^2} = 10$. Opposite sides are equal, so PQRS is a parallelogram. The slope QS is $\frac{5-(-9)}{-2-0} = -7$ and the slope PR is $\frac{-1-(-3)}{6-(-8)} = \frac{1}{7}$, so the slopes of the diagonals are ⊥. This shows that PQRS is a rhombus. $QS = \sqrt{(-2+0)^2 + (5-(-9))^2} = \sqrt{200}$ and $PR = \sqrt{(-8-6)^2 + (-3-(-1))^2} = \sqrt{200}$. Therefore, the lengths of diagonals are equal, so PQRS is a rectangle. Since PQRS is a rectangle and a rhombus, PQRS is a square.

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Math Background

In this lesson, students prove theorems about rhombi and squares. Many of the statements students will prove involve lengths and angles. When working with coordinates, students will find the Slope Formula key for identifying parallel and perpendicular lines and the Distance Formula useful for verifying equal length.

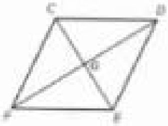
There are often multiple approaches that may be used to prove the properties of a special quadrilateral. Encourage students to consider different strategies.

c. **CRITIQUE REASONING** Mohammed believes quadrilateral *PQRS* is a square if the diagonals are congruent and perpendicular, and a rhombus if the diagonals are perpendicular but not congruent. Bashir believes this information is not sufficient to classify the quadrilateral. Who is correct? Explain your answer.

Sample answer: Bashir is correct. Mohammed's claims are only true if *PQRS* is a parallelogram. *PQRS* could be a kite or isosceles trapezoid with perpendicular or perpendicular and congruent diagonals.

EXAMPLE 2 Proving a Parallelogram is a Rhombus

CONSTRUCT ARGUMENTS Prove that if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.



Given: CDEF is a parallelogram, $\overline{CE} \perp \overline{DF}$

Prove: CDEF is a rhombus

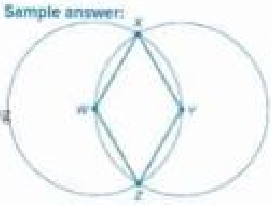
Statements	Reasons
1. CDEF is a parallelogram, $\overline{CE} \perp \overline{DF}$	1. **Given**
2. $\overline{DG} \cong \overline{FG}$	2. **Diagonals of a parallelogram bisect each other.**
3. $\angle CGF$ and $\angle CGD$ are right angles.	3. **Definition of $\perp$**
4. **$\angle CGF \cong \angle CGD$**	4. All right angles are congruent.
5. $\triangle CGF \cong \triangle CGD$	5. **SAS**
6. $\overline{CF} \cong \overline{CD}$	6. **CPCTC**
7. **$\overline{CF} \cong \overline{DE}$, $\overline{CD} \cong \overline{EF}$**	7. Opposite sides of a parallelogram are congruent.
8. **$\overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{CF}$**	8. **Transitive Property of $\cong$**
9. **CDEF is a rhombus**	9. **Definition of a rhombus**

EXAMPLE 3 Constructing a Rhombus

a. **USE TOOLS** Follow these steps to construct rhombus WXYZ.

- In the space to the right, use your compass to construct circle W containing point Y.
- With the compass at point Y, construct circle Y containing point W.
- Label the points of intersection X and Z.
- Draw $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, and $\overline{WZ}$.

Sample answer:



b. **COMMUNICATE PRECISELY** Write a paragraph proof to prove WXYZ is a rhombus.

Sample answer: Circle *W* and circle *Y* are congruent circles because they both have a radius of length *WY*. The four sides of *WXYZ* are each radii of the two congruent circles so these sides are congruent to each other. This makes quadrilateral *WXYZ* a rhombus.

Scaffolding Questions

- Which is true: "All rhombi have perpendicular diagonals" or "All quadrilaterals with perpendicular diagonals are rhombi"? Justify your answer. **The first is true by definition. The second is not true; kites and isosceles trapezoids have perpendicular diagonals.**
- How do we know that a figure that is a rhombus and a rectangle is a square? **If a figure is a rectangle, then each angle is a rt. ∠, and if the figure is a rhombus, then we know that all sides are ≅. Since all sides and ∠s are ≅, the quadrilateral is a square.**

EXAMPLE 2

Teaching Tip SMP 3

In **Example 2**, more statements are provided toward the beginning of the proof so students can focus on providing the reasons at first. There is less support toward the end. Before students start the proof, help them identify what the last statement will be. Have them state the problem in their own words.

Scaffolding Questions

- What can you say about any parallelogram with perpendicular diagonals? **It is a rhombus.**
- Since the figure is a parallelogram, what can we say about the diagonals? **The diagonals bisect each other.**

Emphasizing the Standards for Mathematical Practice

SMP 6 (Attend to precision) is not only a key component of coordinate poofs, it is also a significant part of any answer explanation. Whether justifying their answers with a single sentence, writing paragraph proofs, or formulating two-column proofs, students must be careful to use correct language and notation.

Remind students that mathematics is a language and that being able to express ideas using words and numbers is an essential part of communicating precisely.

EXAMPLE 3

Teaching Tip SMP 3

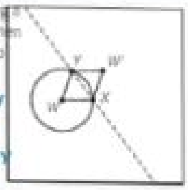
Example 3 provides an excellent opportunity for differentiated instruction. Some students may recognize similarities between the construction in **part a** and the construction of the perpendicular bisector of a segment. Encourage those students to develop a proof in **part b** that makes use of the fact that diagonal $\overline{XZ}$ is a perpendicular bisector of diagonal $\overline{WY}$.

Some students may find it helpful to use patty paper and a compass to reproduce Jamal's construction in **part c**.

Scaffolding Questions

- How are circle *W* and circle *Y* related? How do you know? **They are congruent because they have the same radius.**
- What property of reflections allows us to use reflections to prove congruence? **A reflection is a rigid transformation, so the preimage and image are congruent.**
- In **part d**, what construction must you perform to guarantee that the quadrilateral is a square? **You must construct the perpendicular bisector of a diameter of the circle. This guarantees that the measures of the angles of the quadrilateral will be 90 and the sides will be congruent.**

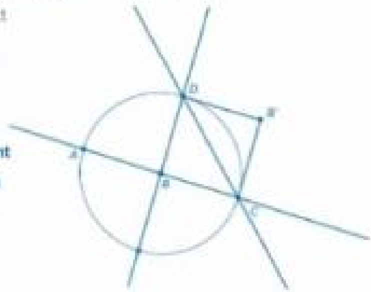
c. **CRITIQUE REASONING** Jamal says that he can construct a rhombus using a circle drawn on patty paper. He constructs circle *W* through point *X* and then draws chord $\overline{XY}$, where *Y* is a point on the circle. He then folds the paper to reflect *W* across $\overline{XY}$. Is *WXW'Y* a rhombus? Explain.



Sample answer: Yes. $\overline{WX}$ and $\overline{WY}$ are radii of the same circle, so they are congruent. A reflection is a rigid transformation, so $\triangle WXY \cong \triangle W'XY$. Therefore $\overline{WX} \cong \overline{W'X}$ and $\overline{WY} \cong \overline{W'Y}$. The four sides of *WXW'Y* are congruent to each other, so *WXYZ* a rhombus.

d. **USE TOOLS** Use Jamal's process to construct a square. Explain.

Sample answer: Construct circle *B*. Draw diameter *AC* through *B* and construct a perpendicular bisector to $\overline{AC}$. Label the intersection of this line and the circle point *D*. Draw a line that passes through points *D* and *C*. Reflect $\triangle BCD$ through $\overline{DC}$ to form square *BCB'D*.



PRACTICE

1\. a. **CRITIQUE REASONING** Shamsah is using coordinate geometry to classify quadrilateral *ABCD*. She finds $AB = BC = CD = AD$ and decides *ABCD* is a rhombus, but not a square. Do you agree with her conclusion? Explain your answer.

Sample answer: No. Shamsah is correct that *ABCD* is a rhombus because it has four congruent sides, but *ABCD* could also be a square. She needs to compare the slopes of a pair of adjacent sides or the lengths of the diagonals.

b. **CRITIQUE REASONING** Shamsah is attempting to classify another quadrilateral, *EFGH*. She finds that the diagonals $EG = FH = 5$. Is it possible for a quadrilateral to be both a rectangle and a rhombus?

Yes it is possible. If *EFGH* is a parallelogram, then by Theorem 11.14 it is a rectangle because its diagonals are congruent. If the diagonals bisect each other, then *EFGH* is also a rhombus.

2\. **COMMUNICATE PRECISELY** The vertices of parallelogram *QRST* are $Q(-4, 7)$, $R(1, 9)$, $S(6, 7)$, and $T(1, 5)$. Determine whether *QRST* is a rectangle, rhombus, or square. List all that apply and explain your answer.

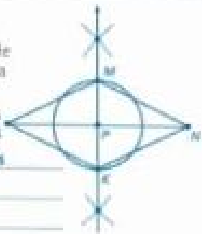
Sample answer: *QRST* is a rhombus: $QR = RS = ST = QT$ so the figure has four congruent sides. However, *QRST* is not a rectangle or a square because the diagonals are not congruent: $QS = 10$ and $RT = 4$.

Common Errors

In **Exercise 1**, some students might consider rhombi and squares to be exclusive sets. Remind students that every square is a rhombus, but not every rhombus is a square.

In **Exercise 2**, students may misidentify the slopes of the sides of *QRST* as being perpendicular. Point out that slopes of perpendicular lines are opposite and reciprocal, but the slopes of the adjacent sides here are only opposites.

3. **COMMUNICATE PRECISELY** Abdulrahman drew segment $\overline{LN}$ and constructed its perpendicular bisector. He labeled the intersection P. Then he constructed circle PM, where M is on the perpendicular line. He labeled the other intersection of the circle with the line point K. Write a paragraph proof that quadrilateral KLMN is a rhombus. Following Abdulrahman's method, make your own construction using a compass and straightedge.



Sample answer: *KLMN* is a parallelogram because the diagonals bisect each other. *P* is the midpoint of $\overline{LN}$ by construction and the midpoint of $\overline{MK}$ because $\overline{MK}$ is a diameter of circle *P*. By construction $\overline{MK}$ is perpendicular to $\overline{LN}$; therefore, *KLMN* is a rhombus.

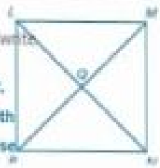
4. **CONSTRUCT ARGUMENTS** Prove that if the triangle formed by the diagonals and a side of a parallelogram is isosceles, then the parallelogram is a rectangle.

Given: ACDE is a parallelogram, and $\triangle ACB$ is an isosceles triangle with $\overline{AC}$ base.

Prove: ACDE is a rectangle.

Statements	Reasons
1. **ACDE is a parallelogram; $\triangle ACB$ is isosc.**	1. Given
2. $\overline{AB} \cong \overline{CB}$	2. Def. of isosceles
3. **$\overline{EB} \cong \overline{BC}$ and $\overline{AB} \cong \overline{BD}$**	3. Diags. of a parallelogram bisect each other
4. $AB = CB = EB = BD$	4. Definition of congruence and Transitive Prop.
5. **$AB + BD = EB + CB$**	5. Addition Property of Equality
6. ACDE is a rectangle	6. If diagonals of par. are equal, it is a rectangle.

5. **USE STRUCTURE** If the diagonals of quadrilateral LMNP form congruent triangles, prove that the quadrilateral is a square. Draw and label a figure and write a paragraph proof.



Sample answer: Given *LMNP* and $\triangle PQN \cong \triangle NQM \cong \triangle MQL \cong \triangle LQP$, then $\overline{PN} \cong \overline{NM} \cong \overline{ML} \cong \overline{LP}$ by CPCTC. *LMNP* is a parallelogram since both pairs of opposite sides are congruent. Then *LMNP* is a rhombus because you can pick any one pair of consecutive sides and they are congruent. $\overline{PQ} \cong \overline{NQ} \cong \overline{MQ} \cong \overline{LQ}$ by CPCTC, which means these segments have the same length. Therefore, $PQ + MQ = NQ + LQ$, so $PM = NL$ and $\overline{PM} \cong \overline{NL}$. Thus, *LMNP* is a rectangle, and, therefore, a square.

11.5 Rhombi and Squares

PRACTICE

In **Exercises 1** and **2**, students use coordinates and algebra to prove simple theorems. Specifically, **Exercise 2** asks students to classify the given quadrilateral using only coordinates.

In **Exercise 3**, students must perform a construction by compass and straightedge to construct a proof.

In **Exercises 4** and **5**, students prove that a parallelogram is a rectangle.

Addressing the Standards

Exercise	SMP
1	3
2	6
3	6
4	3
5	7

Common Errors

In **Exercises 4** and **5**, there are many possible sequences in which the statements can be ordered, but not every sequence is logically sound. Remind students that a reason can only use information already stated in the proof.

Emphasizing the Standards for Mathematical Practice

Exercise 2 gives students practice with **SMP 1 (Make sense of problems and persevere in solving them)** as they must choose an appropriate strategy for classifying the figure.

Have students share their work at the board or in small groups as an opportunity for differentiated instruction. Encourage students to use the properties of special parallelograms to check their answers.

11.6 Trapezoids and Kites

STANDARDS

Standards for Mathematical Practice: 1, 2, 3, 6, 7

PREREQUISITES

- Use distance and slope formulas to solve problems
- Write and solve equation in one variable
- Solve a system of linear equations
- Use properties of parallelograms

MATERIALS

- Patty paper

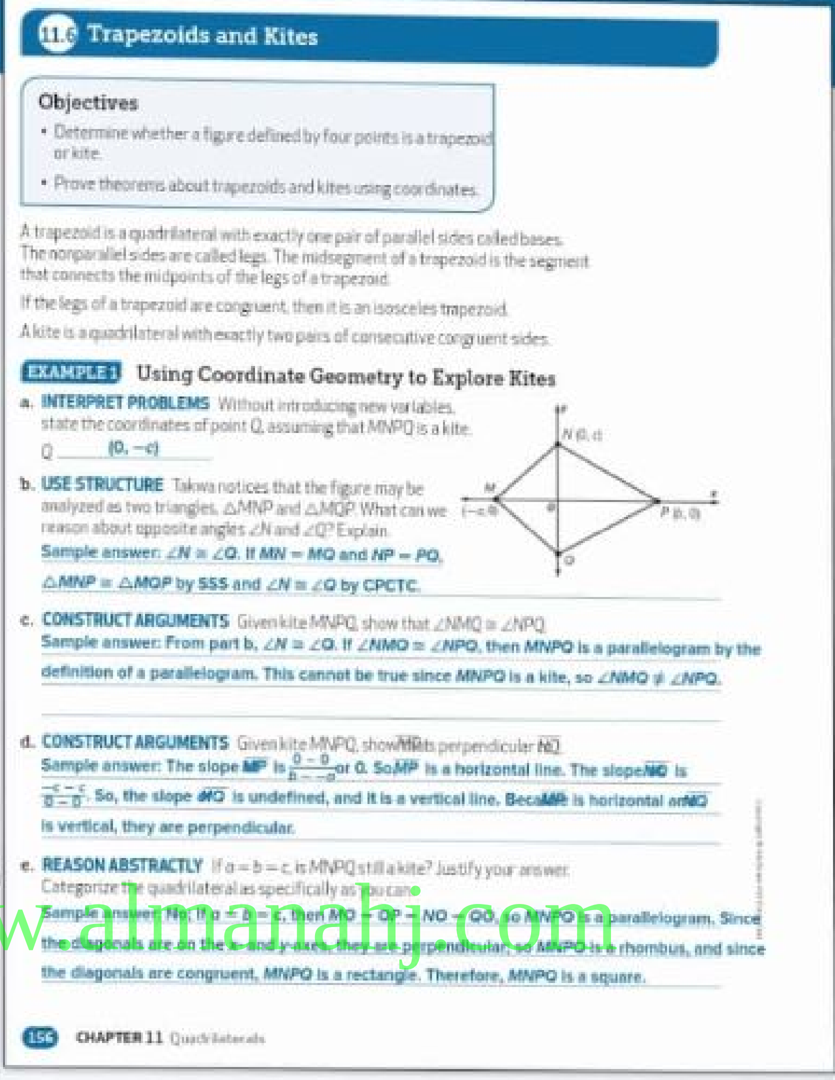
EXAMPLE 1

Teaching Tip SMP 1

Kinesthetic learners may benefit from tracing *MNPQ* and the axes onto patty paper and folding the figure along each axis to check for symmetry. Note to students that the figure is made by reflecting a triangle. This insight is useful in **part b**.

Scaffolding Questions

- Are kites a subset of any other type of special quadrilateral? **No; although they share characteristics with various other special quadrilaterals, they are not a subset of any other class of quadrilaterals.**
- If $a = c$ but $a \neq b$, is *MNPQ* still a kite? **Yes; this still allows for exactly two sets of consecutive congruent sides; $MN = MQ$ and $PN = PQ$, but $MN \neq PN$.**



11.6 Trapezoids and Kites

Objectives

- Determine whether a figure defined by four points is a trapezoid or kite.
- Prove theorems about trapezoids and kites using coordinates.

A trapezoid is a quadrilateral with exactly one pair of parallel sides called bases. The nonparallel sides are called legs. The midsegment of a trapezoid is the segment that connects the midpoints of the legs of a trapezoid.

If the legs of a trapezoid are congruent, then it is an isosceles trapezoid.

A kite is a quadrilateral with exactly two pairs of consecutive congruent sides.

EXAMPLE 1 **Using Coordinate Geometry to Explore Kites**

a. **INTERPRET PROBLEMS** Without introducing new variables, state the coordinates of point Q, assuming that MNPQ is a kite.
Q **(0, −c)**

b. **USE STRUCTURE** Takwa notices that the figure may be analyzed as two triangles, $\triangle MNP$ and $\triangle MQP$. What can we reason about opposite angles $\angle N$ and $\angle Q$? Explain.
Sample answer: $\angle N \cong \angle Q$. If $MN = MQ$ and $NP = PQ$, $\triangle MNP \cong \triangle MQP$ by SSS and $\angle N \cong \angle Q$ by CPCTC.

c. **CONSTRUCT ARGUMENTS** Given kite MNPQ, show that $\angle NMQ \cong \angle NPQ$.
Sample answer: From part b, $\angle N \cong \angle Q$. If $\angle NMQ \cong \angle NPQ$, then MNPQ is a parallelogram by the definition of a parallelogram. This cannot be true since MNPQ is a kite, so $\angle NMQ \not\cong \angle NPQ$.

d. **CONSTRUCT ARGUMENTS** Given kite MNPQ, show $\overline{MP}$ is perpendicular $\overline{NQ}$.
Sample answer: The slope $\overline{MP}$ is $\frac{0-0}{b-(-a)}$ or 0. So $\overline{MP}$ is a horizontal line. The slope $\overline{NQ}$ is $\frac{-c-c}{0-0}$. So, the slope $\overline{NQ}$ is undefined, and it is a vertical line. Because $\overline{MP}$ is horizontal and $\overline{NQ}$ is vertical, they are perpendicular.

e. **REASON ABSTRACTLY** If $a = b = c$, is MNPQ still a kite? Justify your answer. Categorize the quadrilateral as specifically as you can.
Sample answer: No; If $a = b = c$, then $MO = OP = NO = QO$, so MNPQ is a parallelogram. Since the diagonals are on the x- and y-axes, they are perpendicular, so MNPQ is a rhombus, and since the diagonals are congruent, MNPQ is a rectangle. Therefore, MNPQ is a square.

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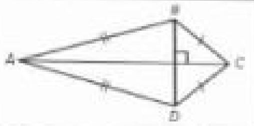
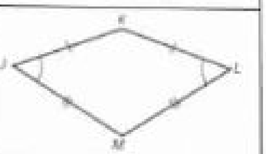
Math Background

In this lesson, students investigate and prove theorems about trapezoids and kites. Because this lesson has students use coordinates to prove geometric theorems algebraically, the emphasis is on properties of sides and diagonals, but the angles of trapezoids and kites have many interesting properties worth exploring.

When working with coordinates, students will need to know the slope formula for identifying parallel and perpendicular lines and the distance formula for determining lengths.

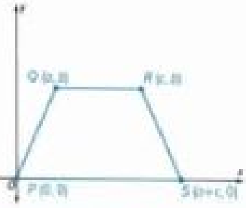
Theorems Kites

11.25	If a quadrilateral is a kite, then its diagonals are perpendicular. **Example** If quadrilateral ABCD is a kite, then $\overline{AC} \perp \overline{BD}$.	
11.26	If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent. **Example** If quadrilateral JKLM is a kite, $\overline{JK} \cong \overline{KL}$, and $\overline{JM} \cong \overline{LM}$, then $\angle J \cong \angle L$ and $\angle K \not\cong \angle M$.	

EXAMPLE 2 **Using Coordinate Geometry to Classify and Prove Theorems About Trapezoids**

a. PLAN A SOLUTION Plot quadrilateral PQRS with vertices $P(0, 0)$, $Q(a, b)$, $R(c, b)$, and $S(a + c, 0)$, where $a > 0$, $b > 0$, and $c > 0$, on the axes to the right.

Sample answer:



b. CALCULATE ACCURATELY Rayan says PQRS is an isosceles trapezoid with bases $\overline{QR}$ and $\overline{PS}$. Do you agree with Rayan? Justify your answer.

Sample answer: Yes; I can show that $\overline{QR}$ and $\overline{PS}$ are parallel (same slope) but have different lengths, while $\overline{PQ}$ and $\overline{RS}$ have different slopes but equal lengths.

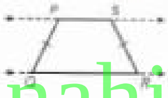
$PQ = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$, slope of $\overline{PQ} = \frac{b-0}{a-0} = \frac{b}{a}$

$QR = \sqrt{(c-a)^2 + (b-b)^2} = c - a$, slope of $\overline{QR} = \frac{b-b}{c-a} = \frac{0}{c-a} = 0$

$RS = \sqrt{((a+c)-c)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$, slope of $\overline{RS} = \frac{0-b}{(a+c)-c} = -\frac{b}{a}$

$PS = \sqrt{((a+c)-(0))^2 + (0-0)^2} = a + c$, slope of $\overline{PS} = \frac{0-0}{(a+c)-0} = \frac{0}{a+c} = 0$

c. CONSTRUCT ARGUMENTS Show that if a trapezoid is isosceles, then its diagonals are congruent.



Given: PQRS is an isosceles trapezoid with bases $\overline{QR}$ and $\overline{PS}$.

Prove: $\overline{QS} \cong \overline{PR}$

$\overline{QP} \cong \overline{RS}$, since PQRS is an isosceles trapezoid. $\overline{PS} \cong \overline{PS}$ by the Reflexive Property. $\angle QPS \cong \angle RSP$ since if a trapezoid is isosceles, then each pair of base angles is congruent. Therefore, $\triangle QPS \cong \triangle RSP$ by SAS. $\overline{QS} \cong \overline{RP}$ by CPCTC.

11.6 Trapezoids and Kites

EXAMPLE 2

Teaching Tip SMP 6

Example 2 requires students to make use of both the Slope and Distance Formulas. Encourage students to take extra care with negative signs and subtraction.

Scaffolding Questions

- Is it necessary to verify that $QR \neq PS$ in **part b** so that we know PQRS is not a parallelogram? **No; if $\overline{PQ}$ and $\overline{RS}$ are not parallel, then it is impossible for PQRS to be a parallelogram.**
- Could PQRS be a trapezoid if $QR = PS$? **No; if one pair of opposite sides is parallel and congruent, then the quadrilateral is a parallelogram and cannot be a trapezoid.**

Teaching Tip SMP 3

Encourage students to look at many of the properties of trapezoids and kites and determine whether they are sufficient conditions to classify a quadrilateral.

Scaffolding Question

- Which quadrilaterals have congruent diagonals? **rectangles, squares, isosceles trapezoids**

Emphasizing the Standards for Mathematical Practice

As students use coordinate geometry to prove statements about kites and isosceles trapezoids, they will be applying **SMP 2 (Reason abstractly and quantitatively)**. For example, they will need to manipulate formulas for slope and distance using more than one unknown.

PRACTICE

Exercises 1–4 each require students to use coordinates to prove geometric theorems, satisfying **G.GPE.4.** In particular, in **Exercise 1** students prove that a particular figure drawn by a student is a kite and that another figure drawn by the student is an isosceles trapezoid. Similarly, in **Exercise 2** students write a proof showing that a given figure is a kite, and in **Exercise 3** students prove that the midsegment of an isosceles trapezoid is parallel to the bases.

In **Exercise 4,** students must prove that a quadrilateral is a trapezoid but not isosceles.

Addressing the Standards

Exercise	SMP
1	2
2	3
3	7
4	3

d. CRITIQUE REASONING Malak says quadrilateral $PQRS$ is an isosceles trapezoid because the diagonals are congruent. Would this be enough information to classify $PQRS$ as an isosceles trapezoid? Explain.

No; Sample answer: It would be enough to classify a trapezoid as isosceles, but it is not enough information to classify any quadrilateral as an isosceles trapezoid: squares, rectangles, and some non-special quadrilaterals also have congruent diagonals.

PRACTICE

1. **REASON QUANTITATIVELY** $W(-1, -11)$ and $X(1, 1)$ are two vertices of quadrilateral $WXYZ$.

   a. Find coordinates Y and Z that will make $WXYZ$ a kite. Justify your answer.

   Sample answer: Let Z be $(-11, 1)$ and Y be $(-1, 13)$ so that $\overline{XZ}$ is horizontal and $\overline{WY}$ is vertical. Make the diagonals perpendicular so $\overline{XZ}$ bisects $\overline{WY}$.

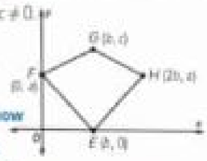
   b. Find coordinates Y and Z that will make $WXYZ$ an isosceles trapezoid. Justify your answer.

   Sample answer: Let $\overline{WX}$ be one leg. The slope of $\overline{WX}$ is 6, and the length of $\overline{WX}$ is $\sqrt{(-11-1)^2+(-1-1)^2}=\sqrt{148}$. Make the bases parallel to the x-axis. Let Y be $(6, 1)$. Then Z will be $(x, -11)$. Find the value of x by setting YZ equal to WX and solving for x: $\sqrt{148}=\sqrt{(x-6)^2+(-11-1)^2}$, $\sqrt{148}=\sqrt{(x-6)^2+144}$, so $(x-6)^2=4$. Therefore, $x=8$ or 4. If Z is $(4, -11)$, then $WXYZ$ will be a parallelogram. So, Z must be $(8, -11)$.

   c. If the coordinates of Y and Z are $(4, 1)$ and $(4, -11)$, identify the shape of $WXYZ$.

   $WXYZ$ is a trapezoid; it has two parallel sides with legs that are not congruent.

2. **CONSTRUCT ARGUMENTS** $EFGH$ is shown to the right, with $a \neq b \neq c \neq 0$. Use coordinate geometry to prove that $EFGH$ is a kite.



Sample answer: First note that $\overline{FH}$ is a horizontal line segment $\left(m=\frac{a-a}{2b-b}=\frac{0}{b}=0\right)$, while $\overline{GE}$ is a vertical line segment $\left(m=\frac{c-0}{b-b}=\frac{c}{0}\right)$. Therefore, the two diagonals are perp. Then show that one of the diagonals bisects the other. The line containing $\overline{GE}$ is $x=b$, and the line containing $\overline{FH}$ is $y=a$. Those two lines intersect at (b, a). Call this M. Then $FM=\sqrt{(0-b)^2+(a-a)^2}=\sqrt{b^2}=b$ and $MH=\sqrt{(2b-b)^2+(a-a)^2}=\sqrt{b^2}=b$, so $\overline{GE}$ bisects $\overline{FH}$ at M. Further, consecutive sides are congruent since $FG=\sqrt{(b-0)^2+(c-a)^2}=\sqrt{b^2+(c-a)^2}=\sqrt{(b-2b)^2+(c-a)^2}=GH$ and $FE=\sqrt{(b-0)^2+(0-a)^2}=\sqrt{b^2+a^2}=\sqrt{(b-2b)^2+(0-a)^2}=EH$.

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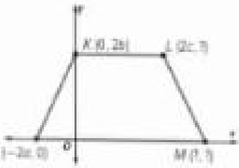
Common Errors

In **Exercise 2**, students may believe the slopes of the two diagonals have the same sign because neither is identified as negative. They may be looking for confirmation that the slope of one of the lines is the negative reciprocal of the other. However, this is not the case when one line is horizontal and the other is vertical because the slope of a vertical line is undefined. Since horizontal lines are perpendicular to vertical lines, these diagonals are perpendicular.

3. **USE STRUCTURE** Isosceles trapezoid *JKLM* is shown to the right.



a. Without introducing new variables, state the coordinates of points *L* and *M*.

L $(2c, 2b)$ *M* $(2c + 2a, 0)$

b. Let point *P* be the midpoint $\overline{JK}$ and *Q* be the midpoint $\overline{LM}$. Use coordinate geometry to show that the midsegment of *JKLM* is parallel to the bases of *JKLM* and equal to half the sum of their lengths.

Sample answer: Let $P = (-a, b)$; $Q = (2c + a, b)$.

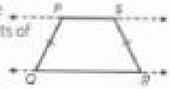
$KL = \sqrt{(2c - 0)^2 + (2b - 2b)^2} = 2c$; slope $\overline{KL} = \frac{2b - 2b}{2c - 0} = \frac{0}{2c} = 0$

$JM = \sqrt{((2c + 2a) - (-2a))^2 + (0 - 0)^2} = 2c + 4a$; slope $\overline{JM} = \frac{0 - 0}{(2c + 2a) - (-2a)} = 0$

$PQ = \sqrt{((2c + a) - (-a))^2 + (b - b)^2} = 2c + 2a$; slope $\overline{PQ} = \frac{b - b}{(2c + a) - (-a)} = 0$

Since $\overline{PQ}$, $\overline{KL}$, and $\overline{JM}$ have the same slope, they are parallel. Also, since $2c + 2a = \frac{1}{2}[(2c) + (2c + 4a)]$, *PQ* is equal to half the sum of *KL* and *JM*.

c. Let point *R* be the midpoint $\overline{JM}$ and *S* be the midpoint $\overline{KL}$. Use coordinate geometry to show that *PSQR*, the quadrilateral connecting the midpoints of *JKLM*, is a rhombus.



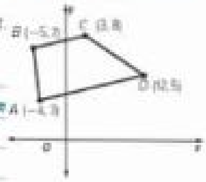
Sample answer: Let $R = (c, 0)$; $S = (c, 2b)$. To show *PSQR* is a rhombus, we must show that it is a parallelogram, which we can do by showing that $\overline{PS}$ and $\overline{QR}$ are parallel and congruent. We must show $\overline{PQ}$ and $\overline{RS}$ are perpendicular.

$PS = \sqrt{(c - (-a))^2 + (2b - b)^2} = \sqrt{(c + a)^2 + b^2}$; slope $\overline{PS} = \frac{2b - b}{c - (-a)} = \frac{b}{c + a}$;

$QR = \sqrt{(2c + a - c)^2 + (b - 0)^2} = \sqrt{(c + a)^2 + b^2}$; slope $\overline{QR} = \frac{b - 0}{2c + a - c} = \frac{b}{c + a}$

slope of $\overline{PQ} = \frac{b - b}{2c + a - (-a)} = \frac{0}{2c + 2a} = 0$; and slope of $\overline{RS} = \frac{2b - 0}{c - c} = \frac{2b}{0}$, which is undefined.

4. **CONSTRUCT ARGUMENTS** Quadrilateral *ABCD* is shown to the right.



a. Show that *ABCD* is a trapezoid.

The slope of $\overline{BC}$ is $\frac{8 - 7}{3 - (-5)} = \frac{1}{8}$, and the slope of $\overline{AD}$ is $\frac{5 - 3}{12 - (-4)} = \frac{1}{8}$. This means that *ABCD* has one pair of opposite sides that are parallel. Therefore, it is a trapezoid.

b. Prove that *ABCD* is not an isosceles trapezoid.

The legs of the trapezoid are $\overline{AB}$ and $\overline{CD}$. Using the distance formula, $AB = \sqrt{17}$ and $CD = 3\sqrt{10}$. This means that $\overline{AB}$ is not congruent to $\overline{CD}$, so *ABCD* cannot be an isosceles trapezoid.

Common Errors

In **Exercise 3**, students may have difficulty finding the *x*-coordinate of point *M*. Encourage students to draw the altitudes from *K* and *L*, dividing the trapezoid into a rectangle and two triangles so that they can see the length of *JM*.

Emphasizing the Standards for Mathematical Practice

Exercise 3 offers a chance to work with **SMP 7 (Look for and make use of structure)** as students may recognize that the midsegment theorem for triangles results when one of the bases in a trapezoid has a length of 0.

Point out to students that a coordinate proof, like a two-column proof, starts with a set of givens. The fact that *JKLM* is isosceles determined the coordinates of the vertices, and as a result they are only proving its validity for isosceles trapezoids in **Exercise 3**. If *JKLM* was not known to be isosceles, at least one additional variable would have had to be used in the coordinates. Ask students to make conjectures about how such a proof would be similar to or different from their work in **Exercise 3**.

11 Performance Task

Identifying a Quadrilateral

Students will use a compass and straightedge to construct a parallelogram and prove when certain requirements guarantee a parallelogram or rhombus.

STANDARDS

Standards for Mathematical Practice: The Chapter 11 Performance Task reinforces Mathematical Practices **SMP 3**, **SMP 5**, and **SMP 6**.

Jump Start

Before students attempt to construct a parallelogram, have them recall the conditions for which a quadrilateral qualifies as a parallelogram.

- What do you need to know about a quadrilateral to determine if it is a parallelogram? **Parallelograms have opposite sides that are congruent, opposite angles that are congruent, consecutive angles that are supplementary, and diagonals that bisect each other.**
- Do you need to establish that the quadrilateral is a parallelogram before you can decide if it is a rhombus? Explain. **Yes. A quadrilateral must be a parallelogram before you can use additional theorems to decide if it is a rhombus.**

Performance Task

Identifying a Quadrilateral

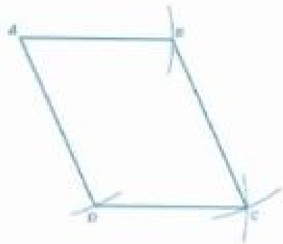
Provide a clear solution to the problem. Be sure to show all of your work, include all relevant drawings, and justify your answers.

You can identify a quadrilateral using the theorems you have learned.

Part A

Construct a parallelogram *ABCD* using a compass and straightedge. Explain your construction and prove why the construction resulted in a parallelogram.

Sample answer:



160 **CHAPTER 11** Quadrilaterals

Emphasizing the Standards for Mathematical Practice

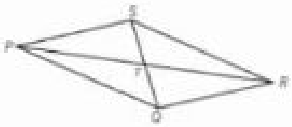
This Performance Task provides a natural connection to **SMP 6 (Attend to precision)**. The standard describes how mathematically proficient students are able to communicate precisely to others and make explicit use of definitions. Here, students use a compass and straightedge to construct a parallelogram. Each student may approach the construction differently. It may be that they try to construct opposite sides to be congruent or they may try to construct diagonals that bisect each other. Each student must be able to justify their construction by method of proof.

Part B

Is your construction a rhombus? Why or why not? If not, how could you alter your construction so that it is a rhombus?

Part C

Quadrilateral $PQRS$ is shown. Prove that if $\triangle PTQ \cong \triangle STR$, then $PQRS$ is a parallelogram.



Part D

Using the same figure from **Part C**, prove that if $PQRS$ is a parallelogram and $\triangle PTQ \cong \triangle PTS$, then $PQRS$ is a rhombus.

Teaching Tip

SMP 3

If students have difficulty proving that a quadrilateral is a parallelogram or rhombus, have them refer back to the definitions and constructions in earlier chapters. Using definitions and previously established results in constructing arguments is part of SMP 3.

Scoring Rubric

Part	Max Points	Full Credit Response
A	3	See *Interactive Student Guide* for drawing. Sample answer: In a parallelogram, each pair of opposite sides is congruent. So, use the compass to find the length $\overline{AB}$. Place the point of the compass at D and draw an arc. Set the compass to the length $\overline{AD}$. Place the point at B and draw an arc. The intersection of the two arcs is point C. Quadrilateral $ABCD$ is a parallelogram because both pairs of opposite sides are congruent.
B	1	Sample answer: It is not a rhombus because $\overline{AB} \neq \overline{AD}$ in my construction. I can alter my construction by first constructing two congruent segments sharing one endpoint, then the parallelogram will have four congruent sides.
C	2	$PR = PT + TR$ and $QS = QT + TS$ by the segment addition postulate. $\overline{PT} \cong \overline{TR}$ and $\overline{QT} \cong \overline{TS}$ by CPCTC. This means the diagonals bisect each other, so $PQRS$ is a parallelogram.
D	2	Because $\triangle PTQ \cong \triangle PTS$, $\overline{PQ} \cong \overline{PS}$ by CPCTC. Also $\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$ because $PQRS$ is a parallelogram, so $\overline{PQ} \cong \overline{PS} \cong \overline{QR} \cong \overline{SR}$. Therefore, $PQRS$ is a rhombus.
Total	8	

11 Performance Task

Using Triangles to Make a Quadrilateral

Students will make two pairs of congruent triangles and use the triangles to make quadrilaterals. They will use their knowledge of quadrilaterals to classify the quadrilaterals.


Performance Task

Using Triangles to Make a Quadrilateral

Provide a clear solution to the problem. Be sure to show all of your work, include all relevant drawings, and justify your answers.

You can make a quadrilateral by combining two triangles.

Part A

Fold a piece of paper, and cut out the corner to create two triangles. Then cut two more triangles from the same corner, starting the cut from the same point. Describe the triangles you made.

Part B

In the space below trace both triangles. Try to make as many different quadrilaterals as you can, without folding or overlapping the triangles. Indicate whether any sides or angles are congruent with appropriate symbols.

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STANDARDS

Standards for Mathematical Practice: The Chapter 11 Performance Task reinforces Mathematical Practices **SMP 3**, **SMP 5**, and **SMP 6**.

Jump Start

Before students make their pair of congruent triangles, have them review classifying triangles by side length and angle measure.

- What do you know about the side lengths and angle measures of a right triangle? **I know that one angle is 90°, and the hypotenuse is the longest side.**
- What do you know about the side lengths and angle measures of a scalene triangle? **Each side has a different length; therefore, each angle must be different. The angles must still add up to 180 degrees.**

Emphasizing the Standards for Mathematical Practice

This Performance Task provides a natural connection to **SMP 5 (Use appropriate tools strategically)**. The standard describes how mathematically proficient students are able to us mathematical tools to solve a mathematical problem. In this performance task, students use paper folding and cutting to form triangles and assemble them into quadrilaterals. You may also elect to have students use a ruler or protractor to confirm measurements of the quadrilaterals formed.

Part C

Classify each quadrilateral that you made. Justify each classification with theorems or definitions. Explain your reasoning.

Part D

Do you think the type of triangle you start with limits the types of quadrilaterals you can make? Justify your answer with drawings or sentences.

Differentiating Instruction

You may wish to have students consider how their answers change if they are allowed (or not allowed) to flip the triangles over. The task as written allows for the students to explore this on their own.

Common Errors

Students may assume information from looking at the quadrilateral. Students must instead apply theorems based on parallel lines or congruent angles and sides. Check that students are aligning the same sides of each triangle, and not two sides that are approximately equal, due to the way the triangles were created. It cannot be assumed, for example, that one of the pairs of triangles is isosceles.

Scoring Rubric

Part	Max Points	Full Credit Response
A	2	Two right triangles; two obtuse triangles.
B	2	Students have quadrilaterals drawn on their paper with the correct sides or angles marked.
C	2	Students have correctly identified each quadrilateral as a quadrilateral, parallelogram, rectangle, or kite. Students have justified their classifications.
D	2	Students have justified what types of triangles can be used to make different types of quadrilaterals. Students should not conclude that a rhombus (or square) can be formed, because no triangles were proven to be isosceles.
Total	8	

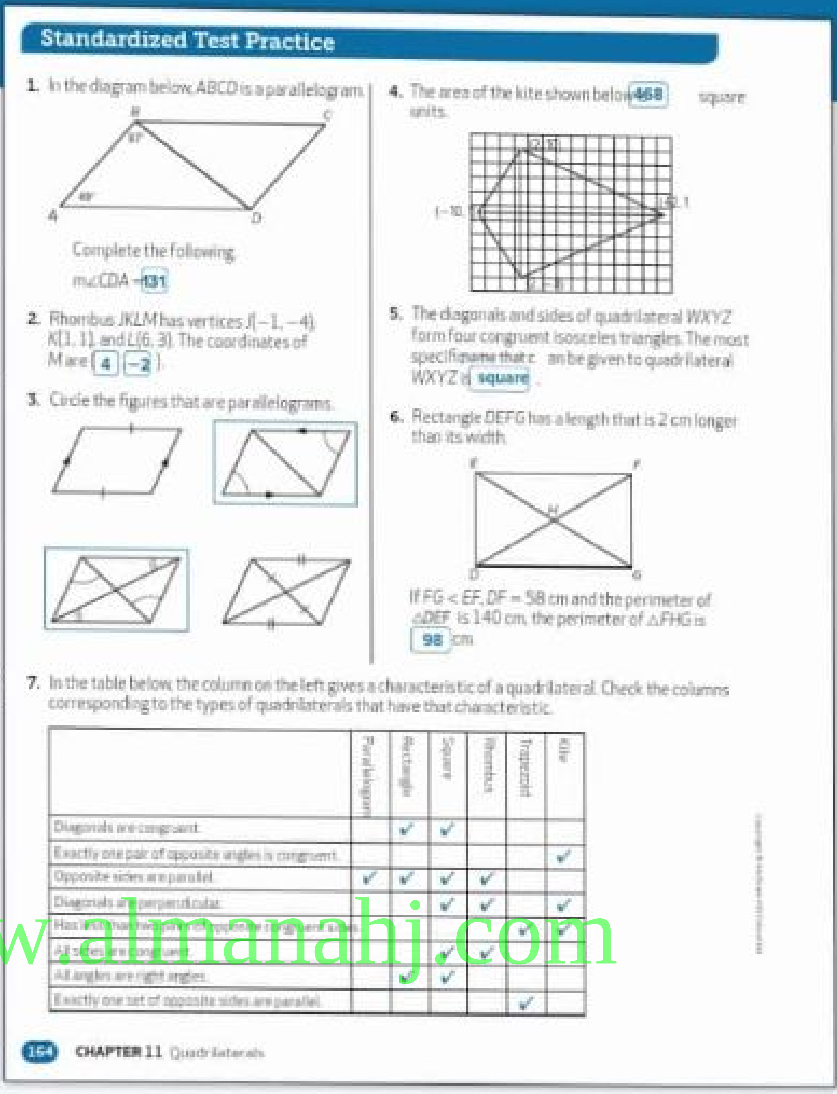
Standardized Test Practice

Diagnosing Errors

Students who select the first quadrilateral in **Item 3** may believe that having one set of opposite sides parallel and another set of opposite sides congruent is enough to prove that a quadrilateral is a parallelogram. Demonstrate that an isosceles trapezoid also has these characteristics.

Students who give an answer of 420 for **Item 4** may have used the x-coordinate of the rightmost vertex and the y-coordinate of the top vertex as the lengths of the diagonals, rather than subtracting the coordinates of the endpoints of the diagonals to determine their length.

Students who check Trapezoid for "Diagonals are congruent" in **Item 7** may be thinking of an isosceles trapezoid. Remind them that in order to place a check mark under the name of a shape, that characteristic must be true for all instances of that shape.



Standardized Test Practice

1. In the diagram below, ABCD is a parallelogram.

Complete the following.

m∠CDA = 131

2. Rhombus JKLM has vertices J(−1, −4), K(1, 1), and L(6, 3). The coordinates of M are (4, −2).

3. Circle the figures that are parallelograms.

4. The area of the kite shown below is 468 square units.

5. The diagonals and sides of quadrilateral WXYZ form four congruent isosceles triangles. The most specific name that can be given to quadrilateral WXYZ is square.

6. Rectangle DEFG has a length that is 2 cm longer than its width.

If FG < EF, DF = 58 cm and the perimeter of △DEF is 140 cm, the perimeter of △FHG is 98 cm.

7. In the table below, the column on the left gives a characteristic of a quadrilateral. Check the columns corresponding to the types of quadrilaterals that have that characteristic.

	Parallelogram	Rectangle	Square	Rhombus	Trapezoid	Kite
Diagonals are congruent.		✓	✓			
Exactly one pair of opposite angles is congruent.						✓
Opposite sides are parallel.	✓	✓	✓	✓		
Diagonals are perpendicular.			✓	✓		✓
Has [illegible]					✓	✓
All sides are congruent.			✓	✓		
All angles are right angles.		✓	✓			
Exactly one set of opposite sides are parallel.					✓	

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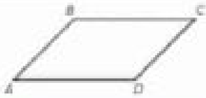
Test-Taking Strategy

For **Item 2**, students will find the problem much simpler if they graph the given points. Remind them that a rhombus has four congruent sides and opposite sides parallel. They can use these characteristics and what they know about slope to find the missing vertex.

8. Complete the steps and reasons in the following proof.

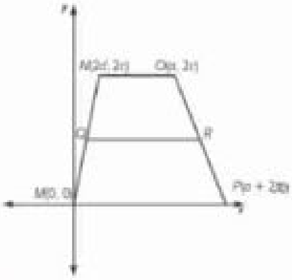


Given: $\angle A$ is supplementary to $\angle B$.
$\angle A$ is supplementary to $\angle D$.

Prove: $ABCD$ is a parallelogram

Statement	Reason
$\angle A$ is supplementary to $\angle B$.	**Given**
$\overline{AD} \parallel \overline{BC}$	Converse of the Consecutive Interior Angles Theorem
$\angle A$ is supplementary to $\angle D$.	Given
$\overline{AB} \parallel \overline{CD}$	**Converse of the Consecutive Interior Angles Theorem**
$ABCD$ is a parallelogram	**Definition of Parallelogram**

9. In the diagram below, $MNOP$ is a trapezoid. Q is the midpoint of $\overline{MN}$ and R is the midpoint of $\overline{OP}$.



a. What are the coordinates of Q? Show your work.
(d, c); Using Midpoint Formula,
$\left(\frac{0 + 2d}{2}, \frac{0 + 2c}{2}\right) = (d, c)$.

b. What are the coordinates of R? Show your work.
$(a + b, c)$; Using Midpoint Formula,
$\left(\frac{a + 2b + a}{2}, \frac{0 + 2c}{2}\right) = (a + b, c)$

c. Show that $QR = \frac{NO + MP}{2}$.
$NO = a - 2d$, $QR = a + b - d$, $MP = a + 2b$
$\frac{NO + MP}{2} = \frac{a - 2d + a + 2b}{2} = a - d + b =$
$a + b - d = QR$

10. The coordinates of the vertices of quadrilateral $LMNP$ are $L(1, 7)$, $M(4, 3)$, $N(3, 1)$, and $P(1, 2)$.

a. Find the length and slope of each side of $LMNP$.
$LM = 5$ and has slope $-\frac{4}{3}$, $MN = \sqrt{5}$ and has slope 2, $NP = \sqrt{5}$ and has slope $-\frac{1}{2}$, $LP = 5$ and has undefined slope.

b. Classify $LMNP$ as a parallelogram, rhombus, trapezoid, kite, or square. Explain your reasoning.
$LMNP$ is a kite because it has exactly two pairs of congruent consecutive sides.

c. Verify that the diagonals of $LMNP$ intersect at right angles by finding the slope of each diagonal and confirming that the two diagonals are perpendicular.
The slope of $\overline{LN}$ is -3 and the slope of $\overline{MP}$ is $\frac{1}{3}$. Because the slopes are negative reciprocals, the diagonals are perpendicular, which means they intersect at right angles.

Test-Taking Strategy

For **Item 9**, students should write down the midpoint formula in the margin to help them focus on the problem. If they have forgotten the exact formula, remind them that the midpoint is the average of the x and y coordinates of the two endpoints, and help them derive the formula.

Diagnosing Errors

Students who identify the wrong sides as parallel in the second step in **Item 8** might find it helpful to extend the sides of the parallelogram. This will help them identify which sides are acting as the parallel lines, and which side is acting as the transversal.

Students who incorrectly calculate the segment lengths for **Item 9c** may not have realized that since the endpoints have the same y-coordinate, the segments are horizontal. Therefore, they can find the length by simply subtracting the x-coordinates.

Rubrics

Item 9

[5] Coordinates of Q are R are correct with work shown. QR, NO, and MP are correctly computed and used in **part c**.
[4] Minor error in work in one of the three parts.
[3] Coordinates of Q and R are correct with work shown, but **part c** is incorrect.
[2] Coordinates of Q and R are correct
[1] Coordinates of Q or R are correct OR NO and MP are correctly computed in **part c**.
[0] no response OR incorrect answer and work

Item 10

[4] All answers correct using correct reasoning.
[3] Incorrectly calculated slope or side length on **part a** but used correctly in **part b** OR incorrect reasoning for **part b**.
[2] One part incorrect
[1] At least one correct component
[0] no response OR incorrect answer