27

FILTRATION

27.1 INTRODUCTION

Filtration is one of the most common applications of the flow of fluids through packed beds. As carried out industrially, it is similar to the filtration carried out in the chemical laboratory using a filter paper in a funnel. The object is still the separation of a solid from the fluid in which it is carried and the separation is accomplished by forcing the fluid through a porous filter. The solids are trapped within the pores of the filter and (primarily) build up as a layer on the surface of this filter. The fluid, which may be either gas or liquid, passes through the bed of solids and through the retaining filter.

As noted above, solid particles are removed from a slurry in a filtration process by passing it through a filtering medium. (According to Webster, a slurry is defined as a "watery mixture of insoluble matter such as mud, lime or plaster of Paris.") The solids are deposited on the filtering medium, which is normally referred to as the filter. Filtration may therefore be viewed as an operation in which a heterogeneous mixture of a fluid and particles of a solid are separated by a filter medium that permits the flow of the fluid but retains the particles of the solid. Therefore it primarily involves the flow of fluids through porous media (see Chapter 25).

In all filtration processes, the mixture or slurry flows as a result of some driving force, that is, gravity, pressure (or vacuum), or centrifugal force. In each case, the filter medium supports the particles as a porous cake. This cake, supported by the filter medium, retains the solid particles in the slurry with successive layers added to the cake as the filtrate passes through the cake and medium.

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The several procedures for creating the driving force on the fluid, the different methods of cake deposition and removal, and the different means for removal of the filtrate from the cake subsequent to its formation, result in a great variety of filter equipment. In general, filters may be classified according to the nature of the driving force supporting filtration. The various equipment are described in the following section.

27.2 FILTRATION EQUIPMENT

There are various types of filtration equipment employed by industry. Included in this list are gravity filters, plate-and-frame filters, leaf filters, and rotary vacuum filters. Gravity filters are the oldest and simplest type. These filters consist of tanks with perforated bottoms filled with porous sand through which the slurry passes. The most common type, however, is the plate-and-frame filter where plates are held rigidly together in a frame. More details are provided in subsequent paragraphs. Leaf filters are similar to the plate-and-frame filters in that a cake is deposited on each side of the leaf and the filtrate flows to the outlet in the channels provided by the coarse drainage screen in the leaf between the cakes. The leaves are immersed in the slurry. Rotary vacuum filters are used where a continuous operation is desirable, particularly for large-scale operations. The filter drum is immersed in the slurry where a vacuum is applied to the filter medium that causes the cake to deposit on the outer surface of the drum as it passes through the slurry.

The plate-and-frame filter press is perhaps the most widely used type of filtering devices in the chemical industry. Plate-and-frame filter presses are used in a variety of industries. The chemical industry uses the filter press in order to separate the solid portion of slurry from the liquid. A chemical, for example zinc, builds up on the frames. The filter press is then opened and the wet cake, containing solid zinc, can be collected, removed and dried. The pharmaceutical industry also uses the filter press in similar applications. The solids collected on the inside of the frames can be dried and later sold as medication. The sugar industry also employes the filter press to separate solid sugar from a solution. A slurry is sent through a filter press and the solid cake is collected on the frames. This solid is later dried, crystallized, and sold as sugar. In the pottery industry, the filter press is used in order to make ceramic pieces. A slurry is sent though the filter press and the solid cake is collected. This cake is then used for the production of various pottery products. The filter press is also used in the wastewater treatment industry. A waste stream containing sludge is passed through a filter press. Once the solids are removed, a smaller volume of "liquid" will have to be disposed of. There are stringent rules that govern the disposal of liquid waste (see Chapter 28). However, it is easier to dispose of solid waste since it can be sent to a landfill. Therefore, the filter press reduces the amount of liquid waste that needs to be treated before it can be disposed of.(1,2)

Regarding the filter press, feed slurry is pumped to the unit under pressure and flows in the press and into the bottom-corner duct (see Fig. 27.1). This duct has outlets into each of the frames, so the slurry fills the frames in parallel. The plates

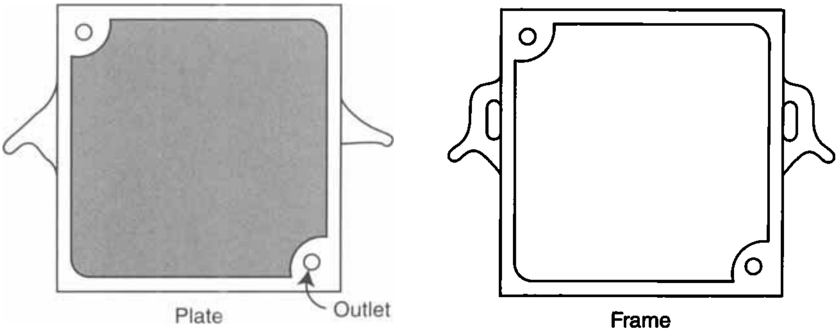

Figure 27.1 Plate and frame schematics.

and frames are assembled alternately with filter cloths over each side of each plate. The assembly is held together as a unit by mechanical force applied hydraulically or by a screw. The solvent, or filtrate, then flows through the filter media while the solids build up in a layer on the frame side of the media. The filtrate flows between the filter cloth and the face of the plate to an outlet duct. As filtration proceeds, the cakes build up on the filter cloths until the cake being formed on each face of the frame meet in the center. When this happens, the flow of filtrate, which has been decreasing continuously as the cakes build up, drops off abruptly to a trickle. Usually filtration is stopped well before this occurs.$^{(3)}$

A photograph of a plate and frame filter located in the chemical engineering lab on the fourth floor of the Manhattan College Leo Engineering Building is provided in Figs. 27.2 and 27.3. It has a total of 18 plates and frames. The layout of these frames and plates can be seen in Fig. 27.4.

Figure 27.2 Filter press experiment: front view.

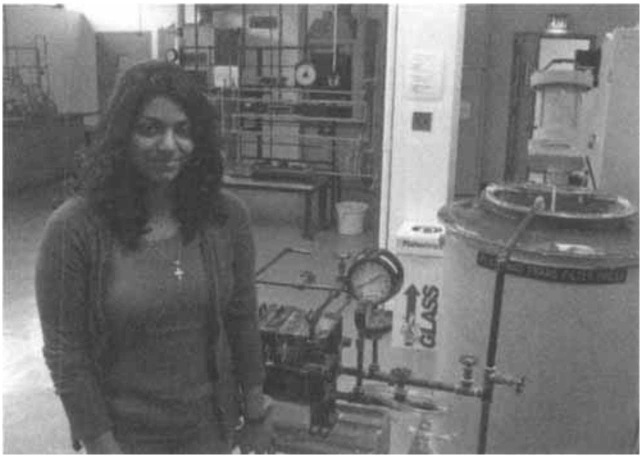

Figure 27.3 Filter press experiment: side view.

Figure 27.4 Frame and plate layout.

The process where the slurry flows through the press can be seen in Fig. 27.5. The slurry enters the lower right-hand side of frame 18. When frame 18 is filled with slurry, the excess slurry is forced through the filter media to the upper right-hand side of plate 17. When the slurry is in frame 18, cake builds up on the filter media. It then flows into plate 16. When in frame 16, the slurry flows down because of gravity. When the frame is filled with slurry, the cake forms and the filtrate is sent to the upper right-hand side of frame 16 and out through plate 15. Plate 15 leads to the filtrate collecting drum outside of the press.

A filter operation can be carried out using a centrifugal force rather than the pressure force used in the equipment described above. Filters using centrifugal

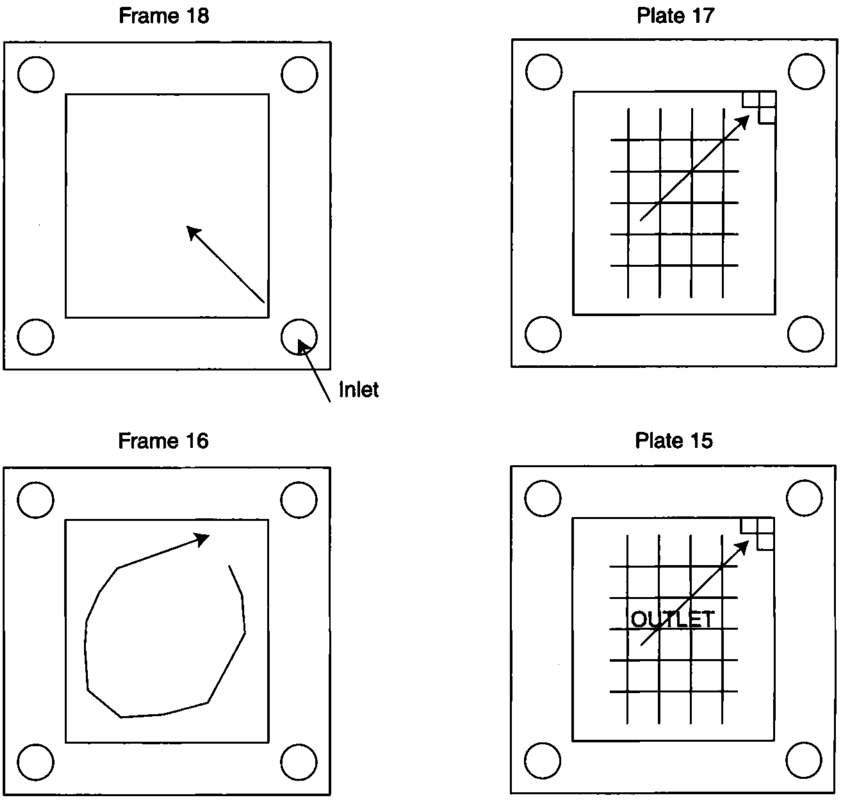

Figure 27.5 Flow of the slurry through the filter press.

force are usually used in the filtration of coarse granular or crystalline solids and are available primarily for batch operation.

Batch centrifugal filters most commonly consist of a basket with perforated sides rotated around a vertical axis. The slurry is fed into the center of the rotating basket and is forced against the basket sides by centrifugal force. There, the liquid passes through the filter medium, which is placed around the inside surface of the basket, and is caught in a "shielding" vessel, referred to as a curb, within which the basket rotates. The solid phase builds up a filter cake against the filter medium. When this cake is thick enough to retard the filtration to an uneconomical rate or to endanger the balance of the centrifuge, the operation is halted, and the cake is scraped into a bottom discharge or is scooped out of the centrifuge. In the *automatically discharging batch centrifugal filter*, unloading occurs automatically while the centrifuge is rotating, but the filtration cycle is still a batch one.(3)

Another process device that some have classified as a "hybrid" filtration unit is the mist eliminator. This class of separator allows gas (usually air) to pass through the unit while captured liquid droplets are returned to the emitting process equipment. These units are widely used to prevent the emission of undesirable amounts of liquid droplets from scrubbers and absorbers. An unfortunate consequence of intimate and vigorous liquid–gas contacting in the scrubber is that some of the scrubbing liquor is atomized, entrained by the gas that has been cleaned, and discharged from the unit. The droplets carried over by entrainment generally contain both suspended and dissolved solids. In many cases, excessive entrainment imposes a limitation upon the capacity of the scrubber. The describing equations to follow for the traditional filtration equipment bear no resemblence to those for mist eliminators.(4)

Irrespective of the type of equipment employed, washing and dewatering operations are usually accomplished with the filter cake in place. Since the cake thickness is now unchanged, the wash rate usually varies directly with the pressure drop. If the wash water follows the same path as the slurry and is fed at the same pressure, the wash rate will then be approximately equal to the final filtration rate. In the dewatering part of a filtration cycle, fluid is drawn through the filter cake, pulling the filtrate or wash water remaining in the pores of the cake out ahead of it.

As discussed earlier, filter media consisting of cloth, paper, or woven or porous metal may be used. The criteria upon which a filter medium is selected must include ability to remove the solid phase, high liquid throughput for a given pressure drop, mechanical strength, and chemical inertness to the slurry to be filtered and to any wash fluids.

Each of these considerations is tempered by the economics involved, so that the design engineer attempts to choose a medium that meets the required filtration requirement while contributing to the lowest possible overall filtration cost.

Filter-cake solids usually penetrate the filter medium and fill some of the pores. As filtration continues, these particles are thought to bridge across the pores as the cake begins to form on the face of the medium. As a result, the resistance to flow through the medium increases sharply. In some cases, the solids fill the filter medium to such an extent that the filtration rate is seriously reduced.

Of all the various filtration equipment, the plate-and-frame filter press is probably the cheapest per unit of filtering surface and requires the least floor space. However, the cost of labor for opening and dumping such presses is high, particularly in the large sizes. For this reason, they are not chosen when a large quantity of worthless solid is to be removed from the filtrate. If the solids have high value as in the pharmaceutical industry, the cost of labor per unit value of product is relatively low and the plate-and-frame filter press proves satisfactory. It has a high recovery of solids, and the solid in the form of a cake may be readily handled in a tray or shelf drier, which is frequently used for valuable products. The leaf filter offers the advantages of ease of handling, minimum labor with efficient washing, and discharge of cake without removing any leaves from the filter. The rotary continuous filter offers the additional advantages of continuous and automatic operation for feeding, filtering, washing, and cake discharge.

More recently, membranes have arrived on the scene. Although a detailed discussion on this subject is beyond the scope of this book, a brief introduction to membranes follows. A membrane can be described as a physical barrier between two fluids. In current industrial practice, membranes are typically made from polymers. Membrane separation is an integral technique among the various methods employed in industry for the separation of various materials and fits in well at the molecular level in food and drug processing applications. Ultrafiltration, reverse osmosis, and electrolyte dialysis are the membrane separation techniques that have been efficiently employed by industry to purify and concentrate desired products at low temperatures while maintaining basic qualities. For example, ultrafiltration is a membrane separation technique whereby a solution is introduced on one side of a membrane barrier, and water, salts, and low molecular weight materials pass through the unit under an applied pressure. Membrane separation processes can also be used to concentrate single solutes or mixtures of solutes. The most promising area for the expansion of ultrafiltration process applications is in the biochemical area (see Chapter 34). Some of its usage in this area includes purifying vaccines and blood fractions.

27.3 DESCRIBING EQUATIONS

Regarding the filtration process, two important considerations need to be addressed: pressure drop and filtration efficiency. Industry normally relies on certain simple guidelines and calculations to insure satisfactory separation efficiency. Although not discussed in this section, it will receive some treatment in the following chapter (Chapter 28—Environmental Concerns). Pressure drop, however, is another matter.

Perhaps the most convenient starting point to describe flow through porous media is with the classic Darcy equation:

$$\frac{dP}{dz} = \frac{-\mu v_s}{K} \tag{27.1}$$

where v_s is the superficial velocity in the filter and μ is the slurry viscosity. This may be integrated (for constant coefficients) to give

$$\frac{\Delta P}{L} = \frac{\mu v_s}{K} \tag{27.2}$$

where L is the filter thickness. The term K is defined as the permeability coefficient for the filtering medium in consistent units. See the permeability section (Section 26.3) in Chapter 26 for additional details.

Darcy's equation may be rearranged and written as

$$v_s = -\frac{K}{\mu}\frac{dP}{dz} \tag{27.3}$$

Multiplying both sides of this equation by the approach (face) area of the filter, S, gives

$$v_s S = q = -\frac{SK}{\mu}\frac{dP}{dz} \tag{27.4}$$

where q is the volumetric flow rate of the slurry passing S. Integrating yields,

$$\Delta P = \left(\frac{\mu \Delta z}{KS}\right) q \tag{27.5}$$

The term in parentheses represents the resistance to flow. It is a constant for the fabric since Δz is simply the filter thickness, L. The pressure drop across the fabric medium is then

$$\Delta P = \left(\frac{\mu L}{KS}\right) q \tag{27.6}$$

The development for the pressure drop across the filter cake is similar to that for the fabric. The bracketed term in Equation (27.5) is not a constant for the deposited particles since Δz is a variable. If the collection efficiency is close to 100%, which is usually the case in an industrial operation, then

$$\Delta z = \frac{V_s c}{\rho_p (1 - \varepsilon) S} = \frac{V_s c}{\rho_B S} \tag{27.7}$$

where V_s is the volume of slurry filtered for deposit thickness Δz, c is the inlet solids loading or concentration (mass/volume), ε is the void volume or void fraction, ρ_p is the true density of the solid and ρ_B is the bulk density of the cake of deposited solids.

Substituting Equation (27.7) into Equation (27.5) yields an equation for the pressure drop across the cake

$$\Delta P = \left(\frac{\mu c}{\rho_B K S^2}\right) V_s q \tag{27.8}$$

It is assumed that Equations (27.6) and (27.8) are additive (i.e., the solids and fabric do not interact). Thus, the total or overall pressure drop across the filter system is obtained by combining these two equations:

$$\Delta P = \left(\frac{\mu c}{\rho_B K S^2}\right) V_s q + \left(\frac{\mu L}{KS}\right) q \tag{27.9}$$

Setting

$$B = \left(\frac{\mu c}{\rho_B K S^2}\right)$$

$$C = \left(\frac{\mu L}{KS}\right)$$

then

$$\Delta P = (B V_s + C) q \tag{27.10}$$

This represents the general relationship between ΔP, q, and V_s.

During a constant (overall) pressure (drop) operation, the flow rate is a function of time

$$q(t) = \frac{\mathrm{d}V_s}{\mathrm{d}t} \tag{27.11}$$

Substituting $q(t)$ into Equation (27.10) yields

$$\Delta P = (BV_s + C)\frac{\mathrm{d}V_s}{\mathrm{d}t}$$

or

$$(\Delta P)\mathrm{d}t = (BV_s + C)\mathrm{d}V_s \tag{27.12}$$

Integrating this from 0 to t and from 0 to V_s and solving for t leads to

$$t = \left(\frac{BV_s^{\,2}}{2\Delta P}\right) + \left(\frac{CV_s}{\Delta P}\right) \tag{27.13}$$

This equation can be rearranged so that V_s is an explicit function of t

$$V_s = \sqrt{\frac{2(\Delta P)t}{B} + \left(\frac{C}{B}\right)^2} - \left(\frac{C}{B}\right) \tag{27.14}$$

Also, V_s can be shown to be related to an instantaneous value of $q(t)$ by rearrangement of Equation (27.10)

$$V_s = \left(\frac{(\Delta P)t}{q(t)}\right) - \left(\frac{C}{B}\right) \tag{27.15}$$

Equating Equations (27.14) and (27.15) and solving for q yields

$$q = \frac{\Delta P}{B\sqrt{\frac{2(\Delta P)t}{B} + \left(\frac{C}{B}\right)^2}} \tag{27.16}$$

Numerical values for design coefficients B and C are usually obtained from experimental data. If Equation (27.13) is rewritten as

$$\frac{t}{V_s} = \left(\frac{BV_s}{2\Delta P}\right) + \left(\frac{C}{\Delta P}\right) \tag{27.17}$$

a plot of t/V_s vs. V_s will yield a straight line of slope $(B/2\Delta P)$ and an intercept $(C/\Delta P)$. Note that only two (V_s, t) data points are necessary to provide a first approximation of B and C.

Some industrial filter operations are conducted in a manner approaching constant flow rate. For this condition,

$$dq/dt = 0$$

and

$$q = dV_s/dt = \text{constant}$$

so that

$$V_s = qt$$

Equation (27.10) now becomes

$$\Delta P = Bq^2t + Cq \tag{27.18}$$

Thus, a plot of ΔP vs. t yields a straight line of slope Bq^2 and intercept Cq. At $t = 0$, the only resistance to flow is that of the filter medium; the pressure drop, however, is a linear function of time, and as time increases, the resistance due to the cake may predominate.

Some filter operations have both a constant pressure and constant rate period. At the beginning of a normal cycle, the pressure drop is held constant until the flow rate increases to a maximum value that is obtained by experiment. The flow rate is then maintained constant until the pressure drop increases above a predetermined limit that may be dictated by economics.

The coefficients B and C above have appeared in revised form in the literature in an attempt to relate them to physically measurable quantities. Equation (27.17) has been rewritten as:

$$\frac{t}{V_s} = \frac{K_c}{2}V_s + \frac{1}{q_0} \tag{27.19}$$

where K_c and q_0 are constants and (in English units), with

$$K_c = \frac{\mu c \alpha}{S^2 g_c \Delta P} \tag{27.20}$$

and

$$\frac{1}{q_0} = \frac{\mu R_m}{S g_c \Delta P} \tag{27.21}$$

Note that the S (once again) is the total surface of filtration cakes in the system (ft^2), α is the specific cake resistance (ft/lb) and R_m is the filter medium resistance (ft^{-1}). As before, a plot of t/V_s vs. V_s will give a straight line with a slope of $K_c/2$. This will allow the calculation of the specific cake resistance, α. Also, the y-intercept of the plot will be $1/q_0$, which means that filter medium resistance, R_m, can be calculated.

Theodore and Buonicore have developed equations to predict coefficients B and C (or α and R_m) from basic principles.(2) Despite the progress in developing pure filtration theory, and in view of the complexity of the phenomena, the most common methods of correlation are based on predicting a form of a final equation that can be verified by experiment.

Illustrative Example 27.1 Qualitatively, explain why the flow rate decreases during constant pressure filtration.

Solution The flow rate is a function of both the pressure drop and the resistance to flow. During filtration, the solids build up within the filter and form a cake, which increases the resistance to flow. In addition, because of the buildup within the filter, the porosity decreases, further increasing the resistance to flow.

Illustrative Example 27.2 A plate-and-frame filter press is to be employed to filter a slurry containing 10% by mass of solids. If 1 ft^2 of filter cloth area is required to treat 5 lb/h of solids, what cloth area, in ft^2, is required for a slurry flowrate of 600 lb/min?

Solution Convert the slurry flowrate, $\dot{m}$, to lb/h:

$$\dot{m}\ (\text{slurry}) = (600\,\text{lb/min})(60\ \text{min/h}) = 36{,}000\,\text{lb/h}$$

Calculate the solids flowrate in the slurry:

$$\dot{m}\ (\text{solids}) = (0.1)(36{,}000\,\text{lb/h}) = 3600\,\text{lb/h}$$

Calculate the filter cloth area, A, requirement:

$$A = (3600\,\text{lb/h})\left(\frac{1}{5}\frac{\text{h}\cdot\text{ft}^2}{\text{lb}}\right) = 720\,\text{ft}^2$$

Illustrative Example 27.3 Engineers have designed a plate-and-frame filter press for a new plant. The design is based on pilot plant data, and the unit in the pilot plant was operated under conditions identical to those intended for the large plant. Engineers assumed that the filtration cycle would allow for cleaning and operating at constant pressure. Their design will provide an average filtration rate approximately equal to the required capacity. To add a factor of safety, Engineer A wants to use 25% more frames; B wants to make each frame 25% longer; C wants to make each frame with 25% more area; D wants to use 25% more filtration pressure. Rank these proposals in the order in which you consider they would increase the average hourly capacity of the filter. Explain and justify your rank by the use of appropriate theory or equations. For purposes of analysis, neglect the resistance of the filter.

Solution Refer to Equations (27.17)–(27.21) and note that $C = 0$. Engineer A's plan to use 25% more frames will increase the collection area by a factor of 1.25. The time of filtration will be decreased by a factor of $(1.25)^2$ since B is inversely proportional to the area squared. If the filtration capacity is

$$q = \frac{V_s}{t + t_c} \qquad t_c = \text{cleaning time}$$

The improved rate, QS, to insure a factor of safety, is then

$$QS = \frac{V_s}{[t/(1.25)^2] + t_c}$$

If cleaning time is neglected, one notes the rate is increased by a factor of 1.56, that is,

$$QS = 1.56\,q$$

The same improved rate is obtained using the plans of engineers B and C. For plan D, the time of filtration is decreased by 1.25. The improved rate is

$$QS = \frac{V_s}{(t/1.25) + t_c} = 1.25q$$

The proposals are therefore ranked (1) a, b, c (tied) and (2) d.

Illustrative Example 27.4 The following data were obtained during two constant pressure runs conducted on a plate-and-frame filter press. Calculated values for t/V are also included. The filtration area values for t/V are also included. The filtration area of the unit is 0.35 ft^2 and the slurry concentration (of solids) per volume of filtrate was previously calculated to be 4.142 lb/ft^3. See Table 27.1. Calculate the coefficients K_c and $(1/q_0)$.

Solution A plot of t/V vs. V (for the run at 20 psig) is shown in Fig. 27.6.

The slope of this graph is $K_c/2$. Thus,

$$\begin{aligned} \frac{K_c}{2} &= 2.4285\ \text{s/L}^2 \\ &= 1947\ \text{s/ft}^6 \\ K_c &= 2(1947) \\ &= 3894\ \text{s/ft}^6 \end{aligned}$$

Also, the y-intercept is $1/q_0$. By extrapolation,

$$\begin{aligned} \frac{1}{q_0} &= 7.6715\ \text{s/L} \\ &= 217\ \text{s/ft}^3 \end{aligned}$$

Table 27.1 Illustrative Example 27.4

Filtrate		Run #1; 20 psig	Run #2; 15 psig
Weight, kg	Volume, L	Time, s	Time, s
0.0	0.0	0	0
0.5	0.5	7	9
1.0	1.0	13	14
1.5	1.5	19	20
2.0	2.0	27	27
2.5	2.5	37	36
3.0	3.0	47	46
3.5	3.5	59	57
4.0	4.0	72	69
4.5	4.5	86	83
5.0	5.0	101	98
5.5	5.5	118	115
6.0	6.0	135	133
6.5	6.5	154	152
7.0	7.0	175	172
7.5	7.5	202	194
8.0	8.0	229	220
8.5	8.5	259	
9.0	9.0	291	

[a] Calculate R_m and α for the 20 psig run.

Illustrative Example 27.5. Refer to the previous example. Calculate the filtration coefficients R_m and α. For water at 2°C, the viscosity (see Table A.2 in the Appendix) is converted to

$$\mu = 5.95 \times 10^{-4}\ \text{lb/ft} \cdot \text{s}$$

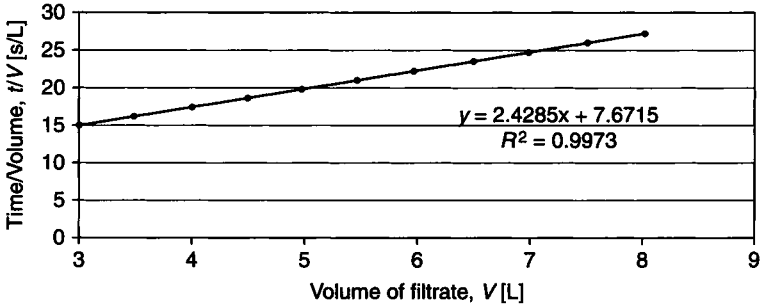

Figure 27.6 Relationship between t/V and V for a Pressure Drop of 20 psig; Illustrative Example 27.4.

Equation (27.21) is rearranged and solved for R_m

$$R_m = \frac{1}{q_0}\frac{Sg_c\Delta P}{\mu}$$

Substitution yields

$$R_m = \frac{1}{q_0}\frac{Sg_c\Delta P}{\mu} = (217)\frac{(0.35)(32.174)(20)(144)}{5.95 \times 10^{-4}} = 1.183 \times 10^{10}\,\text{ft}$$

Similarly, Equation (27.20) is solved for α

$$\alpha = \frac{K_c S^2 g_c \Delta P}{c\mu} = \frac{(3894)(0.35)^2(32.174)(20)(144)}{(4.142)(5.95 \times 10^{-4})} = 1.79 \times 10^{10}\,\text{ft/lb}$$

Illustrative Example 27.6 A filter unit is run at a constant rate for 30 min during which time 3000 ft^3 of slurry from a cement kiln operation is processed. The initial and final pressure in the unit is 0.5 and 5.0 in H_2O gauge, respectively. If the filter is further operated for 30 min at the final pressure, calculate the quantity of additional slurry treated.

Solution For constant rate filtration, Equation (27.18) is employed:

$$\Delta P = Bq^2t + Cq$$

At time zero, the data indicates

$$0.5\,\text{in H}_2\text{O} = 0 + C(100\,\text{ft}^3/\text{min})$$

so that the value of C is 0.005 in $H_2O \cdot \text{min/ft}^3$.

Substituting the data at t equal to 30 min gives:

$$5.0 = B(100)^2(30) + 0.005(100) \quad \text{or}$$

$$B = 1.5 \times 10^{-5}\,\text{in H}_2\text{O} \cdot \text{min/ft}^3$$

At constant pressure, the describing equation is given by Equation (27.13):

$$t = (B/2\Delta P)V_s^2 + (D/\Delta P)V_s$$

where D represents not only the resistance of the filter but also the resistance of the solids deposited during the constant rate period. The coefficient B remains the same since it is still proportional to the quantity of solids deposited during this constant pressure period. Coefficient D is calculated from Equation (27.10):

$$\Delta P = (BV_s + D)q$$

Since $\Delta P = 5$ in H_2O at $V_s = 0$, one obtains

$$D = 0.05 \text{ in } H_2O \cdot \text{min}/\text{ft}^3$$

After 30 min of constant pressure filtration

$$30 = (1.5 \times 10^{-5}/10)V_s^2 + (0.05/5)V_s$$

Solving for the positive root,

$$V_s = 2200 \text{ ft}^3$$

Therefore, the total quantity of slurry filtered is

$$(100)(30) + 2200 = 5200 \text{ ft}^3$$

27.3.1 Compressible Cakes

In actual practice, the deposited cake is usually assumed to be incompressible. However, all cakes are compressible to some degree. For large pressure drops, these effects can become important. These large changes in pressure tend to force the solids further into the interstices in the filtering medium, thereby increasing the resistance to flow and the value of α. If α is not constant, but is a function of ΔP, the cake is referred to as compressible. The cake resistance, α, is one of the more important variables in filtration applications; it is dependent on a host of factors including the filter area, pressure drop, viscosity, etc.

For the above situation, the following experimental relationship between α and ΔP is often assumed to apply:

$$\alpha = \alpha_0 \Delta P^b \qquad (27.22)$$

where a and b are empirically assumed constants that can be obtained from a best-fit straight line on a log-log plot of α vs. ΔP. The term α_0 is usually referred to as the specific cake resistance at zero pressure and b is the compressibility factor for the cake. Note that the term b is zero for incompressible (i.e., the cake resistance does not vary with pressure) sludges and is positive for compressible ones.

Constant pressure experiments are often used to determine the two coefficients of the cake. The first step in this process is to generate a logarithmic graph of α vs. ΔP. Note that the logarithmic form of Equation (27.22) indicates that if the data is regressed linearly, the slope of the regression equation will equal the value of b.

$$\log(\alpha) = \log(\alpha_0) + b \log(\Delta P) \qquad (27.23)$$

If a pressure drop and corresponding value of cake resistance (α) are obtained from the graph then a can then be solved mathematically since it is the only unknown.

Illustrative Example 27.7 The following results were obtained during the running of a filtration experiment in the Unit Operations Laboratory at Manhattan College. The equation of the linear regression line for the plot of cake resistance vs. pressure drop of the experimental filter cake was determined to be:

$$y = 0.210x + 10.99$$

where

$$y = \log \alpha$$
$$10.99 = \log \alpha_0$$
$$0.210x = b \log \Delta P$$

The units for α and ΔP are ft/lb and $\mathrm{lb_f/ft^2}$, respectively. Based on the above results, calculate the specific cake resistance in ft/lb if the cake resistance is 4.57×10^{11} ft/lb at a pressure drop of 1554 $\mathrm{lb_f/ft^2}$.

Solution Refer to Equation 27.22 and solve for α_0. Note that the exponent b was obtained from the regression.

$$\alpha_0 = \alpha/(\Delta P)^{0.21}$$

Substituting

$$\alpha_0 = 4.57 \times 10^{11}/(1554)^{0.21}$$
$$= 9.73 \times 10^{10}\ \mathrm{ft/lb}$$

Illustrative Example 27.8. Describe the factors that affect the choice of a filtration unit.

The choice of filter equipment depends largely on economics, but the economic advantages will vary depending on:

1. Fluid viscosity, density, and chemical reactivity, etc.?
2. Solid particle size, size distribution, and shape.
3. Flocculation tendencies.
4. Deformability.
5. Feed slurry concentration.
6. Slurry temperature.
7. Amount of material to be handled.
8. Absolute and relative values of liquid and solid products.
9. Completeness of separation required.
10. Relative costs of labor, capital, and power.

Illustrative Example 27.9 A filter press operates at a constant pressure of 50 psig. The initial rate of collection is 10 $\mathrm{ft^3/min}$ and 1 hr is needed to collect 100 $\mathrm{ft^3}$ of filtrate. What is the hourly capacity of this filter if 15 ft^3 of wash water is used (at the same operating pressure) and the time for dumping and cleaning is 30 min?

Solution Apply Equation (27.12):

$$q = \frac{\Delta P}{BV_s + C}$$

When $V_s = 0$,

$$q = \frac{P}{C}$$

so that

$$C = \frac{P}{q} = \frac{50}{10}$$
$$= 5$$

For the constant pressure process, apply Equation (27.13):

$$t = \frac{B}{2\Delta P} V_s^2 + \frac{C}{\Delta P} V_s$$

When $t = 60$ min,

$$V_s = 100;\ \Delta P = 50$$

Substituting

$$60 = \frac{B}{2(50)} (100)^2 + \frac{(5)(100)}{50}$$
$$B = 0.5$$

During the washing cycle,

$$t_w = \frac{V_w}{q_w}$$

and (with B and C remaining the same)

$$\Delta P_w = (BV_s + C)q_w$$

Therefore,

$$t_w = \frac{V_w(BV_s + C)}{\Delta P} = \frac{V_w}{q_w}$$
$$= \frac{(15)[(0.5)(100) + 5]}{50} = 16.5\,\text{min}$$

Thus,

$$\begin{aligned} t_c &= t_f + t_w + t_d \\ &= 60 + 16.5 + 30 \\ &= 106.5 \text{ min} = 1.775 \text{ hr} \end{aligned}$$

Finally,

$$q = \frac{V_s}{t_c} = \frac{100}{1.775} = 56.3 \text{ gal/hr}$$

27.4 FILTRATION EXPERIMENTAL DATA AND CALCULATIONS

One of the experiments conducted in the Chemical Engineering Laboratory of Manhattan College is concerned with filtration. Students perform the experiment and later submit a report. In addition to theory, experimental procedure, discussion of results, and so on, the report contains sample calculations. The following is an (edited) example of these calculations that covers a wide range of filtration principles and applications.

During the experiment, two important measurements were determined: the mass increments of the filtrate, and the time for each increment. The scale measured the mass of the filtrate in grams and the time was recorded in seconds.

Four runs were carried out for this experiment. A sample of the cake from each frame was taken and placed in a previously measured dish after each run was stopped. Then, the new mass for the wet cake was obtained. The first run was performed at a constant pressure of 5 psig and is discussed below.

Two frames were used in this experiment: frame 16 and frame 18. The dishes corresponding to each frame were measured prior to addition of sample. Table 27.2 shows the mass of the dishes, wet cake and dry cake.

Table 27.2 Run 1 weights

Run–Frame	Dish (g)	Wet Cake + Dish (g)	Dry Cake + Dish (g)
1–16	1.0716	65.2296	37.4754
1–18	1.0679	66.0205	38.3801

The wet and dry cake masses were determined as follows:

$$\text{Wet cake}_{1-16} = (\text{Wet cake} + \text{Dish})_{1-16} - \text{Dish}_{1-16}$$
$$\text{Wet cake}_{1-16} = 65.2296\text{ g} - 1.0716\text{ g} = 64.158\text{ g}$$

$$\text{Dry cake}_{1-16} = (\text{Dry cake} + \text{Dish})_{1-16} - \text{Dish}_{1-16}$$
$$\text{Dry cake}_{1-16} = 37.4754\text{ g} - 1.0716\text{ g} = 36.4038\text{ g}$$

The same equations are used to determine the wet and dry cake masses for the subsequent runs. The masses are then converted to kilograms:

$$\begin{aligned}\text{Dry cake}_{1-16} &= 36.4038\,\text{g}\\ &= 0.0364\,\text{kg}\end{aligned}$$

The moisture content of each filter cake sample is determined using the wet and dry cake masses calculated previously.

$$\text{Moisture}_{1-16} = \text{Wet cake}_{1-16} - \text{Dry cake}_{1-16}$$
$$\text{Moisture}_{1-16} = 0.0642\,\text{kg} - 0.0364\,\text{kg} = 0.0278\,\text{kg}$$

$$\%\text{Moisture}_{1-16} = \left(\frac{\text{Moisture}_{1-16}}{\text{Wet cake}_{1-16}}\right)100\% = \left(\frac{0.0278\,\text{kg}}{0.0642\,\text{kg}}\right)100\% = 43.3\%$$

One of the important results to be determined for this experiment is the mass of the particles deposited in the filter per unit volume of filtrate (c) for each run. The following equation is used to obtain this concentration:

$$c = \frac{c_s}{1 - [(m_F/m_c) - 1]c_s/\rho}$$

where c_s = mass fraction of solids in filter cake
m_F = mass of filter cake
m_C = mass of solids in filter cake

For run 1, c_s is calculated as follows:

$$c_s = \left(\frac{\text{kg}_{\text{solids}}}{\text{kg}_{\text{solids}} + \text{kg}_{\text{water}}}\right) = \frac{0.0364\,\text{kg}}{(0.0364 + 0.0278)\,\text{kg}} = 0.5669$$

The terms m_F and m_C are the mass of wet cake and mass of dry cake, respectively. Substituting gives:

$$\begin{aligned}c_{1-16} &= \frac{c_s}{1 - [(m_F/m_C) - 1]c_s/\rho} = \frac{0.5669}{1 - [(0.0642/0.0364) - 1]0.5669/1000}\\ &= 0.5671\,\text{kg/m}^3\end{aligned}$$

An important value that needs to be calculated in this experiment is the filter medium resistance, R_m, α. A plot of time over volume vs. volume was constructed to obtain the necessary data to estimate R_m.

After plotting the data for the run, the slope and intercept of the line were obtained and used to calculate the filter medium resistance and the average specific cake resistance. Table 27.3 shows the values obtained from the plot.

Table 27.3 Slope and intercept for Run 1

ΔP (psig)	$K_C/2$ (s/L^2)	$1/q_0$ (s/L)
5	7.5823	13.974

The area for filtration is given by the two faces of the frames. This area S is calculated as follows:

$S = 2LW - 2\pi r^2$; $r =$ hole opening
$L = 16\,\text{cm}$
$W = 16.2\,\text{cm}$
$r = 2.5\,\text{cm}$
$S = 2[(16\,\text{cm})(16.1\,\text{cm}) - (\pi(2.5\,\text{cm})^2)] = 475.9\,\text{cm}^2$

The values of R_m and α are obtained by first converting key terms to English units:

$$K_C = (2)(7.5823)(28.31)^2 = 1.215 \times 10^4\,\text{s/ft}^6$$

$$1/q_0 = (13.974)(28.31) = 395.7\,\text{s/ft}^3$$

$$\text{Area} = \frac{475.9}{929.03} = 0.5157\,\text{ft}^2$$

$$\Delta P = 5(144) = 720\,\text{lb}_\text{f}/\text{ft}^2$$

$$g_c = 32.2\,\text{lb} \cdot \text{ft}/\text{lb}_\text{f} \cdot \text{s}^2$$

$$\mu_{H_2O} = 5.94 \times 10^{-4}\ \text{lb}/\,\text{ft} \cdot \text{s}$$

Substituting into the following equation gives

$$\begin{aligned} R_m &= \frac{(S)(\Delta P)(g_c)(1/q_0)}{\mu_{H_2O}} \\ &= \frac{(0.5157)(720)(32.2)(395.7)}{5.94 \times 10^{-4}} \\ &= 7.966 \times 10^9\ (\text{ft})^{-1} \end{aligned}$$

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